Measuring the Electric Dipole Moment of the Electron Using Resonant Longitudinal Polarimetry

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- Measuring the EDM of the electron is discussed, both on its own merits and as prototype for proton EDM measurement.
- An all-electric lattice for measuring the electron EDM at its "magic" globally frozen spin kinetic energy of $K = 14.5\,\mathrm{MeV}$ is described.
- More to be emphasized is a new scheme using RF excitation in a magnetic ring, with $K = 440\,\mathrm{MeV}$ electrons and resonant polarimetry. With spin tune $Q_s = 1$, the beam beam is frozen locally, but not globally.

The All-Electric Brookhaven AGS Analogue Ring

- While building the AGS at BNL, an all-electric "Electron Analogue" ring was built (to study passage through transition).
- Applying for funds in mid-1953, the approval, commissioning, construction and machine studies had been accomplished in less than two years.
- This is the closest prototype there has been to the all-electric ring needed to "trap" protons to measure their electric dipole moments (EDM).
- I have reverse engineered the lattice design and simulated its performance using the new ETEAPOT code. Results are compared with measurements performed on the ring in 1955.

- The original motivation for studying the BNL electric ring was to develop a test bed for simulations designed to handle electric (as contrasted with magnetic) elements.
- But the study has suggested a more substantial application.
- The AGS Analogue used *electrons* instead of protons, and was limited to achievable electric field. Cost minimization led to 10 MeV as maximum energy and bend radius of 4.7 m.
- These are the same considerations that will fix the parameters of an all-electric proton EDM ring.

- The “Conceptual Design Report” for the AGS Analogue electron ring was a four page letter from BNL Director Haworth to A.E.C. (predecessor of D.O.E) Director of Research Johnson, applying for funding. The first three pages are reproduced next.
- Then a 1955 report by Ernest Courant contains the experimental data to be simulated.
- New (ETEAPOT) code has been used to simulate (old) Courant observations.

Historical BNL Documents

Dr. T.H. Johnson, Director
Division of Research
U.S. Atomic Energy Commission
Washington 25, D.C.

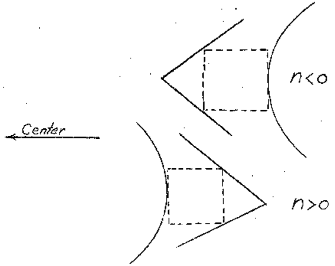
Dear Tom:

This letter concerns certain aspects of our accelerator development program, particularly the proposed electron model.

As you know, the general development of a very high energy alternating gradient synchrotron is proceeding actively at Brookhaven, utilizing operating funds allocated to Basic Physics Research. As I explained in my letter of August 12, however, these funds are insufficient to carry forward the development as rapidly as desirable. Also, there are certain steps which should be taken for which the expenditure of operating funds is not appropriate. The first and most important of these is the construction of an electron model intended to provide final assurance of the technical feasibility of the chosen machine and, more importantly, to provide information enabling us to design in the most effective and economical manner. (We have no doubt of the general feasibility of accelerators of this type.)

We have given considerable thought to the requirements for such a model and to the philosophy which should guide us in designing and building it. In the alternating gradient synchrotron, two problems require especially careful exploration by extensive calculation and experimental modelling. These are the close-spaced resonances in the betatron oscillations and the shift of phase stability at intermediate energies. It seems best to study these problems with an electron accelerator which would be essentially an analogue rather than an exact model. This device should, in our opinion, be designed to yield the maximum of orbital data with a minimum of engineering complications, especially those not applicable to a final machine. After considerable thought we have arrived at a tentative description and list of parameters which follow.

The device would consist of an accelerator having an orbital radius of 15 feet and an overall diameter, including the straight sections, of approximately 45 feet; the guide and focussing fields would be electrostatic, with electrode shapes as indicated in the sketch (full scale).



