TMDs and Spin Asymmetries in Drell-Yan

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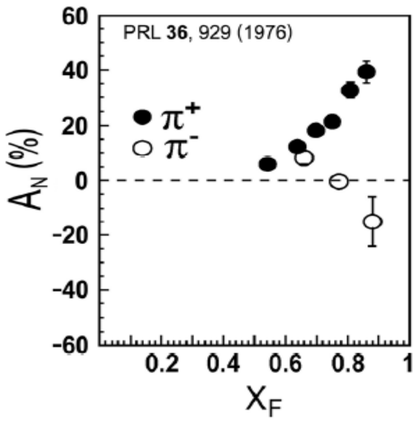
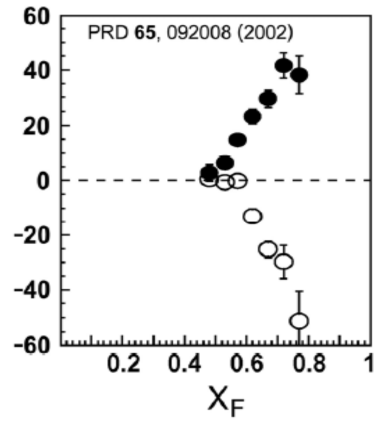
Transverse single-spin asymmetry (SSA)

❑ Consistently observed for almost 40 years!

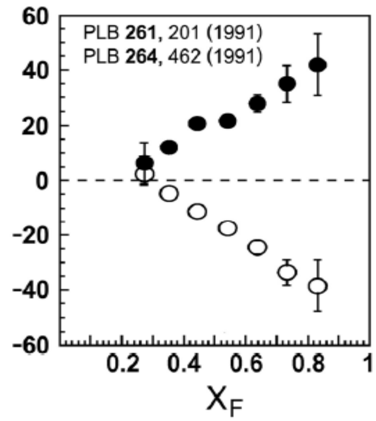
ANL – 4.9 GeV



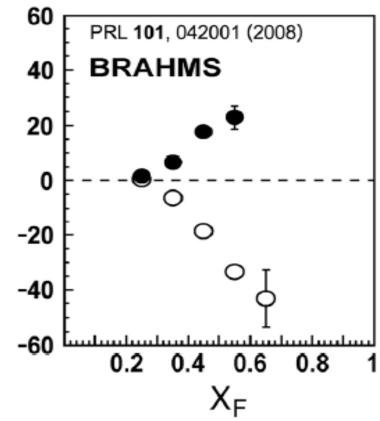
BNL – 6.6 GeV



FNAL – 20 GeV

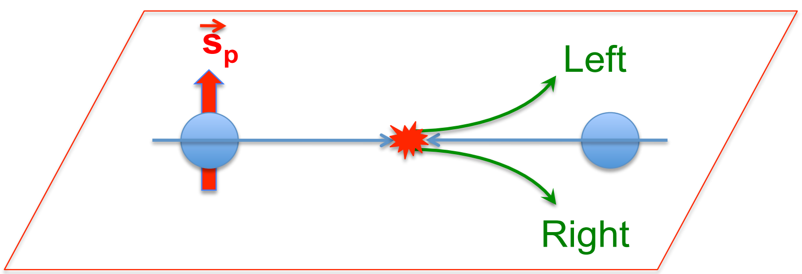


BNL – 62.4 GeV



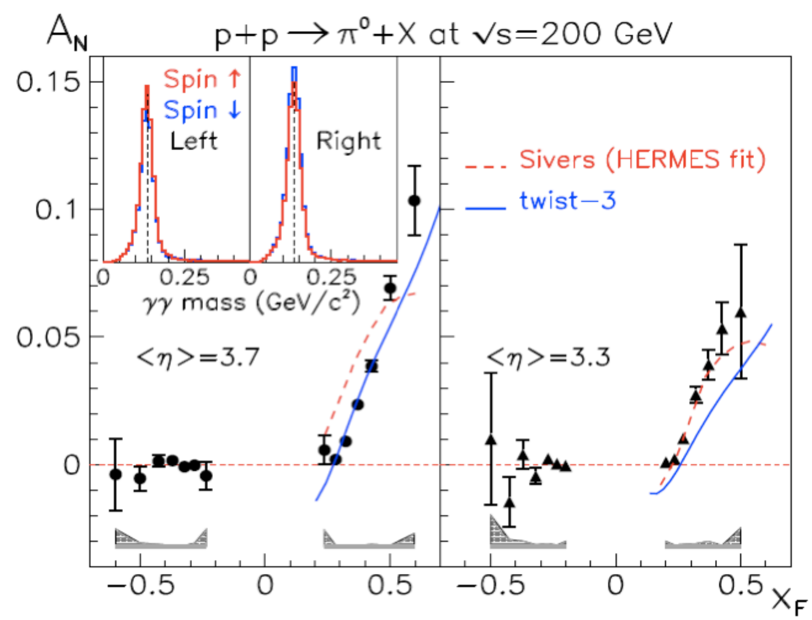
❑ Definition:

$$p(\vec{s}_\perp) + p \to h(\pi^\pm, \pi^0, ...) + X$$



$$A_N \equiv \frac{\Delta\sigma(\ell, \vec{s})}{\sigma(\ell)} = \frac{\sigma(\ell, \vec{s}) - \sigma(\ell, -\vec{s})}{\sigma(\ell, \vec{s}) + \sigma(\ell, -\vec{s})}$$

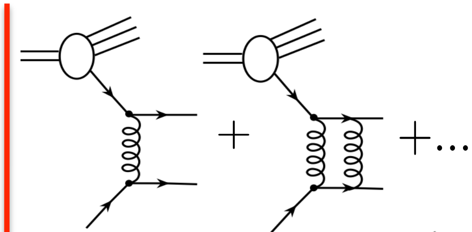
BNL – 200 GeV

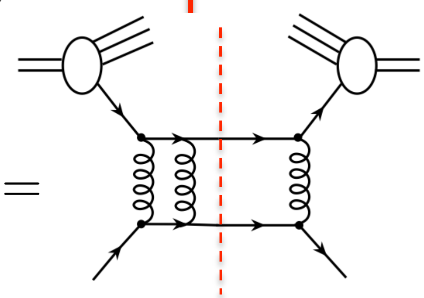


Do we understand it?

Kane, Pumplin, Repko, PRL, 1978

❑ Early attempt:

Cross section: $\sigma_{AB}(p_T, \vec{s}) \propto \Big|$  $\Big|^2$

Asymmetry: $\sigma_{AB}(p_T, \vec{s}) - \sigma_{AB}(p_T, -\vec{s}) =$  $\propto \alpha_s \dfrac{m_q}{p_T}$

Too small to explain available data!

❑ What do we need?

$$A_N \propto i\vec{s}_p \cdot (\vec{p}_h \times \vec{p}_T) \Rightarrow i\epsilon^{\mu\nu\alpha\beta} p_{h\mu} s_\nu p_\alpha p'_{h\beta}$$

Need a phase, a spin flip, enough vectors

❑ Vanish without parton's transverse motion:

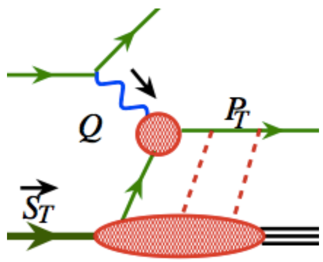
➡ **A direct probe for parton's transverse motion,**

Spin-orbital correlation, QCD quantum interference

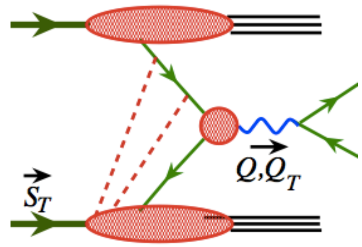
Current understanding of SSAs

Talks by Collins, Sivers, Yuan, …

- Two scales observables – $Q_1 >> Q_2 \sim \Lambda_{QCD}$:



SIDIS: $Q>>P_T$

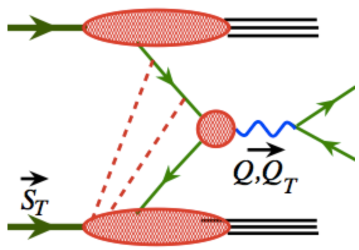


DY: $Q>>Q_T$

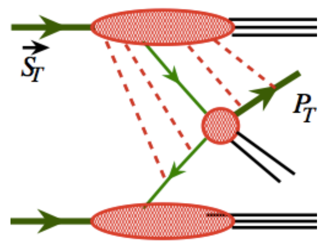
TMD factorization
TMD distributions

Direct information on parton k_T

- One scale observables – $Q >> \Lambda_{QCD}$:



DY: $Q \sim Q_T$

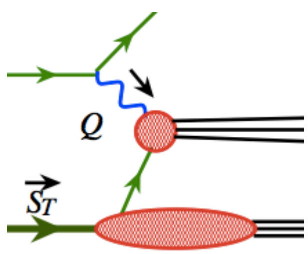


Jet, Particle: P_T

Collinear factorization
Twist-3 distributions

Information on moments of parton k_T

- Symmetry plays important role:



