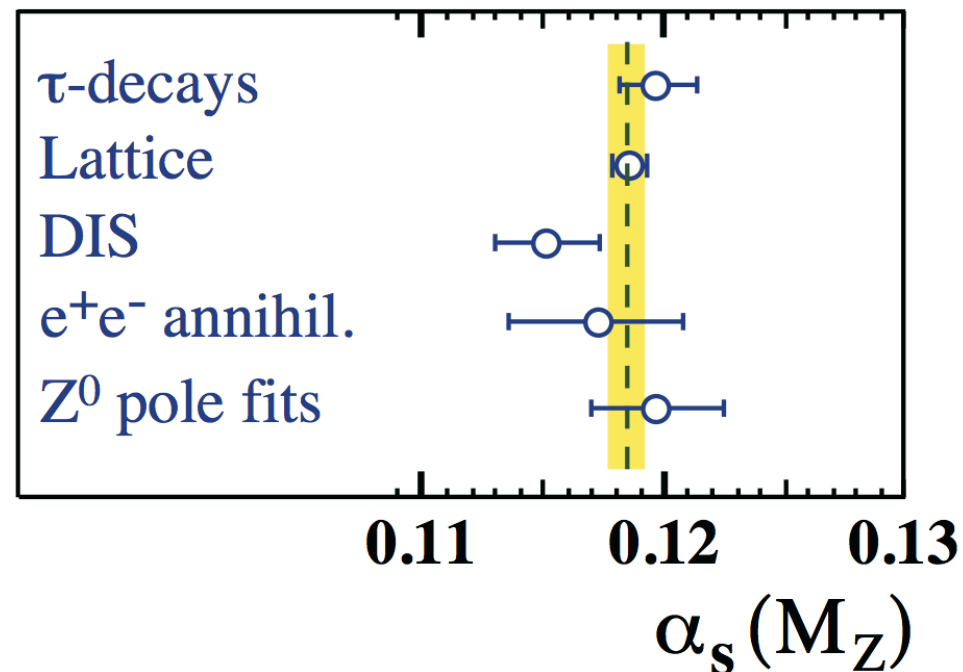# α_s at an e$^+$e$^-$ Z factory

some personal thoughts, plus input from discussion

with Th. Gehrmann (UZH)

G. Dissertori
ETH Zürich

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Recent summary from the PDG

see Bethke, Dissertori, Salam: http://pdg.lbl.gov/2012/reviews/rpp2012-rev-qcd.pdf

τ-decays
Lattice
DIS
e^+e^- annihil.
Z^0 pole fits

0.11   0.12   0.13
$\alpha_s(M_Z)$

current world average:

$\alpha_S(M_Z) = 0.1184 \pm 0.0007$ (0.6 % rel.)

- central value rather insensitive to choice of input
- uncert. dominated by Lattice results (~0.6% rel.)

Question:
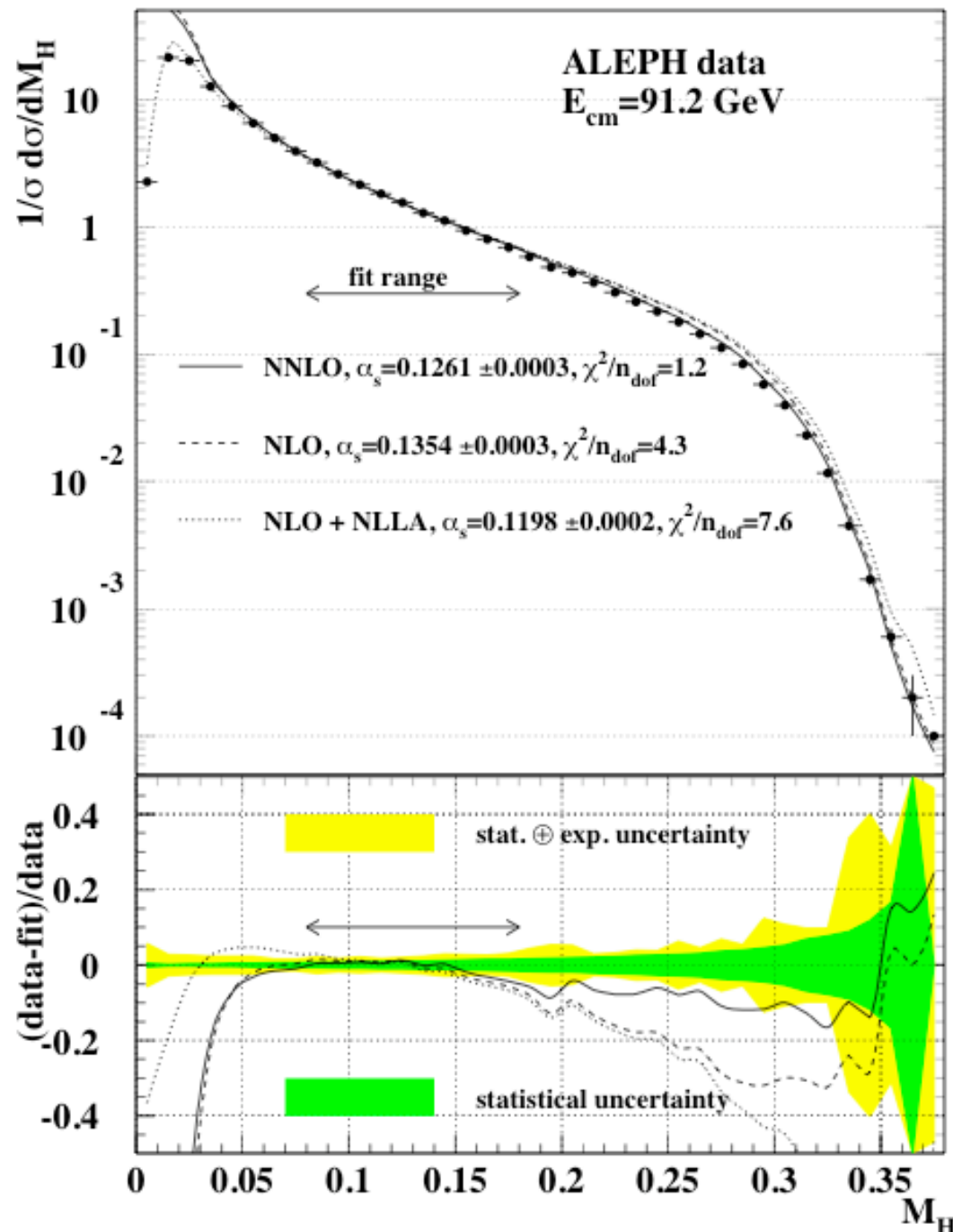what is interesting (or realistic) goal to take as reference for this discussion?

