m_c, m_b, and α_s
Lattice status and prospects

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Parametric uncertainties in BSM searches

Channel	M_H [GeV]	Γ [MeV]	$\Delta\alpha_s$	Δm_b	Δm_c	Δm_t	THU
$H \to bb$	122	2.30	−2.3% / +2.3%	+3.2% / −3.2%	+0.0% / −0.0%	+0.0% / −0.0%	+2.0% / −2.0%
	126	2.36	−2.3% / +2.3%	+3.3% / −3.2%	+0.0% / −0.0%	+0.0% / −0.0%	+2.0% / −2.0%
	130	2.42	−2.4% / +2.3%	+3.2% / −3.2%	+0.0% / −0.0%	+0.0% / −0.0%	+2.0% / −2.0%
$H \to \tau^+\tau^-$	122	$2.51 \cdot 10^{-1}$	+0.0% / +0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.1%	+2.0% / −2.0%
	126	$2.59 \cdot 10^{-1}$	+0.0% / +0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.1% / −0.1%	+2.0% / −2.0%
	130	$2.67 \cdot 10^{-1}$	+0.0% / +0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.1% / −0.1%	+2.0% / −2.0%
$H \to \mu^+\mu^-$	122	$8.71 \cdot 10^{-4}$	+0.0% / +0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.1% / −0.1%	+2.0% / −2.0%
	126	$8.99 \cdot 10^{-4}$	+0.0% / +0.0%	+0.0% / −0.0%	−0.1% / −0.0%	+0.0% / −0.1%	+2.0% / −2.0%
	130	$9.27 \cdot 10^{-4}$	+0.1% / +0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.1% / −0.0%	+2.0% / −2.0%
$H \to c\bar{c}$	122	$1.16 \cdot 10^{-1}$	−7.1% / +7.0%	−0.1% / −0.1%	+6.2% / −6.0%	+0.0% / −0.1%	+2.0% / −2.0%
	126	$1.19 \cdot 10^{-1}$	−7.1% / +7.0%	−0.1% / −0.1%	+6.2% / −6.1%	+0.0% / −0.1%	+2.0% / −2.0%
	130	$1.22 \cdot 10^{-1}$	−7.1% / +7.0%	−0.1% / −0.1%	+6.3% / −6.0%	+0.1% / −0.1%	+2.0% / −2.0%
$H \to gg$	122	$3.25 \cdot 10^{-1}$	+4.2% / −4.1%	−0.1% / −0.1%	+0.0% / −0.0%	−0.2% / +0.2%	+3.0% / −3.0%
	126	$3.57 \cdot 10^{-1}$	+4.2% / −4.1%	−0.1% / −0.1%	+0.0% / −0.0%	−0.2% / +0.2%	+3.0% / −3.0%
	130	$3.91 \cdot 10^{-1}$	+4.2% / −4.1%	−0.1% / −0.2%	+0.0% / −0.0%	−0.2% / +0.2%	+3.0% / −3.0%
$H \to \gamma\gamma$	122	$8.37 \cdot 10^{-3}$	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+1.0% / −1.0%
	126	$9.59 \cdot 10^{-3}$	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+1.0% / −1.0%
	130	$1.10 \cdot 10^{-2}$	+0.1% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+1.0% / −1.0%
$H \to Z\gamma$	122	$4.74 \cdot 10^{-3}$	+0.0% / −0.1%	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.1%	+5.0% / −5.0%
	126	$6.84 \cdot 10^{-3}$	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.1%	+0.0% / −0.1%	+5.0% / −5.0%
	130	$9.55 \cdot 10^{-3}$	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+5.0% / −5.0%
$H \to WW$	122	$6.25 \cdot 10^{-1}$	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.5% / −0.5%
	126	$9.73 \cdot 10^{-1}$	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.5% / −0.5%
	130	1.49	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.5% / −0.5%
$H \to ZZ$	122	$7.30 \cdot 10^{-2}$	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.5% / −0.5%
	126	$1.22 \cdot 10^{-1}$	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.5% / −0.5%
	130	$1.95 \cdot 10^{-1}$	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.0% / −0.0%	+0.5% / −0.5%

Uncertainties in standard model parameters limit possible precision in searches for new physics.

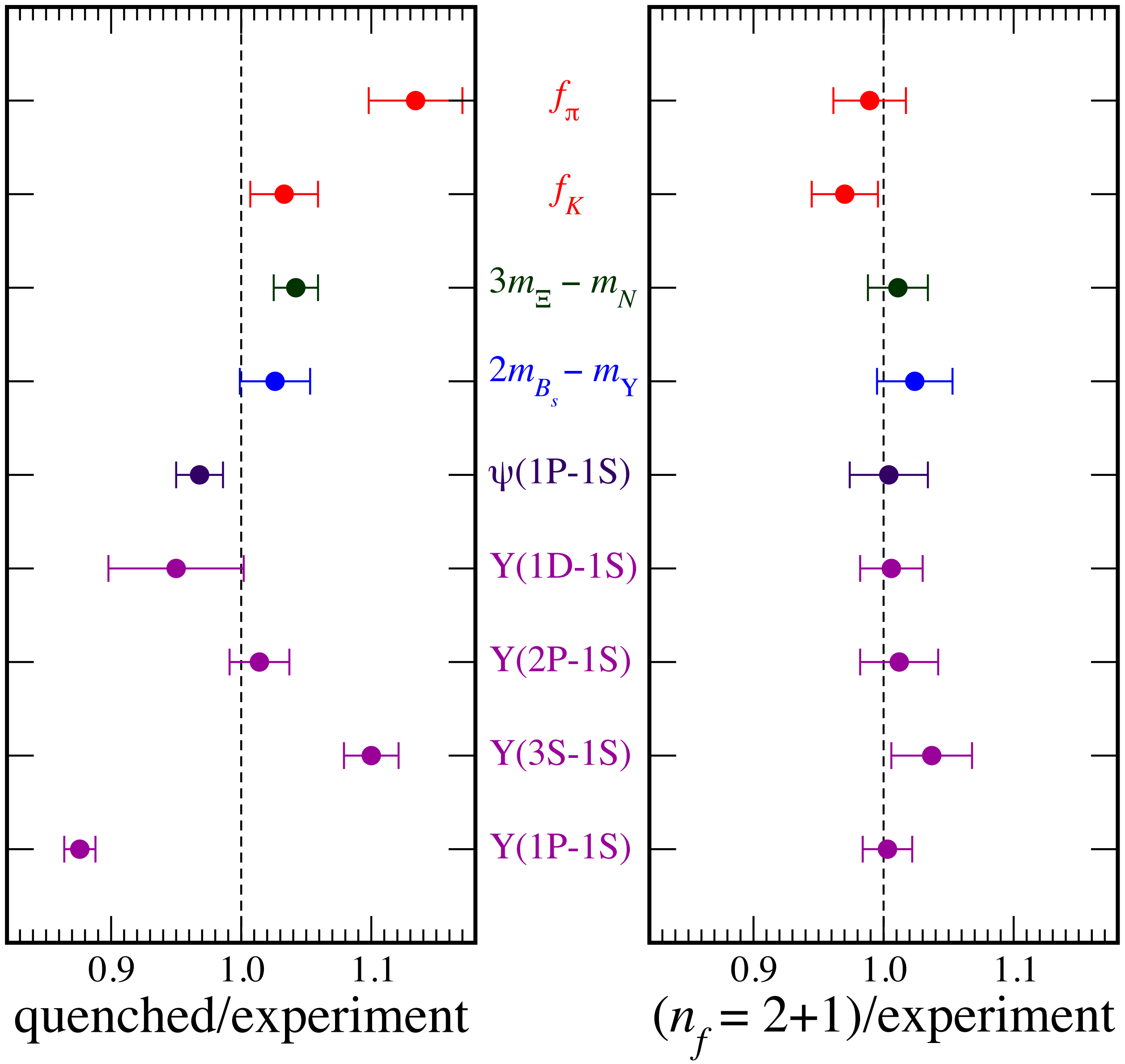
Partial widths into $b\bar{b}$, $c\bar{c}$, and gg are more dependent on parametric uncertainties than on other theory. Since the total width is dominated by the $b\bar{b}$ channel, almost all branching fractions are strongly dependent on m_b, as well.

Lattice QCD can provide the most precise determinations of the parameters α_s, m_c, and m_b.

arXiv:1307.1347v1

Table 1: SM Higgs partial widths and their relative parametric (PU) and theoretical (THU) uncertainties for a selection of Higgs masses. For PU, all the single contributions are shown. For these four columns, the upper percentage value (with its sign) refers to the positive variation of the parameter, while the lower one refers to the negative variation of the parameter.

Lattice in the 21st century



Phys.Rev.Lett. 92 (2004) 022001

Lattice/experiment without (L) and with (R) quark-antiquark pairs.

For the past ~ten years, it has been possible to use lattice QCD Monte Carlo methods to calculate simple quantities with understood error budgets that are complete, including the effects of quark-antiquark pairs.

