Why Search for Primordial Non-Gaussianity?

Daniel Green
KIPAC & Stanford ITP

Courtesy of thecmb.org
Outline

What are we testing?
What are the limits after Planck?
What does this mean for Inflation?
What is the goal?

Courtesy of thecmb.org
What are we testing?
Inflation: the conventional picture

A rolling scalar field

\[ \mathcal{L} = -\frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - V(\phi) \]

\[ \phi(t) : \dot{\phi}^2 \ll V(\phi) \]
Perturbations: the conventional picture

The scalar field fluctuates: \( \phi(x, t) = \phi(t) + \delta \phi(x, t) \)

Source of metric perturbations: \( \zeta = \frac{\delta a}{a} \sim \frac{H \delta \phi}{\dot{\phi}} \)
Inflation: a modern view

There are lots of mechanisms beyond slow-roll

Armendáriz-Picón et al., Silverstein & Tong; Alishahiha et al.; ...

They have two things in common:

(1) Near de Sitter geometry: $H^2 \gg |\dot{H}|$

(2) A clock that defines “end of inflation”

“clock” = Spontaneously broken time-translations

Does not require a scalar field (in principle)
Inflation

Perturbations: a modern view

Fluctuations describe goldstone boson $\pi$

$$\mathcal{L}_\pi = F(t + \pi, \nabla^\mu, g^{\mu\nu})$$

Effective field theory (EFT) of inflation

Goldstone describes fluctuations of the clock

Goldstone is “eaten” by the metric: $\zeta = \frac{\delta a}{a} = -H\pi$

Creminelli et al.  
Cheung et al.
The Power Spectrum

The power spectrum is controlled by two scales:

1. Scale of symmetry breaking: \( f_\pi^2 \)
   
e.g. for slow-roll: \( f_\pi^2 = \dot{\phi} \)

2. Hubble scale \( (H) \): energy scale of fluctuations
   \[
   \langle H^2 \pi^2 \rangle \sim (4\pi^2) \Delta^2 \zeta = \frac{H^4}{f_\pi^4}
   \]
   \[
   \Delta^2 \zeta = 2.2 \times 10^{-9}
   \]
The power spectrum is controlled by two scales:

\[ f_\pi = 57H \]
\[ (2\pi)\Delta\zeta = \left( \frac{H}{f_\pi} \right)^2 \]

\[ H_{\text{inflation}} \]
Non-Gaussanity

Effective action for goldstone contains interactions:

\[ S_{\pi}^{\text{int}} = \int d^4 x \sqrt{-g} \left[ M_2^4 \left( \dot{\pi}^3 - \dot{\pi} \left( \frac{\partial_i \pi}{a^2} \right)^2 \right) + M_3^4 \dot{\pi}^3 + \ldots \right] \]

Interactions give rise to non-Gaussian correlators

These coefficients are model dependent

Gaussian correlation functions as \( H \to 0 \)

(holding the coefficients fixed)
Goldstone can also interact with other fields:

\[ S^{\text{mix}} = \int d^4 x \sqrt{-g} \left[ (-2 \dot{\pi} + \partial_{\mu} \pi \partial^{\mu} \pi) O + \ldots \right] \]

All field with \( m \lesssim H \) are excited during inflation.

We observe the “decays to \( \pi \)”
Non-Gaussanity

What is the point?

Non-Gaussanity tests particle physics at the scale $H$

Probes self-interactions of the “inflaton”

Sensitive to any extra degrees of freedom (e.g. we can test for SUSY at these scales)  

This can be a very high scale:  $H \lesssim 10^{14} \text{ GeV}$
Limits after Planck
Planck Bounds

Most constraints are on the 3-point function

Constraint given in terms of individual templates

\[ \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = B(k_1, k_2, k_3)(2\pi)^2 \delta^3(k_1 + k_2 + k_3) \]

For a given template, bound \( f_{NL} \equiv \frac{5}{18} \frac{B(k, k, k)}{P_{\zeta}(k)^2} \)

