Electroweak Phase Transition:

Towards a gauge-independent approach

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SNOWMASS Workshop 2013

Hiren Patel
hhpatel@wisc.edu

THE UNIVERSITY of WISCONSIN MADISON

THEORETICAL NUCLEAR, PARTICLE, ASTROPHYSICS AND COSMOLOGY

Agenda

- Address issue of gauge-invariance
  Propose possible resolution

  HP, MJRM
  arXiv: 1101.4665 (2011)

- Application to novel patterns of EW symmetry breaking

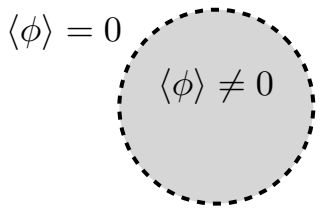
  HP, MJRM
  arXiv: 1212.5652 (2012)

  HP, MJRM, M. B. Wise
  arXiv: 1303.1140 (2013)

Electroweak Baryogenesis

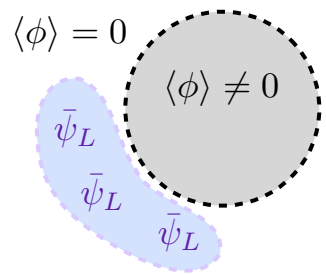
...in a nutshell

First order electroweak phase transition

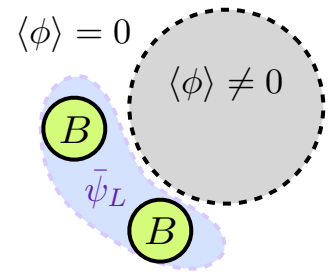


via bubble nucleation

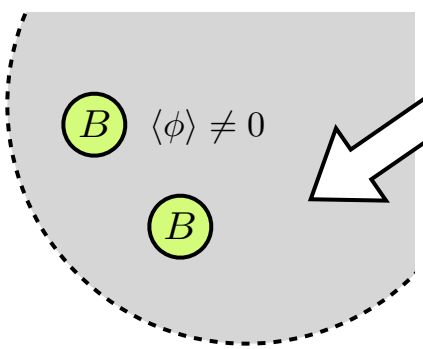
CP-violation:
Generation of particle/ antiparticle asymmetry



B-violation:
Generation of baryon asymmetry

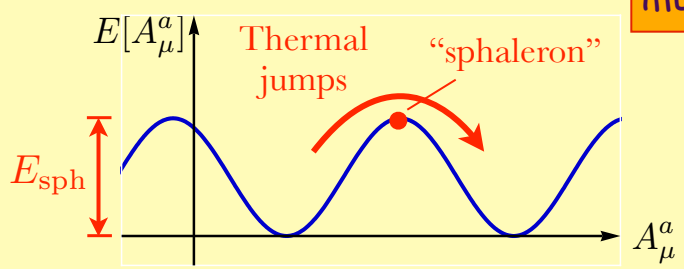


baryons captured, and preserved.



B+L-violating EW Sphalerons convert baryons back to antileptons.

Sphaleron proc. must be quenched!



Electroweak Baryogenesis

...in a nutshell

Kinetic theory: sphaleron rate related to its <u>mass</u> (energy)

$$\Gamma_{\text{sph.}} \sim (gT)^4 e^{-E_{\text{sph}}/T}$$

Sphaleron <u>mass</u> dependent on Higgs field value inside bubble

$$E_{\text{sph}} \sim 4\pi\langle\phi\rangle/g$$

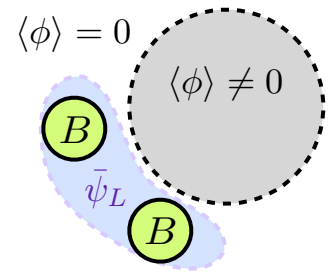
At phase transition, need ratio to be large.

$$\frac{\langle\phi\rangle}{T_c} \gtrsim 1$$

Baryon number preservation criterion on strength of phase transition.

<u>B-violation:</u>

Generation of baryon asymmetry



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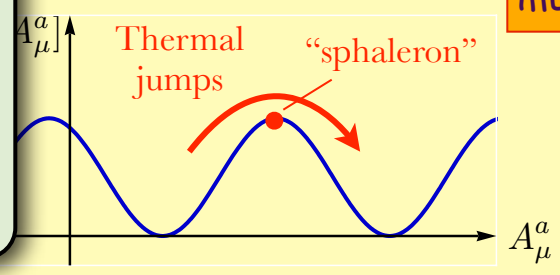

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Baryon number preservation criterion on strength of phase transition.

