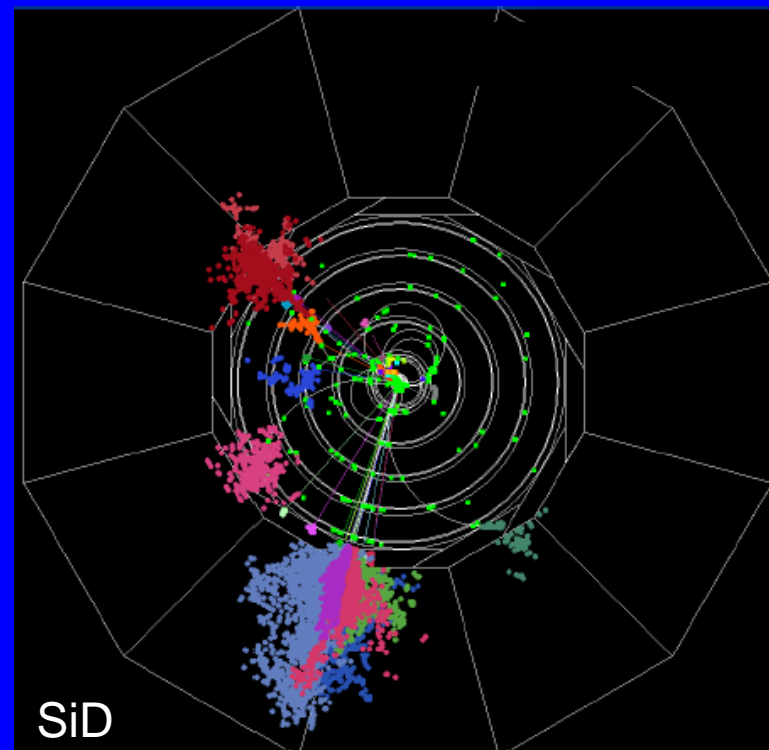
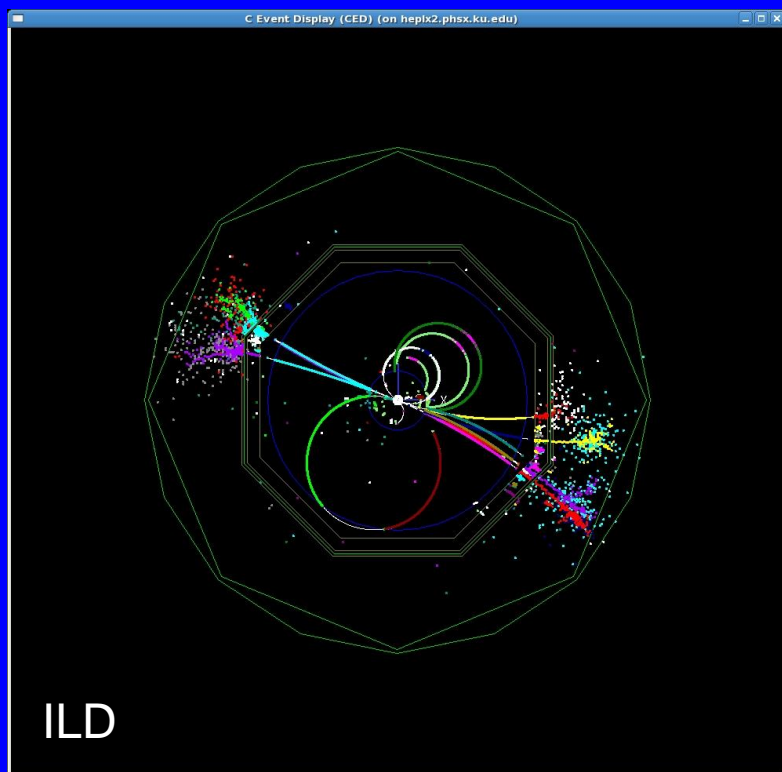


Update on Precision m_W Measurements at ILC

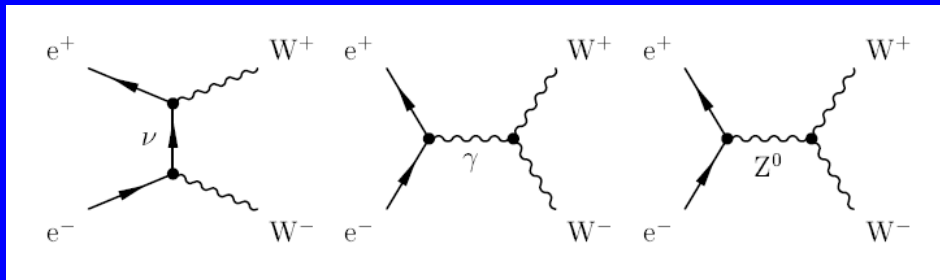


Graham W. Wilson, University of Kansas,
Snowmass EF Workshop, Seattle, July 2nd 2013

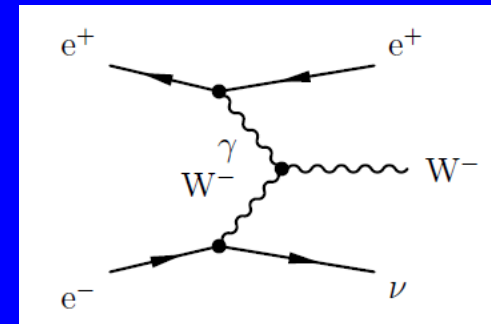
Introduction

- See talks at BNL and Duke for more specific details.
- This talk:
 - Short intro
 - Threshold scan revisited
 - Accelerator issues
 - Tracker-based Beam Energy Measurement study

W Production in e^+e^-

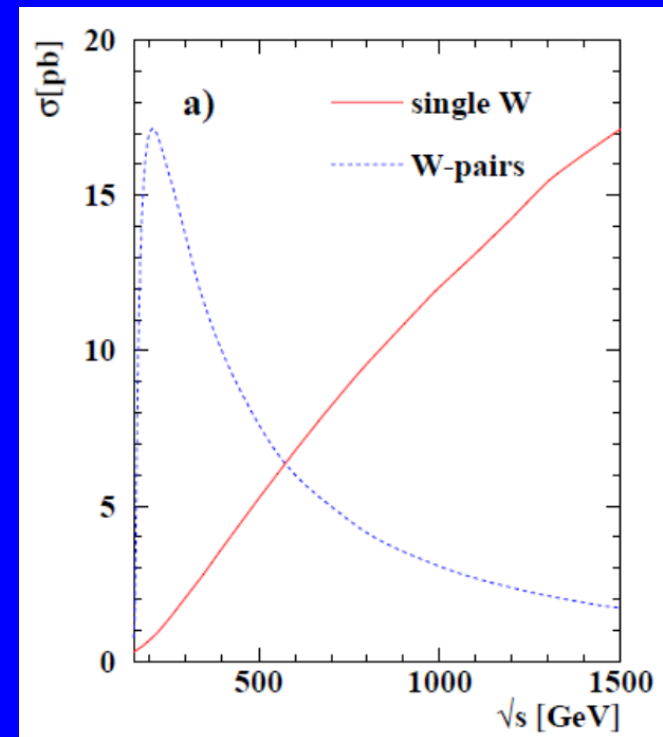
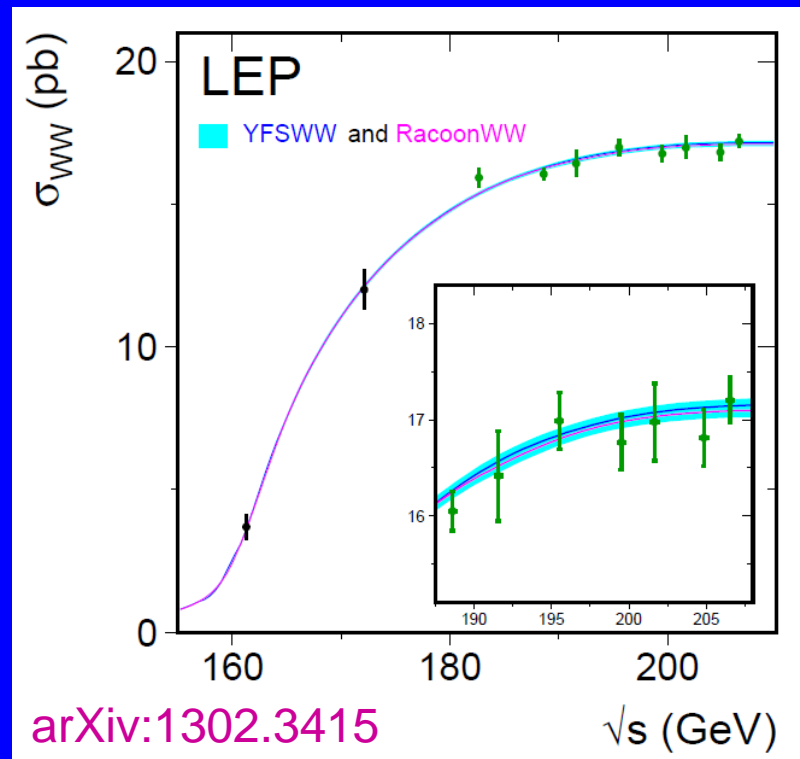


$e^+e^- \rightarrow W^+W^-$



etc ..

$e^+e^- \rightarrow W e \nu$



unpolarized cross-sections

Primary Methods

- 1. Polarized Threshold Scan
 - All decay modes
 - Polarization => Increase signal / control backgrounds
- 2. Kinematic Reconstruction using (E,p) constraints
 - $q q l \nu$ ($l = e, \mu$)
- 3. Direct Hadronic Mass Measurement
 - In $q q \tau \nu$ events and
hadronic single- W events (e usually not detected)

ILC may contribute to W mass measurements over a wide range of energies.
ILC250, ILC350, ILC500, ILC1000, ILC161 ...

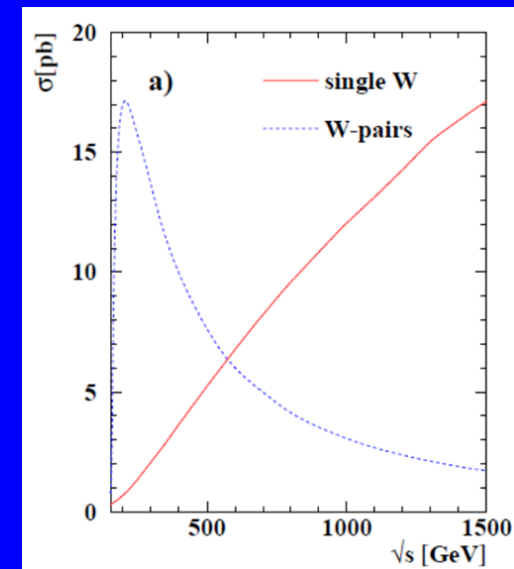
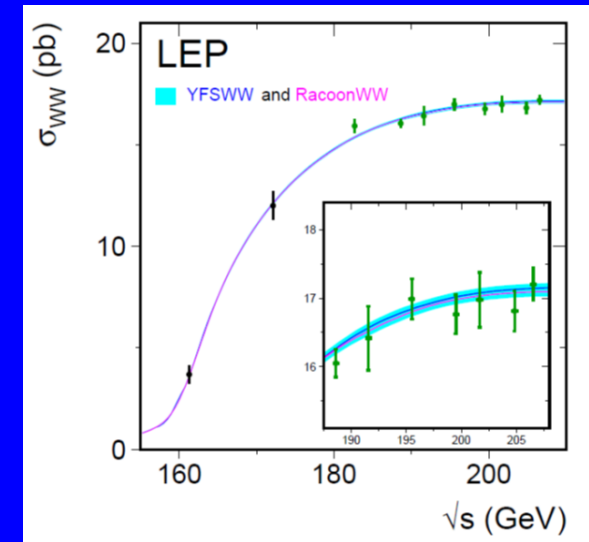
Threshold scan is the best worked out.

W Mass Measurement Strategies

- W^+W^-
 - 1. Threshold Scan ($\sigma \sim \beta/s$)
 - Can use all WW decay modes
 - 2. Kinematic Reconstruction
 - Apply kinematic constraints
- $W e \nu$ (and $WW \rightarrow qq\tau\nu$)
 - 3. Directly measure the hadronic mass in $W \rightarrow q q'$ decays.
 - e usually not detectable

Methods 1 and 2 were used at LEP2. Both require good knowledge of the absolute beam energy.

Method 3 is novel (and challenging), very complementary systematics to 1 and 2 if the experimental challenges can be met.



Statistics

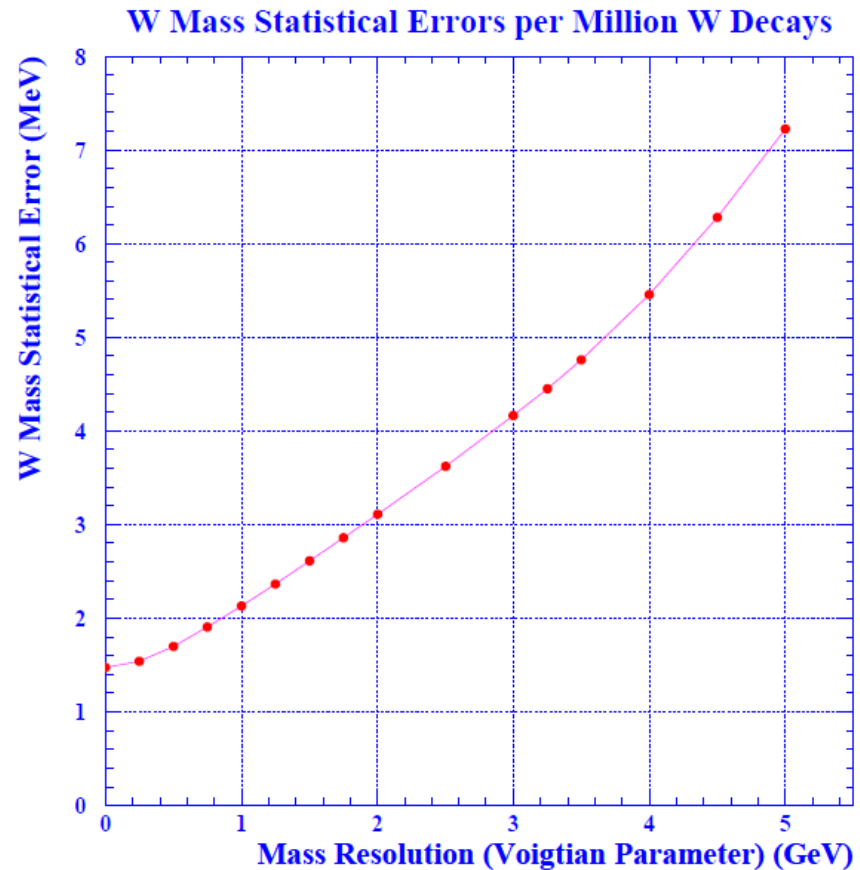
ILC will produce 10-100M W's

Polarization very helpful.

For statistical errors, W width leads to following error per million reconstructed W decays

Can envisage mass resolution in the 1-2 GeV range.

Statistics for below 1 MeV error.



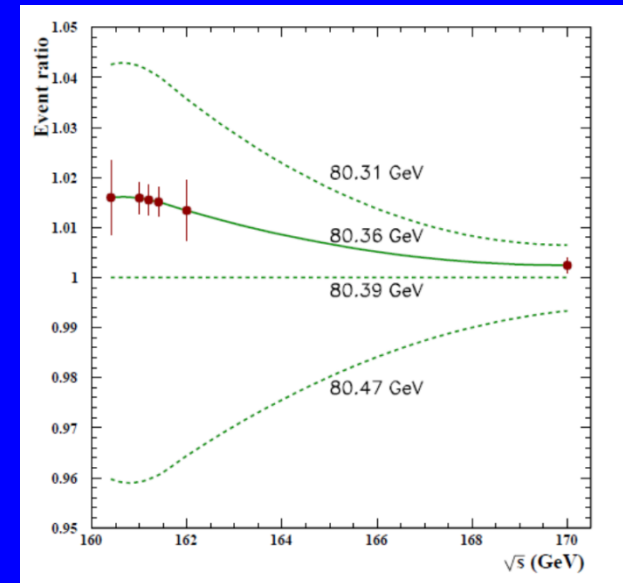
m_W Measurement Prospects Near Threshold

PRECISION MEASUREMENT OF THE W MASS WITH A POLARISED THRESHOLD SCAN AT A LINEAR COLLIDER

Graham W. Wilson, LC-PHSM-2001-009, 21st February 2001

Department of Physics, Schuster Laboratory, The University, Manchester M13 9PL, UK

Threshold scans potentially offer the highest precision in the determination of the masses and widths of known and as yet undiscovered particles at linear colliders. Concentrating on the definite example of the WW threshold for determining the W mass (M_W), it is shown that the currently envisaged high luminosities and longitudinal polarisation for electrons **and positrons** allow M_W to be determined with an error of 6 MeV with an integrated luminosity of 100 fb^{-1} (One 10^7 s year with TESLA). The method using polarised beams is statistically powerful and experimentally robust; the efficiencies, backgrounds and luminosity normalisation may if needed be determined from the data. The uncertainties on the beam energy, the beamstrahlung spectrum and the polarisation measurement are potentially large; required precisions are evaluated and methods to achieve them discussed.



LEP2 numbers

Channel (j)	Efficiency (%)	Unpolarised σ_{bkgd} (fb)	WW fraction (%)
$\ell\ell$	75	20	10.5
ℓh	75	80	44.0
$h h$	67	400	45.5

\sqrt{s} (j)	Luminosity weight
160.4	0.2
161.0	1.0
161.2	1.0
161.4	1.0
162.0	0.2
170.0	1.2

Measure at 6 values of \sqrt{s} , in 3 channels, and with up to 9 different helicity combinations.

Estimate error of 6 MeV (includes Eb error of 2.5 MeV from $Z \gamma$) per 100 fb^{-1} polarized scan (assumed 60% e+ polarization)

Use RR (100 pb) cross-section to control polarization

Accelerator Issues

$$L = (P/E_{\text{CM}}) \sqrt{(\delta_E / \varepsilon_{y,N})} H_D$$

$$P \sim f_c N \quad \delta E \sim (N^2 \gamma) / (\varepsilon_x, N \beta_x \sigma_z) U_1 (\Psi_{\text{av}})$$

Machine design has focussed on 500 GeV baseline

\sqrt{s}	$\mathcal{L}[10^{34}]$	dE [%]	(dp/p)(+) [%]	(dp/p)(-) [%]
200	0.56	0.65	0.190	0.206
250	0.75	0.97	0.152	0.190
350	1.0	1.9	0.100	0.158
500	1.8/3.6	4.5	0.070	0.124
1000	4.9	10.5	0.047	0.085

dp/p same as
LEP2 at 200 GeV

dp/p MUCH better
than an e+e- ring

Scope for improving luminosity performance.

1. Increase number of bunches (more power). (fc)
2. Decrease vertical emittance
3. Increase N
4. Decrease σ_z
5. Decrease β_x^*

3,4,5 => L, BS trade-off
Can trade more BS for more L
or lower L for lower BS.

Polarized Threshold Scan Errors

- conservative – viewed from + 14 years
- Non-Ebeam experimental error (stat + syst)
 - 5.2 MeV

	Scenario 0	Scenario 1	Scenario 2	Scenario 3
L (fb ⁻¹)	100	160*3	100	100
Pol. (e ⁻ / e ⁺)	80/60	90/60	90/60	90/60
Inefficiency	LEP2	0.5*LEP2	0.5*LEP2	0.5*LEP2
Background	LEP2	0.5*LEP2	0.5*LEP2	0.5*LEP2
Effy/L syst	0.25%	0.25%	0.25%	0.1%
Δm_W (MeV)	5.2	2.0	4.3	3.9

BeamStrahlung

Average energy loss of beams is not what matters for physics.

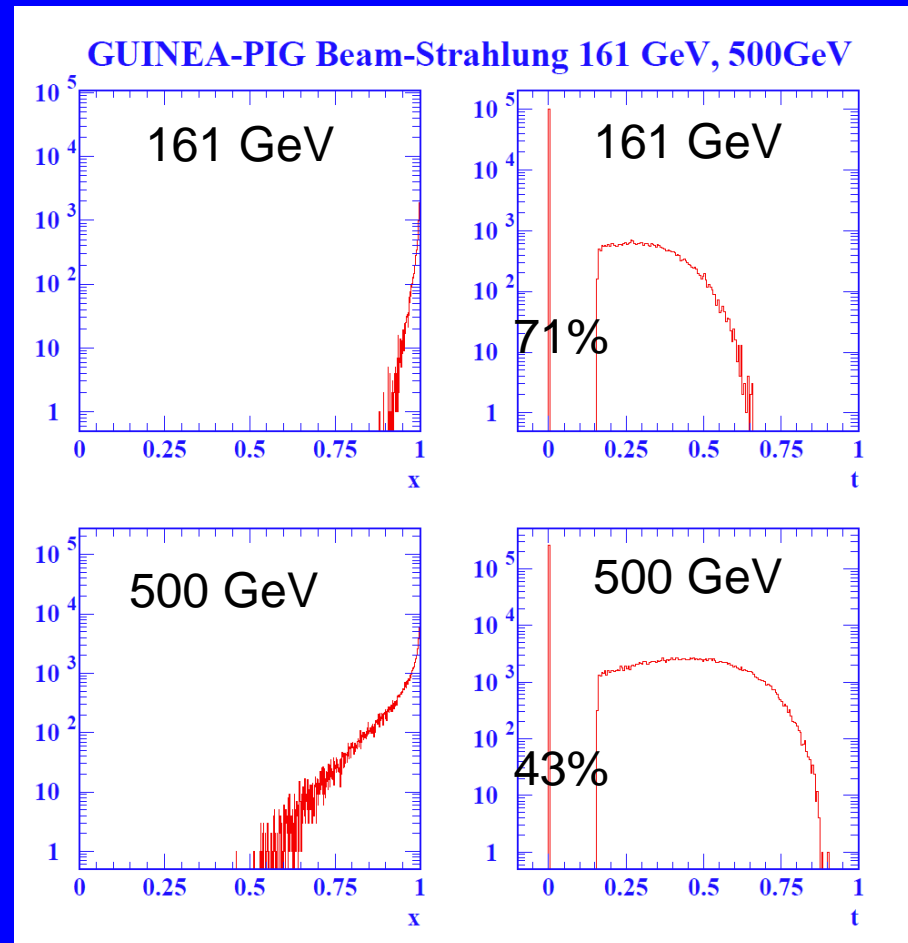
Average energy loss of colliding beams is factor of 2 smaller.

Median energy loss per beam from beamstrahlung typically ZERO.

Parametrized with CIRCE functions.

$f \delta(1-x) + (1-f) \text{Beta}(a2, a3)$

Define $t = (1 - x)^{1/5}$



In general beamstrahlung is a less important issue than ISR for kinematic fits

$t=0.25 \Rightarrow x = 0.999$

In-Situ \sqrt{s} Determination with $\mu\mu(\gamma)$

- ILC physics capabilities will benefit from a well understood centre-of-mass energy
 - Preferably determined from collision events.
- Measure precisely W, top, Higgs masses. (and Z ?)
- Two methods using $\mu\mu(\gamma)$ events have been discussed:
 - Method A: Angle-Based Measurement
 - Method P: Momentum-Based Measurement

See my talk at ECFA LC2013 Hamburg for more details of recent studies on Method P.

Method A) Use angles only, measure m_{12}/\sqrt{s} . Use known m_Z to reconstruct \sqrt{s} .

Hinze & Moenig

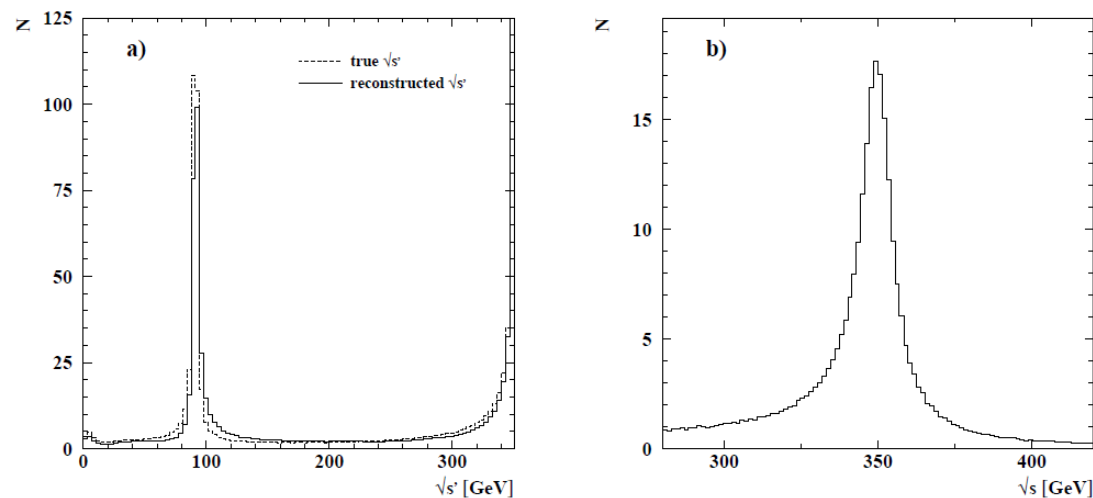


Figure 2: True and reconstructed $\sqrt{s'}$ (a) and reconstructed \sqrt{s} for $e^+e^- \rightarrow Z\gamma \rightarrow \mu^+\mu^-\gamma$ at $\sqrt{s} = 350$ GeV

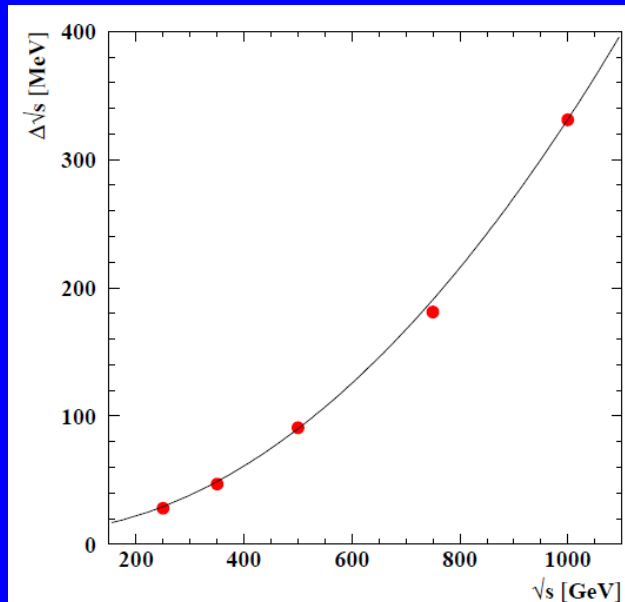


Figure 3: Energy dependence of $\Delta\sqrt{s}$ for $\mathcal{L} = 100 \text{ fb}^{-1}$.

$$\sqrt{s} = m_Z \sqrt{\frac{\sin \theta_1 + \sin \theta_2 - \sin(\theta_1 + \theta_2)}{\sin \theta_1 + \sin \theta_2 + \sin(\theta_1 + \theta_2)}}$$

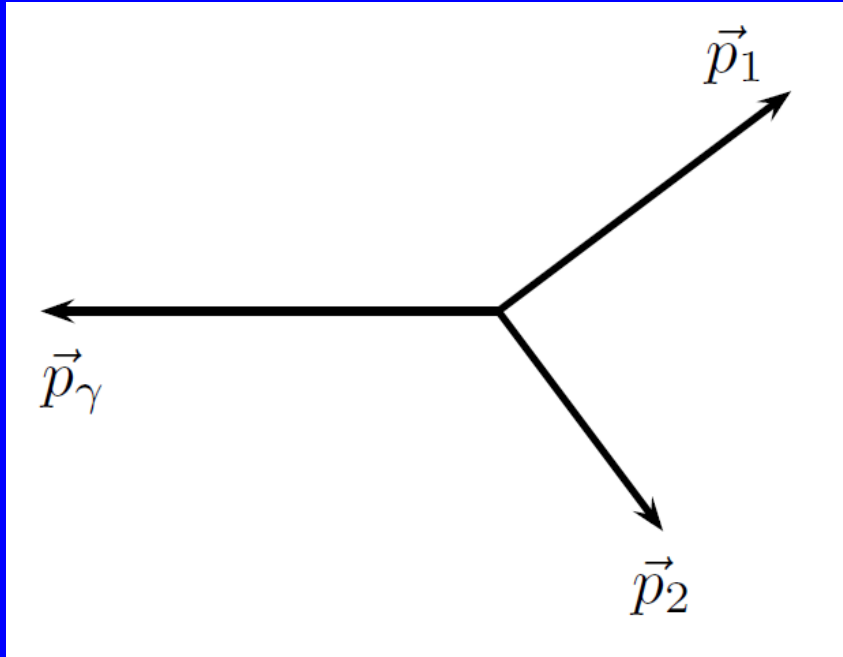
(Note. At 161 GeV my error estimate (ee, $\mu\mu$) on \sqrt{s} is 5 MeV: 31 ppm)

1. Statistical error per event of order $\Gamma/M = 2.7\%$
2. Error degrades fast with \sqrt{s} .

Method P

Use muon momenta. Measure $E_1 + E_2 + |\mathbf{p}_{12}|$.

Proposed and
studied initially by
T. Barklow



Under the assumption of a massless photonic system balancing the measured di-muon, the momentum (and energy) of this photonic system is given simply by the momentum of the di-muon system.

So the center-of-mass energy can be estimated from the sum of the energies of the two muons and the inferred photonic energy.

$$(\sqrt{s})_P = E_1 + E_2 + |\mathbf{p}_1 + \mathbf{p}_2|$$

In the specific case, where the photonic system has zero p_T , the expression is particularly straightforward. It is well approximated by

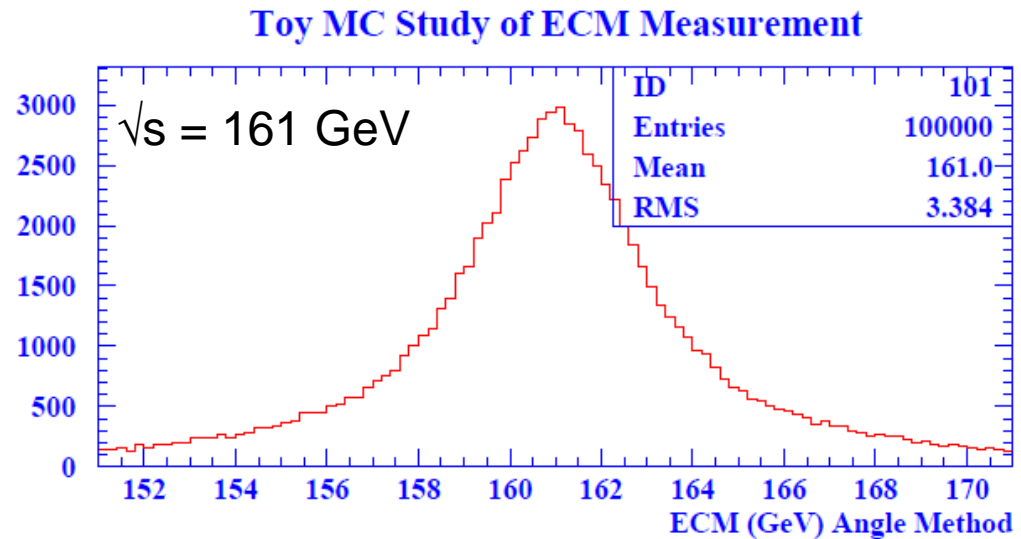
where p_T is the p_T of each muon. Assuming excellent resolution on angles, the resolution on $(\sqrt{s})_P$ is determined by the θ dependent p_T resolution.

$$\sqrt{s}_P = p_T \left(\frac{1 + \cos \theta_1}{\sin \theta_1} + \frac{1 + \cos \theta_2}{\sin \theta_2} \right)$$

Method can also use non radiative return events with $m_{12} \gg m_Z$

Method A (Angles)

(Absolute scale driven by m_Z – known very well)

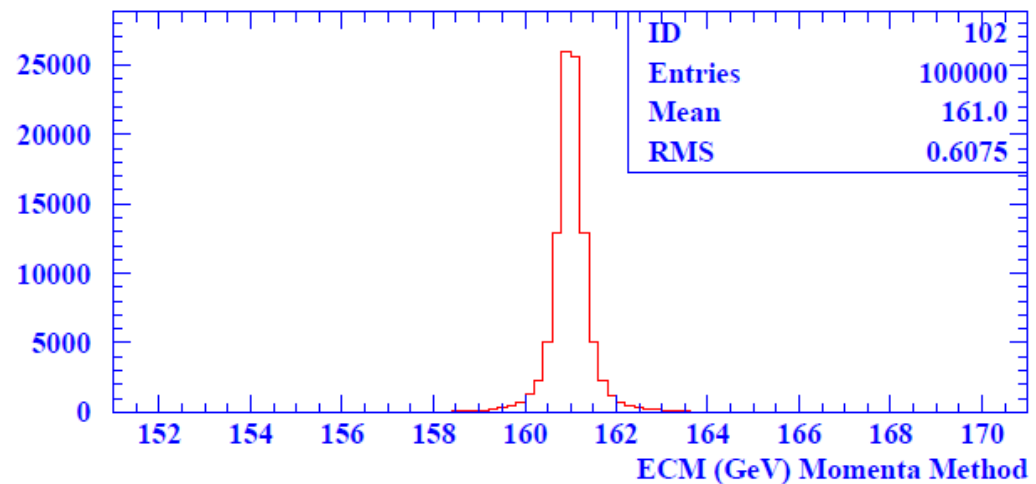


Method P (Momenta)

(Absolute scale driven by tracker momentum scale).

Momenta smeared.

Resolution is effectively 10 times better !



Momentum Resolution

Use the standard parametrization fitted to single muons from the ILD DBD.

$$\sigma_{1/p_T} = a \oplus b/(p_T \sin \theta)$$

Where typically

$$a = 2 \times 10^{-5} \text{ GeV}^{-1} \text{ and } b = 1 \times 10^{-3}$$

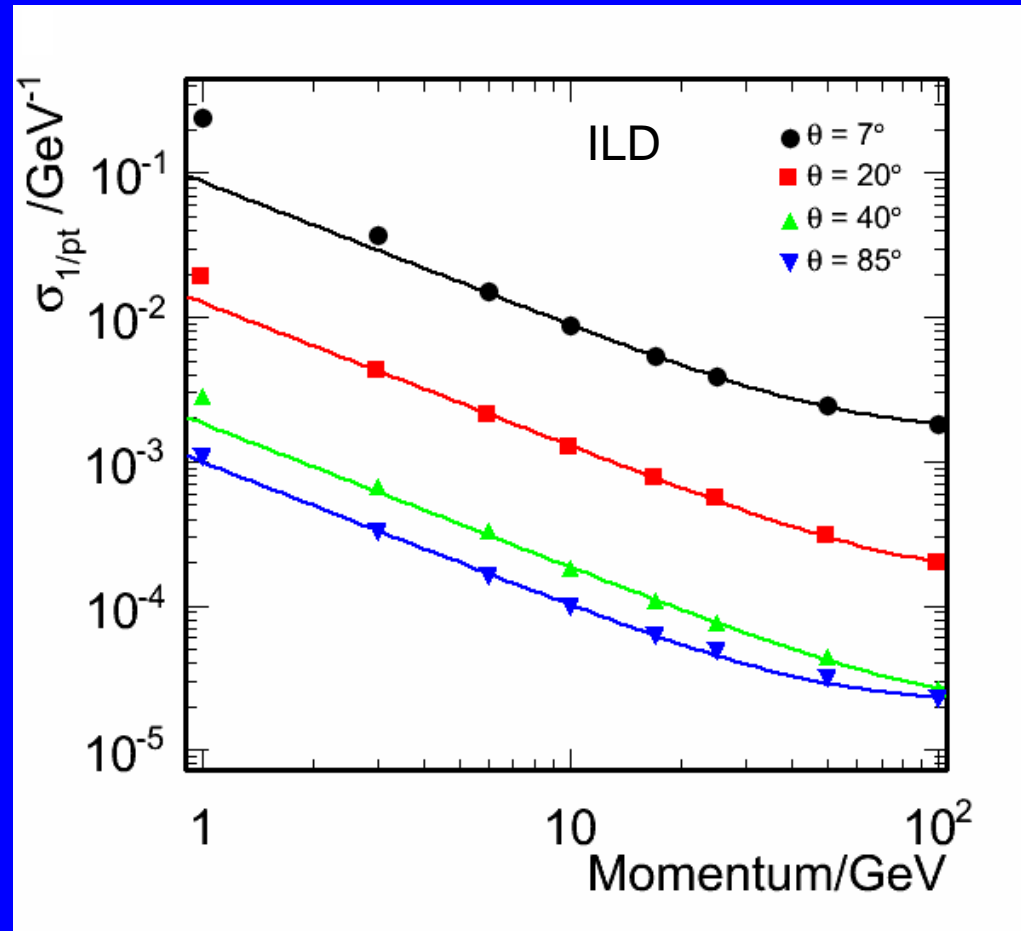
for the full TPC coverage
($\theta > 37^\circ$)

Fit momentum resolution in the $p \geq 10$ GeV range.

Superimposed curves are fits for the a,b parameters at 4 polar angles.

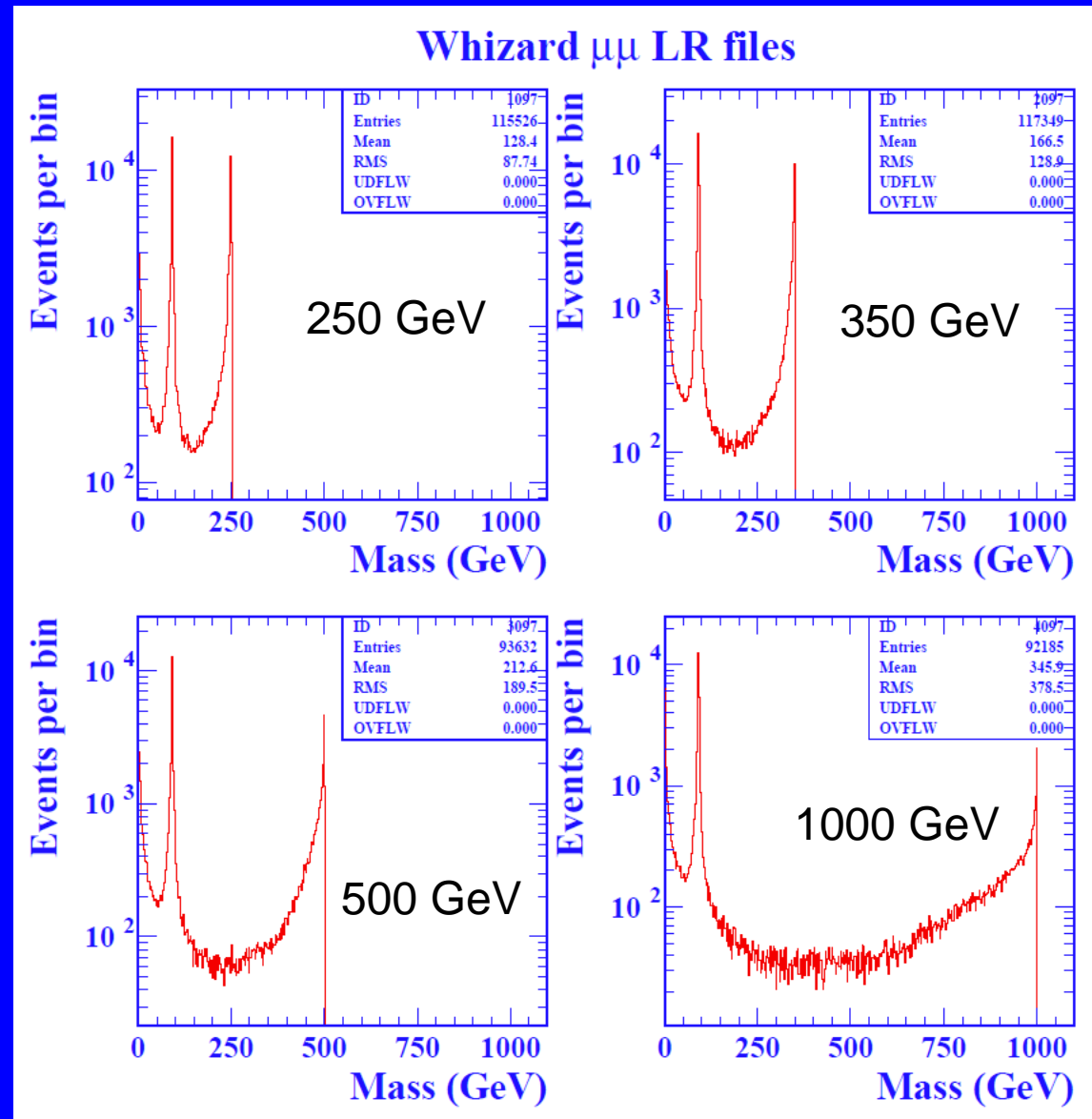
Maximum deviation from fit with this simple parametric form is 6%.

Interpolate between polar angles in endcap (use R^2 scaling for the a term).



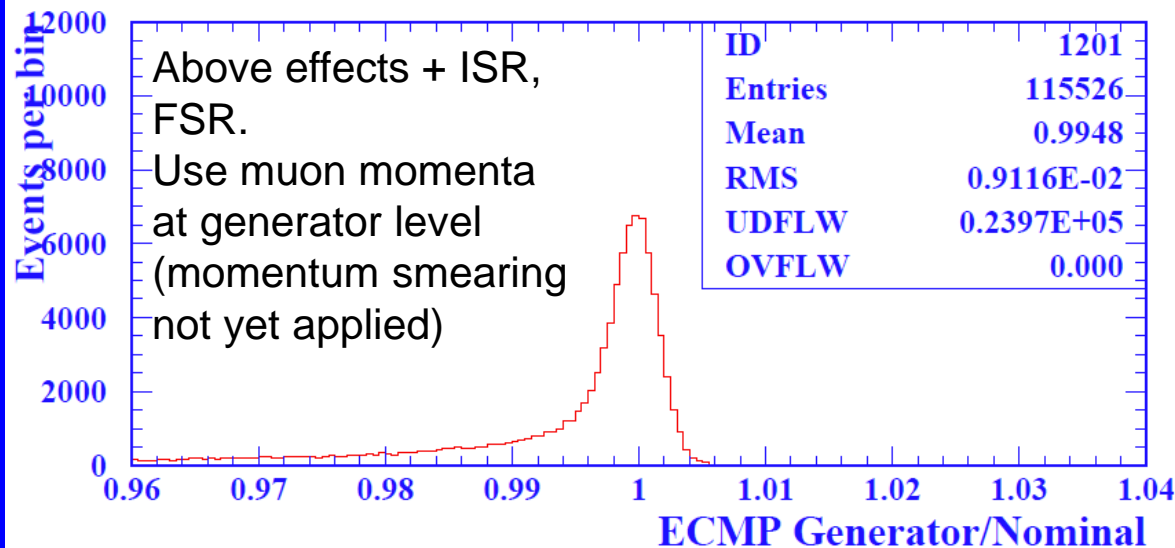
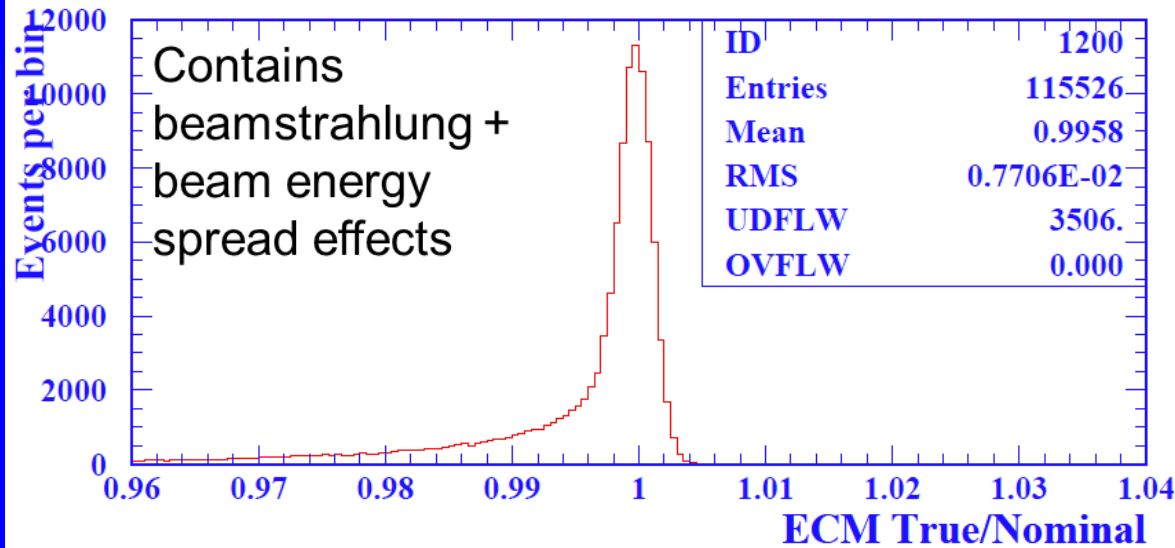
Generator Data-sets

- Use Whizard 4-vector files.
- At ECM=250, 350, 500, 1000 GeV.
- Use 1 stdhep file per energy. (e^-_L, e^+_R).
- Lumis are 10.4, 20.1, 32.2, 109 fb⁻¹.
- Events of interest have a wide range of di-muon mass values.



ECMP as an estimator of ECM

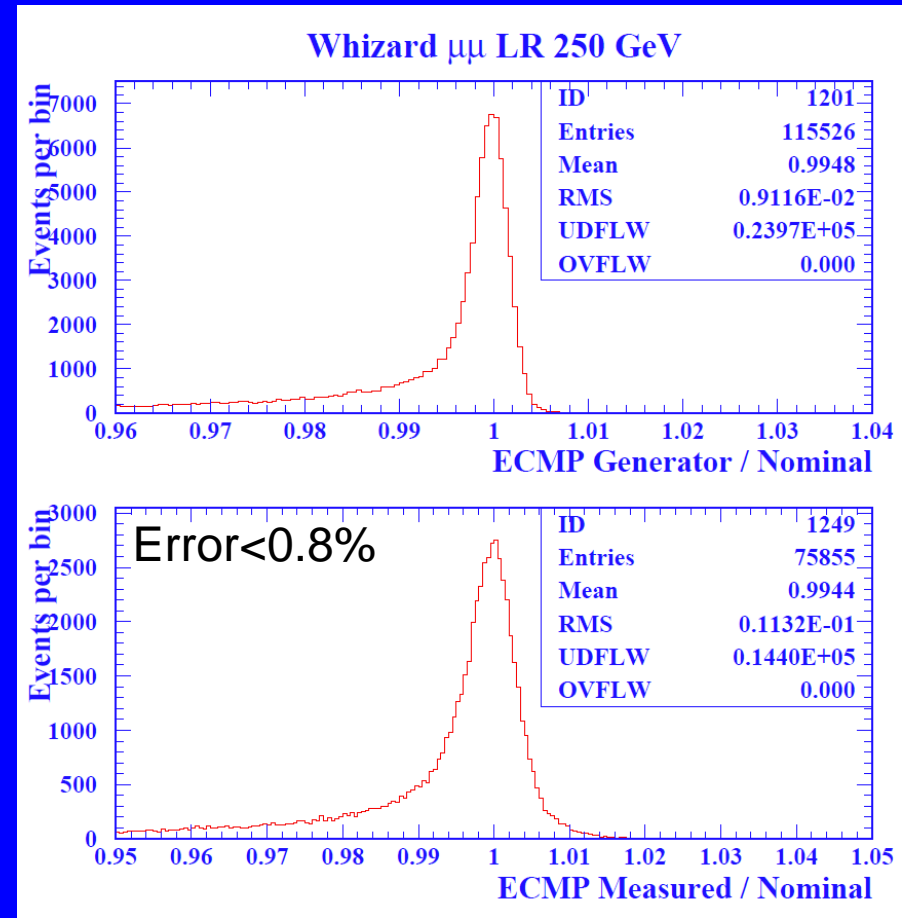
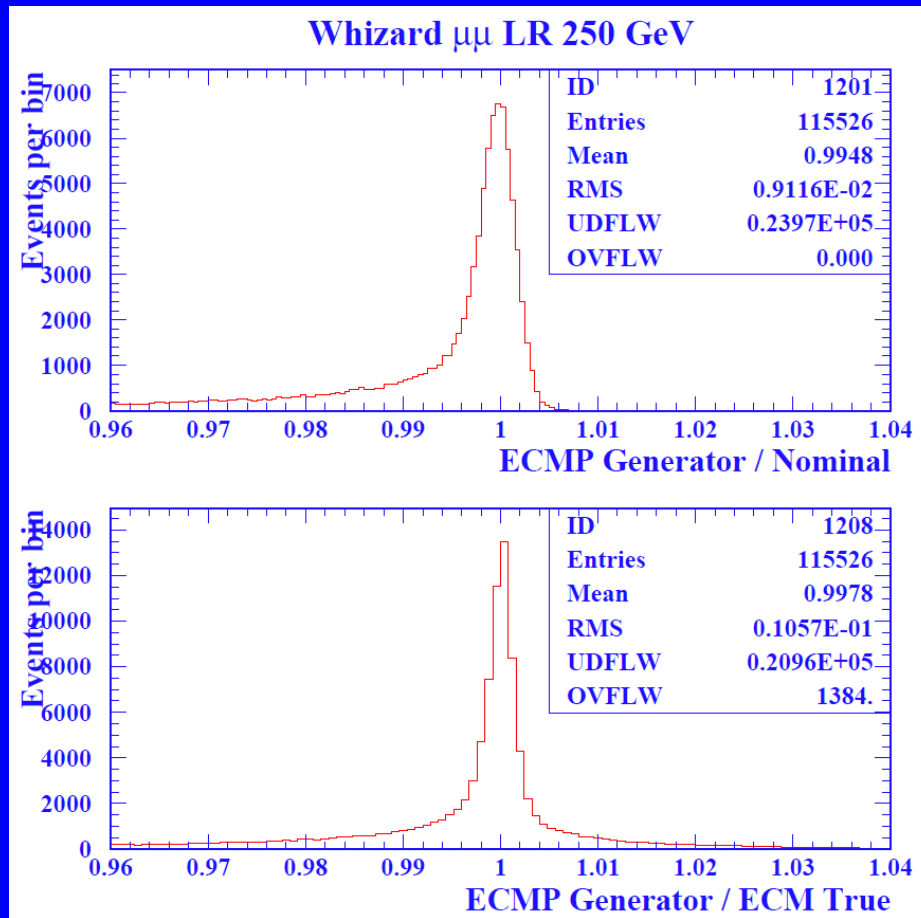
Whizard $\mu\mu$ LR 250 GeV



Full energy peak is wider – but still contains a lot of information on the absolute center-of-mass energy.

Opposite-beam double ISR off-stage left.

ECMP as an estimator of ECM



ECMP often is very well correlated with ECM. But long tails : eg hard ISR from BOTH beams

ECMP measured has additional effects from momentum resolution

Error on $\sqrt{s_p}$

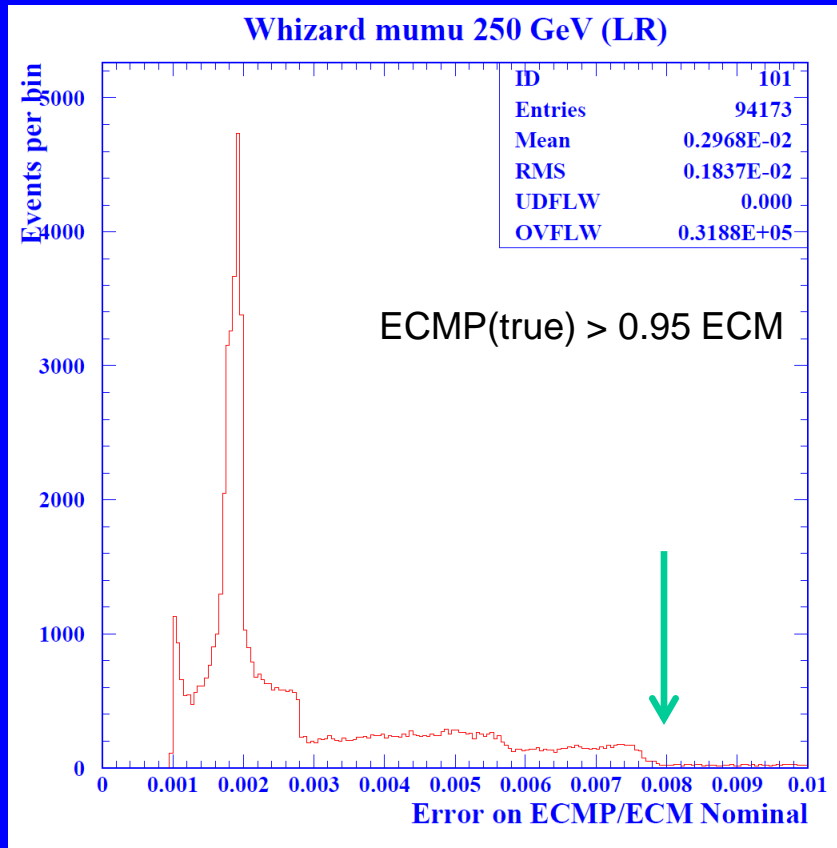
- Can write

$$\begin{aligned}\sqrt{s_p} &= E_1 + E_2 + |\mathbf{p}_{12}| \\ &= \sqrt{(\mathbf{p}_1^2 + m^2)} + \sqrt{(\mathbf{p}_2^2 + m^2)} \\ &\quad + \sqrt{(\mathbf{p}_1^2 + \mathbf{p}_2^2 + 2\mathbf{p}_1\mathbf{p}_2\cos\psi_{12})}\end{aligned}$$

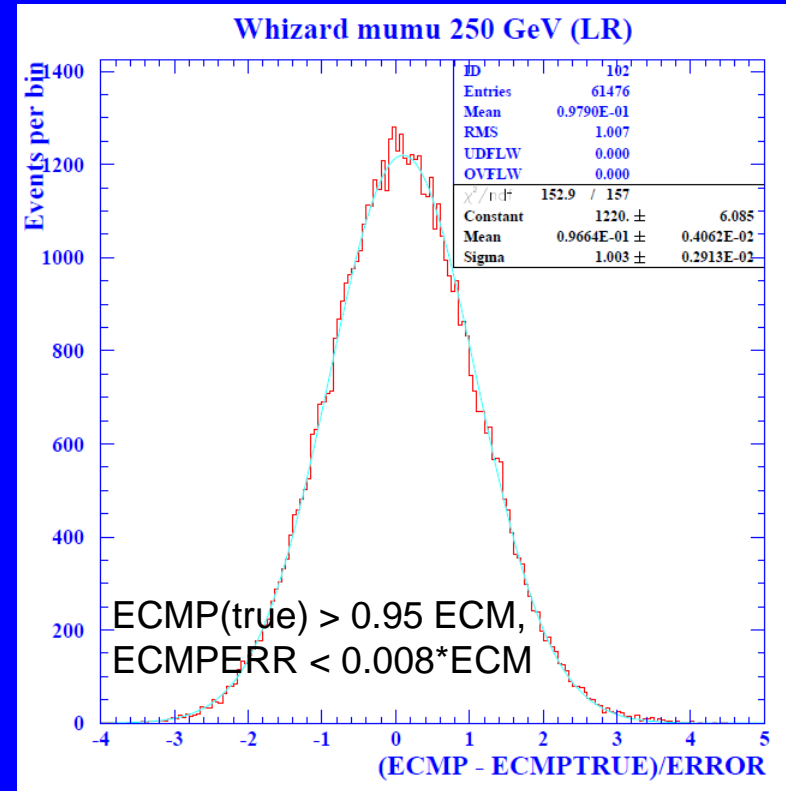
- Write $p_1 = \csc\theta_1/\kappa_1$ with $\kappa_1 = 1/pT_1$ and similarly for p_2 . Use errors on κ from ILD.
- Do error propagation (neglecting angle errors).

Error on \sqrt{s}_p estimator from momentum resolution

- Using general expression with error propagation. Does not use zero pT approximation. Assumes angle errors negligible.



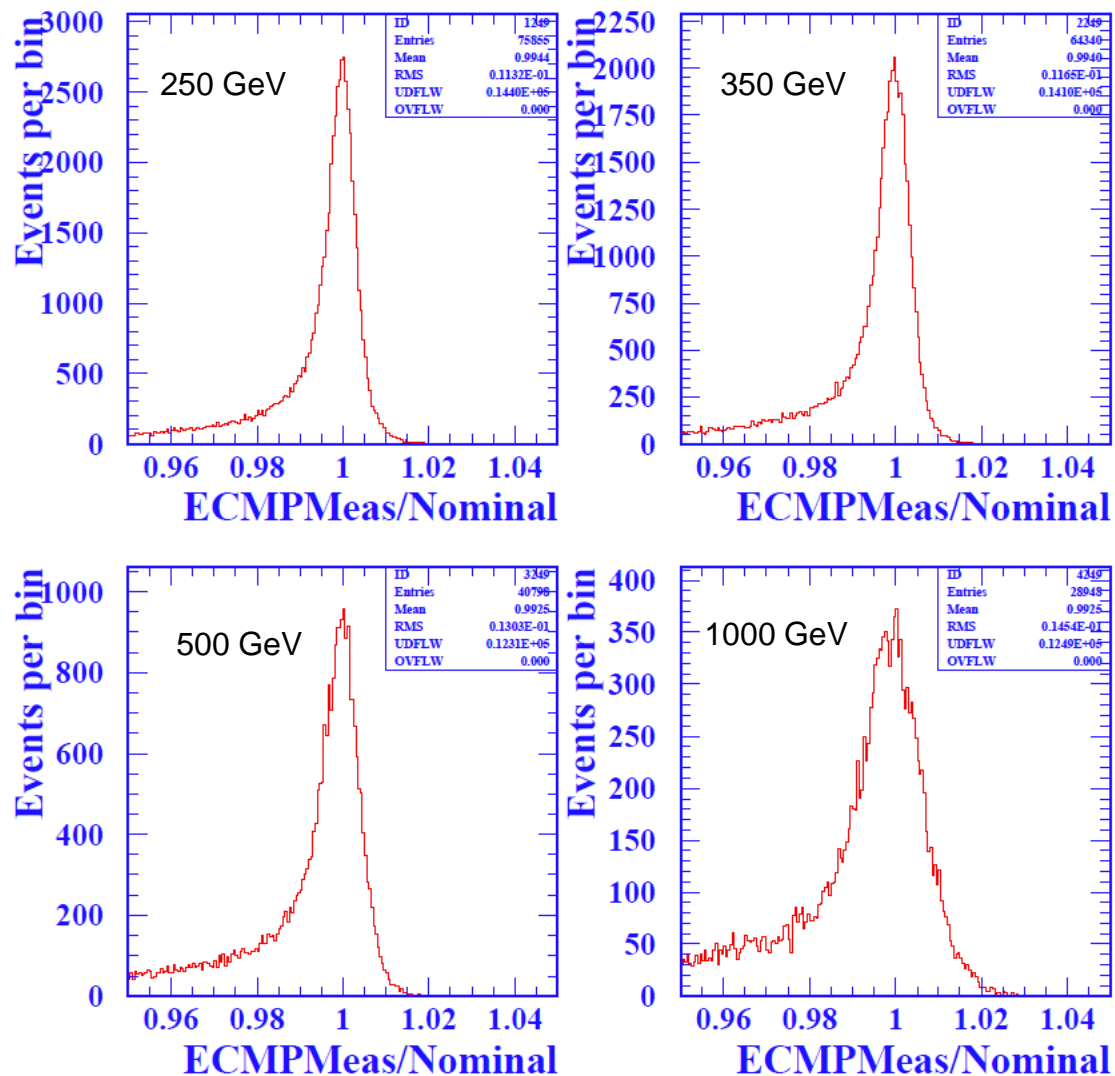
Error distribution is complicated. Reflects the kinematics, beamstrahlung, ISR, FSR, polar angles and p resolution.



Pull distribution has correct width. 10% +ve bias presumably due to errors being Gaussian in curvature ($1/p_T$) not in p.

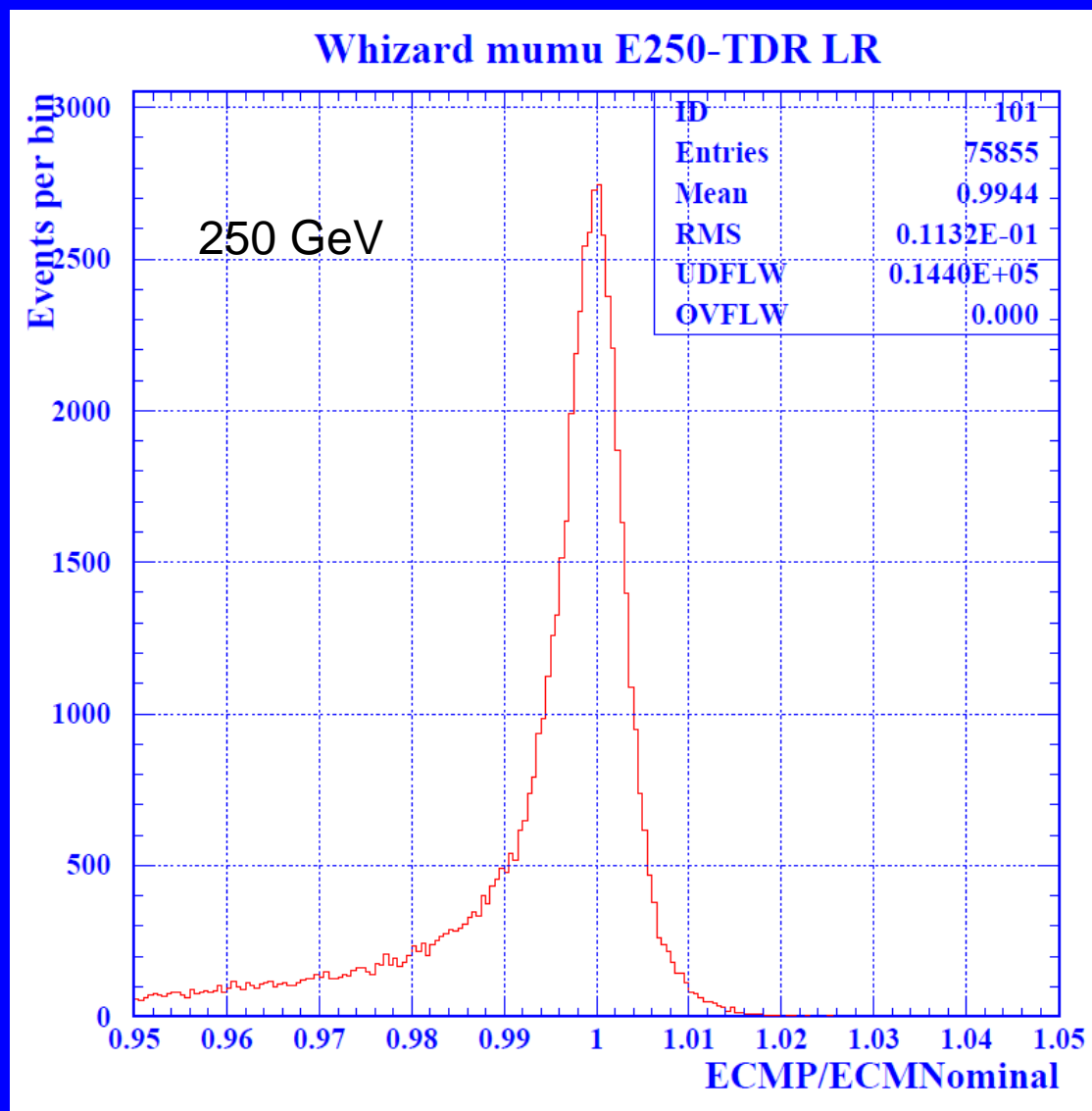
ECMP Distributions (error<0.8%)

Whizard $\mu\mu$ LR



Basic selection at 250 GeV: require error < 0.8%

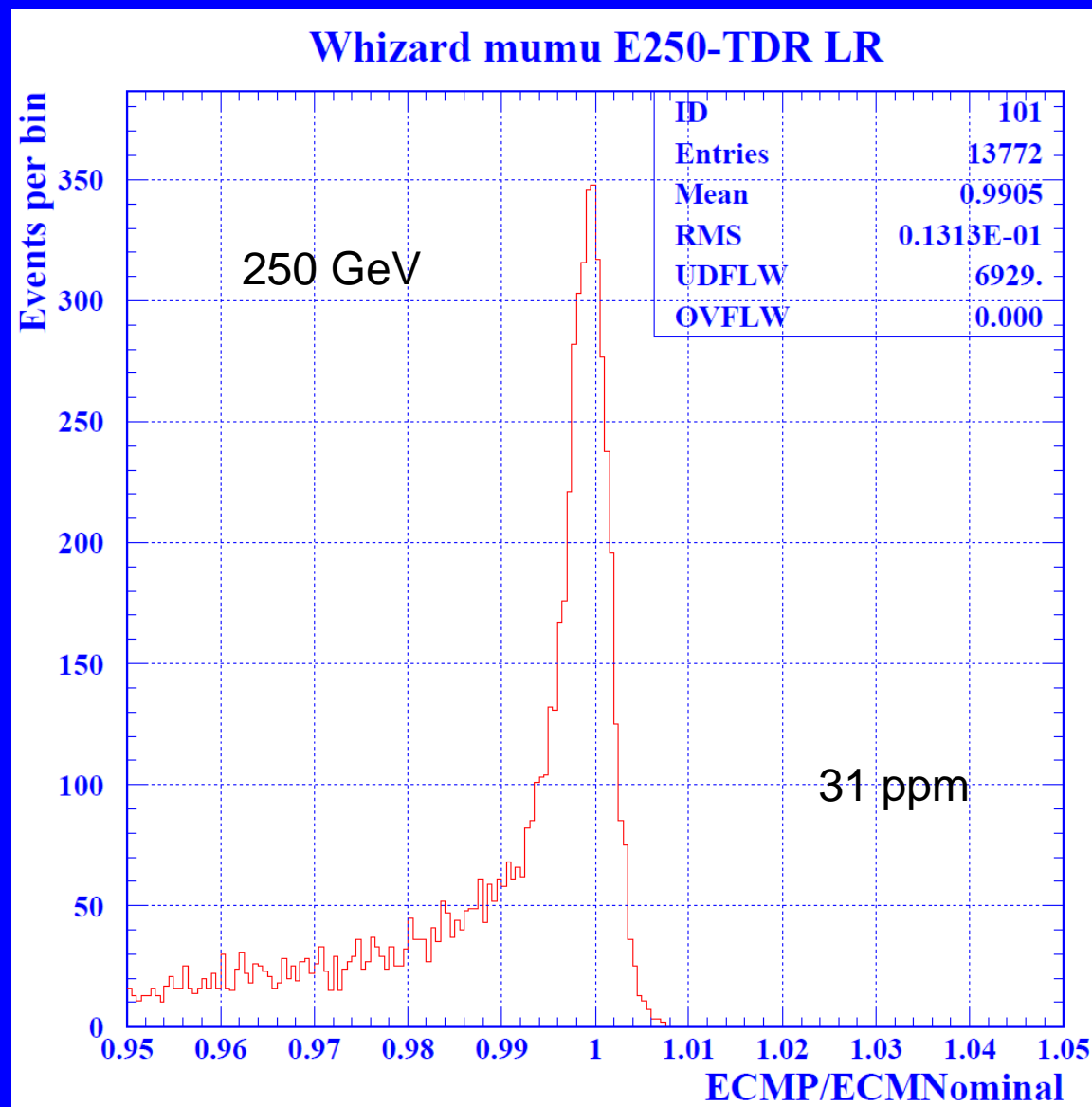
- Beam energy spread contributes 0.122% at 250 GeV.
- ECMP is well measured experimentally when the muons are in the acceptance.



Error < 0.15%

RMS width of peak is less than 0.20%.
As expected from convolving 0.12% with something like 0.13%.

Estimate error of 31 ppm for this sample based on 0.20% error and 60% of these events contributing to a measurement of the peak position.

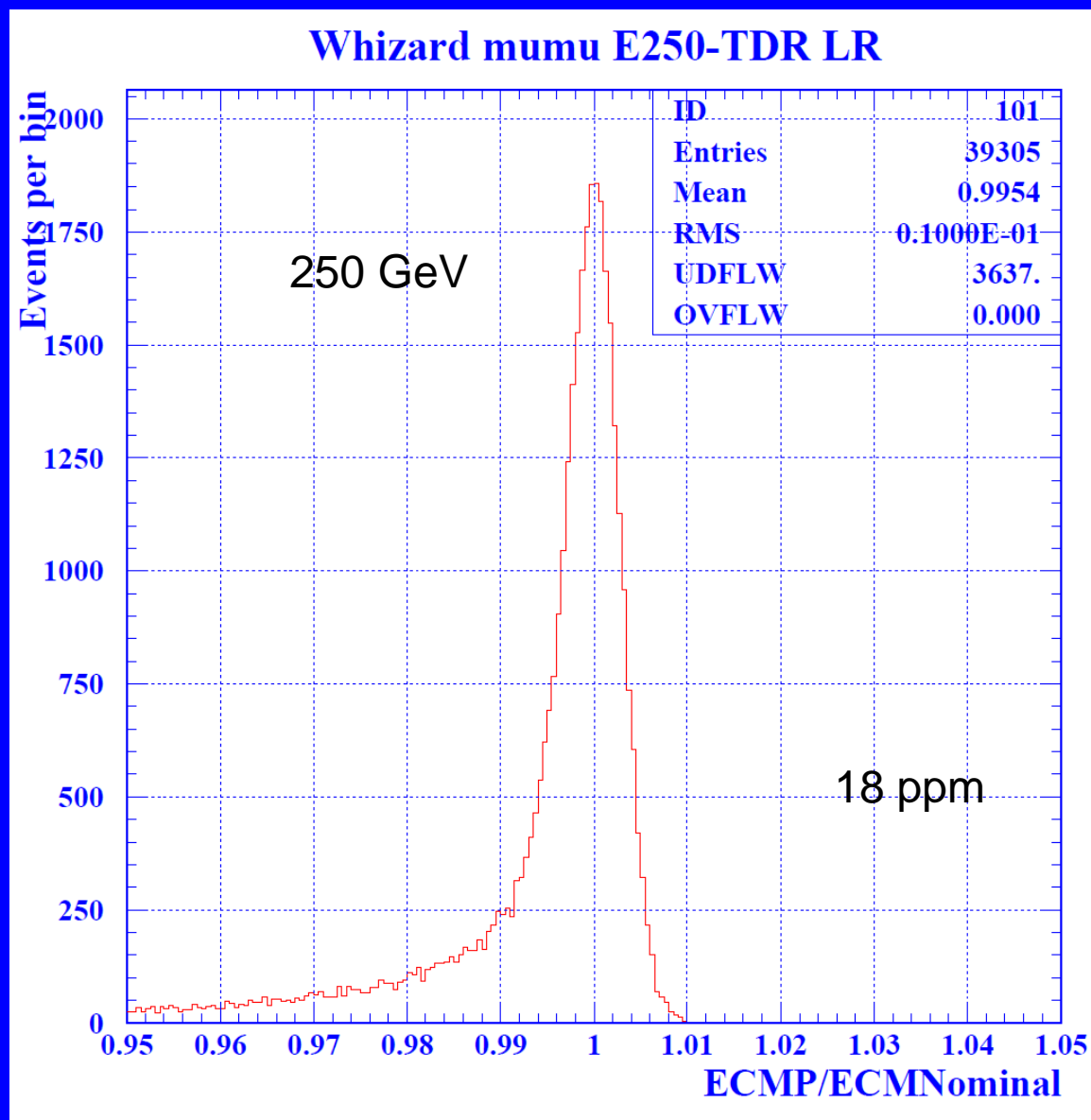


$$0.15\% < \text{Error} < 0.30\%$$

RMS width of peak is about 0.30%.

As expected from convolving 0.12% with something like 0.23%.

Estimate 80% in peak.

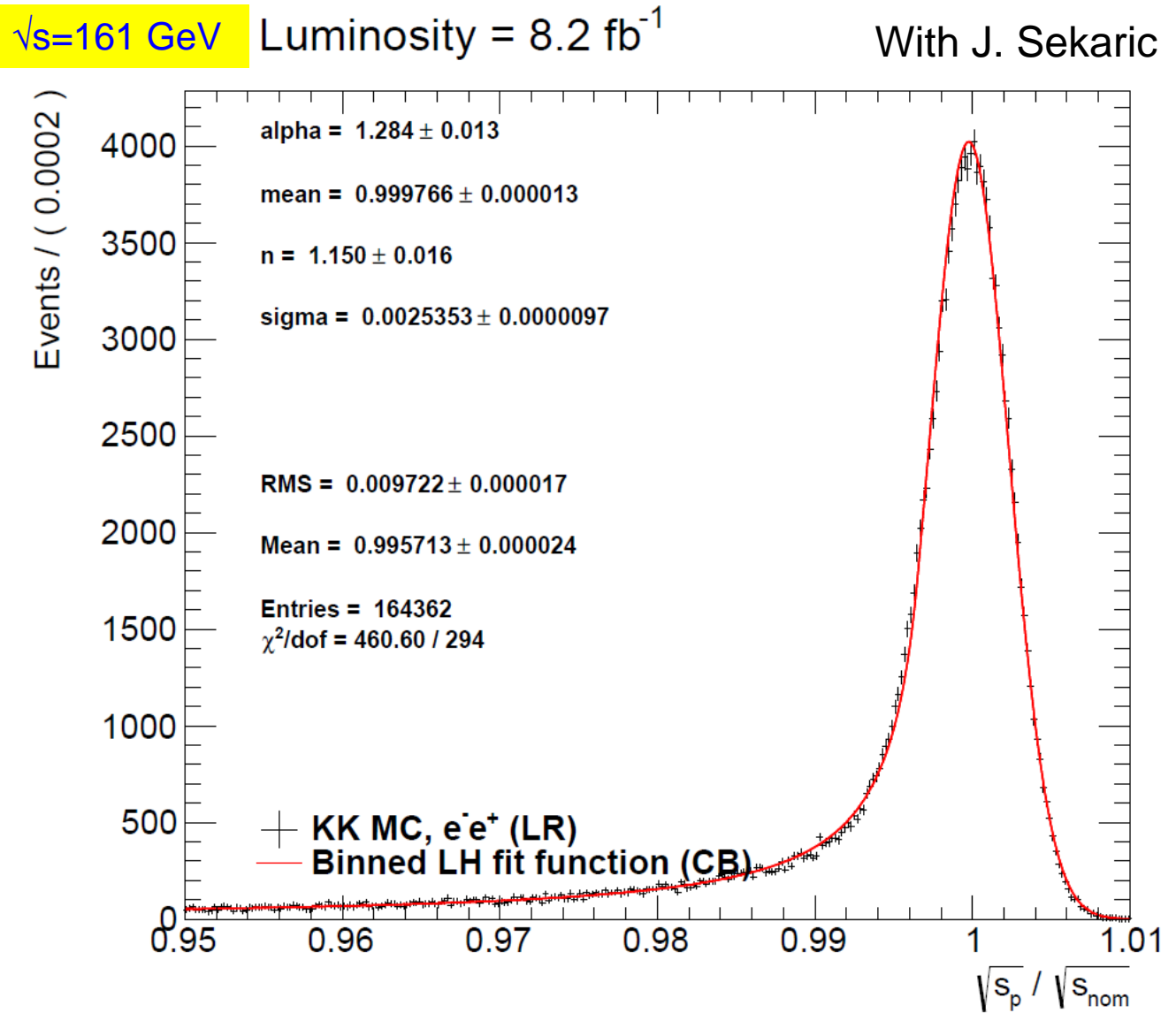


Statistical Error Estimation (in Progress)

Distributions before momentum resolution fit quite well to empirical function (Crystal Ball function)

Here error on scale parameter is 13 ppm.

One approach is to do a convolution fit, assuming that this distribution can be modelled.



Statistical Error Estimation

$\sqrt{s}=161 \text{ GeV}$

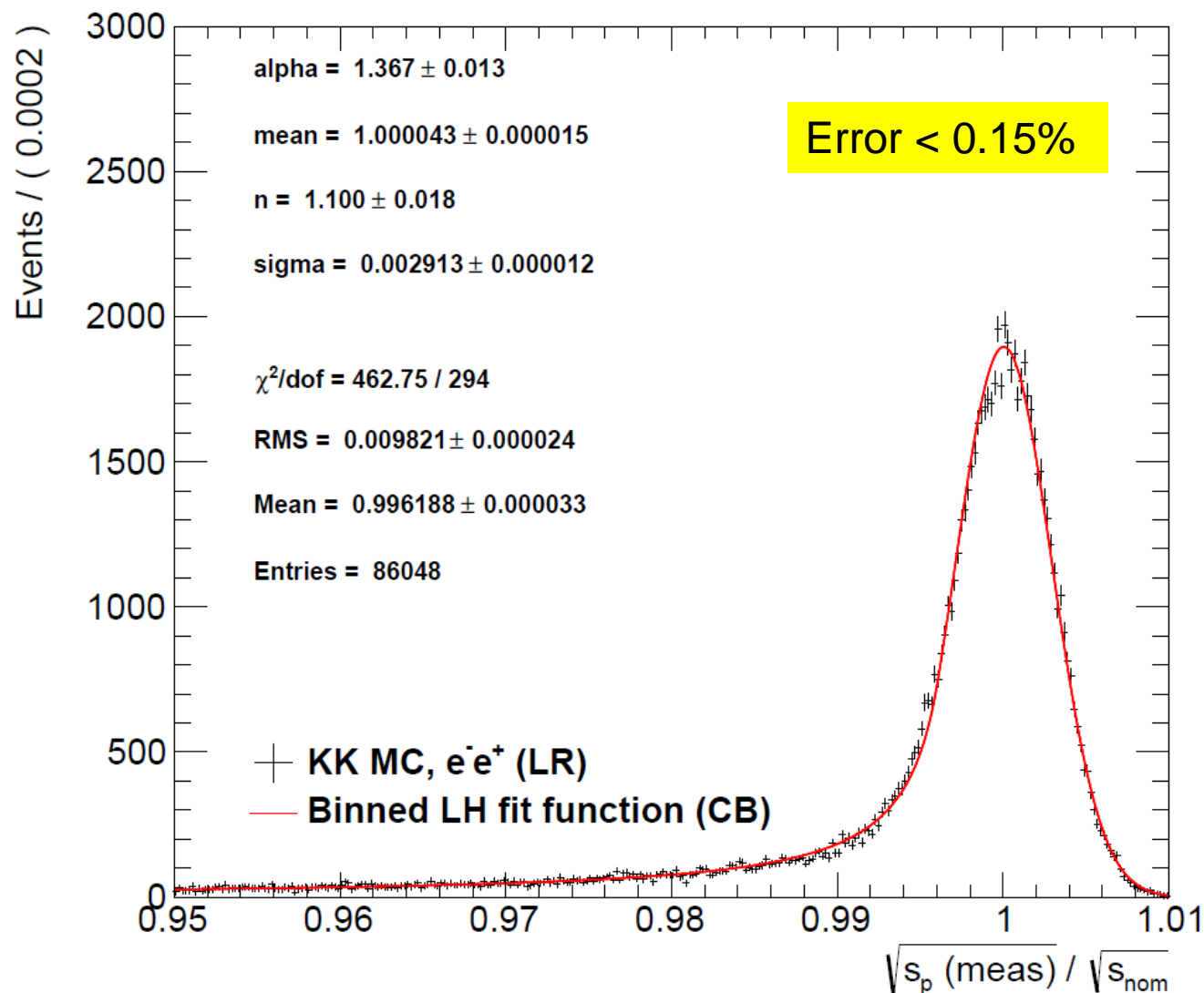
Luminosity = 8.2 fb^{-1}

With J. Sekaric

Distribution after momentum resolution also fits quite well to empirical function (Crystal Ball function)

Here error on scale parameter is 15 ppm.

Eventually may also measure the luminosity spectrum with this channel (dL/dx_1dx_2)



Summary Table

ECMP errors based on estimates from weighted averages from various error bins up to 2.0%. Assumes (80,30) polarized beams, equal fractions of +- and -+.

Preliminary

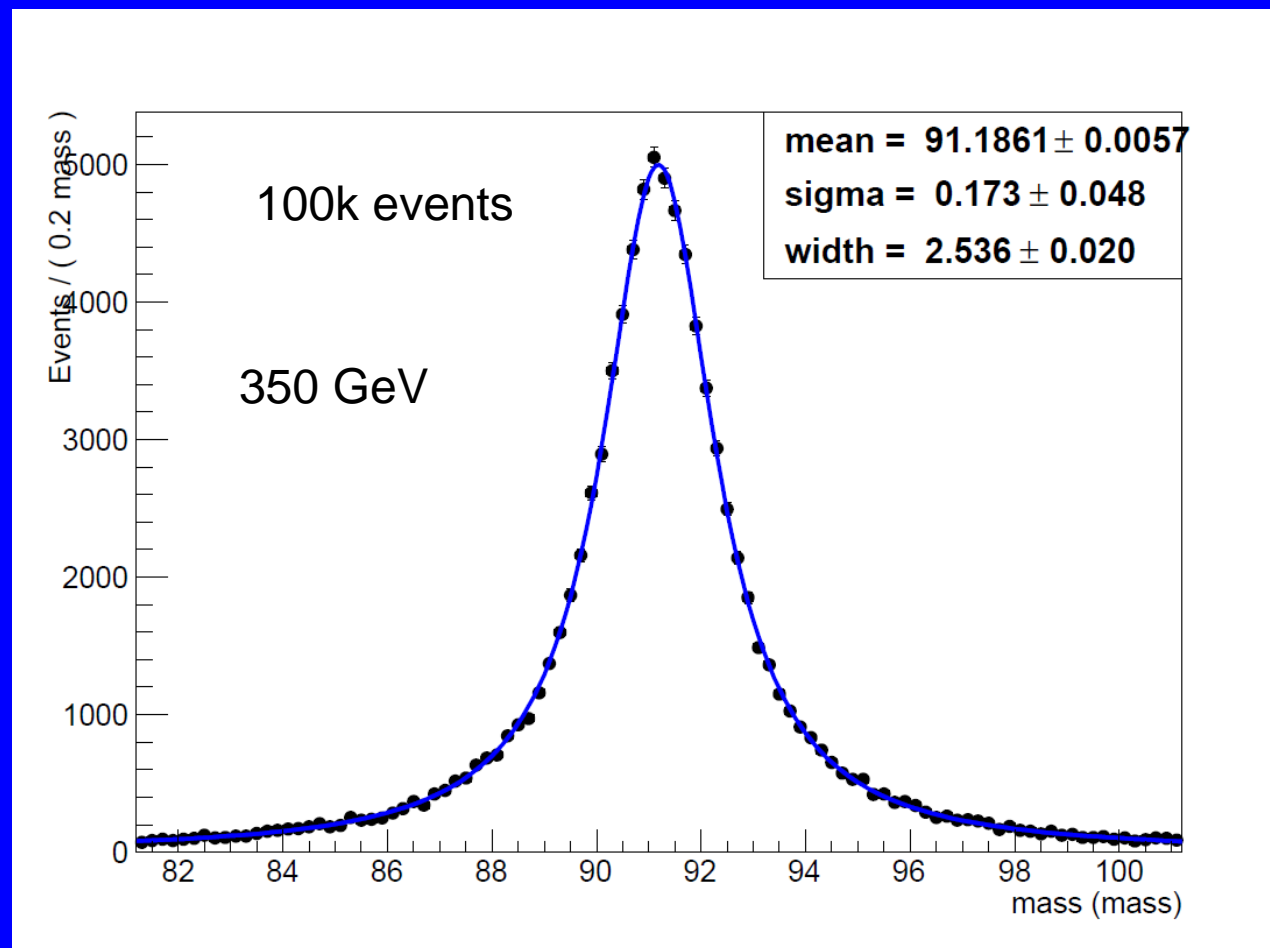
(Statistical errors only ...)

ECM (GeV)	L (inv fb)	$\Delta(\sqrt{s})/\sqrt{s}$ Angles (ppm)	$\Delta(\sqrt{s})/\sqrt{s}$ Momenta (ppm)	Ratio
161	161	-	4.3	
250	250	64	4.0	16
350	350	65	5.7	11.3
500	500	70	10.2	6.9
1000	1000	93	26	3.6

< 10 ppm for 150 – 500 GeV CoM energy

161 GeV estimate using KKMC.

Can control for p-scale using measured di-lepton mass



This is about 100 fb^{-1} at $\text{ECM}=350 \text{ GeV}$.

Statistical
sensitivity if one
turns this into a
Z mass
measurement (if
p-scale is
determined by
other means) is

$$1.8 \text{ MeV} / \sqrt{N}$$

With N in
millions.

Alignment ?
B-field ?
Push-pull ?
Etc ...

Conclusions

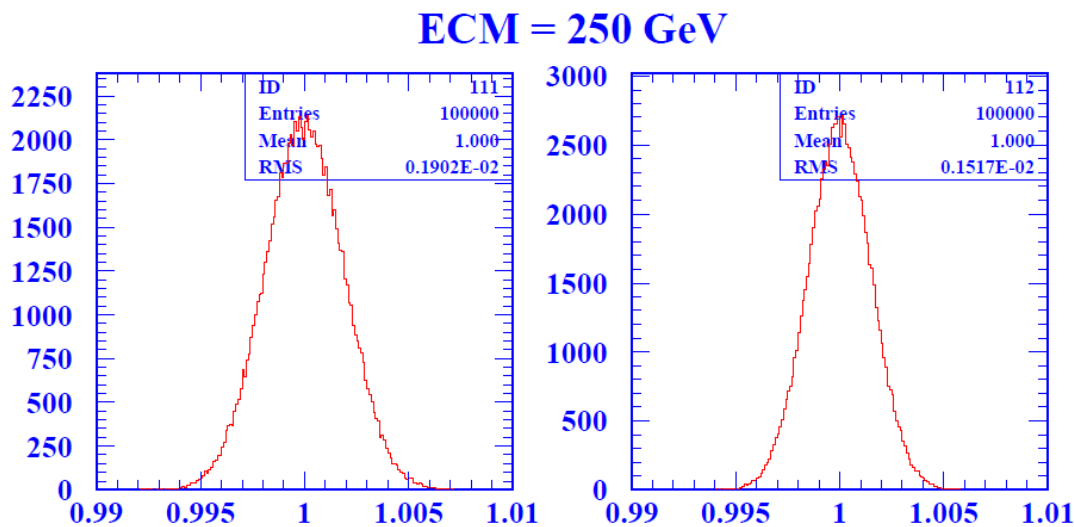
- Beam Energy
 - Statistical contribution < 0.5 MeV for mW
 - New P method works statistically for $\sqrt{s}=161$ -500 GeV
 - Systematics depend on measurement of p-scale
 - Z-based method: limited by Z statistics and Z mass
 - Ultimately 23 ppm (mZ based) $\Rightarrow \Delta m_W = 1.85$ MeV
 - Other methods? Lambda? 19 ppm limit. J/psi (need 91 GeV ?).
- mW measurement prospects
 - 3 methods each with scope to get below 5 MeV.
 - Complementary systematics
 - Important measurement worth measuring as well as possible
 - Ultimately 2.5 MeV error not out of the question.
 - Need more work on all 3 methods.
 - Current position “3-4 MeV” should be achievable.

Backup Slides

Check intrinsic resolution for Method P

$p(e^-) / 125.0$

0.19%

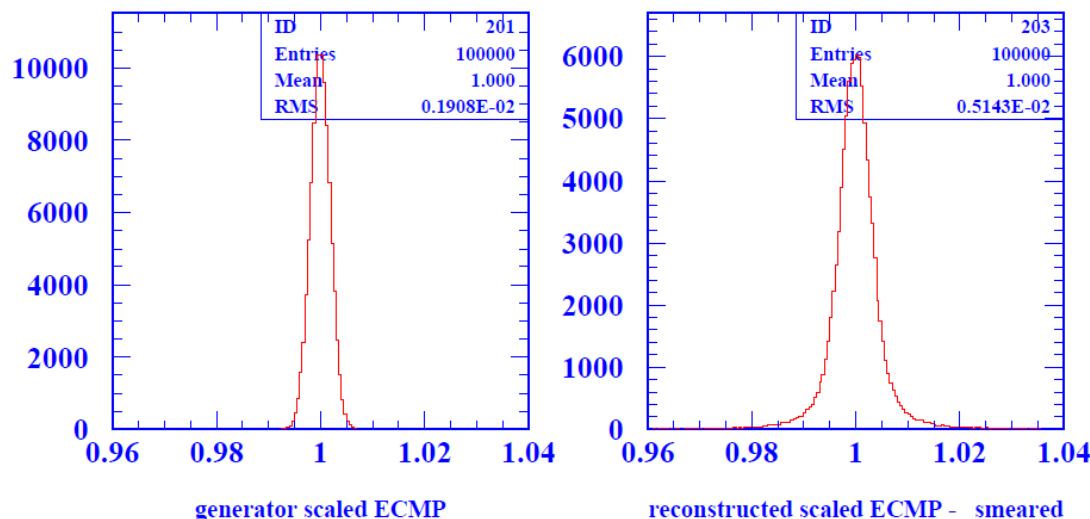


Electron momentum scaled to beam energy

Positron momentum scaled to beam energy

$p(e^+) / 125.0$

0.15%



$(E_1 + E_2 + p_{12})/250$

$(E_1 + E_2 + p_{12})/250$

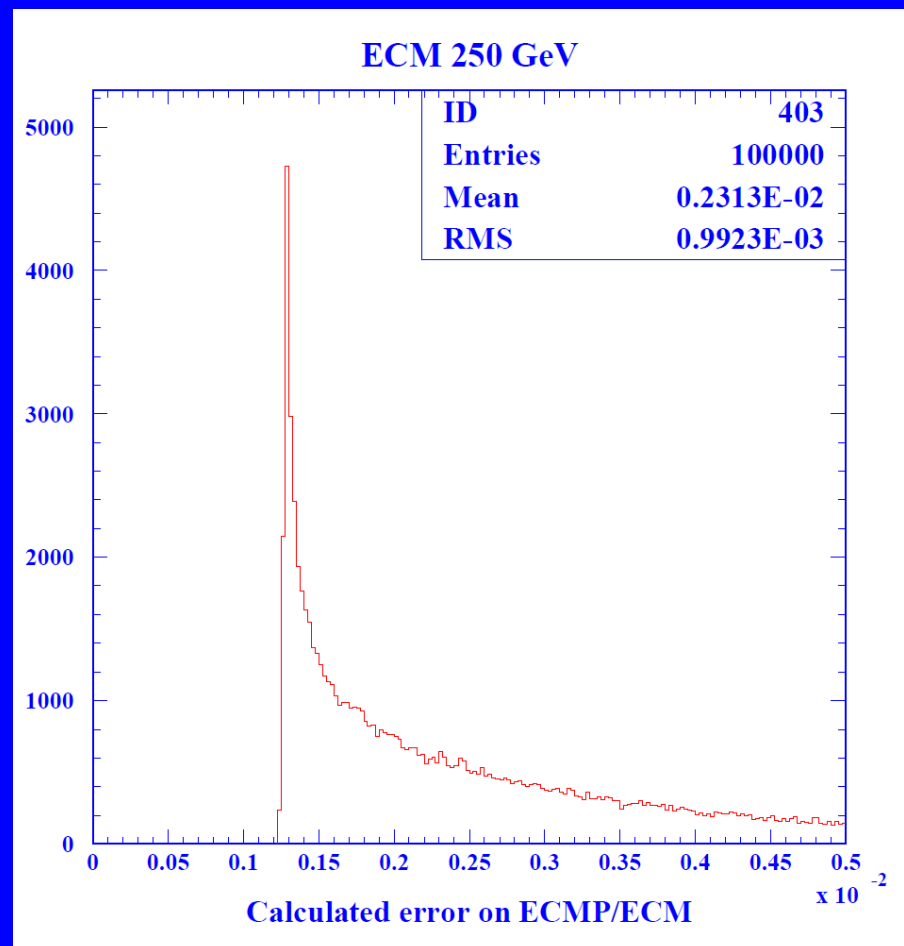
0.51%
(0.34%
central
part)

Contribution from Momentum Resolution.

Calculate error from the measured p_T 's and polar angles of each muon.

Combined this gives a range of errors from event-to-event with symmetric events having an error of around 0.14%.

Can also use this information to improve the statistical power.



Momentum Resolution

Currently use the large polar angle parametrization from ILD LOI (blue line).

$$\sigma_{1/p_T} = a \oplus b/(p_T \sin \theta)$$

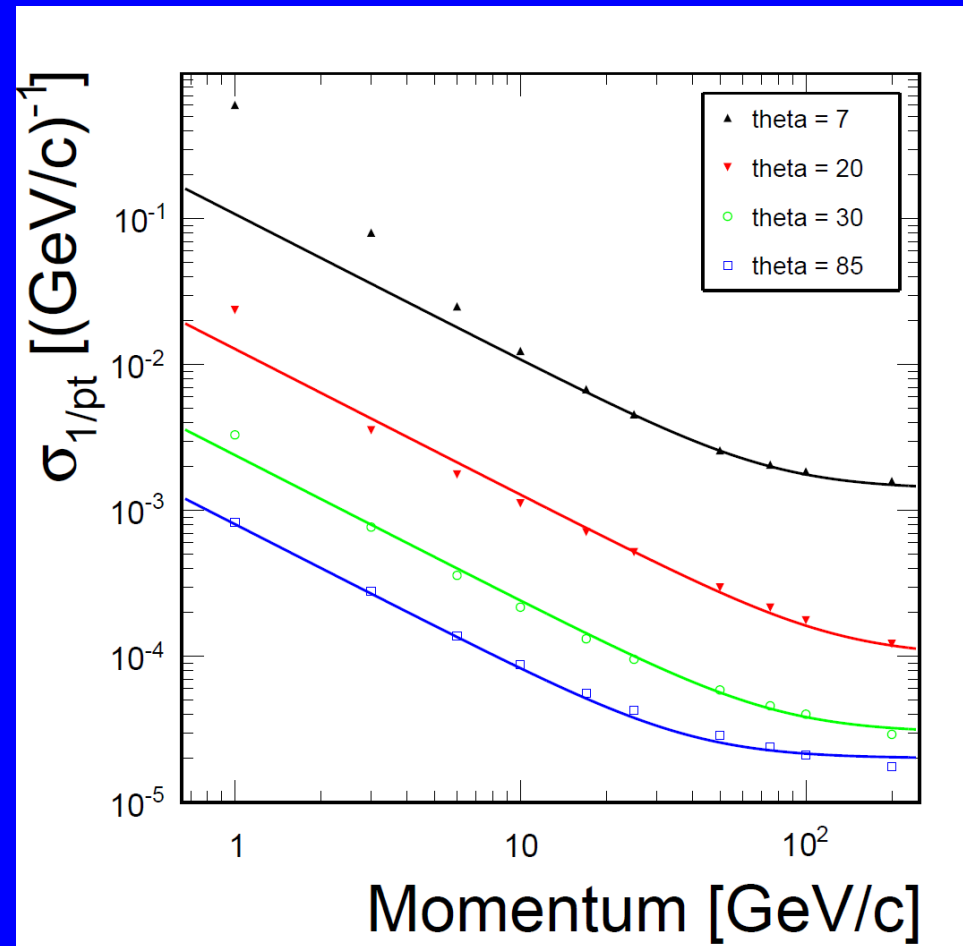
Where

$$a = 2 \times 10^{-5} \text{ GeV}^{-1} \text{ and } b = 1 \times 10^{-3}$$

Should be OK for the full TPC coverage ($\theta > 37^\circ$)

Plot is data from Steve Aplin's macro. Superimposed curves have a,b parameters tweaked for $\theta=7^\circ, 20^\circ, 30^\circ$ to give a decent fit for $p > 10 \text{ GeV}$.

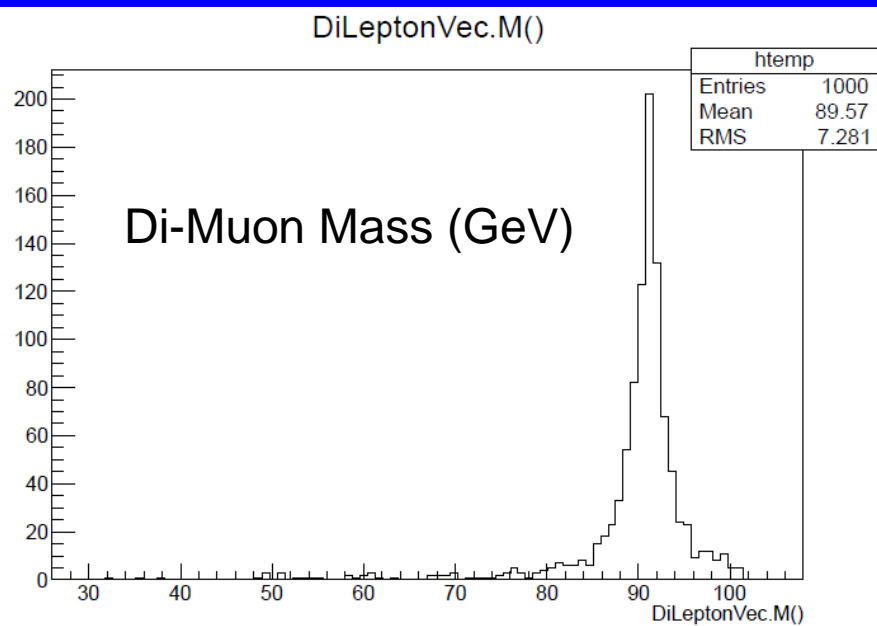
Will need good parametrized description of this and/or use SGV particularly for high \sqrt{s} (for highly boosted di-muons).



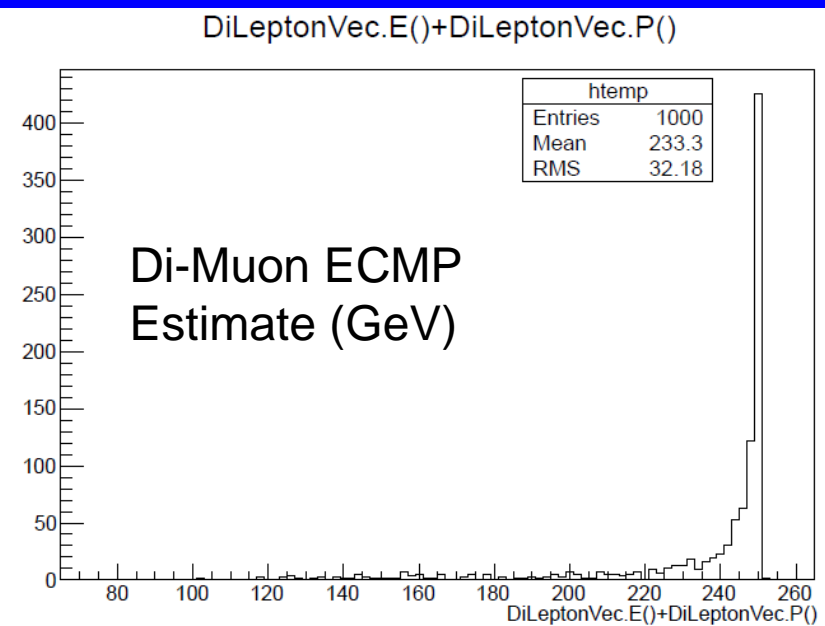
Whizard Generator Level Studies

ECM = 250 GeV. $e^-_L e^+_R \rightarrow \mu^- \mu^+$

Require $81.2 < M < 101.2$ GeV. $\sin\theta > 0.12$. $\sigma = 3.84 \pm 0.02$ pb



Tail to low mass from FSR

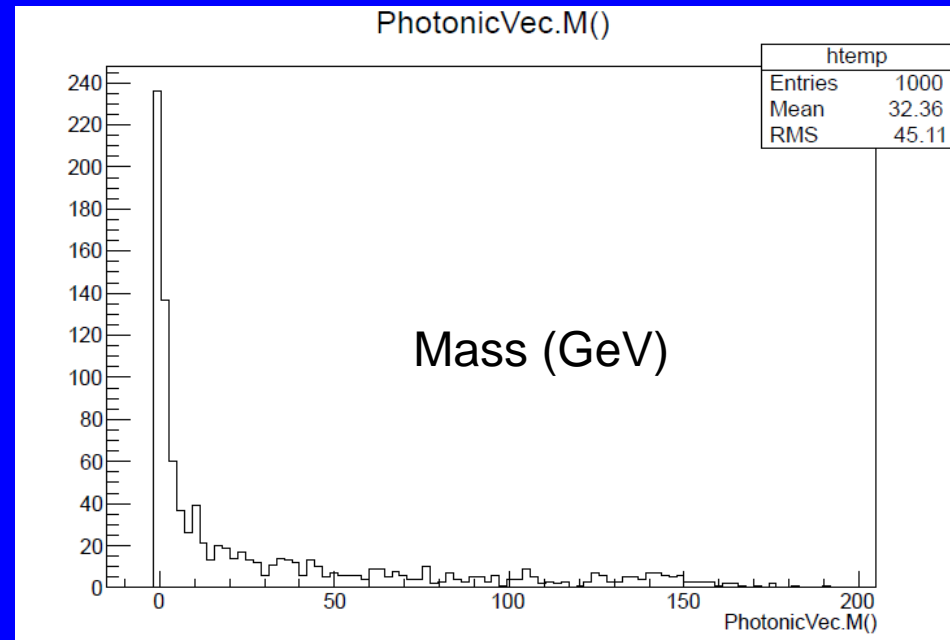
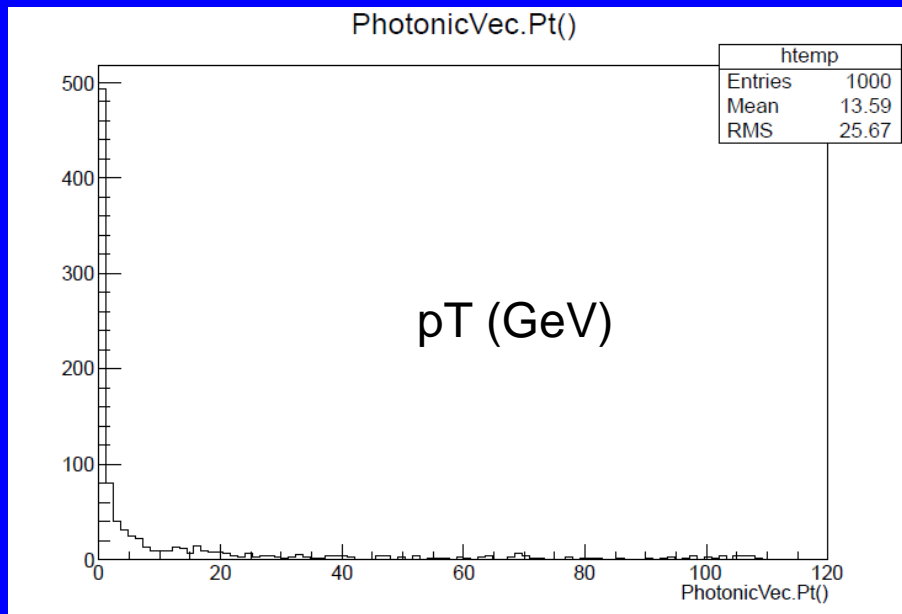


Distribution is sensitive to luminosity spectrum. Not clear to me if beam energy spread is properly included.

