# LEBT Simulations Recent Work

J.-Francois Ostiguy

APC/Fermilab
ostiguy@fnal.gov

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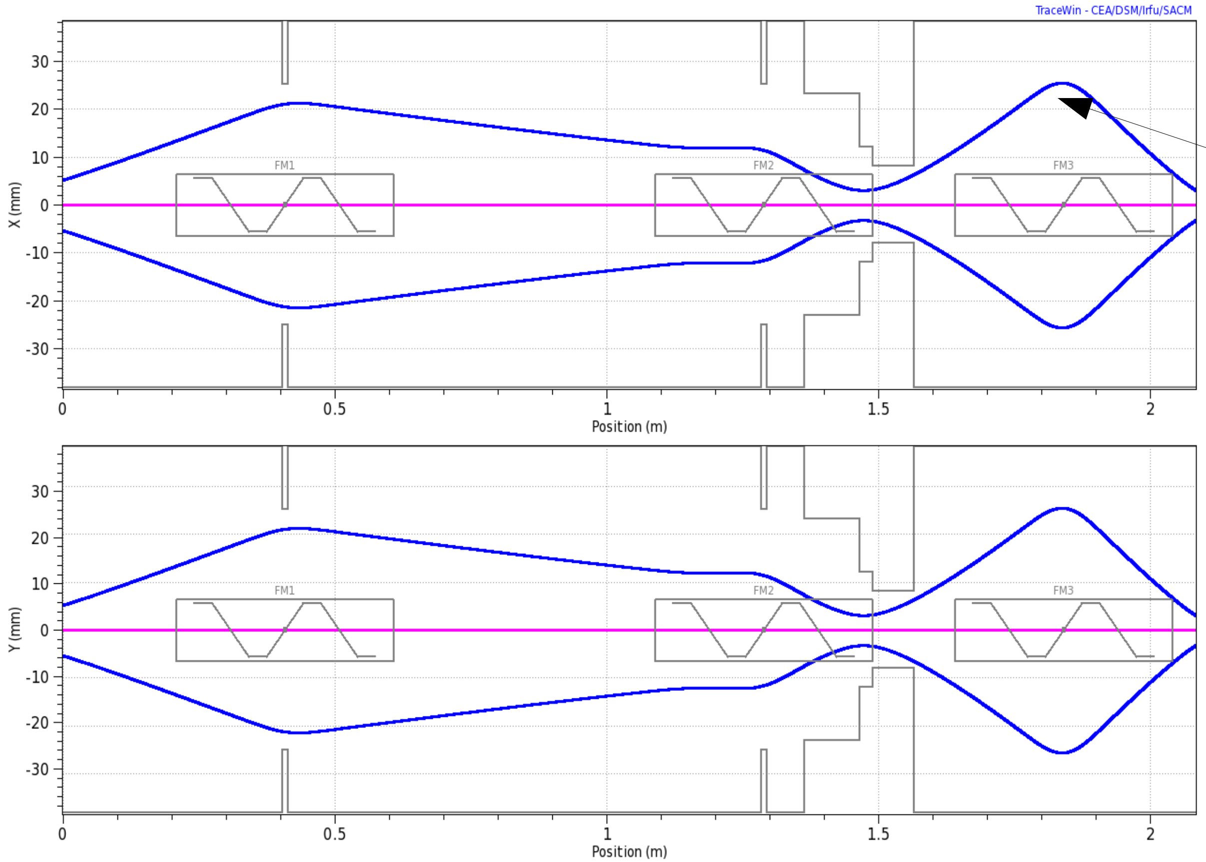
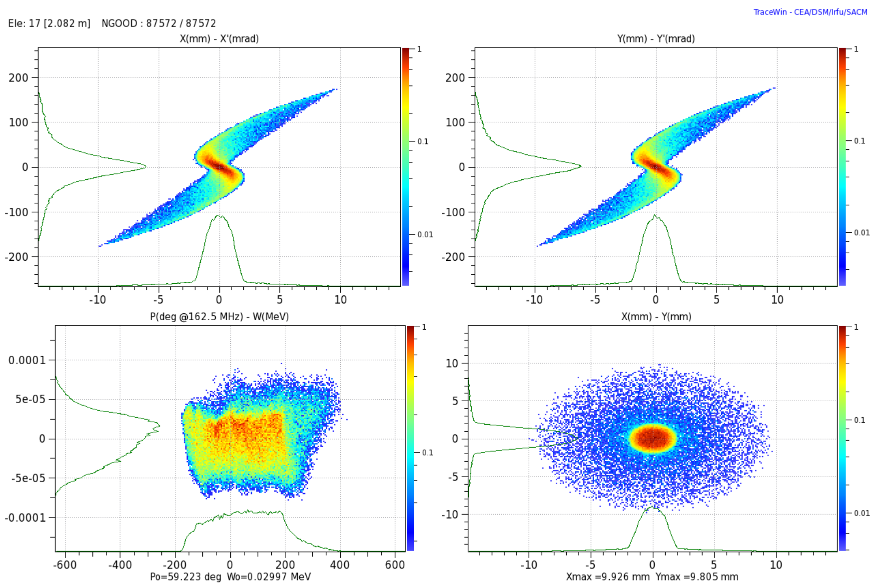
Project X