Let's choose: **0.0001 (abs)   or ~0.1 % (rel)**

let's focus at **Z peak measurements**, in order to have an independent
$\alpha_S(M_Z)$ as input, eg., for m_{top} determination at ttbar threshold of an e^+e^- machine.

Jet rates, event shapes

"Classical" method, theory known at NNLO+NNLL (NNLO obtained only a few years ago). Current status, typical values:

ALEPH data
E_{cm}=91.2 GeV

fit range

NNLO, α_s=0.1261 ±0.0003, χ^2/n_{dof}=1.2

NLO, α_s=0.1354 ±0.0003, χ^2/n_{dof}=4.3

NLO + NLLA, α_s=0.1198 ±0.0002, χ^2/n_{dof}=7.6

stat. ⊕ exp. uncertainty

statistical uncertainty

Experimental Uncertainties

typically ~1% (improvements should be possible)

Hadronization Uncertainties

difference between various models for hadronization,

typically around 0.7 - 1.5 %

going well below 1% seems unrealistic

Theoretical Uncertainties (pQCD)

renormalization scale variation, matching of (N)NLO with resummed calculation, quark mass effects

typically 3 - 5 %

going well below 1% seems unrealistic

my conclusion: this is not the way to go

α_S from inclusive Z decays

🖈 Advantage of inclusive observables:

 ⦿ by now known to NNNLO !

 ⦿ non-perturbative effects strongly suppressed

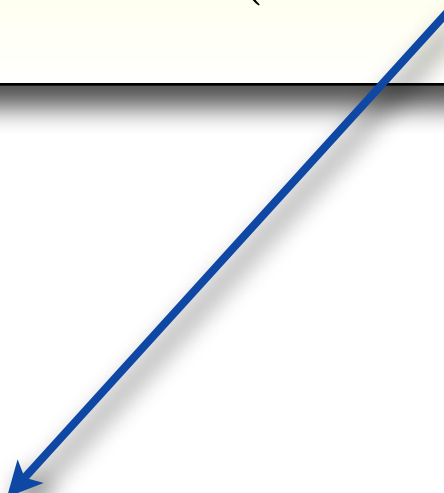$$R_{\mathrm{exp}} = \frac{\Gamma(Z \to \mathrm{hadrons})}{\Gamma(Z \to \mathrm{leptons})} = R_{EW}\left(1 + \delta_{QCD} + \delta_m + \delta_{np}\right)$$

$$\frac{R_{\mathrm{exp}}}{R_{EW}} = \mathcal{O}(1)$$

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$$\delta_{QCD} = \sum_{n=1}^{4} c_n \left(\frac{\alpha_s}{\pi}\right)^n + \mathcal{O}(\alpha_s^5)$$

$$c_1 = 1.045 \Rightarrow c_1 \frac{\alpha_s(M_Z)}{\pi} \sim 0.04 = \mathcal{O}(1/25)$$

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$$\sim \mathcal{O}\left(\frac{m_q^2}{M_Z^2}\right) \qquad \frac{\Delta\alpha_s}{\alpha_s} \approx \mathcal{O}(\mathrm{few}\,\%) \cdot \frac{\Delta\delta_m}{\delta_m}$$