With this definition: non-gaussian = \( f_{NL} \sim 10^5 \)
Planck Bounds

Planck reports limits on 3 templates:

\[ f_{\text{NL}}^{\text{local}} = 2.7 \pm 5.8 \quad (68\% \text{ C.I.}) \]

Peaked at:
\[ k_1 \ll k_2 \sim k_3 \]

Courtesy of Fergusson & Shellard
Planck reports limits on 3 templates:

\[ f_{\text{NL}}^{\text{equil}} = -42 \pm 75 \ (68\% \text{ C.I.}) \]

Peaked at:
\[ k_1 = k_2 = k_3 \]

Courtesy of Fergusson & Shellard
Planck reports limits on 3 templates:

Peaked at:
\[ k_1 = k_2 = k_3 \]
\&
\[ k_1 = k_2 = \frac{1}{2} k_3 \]

\[ f_{\text{ortho}}^{\text{NL}} = -25 \pm 39 \text{ (68\% C.I.)} \]
Planck Bounds

Common sentiments:

‘Bounds on NG (strongly?) favor a simple mechanism’

‘Data has ruled out exotic models’

Are these statements true?

Is there a model-independent expectation for the size of NG in non-slow roll models?
Implications for Inflation
In single-field Inflation:

NG constrains self-interactions of $\pi$

Soft pion theorems: $f_{\text{NL}}^{\text{local}} = 0$ (aka consistency condition)

Use other bounds like precision electroweak tests

I.e. Bound scale of “new physics” $\mathcal{L} \supset \frac{1}{\Lambda^2} \dot{\pi}_c^3$
Constrain energy of interactions:

\[ \mathcal{L} \supset \frac{1}{\Lambda^{\Delta-4}} \mathcal{O}_\Delta \]

\[ f_\pi = 57H \]

\[ \Lambda \approx \]

\[ H_{\text{inflation}} \]
Single-Field Inflation

The primary constraint comes from equilateral:

\[ \mathcal{L}_3 \supset \frac{1}{\Lambda_1^2} \frac{\pi_c}{a^2} \left( \tilde{\partial} \pi_c \right)^2 \frac{1}{\Lambda_2^2} \dot{\pi}_c^3 \]

\[ f_{\text{NL}}^{\text{equil.}} \]

\[ \frac{85}{324} \left( \frac{2 \pi \Delta_\zeta}{\Lambda_1^2} \right)^{-1} \frac{H^2}{\Lambda_1^2} \frac{20}{729} \left( \frac{2 \pi \Delta_\zeta}{\Lambda_2^2} \right)^{-1} \frac{H^2}{\Lambda_2^2} \]

Planck (68%)

\[ \Lambda_1 \gtrsim 3.5 H \]

\[ \Lambda_2 \gtrsim 1.1 H \]
The primary constraint comes from equilateral:

\[ L_3 \supset \frac{c_1}{f^2_{\pi}} \frac{\pi_c}{a^2} \left( \tilde{\partial} \pi_c \right)^2 \]

\[ \frac{c_2}{f^2_{\pi}} \pi_c^3 \]

\[ f_{\text{equil.}} \]

\[ \frac{85}{324} c_1 \]

\[ \frac{20}{729} c_2 \]

Planck (68%)

\[ c_1 = 30 \pm 280 \quad c_2 = 690 \pm 2100 \]
Single-Field Inflation

Places lower bound on “strong coupling scale”

\[ f_\pi = 57H \]

\[ \sqrt{4\pi} \Lambda_{1,2} \gtrsim (4 - 12) H \]

Energy

Background

Strong Coupling

Freeze-out
What would we expect from slow roll?

\[ \mathcal{L} = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + \frac{1}{\Lambda^4} (\partial_\mu \phi \partial^\mu \phi)^2 \]

For this to be slow-roll: \( \Lambda^2 > \dot{\phi} \)

In slow-roll, we have a bound on equilaterial

\[ f_{\text{equil.}}^{\text{NL}} = \frac{\dot{\phi}^2}{\Lambda^4} < 1 \]
What would we expect from slow roll?