Electrons of about 1 MeV energy would be injected from a small horizontal Van de Graaff generator (of the 2 MeV type manufactured by the High Voltage Engineering Corporation) so that 5% to 6% frequency modulation would be required.

Use of a reasonably large radius helps the radio frequency and observing equipment in frequency range where good techniques exist, and permits high n-values which are necessary for strong alternating-gradient focussing. (This, and phase transition, will not be modeled in the Cornell machine.) A moderate rise rate, consistent with attainable vacuum requirements, still permits the use of small, air-cooled amplifier tubes and a heavily loaded low-Q rf cavity.

A tentative list of parameters is:

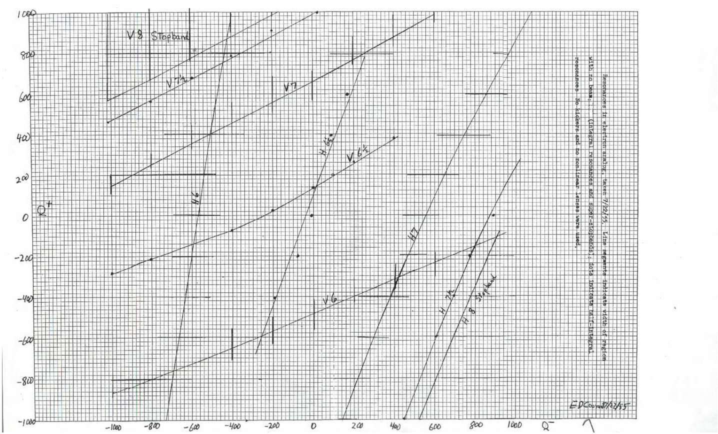
Radius of curvature	15 ft
Over-all diameter	43 ft
n	200
No. of periods	37
No. of straight sections	74
No. of lenses per period	4
Length of lens	7.6 in.
Length of straight section	7.6 in.

Field strength (magnetic type)	
at injection	10.5 gauss
at 10 MeV	74 gauss
Field strength (electrostatic type)	
at injection	3 kV/cm
at 10 MeV	22 kV/cm
Rise time	.01 sec
Phase transition energy	2.8 MeV
Frequency (final)	7 mc
Frequency change	54 %
Volts/turn	150 V
RF power	about 1 kw
No. of betatron wavelengths	about 6.2
aperture	1 X 1 in.
Betatron amplitude for 10^{-3} rad. error	0.07 in.
Maximum stable amplitude, synchrotron osc.	-0.16 in.
Radial spacing of betatron resonances	about 0.4 in.
Vacuum requirement	about 10^{-6} mm Hg

Total power requirements will be small and available with existing installations. The test shack seems to be a suitable location since the ring will be erected inside a thin magnetic shield which can be thermally insulated and heated economically.

We estimate the cost to be approximately $600,000, distributed as shown in the following table:

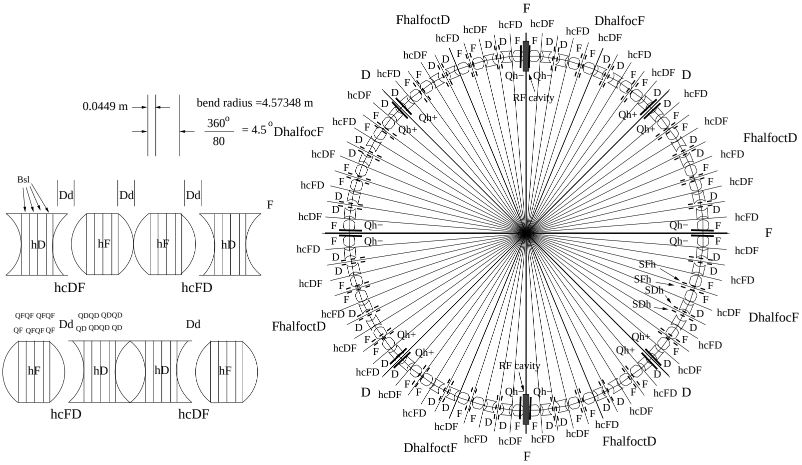
Model	Direct	Overhead	Total
Staff S. & W.	$135,000	$ 65,000	$200,000
Van de Graaff	70,000	-	70,000
Other E. & S.	130,000	-	130,000
Shops	135,000	65,000	200,000
	$470,000	$130,000	$600,000



