Accelerator Physics and Technology Seminar

Small Amplitude Dynamics and Nonlinear Detuning

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Date: Tuesday, July 30
When: 4:00 pm CDT
Where: One West (WH1W) and Zoom

First in a series of three talks aiming to provide a deeper understanding of the dynamics caused by sextupole and octupole magnets installed in accelerator rings. The series is divided into three parts: (I) "Small Amplitude Dynamics and Nonlinear Detuning," (II) "Exact Analytical and Numerical Solutions," and (III) "Large Amplitude Dynamics and Dynamic Aperture."

Abstract: In this part, we address the problem of nonlinear detuning (the dependence of betatron tune on amplitude) for small amplitudes caused by infinitely thin and infinitely thick sextupole/octupole magnets. To gain a qualitative understanding, we use two special model magnets that allow for integrable dynamics based on McMillan sextupole and octupole mappings. By establishing connections between these maps and general chaotic maps in standard form, our investigation reveals that the McMillan sextupole and octupole serve as universal first-order approximations of the dynamics around the fixed point. This is akin to the linear map and quadratic invariant (known as the Courant-Snyder invariant in accelerator physics), which represent zeroth-order approximations (often referred to as linearization). Furthermore, we show that this novel formalism accounts for the dependence of the rotation number on amplitude, providing an accurate prediction of detuning for regular sextupole/octupole magnets. Additionally, we demonstrate the capability of predicting the dynamic aperture around low-order resonances for flat beams—a critical aspect in beam injection/extraction scenarios. This is illustrated using horizontal dynamics within the Fermilab delivery ring, which is employed for third-integer resonant extraction in the Mu2e experiment.