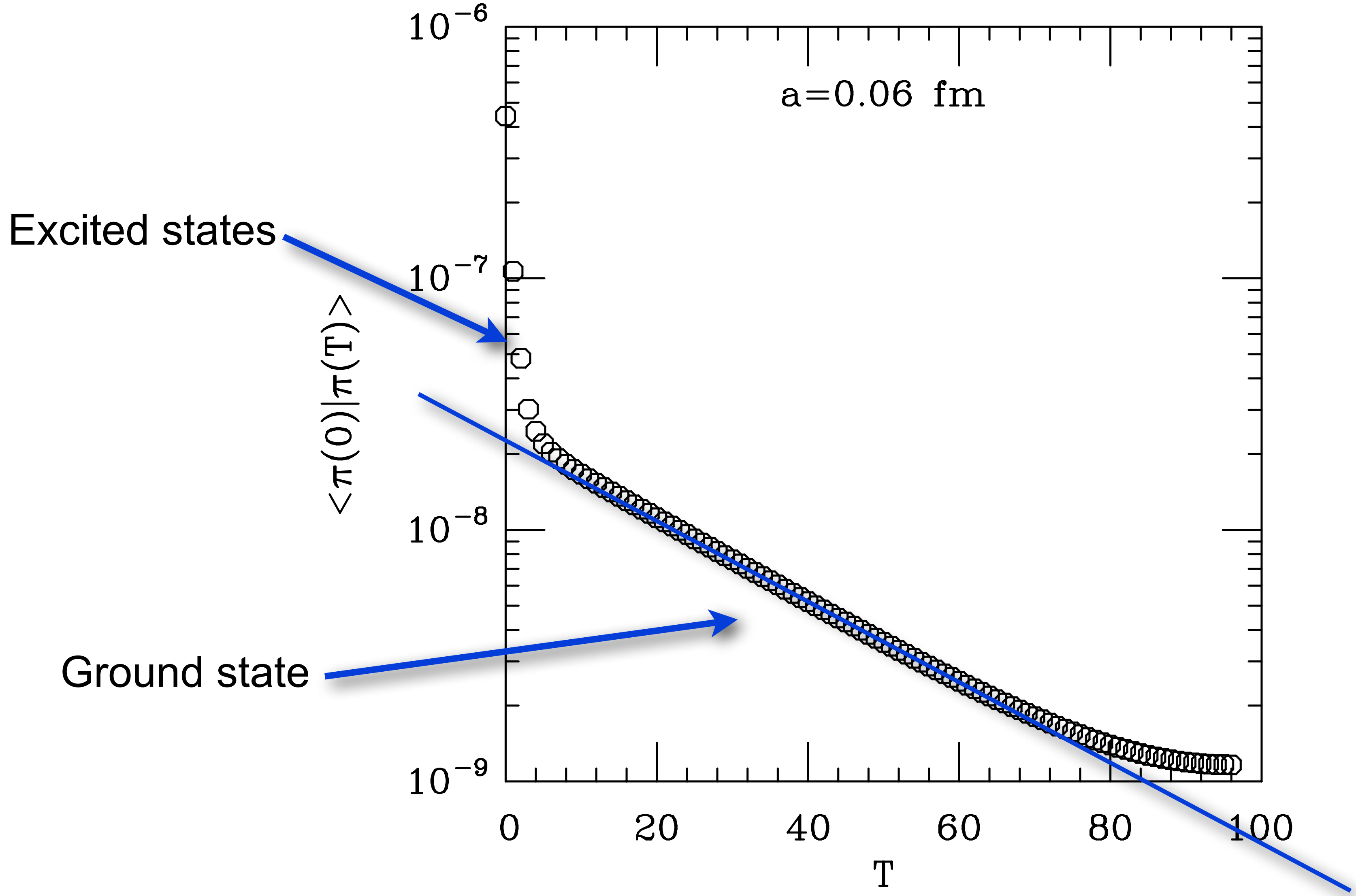
What is “simple”?

- Simplest: stable mesons.
  - Over the last ten years, many key quantities. Hadronically stable mesons, especially:
    - Heavy and light meson decay constants,
    - Semileptonic decays,
    - Meson-antimeson mixing.
  - Make possible important determinations of 8 CKM matrix elements, 5 quark masses, the strong coupling constant.
- Now: $\pi\pi$ systems, nucleons

Coming US experimental program

- Next five years: lattice calculations are needed *throughout* the entire future US experimental program.
  - g-2
  - mu2e, LBNE, Nova: nucleon matrix elements.
  - Underground LBNE: proton decay matrix elements.
  - LHCb, Belle-2: continued improvement of CKM results
  - LHC, Higgs decays: lattice provides the most accurate α_s and m_c now, and m_b in the future

How?

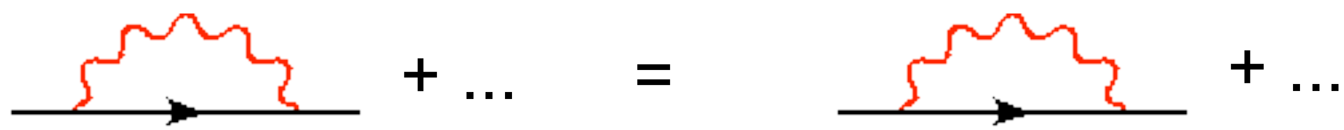


$$\langle \overline{\psi}\gamma_5\psi(t=0) \mid \overline{\psi}\gamma_5\psi(t)\rangle = C\exp(-Mt) + \text{excited states.}$$

If the two quarks were a *u* and a $\overline{u}$, the slope would give M_π, *C* would be proportional to $F_\pi{}^2$.

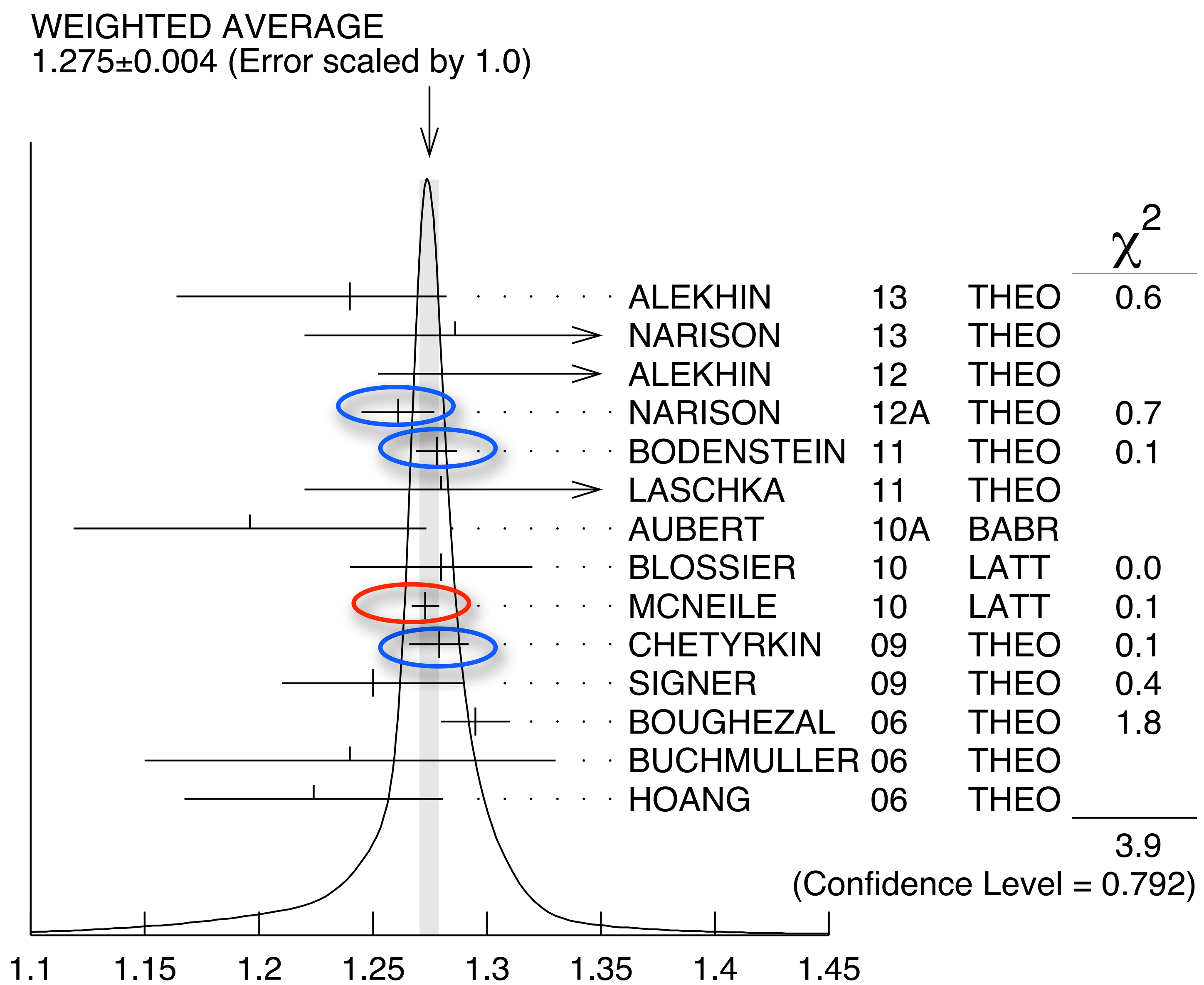
To obtain α_s, m_c, or m_b via the lattice