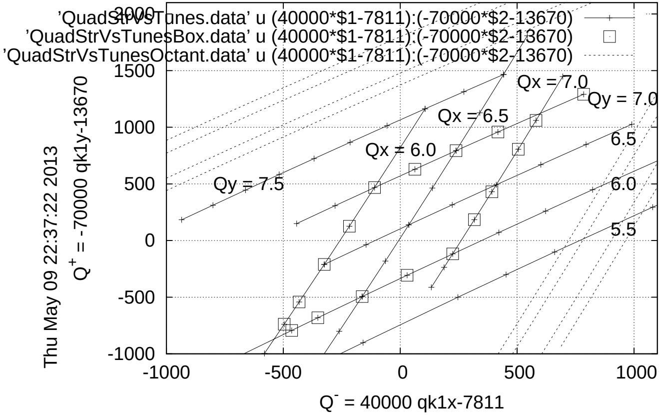
- Tune scans, Courant, BNL report EDG-20, July, 1955
- Heavy lines.....regions with no beam.......integer resonance
- Dots............narrow disruption..........half integer resonance
- Qx=8/Qy=8;......stop bands.................superperiodicity 8
- The nominal central tunes values are $(Q_x, Q_y) = (6.5, 6.5)$.
- Remember this figure!

Reconstructed AGS-Analogue Lattice

The lattice description file `E AGS Analogue2.adxf` for this lattice is in Appendix C.3, “The AGS-Analogue Lattice File in ADXF Form”, of the full report.



Simulation of 1955 Machine Studies Tune Plane Scan



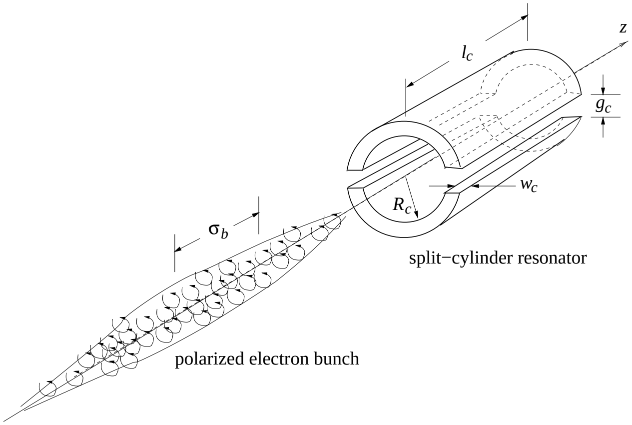
- Boxes mark points on stable diamond about $(Q_x, Q_y) = (6.5, 6.5)$.
- Points lying on $1/2$ integer resonance lines are indicated by dots.
- Superperiodicity bands at $Q_x = 8$ or $Q_y = 8$
- This figure is to be compared with the Courant tune scan figure.

- Dead-reckoned tunes came out within 10% of nominal.
- For the eventual proton EDM ring the vertical tune has to be reduced to $Q_y \approx 0.2$. This will amplify the electric/magnetic differences.
- Comparison between all-magnetic and, otherwise identical, all-electric lattices are contained in the full report, for tune values $(Q_x, Q_y) = (6.2, 2.25)$.
- The electric/magnetic difference is small, but big enough for $Q_y = 2.25$ to be the lowest I have obtained so far. Just switching from magnetic to electric without compensating typically causes a stable lattice to become unstable.