Th. Gehrmann:
calculations can be
improved if necessary

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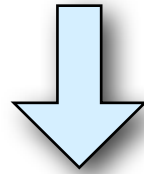
$$\mathcal{O}\left(\frac{\Lambda^4}{M_Z^4}\right)$$

$$c_1 = 1.045 \Rightarrow c_1 \frac{\alpha_s(M_Z)}{\pi} \sim 0.04 = \mathcal{O}(1/25)$$

<< 0.0001, no problem

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$$\frac{\Gamma(Z \rightarrow \text{hadrons})}{\Gamma(Z \rightarrow \text{leptons})} = 20.767 \pm 0.025 \quad (\; 0.12 \text{ \% rel.} \;)$$

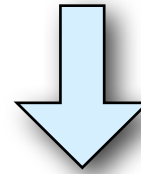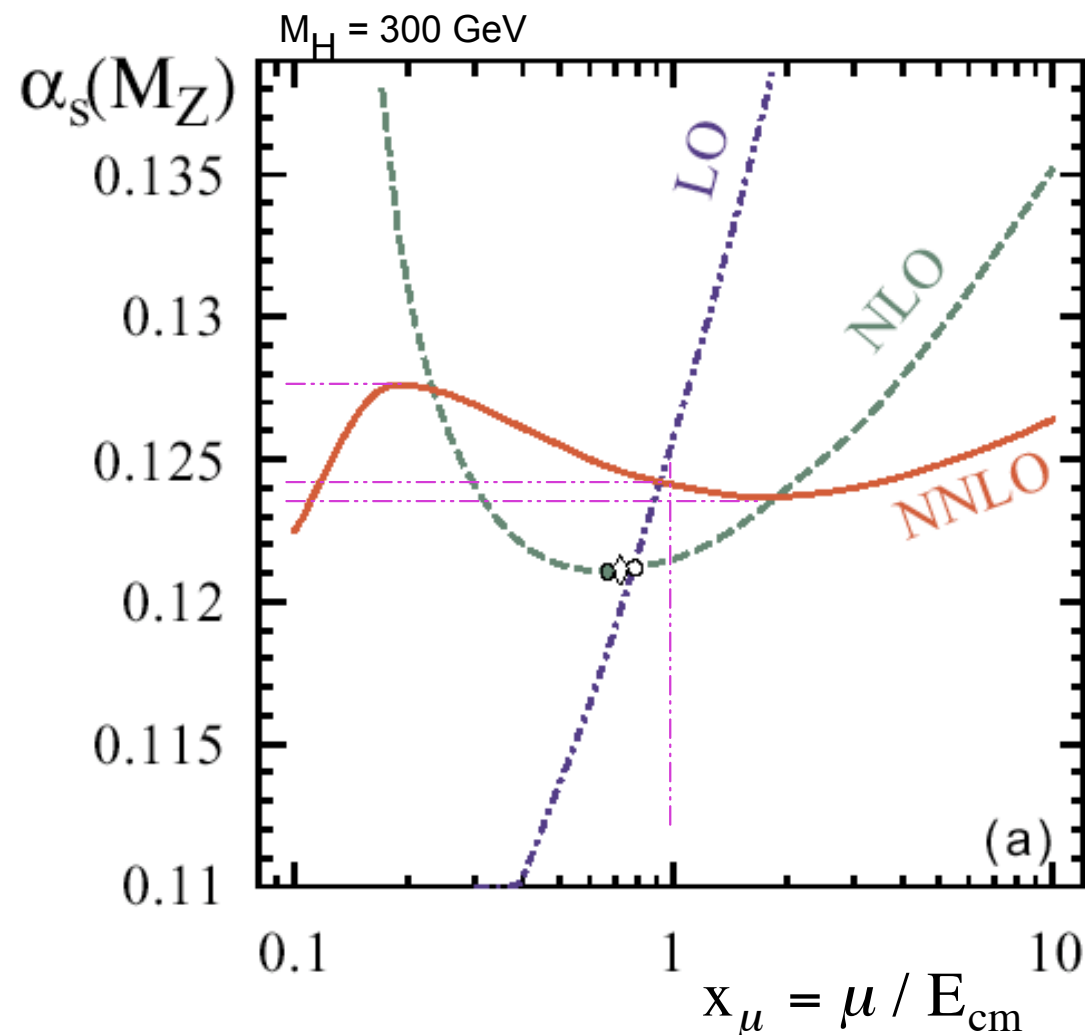# Example (using NNLO)

$$\frac{\Gamma(Z \to \text{hadrons})}{\Gamma(Z \to \text{leptons})} = 20.767 \pm 0.025 \quad (\ 0.12\ \%\ \text{rel.}\ )$$

see next slide

$$\alpha_s(M_Z) = 0.1226 \pm 0.0038 \ \text{exp.}$$

$$\begin{array}{ll} +0.0028 \\ -0.0005 \end{array} \quad \mu = {}^{2}_{0.25}\ M_Z$$

