KANE-PUMPLIN-REPKO (KPR) factorization and PRECISON TESTS of COLLINS CONJUGATION

$\langle \delta k_{TN}(x, \mu_0^2) \rangle$ dst'ns (process dependent)

$\langle \delta k_{TN}(z, \mu_0^2) \rangle$ frag. (universal)

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REPORT from QCD EVOLUTION 2013

Collins Schwartz Schnell Boglione

(A_N - $pp \to jets$)

Mulders Bland Burkardt Metz Gamberg

TMD FACTORIZATION EVOLUTION

CSS EIS CONSISTENT

LONG RANGE PLAN EHPI33

= COLLINS CONJUGATION

< Significant phenomenological process in implementing TMD evolution >

Sivers Vitev Schlegal Idilbi Boer Prokutin

Yuan Chen Rogers Weiss Siebert

JLAB 2013 ⇒ Santa Fe 2014

OUTLINE

I. Introduction to KPR factorization

symmetries, projection operators &

QM superselection

A_γ-odd dynamics of the pion tornado in QFT

II. Spin-Directed momentum transfers & qcd evolution

$\langle \delta k_{TN}(x, M_0^2) \rangle$ a theoretically rigorous alternative to A_N, P no TMD evolution

III. Comparing SIDIS v. DY: what does HP13 mean?

Color factors, K factors, $\langle k_T^2 \rangle$ v. Q^2

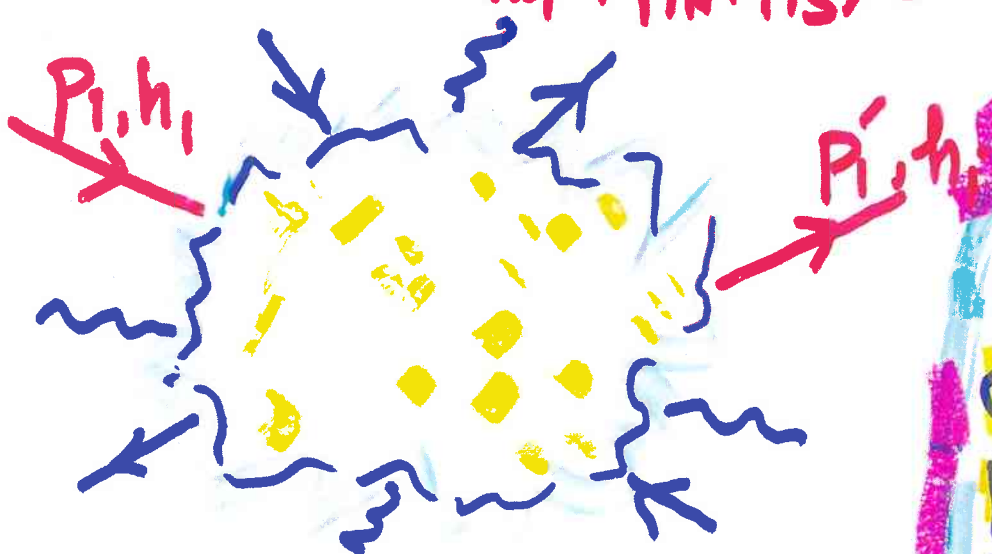
equality $\langle \delta k_{TN}(x, M_0^2) \rangle_{DY} = -\langle \delta k_{TN}(x, M_0^2) \rangle_{SIDIS}$

I. Introduction to KPR factorization

Kane, Pumplin & Repko PRL 41, 1689 (1978)

$$A_N|_{qq\uparrow \Rightarrow qX} = \left[\frac{d\sigma(qq\uparrow \Rightarrow q) - d\sigma(qq\downarrow \Rightarrow q)}{d\sigma(qq\uparrow \Rightarrow q) + d\sigma(qq\downarrow \Rightarrow q)}\right] = \alpha_s(Q^2)\frac{m_q}{\sqrt{s}}\, f(\theta_{cm})(1+\ldots$$

where $f(\theta_{cm}) = \frac{P_{TN}}{(\mu^2 + P_{TN}^2 + P_{TS}^2)^{1/2}}$ $f(\theta_{cm})$ vanishes at $\theta_{cm} = 0$



quark helicity preserved
$O(m_q/\sqrt{s})$

quark helicity conservation in QCD pert. theory implies an additional symmetry in pqcd that is broken in the full theory furthur studies subsequently confirmed that TSSA's in qcd perturbation theory are not enhanced by large logarithms at higher orders

KPR factorization in QCD

In the limit $m_q \to 0$ there exists a symmetry in QCD perturbation theory that ensures that <u>all</u> transverse spin asymmetries in hard scattering processes can be absorbed into the transverse momentum dependence of hadronic distribution functions or fragmentation functions

Sivers 1990, 1991

quite distinct from TMD factorization

$m_u = 1.9 \pm 0.2\ 10^{-3}$ $m_d = 4.6 \pm 0.3\ 10^{-3}$ $m_s = 86 \pm 5\ 10^{-3}$ GeV

To appreciate the value of KPR factorization it is appropriate to incorporate the power of superselection rules and idempotent projection operators in QFT and quantum mechanics

all single-spin asymmetries

$$A(\vec{\sigma}) = [N(\vec{\sigma}) - N(-\vec{\sigma})]/[N(\vec{\sigma}) + N(-\vec{\sigma})]$$

$\vec{k}_i$ = 3 vectors

$\vec{\sigma}_j$ = axial 3 vectors

are odd under an operator Θ

$$\boxed{\Theta\{\vec{k}_i\,;\vec{\sigma}_j\}\Theta^{-1} = \{\vec{k}_i\,; -\vec{\sigma}_j\}}$$

Θ serves as a 3-D Hodge dual of the parity operator

$$\boxed{P\{\vec{k}_i\,;\vec{\sigma}_j\}P^{-1} = \{-\vec{k}_i\,; \vec{\sigma}_j\}}$$

the product $A_\tau = P\Theta$ has the action

$$\boxed{A_\tau\{\vec{k}_i\,;\vec{\sigma}_j\}A_\tau^{-1} = \{-\vec{k}_i\,; -\vec{\sigma}_j\}}$$

A_τ "naive time reflection" (Joffe, 1994 ; Sivers, 1994)

FINITE SYMMETRIES

	T	C	P	(CPT)	Θ	A_τ	A_σ	A_c
Σ_x	−	+	−	+	−	+	−	+
Σ_y	+	+	+	+	−	−	−	−
Σ_z	+	−	−	+	−	+	+	−
					$(*P)*$	ΘP	$A_\tau T$	$A_\tau C$

$*$ HODGE DUAL OPERATOR

$P: (V, A) = (-V, A)$

$*: (V, A) = (\tilde{A}, \tilde{V})$

$P*: (V, A) = (\tilde{A}, -\tilde{V})$

$*P*: (V, A) = (V, -A)$

$\Theta = *P*$ "Snake Operator"

Changes sign of spins without changing momenta

$A_\tau: (\hat{P}, \hat{\sigma}) = (-\hat{P}, -\hat{\sigma})$

$\boxed{\Theta = PA_\tau = - \text{ for } \underline{\text{all}} \text{ single-spin observables}}$

The QFT of the Pion Tornado - A_τ-odd dynamics in confined boundary conditions

by construction $A_\tau = \Sigma_y$

$A_\tau = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ transversity basis

$A_\tau = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$ helicity basis

transversity $|M|^2\uparrow = |M^+|^2 + |M^-|^2$

$|M|^2\downarrow = |M^+|^2 - |M^-|^2$

$A_N = \dfrac{|M^-|^2}{|M^+|^2}$

helicity $|M|^2\uparrow = |M^0|^2 + |F|^2 + \mathrm{Im}(FM^{0*})$

$|M|^2\downarrow = |M^0|^2 + |F|^2 - \mathrm{Im}(FM^{0*})$

$A_N = \dfrac{\mathrm{Im}(FM^{0*})}{|M_0|^2 + |F|^2}$

Feynman diagram calculations often use Im[ ] as a projection operator (cutting rules) and drop $|F|^2$

since I will only be discussing nonperturbative spin-orbit dynamics - stick to transversity basis