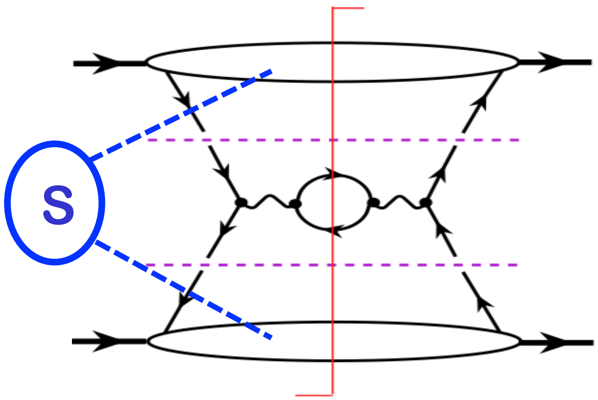
Inclusive DIS
Single scale
Q

Parity
Time-reversal

$A_N = 0$

Drell-Yan mechanism in QCD

❑ Leading order in QCD:



$\Leftarrow$ all γ structure: γ^α, $\gamma^\alpha\gamma^5$, $\sigma^{\alpha\beta}$(or $\gamma^5\sigma^{\alpha\beta}$), I, γ^5

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❑ Parity and Time-reversal:

$$\langle P, s_T|\hat{\mathcal{O}}(\psi, A_\mu)|P, s_T\rangle = \langle P, -s_T|\mathcal{PT}\hat{\mathcal{O}}(\psi, A_\mu)^\dagger\mathcal{T}^{-1}\mathcal{T}^{-1}|P, -s_T\rangle$$

❑ transversity distribution:

$$\langle P, S_\perp|\overline{\psi}(0)\,\frac{\gamma\cdot n\,\gamma_\perp^\sigma}{P\cdot n}\,\psi(yn)|P, S_\perp\rangle \quad \overset{\textbf{PT}}{\Longrightarrow} \quad -\,\langle P, -S_\perp|\overline{\psi}(0)\,\frac{\gamma\cdot n\,\gamma_\perp^\sigma}{P\cdot n}\,\psi(yn)|P, -S_\perp\rangle$$

❑ Asymmetries – collinear factorization:

$$A_{LL} \propto \sum_q e_q^2\,\Delta q(x)\Delta\bar{q}(x') \qquad A_{TT} \propto \sum_q e_q^2\,h_{1q}(x)h_{1\bar{q}}(x') \qquad A_L \propto \sum_q (c_v * c_a)\,\Delta q(x)\,\bar{q}(x')$$

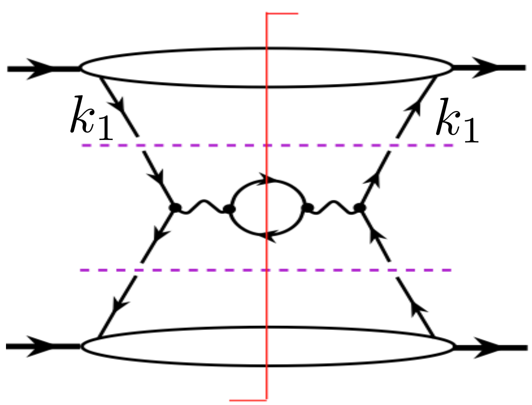
$$A_N \propto \sum_q e_q^2\,T_q(x,x)\,\bar{q}(x') \qquad A_{LT} \propto \sum_q e_q^2\,\Delta q(x)\,\tilde{T}_{\bar{q}}(x')$$

QCD factorization

- **Important facts:**
  - ✧ PDFs, TMDs, GPDs, FFs, and any matrix elements of quark-gluon fields are NOT direct physical observables, like cross-sections!
  - ✧ Any cross section with identified hadrons are NOT perturbative!

- **QCD factorization (is an approximation!):**
  - ✧ Connect cross sections of hadrons/leptons to matrix elements of quark and gluon fields, with calculable coefficients!
  - ✧ Universality of the non-perturbative matrix elements (predictive power)

- **Collinear vs TMD factorization:**



✧ Necessary: "long-lived" active partons

life-time >> time of hard collision

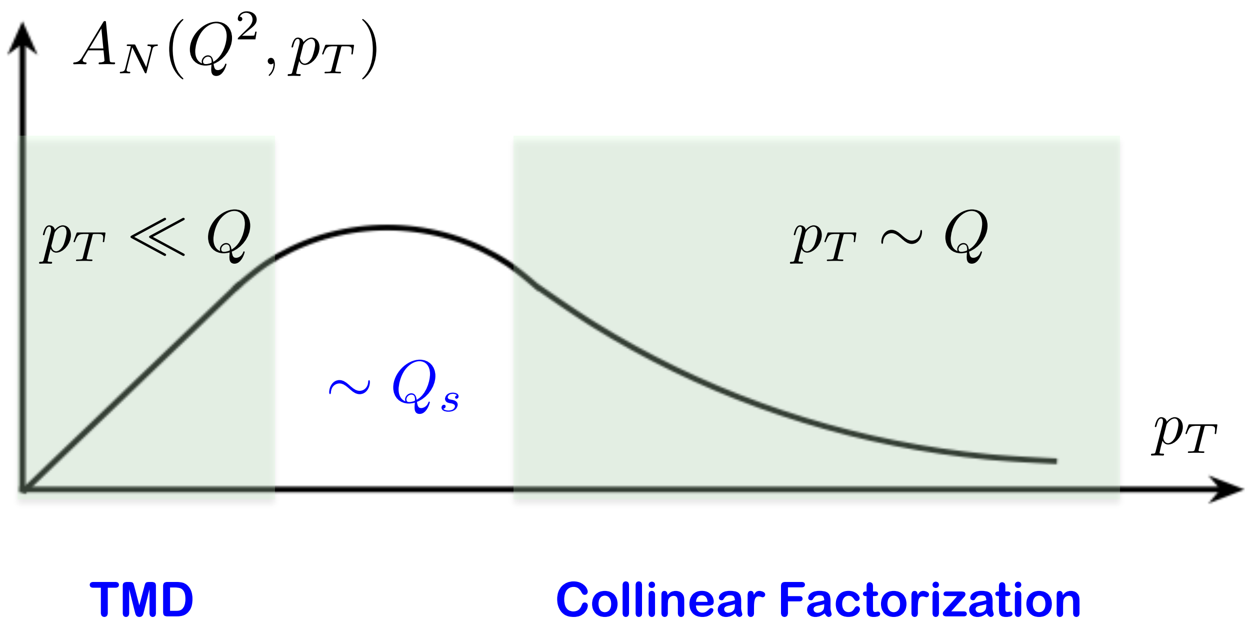
$$Q^2 \gg k_1^2 \approx 0$$

✧ Collinear: All physical scales >> Λ_{QCD}

✧ TMD: $Q_i >> Q_j \sim \Lambda_{QCD}$

Drell-Yan from low p_T to high p_T

- Covers both double-scale and single-scale cases:



- TMD factorization to collinear factorization:

Ji,Qiu,Vogelsang,Yuan,
Koike, Vogelsang, Yuan

Two factorizations are consistent in the overlap region: $\Lambda_{\rm QCD} \ll p_T \ll Q$

A_N finite – requires correlation of multiple collinear partons

No probability interpretation! New opportunities!

Factorized Drell-Yan cross sections

❑ **TMD factorization ($q_\perp \ll Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$

$$+\mathcal{O}(q_\perp/Q) \qquad x_A = \frac{Q}{\sqrt{s}}\, e^{y} \qquad x_B = \frac{Q}{\sqrt{s}}\, e^{-y}$$

The soft factor, $\mathcal{S}$, is universal, could be absorbed into the definition of TMD parton distribution

❑ **Collinear factorization ($q_\perp \sim Q$):**

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a, \mu) \int dx_b\, f_{b/B}(x_b, \mu)\, \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu)$$

$$+\mathcal{O}(1/Q, 1/q_\perp)$$

❑ **Spin dependence:**

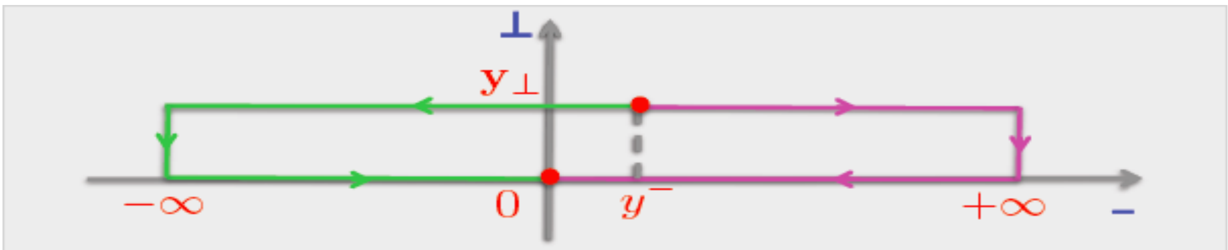
The factorization arguments are independent of the spin states of the colliding hadrons

➡ same formula with different distributions for γ^*, W/Z, H^0…

TMD parton distributions

❑ **TMD distributions with non-local gauge links:**

$$f_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) = \int \frac{dy^- d^2y_\perp}{(2\pi)^3} e^{ixp^+y^- - i\,\mathbf{k}_\perp\cdot\mathbf{y}_\perp} \langle p,\vec{S}|\overline{\psi}(0^-,\mathbf{0}_\perp)\,\boxed{\text{Gauge link}}\,\frac{\gamma^+}{2}\psi(y^-,\mathbf{y}_\perp)|p,\vec{S}\rangle$$



