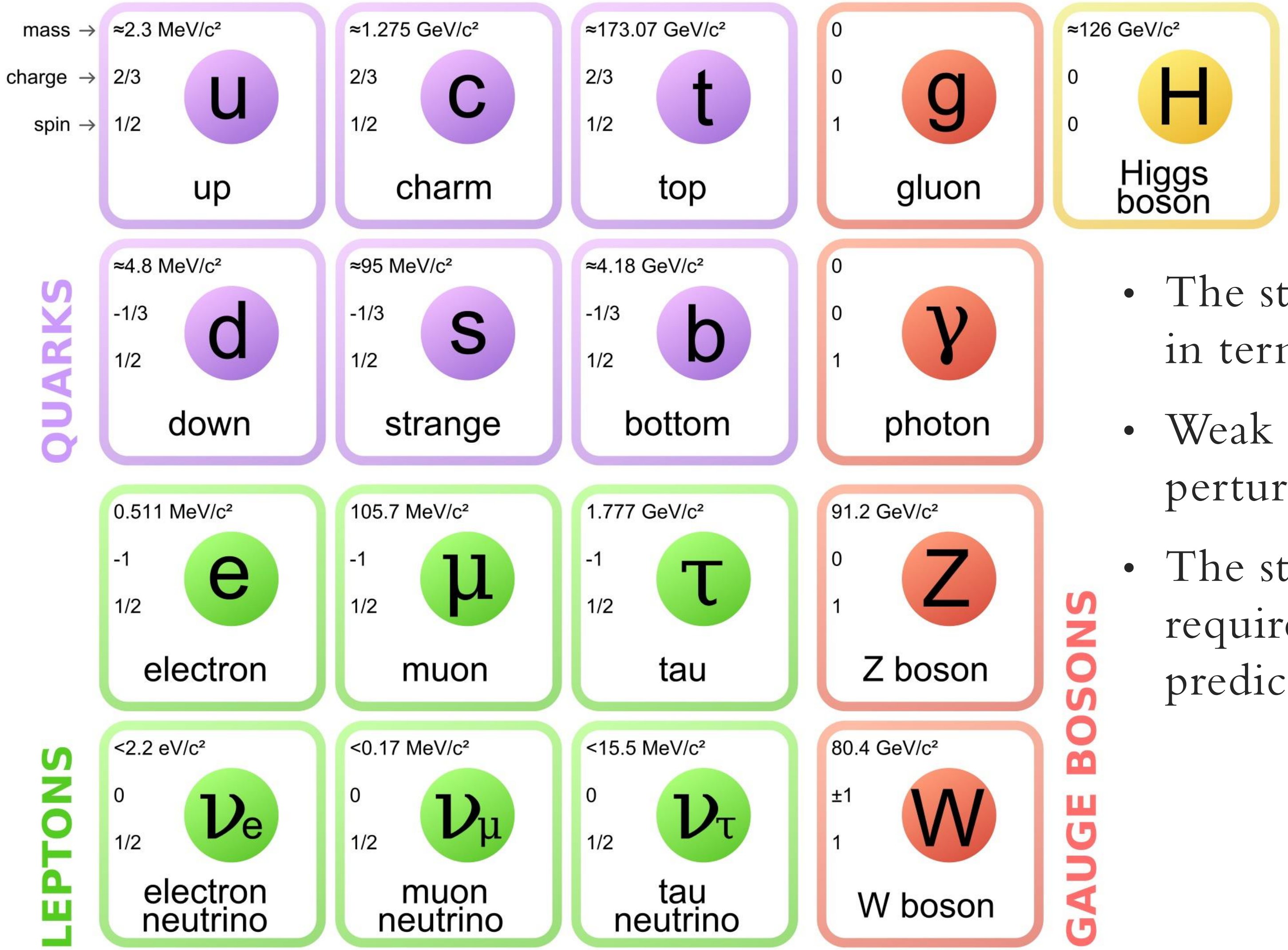
STRING BREAKING IN THE HEAVY QUARK LIMIT WITH SCALABLE CIRCUITS

Anthony Ciavarella

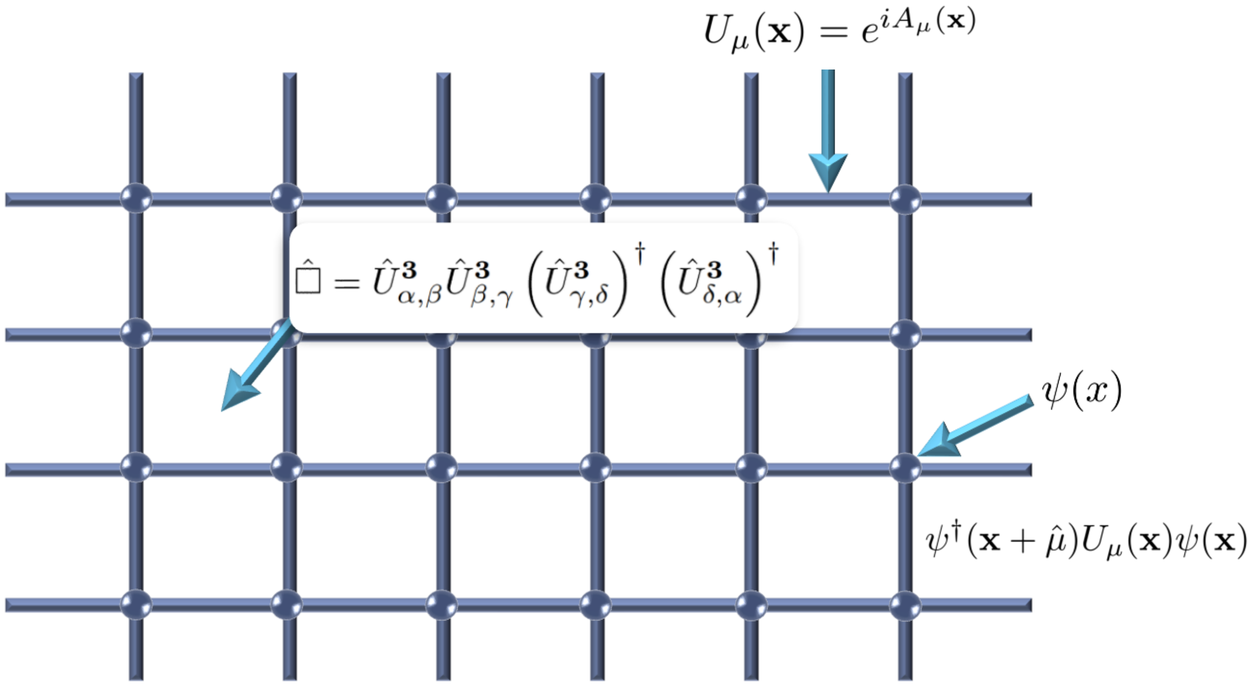
THE STANDARD MODEL



- The standard model describes the universe in terms of 4 fundamental forces
- Weak and electric forces can be described perturbatively
- The strong force is non-perturbative and requires numerical simulation to make predictions at low energy scales