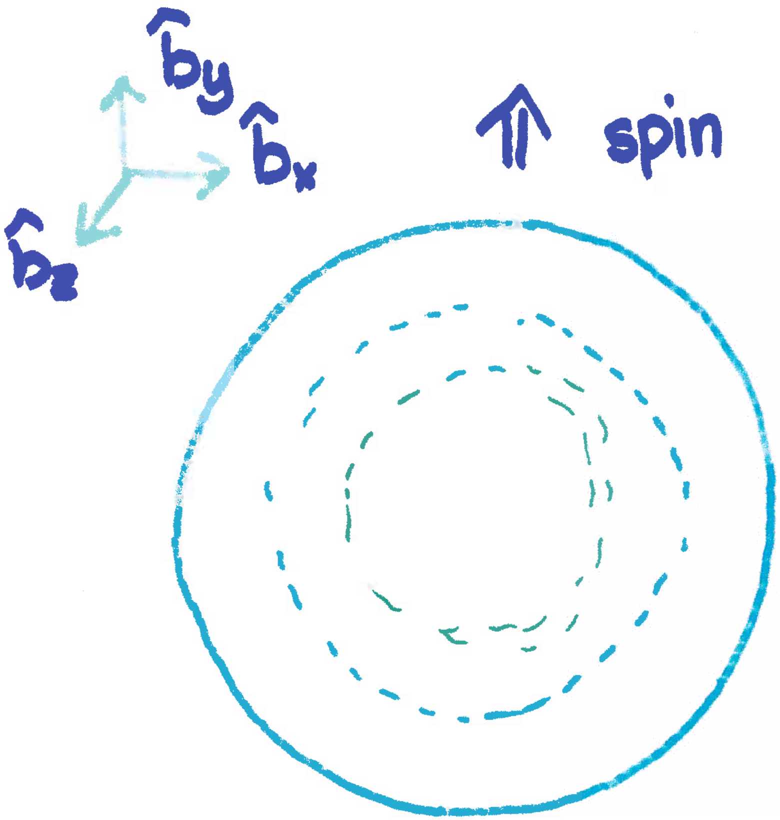
Georgi-Manohar Chiral Quark Model

$|p\uparrow\rangle \Rightarrow [U,D] \otimes U\uparrow$

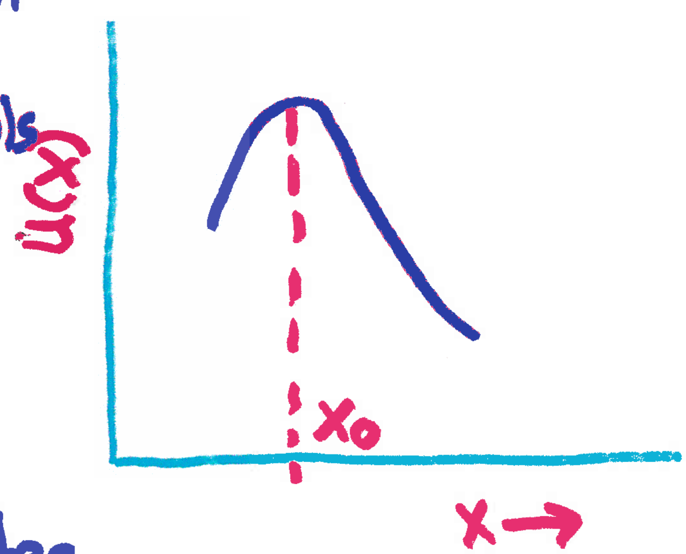
isoscalar 0^+ diquark bound to constituent quark

$P_p = \left(\frac{m_p^2}{P_-}, P_-, 0, 0\right)$ $P_- = m_p$

$P_u = \left(\frac{m_{eff}^2}{xP_-}, xP_-, p_x, p_y\right)$



proton spin controls $u\uparrow$ spin



$x_0 = \frac{m_{eff}(0)}{m_p}$

virtual fluctuations dominated by $\Uparrow$ $(q\bar{q})$ states now "turn on" a set of transitions

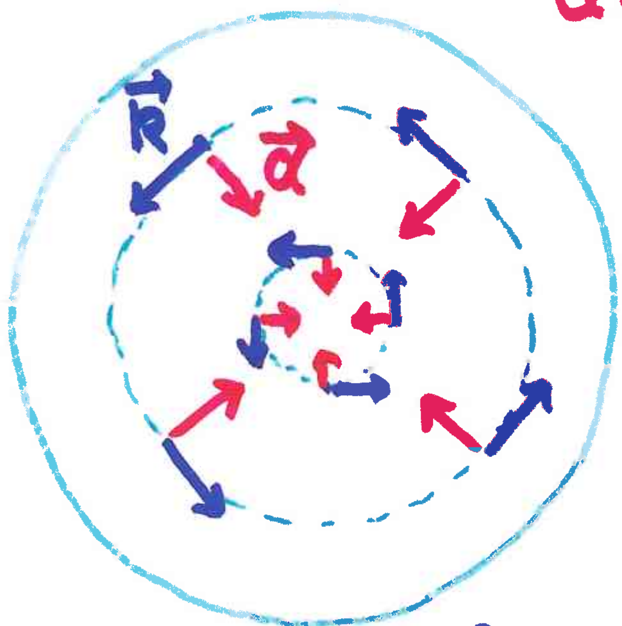
$\left(U\Uparrow \rightleftharpoons \sum_{q=u,d,s} q\Downarrow (\bar{q}\Downarrow u\Uparrow)_{\Uparrow} \right)_{L_y=+1}$ equilibrium with confining boundary conditions

resolve $U\Uparrow$ into massless $q\bar{q}u$ ensemble with $L_y=1$

$L_y = b_z k_x - b_x k_z$

$\vec{a} = \frac{\partial \vec{k}}{\partial t} = \frac{\partial \vec{k}}{\partial s}$

$P_{TN} = P_x \quad -P_x$
$P_{TS} = P_y \quad -P_y$



Evaluate

$P_{TN} = P_{TS} = P_z = 0$

(rest frame of $U\Uparrow$ coincides with rest f pr)

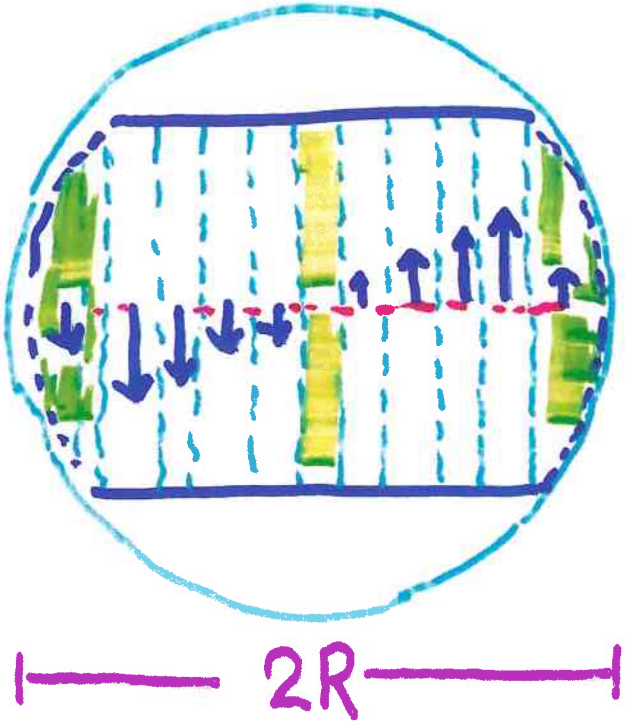
put in P_{TN} P_{TS} P_z fluctuations later

$\vec{k}_q + \vec{k}_{\bar{q}} + \vec{k}_u = 0 \quad \langle k_u \rangle = \frac{1}{3} m_0$