- For a fixed spin state:

$$f^{\text{SIDIS}}_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S}) \neq f^{\text{DY}}_{q/h^\uparrow}(x,\mathbf{k}_\perp,\vec{S})$$

❑ **Parity + Time-reversal invariance:**

$$\mathcal{F}^{\text{SIDIS}}_{q/h}(x,k_T,s_T) = \mathcal{F}^{DY}_{q/h}(x,k_T,-s_T)$$

$$\Longrightarrow \quad f^{\text{Sivers}}_{q/h^\uparrow}(x,k_\perp)^{\text{SIDIS}} = -f^{\text{Sivers}}_{q/h^\uparrow}(x,k_\perp)^{\text{DY}}$$

The sign change is a critical test of TMD factorization approach

Another critical test of TMD factorization

❑ Predictive power of QCD factorization:

- ✧ Infrared safety of short-distance hard parts
- ✧ Universality of the long-distance matrix elements
- ✧ QCD evolution or scale dependence of the matrix elements

❑ QCD evolution:

If there is a factorization/invariance, there is an evolution equation

❑ Collinear factorization – DGLAP evolution:

$$\sigma_{\rm phy}(Q,\Lambda_{\rm QCD}) \approx \sum_f \hat{\sigma}_f(Q,\mu)\otimes\phi_f(\mu,\Lambda_{\rm QCD}) \quad \rightarrow \quad \frac{d}{d\mu}\sigma_{\rm phy}(Q,\Lambda_{\rm QCD}) = 0$$

Scaling violation of nonperturbative functions

Evolution kernels are perturbative – a test of QCD

Evolution equations for TMDs

Boer, 2001, 2009, JI, Ma, Yuan, 2004
Idilbi, et al, 2004, Kang, Xiao, Yuan, 2011
Aybat, Collins, Qiu, Rogers, 2011
Aybat, Prokudin, Rogers, 2012
Idilbi, et al, 2012, Sun, Yuan 2013, …

- Collins-Soper equation:
  - b-space quark TMD with γ^+

$$\frac{\partial \tilde{F}_{f/P^\uparrow}(x, \mathrm{b}_\mathrm{T}, S; \mu; \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T;\mu)\tilde{F}_{f/P^\uparrow}(x, \mathrm{b}_\mathrm{T}, S; \mu; \zeta_F) \qquad \tilde{K}(b_T;\mu) = \frac{1}{2}\frac{\partial}{\partial y_s}\ln\left(\frac{\tilde{S}(b_T; y_s, -\infty)}{\tilde{S}(b_T; +\infty, y_s)}\right)$$

- RG equations:

$$\frac{d\tilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_K(g(\mu)) \qquad \frac{d\tilde{F}_{f/P^\uparrow}(x, \mathrm{b}_\mathrm{T}, S; \mu; \zeta_F)}{d\ln\mu} = \gamma_F(g(\mu); \zeta_F/\mu^2)\tilde{F}_{f/P^\uparrow}(x, \mathrm{b}_\mathrm{T}, S; \mu; \zeta_F).$$

- Evolution equations for Sivers function:

$$F_{f/P^\uparrow}(x, k_T, S; \mu, \zeta_F) = F_{f/P}(x, k_T; \mu, \zeta_F) - F_{1T}^{\perp f}(x, k_T; \mu, \zeta_F)\frac{\epsilon_{ij}k_T^i S^j}{M_p}$$

CS:
$$\frac{\partial \ln \tilde{F}'^{\perp f}_{1T}(x, b_T; \mu, \zeta_F)}{\partial \ln \sqrt{\zeta_F}} = \tilde{K}(b_T;\mu) \qquad \tilde{F}'^{\perp f}_{1T}(x, b_T; \mu, \zeta_F) \equiv \frac{\partial \tilde{F}^{\perp f}_{1T}(x, b_T; \mu, \zeta_F)}{\partial b_T}$$

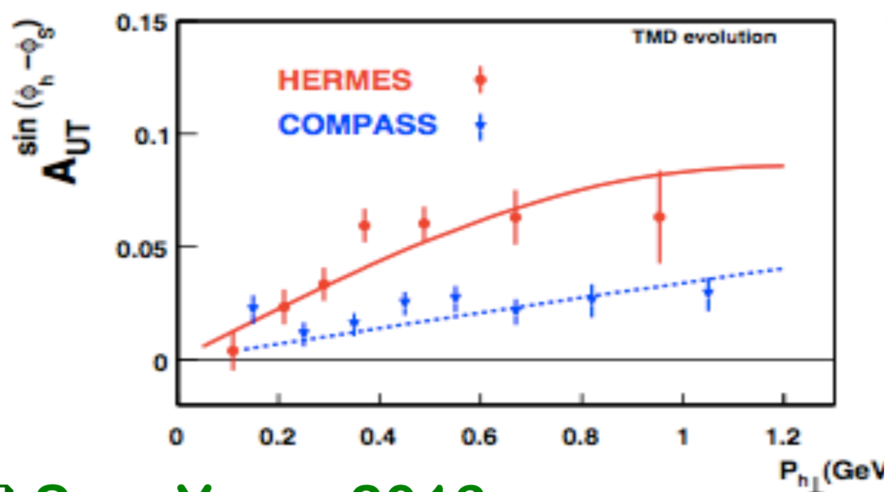
RGs:
$$\frac{d\tilde{F}'^{\perp f}_{1T}(x, b_T; \mu, \zeta_F)}{d\ln\mu} = \gamma_F(g(\mu); \zeta_F/\mu^2)\tilde{F}'^{\perp f}_{1T}(x, b_T; \mu, \zeta_F)$$

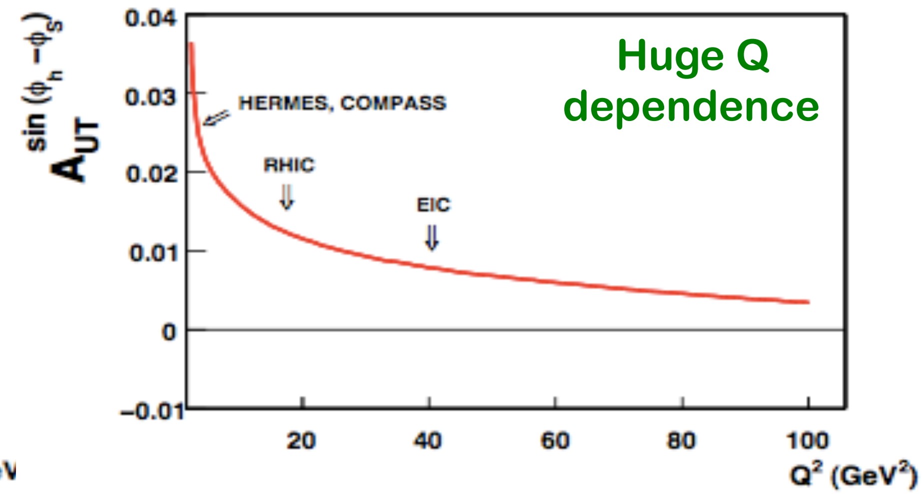
Also see talks by Collins, Yuan, …

$$\frac{d\tilde{K}(b_T;\mu)}{d\ln\mu} = -\gamma_K(g(\mu)) \quad \Longrightarrow \quad \frac{\partial \gamma_F(g(\mu); \zeta_F/\mu^2)}{\partial \ln\sqrt{\zeta_F}} = -\gamma_K(g(\mu)),$$

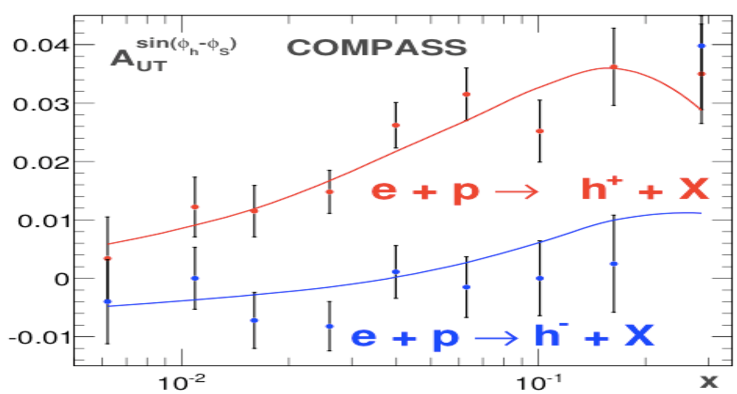
Importance of the evolution

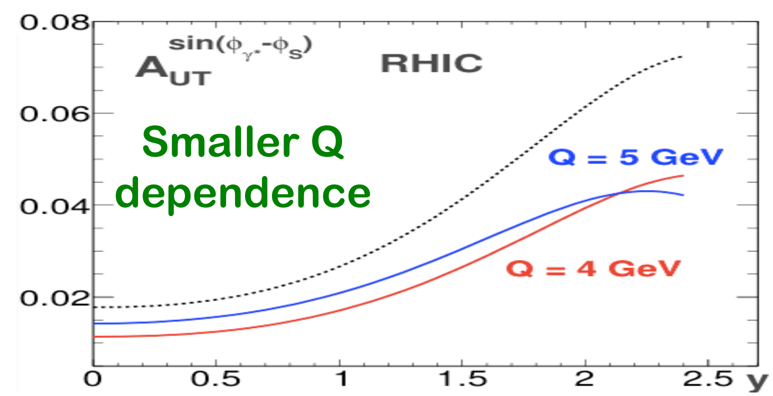
- Aybat, Prokudin, Rogers, 2012:





- Sun, Yuan, 2013:





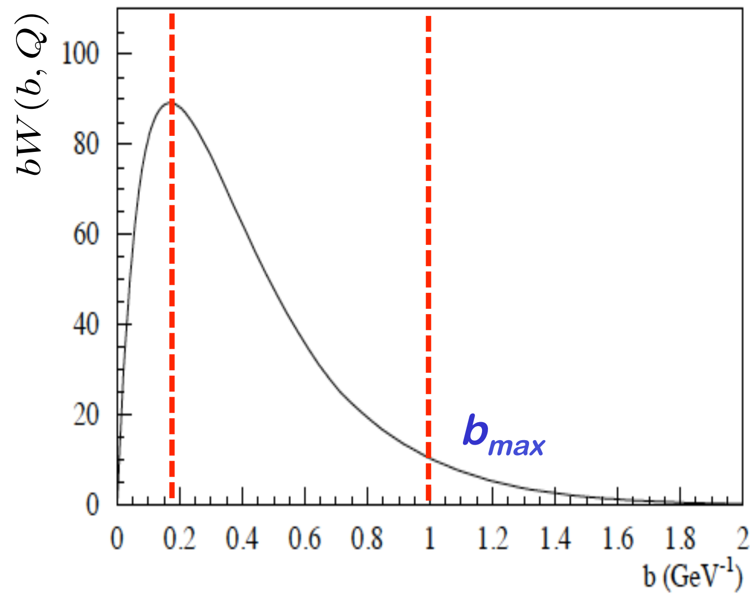
No disagreement on evolution equations!

Issue: extrapolation to non-perturbative large b-region
choice of the Q-dependent "form factor"

Evolution and extrapolation - I

- **Q-evolution is achieved in b-space:**

$$\frac{d\sigma_{AB}}{dQ^2 dQ_T^2} \equiv \frac{1}{(2\pi)^2}\int d^2b\, e^{i\vec{b}\cdot\vec{Q}_T}\tilde{W}_{AB}(b,Q) + Y_{AB}(Q_T^2,Q^2)$$



- **CSS formalism:**

$$\widetilde{W}(b,Q) \approx W^{\rm pert}(b_*,Q)\, e^{-\mathcal{F}^{\rm NP}(b,Q)}$$

✧ $$b_* = \frac{b}{\sqrt{1+b^2/b_{\rm max}^2}} \to b_{\rm max}, \quad \text{as} \quad b\to\infty$$

✧ $$W^{\rm pert}(b,Q) \propto e^{-S_p(b,Q)}[\mathcal{C}\otimes f]_q(x_A,b,\mu)\,[\mathcal{C}\otimes f]_{\bar{q}}(x_B,b,\mu)$$

✧ $$\mathcal{F}^{\rm NP} \approx b^2(a_1 + a_2\ln(Q/Q_0) + a_3\ln(x_A x_B) + \ldots) + \ldots$$

Issues:

Qiu, Zhang, PRL, PRD, 2001

✧ **Predictive power – universality of "form factor":** $\mathcal{F}^{\rm NP}(b,Q)$

$\ln(Q/Q_0)$ **- dependence is only valid when** $\ln(Q/Q_0) \gg (Q_0/Q)$

Not satisfied for HERMES, even COMPASS, data for $Q_0 \sim 2\,{\rm GeV}$ ***!!!***

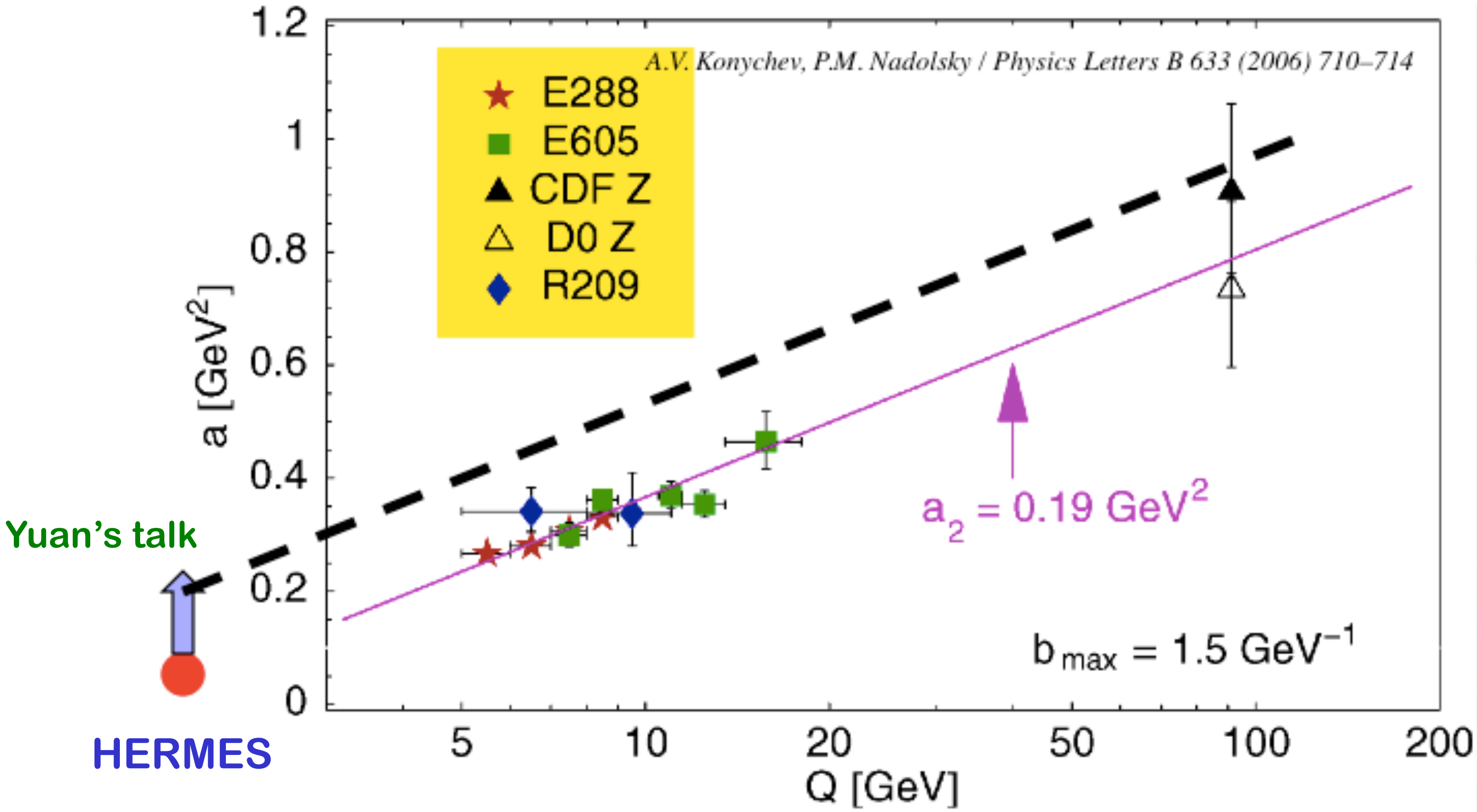
✧ **"Unwanted change" to small-b perturbative contribution**

b_* ***and*** $\mathcal{F}^{\rm NP}(b,Q)$ ***change*** $W^{\rm pert}(b,Q)$ ***in small-b region!!!***

Importance of the evolution - II

- Q-dependence of the "form factor" :

Konychev, Nadolsky, 2006



$$\mathcal{F}^{\rm NP}(b,Q) = a(Q^2)\, b^2$$

At Q ~ 1 GeV, $\ln(Q/Q_0)$ term may not be the dominant one!

$$\mathcal{F}^{\rm NP} \approx b^2(a_1 + a_2\ln(Q/Q_0) + a_3\ln(x_A x_B) + \ldots) + \ldots$$

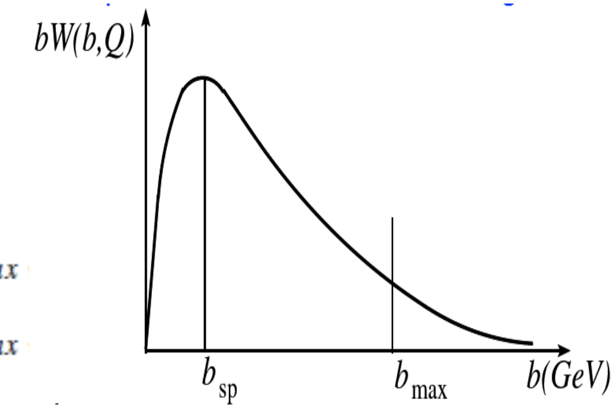
Power correction? $(Q_0/Q)^n$-term? *Better fits for HERMES data?*

Evolution and extrapolation - III

❑ Preserve pQCD calculation at low b:

Qiu, Zhang, PRL, PRD, 2001

$$\frac{d\sigma_{AB}}{dQ^2 dQ_T^2} \equiv \frac{1}{(2\pi)^2}\int d^2b\, e^{i\vec{b}\cdot\vec{Q}_T}\tilde{W}_{AB}(b,Q) + Y_{AB}(Q_T^2,Q^2)$$



$$\tilde{W}^{QZ}(b,Q,x_A,x_B) = \begin{cases} \tilde{W}(b,Q,x_A,x_B), & b \le b_{max} \\ \tilde{W}(b_{max},Q,x_A,x_B)F_{QZ}^{NP}(b,Q,x_A,x_B;b_{max}), & b > b_{max} \end{cases}$$

$$F_{QZ}^{NP}(b,Q,x_A,x_B;b_{max}) = \exp\left\{ -\ln\left(\frac{Q^2 b_{max}^2}{c^2}\right)\{g_1[(b^2)^\alpha - (b_{max}^2)^\alpha] + g_2(b^2 - b_{max}^2)\} - \bar{g}_2(b^2 - b_{max}^2)\right\}$$

❑ Parameters fixing:

- ✧ Continuous in both 1st and 2nd derivatives at b_{max} to fix g_1 and α

  No free parameter if g_2=0 !

