Halo Suppression by Nonlinear Decoherence

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Why do We Use Linear Lattices?

Integrable behavior

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The Strong-Focusing Synchrotron—A New High Energy Accelerator*

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Strong focusing forces result from the alternation of large positive and negative n-values in successive sectors of the magnetic guide field in a synchrotron. This sequence of alternately converging and diverging magnetic lenses of equal strength is itself converging, and leads to significant reductions in oscillation amplitude, both for radial and axial displacements. The mechanism of phase-stable synchronous acceleration still applies, with a large reduction in the amplitude of the associated radial synchronous oscillations. To illustrate, a design is proposed for a 30-Bev proton accelerator with an orbit radius of 300 ft, and with a small magnet having an aperture of 1×2 inches. Tolerances on nearly all design parameters are less critical than for the equivalent uniform-n machine. A generalization of this focusing principle leads to small, efficient focusing magnets for ion and electron beams. Relations for the focal length of a double-focusing magnet are presented, from which the design parameters for such linear systems can be determined.

Courant-Snyder invariant

$$\mathcal{J}_i = \frac{1}{2\beta_i(s)}\left[z_i^2 + (\beta_i(s)z_i' + \alpha_i(s)z_i)^2\right]$$

This gives us...

- Bounded trajectories
- Tunes
- Beta functions
- Dispersion

Controlled Nonlinear Lattices can have Bounded Motion

Normalized coordinates

$$H = \frac{1}{2}\vec{p}^{\,2} + \vec{q}^{\,T}\tilde{K}(s)\,\vec{q} + U(\vec{q}, s)$$

$$\begin{cases} z_N = \dfrac{z}{\sqrt{\beta(s)}} \\ p_N = p\sqrt{\beta(s)} - \dfrac{\beta'(s) z}{2\sqrt{\beta(s)}} \\ \psi'(s) = \dfrac{1}{\beta(s)} \end{cases}$$

canonical transformation

$$\mathcal{H} = \frac{1}{2}\vec{p}_N^{\;2} + \frac{1}{2}\vec{q}_N^{\;2} + \beta(\psi)U\left(\sqrt{\beta_x(\psi)}x_N, \sqrt{\beta_y(\psi)}y_N, s(\psi)\right)$$

Controlled nonlinearity makes the Hamiltonian independent of “time”

$$\beta(\psi)U\left(\sqrt{\beta_x(\psi)}x_N, \sqrt{\beta_y(\psi)}y_N, s(\psi)\right) = V\left(x_N, y_N\right)$$

Hamiltonian becomes a conserved quantity

Now the task is to select a potential with desirable properties that can be realized with free space magnetic fields

Nonlinear Integrable Optics[§]

Bertrand-Darboux Eqn.

$$xy\left(\partial_{xx}U - \partial_{yy}U\right) + \left(y^2 - x^2 + c^2\right)\partial_{xy}U + 3y\,\partial_x U - 3x\,\partial_y U = 0$$