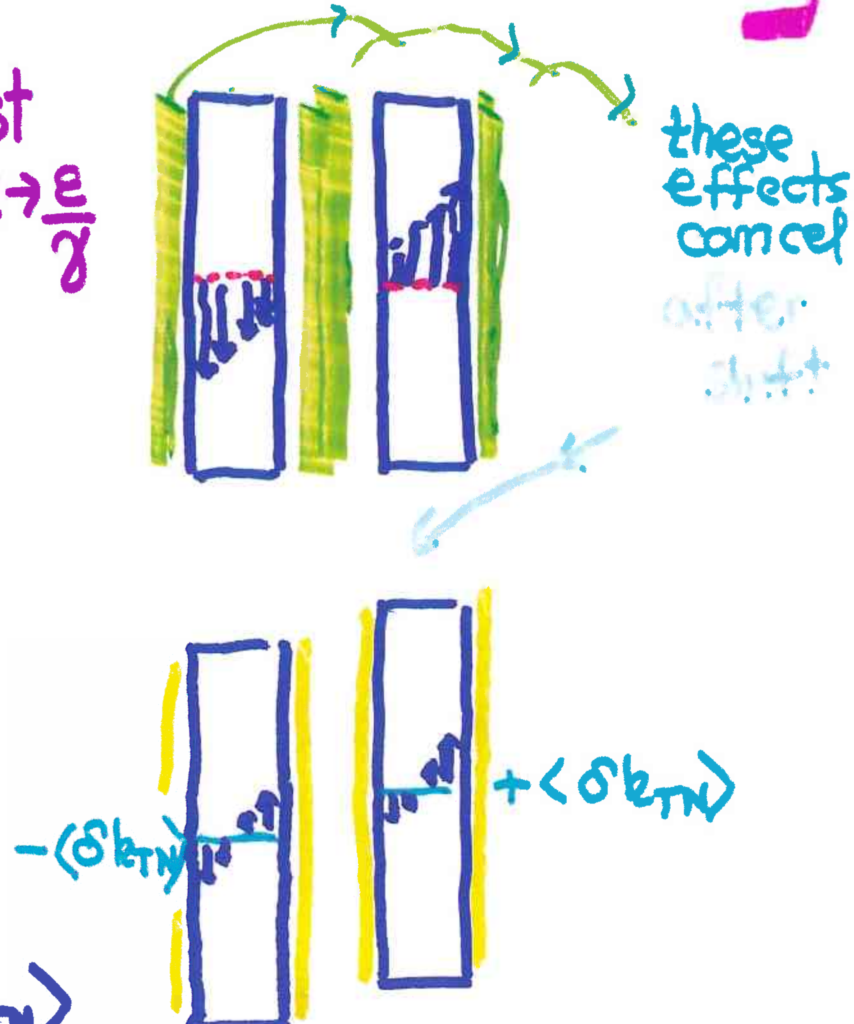
$\langle L_y^u \rangle = \frac{1}{3} \alpha_u \left(\frac{1-\lambda}{2}\right) \rightarrow 0.2 \pm 0.02$

d.sivers
arXiv 0704.1791
(unpublished)

[average over b_y to account for spin precession and split ensemble into bins of $\Delta b_z = \epsilon$ where $\epsilon = R/_{n+2}$]



boost
$\gamma \Rightarrow \epsilon \rightarrow \frac{\epsilon}{\gamma}$

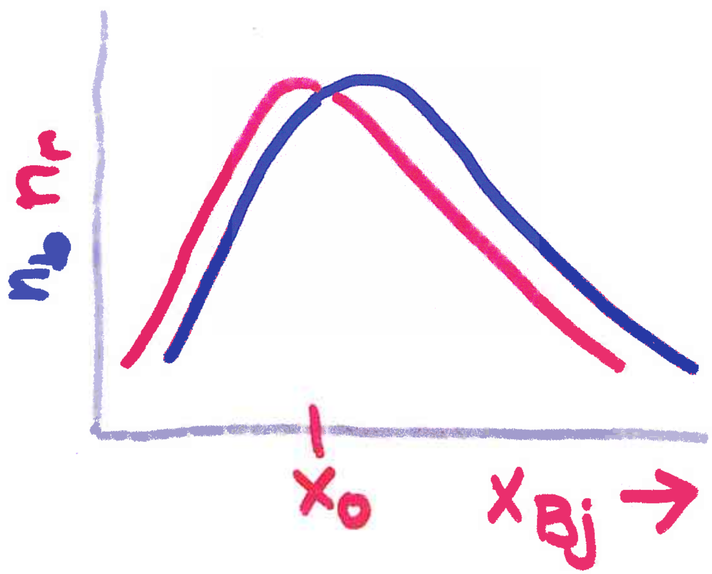


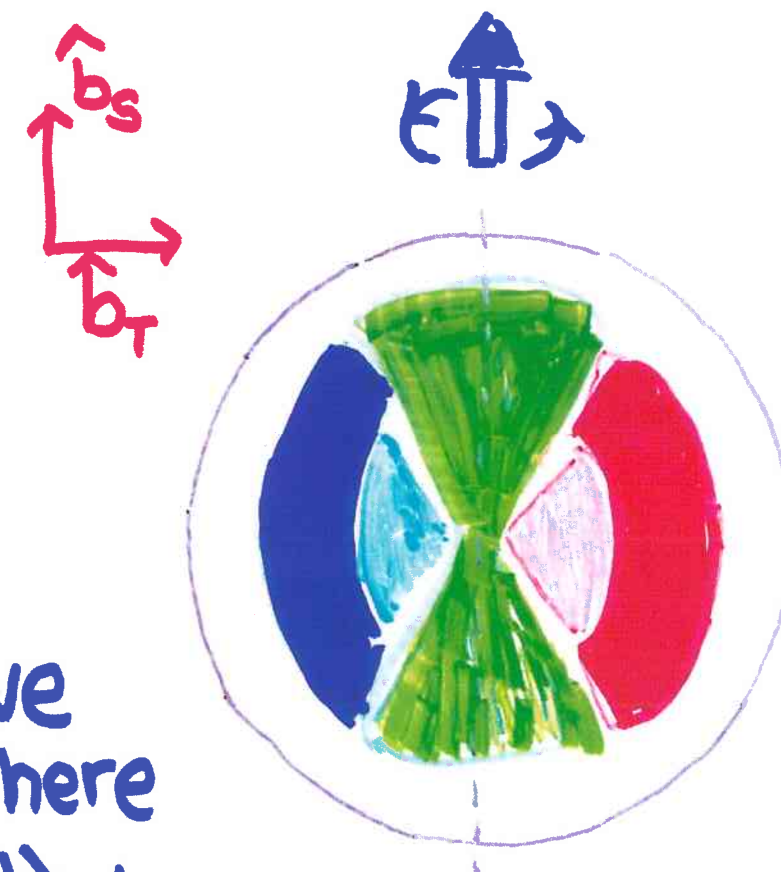
These manipulations result in two ensembles w/t distributions centered $\pm \langle \delta k_{TN} \rangle$

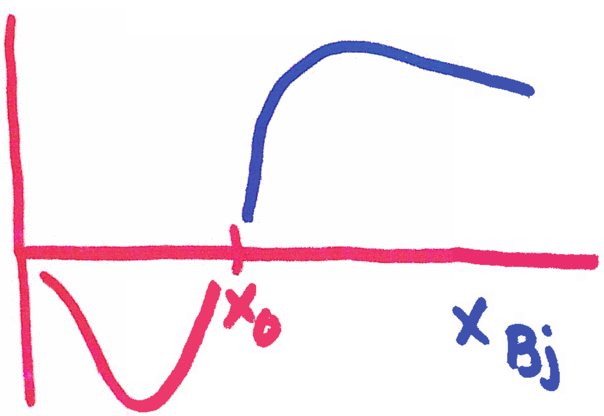
[NOW EXAMINE b_y - b_z projection]

blue shift $(\uparrow, L) + (\downarrow, R)$
red shift $(\uparrow, R) + (\downarrow, L)$

Spin-orbit mechanisms necessarily Bj x-dependent



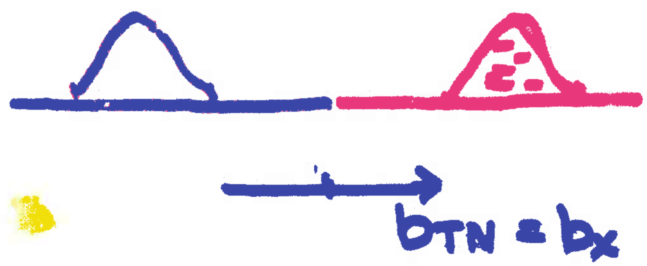




can have node where $\langle \delta k_{TN}(x) \rangle$ changes sign

"x_0 can be Q^2 dependent"

