Resurrecting the 1955 Brookhaven All-Electric AGS Electron Analogue Ring

Workshop on Opportunities for Polarized Physics at Fermilab

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- While building the AGS at BNL, an all-electric "Electron Analogue" ring was built (to study passage through transition).
- Applying for funds in mid-1953, the approval, commissioning, construction and prototyping had been accomplished in less than two years.
- This is the closest prototype there has been to the all-electric ring needed to "trap" protons to measure their electric dipole moments (EDM).
- I have reverse engineered the lattice design and simulated its performance using John Talman's ETEAPOT code.
- Results are compared with measurements performed on the ring in 1955.

- My original project was to develop a test bed for simulations designed to handle electric elements.
- But the study has suggested a more substantial application.
- The AGS Analogue used *electrons* instead of protons, and was limited to achievable electric field. Cost minimization led to 10 MeV as maximum energy and bend radius of 4.7 m.
- These are the same considerations that will fix the parameters of an all-electric proton ring.

- This suggests rebuilding almost the same **electron** ring as a quick, small, inexpensive, and risk-free **prototype** for an eventual, larger, **proton** EDM ring of the similar design.
- As well as having ten times greater bend radius the proton EDM ring will need straight sections not in AGS Analogue.
- High quality electron injection would be easy. See Bazarov[1] report.
- The magic kinetic energy for freezing electrons, which is 15 Mev, is not very different from the 10 MeV of the Electron Analogue ring.
- An added **bonus** of the proposed small prototype electron ring is that it could be used to freeze electron spins and hence measure the electron EDM, as suggested by Morse[2] and others.

Introduction

- I have based the lattice design for the AGS Analogue on the original BNL application for funds from the A.E.C.
- The "Conceptual Design Report" for the AGS Analogue electron ring was a four page letter from BNL Director Haworth to A.E.C. (predecessor of D.O.E) Director of Research Johnson, applying for funding. The first three pages are reproduced next.
- Then a 1955 report by Ernest Courant contains the experimental data to be simulated.

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- Then a 1955 report by Ernest Courant contains the experimental data to be simulated.

- After comparing the new simulation results to the old Courant's observations, I proceed to give other ETEAPOT simulation results and to interpret them theoretically.

Historical BNL Documents

Dr. T.H. Johnson, Director
Division of Research
U.S. Atomic Energy Commission
Washington 25, D.C.

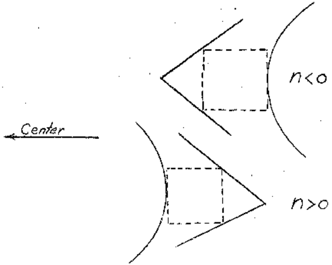
Dear Tom:

This letter concerns certain aspects of our accelerator development program, particularly the proposed electron model.

As you know, the general development of a very high energy alternating gradient synchrotron is proceeding actively at Brookhaven, utilizing operating funds allocated to Basic Physics Research. As I explained in my letter of August 12, however, these funds are insufficient to carry forward the development as rapidly as desirable. Also, there are certain steps which should be taken for which the expenditure of operating funds is not appropriate. The first and most important of these is the construction of an electron model intended to provide final assurance of the technical feasibility of the chosen machine and, more importantly, to provide information enabling us to design in the most effective and economical manner. (We have no doubt of the general feasibility of accelerators of this type.)

We have given considerable thought to the requirements for such a model and to the philosophy which should guide us in designing and building it. In the alternating gradient synchrotron, two problems require especially careful exploration by extensive calculation and experimental modelling. These are the close-spaced resonances in the betatron oscillations and the shift of phase stability at intermediate energies. It seems best to study these problems with an electron accelerator which would be essentially an analogue rather than an exact model. This device should, in our opinion, be designed to yield the maximum of orbital data with a minimum of engineering complications, especially those not applicable to a final machine. After considerable thought we have arrived at a tentative description and list of parameters which follow.

The device would consist of an accelerator having an orbital radius of 15 feet and an overall diameter, including the straight sections, of approximately 45 feet; the guide and focussing fields would be electrostatic, with electrode shapes as indicated in the sketch (full scale).



Electrons of about 1 MeV energy would be injected from a small horizontal Van de Graaff generator (of the 2 MeV type manufactured by the High Voltage Engineering Corporation) so that 5% to 6% frequency modulation would be required.