- In principle,
  - can get $m_{\overline{\mathrm{MS}}}$ from m_{latt} by equating Green's functions calculated in perturbation theory in the two regulators:



- In practice,
  - Calculating short-distance quantities to third order perturbation theory is hard and messy.
  - Calculating some short-distance quantities nonperturbatively is easy and clean.
- The art of determining α_s or m_q via the lattice is finding a quantity as easy to calculate as possible
  - with continuum perturbation theory, *and*
  - nonperturbatively with the lattice.

m_c

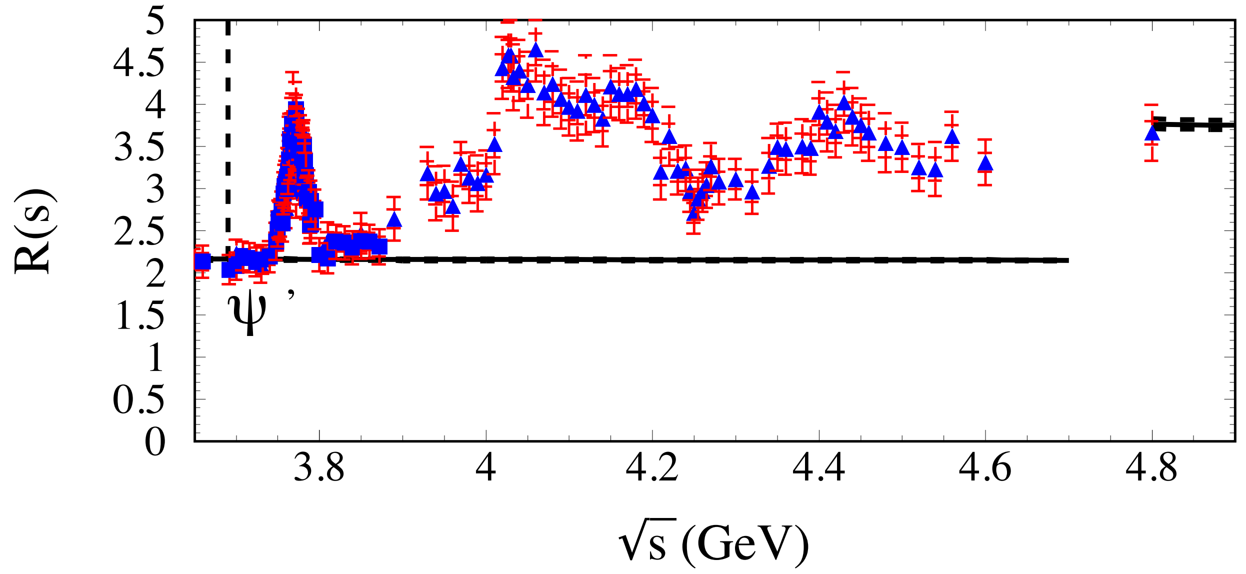


PDG, Beringer et al., 2013.

The most precise non-lattice determinations of m_c use e^+e^- annihilation data and ITEP sum rules. (Karlsruhe group, Chertyrkin et al.)

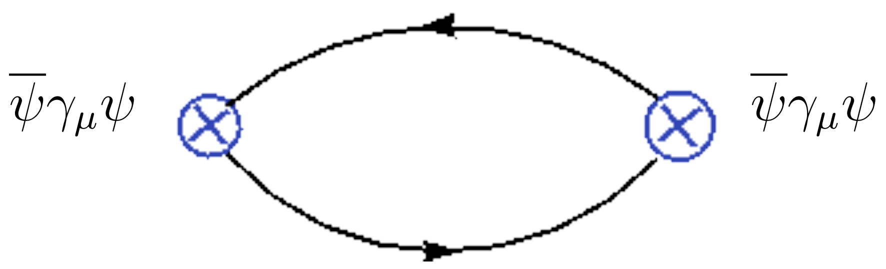
Recent lattice determination of HPQCD uses the same type of perturbation theory, but lattice QCD to supply the correlation functions rather than experiment.

$e^+e^- \to m_c$



$$\mathcal{M}_n \equiv \int \frac{ds}{s^{n+1}} R_Q(s)$$

Moments of the heavy quark production cross section in e^+e^- annihilation can be related to the derivatives of the vacuum polarization at q^2=0.

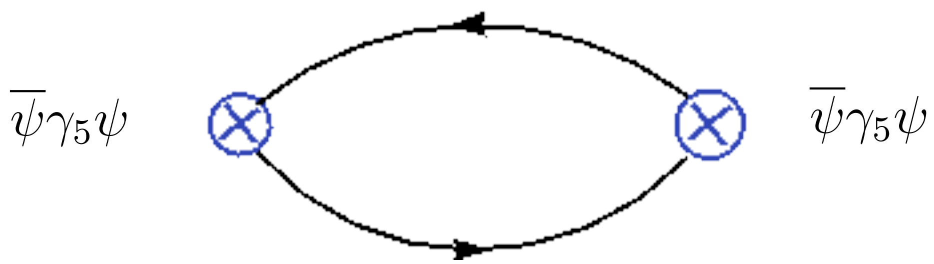


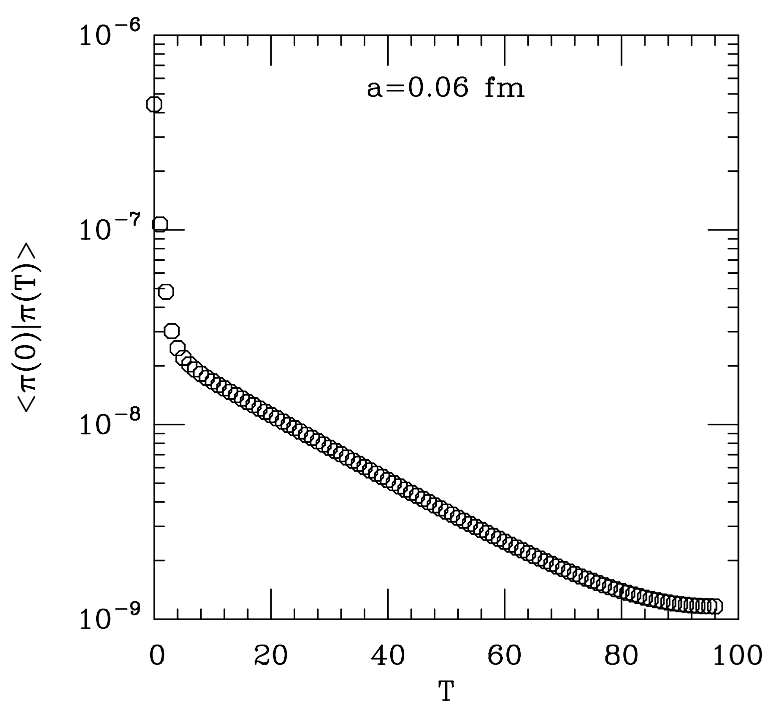
$$\mathcal{M}_n = \frac{12\pi^2}{n!}\left(\frac{d}{dq^2}\right)^n \Pi_Q(q^2)|_{q^2=0}$$

Can be calculated in perturbation theory.
Known to O(α_s^3) (Chetyrkin et. al.)

Lattice QCD

can also compute such correlation functions with high accuracy.





Correlation functions of all currents can be calculated in perturbation theory (and with the lattice). The most precise m_c can be obtained by choosing the one that is most precise on the lattice: the pseudoscalar correlator.

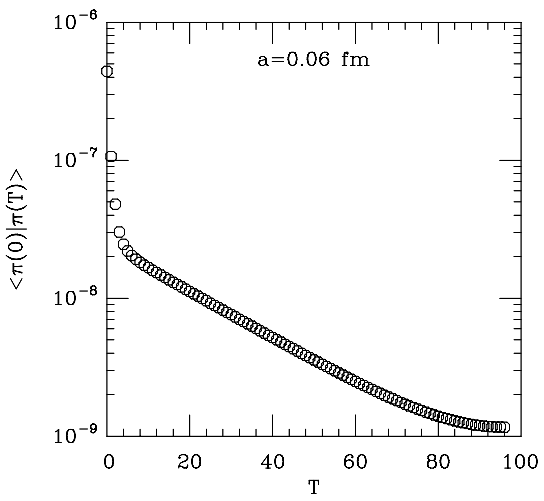
$$G(t) = a^6 \sum_{\mathbf{x}} (am_{0h})^2 \langle 0 | j_5(\mathbf{x}, t) j_5(0,0) | 0 \rangle,$$

$$G_n \equiv \sum_t (t/a)^n G(t),$$

Perturbation theory to α_s^3 from the Karlsruhe group.