Resonant Polarimetry

- *Magnetic Resonance and Beam-Beam Polarimetry for Frozen Spin Storage Rings*, dated October 13, 2012 describes the use of a high-Q resonator to measure the proton EDM.
- Q-value and fields confirmed by Valerie Shemelin using Microwave Studio.
- This polarimetry is challenging for protons, even using very low temperature cryogenics. The signal to noise ratio for a one hundred percent longitudinally polarized beam of nominal intensity would be about 50 to 1 for protons.
- The method will be far more effective for electrons since the electron magnetic dipole moment is a thousand times greater than the proton's.
- For electrons one expects to be sensitive to EDM-induced polarization tilts of less than one milliradian.

- The resonator gap heights are greatly exaggerated
- Though shown as vacuum in the figure, the gaps will actually be filled with low loss saphire.
- Q-values of about 10^{10} (far higher than necessary for electrons, though not for protons) are achievable at low temperature.



- End view of polarimeter and readout using a low temperature pHEMT transistor such as Agilent type ATF-35143 in a source follower circuit.
- Coaxial magnetic read-out is also possible.

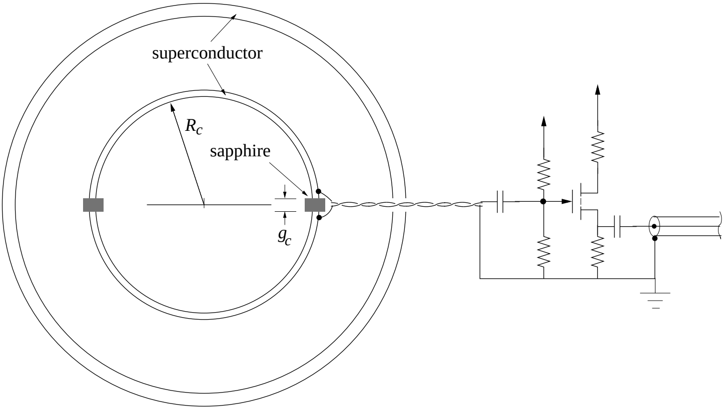


Figure: Polarimeter readout.

- Unlike p-carbon polarimetery, resonant polarimetry is passive, and measures longitudinal rather than transverse polarization.
- A more important distinction is that the resonant polarimeter measures the coherent sum of polarizations of all bunches, in both beams.
- *Deviation from null* does not incur the counting-statistics penalty of scattering asymmetry measurement.

Electron Lattice Spin Tunes

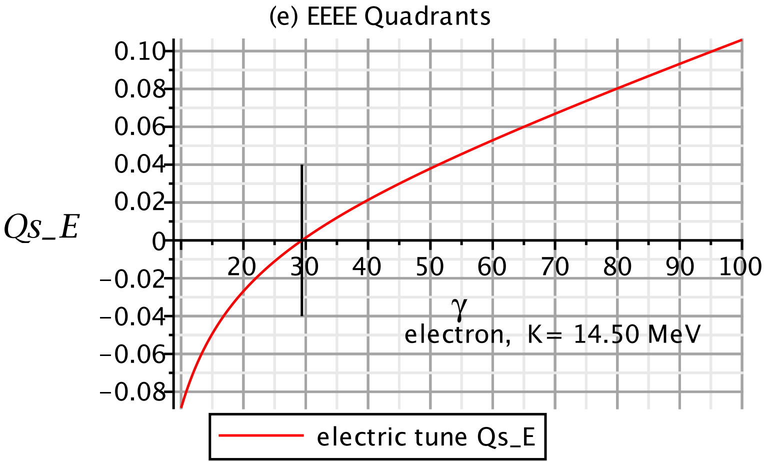


Figure: Electron spin tune Qs_E in electric storage ring.