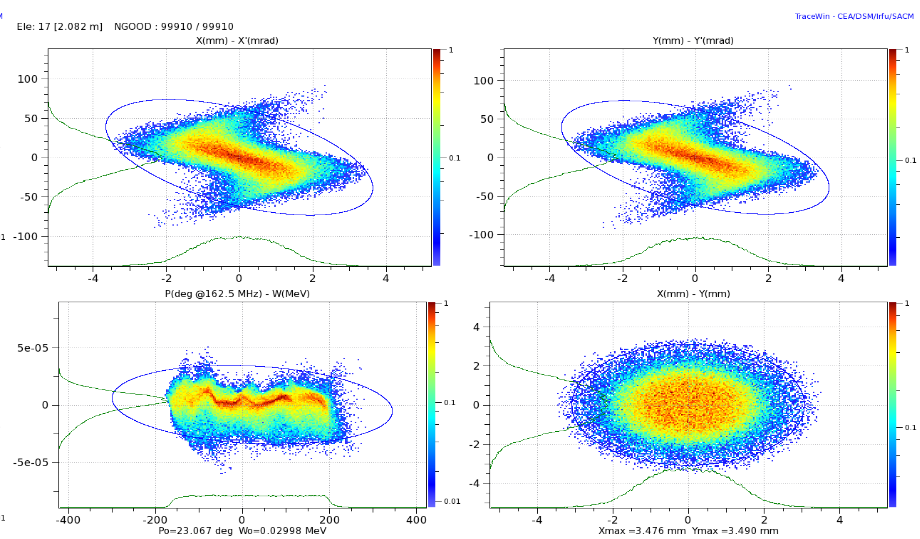
# Beam Size in LEBT




**Full beam size > ½ solenoid aperture in solenoid 3**

# Distribution at