Strong Coupling

\[ \Lambda > \dot{\phi}^{1/2} \]
\[ \dot{\phi}^{1/2} = 57H \]

Freeze-out

\[ H_{\text{inflation}} \]
Single-Field Inflation

Long way to go before data suggests slow-roll

Energy

Strong Coupling

Background

Freeze-out

\[ \Lambda_{1,2} > f_\pi \]

Requires order 10-100 improvement

\[ \sqrt{4\pi \Lambda_{1,2}} \gtrsim (4 - 12) H \]
Multi-field Inflation

Planck constraints still have teeth:
Strong bounds on mixing between sectors

E.g. from slow-roll we might have

\[ \mathcal{L} \supset \frac{1}{\Lambda} (\partial_\mu \phi \partial^\mu \phi) \sigma \]

Planck bounds from local shape \( (f_{\text{NL}}^{\text{local}}) \):

\[ \Lambda \gtrsim 5 \times 10^4 H \]

DG et al.;
Assassi et al.
Multi-field Inflation

Planck constraints still have teeth:
Strong bounds on mixing between sectors

E.g. from slow-roll we might have

\[ \mathcal{L} \supset \frac{1}{\Lambda} (\partial_\mu \phi \partial^\mu \phi) \sigma \]

Planck bounds from local shape \( f_{\text{NL}}^{\text{local}} \):

\[ \Lambda \gtrsim 0.5 \left( \frac{r}{0.01} \right)^{1/2} M_{\text{pl}} \]

DG et al.; Assassi et al.
Generalization

Limits on NG bound couplings between sectors

\[ \mathcal{L} \supset \frac{1}{\Lambda^{\Delta}} (\partial_{\mu} \phi \partial^{\mu} \phi) \mathcal{O}_{\Delta} \]

For moderately NG hidden sectors

\[ \Lambda \gtrsim (10^5)^{1/\Delta} H \]

Origin of the constraint largely insensitive to details

Related to single field bounds when \( \Delta \gtrsim 4 \)
What is the Goal?
What is the Goal?

Back to the sentiments:

‘Bounds on NG (strongly?) favor a simple mechanism’

‘Data has ruled out exotic models’

It seems (to me) like there is a big window left

Can we think of something “exotic”?
Could Inflation be due to strong dynamics?

Energy

Background/ Strong Coupling ??

Freeze-out

\[ f_\pi \sim \Lambda \sim 57H \]

\[ H_{\text{inflation}} \]
Could Inflation be due to strong dynamics? i.e. Is there an analogue of technicolor (or QCD)?

Time translation broken by composite operator

\[ \langle O \rangle = f_{\pi}^{\Delta+1} \times t \]

If the only scale is \( f_{\pi} \), we might expect

\[ \mathcal{L} \supset \frac{O(1-10)}{f_{\pi}^2} \dot{\pi} (\partial \pi)^2 \rightarrow f_{\text{equil.}}^\text{NL} \lesssim 5 \quad ?? \]

\[ (\Delta f_{\text{NL}}^{\text{equil.}})_{\text{Planck}} = 75 \]
Here are some goals:

Single-field slow-roll is ruled out for

\[ f_{\text{NL}}^{\text{equil.}} > 1 \]

A null result at this level would be very informative
(A detection would be spectacular!)

Single field is ruled out with any detection of

\[ f_{\text{NL}}^{\text{local}} > 0 \]

Always useful to improve these bounds
Non-Gaussanity is high energy particle physics

Tests particles and interactions at $H \lesssim 10^{14}$ GeV

Well defined threshold exists for equilateral:

$$f_{\text{NL}}^{\text{equil.}} \sim 1$$

Requires a measurement of the bispectrum in LSS

(much more work is needed but the data will be there!)