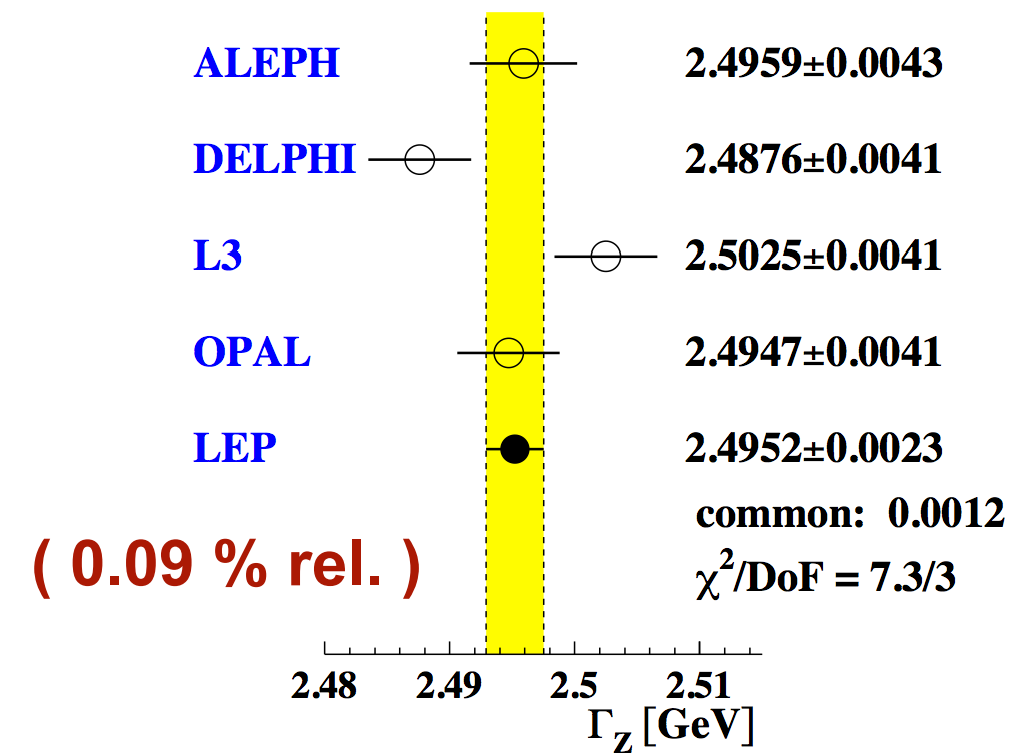$$\begin{array}{ll} +0.0033 \\ -0.0 \end{array} \quad M_H = {}^{900}_{100}\ \text{GeV}$$

$$\pm 0.0002 \quad M_t = \pm 5\text{GeV}$$

$$\pm 0.0002 \quad \text{renormal. schemes}$$

$M_H = 300$ GeV
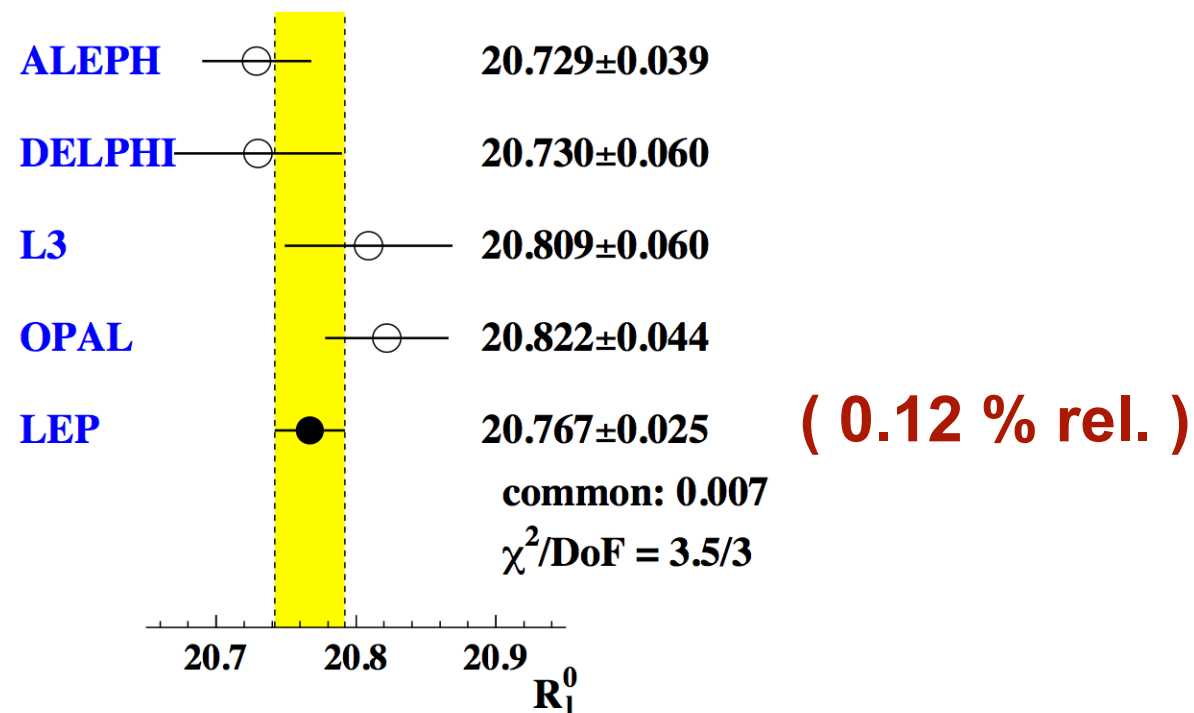
error source	$\Delta\alpha_s(M_Z)$
$\Delta M_{Z^o} = \pm 0.0021$ GeV	± 0.00003
$\Delta M_t = \pm 5$ GeV	± 0.0002
$M_H = 100 \dots 1000$ GeV	± 0.0017
$\mu = (\frac{1}{4} \dots 4)\ M_{Z^o}$	$\begin{array}{l} +\ 0.0028 \\ -\ 0.004 \end{array}$
renormalization schemes	± 0.0002
total	$\begin{array}{l} +\ 0.003 \\ -\ 0.002 \end{array}$