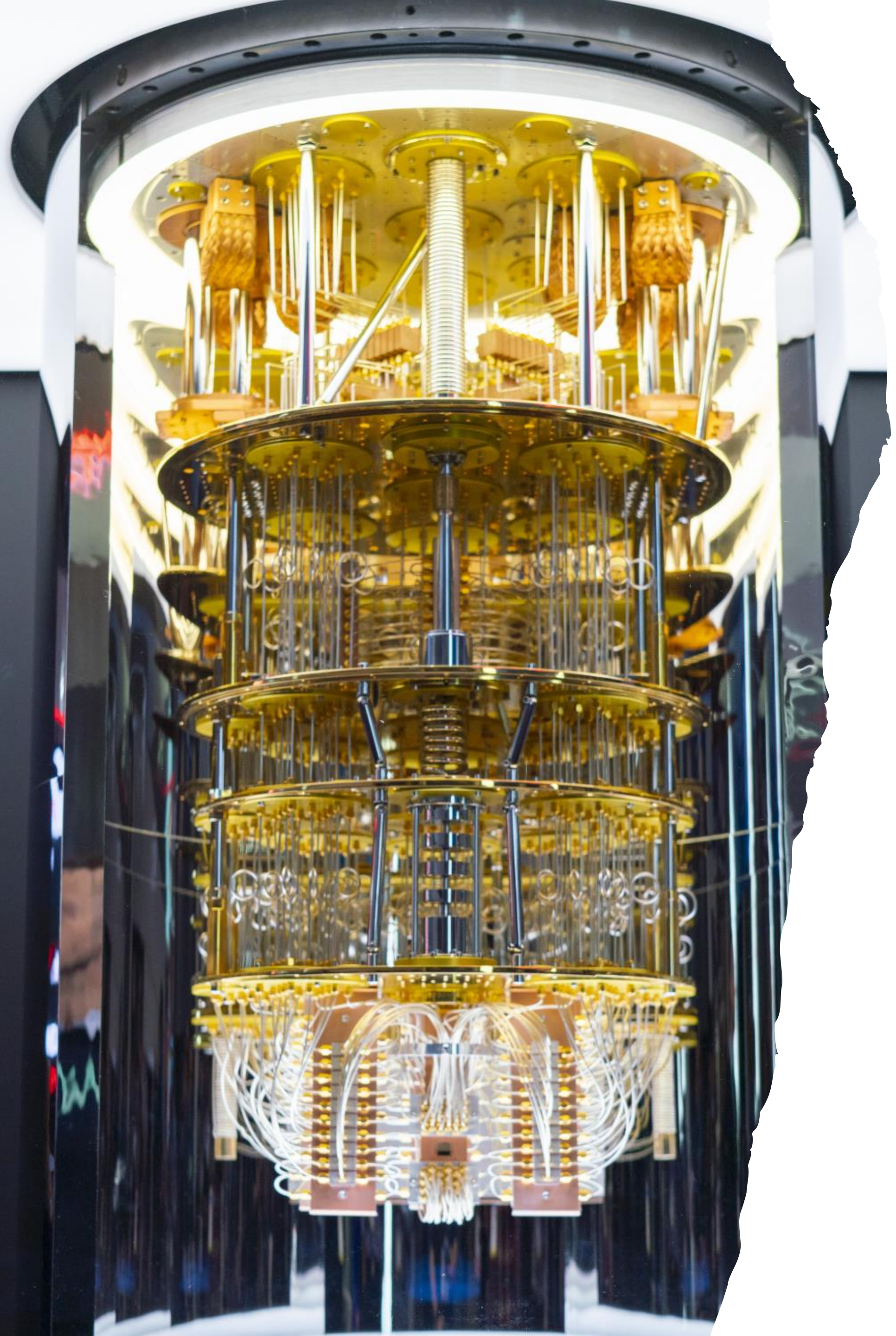
Classical Monte Carlo Simulation

- Hadron masses
- Form factors
- Some Elastic Scattering Amplitudes
- Muon g-2



Sign Problems

- REAL TIME DYNAMICS
- FINITE DENSITY SYSTEMS



WHY QUANTUM SIMULATION?

Simulating Physics with Computers (Richard Feynman 1982)

Classical simulation of quantum dynamics has exponentially scaling costs

Instead, we can engineer quantum systems to simulate the theories we wish to study.

HOW TO REACH FULL QCD?

- Current quantum computers are too small and noisy to do full lattice QCD
- Simulations on noisy hardware can inform the development of techniques to reduce the effects of noise
- 3+1D calculations are limited by hardware connectivity / circuit depth
- Toy models lower dimensions are easier to map onto hardware
- Can be used to develop techniques that will carry over to 3+1D
  - State preparation
  - Constructing physical observables
- Some non-trivial physics can be studied
  - Jet Fragmentation
  - Hadronization

STATE PREPARATION AND MEASUREMENT

- Simulating physics requires preparing physically relevant states
- Also need to perform measurements of the final state

Adiabatic

- Theoretical guarantees
- Potentially deep circuit depths
- Mostly restricted to theoretical studies

Variational

- Heuristic method, depends on circuit ansatz
- Requires optimization of circuits
- Lower circuit depth

- Jordan, Lee and Preskill proposed performing state preparation and measurement by doing adiabatic switching from a free field theory at the beginning and end of a calculation.

PREPARING THE SCHWINGER MODEL VACUUM

PRX Quantum 5 (2), 020315

- QED in 1+1D
- Gapped and translationally invariant
- Confining, like QCD in 3+1D
- We looked at preparing the vacuum state as a step towards studying QCD

Roland Farrell, Marc Illa, Anthony Ciavarella, and Martin Savage

Scalable Circuits for Preparing Ground States on Digital Quantum Computers: The Schwinger Model Vacuum on 100 Qubits

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arXiv: 2308.04481 [quant-ph]