Technical tricks to make the lattice calculation more precise

Choose pseudoscalar (easiest) current correlator. (Easier to calculate than a pion or charmonium mass.)

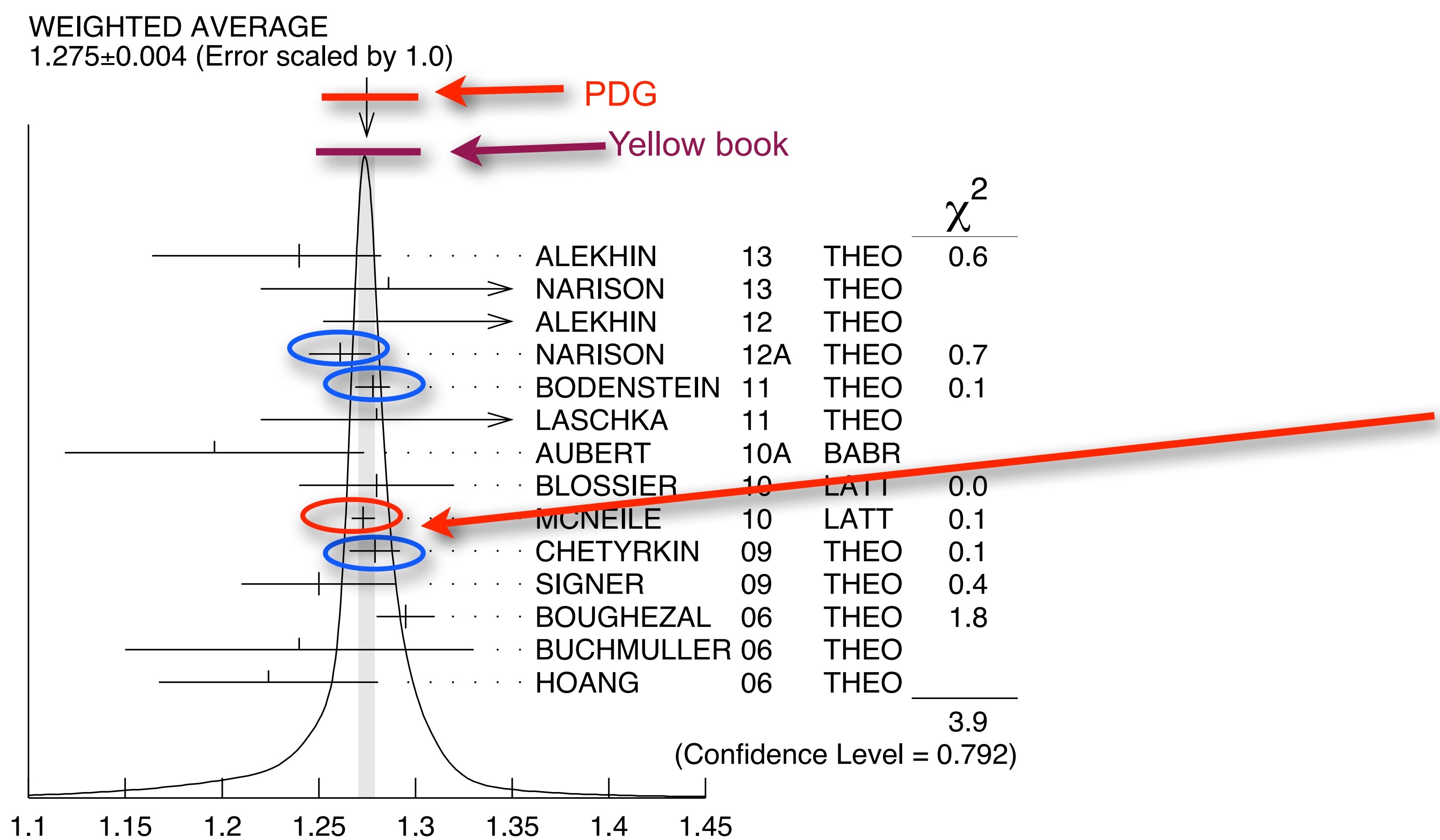


In matching perturbative and nonperturbative results, divide both by the tree level correlator. (Removes leading discretization errors.)

In the lattice calculation of, for example, the charm correlator, use $M\eta_c$ as experimental input to set the energy scale. (Removes sensitivity to the tuning of the lattice mass used.)

$$R_n \equiv \begin{cases} G_4/G_4^{(0)} & \text{for } n = 4 \\ \frac{am_{\eta_h}}{2am_{0h}} (G_n/G_n^{(0)})^{1/(n-4)} & \text{for } n \geq 6 \end{cases}$$

m_c results



PDG, Beringer et al., 2013.

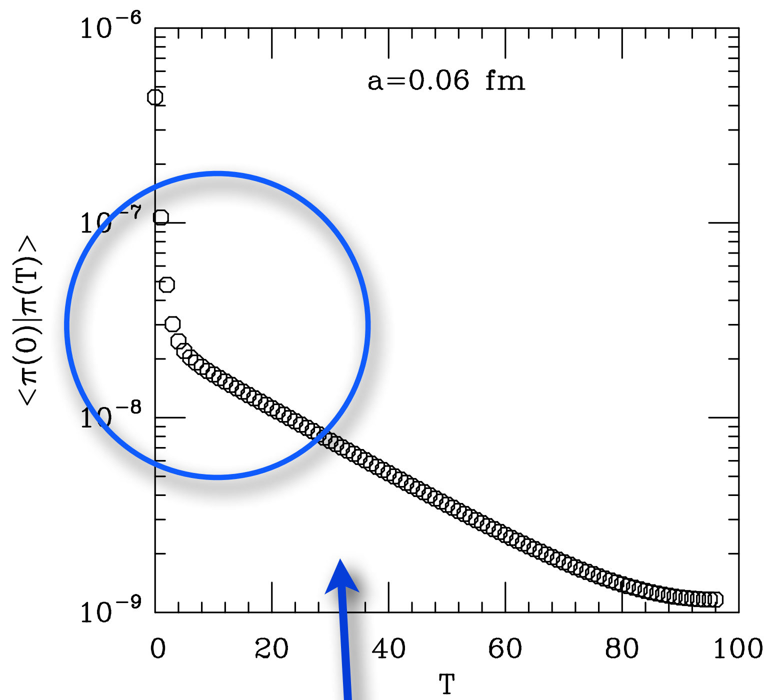
	$m_c(3)$
a^2 extrapolation	0.2%
Perturbation theory	0.5
Statistical errors	0.1
m_h extrapolation	0.1
Errors in r_1	0.2
Errors in r_1/a	0.1
Errors in m_{η_c}, m_{η_b}	0.2
α_0 prior	0.1
Gluon condensate	0.0
Total	0.6%

$$m_c(m_c, n_f = 4) = 1.273(6)\ \text{GeV}$$

HPQCD, McNeile et al.

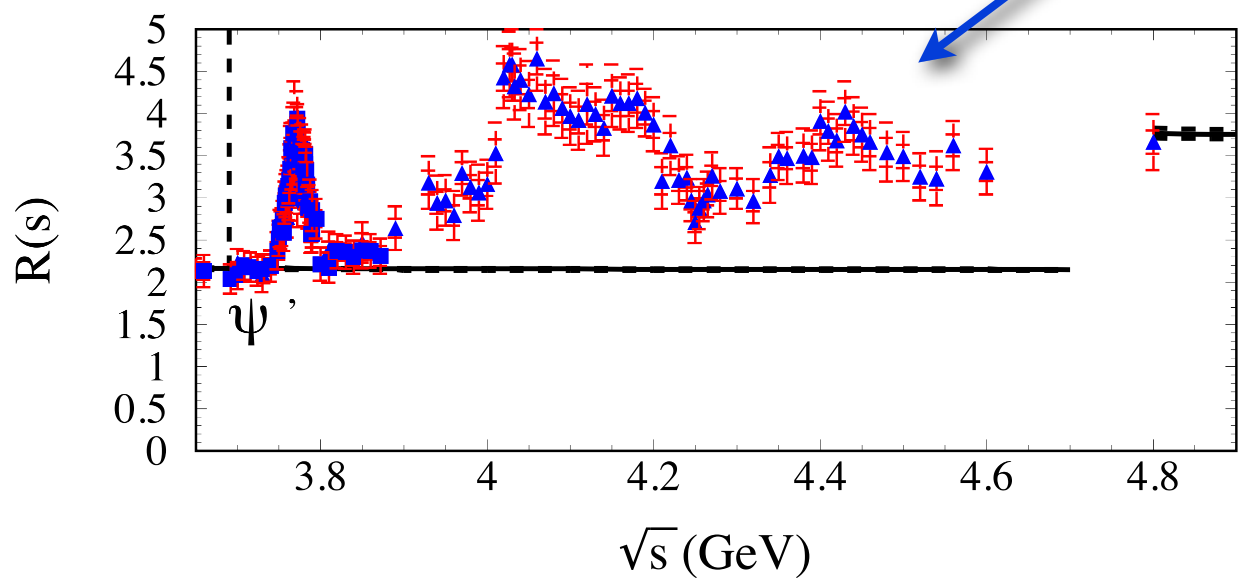
Uncertainty is dominated by the same perturbation theory used in all of the most precise results.