$$= 0.1226 \begin{array}{l} +0.0058 \\ -0.0038 \end{array}$$

Latest results from LEP EW group

Latest results from LEP EW group

ALEPH	20.729±0.039
DELPHI	20.730±0.060
L3	20.809±0.060
OPAL	20.822±0.044
LEP	20.767±0.025

(0.12 % rel.)

common: 0.007
$\chi^2/\text{DoF} = 3.5/3$

R_l^0

ALEPH	2.4959±0.0043
DELPHI	2.4876±0.0041
L3	2.5025±0.0041
OPAL	2.4947±0.0041
LEP	2.4952±0.0023

(0.09 % rel.)

common: 0.0012
$\chi^2/\text{DoF} = 7.3/3$

Γ_Z [GeV]
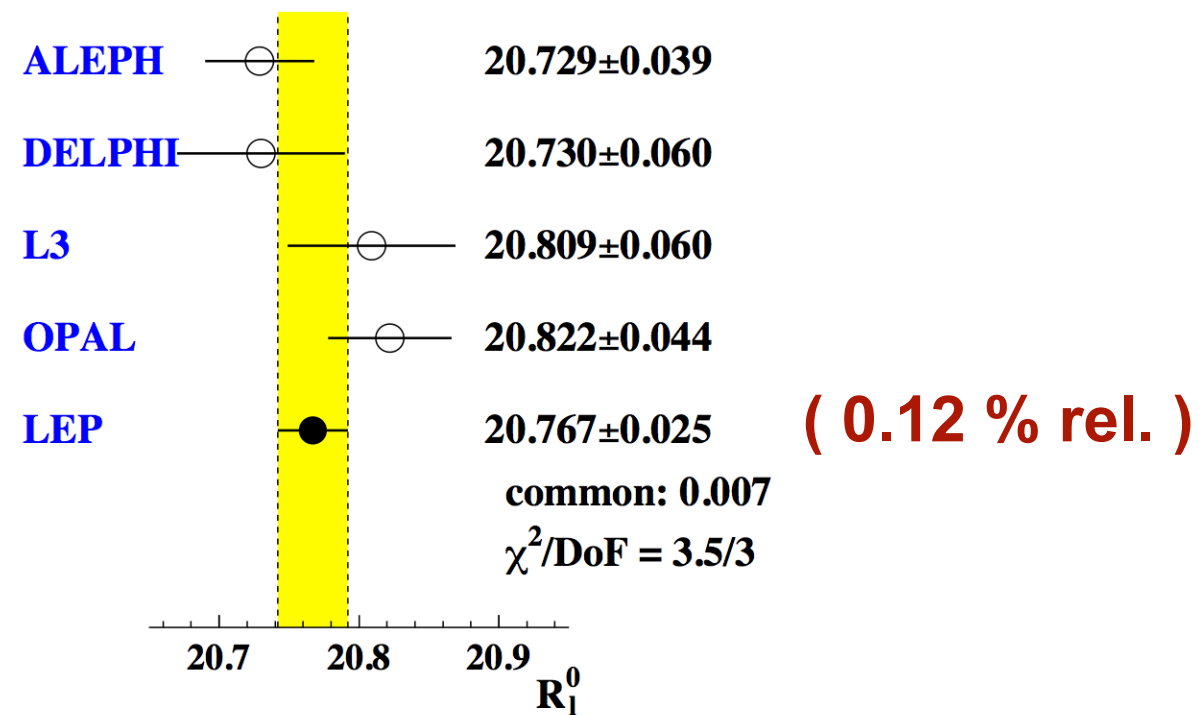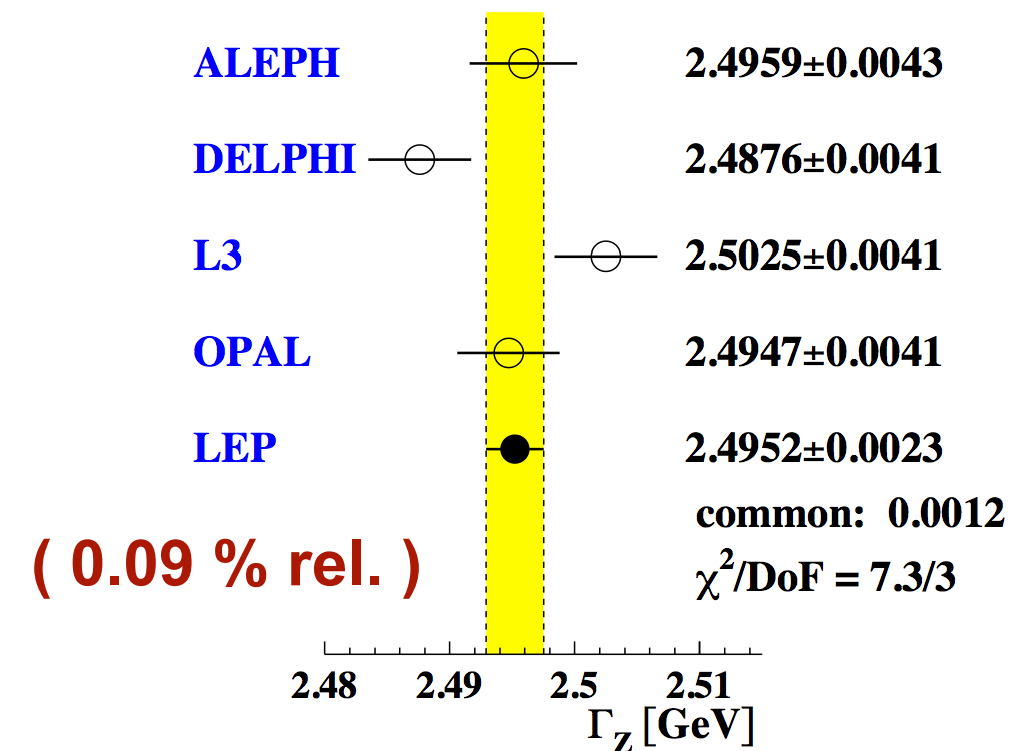
Source	δ	Γ_Z [MeV]	σ_{had}^0 [nb]	R_ℓ^0	R_b^0	ρ_ℓ	$\sin^2\theta_{\text{eff}}^{\text{lept}}$	m_W [MeV]
$\Delta\alpha_{\text{had}}^{(5)}(m_Z^2)$	0.00035	0.3	0.001	0.002	0.00001	—	0.00012	6
$\alpha_S(m_Z^2)$	0.003	1.6	0.015	0.020	—	—	0.00001	2
m_Z	2.1 MeV	0.2	0.002	—	—	—	0.00002	3
m_t	4.3 GeV	1.0	0.003	0.002	0.00016	0.0004	0.00014	26
$\log_{10}(m_H/\text{GeV})$	0.2	1.3	0.001	0.004	0.00002	0.0003	0.00022	28
Theory		0.1	0.001	0.001	0.00002	—	0.00005	4
Experiment		2.3	0.037	0.025	0.00065	0.0010	0.00016	34

from slide 4:

$$\frac{\Delta\alpha_s}{\alpha_s} \approx 25 \cdot \frac{\Delta R_{\text{exp}}}{R_{\text{exp}}}$$

thus: for a rel. prec. of ~0.1% on α_s we need rel. exp. prec. ~25 times better !!

So summarizing

- uncertainty because of m_H gone, m_{top} dep. no problem

- pQCD scale uncertainty, from latest NNNLO calculation:

  - ~**0.0002** (absolute uncertainty on α_s), see arXiv:0801.1821 and 1201.5804

- eg. taking Γ_Z: current uncertainty 2.3 MeV

  - ~ 1.2 MeV from beam energy (dominating contribution)

  - remainder: mostly statistical/experimental

  - **so the question is:**
    can a future Z factory measure Γ_Z at a precision of ~ 0.1 MeV ?
    or R with an absolute precision of ~0.001 ?