Quantum Simulations of Hadron Dynamics in the Schwinger Model using 112 Qubits

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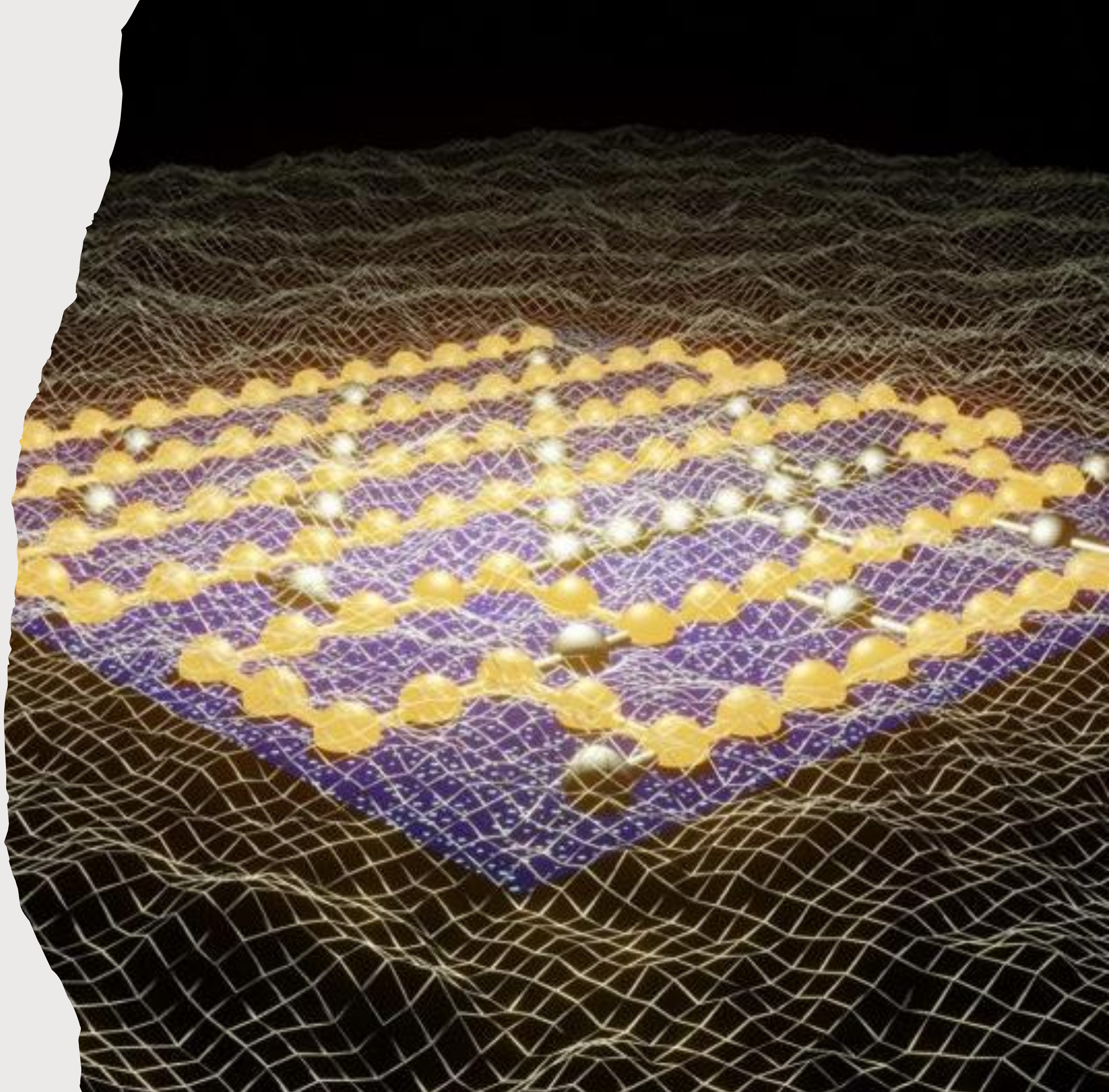
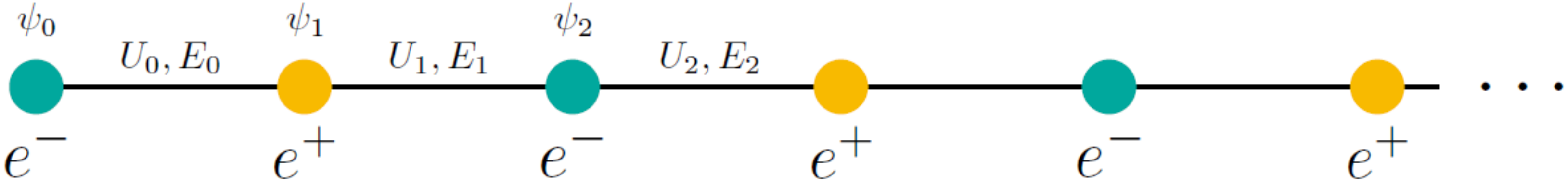


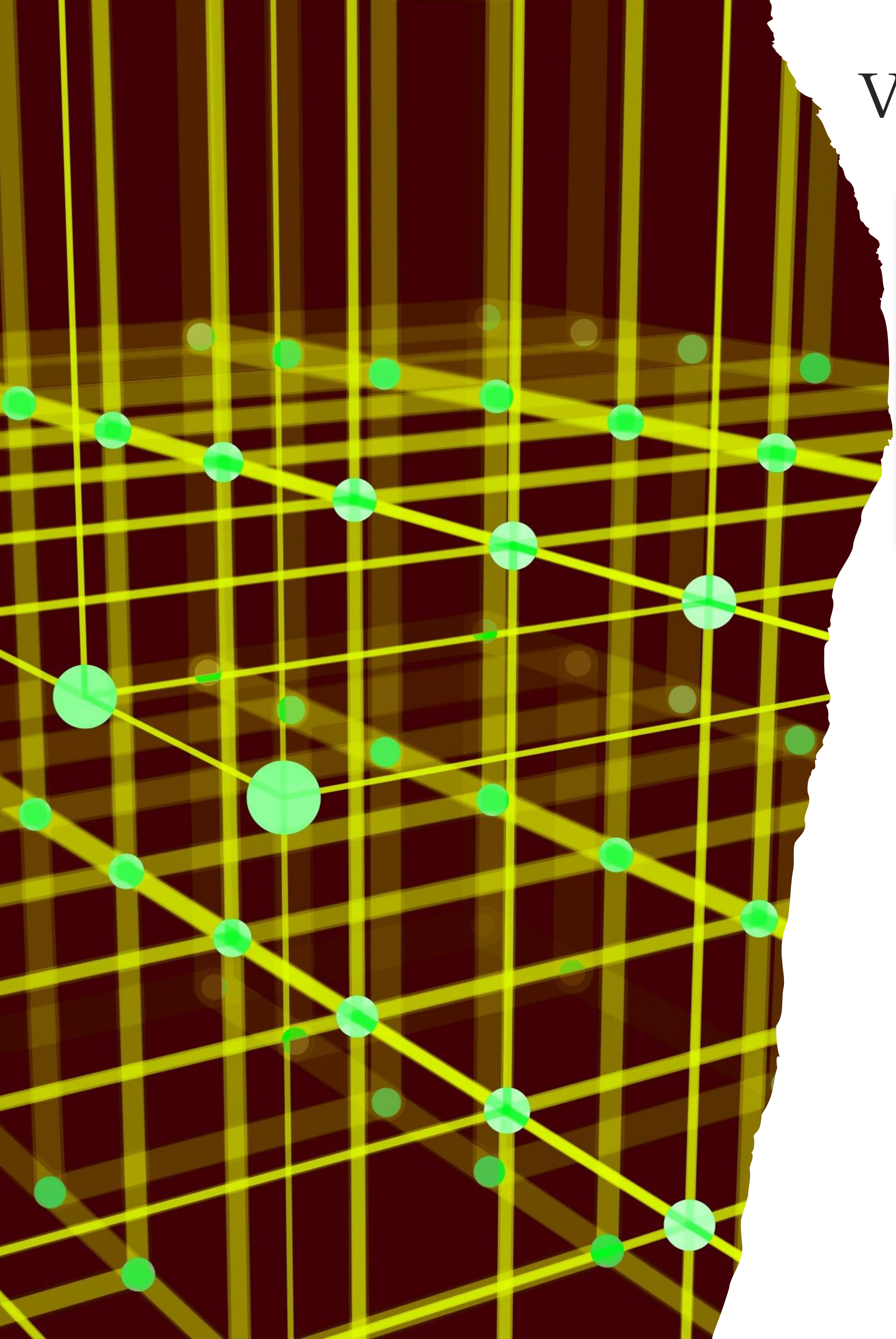
THE LATTICE SCHWINGER MODEL

$$H = \frac{1}{2}\sum_{n=0}^{2L-2}(\psi_n^\dagger U_n \psi_{n+1} + \text{h.c.}) + m\sum_{n=0}^{2L-1}(-1)^n \psi_n^\dagger \psi_n + \frac{g^2}{2}\sum_{n=0}^{2L-2}|E_n|^2$$



$$\hat{H} = \hat{H}_m + \hat{H}_{kin} + \hat{H}_{el} = \frac{m}{2}\sum_{j=0}^{2L-1}\left[(-1)^j \hat{Z}_j + \hat{I}\right] + \frac{1}{2}\sum_{j=0}^{2L-2}(\hat{\sigma}_j^+ \hat{\sigma}_{j+1}^- + \text{h.c.}) + \frac{g^2}{2}\sum_{j=0}^{2L-2}\left(\sum_{k\le j}\hat{Q}_k\right)^2$$