Use of a reasonably large radius helps the radio frequency and observing equipment in frequency range where good techniques exist, and permits high n-values which are necessary for strong alternating-gradient focussing. (This, and phase transition, will not be modeled in the Cornell machine.) A moderate rise rate, consistent with attainable vacuum requirements, still permits the use of small, air-cooled amplifier tubes and a heavily loaded low-Q rf cavity.

A tentative list of parameters is:

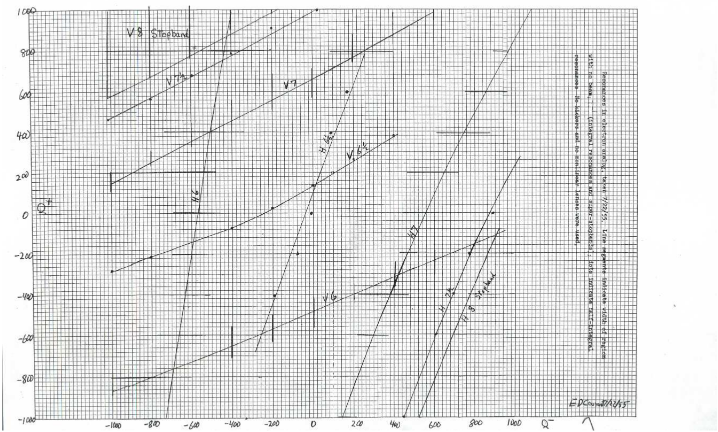
Radius of curvature	15 ft
Over-all diameter	43 ft
n	200
No. of periods	37
No. of straight sections	74
No. of lenses per period	4
Length of lens	7.6 in.
Length of straight section	7.6 in.

Field strength (magnetic type)	
at injection	10.5 gauss
at 10 MeV	74 gauss
Field strength (electrostatic type)	
at injection	3 kV/cm
at 10 MeV	22 kV/cm
Rise time	.01 sec
Phase transition energy	2.8 MeV
Frequency (final)	7 mc
Frequency change	54 %
Volts/turn	150 V
RF power	about 1 kw
No. of betatron wavelengths	about 6.2
aperture	1 X 1 in.
Betatron amplitude for 10^{-3} rad. error	0.07 in.
Maximum stable amplitude, synchrotron osc.	-0.16 in.
Radial spacing of betatron resonances	about 0.4 in.
Vacuum requirement	about 10^{-6} mm Hg

Total power requirements will be small and available with existing installations. The test shack seems to be a suitable location since the ring will be erected inside a thin magnetic shield which can be thermally insulated and heated economically.

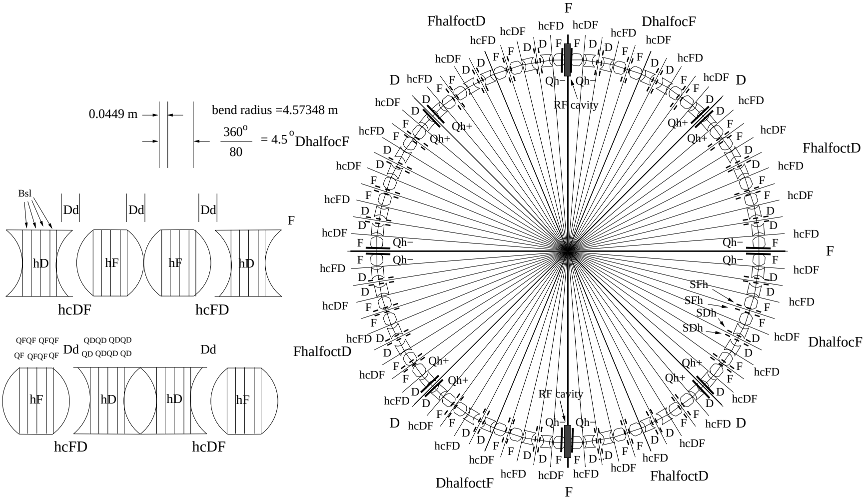
We estimate the cost to be approximately $600,000, distributed as shown in the following table:

Model	Direct	Overhead	Total
Staff S. & W.	$135,000	$ 65,000	$200,000
Van de Graaff	70,000	-	70,000
Other E. & S.	130,000	-	130,000
Shops	135,000	65,000	200,000
	$470,000	$130,000	$600,000