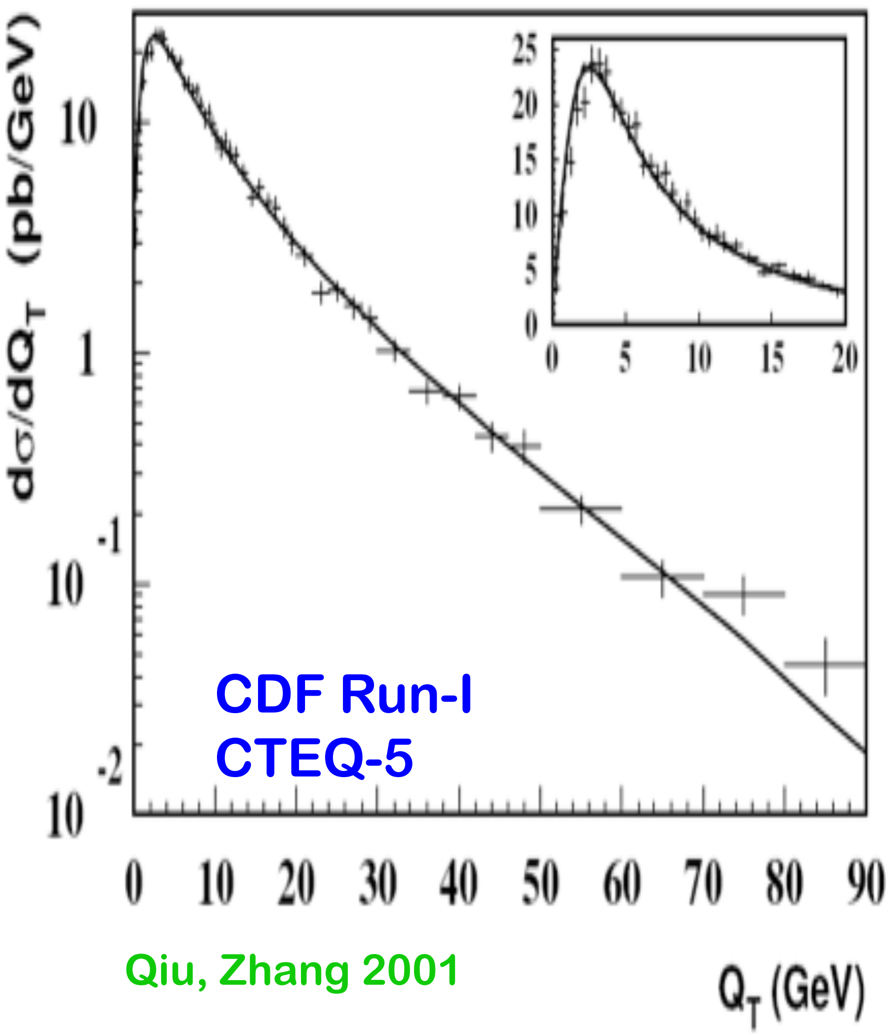
  Consistent with W/Z and ϒ data at Tevatron and LHC
- ✧ Better functional form for low energy Drell-Yan data?

❑ Predictive power:

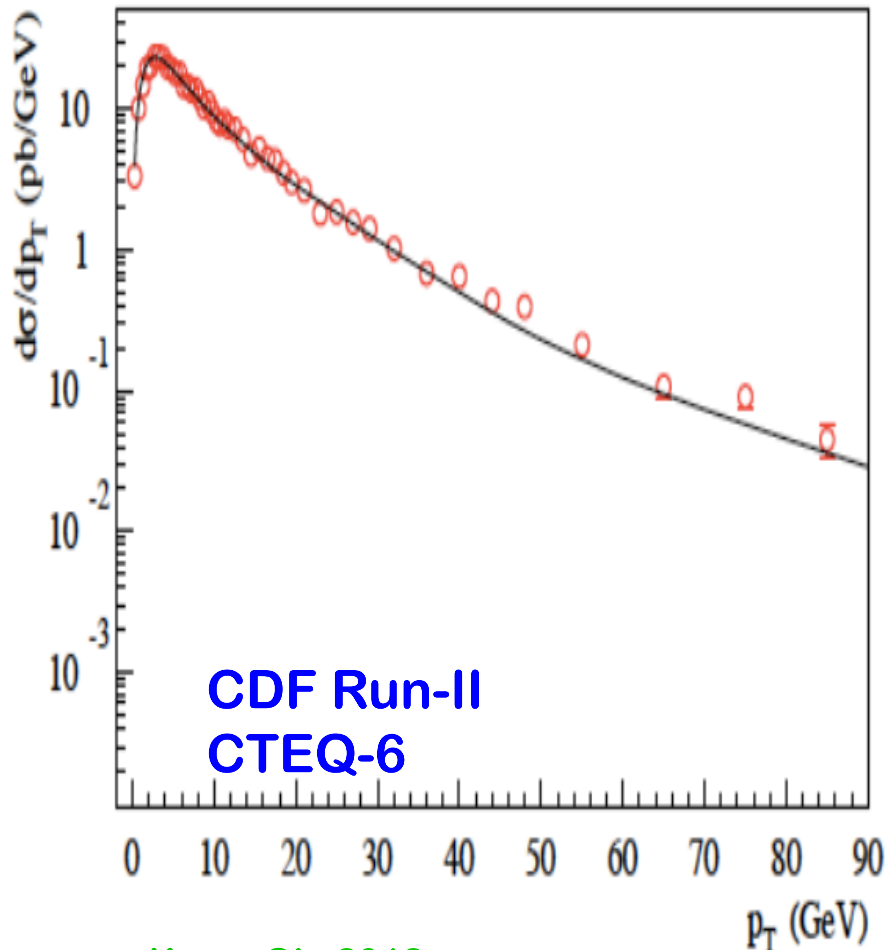
- ✧ Larger Q
- ✧ Larger S

→ Smaller b_{sp} → Better prediction

Phenomenology - Tevatron



Qiu, Zhang 2001

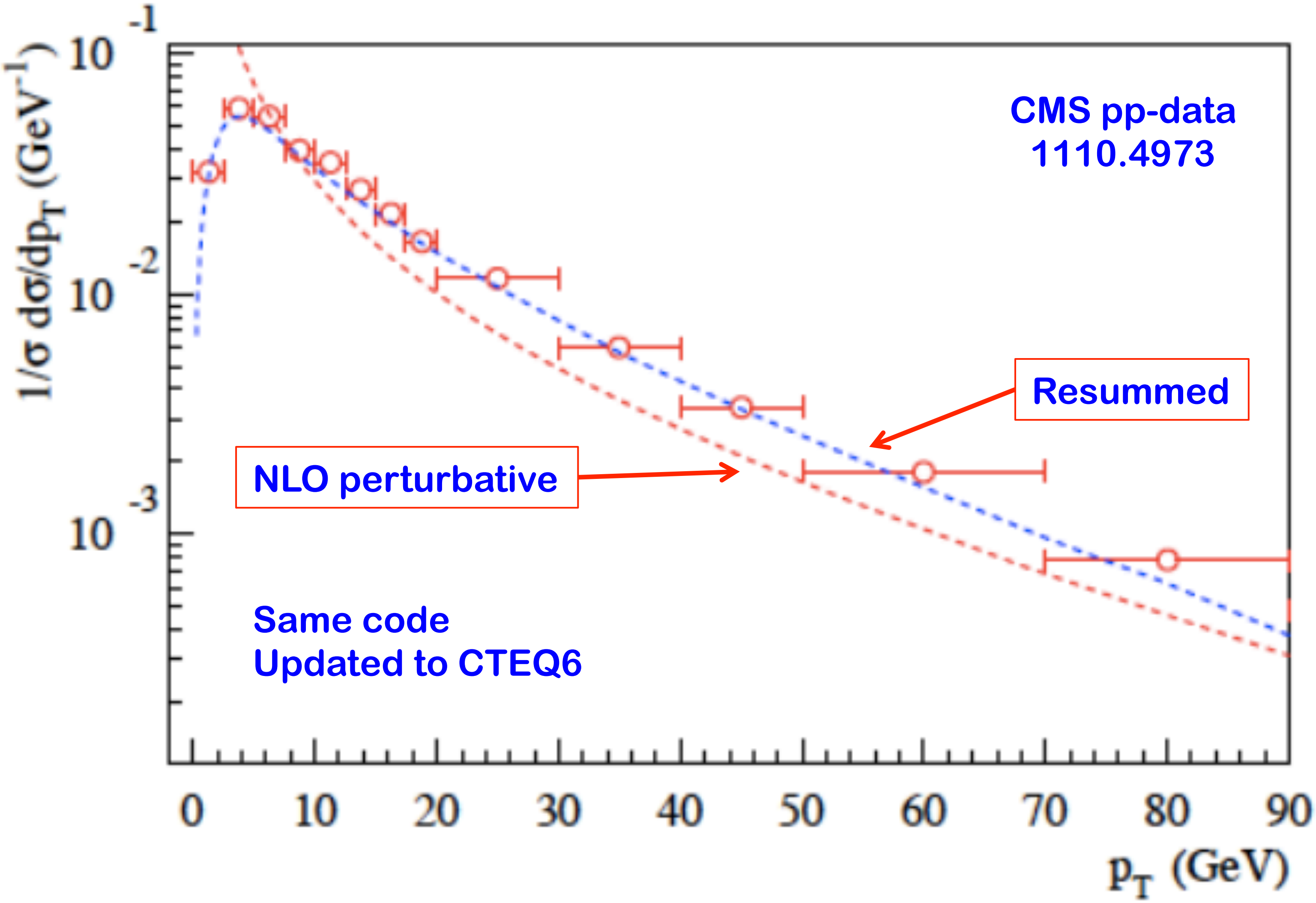


Kang, Qiu 2012

No free fitting parameter!