RFQ Entrance

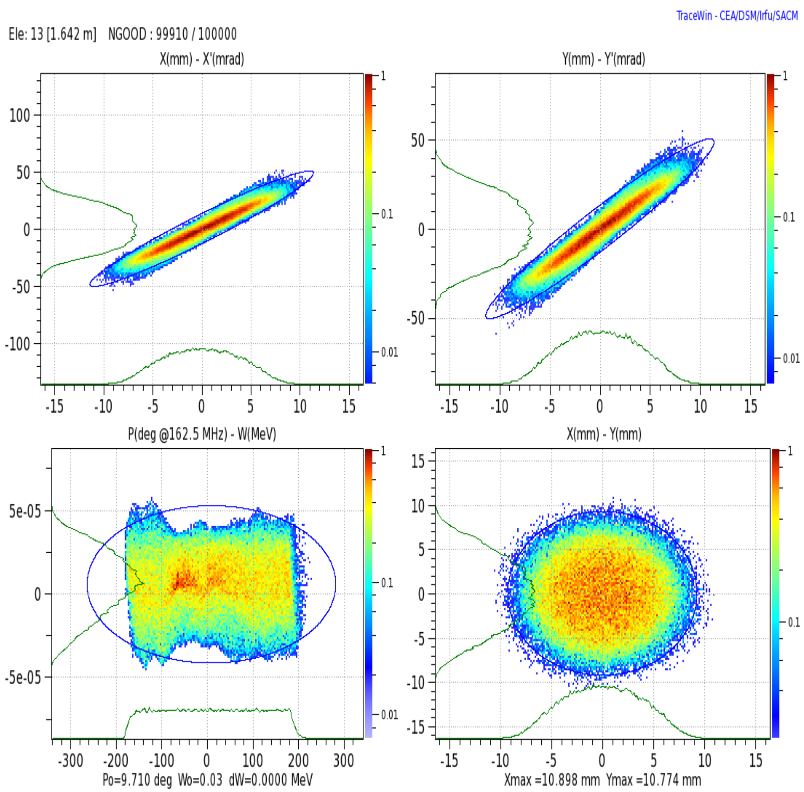


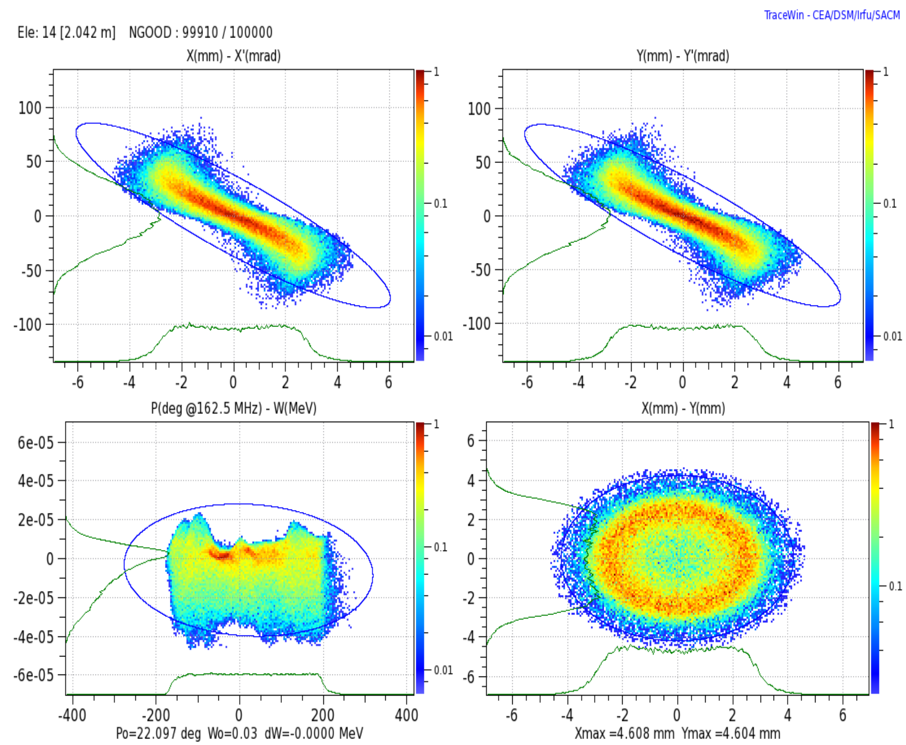




“matched” to RFQ (core parameters)

