

# **Status of predictions for $M_W$ and $Z$ -pole observables**

**A. Freitas**

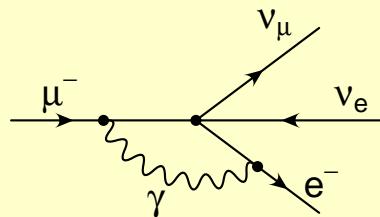
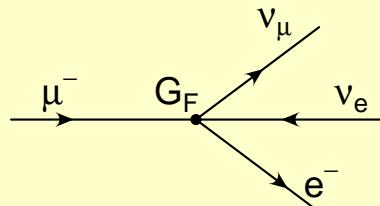
**University of Pittsburgh**

- 1. Overview**
- 2. Current status of SM predictions**
- 3. Conclusions**

# Overview: Electroweak Precision Observables

$W$  mass

$\mu$  decay in Fermi Model

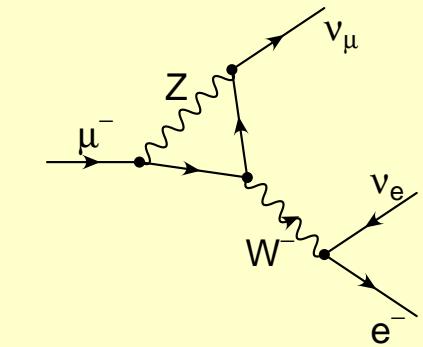
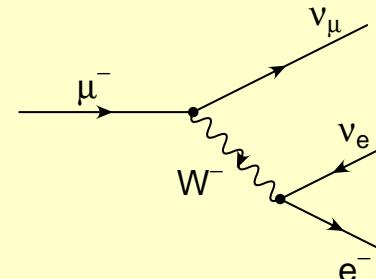


← QED corr.  
(2-loop)

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right) (1 + \Delta q)$$

Ritbergen, Stuart '98

$\mu$  decay in Standard Model



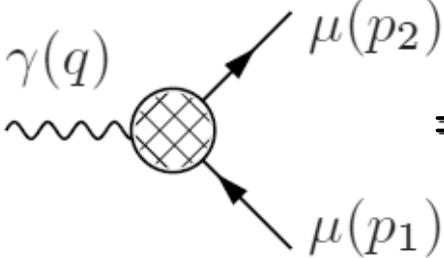
$$\frac{G_F^2}{\sqrt{2}} = \frac{e^2}{8s_W^2 M_W^2} (1 + \Delta r)$$

↑  
electroweak corrections

Experiment:  $M_W = 80.385 \pm 0.015$  GeV

PDG '12

# Muon anomalous magnetic moment



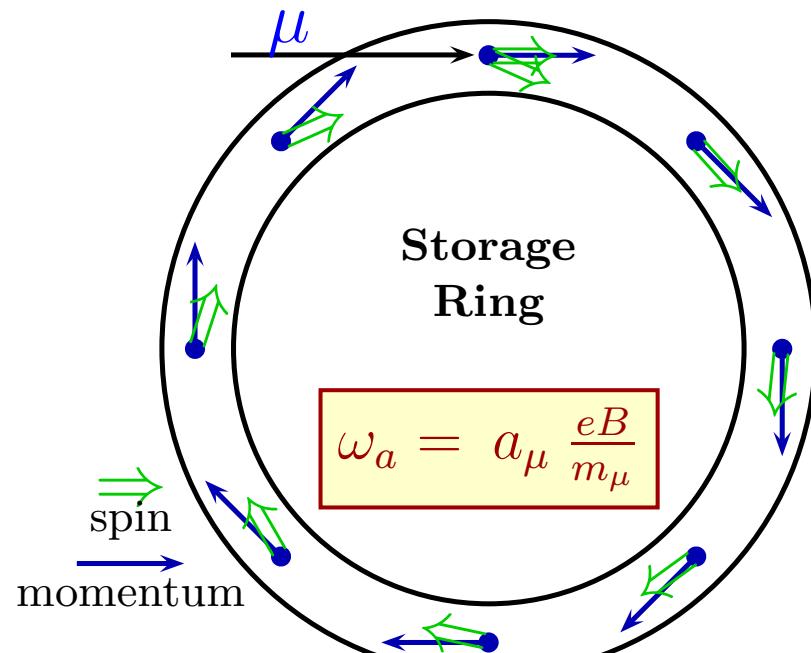
Feynman diagram showing a muon ( $\mu(p_1)$ ) interacting with a virtual photon ( $\gamma(q)$ ) to produce another muon ( $\mu(p_2)$ ). The outgoing muon has momentum  $\mu(p_2)$ .

$$\gamma(q) \sim \mu(p_1) \rightarrow \mu(p_2)$$
$$= (-ie) \bar{u}(p_2) \left[ \gamma^\mu F_E(q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2m_\mu} F_M(q^2) \right] u(p_1)$$

$$a_\mu = F_M(0)$$

Measured at NBL g-2 experiment:

$$a_\mu = (11\,659\,208.0 \pm 6.3) \times 10^{-10}$$



Jegerlehner, Nyfeller '09

actual precession  $\times 2$

## Z-pole observables

$e^+e^- \rightarrow f\bar{f}$  for  $\sqrt{s} \sim m_Z$ :

$$\frac{d\sigma}{d\cos\theta} = \mathcal{R}_{\text{ini}} \left[ \frac{9}{2}\pi \frac{\Gamma_{ee}\Gamma_{ff}[(1 - \mathcal{P}_e\mathcal{A}_e)(1 + \cos^2\theta) + 2(\mathcal{A}_e - \mathcal{P}_e)\mathcal{A}_f \cos\theta]}{(s - m_Z^2)^2 - m_Z^2\Gamma_Z^2} \right. \\ \left. + \sigma_{\text{non-res}} \right],$$

$$\Gamma_{ff} = \mathcal{R}_V^f g_{Vf}^2 + \mathcal{R}_A^f g_{Af}^2, \quad \Gamma_Z = \sum_f \Gamma_{ff},$$

$$\mathcal{A}_f = 2 \frac{g_{Vf}/g_{Af}}{1 + (g_{Vf}/g_{Af})^2} = \frac{1 - 4|Q_f|\sin^2\theta_{\text{eff}}^f}{1 - 4|Q_f|\sin^2\theta_{\text{eff}}^f + 8(|Q_f|\sin^2\theta_{\text{eff}}^f)^2}.$$

## Z-pole observables

$e^+e^- \rightarrow f\bar{f}$  for  $\sqrt{s} \sim m_Z$ :

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# QED/QCD corrections on ext. fermions

- Chetyrkin, Kataev, Tkachov '79
- Dine, Saphirstein '79
- Celmaster, Gonsalves '80
- Gorishnii, Kataev, Larin '88,91

Chetyrkin, Kühn '90                    etc...  
Surguladze, Samuel '91  
Kataev '92  
Chetyrkin '93

## Z-pole observables

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additional initial-state QED corrections

Kuraev, Fadin '85

Berends, Burgers, v. Neerven '88

Kniehl, Krawczyk, Kühn, Stuart '88

Beenakker, Berends, v. Neerven '89

Bardin et al. '89, 91

Montagna, Nicrosini, Piccinini '97  
etc...

## Z-pole observables

$e^+e^- \rightarrow f\bar{f}$  for  $\sqrt{s} \sim m_Z$ :

$$\frac{d\sigma}{d\cos\theta} = \mathcal{R}_{\text{ini}} \left[ \frac{9}{2}\pi \frac{\Gamma_{ee}\Gamma_{ff}[(1 - \mathcal{P}_e\mathcal{A}_e)(1 + \cos^2\theta) + 2(\mathcal{A}_e - \mathcal{P}_e)\mathcal{A}_f \cos\theta]}{(s - m_Z^2)^2 - m_Z^2\Gamma_Z^2} \right. \\ \left. + \sigma_{\text{non-res}} \right],$$

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electroweak corrections

## Z-pole observables

Measured quantities:

- $\sigma_{\text{had}}^0 = \frac{12\pi}{m_Z^2} \frac{\Gamma_{ee}\Gamma_{\text{had}}}{\Gamma_Z^2}$   $\sigma_{\text{had}}^0 = 41.540 \pm 0.037 \text{ nb}$
- $R_f = \Gamma_{ff}/\Gamma_{\text{had}}$   $R_b = 0.21629 \pm 0.00066$
- $\Gamma_Z$   $\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$
- $A_{\text{FB}}^f \equiv \frac{\sigma(\theta < \frac{\pi}{2}) - \sigma(\theta > \frac{\pi}{2})}{\sigma(\theta < \frac{\pi}{2}) + \sigma(\theta > \frac{\pi}{2})} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_f$   $A_{\text{FB}}^b = 0.0992 \pm 0.0016$
- $A_{\text{LR}} \equiv \frac{\sigma(\mathcal{P}_e > 0) - \sigma(\mathcal{P}_e < 0)}{\sigma(\mathcal{P}_e > 0) + \sigma(\mathcal{P}_e < 0)} = \mathcal{A}_e$   $A_{\text{LR}} = 0.1513 \pm 0.0021$

→ Each observable involves several theoretical building blocks,  
in different combinations

## Current status of SM predictions

Methods for theory error estimates:

- Parametric factors, *i.e.* factors of  $\alpha$ ,  $N_c$ ,  $N_f$ , ...
- Geometric progression, *e.g.*  $\frac{\mathcal{O}(\alpha^3)}{\mathcal{O}(\alpha^2)} \sim \frac{\mathcal{O}(\alpha^2)}{\mathcal{O}(\alpha)}$
- Renormalization scheme dependence (use with care!)
- Experience from similar calculations

## $W$ mass

- Complete NNLO corrections Freitas, Hollik, Walter, Weiglein '00  
Awramik, Czakon '02; Onishchenko, Veretin '02
- Partial 3/4-loop corrections Chetyrkin, Kühn, Steinhauser '95  
Faisst, Kühn, Seidensticker, Veretin '03  
Boughezal, Tausk, v. d. Bij '05; Schröder, Steinhauser '05  
Chetyrkin et al. '06; Boughezal, Czakon '06
- Theoretical error  $\delta_{\text{th}} \sim 4$  MeV  
mainly from  $\mathcal{O}(\alpha^2 \alpha_s)$ ,  $\mathcal{O}(N_f^{\geq 2} \alpha^3)$  (3-loop self-energies)  
 $\rightarrow \delta_{\text{th,future}} \lesssim 1$  MeV
- Exp. uncertainty  $\delta_{\text{exp}} \sim 15$  MeV  
 $\delta_{\text{ILC}} \sim 2 \dots 7$  MeV  $\rightarrow$  G. Wilson's talk  
 $\delta_{\text{TLEP}} < 1$  MeV  $\rightarrow$  A. Blondel's talk
- Uncertainty from input parameters:

$\delta m_t = 1$ GeV:	$\sim 6$ MeV	$\delta M_H = 1$ GeV:	$\sim 0.5$ MeV
$\delta(\Delta\alpha) = 3 \times 10^{-4}$ :	$\sim 5.5$ MeV	$\delta \alpha_s = 0.001$ :	$\sim 0.6$ MeV
$\delta M_Z = 2.1$ MeV;	$\sim 2.6$ MeV		

# Muon anomalous magnetic moment

- Complete 5-loop QED  
Aoyama, Hayakawa, Kinoshita, Nio '12
- Complete NNLO  
Czarnecki, Krause, Marciano '96  
Knecht et al. '02  
Czarnecki, Marciano, Vainshtain '03
- Theory error  $\delta_{\text{th}} \sim 6.5 \times 10^{-10}$   
mainly from hadronic contributions
- Exp. error  $\delta_{\text{exp}} \sim 6.3 \times 10^{-10}$

# Effective weak mixing angles

$\sin^2 \theta_{\text{eff}}^\ell$ :

- Complete NNLO corrections

Awramik, Czakon, Freitas, Weiglein '04

Awramik, Czakon, Freitas '06

Hollik, Meier, Uccirati '05,07

- Partial 3/4-loop corrections

Chetyrkin, Kühn, Steinhauser '95

Faisst, Kühn, Seidensticker, Veretin '03

Boughezal, Tausk, v.d. Bij '05; Schröder, Steinhauser '05

Chetyrkin et al. '06; Boughezal, Czakon '06

- Theoretical error  $\delta_{\text{th}} \sim 4.5 \times 10^{-5}$ ; mainly from

$\mathcal{O}(\alpha^2 \alpha_s)$  (3-loop diagrams, doable as large- $m_t$  expansion)

$\mathcal{O}(N_f^{\geq 2} \alpha^3)$  (3-loop vertices with sub-bubbles)

$\rightarrow \delta_{\text{th,future}} \sim 10^{-5}$

- Exp. uncertainty  $\delta_{\text{exp}} \sim 16 \times 10^{-5}$

$\delta_{\text{ILC}} \sim 1.3 \times 10^{-5}$ ,  $\delta_{\text{TLEP}} \sim 2 \times 10^{-6}$ ?

# Effective weak mixing angles

$\sin^2 \theta_{\text{eff}}^\ell$ :

- Complete NNLO corrections

Awi

- Partial 3/4-loop corrections

Fai

Boughezal, Tausk, v.d.  
Chetyrkin et al.

- Theoretical error  $\delta_{\text{th}} \sim 4.5 \times 10^{-5}$

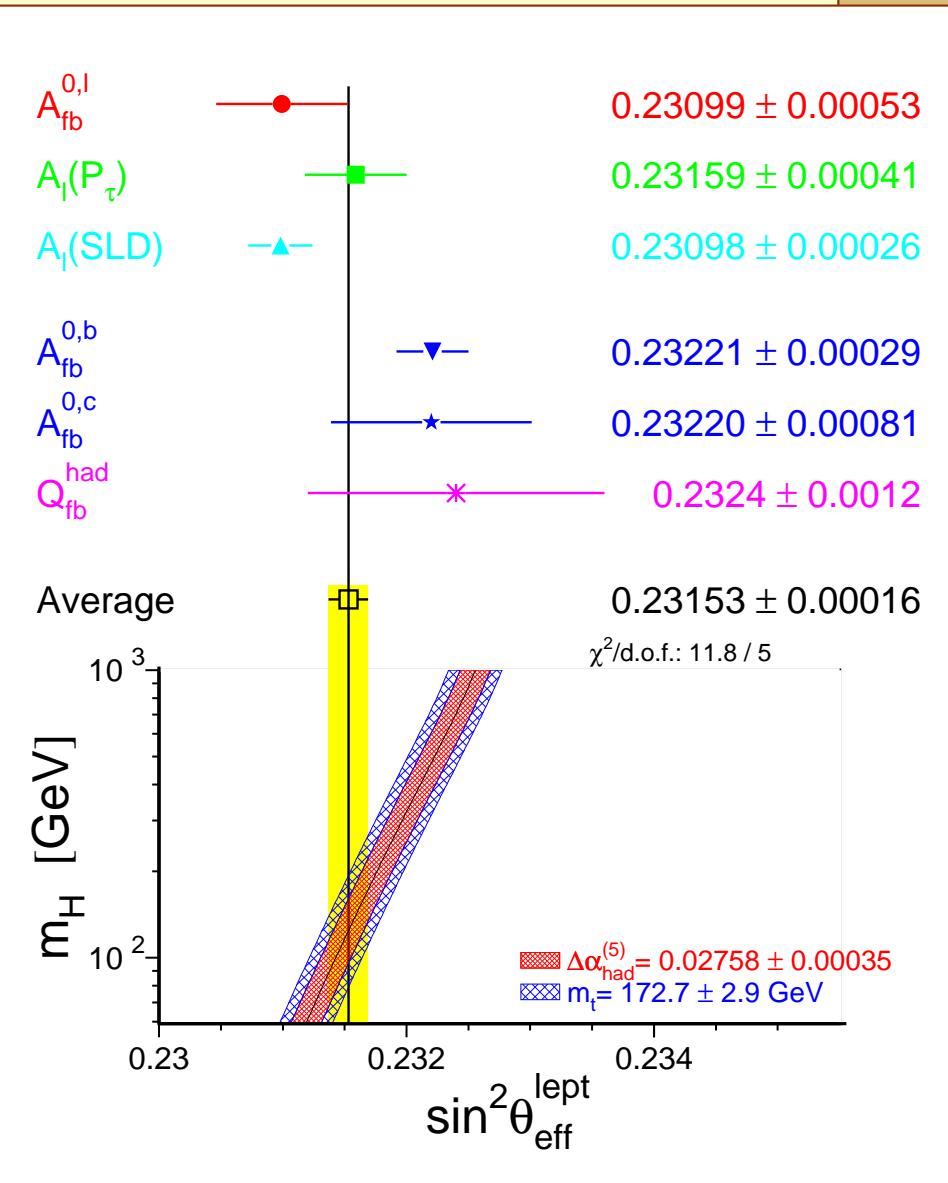
$\mathcal{O}(\alpha^2 \alpha_S)$  (3-loop diagrams, doas)

$\mathcal{O}(N_f^{>2} \alpha^3)$  (3-loop vertices with)

$\rightarrow \delta_{\text{th,future}} \sim 1 \dots 2 \times 10^{-5}$

- Exp. uncertainty  $\delta_{\text{exp}} \sim 16 \times 10^{-5}$

$\delta_{\text{ILC}} \sim 1.3 \times 10^{-5}$



LEP EWWG '05

## Effective weak mixing angles

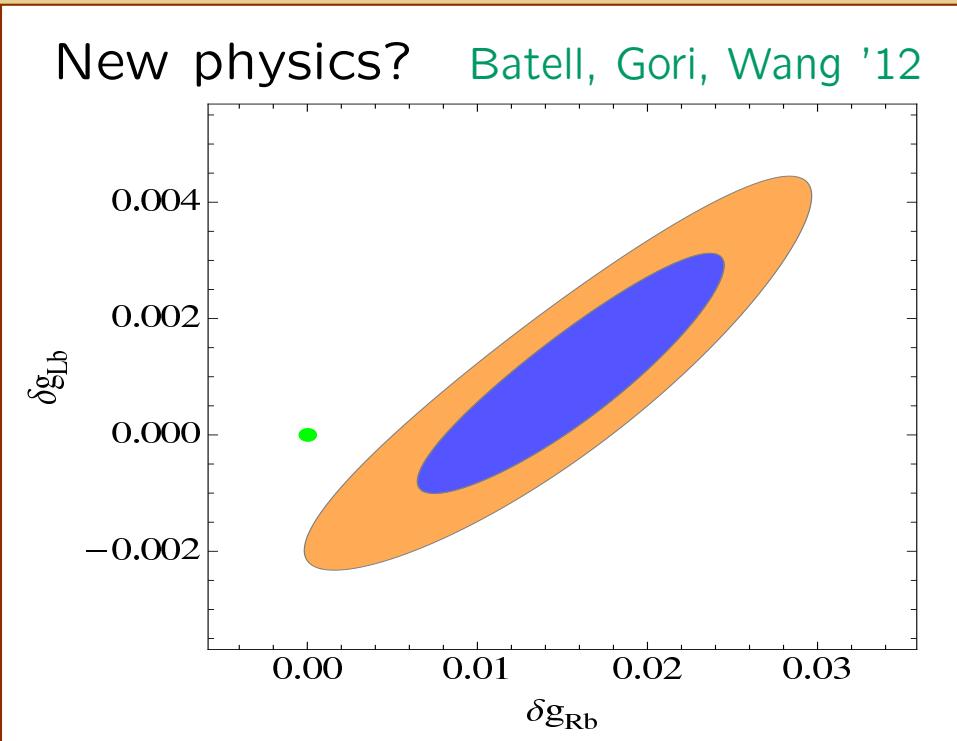
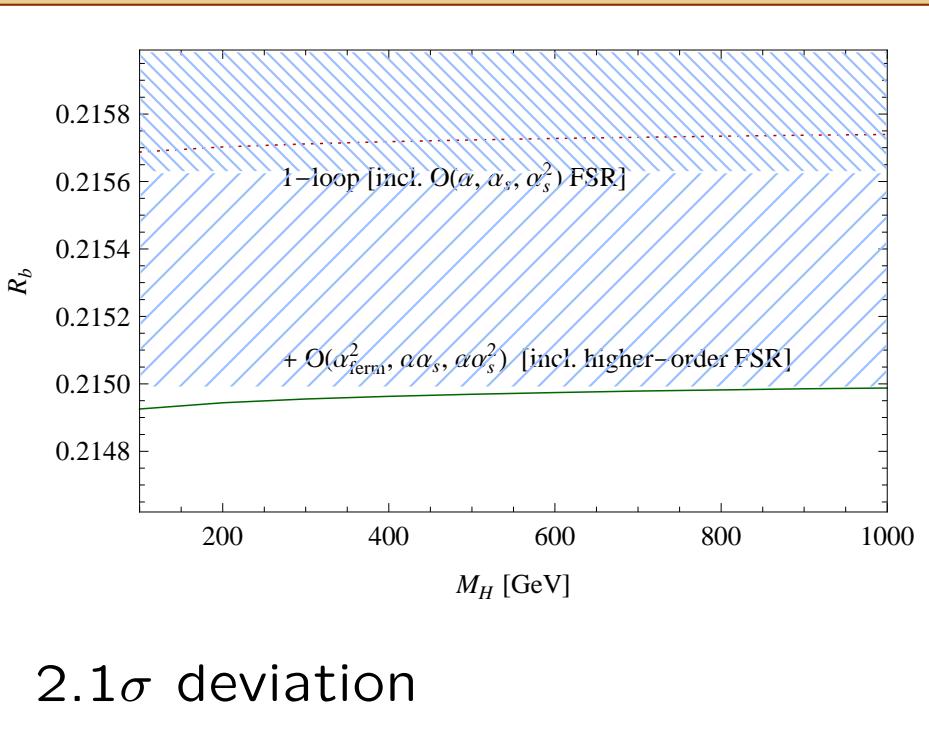
$\sin^2 \theta_{\text{eff}}^q$ :

- Complete *fermionic* NNLO corrections Awramik, Czakon, Freitas '06  
Awramik, Czakon, Freitas, Kniehl '08
- Partial 3/4-loop corrections
- Theoretical error  $\delta_{\text{th}} \gtrsim 5 \times 10^{-5}$   
mainly from  $\mathcal{O}(\alpha \alpha_s^2)$ ,  $\mathcal{O}(\alpha^2 \alpha_s)$   
(3-loop vertex diagrams, doable as large- $m_t$  expansion)
- Exp. uncertainty  $\delta_{\text{exp}} \sim 3 \times 10^{-2}$  ( $q = b$ )  
 $\delta_{\text{exp}} \sim 8 \times 10^{-3}$  ( $q = c$ )  
 $\delta_{\text{LHC}} \sim 3 \times 10^{-4}$  ( $q = u, d$ )

## Branching ratios

$$R_b = \Gamma_b / \Gamma_{\text{had}}$$

- Complete *fermionic* NNLO corrections Freitas, Huang '12
- Th. uncertainty  $\delta_{\text{th}} \sim 2 \times 10^{-4}$   
mainly from  $\mathcal{O}(\alpha\alpha_s^2)$ ,  $\mathcal{O}(\alpha^2\alpha_s)$  (3-loop vertex diag., see above)
- Exp. uncertainty  $\delta_{\text{exp}} \sim 6.6 \times 10^{-4}$   
 $\delta_{\text{ILC}} \sim 1.5 \times 10^{-4}$



## Total width and cross section

1

## Summary

- Experimental precision from LEP/SLC demands SM prediction with **complete 2-loop corrections**
- Much progress during last 10–20 years, but still large theoretical uncertainties in some observables
- **LHC** will provide independent results for  $\sin^2 \theta_{\text{eff}}$  and  $M_W$ , but overall precision not improved
- **ILC** with  $\sqrt{s} \sim M_Z$  will reduce experimental error by  $\mathcal{O}(2 - 10)$   
→ Challenge for theorists!
- Need modular and flexible computer program implementation at fully consistent NNLO order

## Backup slides

# Implementation

ZFITTER:

Bardin et al. '99, Arbuzov et al. '05

- Originally designed for NLO, not NNLO
  - Mismatches between NNLO electroweak and final-state QED/QCD corrections (due to approximations)
  - Expansion about  $Z$ -pole not consistent to NNLO
- Currently no indication for numerically large problems,  
but fully consistent NNLO treatment is desirable

Other programs:

GFITTER

Flächer et al. '08

TOPAZ0 (deprecated)

Montagna, Nicrosini, Passarino, Piccinini '98,01

## Implementation

Elements of consistent and flexible code:

- Define amplitude a Laurent expansion about complex pole  $s = m_Z^2 - im_Z\Gamma_Z$
- Real corrections: only treat soft/collinear pieces analytically, the rest numerically (allows for flexible cuts)
- Include ISR resummation without double-counting
- Modular (object oriented?) code structure