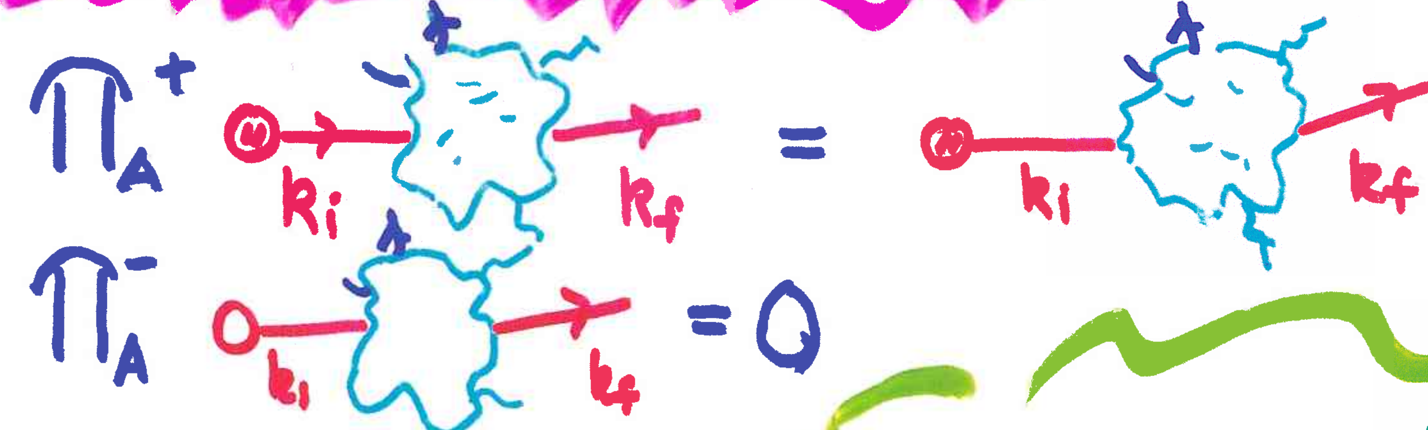
now integrate over k_{TS} set $b_y = 0$



fourier transform and shift

average these shifts with those of other projection

Kane Pumplin Repko Isolation



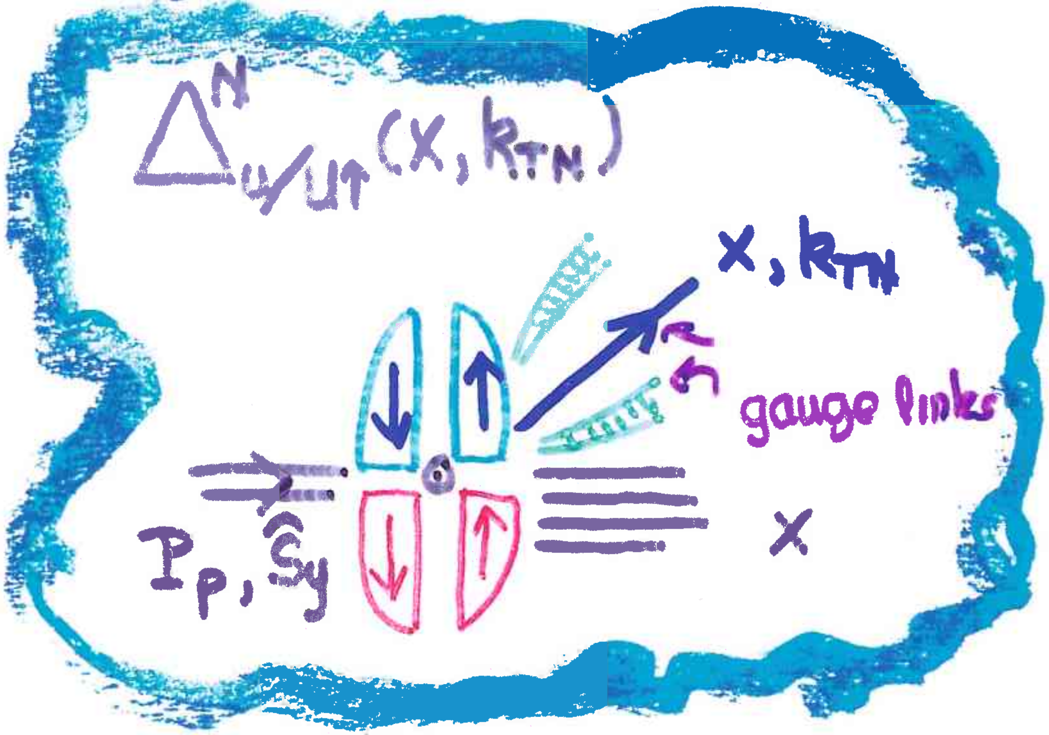
so that all the spin asymmetry in $\pm \delta k_{TN}$

$k_i = (xP_-, 0, 0, 0)$

$k_f = (xP_- \cos\theta, 0, xP_- \sin\theta \sin\phi, xP_- \sin\theta \cos\phi)$

$0 = \langle \sin\phi \rangle = \langle \cos\phi \rangle$

The difference $U\uparrow - U\downarrow$ leads to A_τ-odd ("T"-odd) distribution function



In the absence of ISI or FSI interactions the sampling of these sectors in a hard-scattering process the total contribution will vanish

The result for a specific hard process is sometimes called an "effective" or "dynamical" distribution

GM model also gives $\Delta^N_{d/U\uparrow}(x, k_{TN})$, $\Delta^N_{s/U\uparrow}(x, k_{TN})$, $\Delta^N_{\bar{u}/U\uparrow}(x, k_{TN})$ etc.

In the framework of TMD's there are 4 independent A_T-odd quantum structures

process dependent

$\Delta^N G_{q/p\uparrow}(x, k_{TN}; \mu^2)$

orbital distributions

$\Delta^N G_{q\uparrow/p}(x, k_{TN}; \mu^2)$

Boer-Mulders functions

chiral even

chiral odd

$\Delta^N D_{H\uparrow/q}(z, k_{TN}; \mu^2)$

polarizing fragmentation functions

$\Delta^N D_{H/q\uparrow}(z, k_{TN}; \mu^2)$

Collins functions

factorize

where QM superselection has <u>isolated</u> spin-orbit dynamics of confinement and chiral-symmetry breaking leading to the indicated spin asymmetries

II. Spin-Directed Momentum Transfers and QCD Evolution

KPR factorization describes the isolation of non-perturbative spin/orbit dynamics into fragmentation functions or into "effective" distribution functions

Single-spin asymmetries are rigorously defined by spin-oriented momentum transfers

$\langle \delta k_{TN}(x, \mu_0^2) \rangle$ A_T-odd distributions

$\langle \delta k_{TN}(z, \mu_0^2) \rangle$ A_T-odd fragmentations

Projection Operators $\Pi_A^{\pm}$

$$\frac{d\sigma\uparrow}{dk_{TN}\,dk_{TS}}(x; k_{TN}\, k_{TS}) = K(|M^+|^2 + |M^-|^2)$$

$$\frac{d\sigma\downarrow}{dk_{TN}\,dk_{TS}}(x, k_{TN}, k_{TS}) = K(|M^+|^2 - |M^-|^2)$$

$$\int d\sigma\uparrow k_{TN} = \int |M^-|^2 k_{TN} = \frac{1}{2}\langle\delta k_{TN}\rangle$$

$$\int d\sigma\downarrow k_{TN} = -\int |M^-|^2 k_{TN} = -\frac{1}{2}\langle\delta k_{TN}\rangle$$

don't necessarily assume δk_{TN} positive

Hadronic transverse-momentum distributions are sharply-peaked functions

$\frac{d\sigma\uparrow}{dk_{TN}dk_{TS}}(x;k_{TN},k_{TS})$ peaks at $k_{TN}=\frac{1}{2}\langle\delta k_{TN}\rangle$, $k_{TS}=0$