Phenomenology – Z^0 @ LHC

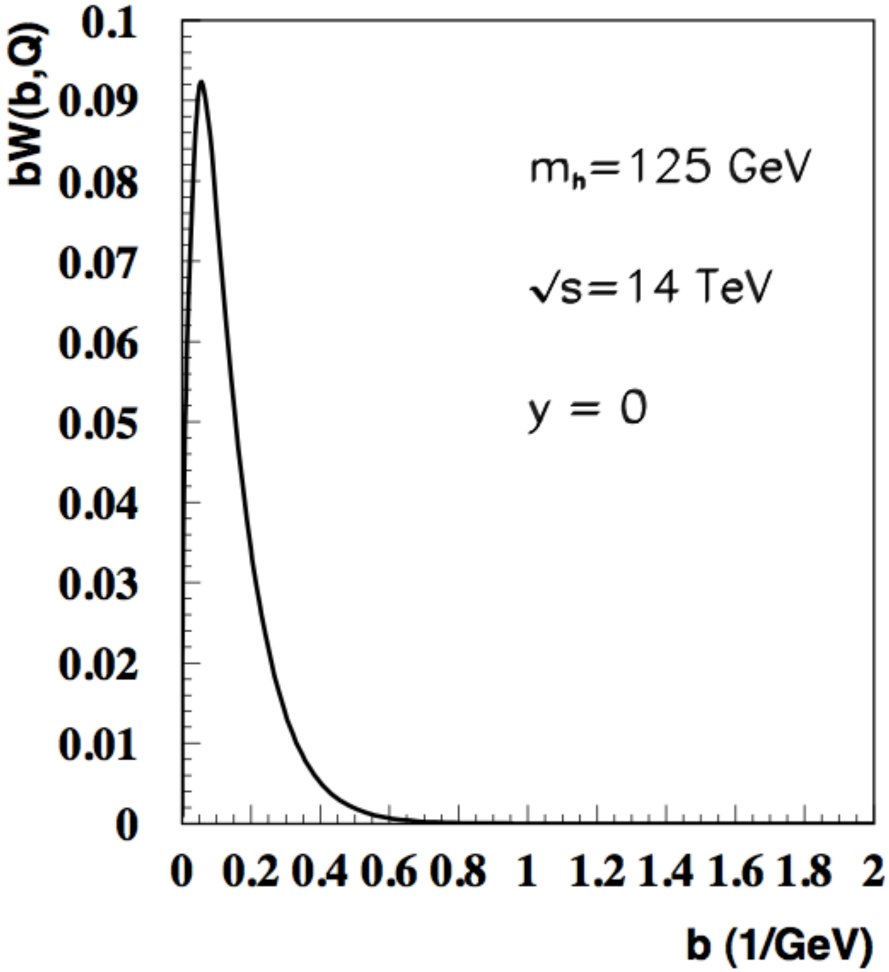
Kang, Qiu, 2012

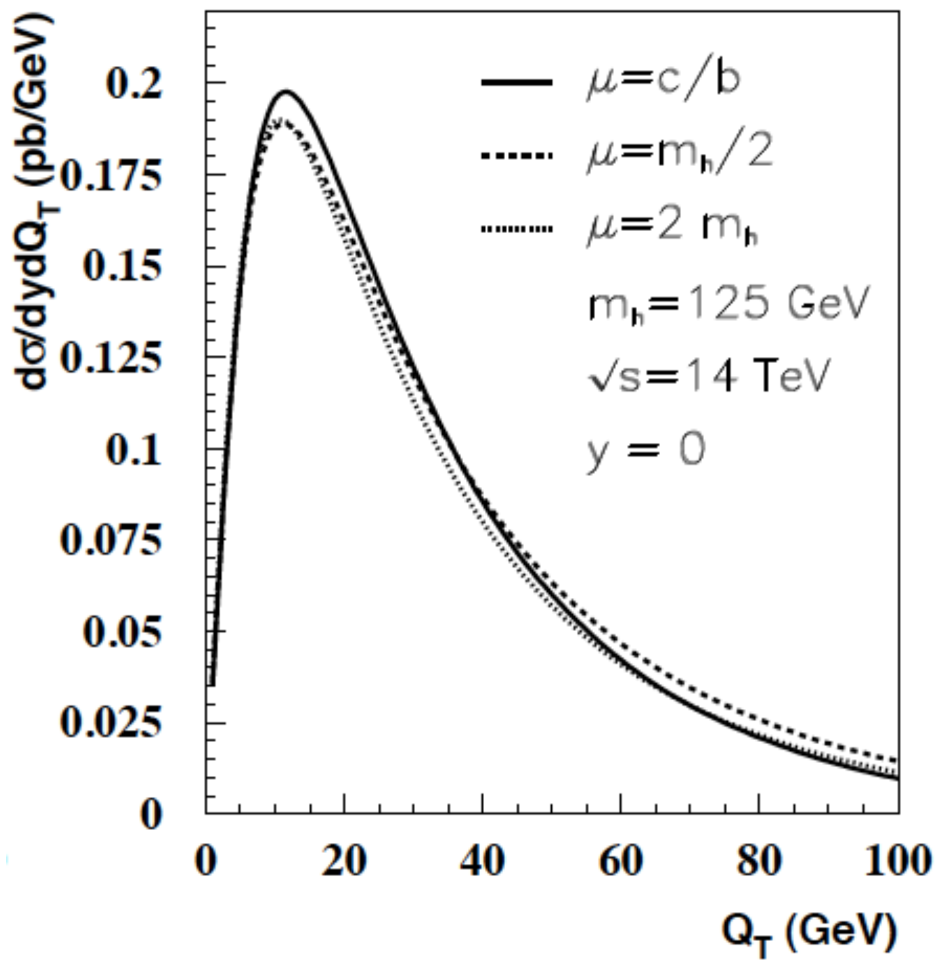


Effectively no non-perturbative uncertainty!