Whizard Generator Level Studies

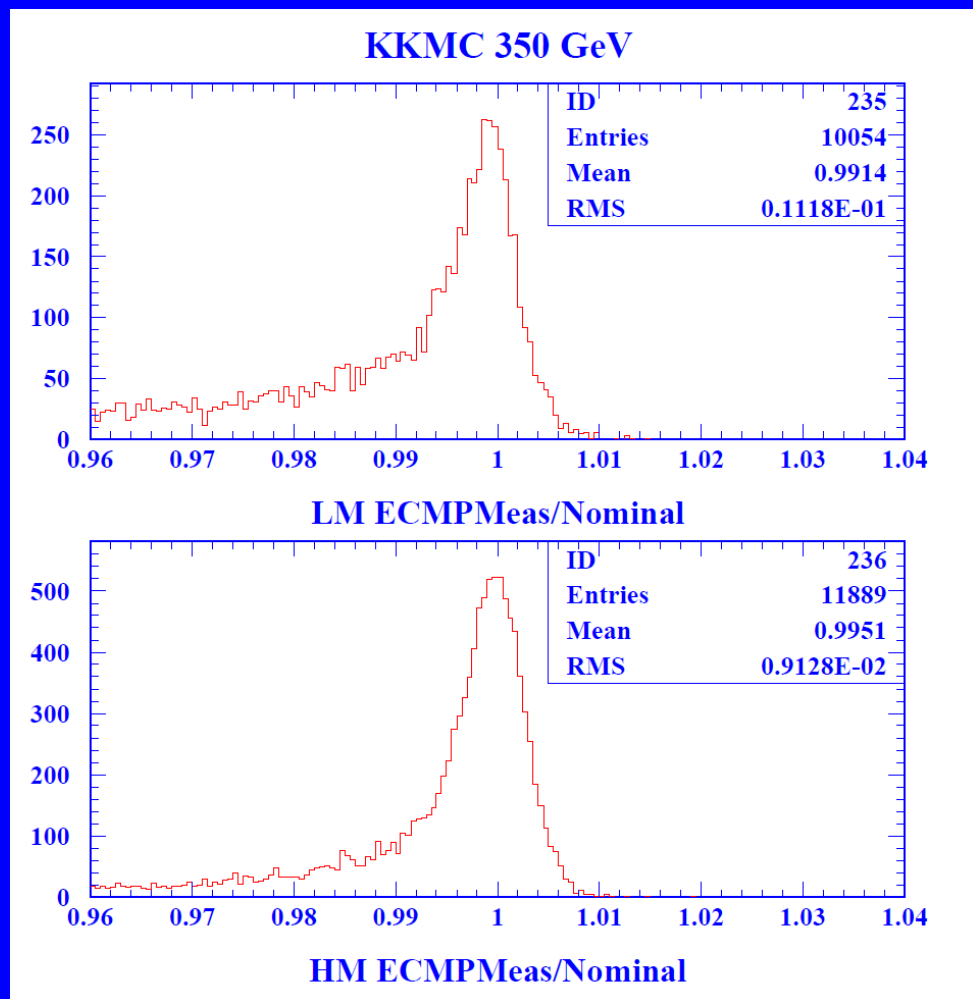
ECM = 250 GeV. $e^-e^+ \rightarrow \mu^+\mu^-$

Check characteristics of photonic system (ISR + FSR).



As expected, photonic system usually has small p_T , and low mass – making 3-body assumption often plausible. But double ISR from opposite beam particles does give long tail to high mass.

KKMC Study contd.



$m_{12} < 200 \text{ GeV}$

$m_{12} > 200 \text{ GeV}$

High mass and low mass have similar sensitivities. High mass – more events in peak, less tail - but worse intrinsic resolution (high p_T).

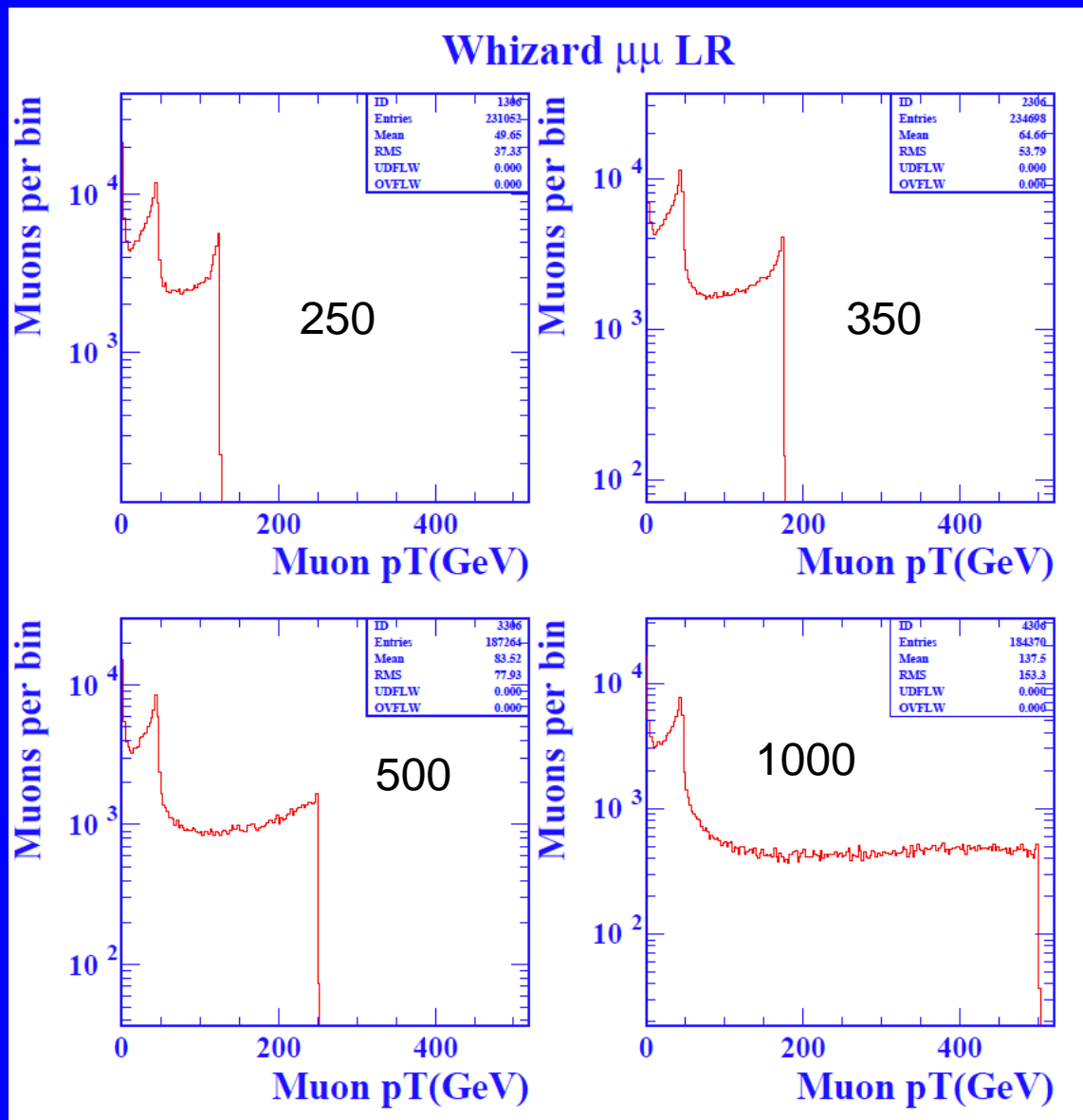
Tim's Conjecture

- Slides from Tim suggest that one can fit for the tracker momentum scale without using the Z peak.
- This does not appear to be the case in my simplified tests with 3-body zero pT photon with $m \approx m_Z$ and no additional complications.
- Tests done with shifted \sqrt{s} and shifted tracker momentum-scale factors
 - see no ability to distinguish a shift in one from a shift in the other.
- Because of the basic 1-1 correspondence between track pT and the \sqrt{s}_p estimate, this seems to me unlikely to be correct.

$$\sqrt{s}_p = p_T \left(\frac{1 + \cos \theta_1}{\sin \theta_1} + \frac{1 + \cos \theta_2}{\sin \theta_2} \right)$$

- This is a pity – but we should have handles on the momentum scale – not least the Z mass.

Muon pT distributions



Note that ILD DBD momentum resolution numbers only verified up to $p = 100$ GeV. But expected to be reliable.

Beam Energy Spread

- Current ILC Design.
- Not a big issue especially at high \sqrt{s}

IP RMS Energy spreads (%)

Centre of mass energy (GeV)		200	230	250
Damping ring @ 5GeV	e+	0,137	0,137	0,137
	e-	0,12	0,12	0,12
RTML @ 15 GeV (assume no z-correlation)	e+	1,23	1,23	1,23
	e-	1,17	1,17	1,17
Main linac	e+	0,185	0,160	0,148
	e-	0,176	0,153	0,140
Long. wakefield contribution		0,046	0,039	0,036
Positron undulator contribution	e-	0,098	0,113	0,123
IP value	e+	0,190	0,165	0,152
	e-	0,206	0,194	0,190

350	500
0,11	0,11
0,12	0,12
1,13	1,13
1,13	1,13
0,097	0,068
0,097	0,068
0,026	0,018
0,122	0,103
0,100	0,070
0,158	0,124

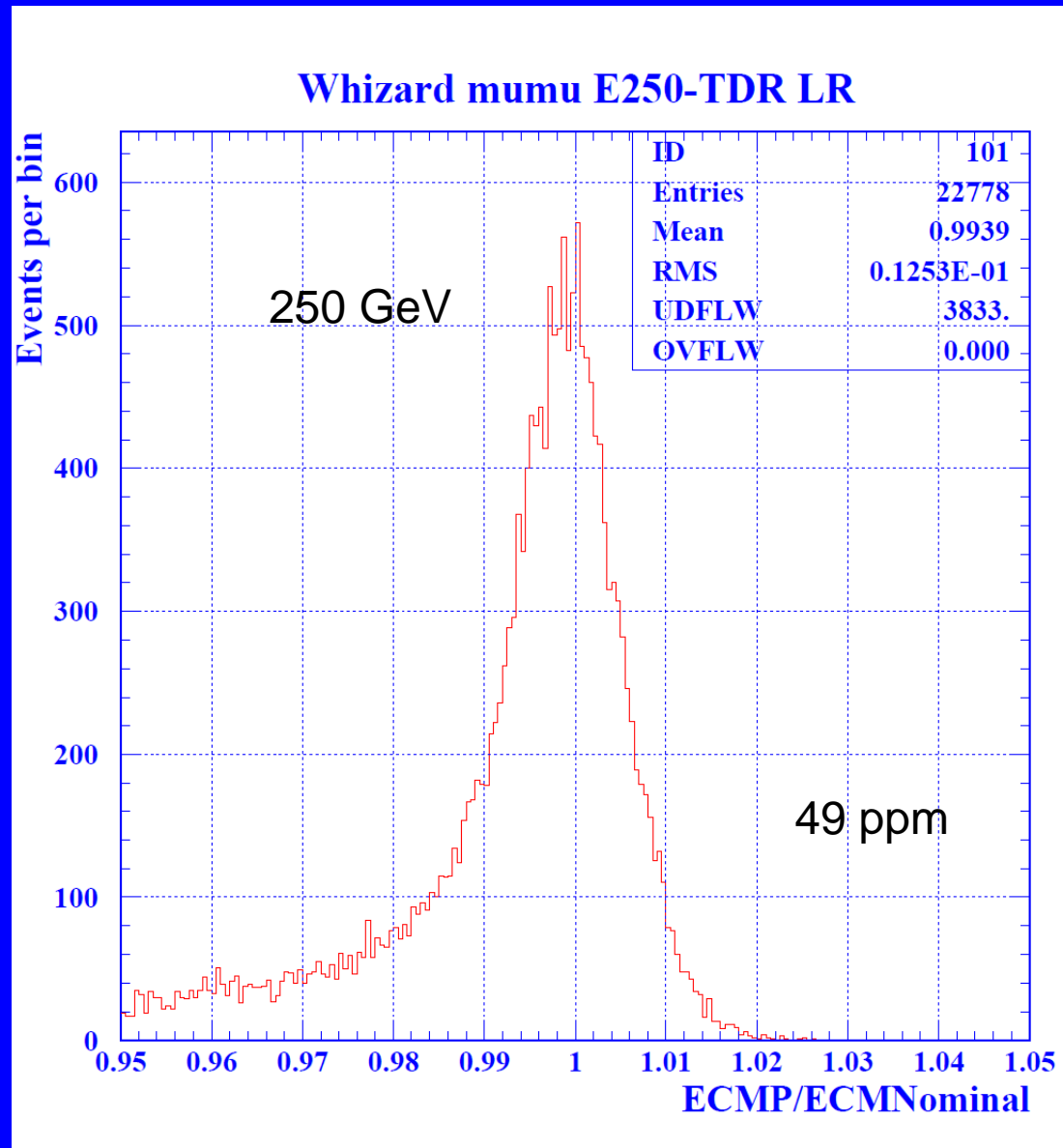
1000	1000
A1	B1B
0,250	0,225
0,109	0,109
1,36	1,51
0,041	0,045
0,014	0,014
0,071	0,071
0,043	0,047
0,083	0,085

LEP2 was 0.19% per beam at 200 GeV.

$$0.30\% < \text{Error} < 0.80\%$$

RMS width of peak is
about 0.6%.

Estimate 80% in
peak



Statistical Errors

- Numbers on 250 GeV slides estimated for the statistics of 1 LR stdhep file (10.4 inv fb).
- Weighted average of the 3 bins – gives 15 ppm on peak \sqrt{s} .
- Canonical 250 inv fb at 250 GeV with equal weights of LR, RL and (80,30) polarization, gives 4 ppm on peak \sqrt{s} .
- (Remember 10 ppm on mW is 0.8 MeV)
 - Good prospects for beam energy precision at a level far better than what is required to make beam energy error for W mass measurements negligible.