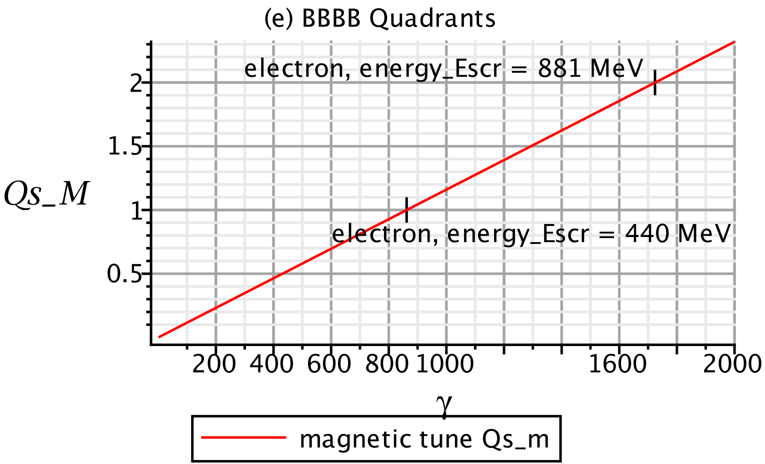


Figure: Electron spin tune Qs_M in a magnetic ring.

Table: Some frozen spin electron EDM storage ring options. “P/UP” stands for “polarized/unpolarized”. There are also extracted beam polarimeter options.

label	strength	bend radius r_0	kinetic energy K_0	spin tune Q_s	magic gamma γ_m	source of EDM torque	minimal bunch pattern	RF period-icity	polarimetry or comment
	MV/m,T	m	MeV						
All Electric, One Beam									
$eE1_0^{\rm res}$	“	5.0	14.5	0	“	“	2e-(P)	“	resonant
$eE1_0^{\rm Mol}$	“	“	“	“	“	“	2e-(P)	“	Möller
All Electric, Two Beams									
$eE2_0^{\rm bb}$	3.0 MV/m	5.0	14.5	0	29.382	E_r	e-(P) + e(P or UP)	arbitrary	beam-beam
$eE2_0^{\rm res}$	“	“	“	“	“	“	e-(P) +e -(P or UP)	“	resonant
All Magnetic, One Beam									
$eM1_1^{\rm res}(1)$	0.734 T	2.0	440	1	862	E_z	e-(P)	2	resonant
$eM1_1^{\rm res}(2)$	“	“	“	“	“	“	2e-(P)	2	resonant
All Magnetic, Colliding Beams									
$epM2_1$unp	0.734 T	2.0	440	1	862	E_z	e-(P) + e+(UP)	2	elec. posit.
$epM2_1$pol	“	“	“	“	“	“	e-(P) + e+(P)	2	pol. posit.

Table: Electron EDM storage ring parameters-continued. Some of the parameter values are little better than guesses.

label	number of particles	ϵ_x injection m	ϵ_x SR equilib. m	damping time s	polar. time s	good luminosity $/\mathrm{cm}^2/\mathrm{s}$	bad luminoscity lifetime s	energy loss per turm keV	critical energy eV
All Electric, One Beam									
$\mathrm{eE1}_0^{\mathrm{res}}$ $\mathrm{eE1}_0^{\mathrm{mol}}$									
All Electric, Two Beams									
$\mathrm{eE2}_0^{\mathrm{bb}}$ $\mathrm{eE2}_0^{\mathrm{Mol}}$									
All Magnetic, One Beam									
$\mathrm{eM1}_1^{\mathrm{res}}(1)$ $\mathrm{eM1}_1^{\mathrm{res}}(2)$		$\sim 10^{-8}$ "	$\sim< 10^{-8}$ "	$\sim$0.033 "	$\sim$47700 "			1.66 "	94 "
All Magnetic, Colliding Beams									
$\mathrm{epM2}_1$unp $\mathrm{epM2}_1$pol				$\sim$0.033 "	$\sim$47700 "				