Phenomenology – Higgs @ LHC

Berger, Qiu, 2003

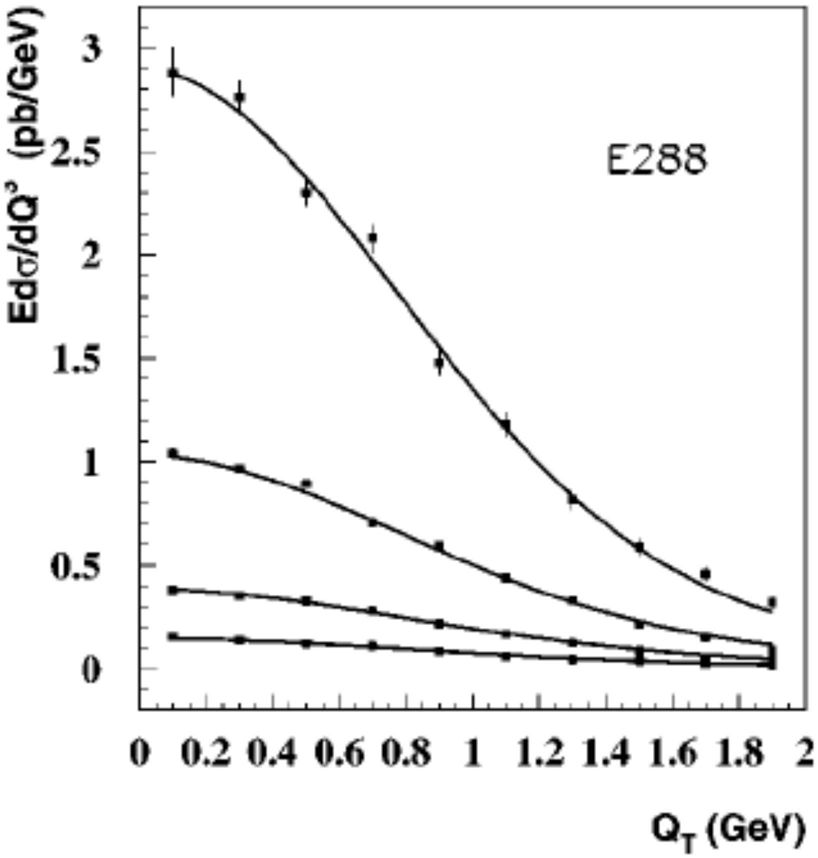


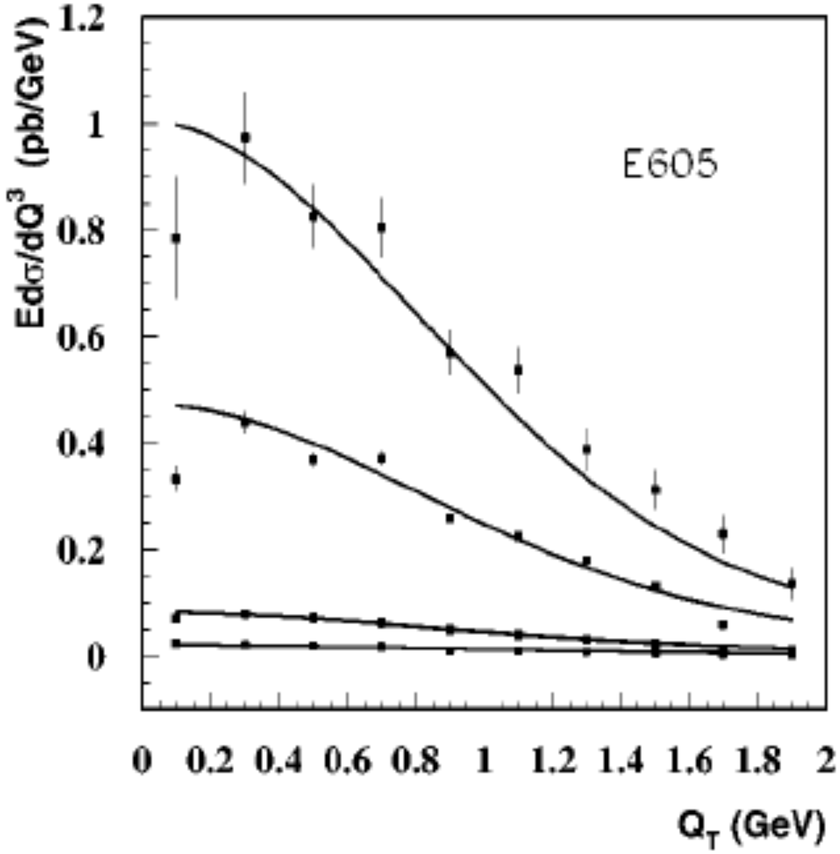


Effectively no non-perturbative uncertainty!

Low energy Drell-Yan data

Qiu, Zhang, PRL, PRD, 2001



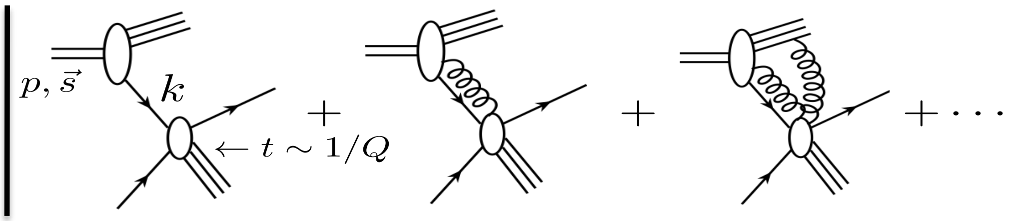


HERMES and COMPASS data?

Much lower Q2, need better non-perturbative functions

How collinear factorization generates SSA?

❑ Collinear factorization beyond leading power:

$\sigma(Q,\vec{s}) \propto \Bigg|$  $\Bigg|^2 \left(\frac{\langle k_\perp \rangle}{Q}\right)^n$ – Expansion

$$\sigma(Q, s_T) = H_0 \otimes f_2 \otimes f_2 + (1/Q)\, H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2)$$

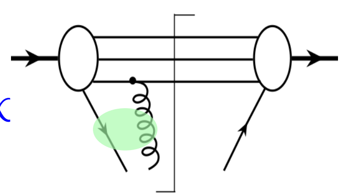
Too large to compete! (→ f_2)

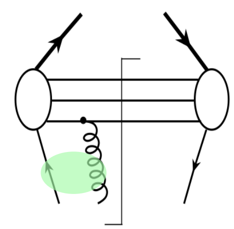
Three-parton correlation (→ f_3)

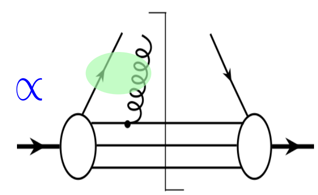
❑ Single transverse spin asymmetry:

Efremov, Teryaev, 82;
Qiu, Sterman, 91, etc.

$$\Delta\sigma(s_T) \propto T^{(3)}(x,x) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x) \otimes \hat{\sigma}_D \otimes D^{(3)}(z,z) + \ldots$$

$T^{(3)}(x,x) \propto$ 

$D^{(3)}(z,z) \propto$ 

$T^{(3\sigma)}(x,x) \propto$ 

Qiu, Sterman, 1991, …

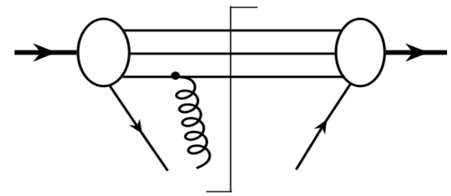
Kang, Yuan, Zhou, 2010

Kanazawa, Koike, 2000

Integrated information on parton's transverse motion!

Twist-3 distributions relevant to A_N

- **Two-sets Twist-3 correlation functions:**



No probability interpretation!

$$\widetilde{T}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P, s_T|\overline{\psi}_q(0)\, \frac{\gamma^+}{2}\, \left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right] \psi_q(y_1^-)|P, s_T\rangle$$

Kang, Qiu, 2009

$$\widetilde{T}_{G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P, s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right] F^{+\lambda}(y_1^-)|P, s_T\rangle(-g_{\rho\lambda})$$

$$\widetilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \langle P, s_T|\overline{\psi}_q(0)\, \frac{\gamma^+\gamma^5}{2}\, \left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right] \psi_q(y_1^-)|P, s_T\rangle$$

$$\widetilde{T}_{\Delta G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\, \frac{1}{P^+}\langle P, s_T|F^{+\rho}(0)\left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right] F^{+\lambda}(y_1^-)|P, s_T\rangle\, (i\epsilon_{\perp\rho\lambda})$$

Role of color magnetic force!

- **Twist-2 distributions:**
  - Unpolarized PDFs:

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$

$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$

  - Polarized PDFs:

$$\Delta q(x) \propto \langle P, S_\parallel|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P, S_\parallel\rangle$$

$$\Delta G(x) \propto \langle P, S_\parallel|F^{+\mu}(0)F^{+\nu}(y)|P, S_\parallel\rangle(i\epsilon_{\perp\mu\nu})$$

- **Twist-3 fragmentation functions:** See Kang, Yuan, Zhou, 2010, Kang 2010

Test QCD evolution at twist-3 level

Kang, Qiu, 2009; Yuan, Zhou, 2009
Vogelsang, Yuan, 2009, Braun et al, 2009

❑ Scaling violation – "DGLAP" evolution:

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \begin{bmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{bmatrix} = \begin{pmatrix} K_{qq} & K_{q\Delta q} & K^{(f)}_{qG} & K^{(d)}_{qG} & K^{(f)}_{q\Delta G} & K^{(d)}_{q\Delta G} \\ K_{\Delta qq} & K_{\Delta q\Delta q} & K^{(f)}_{\Delta qG} & K^{(d)}_{\Delta qG} & K^{(f)}_{\Delta q\Delta G} & K^{(d)}_{\Delta q\Delta G} \\ K^{(f)}_{Gq} & K^{(f)}_{G\Delta q} & K^{(ff)}_{GG} & K^{(fd)}_{GG} & K^{(ff)}_{G\Delta G} & K^{(fd)}_{G\Delta G} \\ K^{(d)}_{Gq} & K^{(d)}_{G\Delta q} & K^{(df)}_{GG} & K^{(dd)}_{GG} & K^{(df)}_{G\Delta G} & K^{(dd)}_{G\Delta G} \\ K^{(f)}_{\Delta Gq} & K^{(f)}_{\Delta G\Delta q} & K^{(ff)}_{\Delta GG} & K^{(fd)}_{\Delta GG} & K^{(ff)}_{\Delta G\Delta G} & K^{(fd)}_{\Delta G\Delta G} \\ K^{(d)}_{\Delta Gq} & K^{(d)}_{\Delta G\Delta q} & K^{(df)}_{\Delta GG} & K^{(dd)}_{\Delta GG} & K^{(df)}_{\Delta G\Delta G} & K^{(dd)}_{\Delta G\Delta G} \end{pmatrix} \otimes \begin{bmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{bmatrix}$$

$(x, x + x_2, \mu, s_T)$ $\quad$ $(\xi, \xi + \xi_2; x, x + x_2, \alpha_s)$ $\quad$ $\int d\xi \int d\xi_2$

❑ Evolution equation – consequence of factorization:

Factorization: $\Delta\sigma(Q, s_T) = (1/Q) H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F)$