$$\hat{Q}_k = -\frac{1}{2}\left[\hat{Z}_k + (-1)^k \hat{I}\right] .$$



VQE

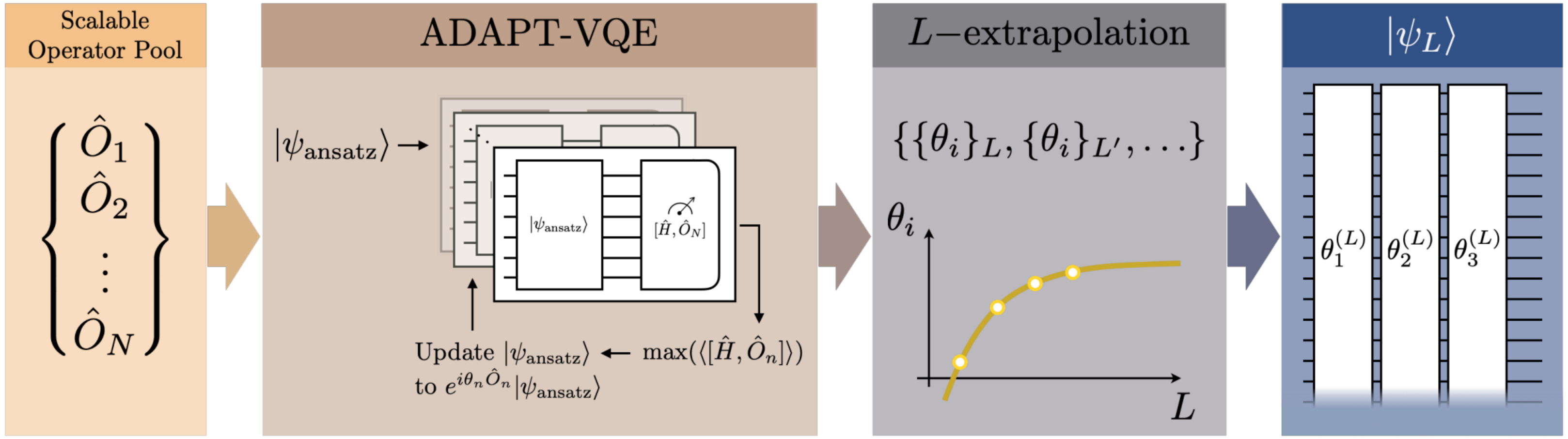
Quantum
Prepare some state $|\psi(\theta)\rangle$
Measure the energy of the state, (and possibly the gradient as well)

⇄

Classical
Use a classical optimization algorithm to update θ

- Hybrid algorithm that can be used to prepare ground states.
- Previously has been applied to studies of the Schwinger model (PhysRevA.98.032331, Nature 569 355–360 (2019), Phys. Rev. Lett. 126, 220501), SU(2) hadrons (Nature Communications 12, 6499 (2021)), …
- Use of VQE at scale will require an appropriate scalable ansatz circuit and optimization algorithm

SCALABLE CIRCUITS



- Construct an operator pool that respects translation invariance and other symmetries of the Hamiltonian
- Perform ADAPT-VQE on several small lattices to optimize a state prep circuit
- Provided the parameters were computed on a lattice larger than multiple correlation lengths, the convergence will be exponentially fast
- Extrapolate the parameters in lattice size to use on a larger lattice

RUNNING ON HARDWARE

Quantum computers are noisy and to perform reliable calculations, this noise needs to be corrected

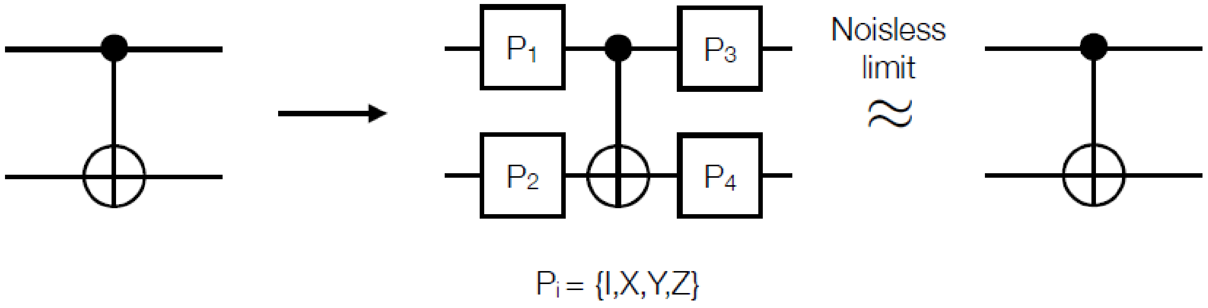
Two types of noise on quantum computers

- Incoherent noise: relaxation and dephasing
- Coherent noise: unitary rotations caused by miscalibration or cross-talk

We can mitigate incoherent noise, but not coherent noise.

However, we can convert coherent noise into incoherent noise

Pauli Twirling (or randomized compiling)

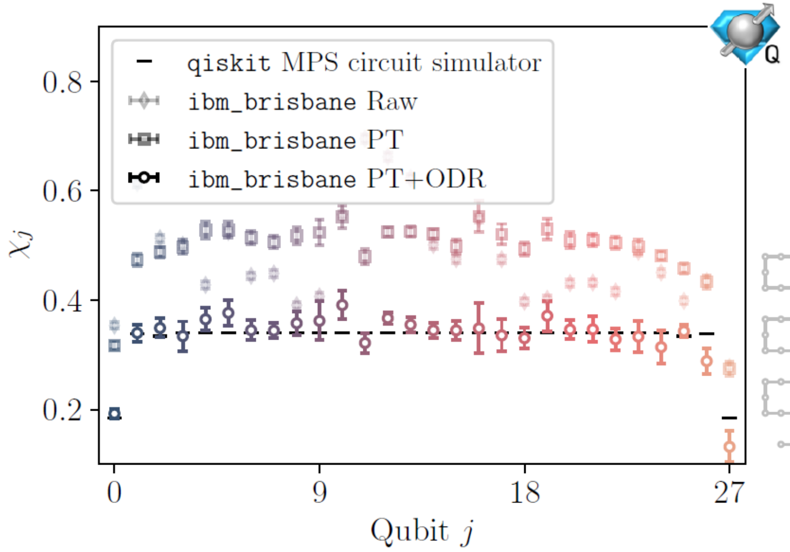


OPERATOR DECOHERENCE RENORMALIZATION

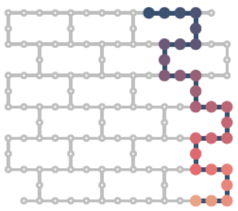
- Remaining errors in the simulation are incoherent
- Measured observable is proportional to noiseless one (assuming a Pauli error channel)

$$\langle \hat{O} \rangle_{\text{meas}} = (1 - \eta_O) \langle \hat{O} \rangle_{\text{pred}}$$

- Measure the noise parameter by running the same circuit with single qubit rotation angles set to 0