- Careful numerical analysis (lattice+molecular dynamics simulation) reveals that phase transition not strong enough (not even first order) at observed Higgs mass.
- Go beyond SM.
- Many unconstrained parameters
- Not feasible to do lattice simulations
- Resort to perturbation theory.

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<u>Problem</u>: This is gauge dependent

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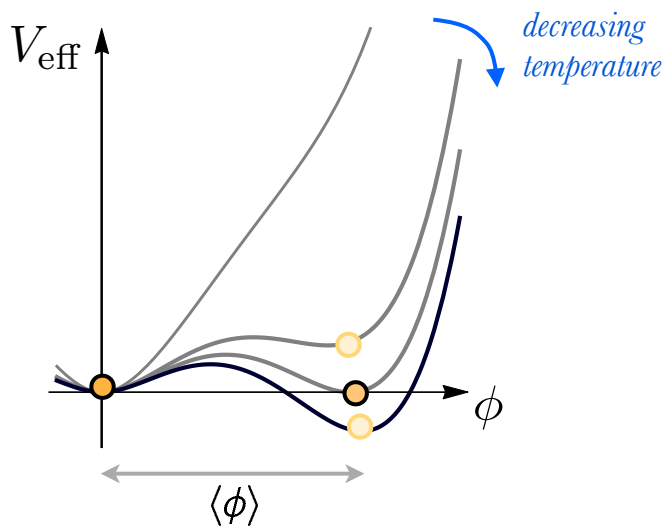
Computation of $\frac{\langle\phi\rangle}{T_c}$

1. Track evolution of minima in V_{eff} as function of temperature

2. Numerically solve minimization and degeneracy condition equations:

   1. $V'_{\text{eff}}(\phi_{\text{min}}, T_c) = 0$
   2. $V_{\text{eff}}(0, T_c) = V_{\text{eff}}(\phi_{\text{min}}, T_c)$



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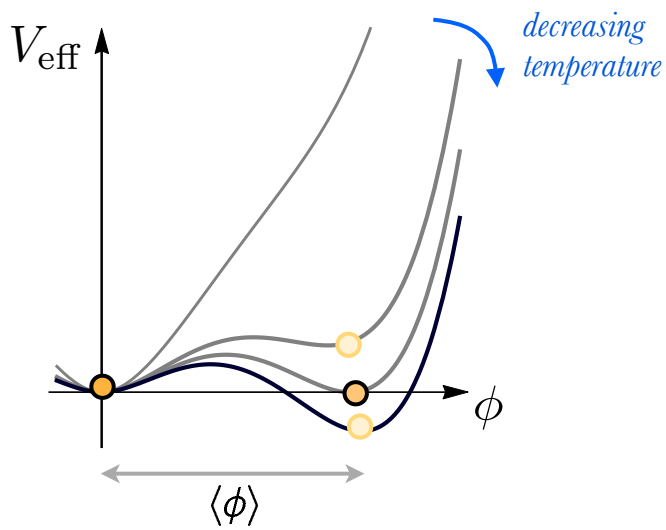
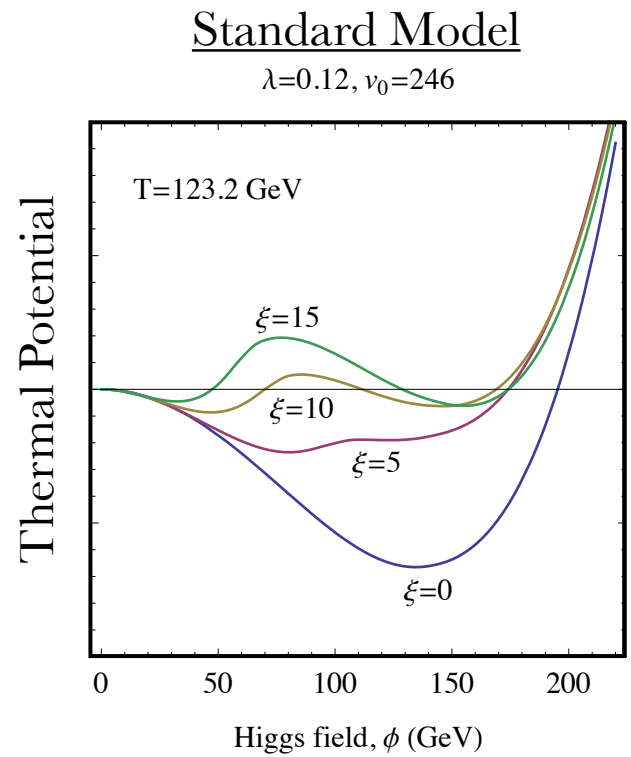


In a gauge theory, effective potential is gauge-dependent:



Computed T_c and $\langle\phi\rangle$ depends on gauge parameter!