Electron EDM Lattice Schematics

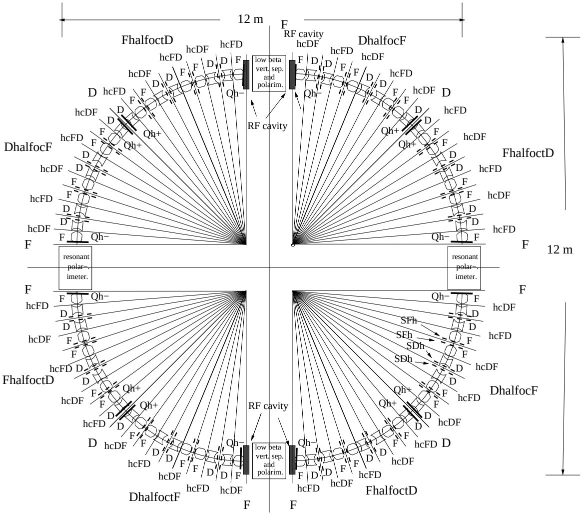


Figure: An all-electric colliding beam storage $\mathtt{eE2}_0$ ring for measuring the electric dipole moment of the 14.5 MeV **electrons**. The radius could be significantly smaller.

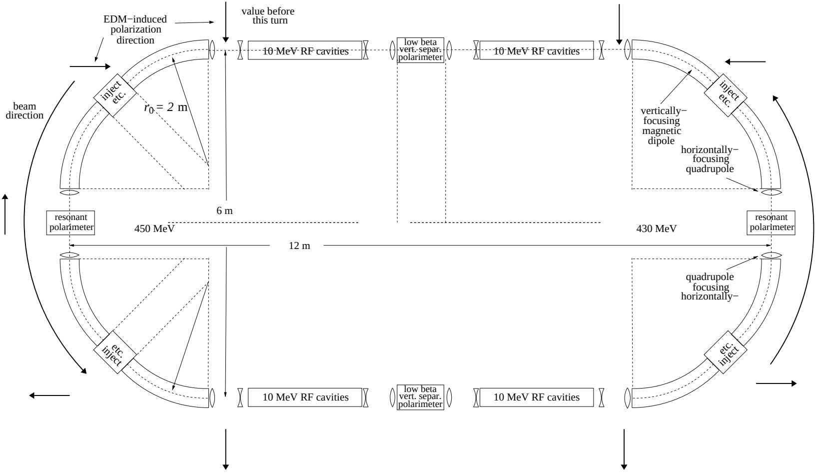


Figure: All-magnetic single beam $\mathtt{eM1}_1^{\mathrm{res}}$ or colliding beam $\mathtt{eM2}_1^{\mathrm{res}}$ storage ring for measuring the electric dipole moments of 440 MeV, $Q_s = 1$ electrons/positrons. The bend fields and beam energies are different in left and right arcs.

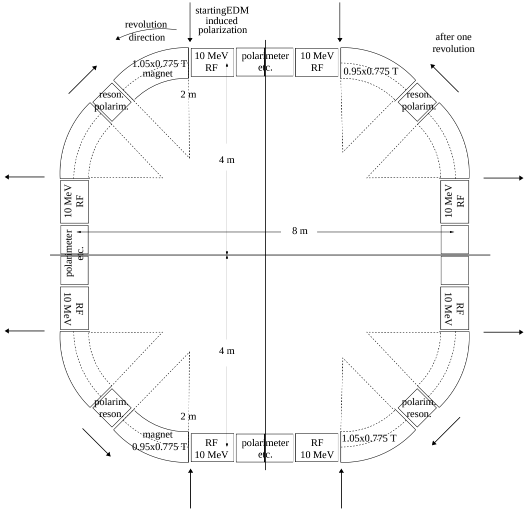


Figure: All-magnetic single beam $\mathtt{pM1}_2^{\mathrm{res}}$ $Q_s = 2$ storage ring appropriate for measuring the electric dipole moment of **protons** of kinetic energy 108.9 MeV or **electrons** with kinetic energy 881 MeV. Beam energy varies quadrant to quadrant.)