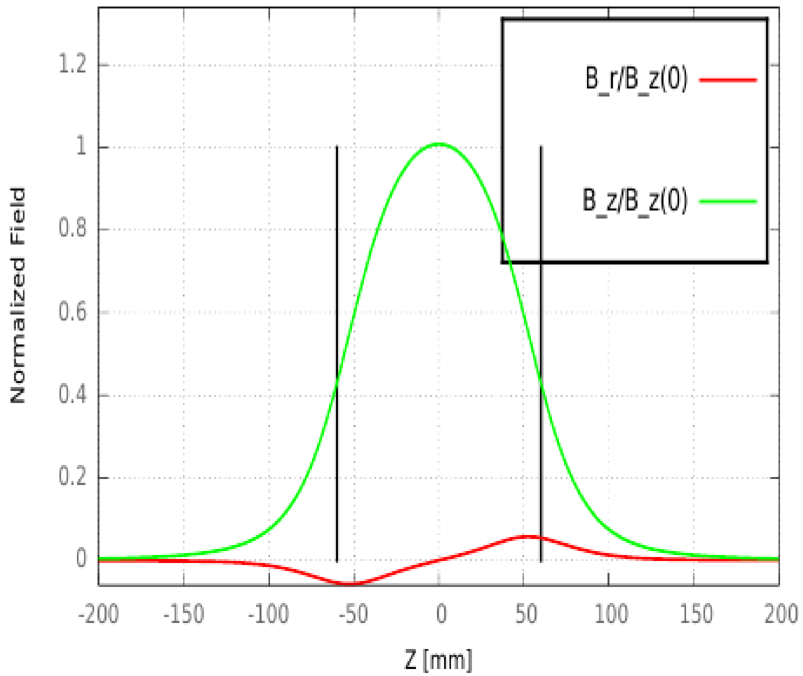
Beta = beta RFQ
alpha = ½ alpha RFQ

# Upstream and Downstream of Sol 3



# PXIE Solenoid




(a) PXIE solenoid field $B_z$ and $B_r$ at an offset $r = 10$ mm. The vertical lines indicate the physical boundaries of the solenoid.

(b) LEBT Solenoid Parameters

| Parameter | Value | Units |
|---|---|---|
| Min inner diameter | 80 | mm |
| Max physical length | 140 | mm |
| Strength $\int B_z^2 dz$ | 0.03 | T-m$^2$ |
| Peak Current | 300 | A |
| Power dissipation | 0.8-1.0 | keV-ns |
| Dipole corrector coils | 4 | |
| Peak Dipole Coil Current | 10 | A |
| Dipole coil field integral | 0.5 | T-m |

# Related Work

- PX DocDB Document 1015-v2 estimates the impact of the spherical aberration in the PXIE Solenoid in a manner similar to what is outlined here.

# Solenoid Field Expansion

The field of an axi-symmetric solenoid can be expanded based on based on the on-axis form of B_z(z)

$$B_z(r,z) = B(z) - \frac{r^2}{2}B''(z) + \frac{r^4}{64}\frac{d^4B(z)}{dz} - \dots$$

$$B_r(r,z) = -\frac{r}{2}B'(z) + \frac{r^3}{16}\frac{d^3B(z)}{dz} - \dots$$

# Radial Kick From a Short Solenoid

By integrating the equations of motion in cylindrical coordinates, one can show that in the short solenoid approximation (r is constant through the solenoid, the the radial kick experienced by a particle is

$$\Delta r' = -\frac{r}{f_0}[1 + C_1 r^2 + C_2 r^4]$$

C_1 and C_2 are known as the spherical aberration coefficients. These coefficients are determined by the field shape, and here the latter depends only on the on-axis shape of B_z.

$$\frac{1}{f_0} = \left(\frac{q}{2\beta_z \gamma c}\right)^2 \int_{-\infty}^{\infty} B^2(z) dz$$

# Aberration Coefficients

$$C_1 = \frac{1}{2} \frac{\int [B'(z)]^2 dz}{\int B^2(z) dz},$$

$$C_2 = \frac{5}{64} \frac{\int [B''(z)]^2 dz}{\int B^2(z) dz}$$

# Emittance Growth due to Spherical Aberrations

$$\Delta\epsilon_{x,y} = \frac{R^4}{2\sqrt{6}f_0}\sqrt{\frac{C_1^2}{12} + \frac{C_1 C_2}{5}R^2 + \frac{C_2^2}{8}R^4}$$

This expression is obtained by computing the emittance increase of a spatially uniform beam of radius R with zero emittance (r'=0) receiving a radially dependent kick from the lens.

# Analytic Forms for B(z)

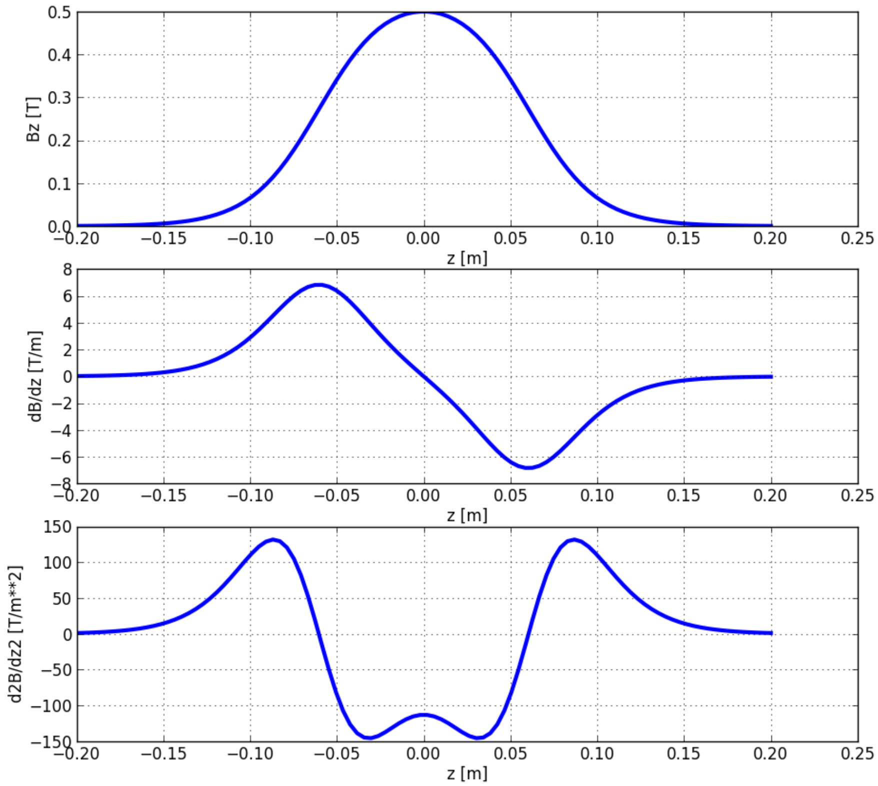
For numerical estimates, a number of sufficiently differentiable analytic forms for B(z) can be used. A popular one is

$$B(z) = \frac{B_0}{2\tanh\left(L/2R\right)}\left[\tanh\frac{z+L/2}{R} - \tanh\frac{z-L/2}{R}\right]$$

Another one, well-suited to short solenoids is the so-called Glaser model

$$B(z) = \frac{B_0}{1+(z/(L/2))^2}$$

# On-axis Fields and Derivatives for a PXIE-Style Solenoid

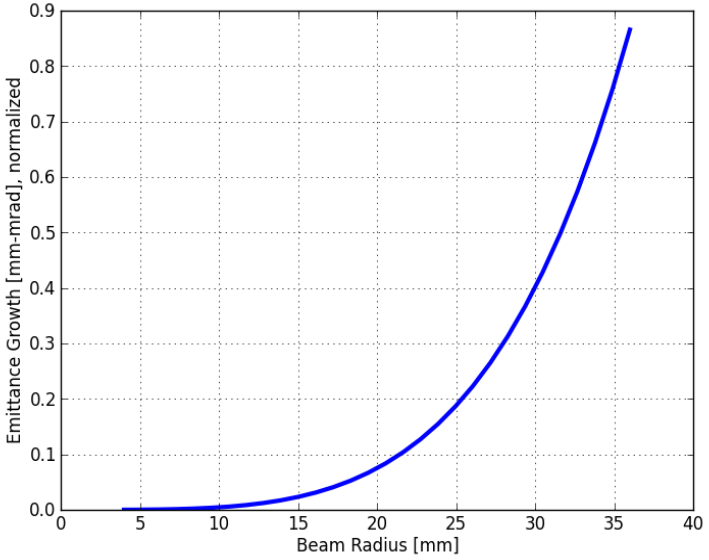



atanh analytic form

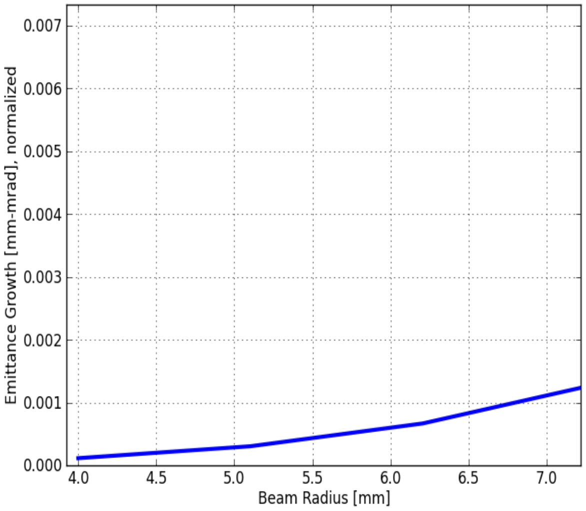
Numerically computed
first and second derivatives

# Predicted Emittance Growth

For PXIE : $\epsilon_{x,y} = 0.116 \ \text{mm-mrad}$



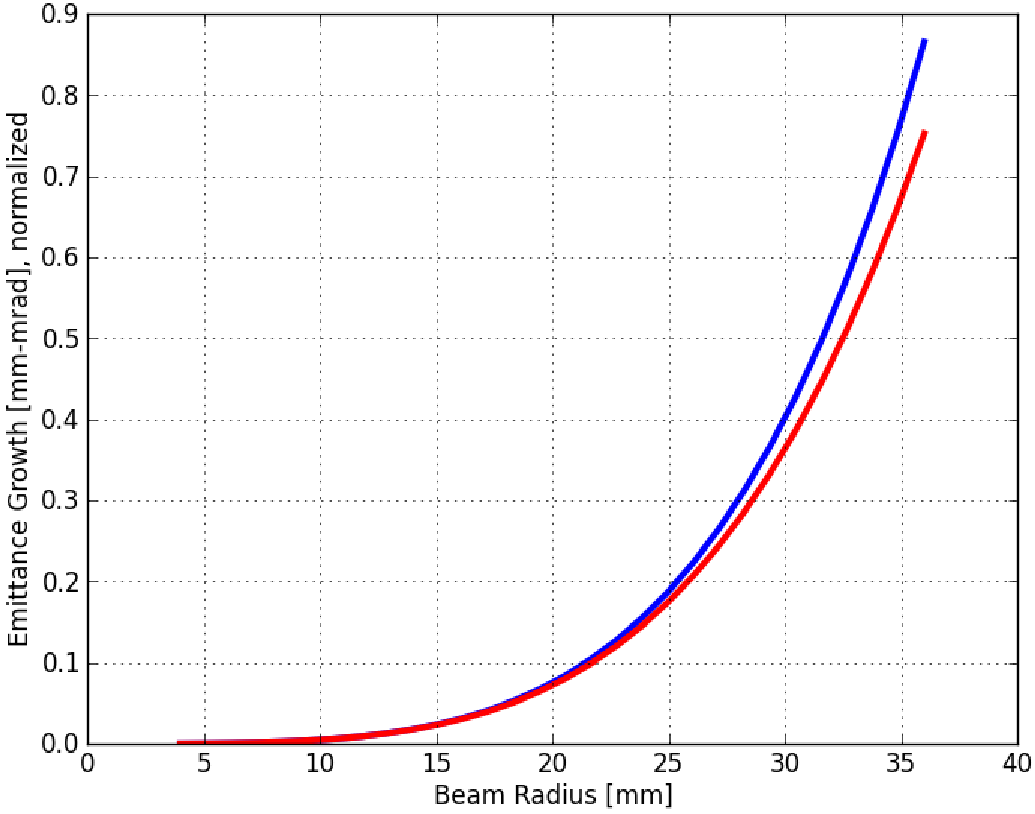

For R = ½ of radial aperture




$$\frac{\Delta\epsilon}{\epsilon} \simeq \frac{0.08}{0.11} = 72\%$$

Probably excessively pessimistic for a Gaussian beam.

# Aberrations : 1st or 2nd Order ?




1st order aberration OK

# Aberration Control

Aberrations can be controlled by tailoring the solenoid excitation profile or the geometry of the external iron enclosure.  Example : FETS

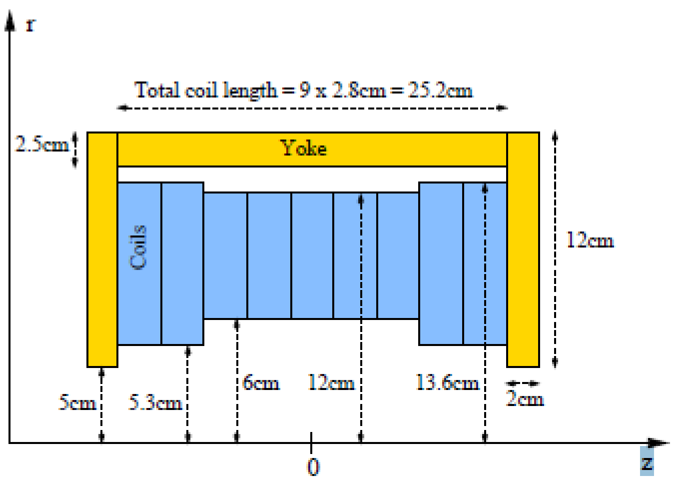



Figure 2: Schematic of the solenoid design.

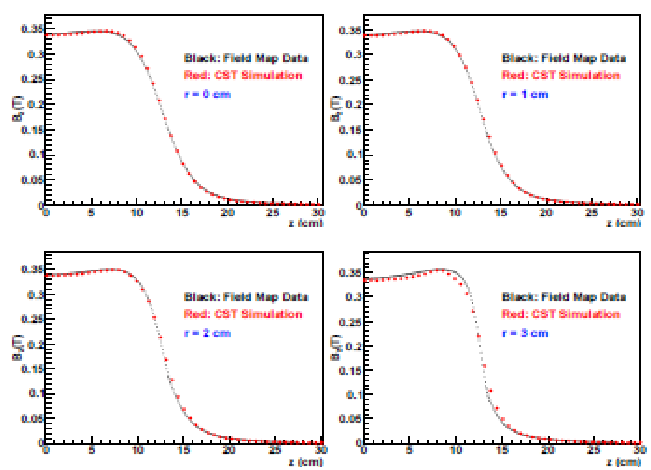



Figure 3: $B_z(z)$ measured and simulated field map distributions for one of the solenoids at different radii.

# Tweaking Solenoid 3 ?

- It appears very difficult to get a good match into the RFQ
- The distance between Sol 3 and the RFQ entrance is critical as it determines the amplitude in Sol 3
- Can we improve phase space distribution and obtain and a better match by tweaking Sol 3 ?

# Next Steps

- confirm that emittance growth observed with TraceWin agrees with prediction for formula (this one or the one from 1015-v2) No space charge.
- Can we mitigate SC with sol aberrations ? Experiment with a few solenoid axial profiles.