Diagnosis and Resolution 1

HP, MJRM
arXiv: 1101.4665 (2011)

Determination of T_c (or T_N) $\rightarrow$ $\dfrac{\langle\phi\rangle}{T_c} \gtrsim 1$

~Diagnosis~

- Nielsen identity $\Longrightarrow$ T_c gauge-independent
- valid order-by-order in loop-expansion
- But, numerical solution to minimization condition

1. $V'_{\text{eff}}(\phi_{\text{min}}, T_c) = 0$
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leads to inconsistent truncation in loop-expansion!

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$$2. \quad V_{\rm eff}(0, T_c) = V_{\rm eff}(\phi_{\rm min}, T_c)$$

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~Resolution~

Minimize by an inversion of series

counts # of loops

$$\begin{cases} V(\phi, T) = V_0 + \hbar V_1 + \hbar^2 V_2 + \ldots \\ \phi_{\rm min} = \phi_0 + \hbar\phi_1 + \hbar^2\phi_2 + \ldots \end{cases}$$

Equation for ea. power of $\hbar$; yields $\phi_{\rm min}$.

Subs. into each side;

$$\boxed{\begin{aligned} V(\phi_{\rm min}, T) &= V_0(\phi_0) + \hbar V_1(\phi_0, T) \\ &+ \hbar^2\left[V_2(\phi_0, T, \xi) - \frac{1}{2}\phi_1^2(\xi)\frac{\partial^2 V_0}{\partial\phi^2}|_{\phi_0}\right] + \ldots \end{aligned}}$$

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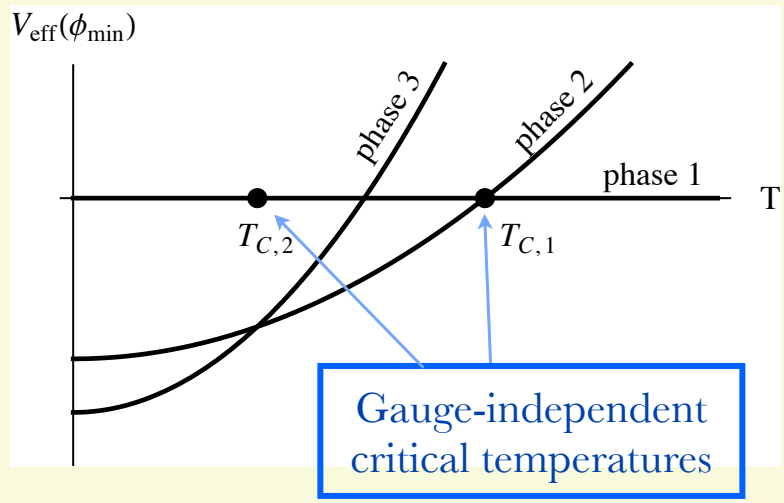
Expression gives gauge independent minima of the effective potential.



consistent with Nielsen identity.
(explicitly checked at 1-loop)

$$+\hbar^2\left[V_2(\phi_0,T,\xi) - \frac{1}{2}\phi_1^2(\xi)\frac{\partial^2 V_0}{\partial\phi^2}|_{\phi_0}\right] + \ldots$$

Diagnosis and Resolution 2

HP, MJRM
arXiv: 1101.4665 (2011)

Determination of $\langle\phi\rangle$. $\rightarrow$ $\dfrac{\langle\phi\rangle}{T_c} \gtrsim 1$

~Diagnosis~

- Nielsen identity $\not\Rightarrow$ $\langle\phi\rangle$ gauge-independent
- minimizing field is an inherently unphysical quantity.
- Sets the sphaleron energy scale.
- Nielsen identity applies to sphaleron energy.