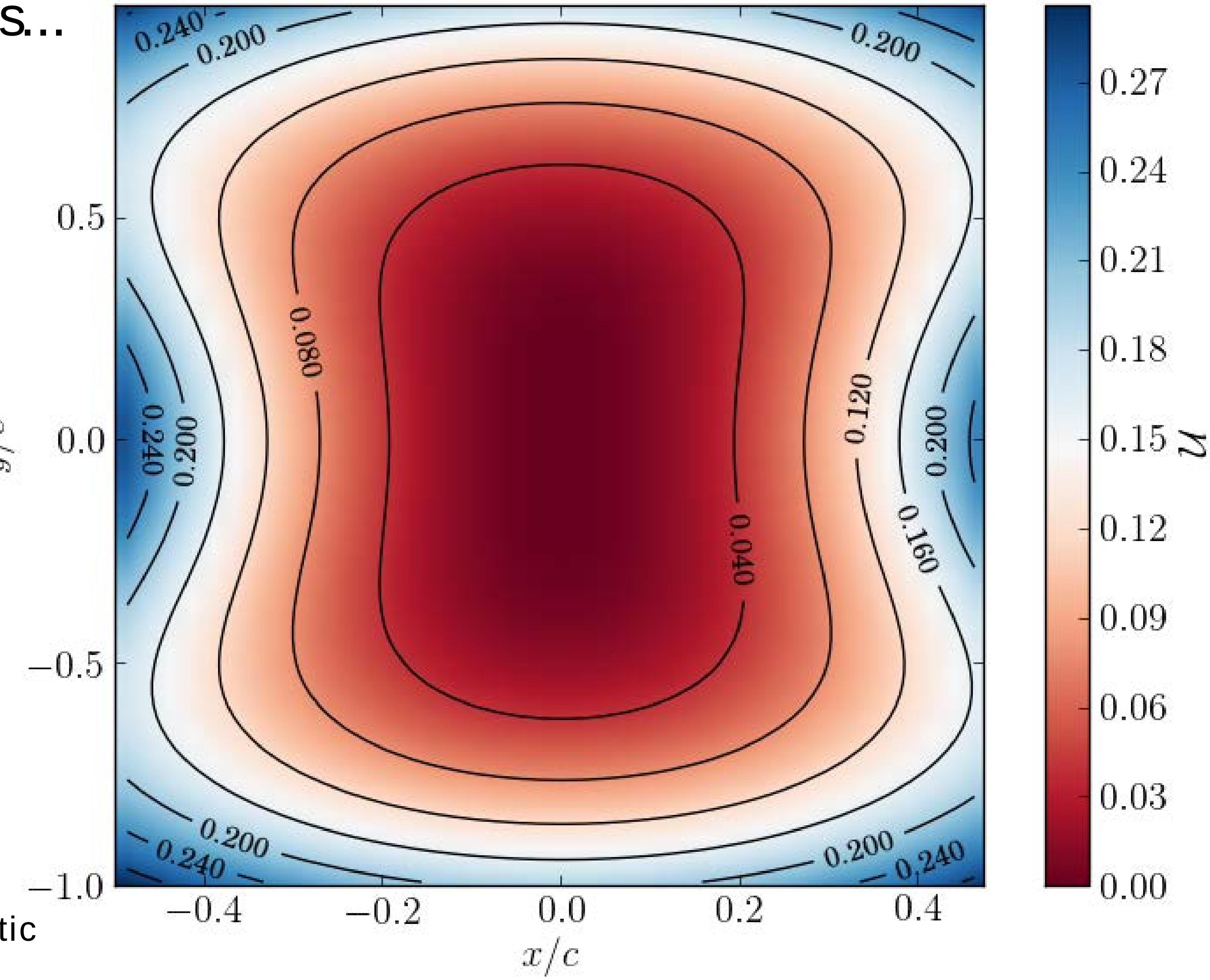
Self-consistently with Maxwell's equations yields...

$$U(x,y) = \frac{f(\xi) + g(\eta)}{\xi^2 - \eta^2}$$

$$\left\{ \begin{array}{c} \xi \\ \eta \end{array} \right\} = \frac{\sqrt{(x+c)^2 + y^2} \pm \sqrt{(x-c)^2 + y^2}}{2c}$$

$$f(\xi) = -\xi\sqrt{\xi^2 - 1}\cosh^{-1}(\xi)$$

$$g(\eta) = \eta\sqrt{1 - \eta^2}\left(\frac{\pi}{2} + \cos^{-1}(\eta)\right)$$



[§] V. Danilov and S. Nagaitsev, "Nonlinear lattices with one and two analytic invariants", Phys. Rev. ST - Acc. Beams 13, 084002 (2010).