Why can lattice determinations of m_c from correlation functions be more precise than those from e^+e^-?

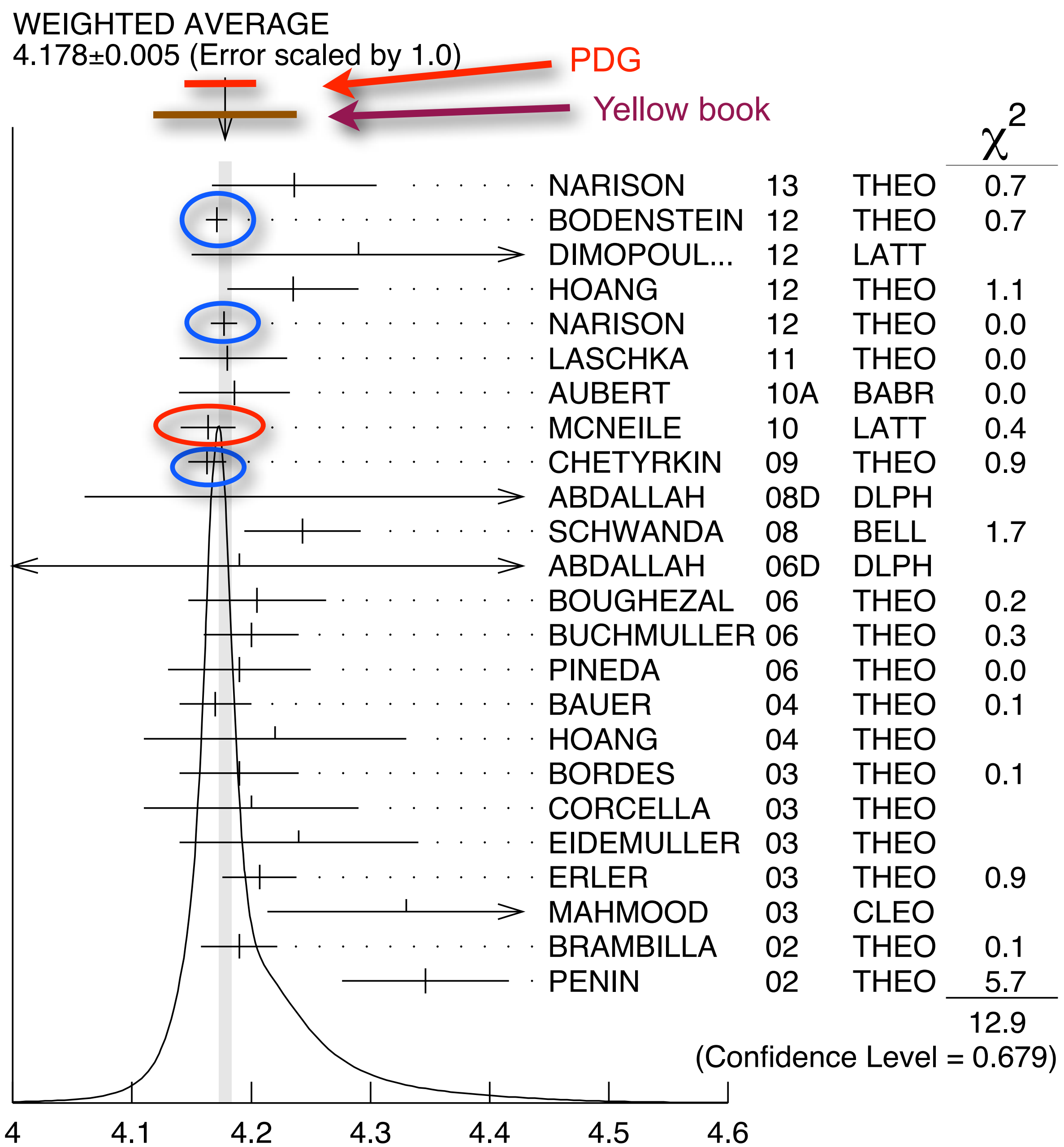


Moments of correlation functions are even easier than what I earlier told you have been considered the easiest quantities for the last ten years.
We need the correlation functions at finite *T*, and not their asymptotic form at large *T*.

Because this is cleaner data than this.



m_b



PDG, Beringer et al., 2013.

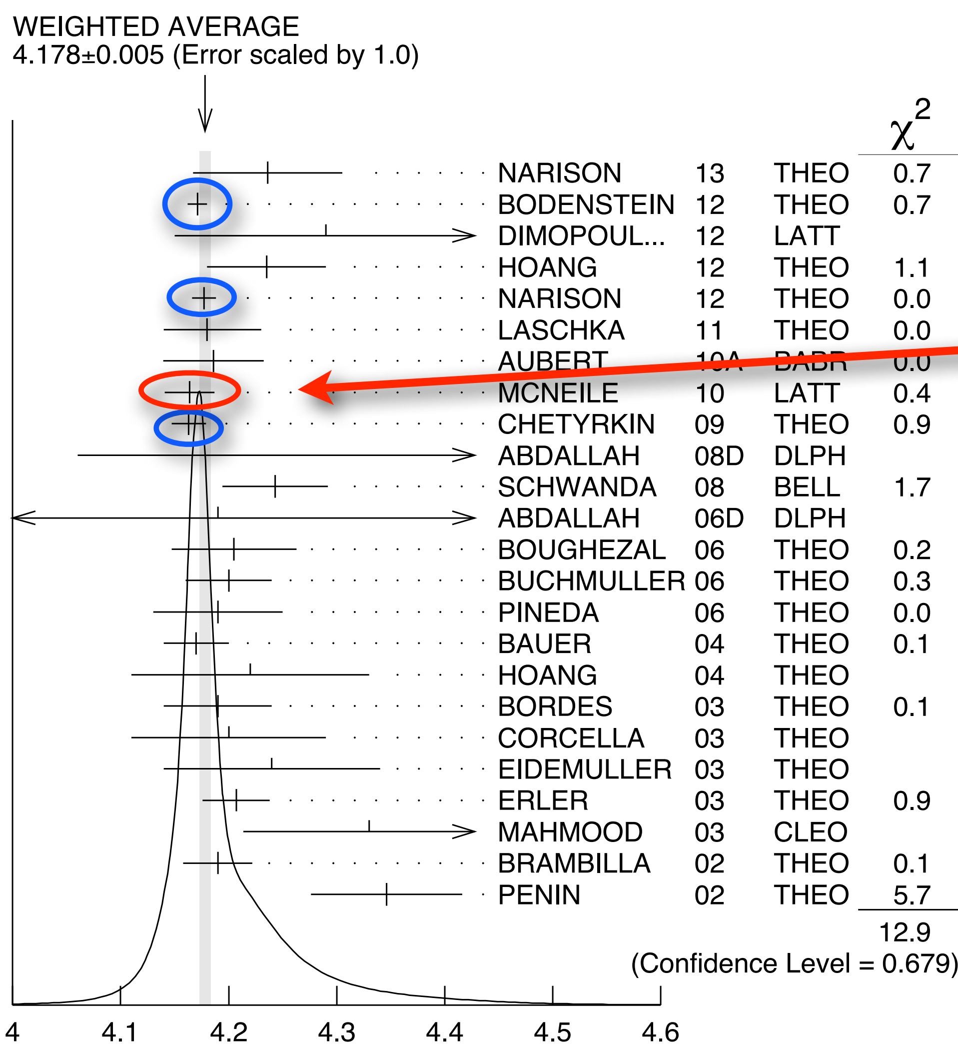
The most precise non-lattice determinations of m_c use e^+e^- annihilation data and ITEP sum rules. (Karlsruhe group, Chertyrkin et al.)

Recent lattice determination uses the same type of perturbation theory, but lattice QCD to supply the correlation functions rather than experiment.

For m_b, perturbative errors are tiny. $(\alpha(m_b)^4 << \alpha(m_c)^4.)$

m_b results

For m_b, these lattice correlator methods are just barely working at a=0.045 fm. (They treat the b as light compared with 1/a.)
Need a=0.03 fm to be comfortable.
Discretization errors and statistics dominate current uncertainties. Both can be attacked with brute force computing power.
Needed configurations are projected to be generated in the next few years.



	$m_b(10)$
a^2 extrapolation	0.6%
Perturbation theory	0.1
Statistical errors	0.3
m_h extrapolation	0.1
Errors in r_1	0.1
Errors in r_1/a	0.3
Errors in m_{η_c}, m_{η_b}	0.1
α_0 prior	0.1
Gluon condensate	0.0
Total	0.7%

$$m_b(m_b, n_f = 5) = 4.164(23)\ \text{GeV}$$

m_b results

For m_b, these lattice correlator methods are just barely working at a=0.045 fm. (They treat the b as light compared with 1/a.)
Need a=0.03 fm to be comfortable.
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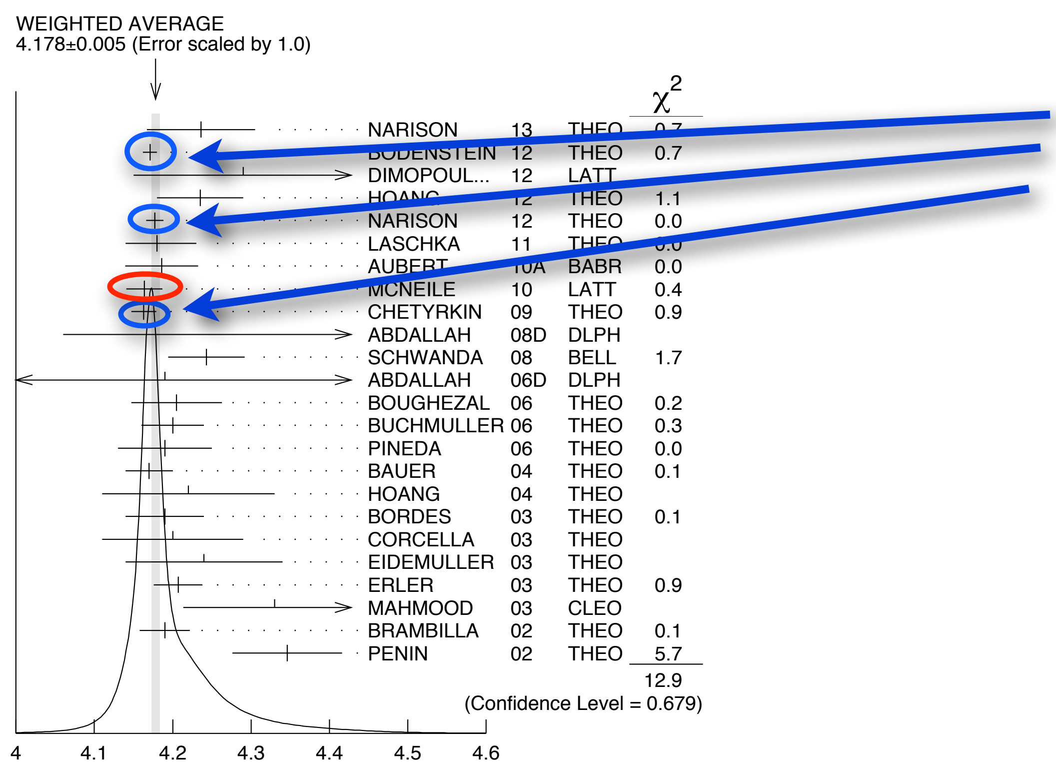
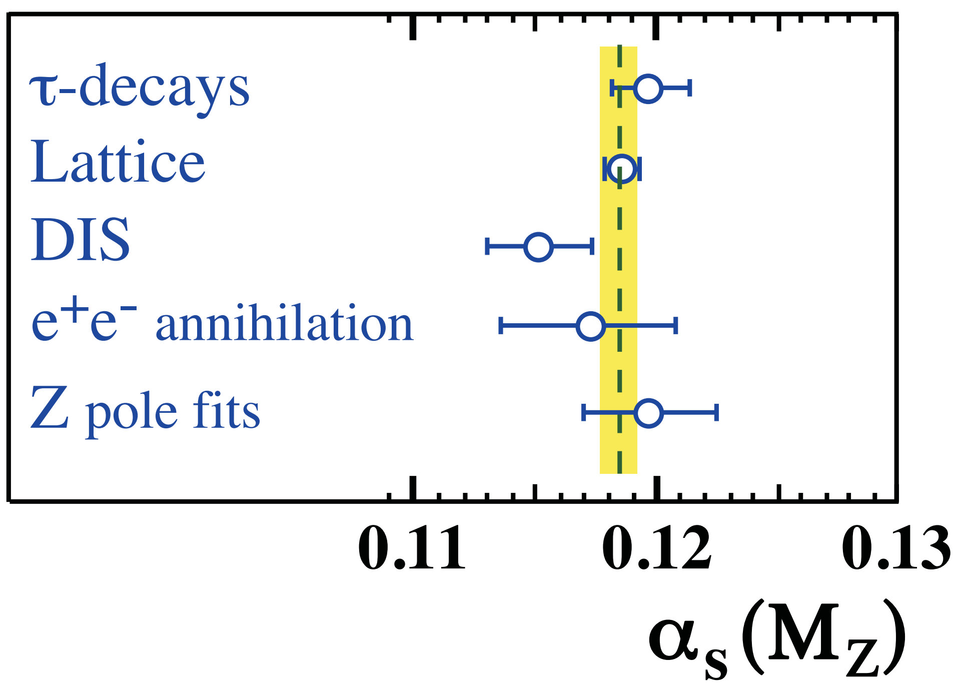


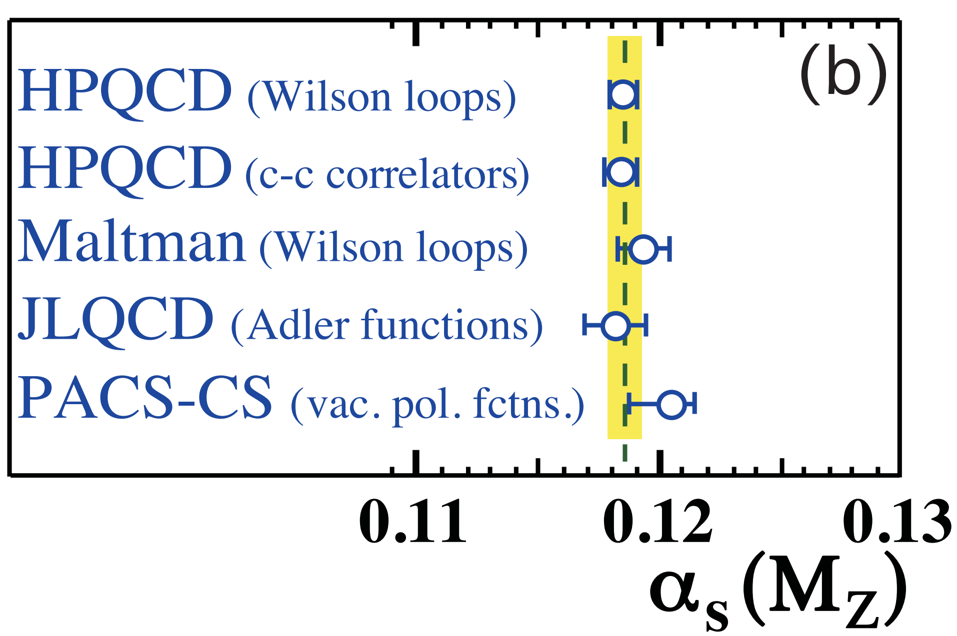
The three most precise determinations of mb using moments of e^+e^- data arrive at different estimates of the precision.

Coming lattice calculations should be able to confirm (or not) the more more precise claims.
Unlike m_c, where the lattice and e^+e^- determinations share the same perturbation theory, perturbative uncertainties are neglible and the lattice and e^+e^- determinations will have totally independent uncertainties.

α_s



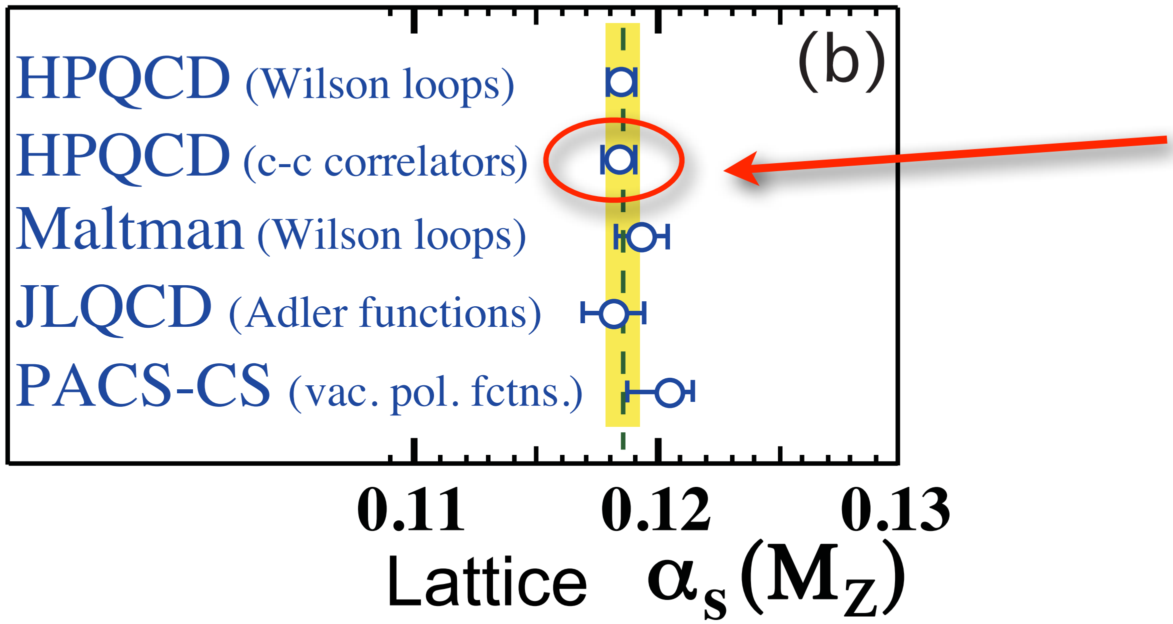
There are multiple ways of determining α_s, both with and without the lattice.



There are several lattice determinations equal to or more precise than all the non-lattice determinations together.

PDG, QCD review, 2012.

α_s results: correlator method

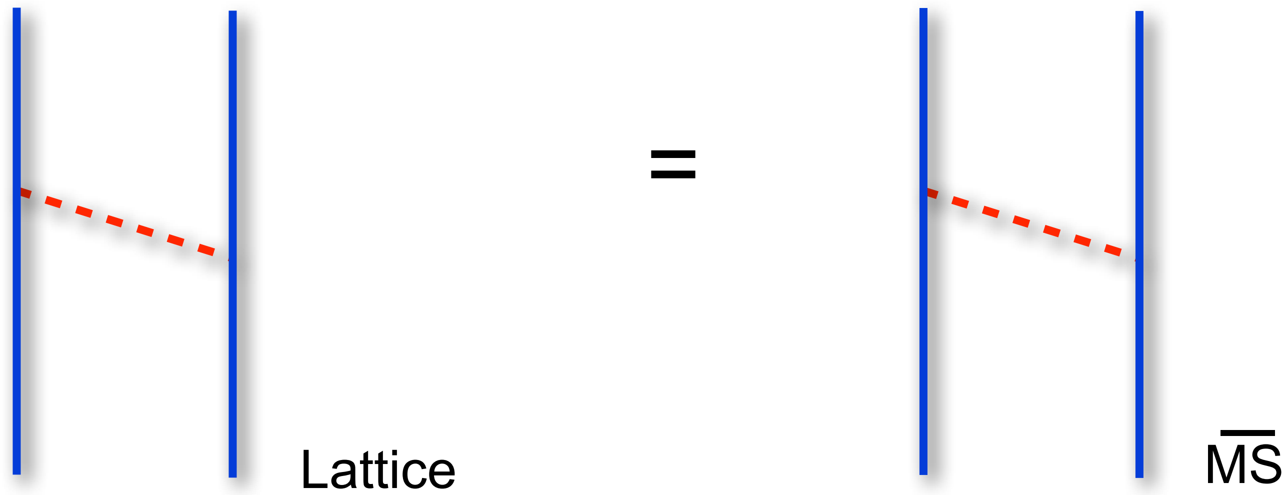


	$\alpha_{\overline{MS}}(M_Z)$
a^2 extrapolation	0.2%
Perturbation theory	0.4
Statistical errors	0.2
m_h extrapolation	0.0
Errors in r_1	0.1
Errors in r_1/a	0.1
Errors in m_{η_c}, m_{η_b}	0.0
α_0 prior	0.1
Gluon condensate	0.2
Total	0.6%

Results are dominated by perturbation theory. May by hard to improve without next term in perturbation theory.

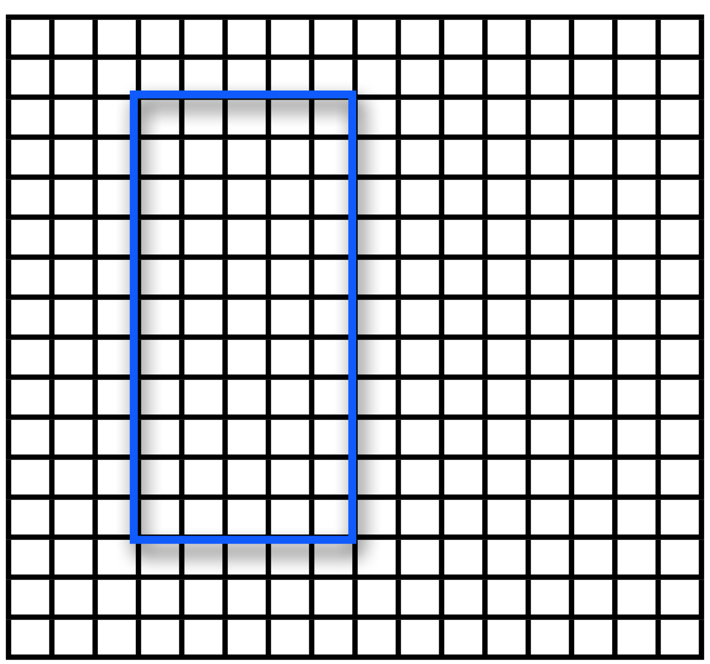
α_s results: Wilson loops

α_s can be determined with lattice calculations of many other quantities, e.g., the heavy quark potential.



to obtain α_s $\overline{\mathrm{MS}}$.

Lattice calculates the heavy quark potential from Wilson loops.



HPQCD has determined α_s directly from Wilson loops.

Result compatible with their correlator result, similar precision:
$\alpha_s = 0.1184(6)$, but
totally different uncertainties, heavy use of lattice perturbation theory.

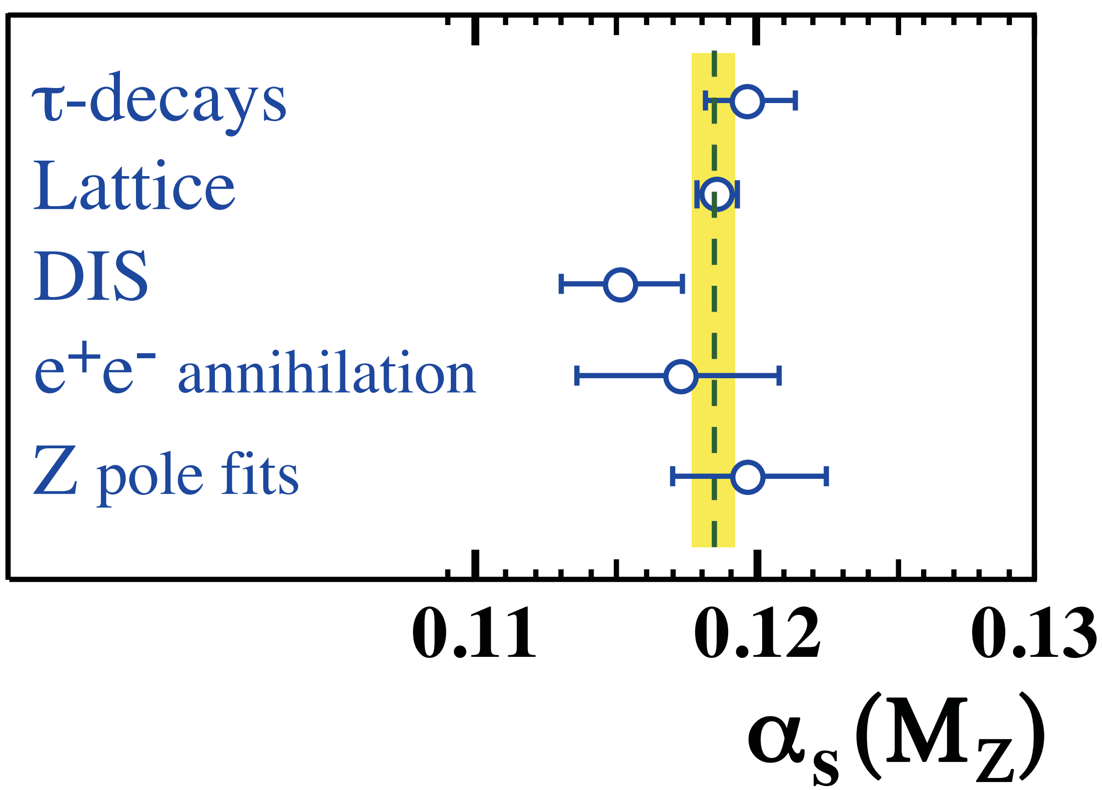
α_s, other lattice results

There are numerous good ways of determining α_s using lattice QCD.

- The Adler function, JLQCD. Phys.Rev. D82 (2010) 074505.
  - $\alpha_s = 0.1181 \pm 0.0003{+}0.0014{-}0.0012$
- The Schrödinger functional, PACS-CS. JHEP 0910:053,2009.
  - $\alpha_s = 0.1205(8)(5)(+0/–17)$
- The ghost-gluon vertex, European Twisted Mass Collaboration (ETM). Phys.Rev.Lett. 108 (2012) 262002.
  - $\alpha_s = 0.1200(14)$

PDG

2012, combined the lattice numbers in a weighted average.
It takes a combined error of the most precise of the inputs.