Calculation of EDM-Induced Spin Precession

- To suppress magnetic dipole moment (MDM) effects it is essential to use **deviation from null** experiments. i.e. Any purely MDM signal cancels in the polarimeter.
- For proton-carbon (which measures transverse polarization) any left-right counting rate asymmetry is ascribed to EDM (or spurious EDM) effect.
- Resonant polarimeter is passive (no particles are wasted) but it measures longitudinal polarization. This limits the storage ring possibilities.

- For spin tunes other than $Q_s = 0$ the EDM-effect over the full ring would vanish for uniform magnetic field B.
- RF acceleration modulates the energy (with no net change) and correspondingly B, synchronized with the frozen spin pattern.
- There is a net torque causing a systematic, EDM-induced precession that can be measured using resonant longitudinal polarimeters.
- The EDM effect is proportional to the RF amplitude $V_{\rm rf}$.

- The EDM-induced precession over one turn is

$$\Delta\Theta_{\mathrm{EDM,B}} = \tilde{d}\ 2\pi r_{0,B}\ B_{\mathrm{ave}}\,\frac{1}{\pi}\,\frac{cp_{\mathrm{max}} - cp_{\mathrm{min}}}{cp_{\mathrm{ave}}},$$

- cp_{max}, cp_{min} and cp_{ave} are maximum, minimum and average momenta in the arcs.
- The precession imbalance is proportional to the magnetic field modulation, which is proportional to the momentum modulation.
- One sacrifices an absolute rate "mismatch penalty",

$$\mathrm{M.P.} = \frac{1}{\pi}\,\frac{cp_{\mathrm{max}} - cp_{\mathrm{min}}}{cp_{\mathrm{ave}}},$$

in exchange for enabling longitudinal polarimetry, and for cancelling spurious MDM-induced precession due to external fields.

- Who says anomalous magnetic moments are "anomalous"?

$$\begin{aligned} \text{electron magneton} \equiv \mu_e &= 5.78838175 \times 10^{-5}\,\mathrm{eV/T}, \\ G_e &= 0.001159652, \\ \frac{G_e \mu_e}{\hbar} &= 1.020 \times 10^{8}\mathrm{s}^{-1}/\mathrm{T} \\ \text{proton magneton} \equiv \mu_p &= 3.1524512 \times 10^{-8}\,\mathrm{eV/T}, \\ G_p &= 1.792847356, \\ \frac{G_p \mu_p}{\hbar} &= 0.859 \times 10^{8}\mathrm{s}^{-1}/\mathrm{T} \end{aligned}$$

- Curiously similar precession rates of e and p spins in magnetic field !

- Define a **nominal EDM of** 10^{-29} *e*-**cm** by the product $d_{\rm nom}c$ (because E and B have different units);

$$\frac{d_{\rm nom}c}{\hbar} = \frac{10^{-29} \times (0.01) \times 3 \times 10^{8}}{6.58 \times 10^{-16}} = 4.56\times10^{-8}\,{\rm eV/T}. \quad (1)$$

- Relative-effectiveness ratio, EDM/MDM is about 0.5×10^{-15}.

- **Relative precession task:** Distinguish EDM-induced precession from spurious, wrong-plane, MDM-induced, precession. (Especially for electrons) this is probably the dominant source of EDM measurement error.
- **Absolute precession task:** For a pure Dirac particle in a magnetic field the precession is 2π per turn. For an $eM2_1$ case with revolution frequency of 10 Mhz, this is 0.63×10^8 r/s.
- Applying the 0.5×10^{-15} ratio mentioned above, for globally frozen spin one has to plan on measuring a "nominal" EDM-induced precession of order 10^{-7} r/s, or about 10 mr/day.
- For anticipated longitudinal polarimetry this is about ten sigma per day, for EDM of 10^{-29} e-cm.