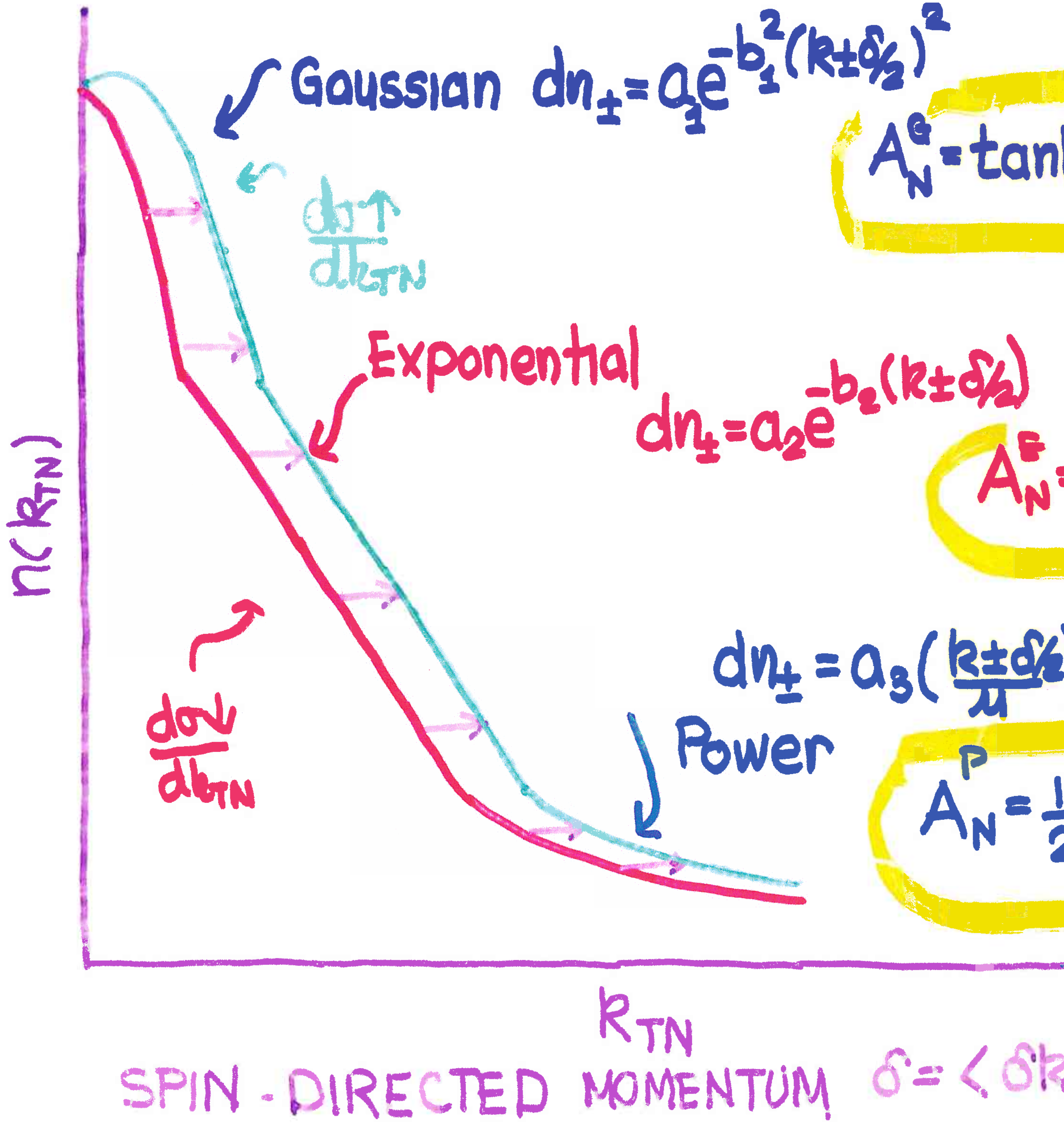
$\frac{d\sigma\downarrow}{dk_{TN}dk_{TS}}(x;k_{TN},k_{TS})$ peaks at $k_{TN}=-\frac{1}{2}\langle\delta k_{TN}\rangle$, $k_{TS}=0$

$$\int dk_{TS}\frac{d\sigma}{dk_{TN}dk_{TS}}(x;k_{TN},k_{TS}) = f(x, k_{TN}-\tfrac{1}{2}\langle\delta k_{TN}(x)\rangle)\left(1+\mathcal{O}\left(\frac{\langle\delta^2\rangle}{m_p^2}+\cdots\right)\right)$$

$$\int dk_{TS}\frac{d\sigma}{dk_{TN}dk_{TS}}(x;k_{TN},k_{TS}) = f(x, k_{TN}+\tfrac{1}{2}\langle\delta k_{TN}(x)\rangle)\left(1+\mathcal{O}\left(\frac{\langle\delta\rangle^2}{m_p^2}+\cdots\right)\right)$$

where $f(x,k_{TN})$ sharply peaked at $k_{TN}=0$

Some fraction of single unit of $\hbar$ representing flip of proton spin $S_y=\frac{1}{2}\hbar \Rightarrow S_y=-\frac{1}{2}\hbar$ converted into a spin-directed momentum shift via spin-orbit correlations



