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Optimal Control for Fourier Transform on Qudits

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Abstract

In this work, we develop a protocol to find the optimal control pulse for implementing the Quantum Fourier Transform (QFT) on qudit-based hardware. We focus on two types of controls: (1) qubit-qudit dispersive coupling with qubit and qudit drives, and (2) qubit-qudit system with tunable coupling and qubit drive. We observe that for both systems, the minimum gate time grows linearly as a function of the size of the logical space. We find that the latter system supports faster pulse, requiring 45% shorter pulse time.

Qudit-based Architecture

Circuit Quantum Electrodynamics (cQED)

- cQED system includes a qubit (transmon) coupling with a qudit (SRF cavity), described by Jaynes-Cummings Hamiltonian ($\hbar$=1)

$$H = \frac{1}{2}\omega_q \sigma^z + \omega_c a^\dagger a + g(a\sigma^+ + a^\dagger \sigma^-)$$

where $\boldsymbol{\omega_q}(\boldsymbol{\omega_c})$ is the qubit (qudit) frequency; $\boldsymbol{\sigma^+}, \boldsymbol{\sigma^-}(\boldsymbol{a^\dagger}, \boldsymbol{a})$ are the qubit's (qudit's) create, destroy operators; $\boldsymbol{\sigma^z}$ is the Pauli-Z operator, $\boldsymbol{g}$ is the qubit- qudit coupling strength

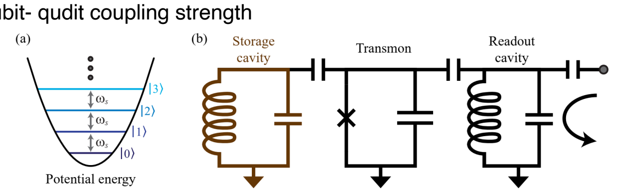


Image from [1]: (a) Cavity as a harmonic oscillator (b) A transmon coupling with a cavity

Mode (1): Qubit & Qudit Controls under dispersive coupling

- Dispersive approximation: $|\Delta| = |\omega_q - \omega_c| \gg g$

$$H_0 \approx \frac{1}{2}\omega_q \sigma^z + \omega_c a^\dagger a + \frac{1}{2}\chi a^\dagger a \sigma^z$$

where $\chi = 2g^2/\Delta$ is called the dispersive shift

- Qubit control: $H^q_{ctrl}(t) = I_q(t)\sigma^x + Q_q(t)\sigma^y$
- Qudit control: $H^c_{ctrl}(t) = I_c(t)(a + a^\dagger) + \mathrm{i}\, Q_c(t)(a - a^\dagger)$

Mode (2): Qubit Control & Tunable Coupling

$$H_0 = \frac{1}{2}\omega_q \sigma^z + \omega_c a^\dagger a$$

- Qubit control: $H^q_{ctrl}(t) = I_q(t)\sigma^x + Q_q(t)\sigma^y$
- g-control: $H^g_{ctrl}(t) = g_I(t)(a\sigma^+ + a^\dagger\sigma^-) + i g_Q(t)(a\sigma^+ - a^\dagger \sigma^-)$

where $\boldsymbol{\sigma^x}$ is the Pauli-X operator; $\boldsymbol{\sigma^y}$ is the Pauli-Y operator

Quantum Optimal Control

- Use optimal control techniques to find the pulses $I_q(t)$, $Q_q(t)$, $I_c(t)$, $Q_c(t)$, $g_I(t)$, $g_Q(t)$ that minimize the gate infidelity

$$1 - \frac{1}{N}\left| Tr\left\{ U^\dagger_{targ}\, \mathbb{T}\, exp\left[\left[\int_0^T -i\left(H_0 + H_{ctrl}(t)\right)dt\right]\right]\right\}\right|$$

where $\boldsymbol{N}$ is the dimension of total Hilbert space; $\boldsymbol{U_{targ}}$ is the target unitary; $\mathbb{T}$ is the time-ordered operator

- Gradient Ascent Pulse Engineering (GRAPE) [2]: gradient-based optimization algorithm that computes the gradient using backward and forward propagation

Quantum Fourier Transform (QFT)

QFT is a crucial element of prime factorization (Shor's algorithm), linear systems solver (Harrow-Hassidim-Lloyd algorithm), etc.