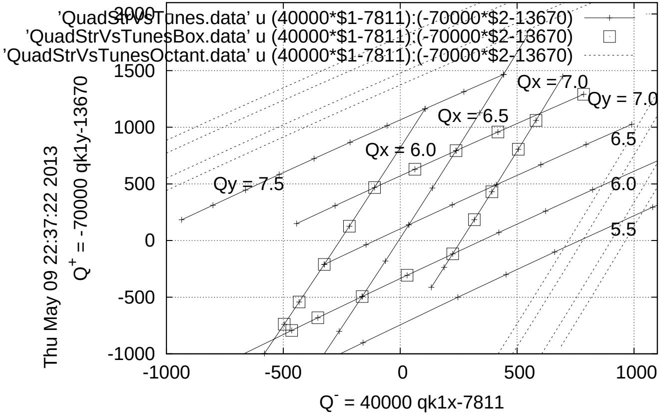


- Tune scans, Courant, BNL report EDG-20, July, 1955
- Heavy lines.....regions with no beam.......integer resonance
- Dots............narrow disruption..........half integer resonance
- Qx=8/Qy=8;......stop bands.................superperiodicity 8
- The nominal central tunes values are $(Q_x, Q_y) = (6.5, 6.5)$.
- Rememeber this figure!

Reconstructed AGS-Analogue Lattice



Simulation of 1955 Machine Studies Tune Plane Scan



- Boxes mark points on stable diamond about $(Q_x, Q_y) = (6.5, 6.5)$.
- Points lying on $1/2$ integer resonance lines are indicated by dots.
- Superperiodicity bands at $Q_x = 8$ or $Q_y = 8$
- This figure is to be compared with the Courant tune scan figure.

- Dead-reckoned tunes came out within 10% of nominal.
- For the eventual proton EDM ring the vertical tune has to be reduced to $Q_y \approx 0.2$. This will amplify the electric/magnetic differences.
- Comparison between all-magnetic and, otherwise identical, all-electric lattices are contained in the following figures, for tune values $(Q_x, Q_y) = (6.2, 2.25)$.
- The electric/magnetic difference is small, but big enough for $Q_y = 2.25$ to be the lowest I have obtained so far. Just switching from magnetic to electric without compensating typically causes a stable lattice to become unstable.

Effects of Change from Magnetic to Electric Elements

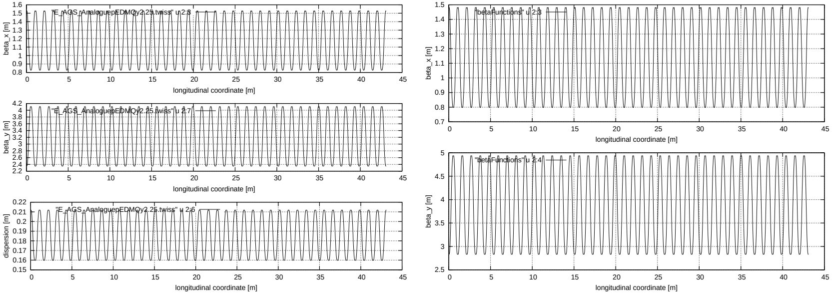


Figure: Lattice functions, magnetic left, electric right; $(Q_x, Q_y) = (6.2, 2.25)$.

- Focusing strengths are adjusted to match the central tunes.
- After this the optics are nearly identical.

Investigation of Synchro-Betatron Coupling

- Transfer matrices produced by ETEAPOT have unity determinants and therefore satisfy Liouville's theorem exactly.
- And they are derived from orbits that are exactly Hamiltonian and symplectic (in the analytic dynamics sense).
- Nevertheless, because the coordinates are not canonical, the transfer maps are not exactly symplectic in the linear algebraic sense; they violate conditions other than unity determinant.
- This complicates, but does not prevent, a quite compact analytic eigenmode and normal mode partioning needed for synchro-betatron analysis.
- This is explained in appendices.

A Numerical Example

- The purpose of this example is to illustrate the numerical extraction of normal mode motions from ETEAPOT-derived transfer matrices.
- The numerical values are chosen for weak coupling and have not been correlated with any particular operating conditions.
- The "tune aliasing" problem is ignored—only fractional parts of tunes are determined.
- Work with the transfer matrix for just half of the ring, with nominal tune advances therefore being, for example, $Q_x = 6.5/2$ and $Q_y = 6.5/2$.

