Reliability of Predictions for the SSA in Drell-Yan

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- Issues in theory of TMD factorization and the Sivers sign-reversal
- Accuracy of phenomenology and quantitative predictions

  Especially about dilution of Sivers asymmetry by evolution to higher Q

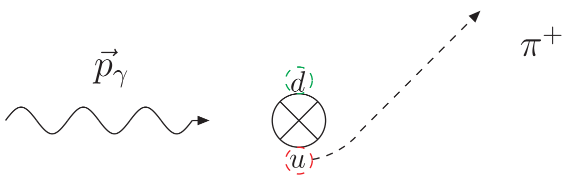
However, you should review history of SSA, e.g., in $pp \to \pi X$ before evaluating theoretical predictions.

Justification of TMD factorization ($q_\mathsf{T} \ll Q$), Sivers sign-reversal

- Full proof with details about Wilson lines: JCC's "Foundations of Perturbative QCD". Check!

- Verification in low-order graphs for SIDIS and DY
  [Brodsky, Hwang, Schmidt, PL B530, 99 (2002); Brodsky, Hwang, Schmidt, NP B642, 344 (2002); Collins, Qiu, PRD 75, 114014 (2007; Brodsky et al., arXiv:1304.5237.]

- Burkardt model gives sign reverse:

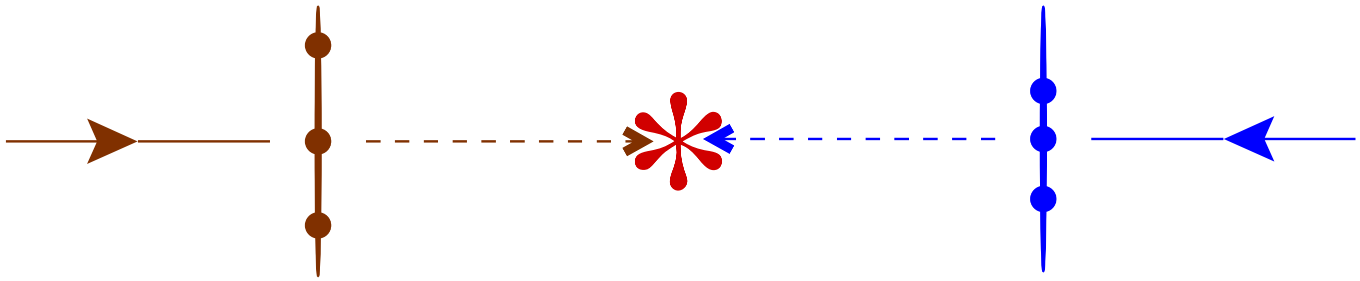


  Force on outgoing quark v. incoming antiquark in color field in spinning proton.
  E.g., Burkardt, arXiv:1009.5442

- Lattice QCD verifies Sivers sign-reversal in suitable correlation function.

→ N.B. Better understanding of size of effects of power corrections is needed

TMD factorization for Drell-Yan with $q_\mathsf{T} \ll Q$



$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q\, \mathrm{d}\Omega} = \frac{2}{s} \sum_j \frac{\mathrm{d}\hat{\sigma}_{j\bar{j}}(Q, \mu, g(\mu))}{\mathrm{d}\Omega} \int \mathrm{d}^2 \boldsymbol{b}_\mathsf{T} \;\; e^{i \boldsymbol{q}_{h\,\mathsf{T}} \cdot \boldsymbol{b}_\mathsf{T}} \;\; \tilde{f}_{j/A}(x_A, \boldsymbol{b}_\mathsf{T}; \zeta_A, \mu) \;\; \tilde{f}_{\bar{j}/B}(x_B, \boldsymbol{b}_\mathsf{T}; \zeta_B, \mu)$$

$+$ poln. terms $+$ high-q_T term $+$ power-suppressed

with: $\zeta \simeq (2 \times \text{parton energy})^2, \quad \zeta_A \zeta_B = Q^4.$

TMD factorization, and evolution

$$\frac{d\sigma}{d^4q\,d\Omega} = \frac{2}{s}\sum_j \frac{d\hat{\sigma}_{j\bar{\jmath}}(Q,\mu,g(\mu))}{d\Omega} \int d^2\boldsymbol{b}_\mathrm{T}\ e^{i\boldsymbol{q}_{h\,\mathrm{T}}\cdot\boldsymbol{b}_\mathrm{T}}\ \tilde{f}_{j/A}(x_A,\boldsymbol{b}_\mathrm{T};\zeta_A,\mu)\ \tilde{f}_{\bar{\jmath}/B}(x_B,\boldsymbol{b}_\mathrm{T};\zeta_B,\mu)$$

$$+ \text{ poln. terms} + \text{high-}q_\mathrm{T}\text{ term} + \text{power-suppressed}$$

CSS evolution:

$$\frac{\partial \ln \tilde{f}_{f/H}(x,b_\mathrm{T};\zeta;\mu)}{\partial \ln\sqrt{\zeta}} = \tilde{K}(b_\mathrm{T};\mu) \qquad (\text{with } \zeta = Q^2)$$

- Evolve to remove logarithms in perturbative regions "resummation".
- Parameterize intrinsically non-perturbative part:
  - Large b_T in TMD pdfs;
  - Large b_T in CSS kernel $\tilde{K}(b_\mathrm{T})$, or corresponding function in other formalisms.
- Non-perturbative region in b_T. Strikman & Weiss [JHEP 01 (2013) 163] argue for two non-perturbative scales:
  $1\,\mathrm{fm} = 5\,\mathrm{GeV}^{-1}$ (confinement); $0.3\,\mathrm{fm} = 1.5\,\mathrm{GeV}^{-1}$ (chiral condensate).
- Also use relation of TMD pdfs to integrated pdfs at small-b_T.

One solution: Factorization with fixed TMD pdfs

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q\,\mathrm{d}\Omega} = \frac{2}{s}\sum_j \frac{\mathrm{d}\hat{\sigma}_{j\bar{j}}(Q,\mu_Q,g(\mu_Q))}{\mathrm{d}\Omega}\int \mathrm{d}^2\boldsymbol{b}_\mathsf{T}\, e^{i\boldsymbol{q}_{h\,\mathsf{T}}\cdot\boldsymbol{b}_\mathsf{T}}\times$$

$$\times\, \tilde{f}_{f/A}(x_A,\boldsymbol{b}_\mathsf{T};m^2,\mu_0)\; \tilde{f}_{\bar{f}/B}(x_B,\boldsymbol{b}_\mathsf{T};m^2,\mu_0)$$

$$\times \left(\frac{Q^2}{m^2}\right)^{\tilde{K}(b_\mathsf{T};\mu_0)} \times \exp\left\{\int_{\mu_0}^{\mu_Q}\frac{\mathrm{d}\mu'}{\mu'}\left[2\gamma(g(\mu');1) - \ln\frac{Q^2}{(\mu')^2}\gamma_K(g(\mu'))\right]\right\}$$

$+$ polarized terms $+$ large $q_{h\,\mathsf{T}}$ correction, Y $+$ p.s.c.

where:
- $\mu_Q \propto Q$
- blue $\Leftrightarrow$ non-perturbative and/or non-"resummed" logarithms

N.B. Perturbative analysis/prediction of TMD pdfs at small b_T also used

What affects shape in q_T at $q_T \ll Q$?

We have q_T-independent factor times

$$\int d^2\boldsymbol{b}_T \, e^{i\boldsymbol{q}_{h\,T}\cdot\boldsymbol{b}_T} \tilde{f}_{j/A}(x_A, \boldsymbol{b}_T; m^2, \mu_0) \, \tilde{f}_{\bar{\jmath}/B}(x_B, \boldsymbol{b}_T; m^2, \mu_0) \left(\frac{Q^2}{m^2}\right)^{\tilde{K}(b_T;\mu_0)}$$

$$= \int d^2\boldsymbol{b}_T \, e^{i\boldsymbol{q}_{h\,T}\cdot\boldsymbol{b}_T} \tilde{f}_{j/A}(x_A, \boldsymbol{b}_T) \, \tilde{f}_{\bar{\jmath}/B}(x_B, \boldsymbol{b}_T) \, e^{\tilde{K}(b_T;\mu_0)\ln(Q^2/m^2)}$$

RG to deal with logarithms, segregation of non-perturbative information:

$$\tilde{K}(b_T; \mu_0) = \tilde{K}(b_T; \mu_b) + \int_{\mu_0}^{\mu_b} \frac{d\mu'}{\mu'} \gamma_K(g(\mu'))$$

$$= \tilde{K}(b_*; \mu_b) + \int_{\mu_0}^{\mu_b} \frac{d\mu'}{\mu'} \gamma_K(g(\mu')) + \tilde{K}(b_T; \mu_b) - \tilde{K}(b_*; \mu_b)$$

$$= \tilde{K}(b_*; \mu_b) + \int_{\mu_0}^{\mu_b} \frac{d\mu'}{\mu'} \gamma_K(g(\mu')) - g_K(b_T; b_{\max})$$

where (CSS):
- $\mathbf{b}_* = \boldsymbol{b}_T / \sqrt{1 + b_T^2/b_{\max}^2}, \quad \mu_b = C_1/b_*,$
- red $\Leftrightarrow$ (treated as) non-perturbative

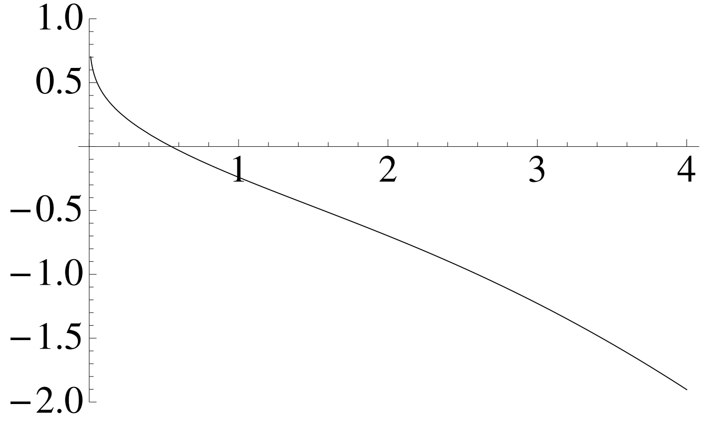
Evolution shifts TMD to smaller b_T as Q increases

We have $\boldsymbol{q}_\mathsf{T}$-independent factor times

$$\int \mathrm{d}^2\boldsymbol{b}_\mathsf{T}\, e^{i\boldsymbol{q}_{h\,\mathsf{T}}\cdot\boldsymbol{b}_\mathsf{T}} \tilde{f}_{j/A}(x_A, \boldsymbol{b}_\mathsf{T}; m^2, \mu_0)\, \tilde{f}_{\bar{\jmath}/B}(x_B, \boldsymbol{b}_\mathsf{T}; m^2, \mu_0) \left(\frac{Q^2}{m^2}\right)^{\tilde{K}(b_\mathsf{T};\mu_0)}$$

$$= \int \mathrm{d}^2\boldsymbol{b}_\mathsf{T}\, e^{i\boldsymbol{q}_{h\,\mathsf{T}}\cdot\boldsymbol{b}_\mathsf{T}} \tilde{f}_{j/A}(x_A, \boldsymbol{b}_\mathsf{T})\, \tilde{f}_{\bar{\jmath}/B}(x_B, \boldsymbol{b}_\mathsf{T})\; e^{\tilde{K}(b_\mathsf{T};\mu_0)\ln(Q^2/m^2)}$$

$\tilde{K}(b_\mathsf{T})$ is a decreasing function of b_T

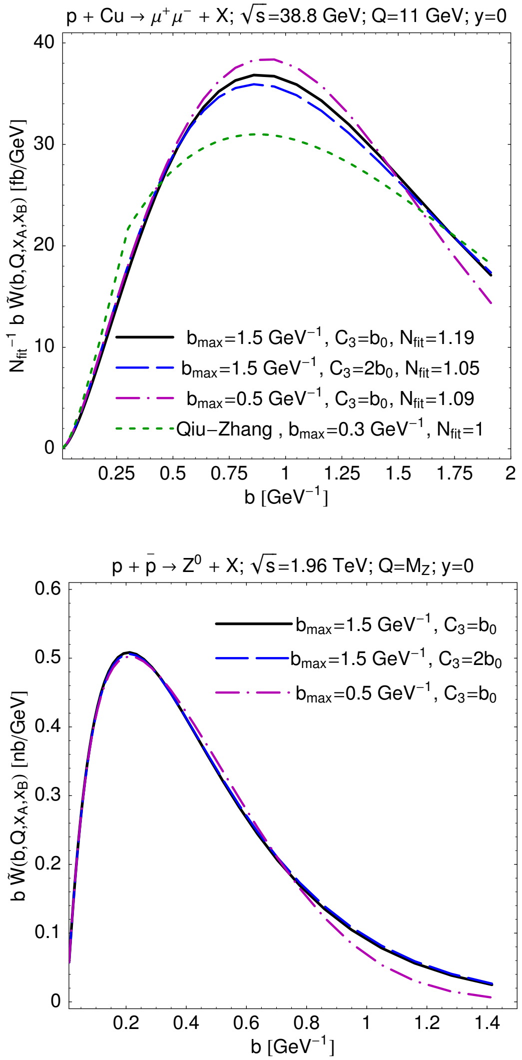


So it shifts TMD to smaller b_T and broadens the q_T distribution . . .

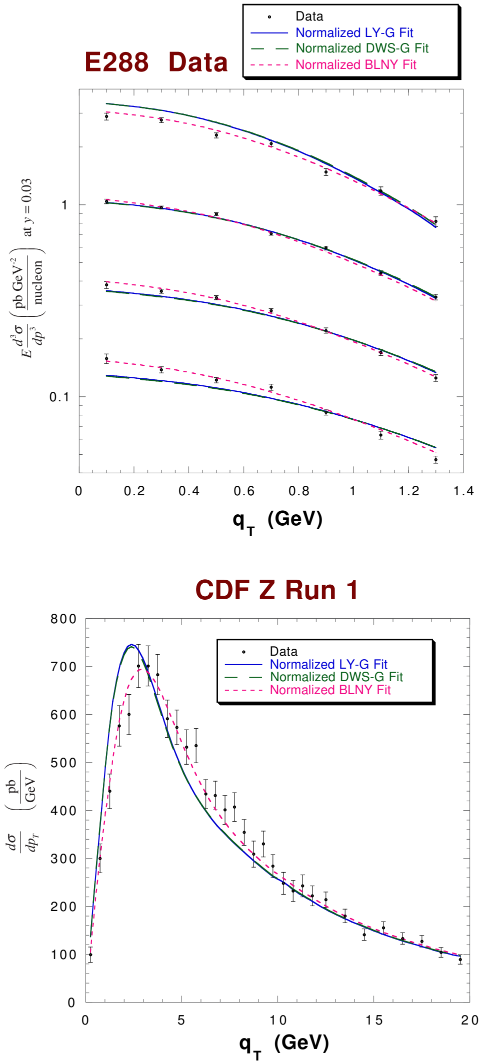
Evolution in q_T v. b_T

$Q = 11\,\mathrm{GeV}$
$\sqrt{s} = 38.8\,\mathrm{GeV}$

$Q = m_Z$
$\sqrt{s} = 1960\,\mathrm{GeV}$



Konychev & Nadolsky, PLB 633, 710 (2006)



Landry et al., PRD 67,073016 (2003)

