Deriving Effective Coupling Strength With Born-Oppenheimer Approx.

Conrad Wichmann, University of Chicago

FERMILAB-SLIDES-24-0197-STUDENT

Introduction

- Tunable couplers have been important in the development of superconducting quantum computers.
- The ability to maintain zero coupling between qubits during operation, a feature of high-fidelity quantum gates, has been shown to be possible with the use of tunable couplers.
- As an example, Sete et al., 2021 (1) showed this was possible with a tunable coupler between transmons (see right): the qubit-coupler and qubit-qubit couplings create a net effective coupling strength between the qubits that can be tuned to zero.

Statement of Purpose

The derivation of an effective coupling strength (see right) between the qubits is quite complex. We propose a simpler method of obtaining this effective coupling with the Born-Oppenheimer approximation.

$$g = g_{12} - g_{\text{eff}}, \quad g_{\text{eff}} = \frac{g_{1c}g_{2c}}{2}\sum_{j=1}^{2}\left(\frac{1}{\Delta_j} + \frac{1}{\Sigma_j}\right)$$

Figure 2

Formula for an effective coupling strength between transmons from (1). Derived using the Schrieffer-Wolff Transformation.

$$H_0 = \omega_c c^\dagger c + e^{-i\omega_a t} a c^\dagger + e^{i\omega_a t} a^\dagger c + e^{-i\omega_b t} b c^\dagger + e^{i\omega_b t} b^\dagger c$$

$$R(t) = U_3^\dagger D_2 U_3 U_1^\dagger D_1 U_1 = \tilde{D}_2 \tilde{D}_1$$

$$H_5 = \omega_c c^\dagger c - \frac{|\alpha|^2}{\omega_c - \omega_a} - \frac{|\beta|^2}{\omega_c - \omega_b} - \frac{\alpha\beta^* e^{i(\omega_b-\omega_a)t} + \alpha^*\beta e^{-i(\omega_b-\omega_a)t}}{\omega_c - \omega_a}$$

$$U_1 = e^{-i\omega_a c^\dagger c t} \qquad D_1 = e^{\frac{\alpha}{\omega_c-\omega_a}c^\dagger - \frac{\alpha^*}{\omega_c-\omega_a}c}$$

$$U_3 = e^{i\omega_b c^\dagger c t} \qquad D_2 = e^{\frac{\beta}{\omega_c-\omega_b}c^\dagger - \frac{\beta^*}{\omega_c-\omega_b}c}$$

Figure 3

On the left, we apply a series of unitary transformations to achieve the desired Hamiltonian. We first rotate the Hamiltonian by the frequency of qubit a (U_1) and then apply a displacement operator (D_1) to remove the coupling terms between qubit a and the coupler. Next, we rotate the Hamiltonian to the frequency of qubit b (U_3) and apply a second displacement operator (D_2) to remove the coupling terms between qubit b and the coupler. Finally, we rotate back by the frequency of qubit b to obtain the final Hamiltonian, H_5.

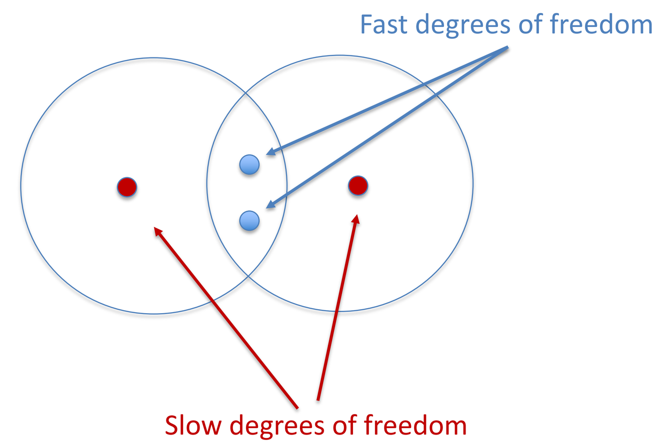

Figure 4

A diagram of two hydrogen atoms with electrons and nuclei marked as fast and slow degrees of freedom, respectively. In our implementation of the Born-Oppenheimer approximation, the coupler takes on the role of the fast degrees of freedom (electrons) and the qubits take on the role of the slow degrees of freedom (nuclei).