- The (horizontal/longitudinal) transfer matrix produced by ETEAPOT for one half of the ring (not including RF) is

$$\mathbf{M_0} = \begin{pmatrix} .44836996 & 1.46977289 & 0 & .32405761 \\ -.54359716 & .44836996 & 0 & .31933866 \\ -0.45599045 & 0.000037764113 & 1. & -129.88420583 \\ 0. & 0. & 0 & 1. \end{pmatrix} \tag{1}$$

- The determinant is equal to 1 to 10 decimal places.
- The matrix for the RF cavity is

$$\mathbf{M}_{\rm rf} = \begin{pmatrix} 1.0 & 0.75 & 0 & 0 \\ 0.0 & 1.0 & 0 & 0 \\ 0.0 & 0.0 & 1.0 & 0 \\ 0.0 & 0.0 & krf & 1.0 \end{pmatrix} \tag{2}$$

where krf=0.00001 for the present example.

- The product is the transfer matrix for one half of the ring.

$$\mathbf{M} = \mathbf{M}_0\mathbf{M}_{\mathrm{rf}}$$
$$= \begin{pmatrix} .44836996 & 1.80605036 & 0.00000324057 & .32405761 \\ -.54359716 & 0.04067208 & 0.00000319338 & .31933866 \\ -0.00004559 & 0.00000356 & .99870115 & -129.88420583 \\ 0 & 0 & 0.00001000 & 1. \end{pmatrix} \quad (3)$$

- The **symplectic conjugate** matrix is

$$\overline{\mathbf{M}} = -\mathbf{S}\mathbf{M}^T\mathbf{S} \quad (4)$$

where

$$\mathbf{S} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \quad (5)$$

- Evaluate

$$\mathbf{M}\overline{\mathbf{M}} = \begin{pmatrix} 1.182799 & 0.150776 & 0.000003240 & 0.108706 \\ -0.221622 & 0.817200 & 0.000003193 & 0.225190 \\ 0.159443 & -0.161791 & 0.998701157 & -0.168693 \\ 0.0000015 & -0.000001 & 0.000010000 & 1.001298 \end{pmatrix} \tag{6}$$

- If $\mathbf{M}$ were algebraically symplectic, this product would be the identity matrix.
- It clearly is **not.**
- It is not because the evolution is non-Hamiltonian.
- It is because the coordinates being evolved by the transfer matrix are not canonically conjugate.

- It is explained in an appendix how to obtain the normal modes of transfer matrix $\mathbf{M}$ using $\mathbf{M}+\overline{\mathbf{M}}$.
- The characteristic polynomial equation of the matrix $\mathbf{M}+\overline{\mathbf{M}}$ is

$$\lambda^4-4.97548640\lambda^3+8.1437640\lambda^2-4.86328373\lambda+.95540635=0, \tag{7}$$

- The (double) roots are 1.3237706 and 0.0360414.
- From these one obtains the two eigentunes in the form $\cos\mu_1=0.99935$, $\cos\mu_2=0.24452$.

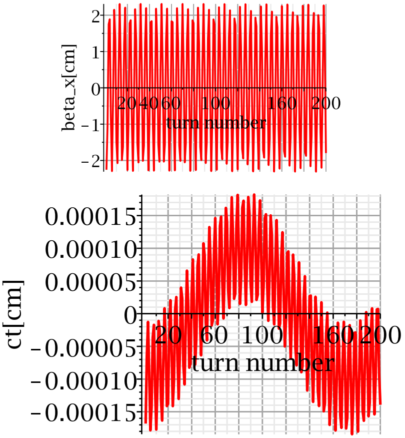


Figure: Synchro-betatron coupled evolution simulated by ETEAPOT transfer matrix evolution. Starting initially in the horizontal plane, the longitudinal position varies on a slow time scale with ± 0.0001 cm amplitude, and on a fast time scale with ± 0.00005 cm amplitude

- I. Bazarov, *Performance of a High Current, Low Emittance Electron Gun*, Cornell Report, 2013
- W. Morse, *All Electric Magic Momentum Electron EDM Precursor Experiment¡* Brookhaven National Lab Internal Report, January 24, 2012
- J. Talman and R. Talman, BNL internal reports, *UAL/ETEAPOT Results for Proton EDM Benchmark Lattices,* April, 2012, *UAL/ETEAPOT Proton EDM Benchmark Comparisons II: Transfer Matrices and Twiss Functions,* August, 2012, and *UAL/ETEAPOT Proton EDM Benchmark Comparisons III: Dispersion, Longitudinal Dynamics and Synchrotron Oscillations,* April, 2012
- M. Plotkin, *The Brookhave Electron Analogue, 1953-1957,* BNL–45058, December, 1991
- E. Courant, *Resonance in the Electron Analogue,* BNL internal report EDC-20, July 28, 1955

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