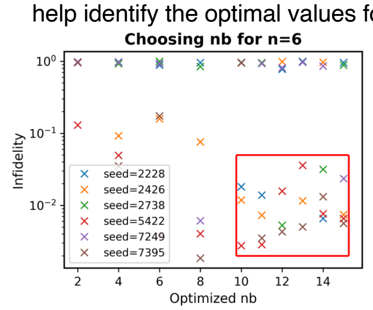
$$QFT_n = \frac{1}{\sqrt{n}}\begin{pmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \omega & \omega^2 & \dots & \omega^{n-1} \\ 1 & \omega^2 & \omega^4 & \dots & \omega^{2(n-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{n-1} & \omega^{2(n-1)} & \dots & \omega^{(n-1)(n-1)} \end{pmatrix} \quad \text{with } \omega = e^{2\pi i/n}$$

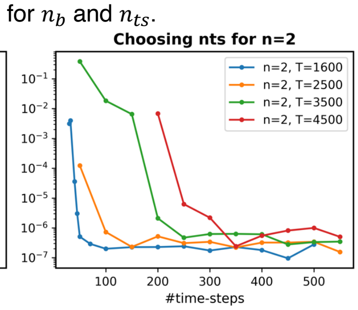
Optimization Setup

- GRAPE is implemented by qutip-qtrl [3]
- Bumper states - high energy levels beyond the logical states - are added to reduce the truncation error.

Quantity	Values
$\omega_q/2\pi$	6 GHz
$\omega_c/2\pi$	4.5 GHz
$\chi/2\pi$	-5 MHz

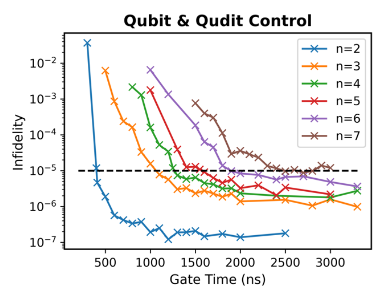
- Careful investigations on the impact of increasing number of bumper states (n_b) and timesteps (n_{ts}) on the optimization results help identify the optimal values for n_b and n_{ts}.

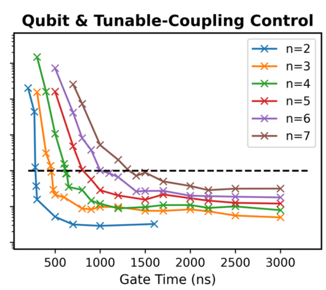


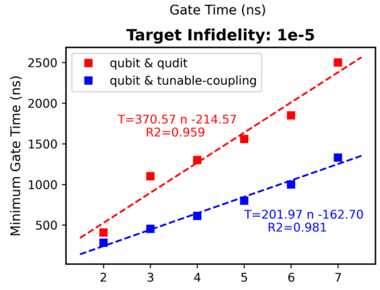


Final optimization parameters: at most 20000 iterations; optimization tolerance of 1e-10; n_{ts} = 250/channel (1000 in total); n_b dependent on n

Results







- Minimum gate time grows linearly with the size of the logical space.
- **Qubit and tunable-coupling control supports pulses that are 45% shorter compared to qubit-qudit controls.**

Acknowledgement

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References

[1] Roy et al., Proc. Sci, LATTICE2023 (**2024**) 127
[2] Khaneja et al., J. Magn. Reson. 172(2):296–305 (2005)
[3] Johansson et al., Comp. Phys. Comm. **184**, 1234 (2013)