- **Note: all this is based on the assumption that there are no BSM effects which affect the Z pole observables at this level of precision.**

R_τ: τ hadronic BR

ALEPH

$\tau \rightarrow$ hadrons ν_τ

s [GeV^2]

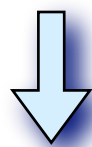s = (invariant mass)^2 of hadronic decay products
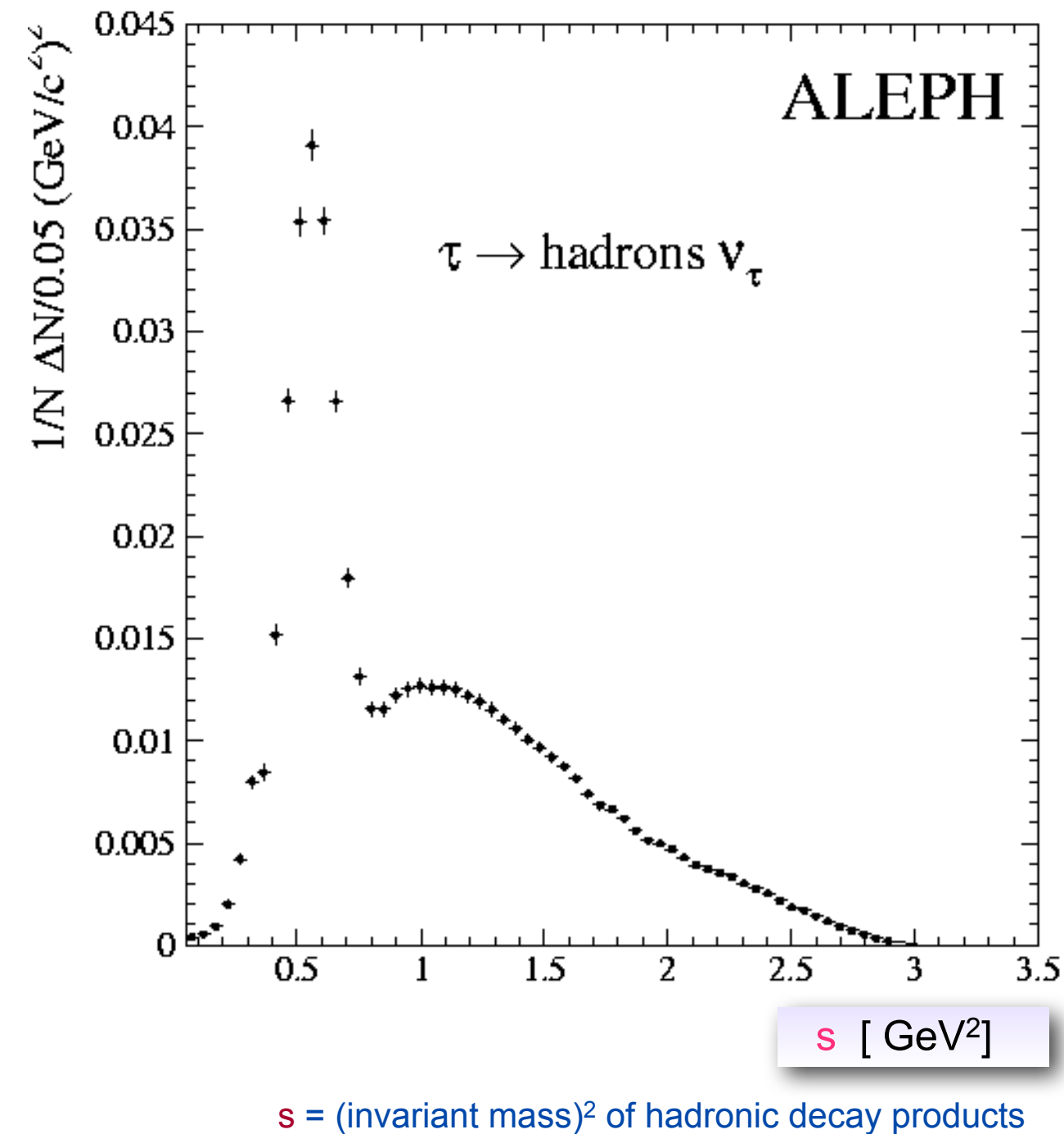
in principle even more inclusive than R at the Z pole, since integrating over hadronic inv. mass spectrum

$$R_\tau = \frac{1}{\pi} \int_0^{m_\tau^2} \frac{ds}{m_\tau^2} (1 - \frac{s}{m_\tau^2})^2 Im\Pi_\tau(s)$$