(1) Sete et al. (2021). Floating Tunable Coupler for Scalable Quantum Computing Architectures. *Phys. Rev. Appl., 15, 064063.*

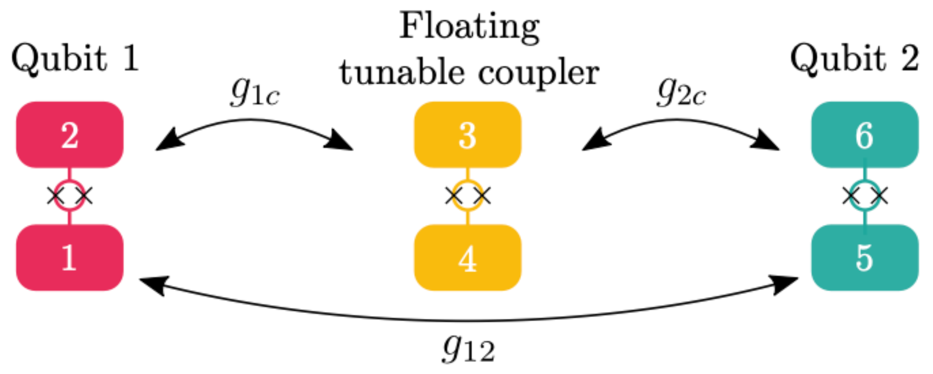

Figure 1

Diagram of a transmon-coupler system from (1) with no direct capacitance between qubits. g_{1c} and g_{2c} are the coupling strengths between the coupler and qubits 1 and 2, respectively, while g_{12} is the direct coupling strength between the qubits.

Methods

- As in the Born-Oppenheimer approximation, we take the tunable coupler and qubits as our fast and slow degrees of freedom, respectively (see Fig. 4). Thus, we replace the qubit creation and annihilation operators with their eigenvalues.
- We rotate the Hamiltonian by the qubit frequencies and apply displacement operators to eliminate the qubit-coupler terms from the Hamiltonian (see Fig. 3).
- We find the propagator to determine the phase and differentiate to yield the coupling strength.

(1) $$U(t) = R^\dagger(t) e^{-i\tilde{H}_5 t} R(t=0) \exp\left(-i\int_0^t f(t')dt'\right)$$

Now, we consider the product of the following two terms of U

(2) $$\exp\left(\frac{\alpha\beta^* e^{i(\omega_b-\omega_a)t} - \alpha^*\beta e^{i(\omega_a-\omega_b)t}}{(\omega_c-\omega_a)(\omega_b-\omega_a)}\right) \cdot \exp\left(\frac{\alpha\beta^* e^{i(\omega_b-\omega_a)t} - \alpha^*\beta e^{i(\omega_a-\omega_b)t}}{2(\omega_c-\omega_a)(\omega_c-\omega_b)}\right)$$

(3) $$\frac{i}{2}\left(\alpha\beta^* e^{i(\omega_b-\omega_a)t} + \alpha^*\beta e^{i(\omega_a-\omega_b)t}\right)\left(\frac{1}{\Delta_{ca}} + \frac{1}{\Delta_{cb}}\right)$$

$$\Delta_{ca} = \omega_c - \omega_a \text{ and } \Delta_{cb} = \omega_c - \omega_b$$

Figure 5

We apply the formula for U(t) given R(t), the total transformation outlined in Fig. 3. We factor out two phase factors from U(t), one of which comes from $R^\dagger$ and the other of which comes from the integral of f(t) and take their product to yield (3).

Results and Conclusions

- We achieve a result in Figure 5 which corresponds to the effective coupling term produced by Sete et al., 2021 (1) with the Schrieffer-Wolff transformation (as shown in Figure 1).
- In general, usage of the Schrieffer-Wolff method requires prior knowledge of the generator of the SW transformation, which is hamiltonian-specific.
- Our method is less case-specific, making it a potential framework to determine effective coupling strengths with other hamiltonians.

(a) This manuscript has been authored by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy, Office of Science, Office of High Energy Physics.

(a) This work was supported in part by the U.S. Department of Energy, Office of Science, Office of Workforce Development for Teachers and Scientists (WDTS) under the Science Undergraduate Laboratory Internships Program (SULI).