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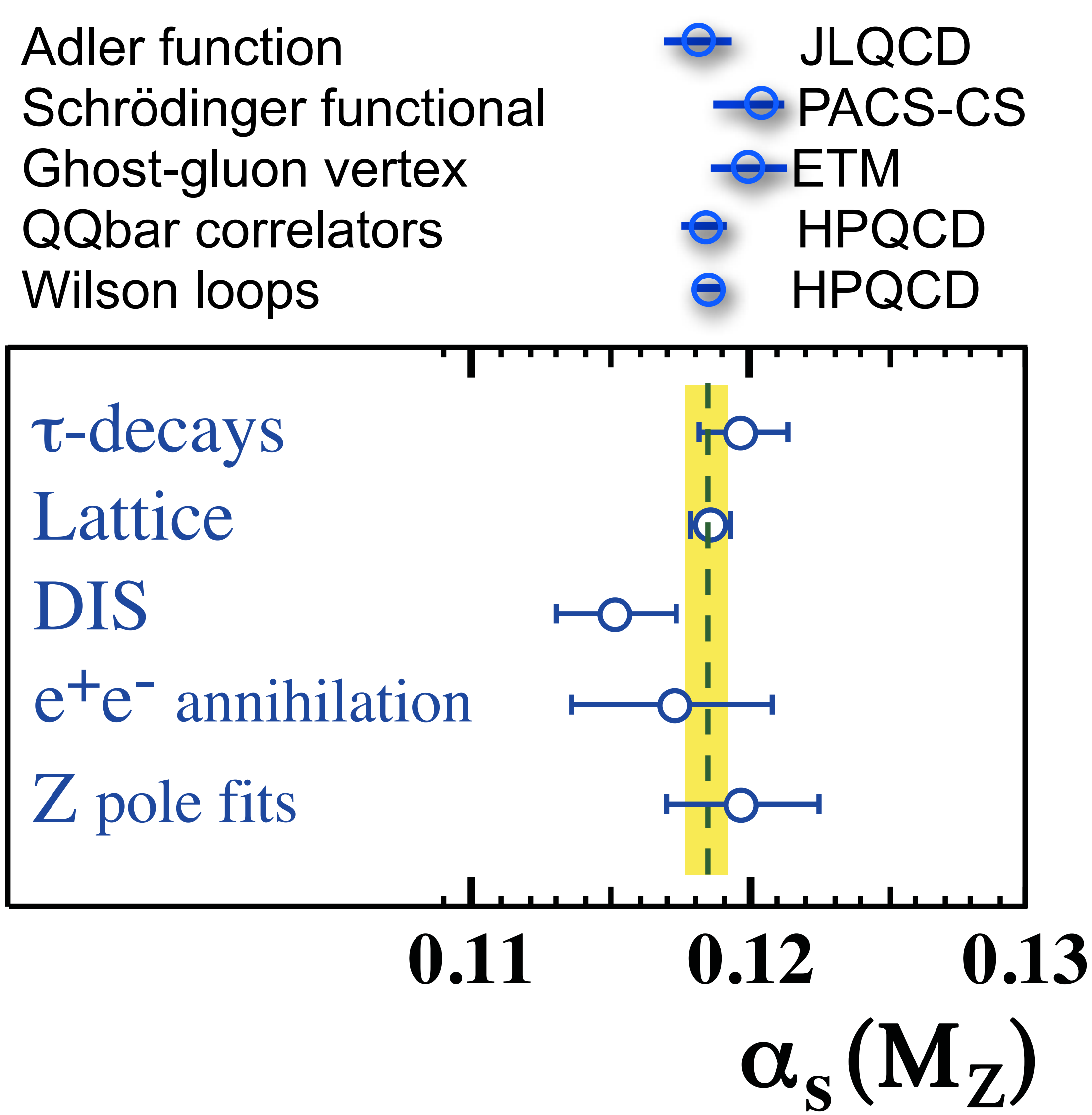

The lattice results (2013) are dominated by the two most precise results from HPQCD, but there are several other lattice results from Europe and Japan, all of which agree with each other and each which is more precise than any non-lattice result.

Prospects: m_c and m_b

- Correlator methods are currently the most precise, both with
- e^+e^- and with lattice methods.
- For m_c, correlator moments are simple to calculate on the lattice
  - Should be checkable by many lattice groups.
  - Results should be of comparable precision to determinations from e^+e^-.
  - Uncertainty will be dominated by perturbation theory.
- For m_b, most precise lattice determination relies on treating *b* quark as light compared to $1/a$.
  - Possible with HISQ fermions, may be hard for other lattice methods.
  - The lattice result should catch up to the most precise of the e^+e^- results with more CPU power.
  - The resulting uncertainties in the e^+e^- determinations and the lattice determinations will be totally independent of each other (unlike the case for m_c); perturbative uncertainty is negligible.

Prospects: α_s

- The uncertainties of the Wilson loop and correlator determinations of α_s are dominated by perturbation theory and will improve somewhat, but probably not dramatically.
- α_s can be determined well from lattice calculations of many different quantities. There is likely to be continued improvement in the apparent robustness of the lattice results as more quantities are calculated with increasing precision.
- As of now there are results from
  - five different quantities,
  - four different groups on three continents,
  - four different fermion discretizations.
  - Results are completely independent and consistent, and each is more precise than the most precise non-lattice determination.

Treatment of parametric uncertainties in Higgs physics

Current discussions of Higgs branching fractions and partial widths use very conservative estimates of parametric precisions.

Table 1: Input parameters and their relative uncertainties, as used for the uncertainty estimation of the branching ratios. The masses of the central values correspond to the 1-loop pole masses, while the last column contains the corresponding $\overline{\mathrm{MS}}$ mass values.

Parameter	Central value	Uncertainty	$\overline{\mathrm{MS}}$ masses $m_q(m_q)$
$\alpha_s(M_Z)$	0.119	± 0.002	
m_c	1.42 GeV	± 0.03 GeV	1.28 GeV
m_b	4.49 GeV	± 0.06 GeV	4.16 GeV
m_t	172.5 GeV	± 2.5 GeV	165.4 GeV

arXiv:1201.3084v1 [hep-ph] 15 Jan 2012

	Higgs X-Section WG	PDG	lattice	Karlsruhe (e^+e^-)	world non-lattice
$\delta\ \alpha_s$	0.002	0.0007	0.0007		0.0012
$\delta\ m_c$ (GeV)	0.03	0.025	0.006	0.013	
$\delta\ m_b$ (GeV)	0.06	0.03	0.023	0.016	

Should interpret as 1 σ errors.

Level of conservatism in assumed uncertainties that is appropriate depends on circumstances, e.g., on whether you're discussing with a postdoc where something funny might be going on or whether you're discussing with the New York Times.

What to expect

- m_c: Uncertainty in leading lattice result will improve somewhat. Correlator moments will be calculated by a number of lattice groups with competing methods. Uncertainty will be dominated by perturbation theory.
- α_s: Uncertainty in leading lattice result will improve somewhat. α_s will be determined by a number of lattice groups using competing methods. Each will be more precise than all the non-lattice determinations put together.
- m_b: The precision of the best lattice result will improve by a factor of two or more, matching the most precise claimed uncertainties from e^+e^-. Uncertainties from lattice and e^+e^- will have nothing to do with each other.

What to expect

P	PDG 2013	Lattice 2013	δ P 2018	Corroboration 2018
m_c	1.275(25) GeV	1.273(6) GeV	<0.006 GeV	Many lattice calculations of the charm moments will exist with completely independent uncertainties.
α_s	0.1184(7)	0.1184(6)	<0.0006	Many lattice calculations of the charm moments will exist with completely independent uncertainties. Many different lattice determinations using different quantities will exist with precisions approaching this value and completely independent uncertainties.
m_b	4.18(3) GeV	4.164(23) GeV	<0.011 GeV	Lattice result and the most precise e+e- results will agree (?) within stated precisions, with completely independent uncertainties.

Conclusion

- Lattice calculations now provide the most precise determinations of α_s and m_c. They soon will also provide the most precise determination of m_b.

- People who wish to really be serious about understanding the partial widths of the Higgs will have to try to understand them

Backup slides

Perturbative coefficients for moments

TABLE III. Perturbation theory coefficients ($n_f = 3$) for r_n [2–6]. Coefficients are defined by $r_n = 1 + \sum_{j=1} r_{nj} \alpha^j_{\overline{\mathrm{MS}}}(\mu)$ for $\mu = m_h(\mu)$. The third-order coefficients are exact for $4 \leq n \leq 10$. The other coefficients are based upon estimates; we assign conservative errors to these.

n	r_{n1}	r_{n2}	r_{n3}
4	0.7427	−0.0577	0.0591
6	0.6160	0.4767	−0.0527
8	0.3164	0.3446	0.0634
10	0.1861	0.2696	0.1238
12	0.1081	0.2130	0.1(3)
14	0.0544	0.1674	0.1(3)
16	0.0146	0.1293	0.1(3)
18	−0.0165	0.0965	0.1(3)

HPQCD take uncalculated coefficients in series for moments $r_{nj} \sim O(0.5\ \alpha_s(m_q)^j)$;
further constrain the possible sizes for coefficients by comparing nonperturbative results for many quark masses with perturbation theory using Baysian priors for higher order terms.