Diagnosis and Resolution 2

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~Resolution~

1. Compute sphaleron energy based on <u>gauge-invariant</u> $\mathcal{O}(T^2)$ effective action.

$$\Gamma(T) \sim -\tfrac{1}{4}F^a_{ij}F^a_{ij} + (D\phi)^2 + V_0(\phi) + \alpha\phi^2T^2$$

2. Extract gauge-invariant scale from $\Gamma(T)$.

$$\bar{v}(T) = v_0\sqrt{1 - T^2/T_0^2}$$

Diagnosis and Resolution 2

HP, MJRM
arXiv: 1101.4665 (2011)

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Bottom line

Gauge-invariant baryon number preservation criterion:

$$\frac{\langle\phi\rangle}{T_c} \longrightarrow \frac{\bar{v}(T_c^{\text{G.I.}})}{T_c^{\text{G.I.}}}$$

1. Use gauge invariant sphaleron scale

2. Determine Tc gauge-invariantly

~Resolution~

1. Compute sphaleron energy based on gauge-invariant $\mathcal{O}(T^2)$ effective action.

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Application: *Novel pattern of Symmetry Breaking*

HP, MJRM
arXiv: 1212.5652
(2012)

Higgs portal-type model:

Higgs doublet	$H \sim (1, 2, \frac{1}{2})$
SU(2) real triplet	$\Sigma \sim (1, 3, 0)$

Physical states:

H^0	$m_H \approx 125$ GeV	(LHC)
$\Sigma^{\pm}, \ \Sigma^0$	$m_\Sigma > 100$ GeV	(LEP)

- Simplest extension with non-trivial SU(2) quantum numbers.

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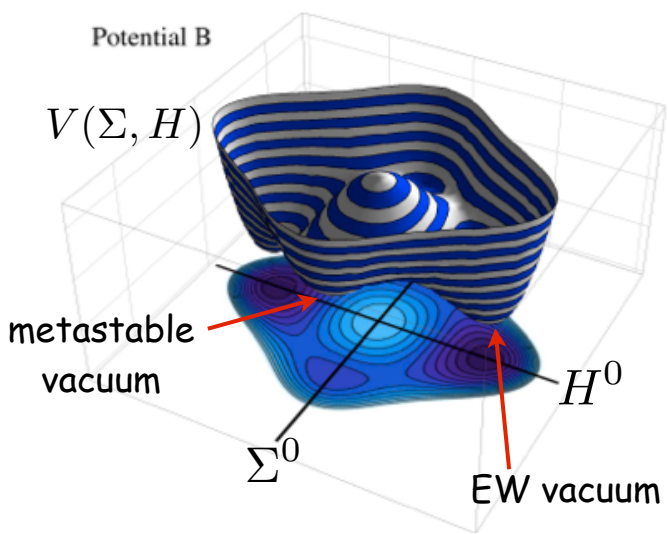
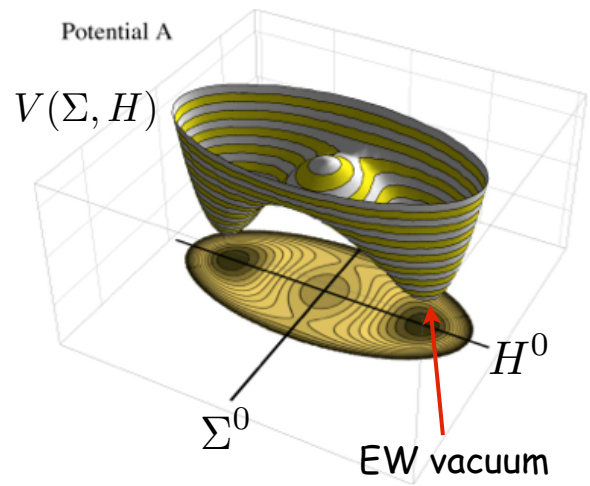
- EWPO ρ-parameter constraint: $\Rightarrow$ Two types of vacuum structures:





Early universe:

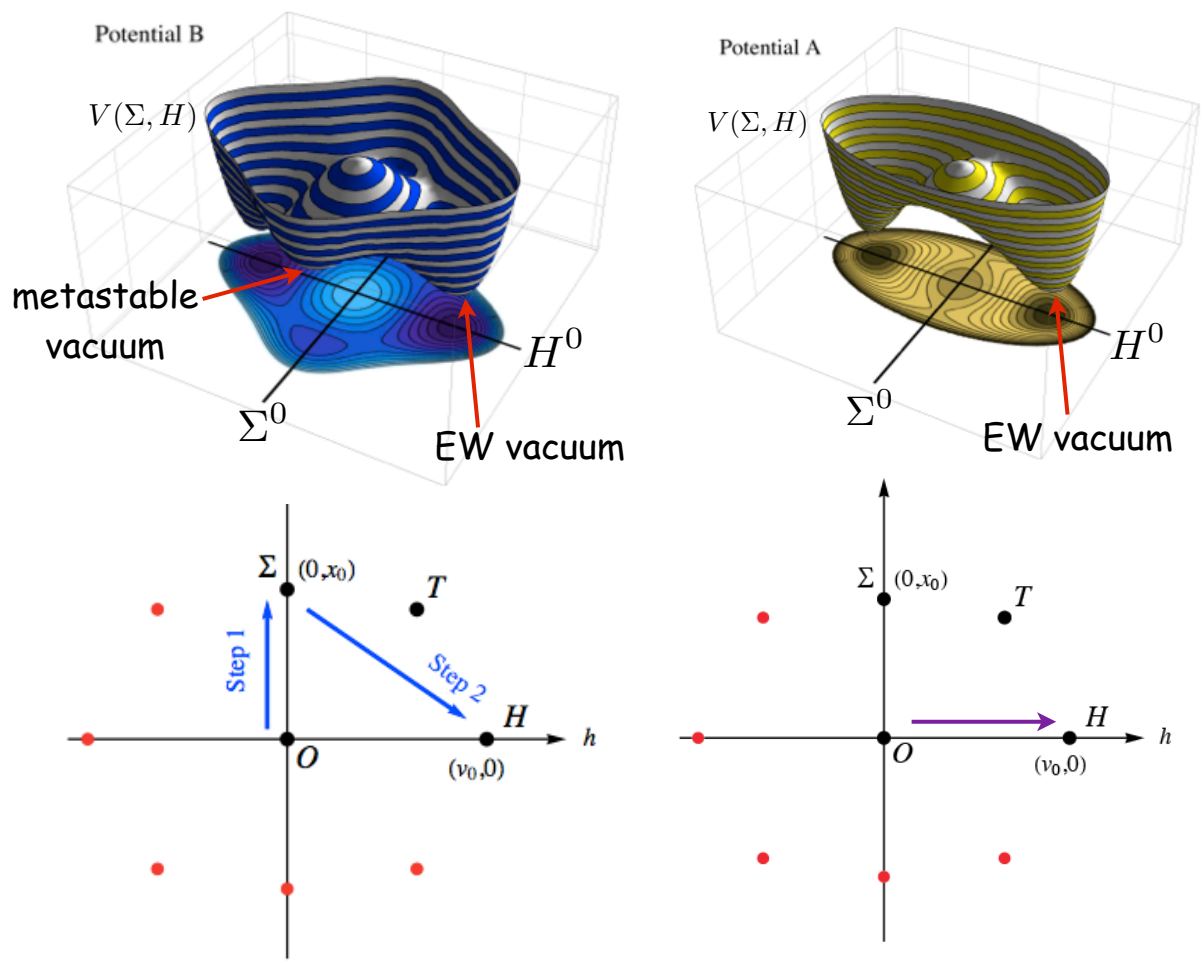
Electroweak phase transition influenced by vacuum structure:

- Gauge-independent method

$$\frac{\langle\phi\rangle}{T_c} \longrightarrow \frac{\bar{v}(T_c^{\text{G.I.}})}{T_c^{\text{G.I.}}}$$

- Two-step phase transition delays transition to EW phase.
- Lower T_c in second step, always strongly first order.
- Application to early universe color-breaking phase transition.

HP, MJRM, M. Wise
arXiv: 1303.1140 (2013)



$$\text{SU}(2)_L \times \text{U}(1)_Y \to \text{U}(1)_L \times \text{U}(1)_Y \to \text{U}(1)_{\text{EM}}$$

$$\text{SU}(2)_L \times \text{U}(1)_Y \to \text{U}(1)_{\text{EM}}$$

Summary

- Issue of gauge-invariance, and possible resolution:

HP, MJRM
arXiv: 1101.4665 (2011)

$$\frac{\langle\phi\rangle}{T_c} \longrightarrow \frac{\bar{v}(T_c^{\text{G.I.}})}{T_c^{\text{G.I.}}}$$

- Application to 2-step EW symmetry breaking.

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