40 mitigation circuits + 40 physics circuits
= 80 circuits
w/ 8000 shots eacl

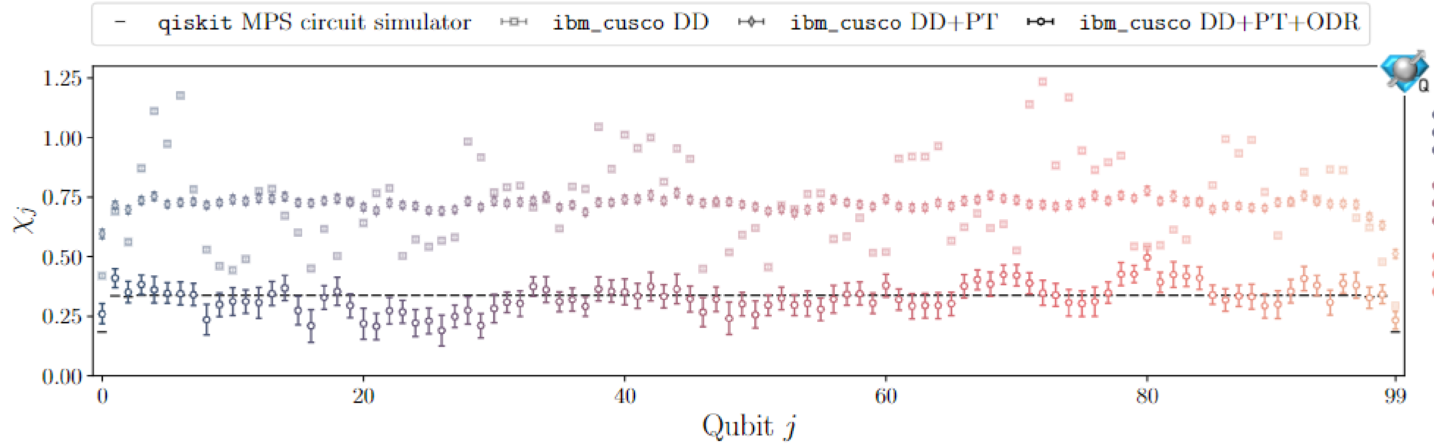


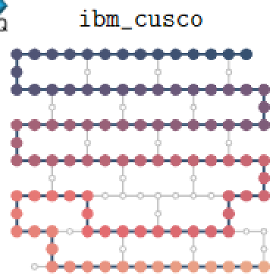
$$\chi = \frac{1}{L}\sum_i \langle \overline{\psi}_i \psi_i \rangle = \frac{1}{2L}\sum_i [(-1)^i Z_i + I] \equiv \frac{1}{2L}\sum_i \chi_i$$

<u>Operator Decoherence Renormalization</u>
Phys. Rev. Lett. 127, 270502
arXiv:2210.11606
PRX Quantum 5, 020315
Phys. Rev. D 109, 114510

IMPLEMENTATION ON UP TO 100 QUBITS

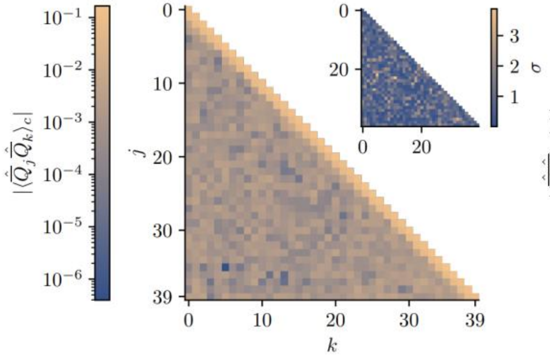
- All circuits were optimized classically with up to L=14 sites (28 qubits)
- Errors were mitigated using operator decoherence renormalization

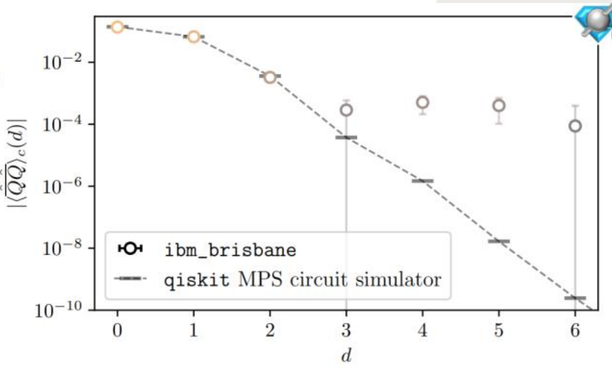




L	Qubits	CNOTs	$\chi^{(\mathrm{SC-IBM})}$ before ODR	$\chi^{(\mathrm{SC-IBM})}$ after ODR	$\chi^{(\mathrm{SC-MPS})}$
14	28	212	0.491(4)	0.332(8)	0.32879
20	40	308	0.504(3)	0.324(5)	0.33105
30	60	468	0.513(2)	0.328(4)	0.33319
40	80	628	0.532(2)	0.334(3)	0.33444
50	100	788	0.737(2)	0.318(8)	0.33524

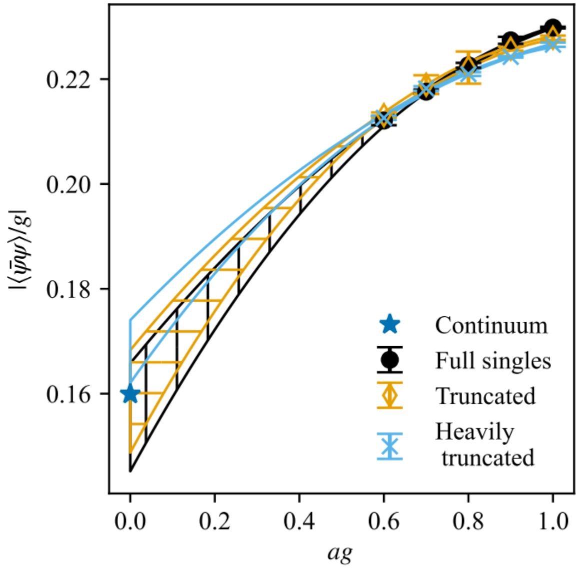
TABLE IV. The chiral condensate in the Schwinger model vacuum obtained from ibm_brisbane ($L \leq 40$) and ibm_cusco ($L = 50$) for large lattices with $m = 0.5, g = 0.3$ using the scaled circuits from 2 steps of ADAPT-VQE. The values before and after applying ODR are given in columns four and five. The last column gives results obtained from the MPS classical simulator.

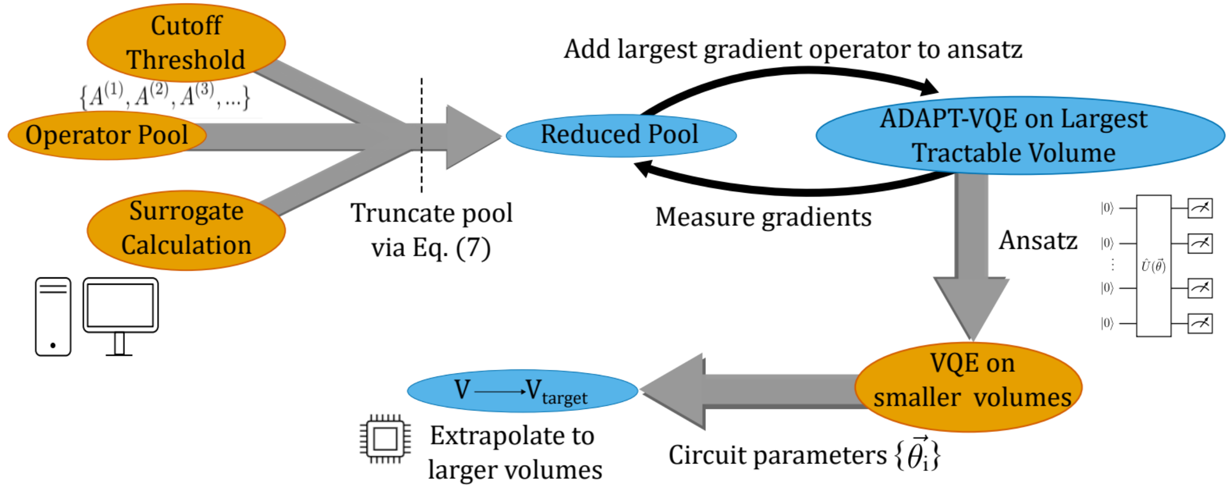




(SC)^2-ADAPT-VQE

- ADAPT-VQE can choose different sequences of operators on different lattice sizes.
- This problem can be avoided by doing ADAPT-VQE on one lattice size and optimizing the same operator sequence on different lattice sizes.
- Optimization doesn't have to minimize energy. One can instead maximize the overlap with a surrogate for a given state, ex. MPS representation of the vacuum.
- Performing the optimization on lattices with up to 16 sites, the authors were able to perform an infinite volume and continuum extrapolation.