Gaussian $dn_{\pm} = a_1 e^{-b_1^2 (k \pm \delta/2)^2}$

$$A_N^G = \tanh(b_1^2 k_{TN} \delta)$$

Exponential $dn_{\pm} = a_2 e^{-b_2 (k \pm \delta/2)}$

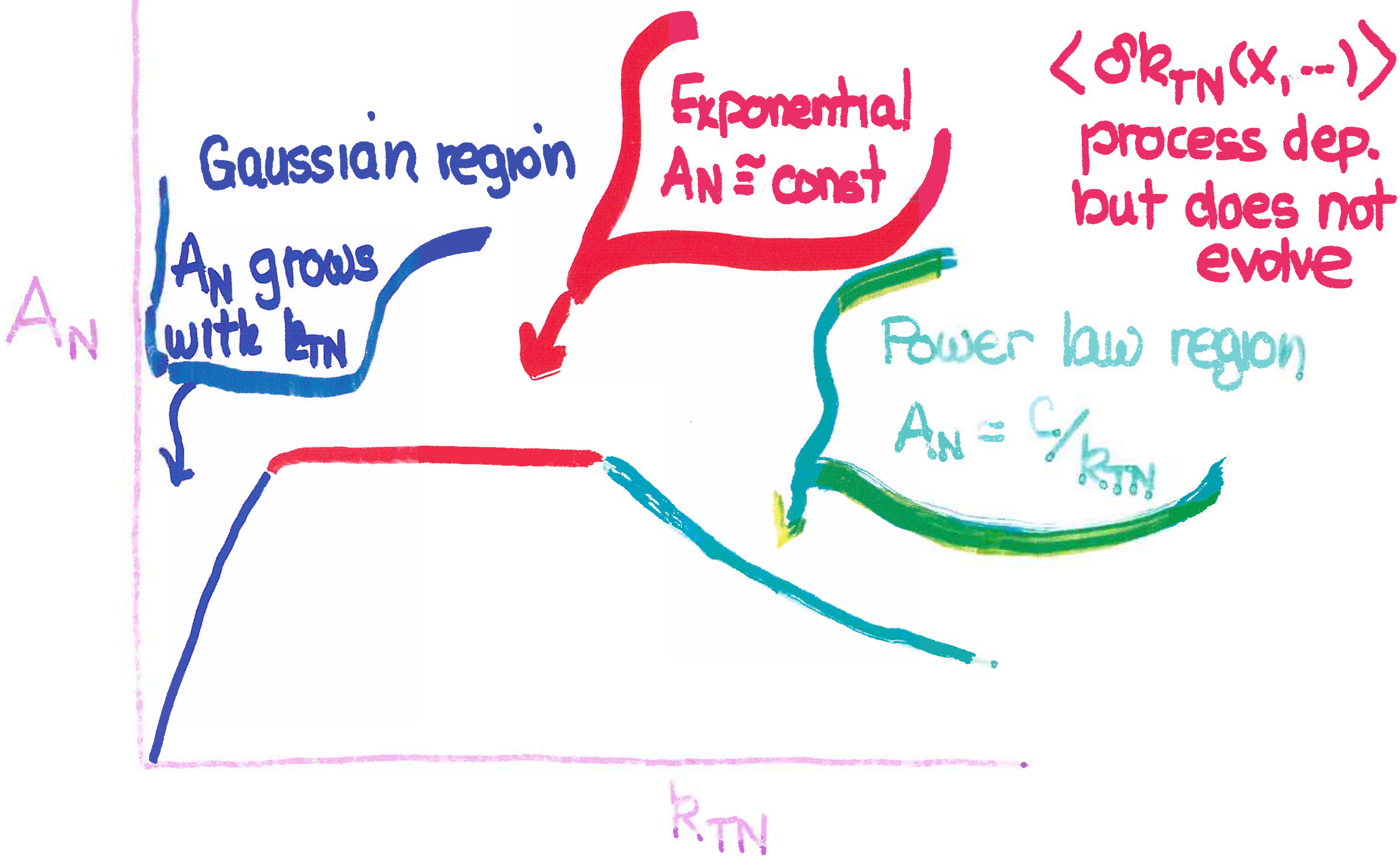
$$A_N^E = \tanh(b_2 \delta)$$

Power $dn_{\pm} = a_3 \left(\frac{k \pm \delta/2}{\mu} \right)^{-N}$

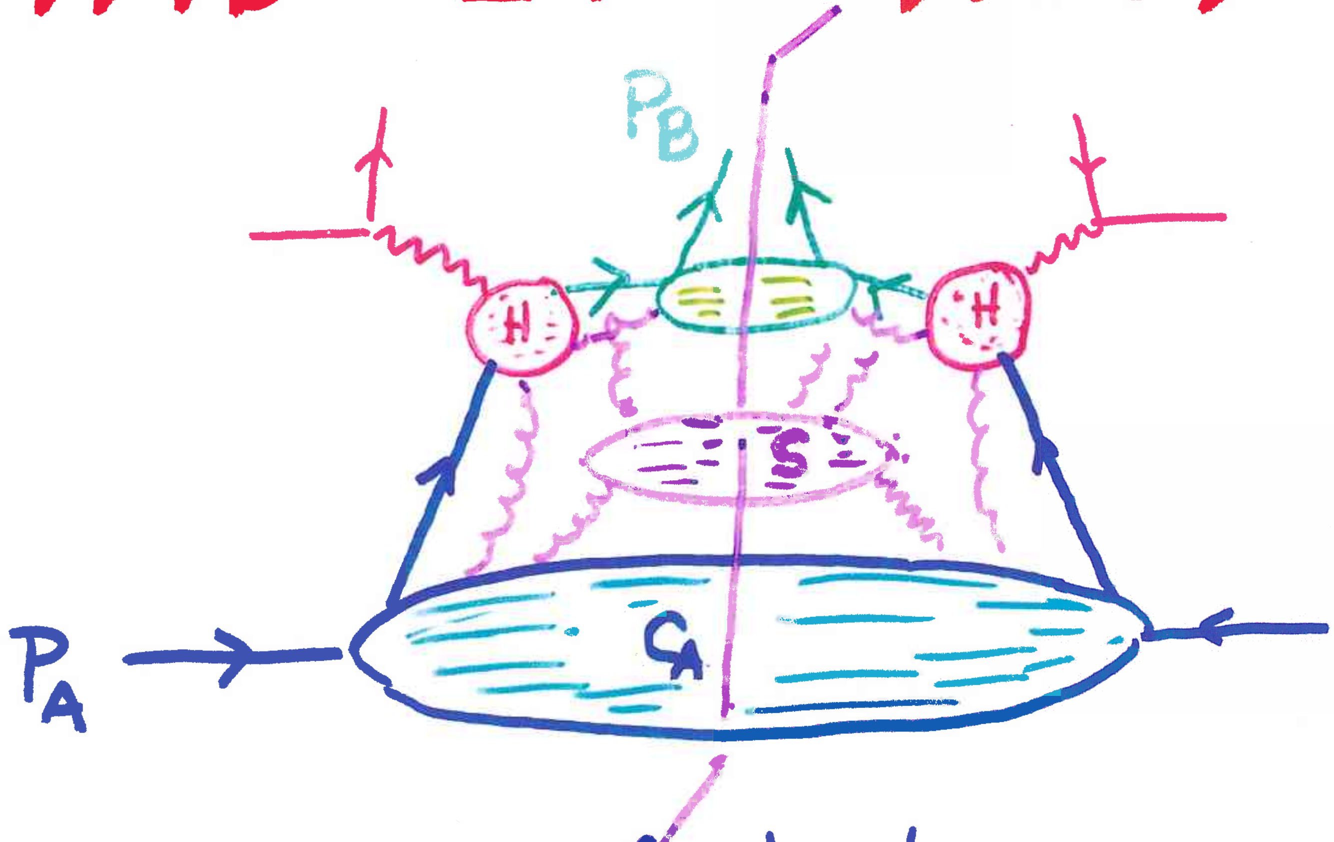
$$A_N^P = \frac{1}{2} \frac{N\delta}{k_{TN}}$$

SPIN - DIRECTED MOMENTUM $\delta = \langle \delta k_{TN}(x, M_0^2) \rangle$

A_N changes dramatically in response to QCD evolution in shape of $d\sigma/dk_{TN}$