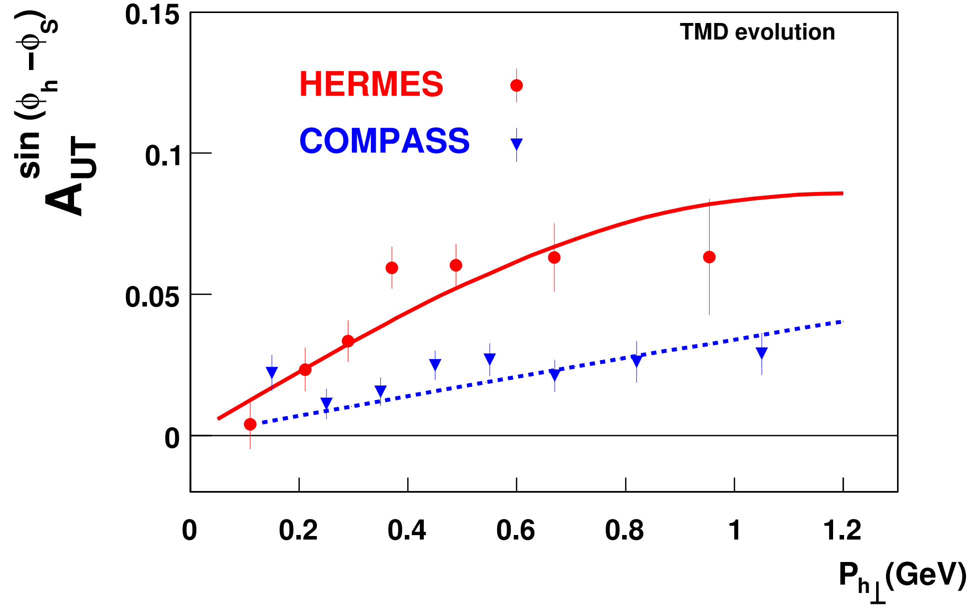
N.B. • x-dependence of k_T shape in TMD pdfs
• Sensitivity to non-perturbative region goes away at large Q

Data used in fits, etc

Data	Q	x	ff	pdf	Sivers	$\tilde{K}$	
HERMES (SIDIS)	$\sqrt{2.4}\,\mathrm{GeV}$	0.04–0.3	✓	✓	✓		Torino
COMPASS (SIDIS)	$\sqrt{3.8}\,\mathrm{GeV}$	0.01–0.3					Predict
E288 (DY)							
$\sqrt{s} = 27.4\,\mathrm{GeV}$	$5–9\,\mathrm{GeV}$	0.18–0.33		✓		✓	KN, BLNY
E605 (DY)							
$\sqrt{s} = 38.8\,\mathrm{GeV}$	$7–18\,\mathrm{GeV}$	0.18–0.46		✓		✓	KN, BLNY
R209 (DY)							
$\sqrt{s} = 62\,\mathrm{GeV}$	$5–11\,\mathrm{GeV}$	0.08–0.18		✓		✓	KN, BLNY
D0, CDF (DY)							
$\sqrt{s} = 1.8\,\mathrm{TeV}$	m_Z	0.05		✓		✓	KN, BLNY

Prediction for COMPASS v. HERMES

[Aybat, Prokudin, Rogers, PRL 108, 242003 (2012)]



On basis of

- Unpolarized DY for: $g_K(b_\mathrm{T})$ in BLNY fit (with $b_\mathrm{max} = 0.5\,\mathrm{GeV}^{-1} = 0.1\,\mathrm{fm}$)
- HERMES (Torino fit) for: fragmentation fns. unpolarized TMD pdf, Sivers
- But *not* non-perturbative unpolarized TMD pdfs of BLNY

N.B. Shape looks inaccurate

Problem with BLNY fit

[Landry et al., PRD 67,073016 (2003)]

- Non-perturbative factor (large b_T):

$$\exp\left\{-b_\mathsf{T}^2\,[\,0.21 + 0.68\ln(Q/3.2\,\mathrm{GeV}) - 0.126\ln(100x_Ax_B)\,]\right\}$$

- Coefficient of b_T^2 becomes negative when Q is small and x_Ax_B large.

  E.g., $Q = 3.2\,\mathrm{GeV}$ and $x_A = x_B = 0.3$,
  or $Q = \sqrt{2.4}\,\mathrm{GeV}$ (HERMES) and $x_A = x_B = 0.1$,

- So fit is not applicable beyond range of fairly small b_T relevant for the fitted data.

- But $b_{\max} = 0.5\,\mathrm{GeV}^{-1} = 0.1\,\mathrm{fm}$

- Konychev & Nadolsky, PLB 633, 710 (2006) use $b_{\max} = 1.5\,\mathrm{GeV}^{-1} = 0.3\,\mathrm{fm}$. They get

$$g_K(b_\mathsf{T}) = \frac{0.16}{2}b^2 = 0.08b^2, \qquad \text{instead of} \qquad g_K(b_\mathsf{T}) = \frac{0.68}{2}b^2 = 0.34b^2$$

  with better fit

- BLNY result $\Longrightarrow$ overestimate of evolution of Sivers from HERMES to COMPASS and Polarized DY @ Fermilab

Sun & Yuan [arXiv:1304.5037v1]

- They use evolution factor

$$\exp\left\{-2C_F\int_{Q_0}^{Q}\frac{\mathrm{d}\mu}{\mu}\frac{\alpha_s(\mu)}{\pi}\left[\ln\left(\frac{Q^2}{\mu^2}\right)+\ln\left(\frac{Q_0^2 b_{\mathsf{T}}^2}{c_0^2}\right)-\frac{3}{2}\right]\right\}$$

[with $c_0 = 2e^{-\gamma_E}$]

- Logarithms of b_{T} not resummed, no non-perturbative function
- Hence effective evolution kernel is

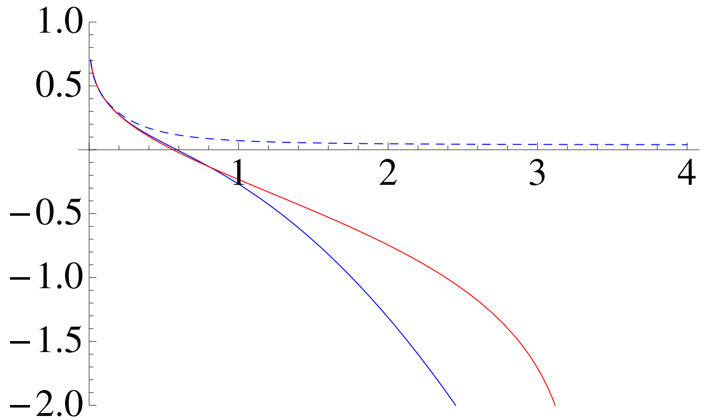
$$\frac{\mathrm{d}\ln\tilde{\sigma}}{\mathrm{d}\ln Q^2} = -C_F\frac{\alpha_s(Q)}{\pi}\ln\left(\frac{Q_0^2 b_{\mathsf{T}}^2}{c_0^2}\right) + \text{terms independent of } b_{\mathsf{T}}$$

which is Q-dependent

Plots of $\tilde{K}(b_{\mathbf{T}}, \mu_0 = 2\,\mathrm{GeV})$ I

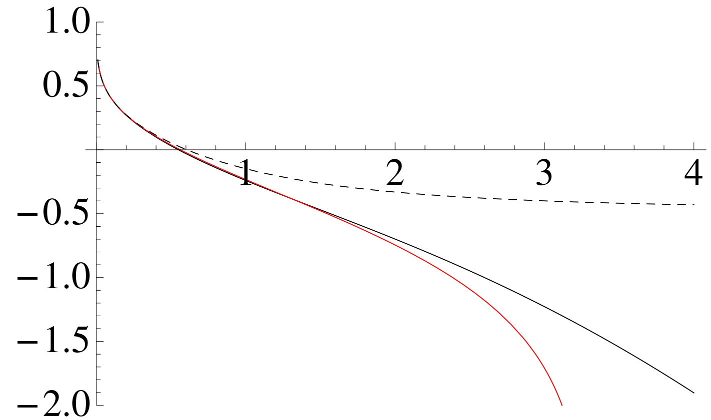
BLNY

$(b_{\mathrm{max}} = 0.5\,\mathrm{GeV}^{-1} = 0.1\,\mathrm{fm})$



KN

$(b_{\mathrm{max}} = 1.5\,\mathrm{GeV}^{-1} = 0.3\,\mathrm{fm})$

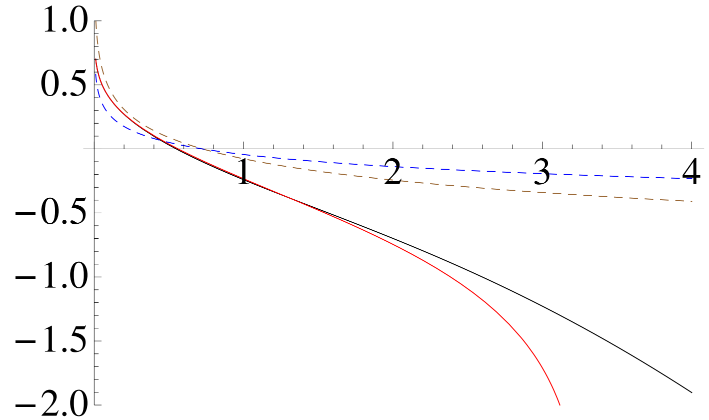


Code:		
	Red	With RG improvement
	Blue	BLNY (Landry et al., PRD 67,073016 (2003))
	Blue dotted	Perturbative part of BLNY
	Black	KN (Konychev & Nadolsky, PLB 633, 710 (2006))
	Black dotted	Perturbative part of KN

N.B. $4\,\mathrm{GeV}^{-1} = 0.8\,\mathrm{fm}$

Plots of $\tilde{K}(b_{\mathbf{T}})$ II

KN v. Sun-Yuan



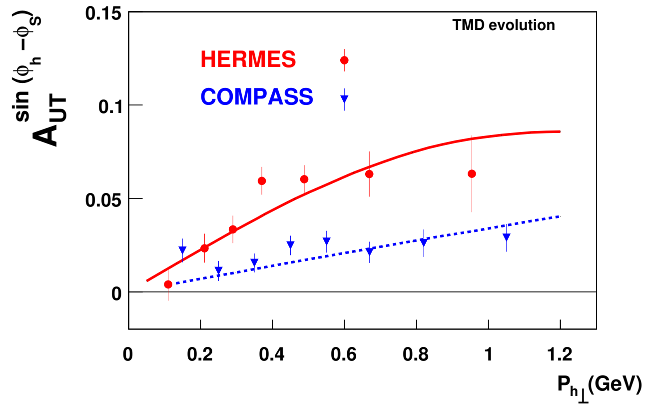
Code:	Red	With RG improvement
	Black	KN
	Brown dashed	Sun-Yuan at $2\,\mathrm{GeV}$
	Blue dashed	Sun-Yuan at $10\,\mathrm{GeV}$

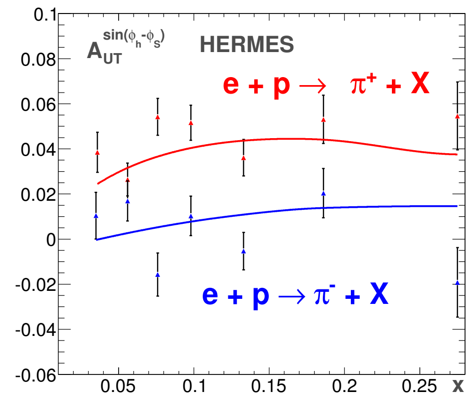
N.B. $4\,\mathrm{GeV}^{-1} = 0.8\,\mathrm{fm}$

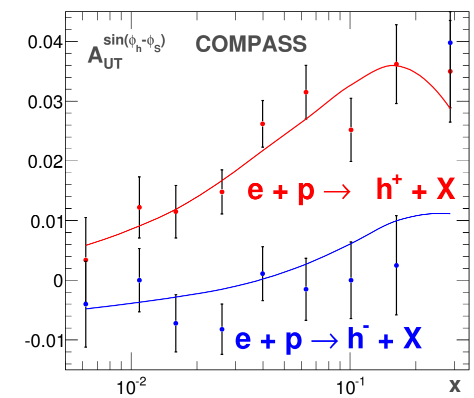
Are the results consistent?

Aybat et al. | Sun & Yuan

Sivers

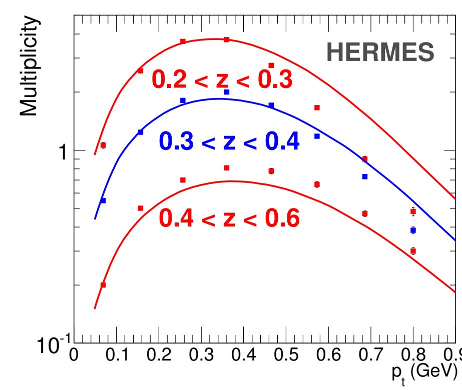


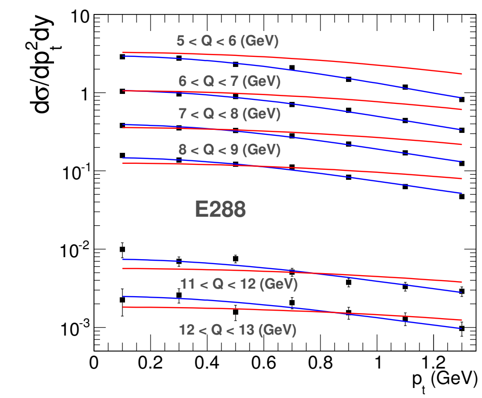




unpol.

Evolution from higher energy fit





- Slower evolution $Q = \sqrt{2.4}\,\mathrm{GeV}$ to $Q = 9\,\mathrm{GeV}$; faster evolution above $Q = 9\,\mathrm{GeV}$
- Roughly: Relevant b_T: 3–$4\,\mathrm{GeV}^{-1}$ for HERMES 1–$2\,\mathrm{GeV}^{-1}$ at $Q = 11\,\mathrm{GeV}$
- Does $\tilde{K}(b_\mathrm{T})$ flatten at $b_\mathrm{T} \gtrsim 2\,\mathrm{GeV}^{-1}$?

Conclusions

- A_N(DY) reduced substantially compared with HERMES
- Evolution, perturbative and non-perturbative, is controlled by $\tilde{K}(b_\mathsf{T}, \mu_0)$
- But it's used at larger b_T than where "non-perturbative" content was fit
- Probably Aybat-Prokudin-Rogers too pessimistic
- But Sun-Yuan probably too optimistic, and don't allow for known physics issues
- We need global fit/calculation of $\tilde{K}(b_\mathsf{T}, \mu_0)$
- New fit mustn't violate agreement with any existing data and principles
- Intrinsic TMD: Probably exponential rather than Gaussian better at large b_T
- Perhaps even constant for $\tilde{K}$ at large b_T
- We also need to evaluate effect of power corrections in factorization

EXTRA SLIDES

Power corrections ("higher twist")

- Factorization derived up to errors suppressed by a power of Q
- SSA (Sivers-type):
  - Graph-by-graph leading power if $q_{\mathrm{T}} \ll Q$
  - Suppressed by M/q_{T} at large q_{T}. ("Twist-3")
  - But gluon radiation (evolution) dilutes small q_{T} SSA as Q increases
  - Need answer to: Does that uncover power corrections, or are these also diluted?
- Need answer to: How accurate is factorization when Q is not large?

What form for large b_T?

- Standard:
  - $e^{-\text{const}\times b_T^2}$ in TMD pdf
  - $e^{-\tilde{K}\ln Q^2} \sim e^{-\text{const}\times b_T^2 \ln Q^2}$ in evolution

  Coefficients significantly non-zero according to fits: Landry et al. PRD 67,073016 (2003), and Konychev & Nadolsky, PLB 633, 710 (2006)

- But: Euclidean correlation functions in QFT are usually $e^{-m b_T}$

- KN & BLNY fits are at relatively large Q ($10\,\mathrm{GeV}$ up), and hence determine non-perturbative functions up to $b_T \lesssim 2\,\mathrm{GeV}^{-1} = 0.4\,\mathrm{fm}$

- But to get evolution from HERMES ($Q \sim \sqrt{2.4}\,\mathrm{GeV}$), we need the non-perturbative functions at larger b_T: Extrapolation v. theoretical motivation, . . .

- Need to retry fits with better forms at large b_T.

- *Un*polarized HERMES v. higher energy DY should be enough.