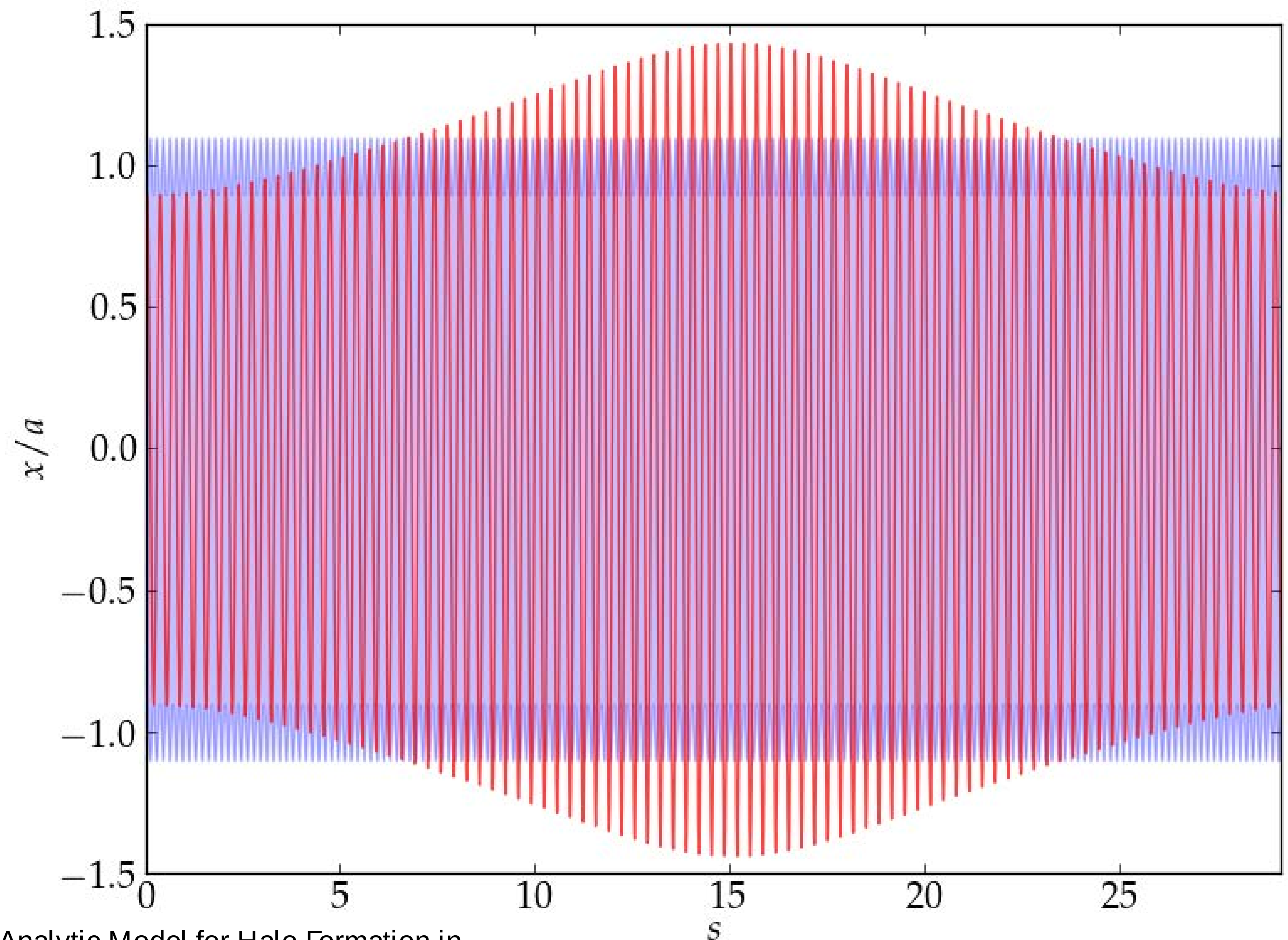
Beam Mismatch Sets Up Space Charge Driven Resonances

QuickTime™ and a
PNG decompressor
are needed to see this picture.

Halo Can Form from Mismatched Beams

Particle-Core Model†

Particle outside core: $$\ddot{x} + \left(\omega_0^2 - \kappa/a^2\right) x = -\frac{\kappa}{a^2}\frac{x^3}{a^2} + 2\frac{\epsilon\kappa}{a^2} x \cos(\omega_{\mathrm{core}} s)$$



† R.Gluckstern, "Analytic Model for Halo Formation in High Current Ion Linacs", Phys. Rev. Lett., **73**, 9 (1994)

Linear Lattice Forming Beam Halo

QuickTime™ and a
GIF decompressor
are needed to see this picture.

Integrable Elliptic Lattice Suppresses Beam Halo

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Conclusion

- Nonlinearities can be your friend, if you know how to talk to them
- There are realizable lattices that are nonlinear and realizable with magnetic elements
- Beam halo is a direct consequence of linear lattices...
- ... but nonlinear optics mitigate the instability

Thank you

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