## **Accelerator Physics and Technology Seminar**

## **Exact Analytical and Numerical Solutions**

## Tim Zolkin, FNAL

Date: Tuesday, September 17When: 4:00 pm CDTWhere: One West (WH1W) and Zoom

Second in a series of three talks aiming to provide a deeper understanding of the dynamics caused by sextupole and octupole magnets installed in accelerator rings. The series is divided into three parts: (I) "Small Amplitude Dynamics and Nonlinear Detuning," (II) "Exact Analytical and Numerical Solutions," and (III) "Large Amplitude Dynamics and Dynamic Aperture."

**Abstract**: Nonlinear magnetic devices, such as sextupoles, octupoles, and RF stations, are integral components in accelerators. Despite their significance, solving the equations of motion analytically, even under the thin lens approximation, is often impossible for general cases, necessitating the use of approximations in beam dynamics calculations.

In this talk, we shift the focus from approximated methods to exact analytical solutions and cutting-edge simulations enabled by advancements in computational power and numerical techniques. We will explore several methods for illustrating stability diagrams, including Frequency Map Analysis (FMA), the Smaller Alignment Index (SALI), and the Reversibility Error Method (REM). Additionally, we will examine the connection to the famous Hénon mapping and highlight the critical role of symmetry lines, symmetric fixed points, and n-cycles in the context of dynamic aperture. To conclude, we will unveil a solution to one of Hénon's longstanding unsolved mysteries, providing fresh insights into the study of single-particle dynamics.