arXiv:2408.12641

Surrogate Constructed Scalable Circuits ADAPT-VQE in the Schwinger model

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Yanzhu Chen[3,4,6] Henry Lamm[7,8] Semeon Valgushev[9] Andreas Weichselbaum[5]
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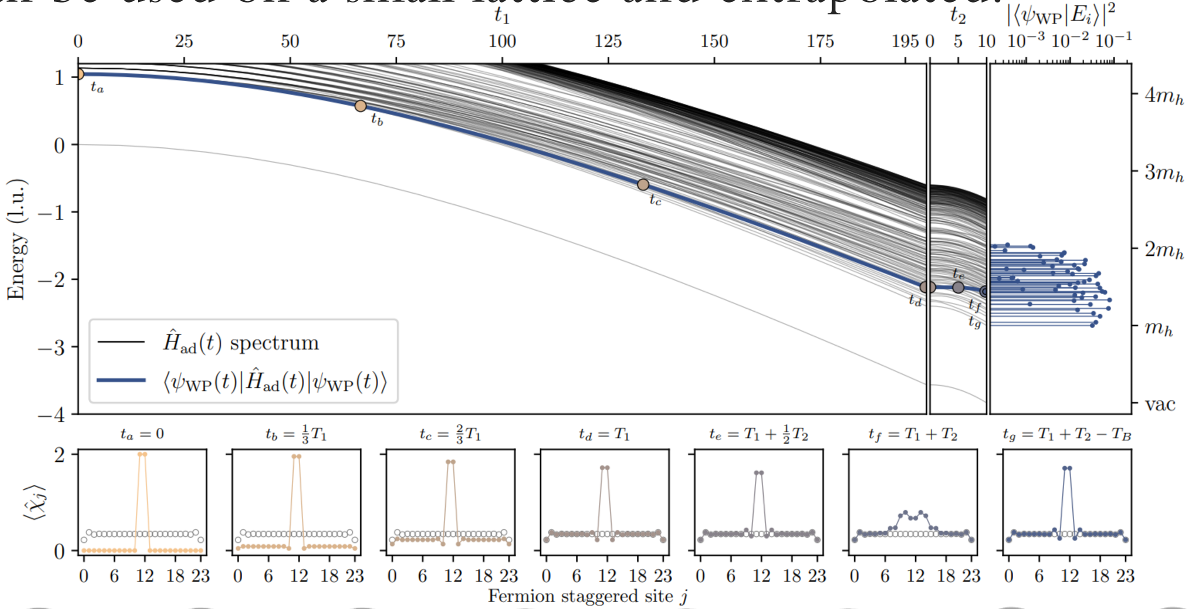
PREPARING HADRON STATES

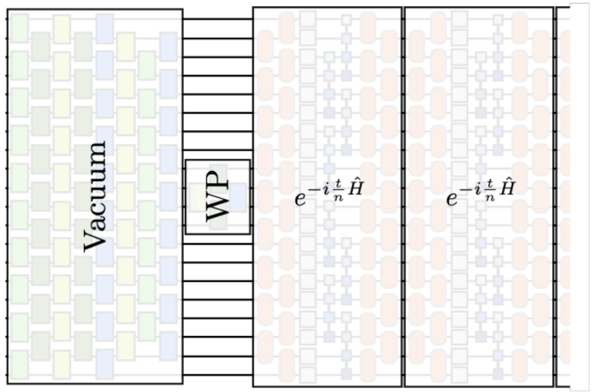
- To study scattering or other dynamics, one needs to be able to prepare a state with hadrons.
- Previous studies of quantum simulations of scalar field theories proposed using adiabatic switching with forward and backwards evolution.
- In the Schwinger model, adiabatic switching can be performed from the strong coupling vacuum.
- Not necessary to use a large lattice or do adiabatics on the quantum computer. The same variational techniques can be used on a small lattice and extrapolated.

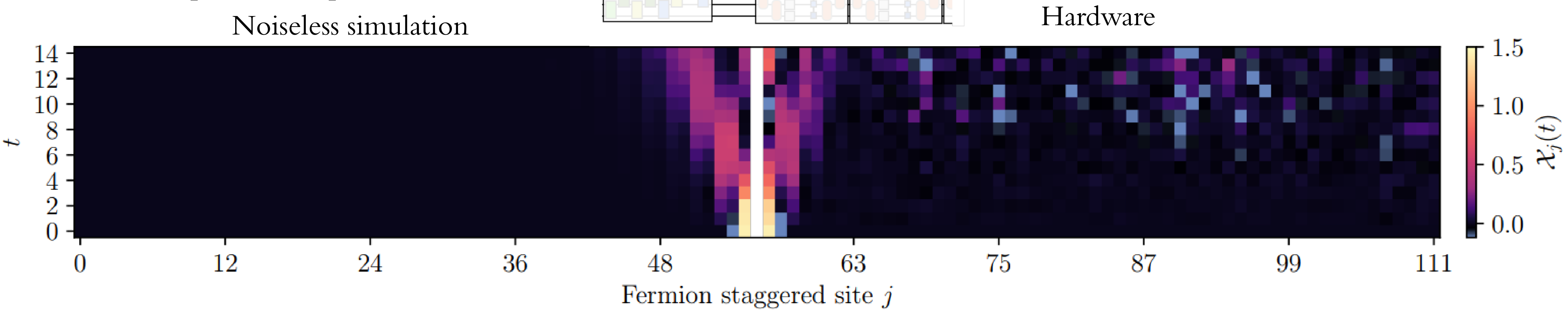


TIME EVOLUTION

- Time evolution on a quantum computer is done by Trotterization, i.e. one breaks up the Hamiltonian into individual pieces that can be implemented in a sequence.
- QQ terms in the Hamiltonian give rise to long range interactions, due to confinement QQ interactions are exponentially suppressed at long distances can be neglected beyond a certain distance.
- Propagation of hadrons was tracked by measuring the disturbance of the chiral condensate from its value in vacuum

112 Qubits on ibm_torino
CNOT Depth 370
13,858 CNOT gates
10^7 shots per time step





WHAT TO MEASURE?

- We would like to be able to measure hadron positions or hadron # throughout our simulation
- Adiabatic switching from the strong coupling limit works in principle, but is impractical to implement
- Instead we can optimize a single circuit to prepare both the vacuum and single hadron states. This can be done using concurrent-VQE.

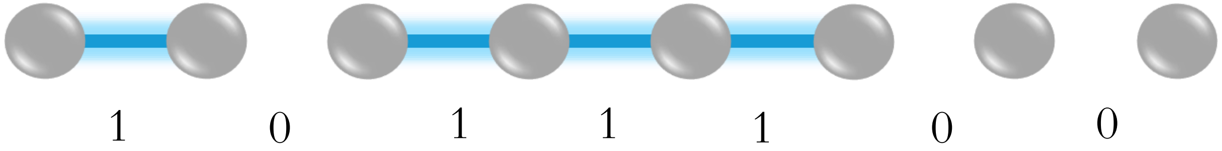
$$|p\rangle = \frac{1}{\sqrt{2L}} \sum_x e^{ipx} |x\rangle \quad O(\vec{\theta}) = \frac{1}{2L+1}\left(\left| \langle \mathrm{Vac} | U(\vec{\theta}) |0\rangle \right|^2 + \sum_p \left| \langle \psi_p | U(\vec{\theta}) |p\rangle \right|^2 \right)$$

- Optimized circuits can be extrapolated to larger system sizes using the scalable circuits formalism. SC^2-VQE
- At long times in a scattering simulation, states should consist of isolated hadrons. Their positions can be determined by undoing U and performing measurements in the electric (strong coupling) basis.

arXiv:2411.05915

HEAVY QUARK LIMIT OF SU(2)

- In the heavy quark limit, we can restrict the quark number to at most one per site.
- Truncating the electric field at j=1/2 allows us to represent states with a single qubit per link.



- The strong coupling vacuum has all qubits = 0 and strong coupling meson states have a single qubit in the 1 state.
- Meson number and position can be tracked by measuring the number of string ends in the simulation.

$$\hat{H} = \hat{H}_{Kin} + \hat{H}_m + \hat{H}_E$$

$$\hat{H}_{Kin} = \sum_{x,a,b} \frac{1}{2} \hat{\psi}^\dagger_{x,a} \hat{U}^{a,b}_{x,x+1} \hat{\psi}_{x+1,b} + \text{h.c.}$$

$$\hat{H}_m = m \sum_{x,a} (-1)^x \hat{\psi}^\dagger_{x,a} \hat{\psi}_{x,a}$$

$$\hat{H}_E = \sum_{x,c} \frac{g^2}{2} \hat{E}^c_{x,x+1} \hat{E}^c_{x,x+1} \quad .$$

$$\hat{H} = \hat{H}_{Kin} + \hat{H}_m + \hat{H}_E$$

$$\hat{H}_{Kin} = \sum_l \frac{1}{\sqrt{2}} \hat{P}_{0,l} \hat{X}_{l+1} \hat{P}_{0,l+2} + \frac{1}{2\sqrt{2}} \hat{P}_{1,l} \hat{X}_{l+1} \hat{P}_{1,l+2}$$

$$\hat{H}_m = m \sum_l \hat{P}_{0,l} \hat{P}_{1,l+1} + \hat{P}_{1,l} \hat{P}_{0,l+1}$$

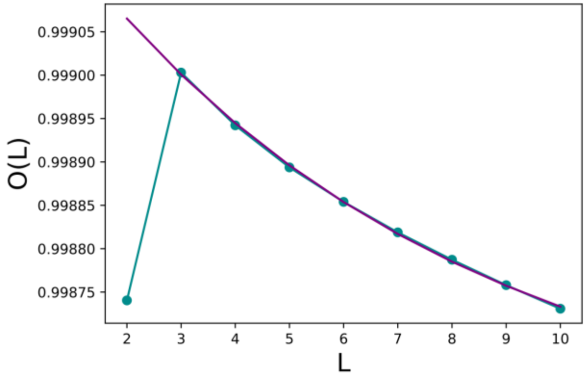
$$\hat{H}_E = \sum_l \frac{3}{8} g^2 \hat{P}_{1,l} \quad , \qquad (2)$$

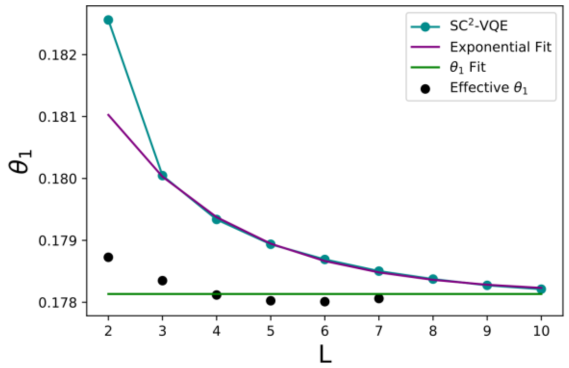
CLASSICAL SIMULATIONS

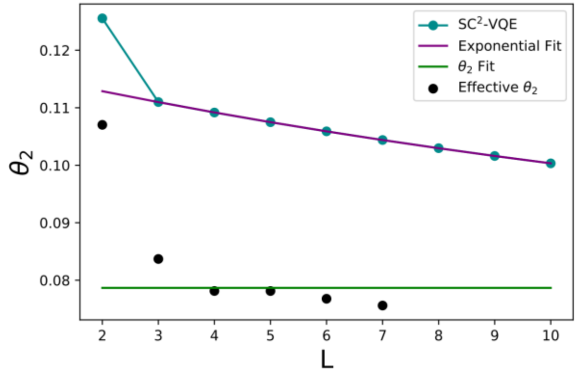
- Circuit Ansatz

$$\hat{U}(\vec{\theta}) = e^{i\sum_{x\ \text{odd}} \theta_0 \hat{P}_{0,x-1}\hat{Y}_x\hat{P}_{0,x+1} + \theta_1 \hat{P}_{1,x-1}\hat{Y}_x\hat{P}_{1,x+1}}$$
$$e^{i\sum_{x\ \text{even}} \theta_0 \hat{P}_{0,x-1}\hat{Y}_x\hat{P}_{0,x+1} + \theta_1 \hat{P}_{1,x-1}\hat{Y}_x\hat{P}_{1,x+1}}$$