- The current upper limit for the electron EDM (obtained from the EDM of thallium atoms, corrected up by a factor of 585 to account for induced polarization) is about 100 in our nominal 10^{-29} e-cm EDM units, giving an EDM-induced precession of order 10^{-5} r/s.
- At the advertised one milliradian polarimeter noise floor a signal of this size would appear in 100 s.

Systematic Error Reduction Reversal Possibilities

- The only important longitudinal electric field is in the RF. It contributes (very weakly) to the foreground EDM precession signal and causes no MDM-induced precession.
- DC electric fields can be strongly suppressed.

- A radial magnetic field B_r acting on the MDM mimics the vertical magnetic field B_y acting on the EDM.
- There are no $\mathrm{intentional}$ radial magnetic fields B_r. Furthermore there can be no time-averaged value $\langle B_r \rangle$, since this would cause secular vertical drift of the beam.
- But $\langle B_r \rangle_{\mathrm{east}}$ and $\langle B_r \rangle_{\mathrm{west}}$ can be equal and opposite to produce $\langle B_r \rangle = 0$.
- The experiment amounts to placing the beam polarization in unstable equilibrium on a spin resonance. Froissart-Stora scans can balance $\langle B_r \rangle_{\mathrm{sector}}$, sector by sector, to cause precession much less than π per spin coherence time to, say 10^{-3} r/s. This is about 100 times as great as the precession caused by 10^{-29} e-cm EDM.

- Reversing RF phase reverses EDM effect, but does not change MDM effect (unless B_r is proportional to B_y, which is likely for internal, but not external, magnetic fields).
- Switching magnet polarity (beam circulation direction from CW to CCW) will separate magnet-corollated from external sources.
- Counter-circulating positrons are separated vertically by $\langle B_r \rangle$. As in the all-electric proton ring, this plus high precision measurement of vertical beam separation gives strong protection against B_r.
- Using partially-polarized positrons for Froissart-Stora scans to cancel $\langle B_r \rangle$ may obviate the need for simultaneous counter-circulating beams, and avoid the need for squid magnetometer BPM's.
- In the end one subtracts $V_{\rm rf} = 0$ data from $V_{\rm rf} \neq$ data.

- The four-fold symmetric lattice (with twice as many RF's) can study both $eM2_2$ at 440 MeV and $eM2_3$ at 881 MeV. Agreement between values at two energies would be powerful.
- Furthermore, with four-fold symmetry, more subtractions are available for suppressing systematic errors.

- There are two, inequivalent initial polarization conditions:
- With initial beam polarization vertical everywhere, longitudinal polarization can grow at the resonant polarimeter. This is favorable for EDM measurement.
- Initial beam polarization horizontal. This is favorable for set-up, but unfavorable for EDM because the EDM-induced polarization is vertical and requires transverse polarimeter which is unfavorable for electrons.

- I. Bazarov, *Performance of a High Current, Low Emittance Electron Gun*, Cornell Report, 2013

- W. Morse, *All Electric Magic Momentum Electron EDM Precursor Experiment¡* Brookhaven National Lab Internal Report, January 24, 2012

- J. Talman and R. Talman, BNL internal reports, *UAL/ETEAPOT Results for Proton EDM Benchmark Lattices,* April, 2012, *UAL/ETEAPOT Proton EDM Benchmark Comparisons II: Transfer Matrices and Twiss Functions,* August, 2012, and *UAL/ETEAPOT Proton EDM Benchmark Comparisons III: Dispersion, Longitudinal Dynamics and Synchrotron Oscillations,* April, 2012

- M. Plotkin, *The Brookhaven Electron Analogue, 1953-1957,* BNL–45058, December, 1991

- E. Courant, *Resonance in the Electron Analogue,* BNL internal report EDC-20, July 28, 1955