DGLAP for f_2: $\frac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F)$

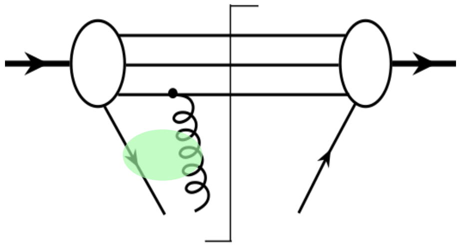
Evolution for f_3: $\frac{\partial}{\partial \ln(\mu_F)} f_3 = \left(\frac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)} \right) \otimes f_3$

"Interpretation" of twist-3 correlation functions

- **Measurement of direct QCD quantum interference:**

Qiu, Sterman, 1991, ...

$$T^{(3)}(x, x, S_\perp) \;\propto$$



Interference between a single active parton state and an active two-parton composite state

- **"Expectation value" of QCD operators:**

$$\langle P, s|\overline{\psi}(0)\gamma^+\psi(y^-)|P, s\rangle \longrightarrow \langle P, s|\overline{\psi}(0)\gamma^+ \left[\epsilon_\perp^{\alpha\beta} s_{T\alpha} \int dy_2^- F_\beta{}^+(y_2^-)\right] \psi(y^-)|P, s\rangle$$

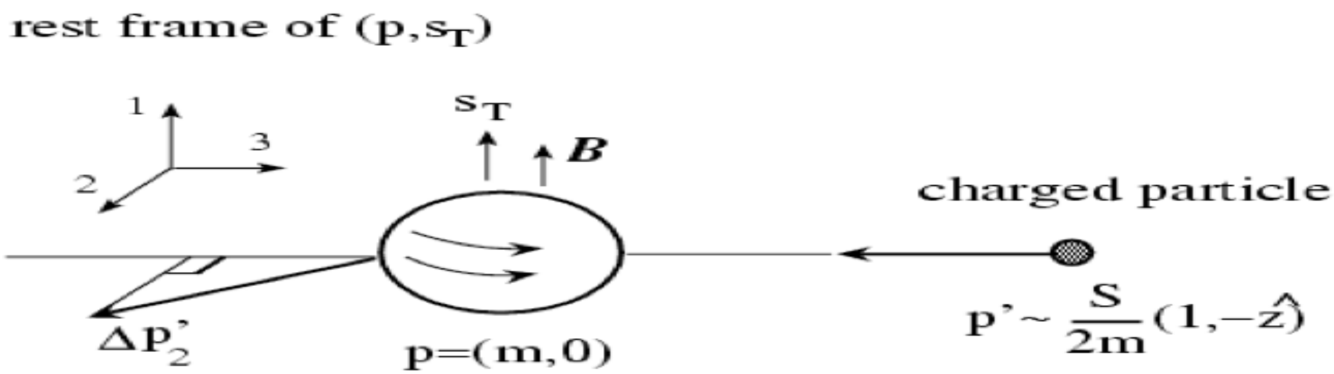
$$\langle P, s|\overline{\psi}(0)\gamma^+\gamma_5\psi(y^-)|P, s\rangle \longrightarrow \langle P, s|\overline{\psi}(0)\gamma^+ \left[i\, g_\perp^{\alpha\beta} s_{T\alpha} \int dy_2^- F_\beta{}^+(y_2^-)\right] \psi(y^-)|P, s\rangle$$

How to interpret the "expectation value" of the operators in RED?

A simple example

- The operator in Red – a classical Abelian case:

Qiu, Sterman, 1998



- Change of transverse momentum:

$$\frac{d}{dt}p_2' = e(\vec{v}' \times \vec{B})_2 = -ev_3B_1 = ev_3F_{23}$$

- In the c.m. frame:

$$(m,\vec{0}) \to \bar{n} = (1,0,0_T), \quad (1,-\hat{z}) \to n = (0,1,0_T)$$

$$\Longrightarrow \quad \frac{d}{dt}p_2' = e\,\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma{}^+$$

- The total change:

$$\Delta p_2' = e\int dy^-\,\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma{}^+(y^-)$$

Net quark transverse momentum imbalance caused by color Lorentz force inside a transversely polarized proton

Summary

- Drell-Yan process is one of the oldest hard process proposed to test QCD – it still a very good one!
- The proof of QCD factorization for Drell-Yan is solid (LP + NLP for collinear, LP for TMD)
- The test of the sign change of the Sivers function is a critical test of TMD factorization!
- Drell-Yan could provide much more than the sign change

Thank you!

Backup transparencies

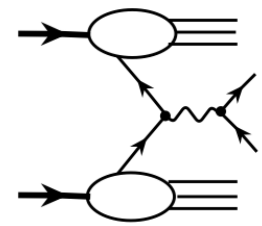
Drell-Yan dilepton production

- **Drell-Yan lepton-pair production:**

$$\frac{d\sigma_{A+B\to\ell\bar{\ell}(Q^2)+X}}{dQ^2} = \sigma_0 \sum_q e_q^2 \int dx\, \phi_{q/A}(x) \int dx'\, \phi_{\bar{q}/B}(x')\, \delta(Q^2 - xx's_{AB}) + q \leftrightarrow \bar{q}$$

$$= \frac{\sigma_0}{s_{AB}} \sum_q e_q^2\, \mathcal{F}_{q\bar{q}}(\tau = Q^2/s_{AB}),$$

$$\sigma_0 = \sigma^{\rm incl}_{q\bar{q}\to\ell\bar{\ell}(Q^2)}$$



Effective flux: $$\mathcal{F}_{q\bar{q}}(\tau) = \int dx\, \phi_{q/A}(x) \int dx'\, \phi_{\bar{q}/B}(x')\, \delta(\tau - xx') + q \leftrightarrow \bar{q}$$

- **Predictions:**

  - ✧ **No free parameter for production rate!**
  - ✧ **Normalized Drell-Yan angular distribution**

$$\frac{dN}{d\Omega} \equiv \left(\frac{d\sigma}{d^4q}\right)^{-1} \frac{d\sigma}{d^4q\,d\Omega} = \frac{3}{4\pi}\left(\frac{1}{\lambda+3}\right)\left[1 + \lambda\cos^2\theta + \mu\sin(2\theta)\cos\phi + \frac{\nu}{2}\sin^2\theta\cos(2\phi)\right]$$

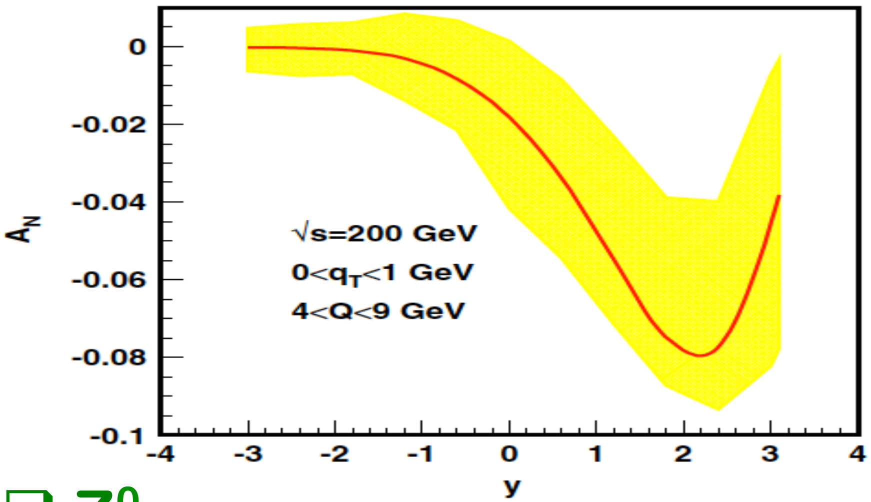
  - ✧ **Transversely polarized virtual photon: 1 + $\cos^2\theta$ distribution**
  - ✧ **Lam-Tung relation:** $1 - \lambda - 2\nu = 0$

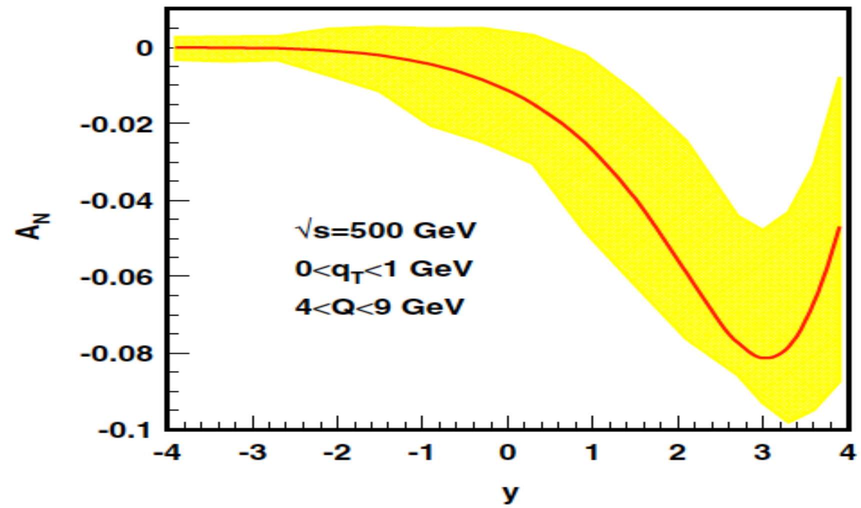
Test of the modified universality

❑ Drell-Yan:

$$A_N^{\sin(\phi-\phi_s)} = -A_N$$

Collins et al. 2006
Kang, Qiu, 2009





❑ Z^0: