

Calculation of Very Large Hadron Collider Parameters in Java

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Abstract

The discovery of a Higgs-like boson of approximately 126 GeV mass has boosted motivation for continued exploration in high-energy physics. Specifically, for physics beyond the Standard Model, a new, larger collider of 100 km circumference will be needed. A program has been generated in Java allowing the user input of design parameters of an accelerator. The program, intended and used for calculation of properties of the theoretical Very Large Hadron Collider, outputs and displays numerous calculated accelerator properties, transverse and longitudinal, as well as the storage time evolution of transverse beam emittance, bunch intensity, beam-beam tune shift, and luminosity. The program considers effects for proton-proton collisions such as general lattice focusing and bending, synchrotron radiation, and intra-beam scattering. Results show an optimal storage time ~5 hours and peak luminosity $\sim 4.5 \cdot 10^{30} \text{ cm}^{-2} \text{ s}^{-1}$, 4.5 times that of the LHC.

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I. Introduction

Prior to the construction of any accelerator, physicists must have a good idea of how it would work. Large particle accelerators and colliders have enabled discoveries of many elementary particles. The discovery of the Higgs boson last year, at the CERN LHC, completes the standard model picture of particle physics, yet many important questions remain. Particularly, the possibility of multiple Higgs bosons or the supersymmetric partners of the quarks and leptons remains unanswered. To accomplish the task of searching for these particles, and for the exploration of possible new physics beyond the standard model, high energy physics requires the proposed Very Large Hadron Collider (VLHC).

My research project at Fermilab, this summer, has been to write a software program in Java to input design parameters for such a machine and generate storage time evolutions of some general design properties such as the beam emittance, bunch intensity, beam-beam tune shift, and collision luminosity. The user of the program is allowed to set various parameters and options at the beginning of the program, before initial calculations are performed. Afterward, the program iterates storage time values to calculate the most important variables across time. The values of these variables determine critical information about operation scenarios of the collider for both accelerator physicists and detector-based experimental particle physicists.

At the time of this document's writing) a letter of intent, "white paper," has been submitted by Dr. Pushpa Bhat, et. al., to build the Very Large Hadron Collider at Fermi National Accelerator Laboratory.¹ The original proposal and design study for a VLHC ring with 233 km circumference, at Fermilab, was developed in 2001.² The current version of the proposal envisions a circular ring of 100 km circumference. The plan is to have the current order of injection; ion source, linear accelerator, Booster ring, Main Injector, Tevatron; upgraded. The linear accelerator and booster ring, the initial accelerators, would be replaced in the sequence by a three-stage Project X designed to supply a high intensity, low emittance 8 GeV beam. The Main Injector will then increase the energy to 150 GeV. Finally, the Tevatron would be re-outfitted with 16 T dipole magnets currently researched in Fermilab's Technical Division. This would allow the Tevatron to inject 3 TeV beam to the VLHC, colliding protons at 100 TeV collision energy. Project X, useful for other, current experiments at Fermilab as well, is necessary for the luminosity L of the VLHC collisions to be high. Luminosity is, after all, a measure of the fraction of collisions resulting in interactions. Since the interactions are what detector-based experimentalists need for their research, the luminosity is a figure of merit of a certain type of efficiency: the efficiency of how much physics research one can get from an accelerator given the amount of inputs to run the accelerator. Figure 1 gives an idea of the size and location of the collider, with the size of the LHC superimposed nearby.

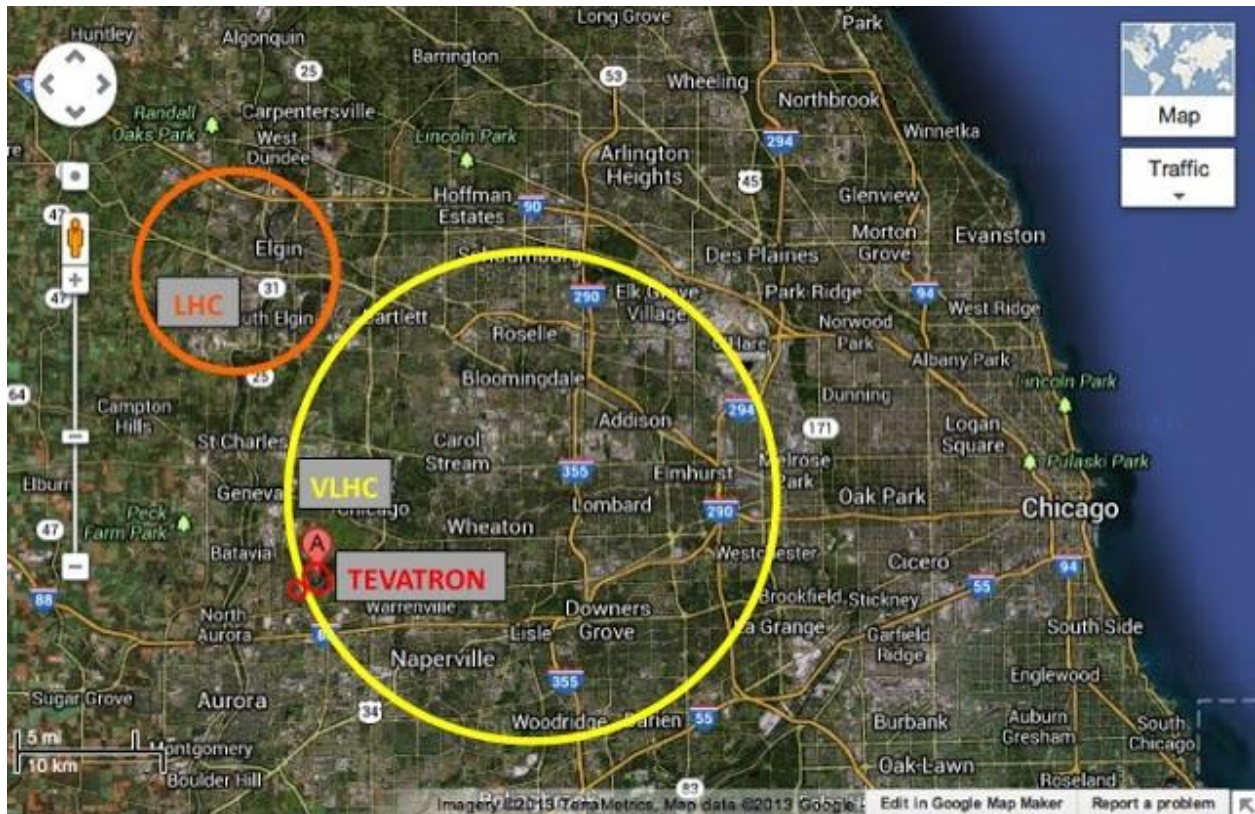


Figure 1. Red ovals place the Tevatron and its Main Injector. The Tevatron would inject to the VLHC, which could be built underneath the map’s yellow circle. The orange circle denotes the size of the 26 km LHC for comparison.

II. Methods

The program, named “proton_ring8.java”, makes key simplifications that should be noted. The most important are that, besides the lattice itself, synchrotron radiation and intra-beam scattering are the dominant effects on transverse emittance and intensity, due to the high energy and small emittance of the beams. Further, the program assumes that, via noise injection into the beams, the collider operators will keep longitudinal emittance constant during the storage time. Such simplifications are necessary to keep the calculations simple in the program. Any so-called

“accelerator simulator,” as the program may be called, must have each major accelerator phenomenon’s set of relevant equations written into the code individually.

Generally, the program’s structure follows this format: variable instantiation; computation functions; Graphical User Interface (GUI) -assisted input; various calculations of initial parameters, organized by accelerator physics phenomenon; calculation output writing to a file; iterative calculation of the time-dependent parameters; constructing chart outputs. The program is not included in an appendix because it would require around 30 pages.

The variable instantiations are high in volume, but otherwise of no note. The second section contains all the formulae used for time evolution computation. The GUI section allows the user several options. The user may change any of 26 design and beam parameters. One may choose not only the initial value of the crossing angle, but also any of three options. The first is to keep the angle constant across time. The second is to vary (decrease) the angle such that the quotient of crossing angle ϕ_c and horizontal beam size σ_x remains fixed. The third is luminosity leveling. The user sets the target luminosity at which the evolution should reach then stay constant instead of rising above it in a peak. The crossing angle is varied by the program to hold the luminosity to this level for as long as possible until the luminosity falls below the target. At this time only the first two options are available, but soon hereafter the third option’s coding will be complete. Some work is also being done on allowing the user the option of inputting several values for a single parameter, so that the program immediately runs one trial for each different value. This option makes gleaning one parameter’s effect on evolution curves easier, but it is merely a convenience. It’s incompleteness at this time is no significant limitation on the program.

The fourth major program section, initial parameter calculation, is divided into sub-sections: kinematic and interaction point; main dipole and FODO cell; tunes, beta, and dispersion in ARC cells; longitudinal parameters; synchrotron radiation power and photon parameters;³ polarization; luminosity, beam energy, and energy deposition; single bunch instabilities--space charge, microwave instability thresholds, loss of Landau damping, and transverse mode coupling threshold; coupled bunch instability from resistive wall;⁴ intra-beam scattering.⁵ The most notable phenomena which the program does not consider are beam-gas scattering, longitudinal coupled bunch instability, single and coupled bunch electron cloud effects, and the change of transverse emittance due to elastic proton-proton collisions.

Therefore, instead of the more complete eqns (1) and (3),⁶ the program relies on (2) and (4).

$$\left(\frac{d\varepsilon}{dt}\right)_{total} = \left(\frac{1}{\tau_{ibs}} - \frac{1}{\tau_{rad}}\right) * \varepsilon + \left(\frac{d\varepsilon}{dt}\right)_{gas} + \left(\frac{d\varepsilon}{dt}\right)_{pp} \quad (1)$$

τ_{ibs} denotes the characteristic time with respect to which ε has an exponential behavior due to intra-beam scattering. τ_{rad} denotes the characteristic time with respect to which ε has an exponential behavior due to synchrotron radiation. The second term on the right side of eqn (1) is change in ε due to residual gas in the beamline from imperfect vacuum. The third term indicates change in ε due to proton-proton collisions at the interaction point or points (IP). The latter terms are smaller than the first.

$$\left(\frac{d\varepsilon}{dt}\right)_{total} = \left(\frac{1}{\tau_{ibs}} - \frac{1}{\tau_{rad}}\right) * \varepsilon \quad (2)$$

$$\left(\frac{dN_b}{dt}\right)_{total} = \frac{L * N_{IP}}{M_b} * \sigma_{tot} + \left(\frac{dN_b}{dt}\right)_{gas} + \left(\frac{dN_b}{dt}\right)_{others} \quad (3)$$

N_{IP} is the number of interaction points in the collider. M is the number of bunches of protons in each beam. σ_{tot} is the total cross section of the proton-proton collisions. The second term on the right side of eqn (3) is change in N_b due to residual gas in the beamline from imperfect vacuum. The final term indicates change due to other factors. The latter terms are smaller than the first.

$$\left(\frac{dN_b}{dt}\right)_{total} = \frac{L * N_{IP}}{M_b} * \sigma_{tot} \quad (4)$$

$$L = \frac{N_b^2 * M_b * f_{rev}}{4\pi * \sigma_x * \sigma_y} * R(\phi_c) \quad (5)$$

f_{rev} is the frequency at which each proton makes a revolution around the ring. σ_x^* and σ_y^* are, respectively, the transverse horizontal and vertical beam sizes at the IP. ϕ_c is the beam-beam crossing angle at collision.

$$R(\phi_c) = 1 / \sqrt{1 + \left(\frac{\phi_c * \sigma_{s,rms}}{2 * \sigma_{x,av}} \right)^2}$$
(6)

$\sigma_{s,rms}$ is the longitudinal beam size root-mean-square value. $\sigma_{x,av}$ is the average horizontal beam size along the entire beamline.

$$\frac{\tau_{ibs}(t)}{\tau_{ibs}(0)} = \left(\frac{\varepsilon(t)}{\varepsilon(0)} \right)^q, \quad q \approx 2.2$$
(7)

$$\varepsilon(t) = \left(\frac{\left(\frac{\varepsilon(0)}{\tau_{ibs}(0)} \right)^q + e^{-q * 1/\tau_{rad} * (t - (\frac{1}{q} * \tau_{rad} * \ln(\frac{\varepsilon(0)}{\tau_{rad}} - \frac{\varepsilon(0)}{\tau_{ibs}(0)}))}}}{1/\tau_{rad}} \right)^{1/q}$$
(8)

The program considers both an approximated solution to eqn (2) as well as the exact solution (8), given (7) as an assumption. In addition to the independence of ε and τ_{rad} , the phenomenological relationship eqn (7) was needed to find (8). In case the user is well-informed that the exponent q is different, the exponent in (7) is left as a changeable input with default value 2.2. This is necessary as q will change with lattice design and must be observed by plotting τ_{ibs} for multiple ε values. The program may be used to test (8) against the approximated solution. Such a comparison may be seen in Figure 2. The approximate emittance values are found by the differential equation approximation method the fourth-order Runge-Kutta. With use of the same

input parameters for each set of values, the choice of solution is significant to the equilibrium emittance value.

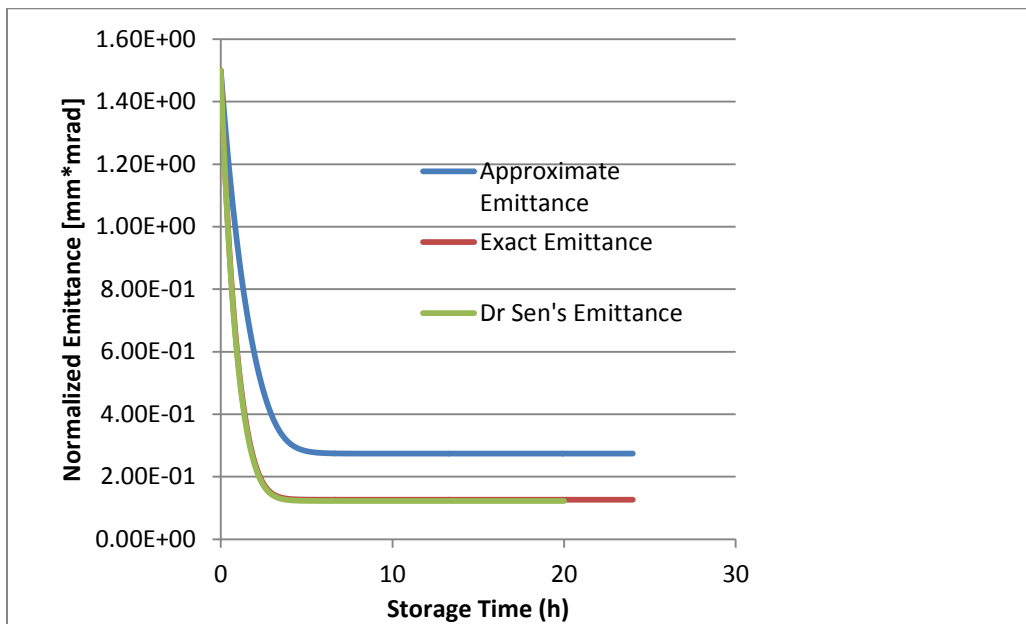
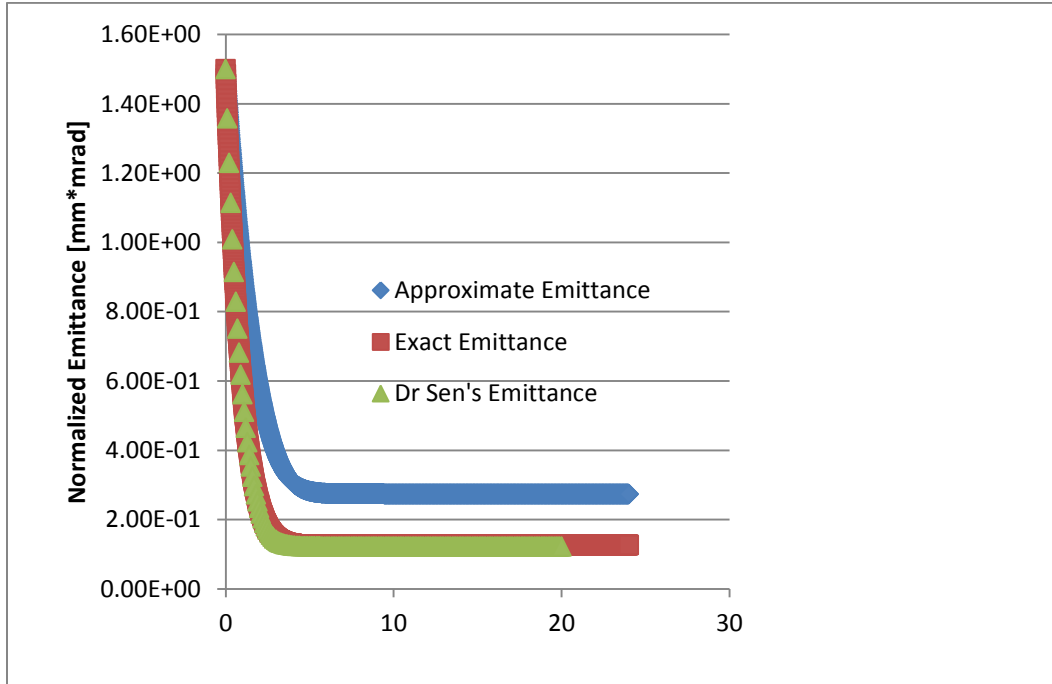


Figure 2. Comparison of approximate Runge-Kutta method-derived values for emittance evolution and values from exact solution (7).

Upgrades are possible to the program, allowing use of eqns (1) and (3). But that may eliminate possibility of an exact solution, which would pose the challenge of interpreting how accurate one should expect the model to be. The calculations of emittance ε and intensity N_b over storage time t may have to become more approximate to allow for more phenomena. Yet it is a favorable result that, despite the simplicity and commonly known poor accuracy of the trapezoidal rule, the use of that simple rule for evolution calculation of N_b compared incredibly closely to an approximate solution to the N_b differential equation (4) performed by Dr. Tanaji Sen on Mathematica. See Figure 3.

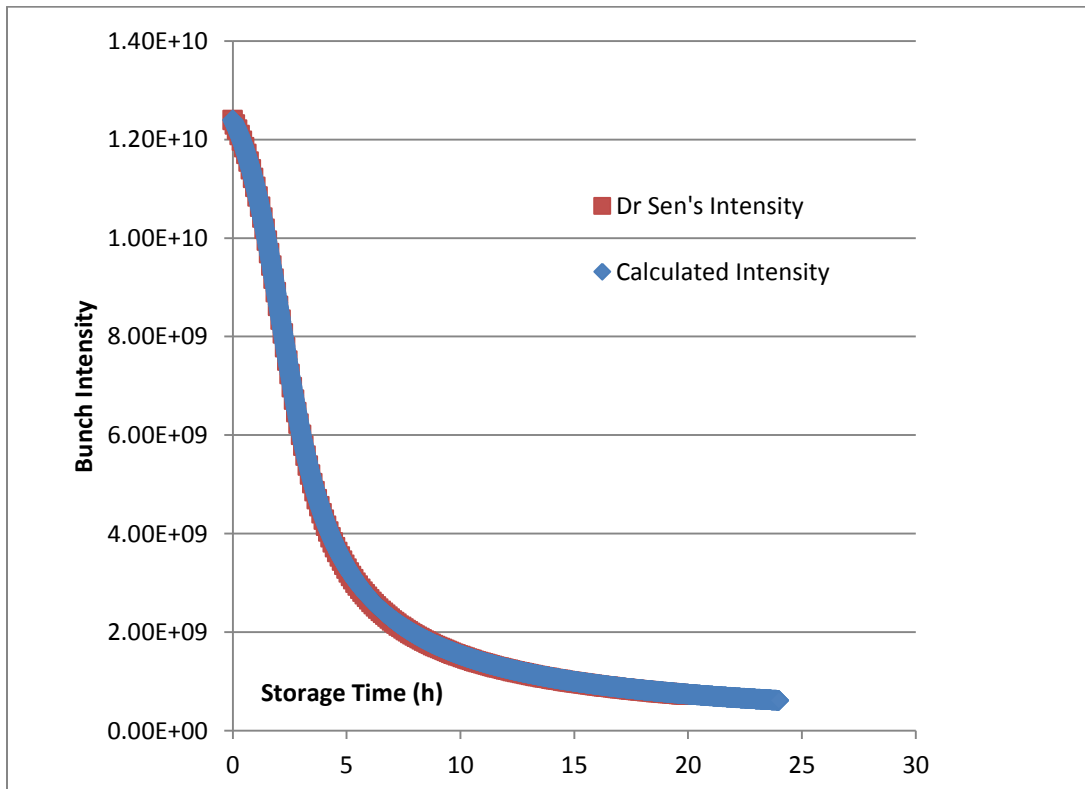


Figure 3. Comparison of two approximations of N_b . One was performed on Mathematica by Dr. Sen. The other was a simple trapezoidal integral computation with a high number of small steps, 2400. If the points are replaced by smooth lines, the red curve is no longer visible under the blue.

A full list of the evolution charts constructed automatically and printed through Java GUI pop-up windows are horizontal emittance, bunch intensity, luminosity, average luminosity, integrated luminosity, horizontal (X) beam-beam tune shift, vertical (Y) beam-beam tune shift, and collision crossing angle.

The last sections of the program deal not only with the management of calculation functions into time evolution data, but also with the writing of data, both initial and evolving values, to output text files. The neatly organized output files are highly convenient for copying to excel documents for comparison of evolutions from different trials in the same graph.

III. Results

Despite a length of over 1300 lines, a single trial of the program schedule at 2400 iterative steps of storage time requires less than a minute beyond completion of user input.

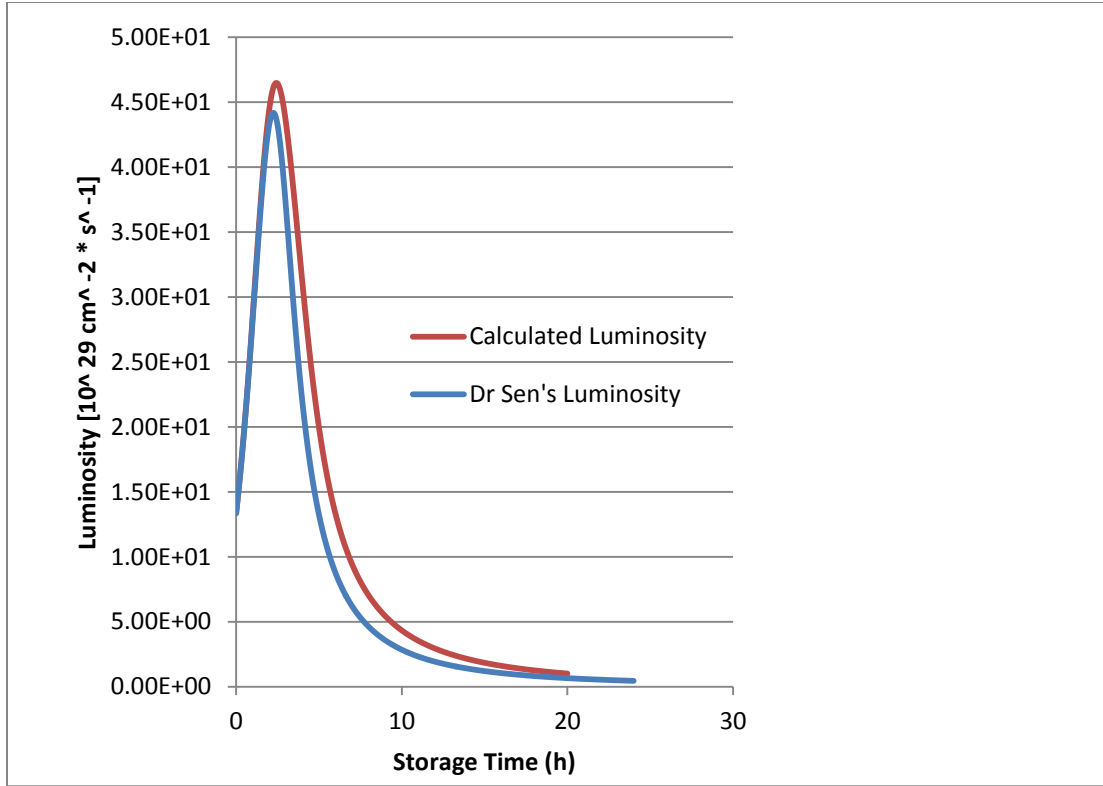


Figure 4. Comparison of two approximations of L . One was performed on Mathematica by Dr. Sen. The other was computed directly from ε exact solution (8) values and the corresponding values from the N_b approximate integral. 2400 storage time steps were used.

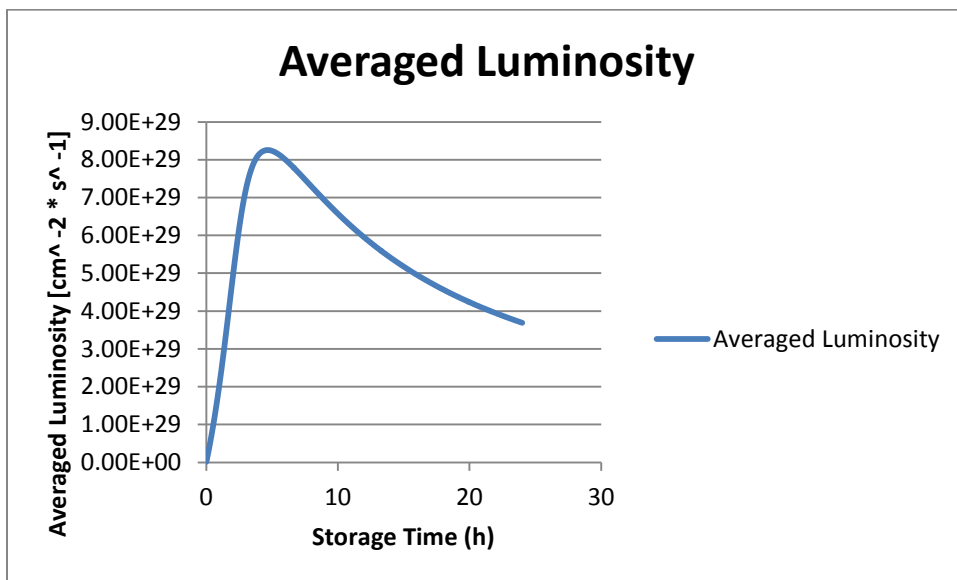
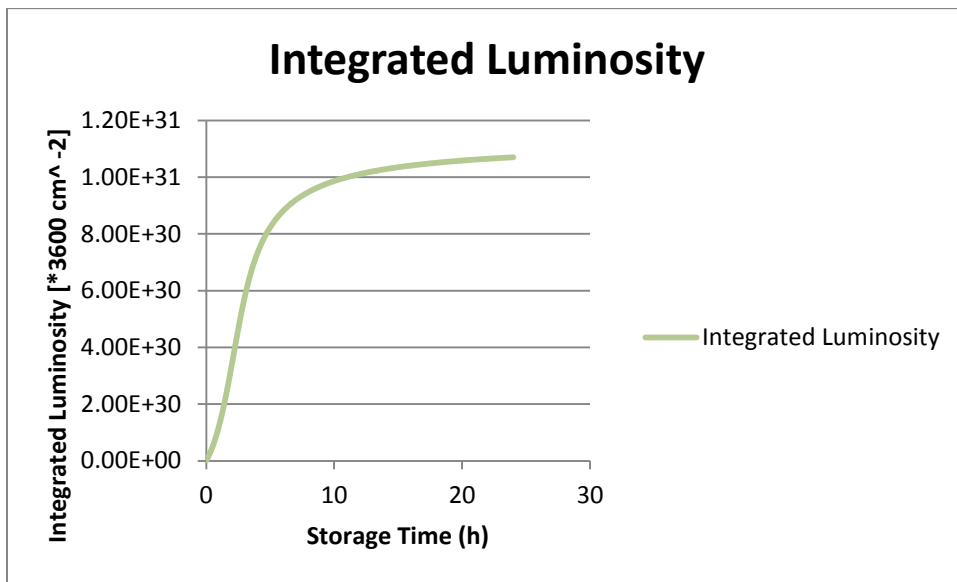
The finding of a peak luminosity of ~ 4.5 times that of the LHC's peak luminosity $1 \cdot 10^{30} \text{ cm}^{-2} \text{ s}^{-1}$ is a successful value expected according to the high intensity, low emittance parameters of the VLHC.

$$L_{int} = \int_0^{t_{st}} L(t') dt'$$

$$L_{av} = \frac{1}{t_{st} + t_{ref}} \int_0^{t_{st}} L(t') dt'$$

(10)

t_{st} is the storage time, in previous equations denoted just t . t' is merely an integration variable. t_{ref} is the refill time, which is a constant with respect to storage time. It denotes the length of time between stores needed to dump the previous beams and inject new beams.



Figures 5 and 6. Because these calculations are directly from L , there is no need for comparison here. Note the differences in shape of the two curves because of the extra factor in L_{av} depending on the time variable. Also note where L_{av} begins to fall, the optimal store time.

The next interesting luminosity values are the integrated and averaged luminosities L_{int} and L_{av} , given by eqns (9) and (10), respectively. The first is of great interest to detector-based physicists. L_{int} is a measure of how many interactions have occurred over the current store time, and those interactions at the detectors are the basis of these scientists' data. L_{av} determines for accelerator physicists the optimal store time period, after which L_{av} falls. Once L_{av} drops, the efficiency value or quality factor of the accelerator, whose purpose is to provide collision interactions, is decreasing. Therefore it is of greatest efficiency to stop and dump the beams and refill. Figure 6 shows that the optimal store time is about 5 hours, despite that it is possible to interpret in Figure 5 that storage time up to or even beyond 10 hours adds very significantly to the integrated luminosity and therefore to the total number of interactions observable.

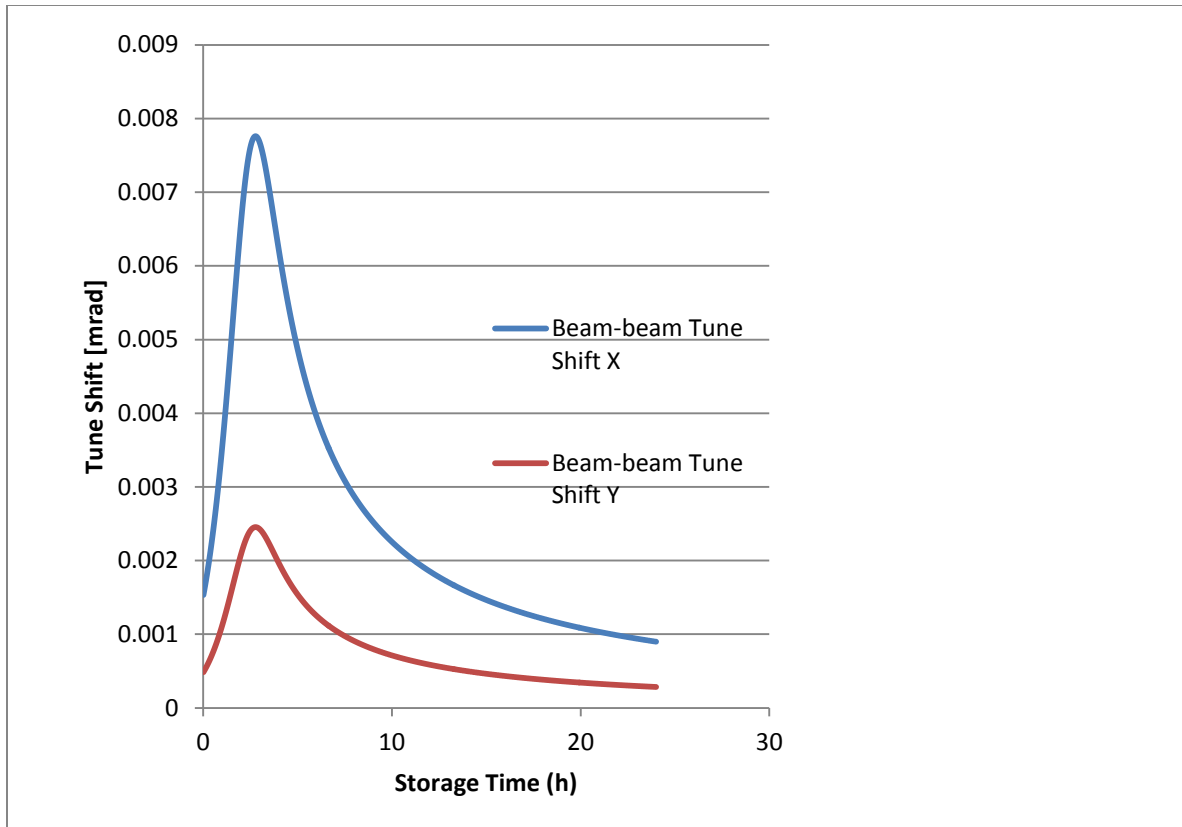


Figure 7. These curves show the usual behavior of beam-beam tune shift. Somewhat similar to L , there is a peak then a gradual fall. Unlike L , but similar to ε , there may be an equilibrium value for each tune shift curve. The shapes will be very similar for round beams because the only difference will be a factor of the square root of the quotient of IP beta function values.

Both beams have oscillatory motion in both transverse directions, and as bunches from different beams approach the interaction point, they interacting by the Coulomb force. Particularly, one bunch leaving the IP and a second from the other beam approaching the IP will reach a minimum separation distance where their interaction is strongest. Their pushes on each other will affect their transverse oscillations, a tune shift in their transverse motions. This beam-beam tune shift is dangerous if it approaches a resonance value or becomes too high; so it is necessary to calculate

across time in both horizontal and vertical directions. Therefore, Figure 7 demonstrates the program's outputs.

IV. Conclusion

The program at this time has nearly every option it was designed to have, and there is some confidence in the program because of the comparison with Mathematica approximations. In discussion with Dr. Sen, it was very clear that every graph plotted has the shape it is supposed to. However, there is still some work left to be completed. Very soon, more work will be done on the program to complete the automatic multi-trial routine for varying a single initial parameter. Likewise, some attempts were made at enabling the program to handle typos by the user in the GUI input table, but these efforts failed to fully complete the task. The program is already a very useful and complex tool, but there is great potential for it, and much work to do to unravel that potential, possibly culminating in an advanced, accurate proton-proton "collider calculator" which could consider all collider effects and determine all values to high precision in practice.

V. Acknowledgments

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