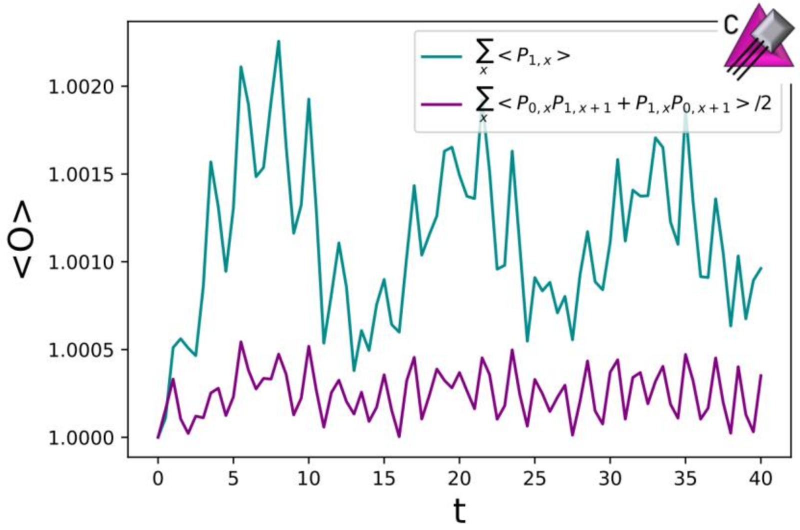
g=2
m=1

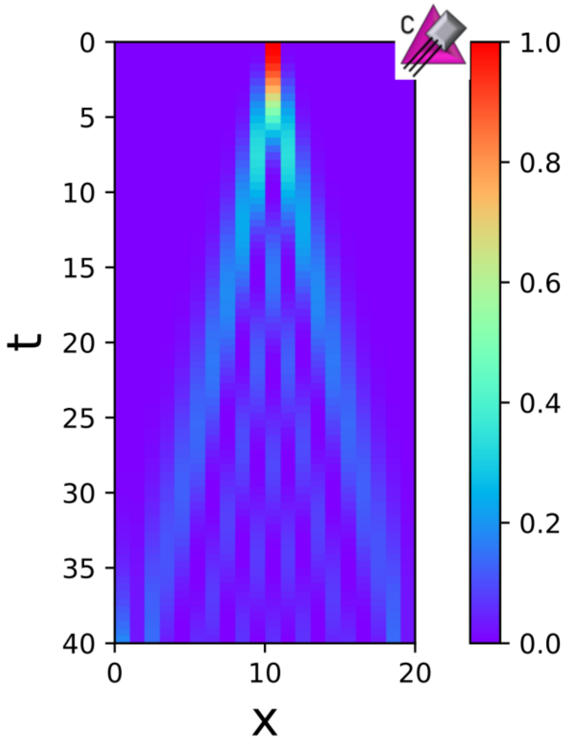






- We can test our procedure with a single meson state

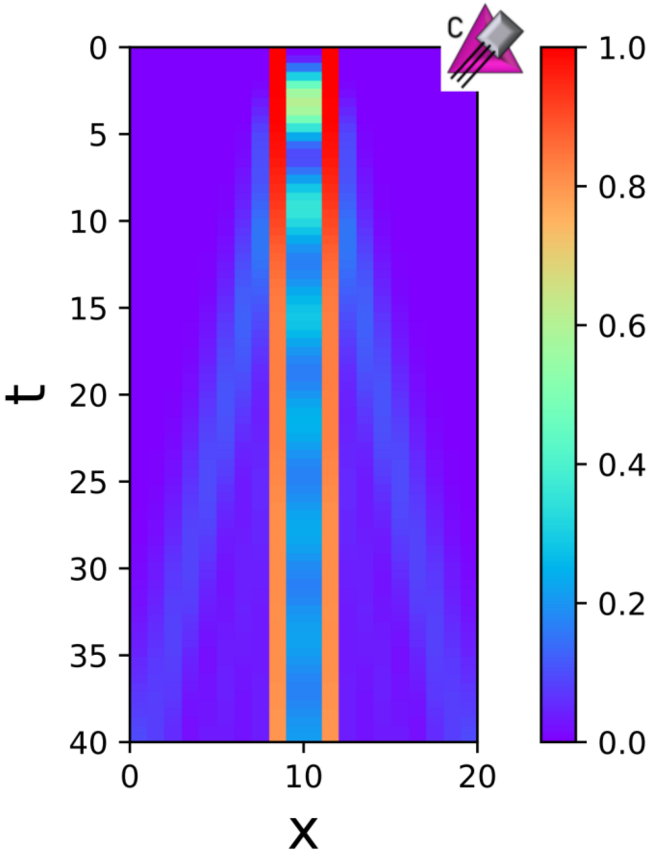


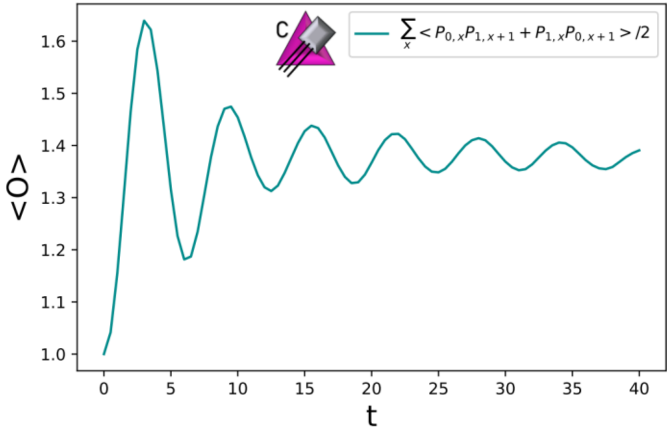


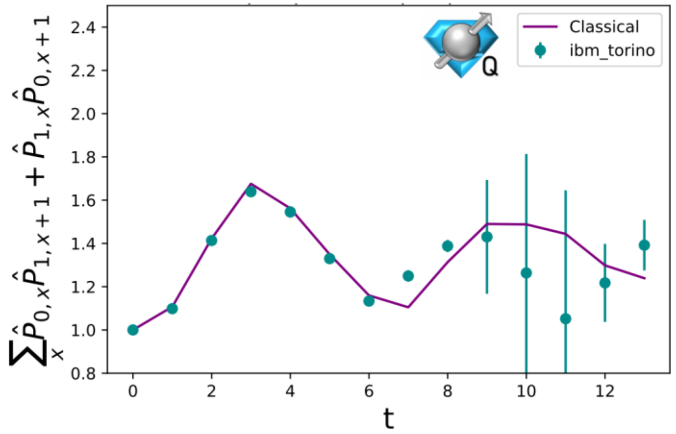
SIMULATING STRING BREAKING

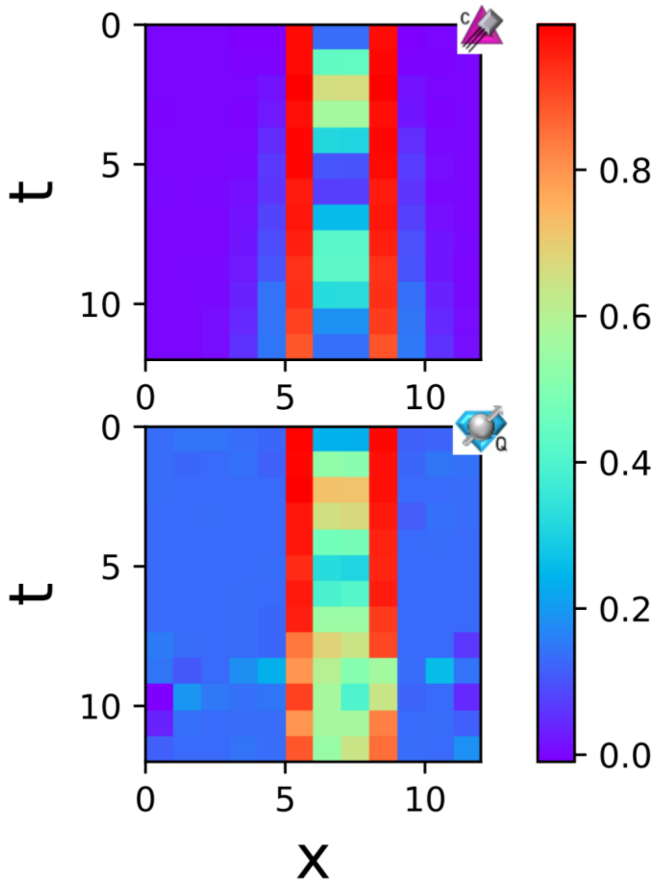
- States with a long string attaching a $q\bar{q}$ can be prepared with $|\psi_S\rangle = U(\vec{\theta}) \prod_{x \in S} \hat{X}_x |0\rangle$

- We expect to see the string break and have hadrons propagate out









SUMMARY

- Variational calculations can be extrapolated to larger system sizes.
- This has enabled preparation of vacuum and single hadron states on quantum computers.
- This approach can also be used to measure the propagation of hadrons.
- These techniques should scale to higher dimensions, and other gauge groups.