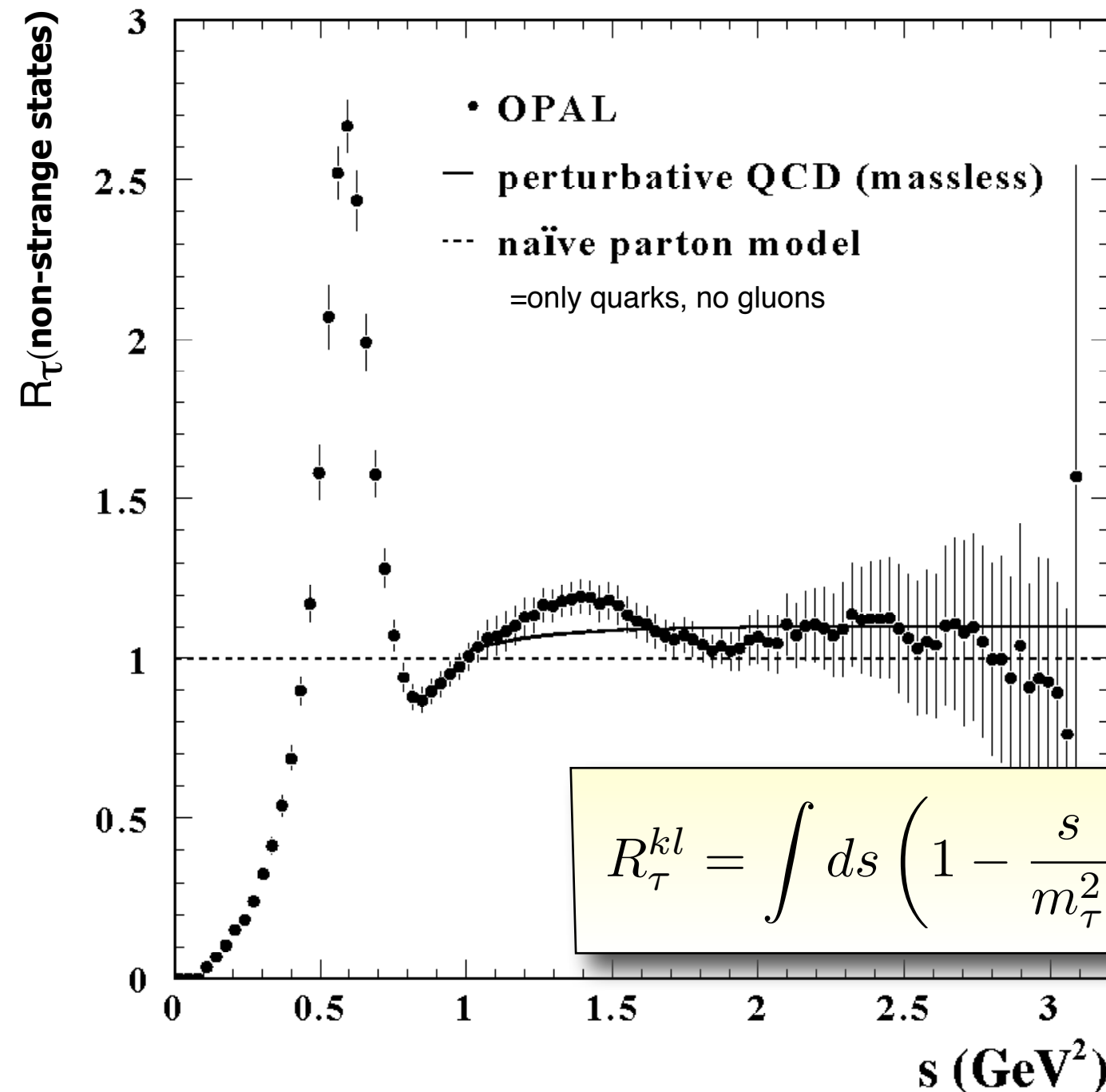interesting "advantage": "shrinking" of uncertainty just due to running of α_s :

$$\alpha_s(m_\tau) = 0.3285 \pm 0.018 \quad \text{(5.5 \% rel.)}$$

$$\alpha_s(m_Z) = 0.1194 \pm 0.0021 \quad \text{(1.8 \% rel.)}$$

Without phase space factor and taking moments, in order to average out resonances:

approach taken at LEP: fit simultaneously α_S and the non-pert. coefficients, by measuring various moments.

opinions vary about the importance of this "ZERO", see eg. Altarelli, arXiv: 1303.6065

$$\delta_{NP} = \frac{ZERO}{m_\tau^2} + c_4 \cdot \frac{<O_4>}{m_\tau^4} + c_6 \cdot \frac{<O_6>}{m_\tau^6} + \cdots$$

$$R_\tau^{kl} = \int ds \left(1 - \frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_\tau}{ds} \propto 3\left(1 + \delta_{QCD} + \delta_{NP}\right)$$

also known at NNNLO

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🔘 # From moments-measurements at LEP:

📌 the non-perturbative contributions turn out to be (surprisingly) small

eg. ALEPH: δ_{NP} = - 0.0059 ± 0.0014 $$\Delta\alpha_s(m_\tau) \approx \pi \cdot \Delta\delta_{NP}$$

📌 it would definitely be interesting to measure such moments again, with much improved precision. Eg. an uncertainty on δ_{NP} of < 0.0005

🔘 # But:

📌 various methods of estimating higher-order terms (see eg. Altarelli:1303.6065, or Pich:1303.2262) differ by >~ 5 % for $\alpha_s(m_{tau})$, ie. leading to >~ 1% at the Z mass scale.

📌 Seems difficult (impossible?) to improve on this?

Conclusion

- Jet-or event-shape based measurements, as well as using tau decays:
  seems difficult (impossible?) to go well below the 1% rel. uncertainty.

- EWK observables at the Z pole, such as hadronic width (branching ratio):
  this could be interesting. Depends on the precision on Z line shape observables, which may be achieved by a Z factory