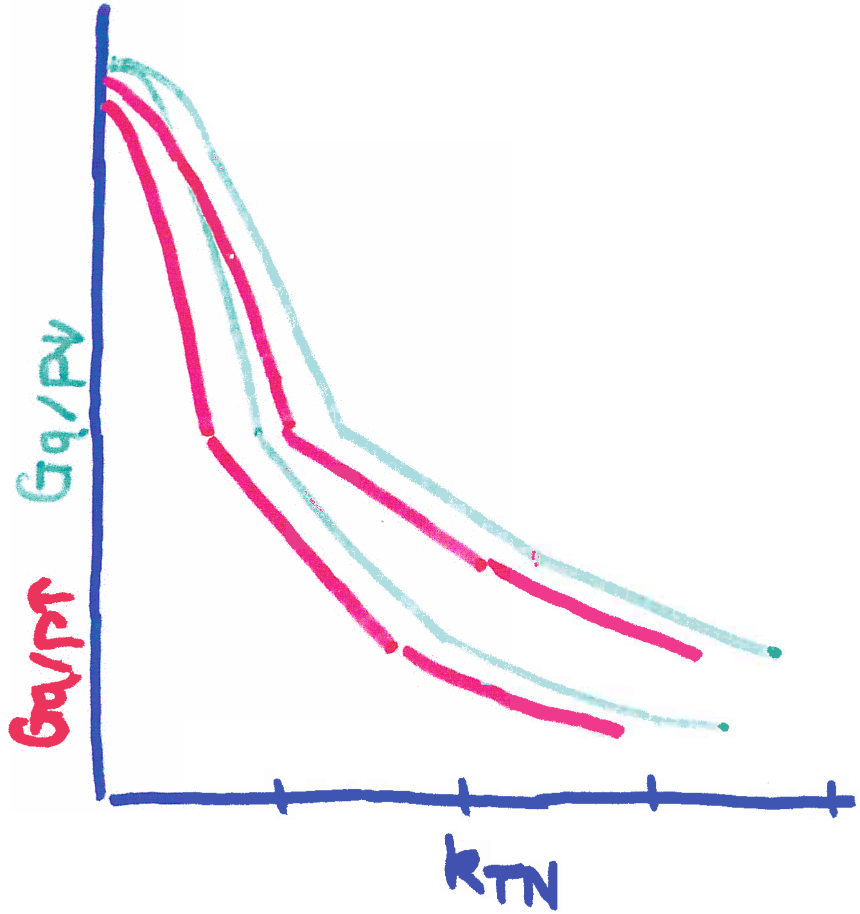


TMD EVOLUTION

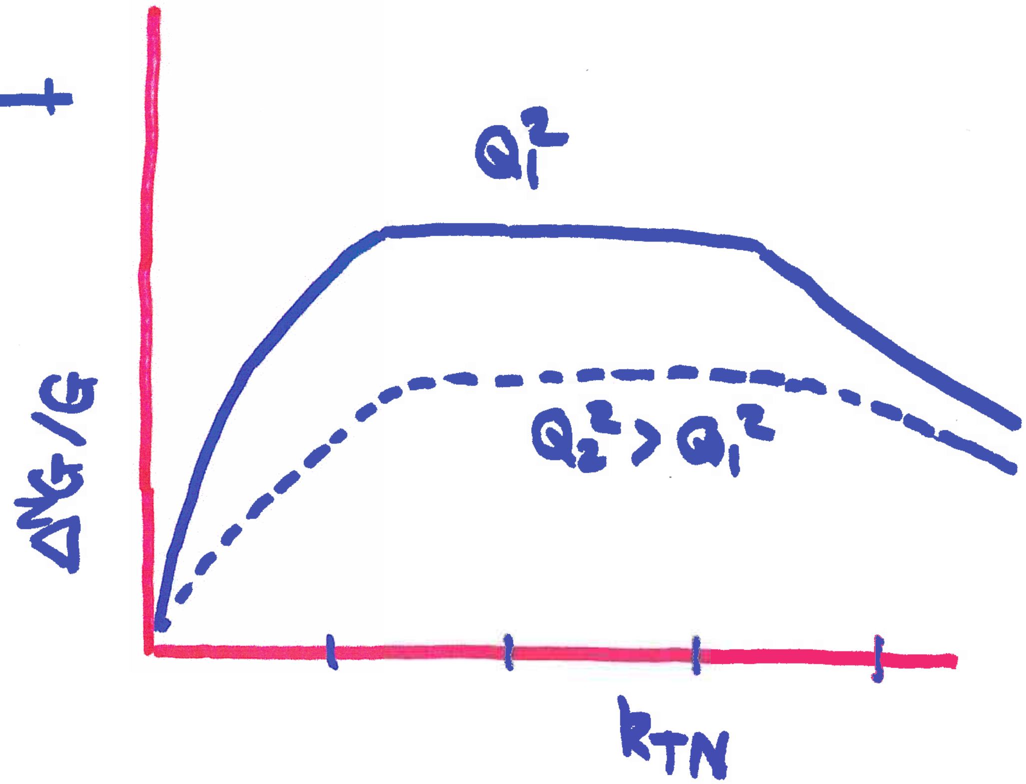


Leading regions for low transverse momentum in SIDIS

Based on Fig 13.11 in <u>Foundations of Perturbative QCD</u> John Collins 2011



$G_{q/p\uparrow}(x, k_{TN}; Q^2)$ and
$G_{q/p\downarrow}(x, k_{TN}; Q^2)$ both evolve
in shape with
$\langle \delta k_{TN}(x, \mu_0^2) \rangle$ constant
KPR factorization



In processes with TMD factorization (SIDIS & DY) spin-directed momentum incorporated into TMDs consequences

1. $\langle \delta k_{TN}(x, \mu_0^2) \rangle$ completely describes spin asymmetry in $d\sigma\uparrow - d\sigma\downarrow$

2. $A_N(x, k_{TN}; Q^2) = |M^{-2}(x, k_{TN}; Q^2)| / |M^{+2}(x, k_{TN}; Q^2)|$ shows significant TMD evolution

3. $\langle \delta k_{TN}(x, \mu_0^2) \rangle$ does respond to DGLAP evolution of $d\sigma\uparrow$ and $d\sigma\downarrow$

4. KPR factorization requeres $\langle \delta k_{TN}(x, \mu_0^2) \rangle$ not altered by perturbative corrections

Now let's turn to HP13 ! !

III. Comparing SIDIS v. DY - Collins conjugation $\langle \delta k_{TN}(x,\mu_0^2)\rangle_{DY} = -\langle \delta k_{TN}(x,\mu_0^2)\rangle_{SIDIS}$

color factors, K-factors, TMD factorization & universality

Drell-Yan	SIDIS	$e^+e^- \to$ hadrons	
[1/3]	[1]	[3]	color factors
γ_H K_{ISI}	γ_H K_S	γ_H $K_{e^+e^-}$	vertex renorm K factors
$G \otimes G$	$G \otimes D$	D	universal dst'ns G & FF D (collinear factorization)
$G \otimes \Delta^N G$ (−1)	$\Delta^N G \otimes D$, $f^\uparrow \otimes \Delta^N D$	$\Delta^N D \otimes \Delta^N D$ (+1)	TMD's, SSI's not universal

The non-universality of SSA's in transverse momentum-dependent "effective" distributions is now familiar. They can be parameterized in terms of gauge-link dependent lensing functions that incorporate the geometrical sampling of the A_T-odd spin-orbit dynamics by a specific process

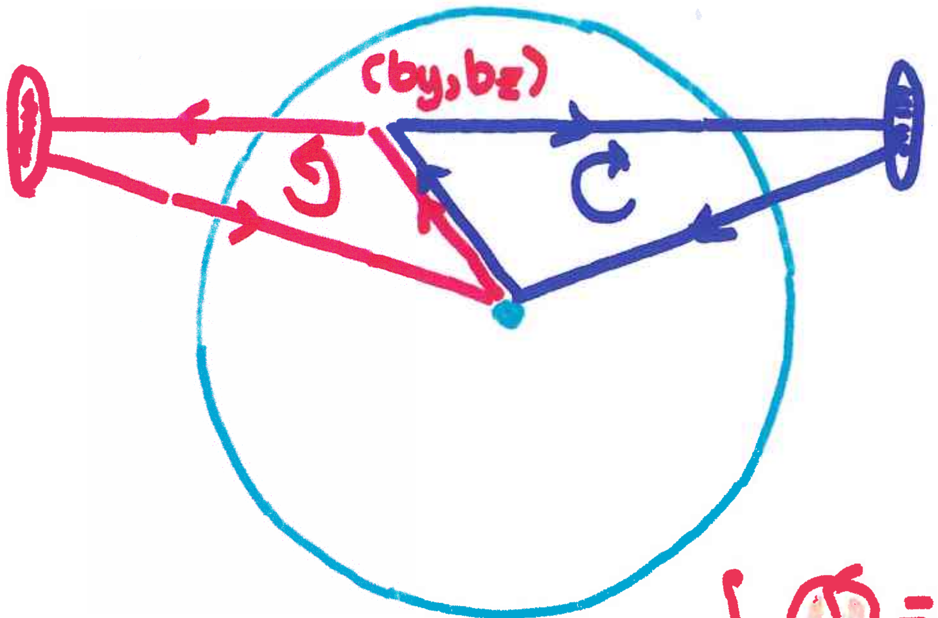
The original prediction (John Collins)

$$f_1^{TN}(x)\Big|_{DY} = -f_1^{TN}(x)\Big|_{SIDIS}$$

derived in gauge-link formalism & confirmed by many models an calculations

A simple and direct confirmation of this result involves non-Abelian Wilson loops

Wilson loops DY and SIDIS



most conveniently evaluated in radial coordinate gauge
$\vec{A}^a \cdot \hat{r} = 0$
radial lines vanish and only horizontal lines survive

$\int$ = $\longleftarrow\!\bullet$ (b_y, b_z) $\Delta k_{TN}(b_y, b_z)$

$\int$ $\bullet\!\longrightarrow$ (b_y, b_z) $\Delta k_{TN}(b_y, b_z)$

Integrating over $b_z \in S$

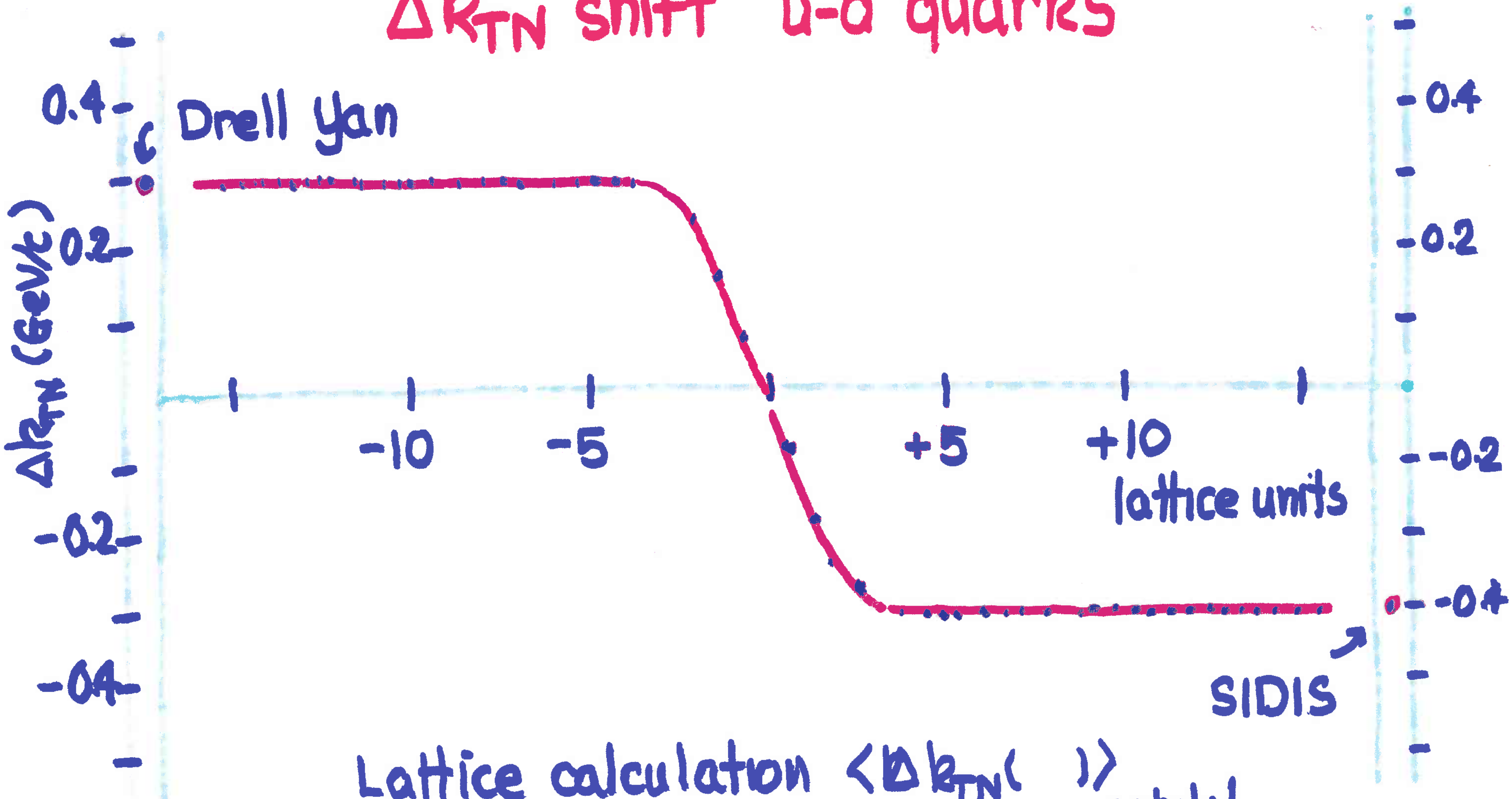
$$0 = \int db_z \{ \Delta k_{TN}(b_y, b_z) + \Delta k_{TN}(b_y, b_z) \}$$

$$\boxed{\Delta k_{TN}(b_y)\Big|_{DY} = -\Delta k_{TN}(b_y)\Big|_{SIDIS}}$$

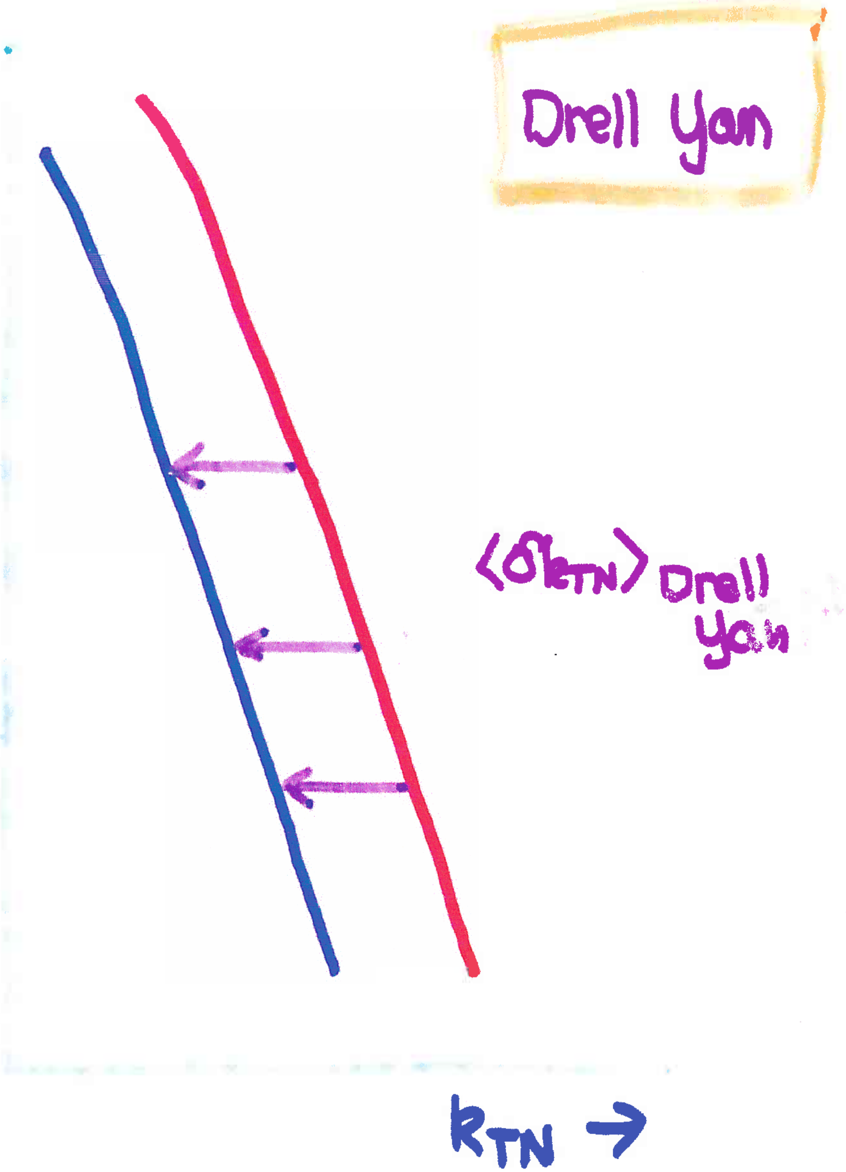
B. Musch, P. Hagler, M. Engelhardt, J.W. Nagle & A. Shäfer

Phys. Rev D85, 094510 (2012) arXiv: 1111.4249 [hep. lat]

Δk_{TN} shift u-d quarks

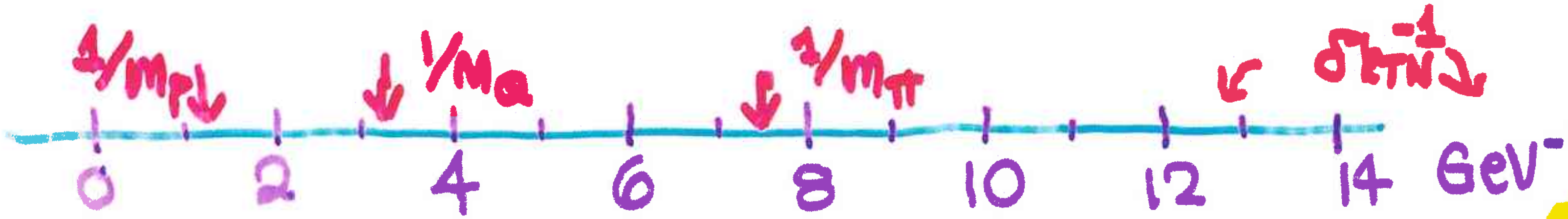


Lattice calculation $\langle \Delta k_{TN}(\ ) \rangle_{orbital}$



high statistics and good momentum resolution in both processes required for test

π- tornado in Georgi-Manohar reveals scales



$$\langle \delta k_{TN}(x,\mu_0^2)\rangle_{DY} = -\langle \delta k_{TN}(x,\mu_0^2)\rangle_{SIDIS}\left(1+\sum_n b_n\left(\frac{\langle \delta k_{TN}\rangle}{m_\pi^2}\right)^{n/2}+\sum_n c_n\left(\frac{m_\pi^2}{m_Q^2}\right)^{n}+\sum_n d_n\left(\frac{m_\pi^2}{m_\rho^2}\right)^{n}+\ldots\right)$$

model-dependent may be small but must all vanish if gauge formulation of QCD valid at these scales

Collins Conjugation tests the gauge formulation of QCD at scales where quarks and gluons not the dominant degrees of freedom. Test supplements lattice calculations, χEFT, ... by direct comparison of 2 independent Δσ's

Calibrates the application of TMD's to observe other quantum structures in proton