

3. Sterile Neutrinos: Theory

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The Allure of Ultrasensitive Experiments

V. Neutrinos

January 14 to February 27, 2014

Outline

1. The Story So Far;
2. More Neutrinos?;
3. Why Are Neutrino Masses Small?;
4. Sample Sterile Neutrino Theory: the Seesaw Mechanism;
5. How Do We Learn More? [See David Schmitz lecture on Thursday]

Questions are ALWAYS welcome!

ν Flavor Oscillations are a Fact

Neutrino oscillation experiments have revealed that **neutrinos change flavor** after propagating a finite distance. The rate of change depends on the neutrino energy E_ν and the baseline L . The evidence is overwhelming.

- $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$ — atmospheric and accelerator experiments;
- $\nu_e \rightarrow \nu_{\mu,\tau}$ — solar experiments;
- $\bar{\nu}_e \rightarrow \bar{\nu}_{\text{other}}$ — reactor experiments;
- $\nu_\mu \rightarrow \nu_{\text{other}}$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_{\text{other}}$ — atmospheric and accelerator expts;
- $\nu_\mu \rightarrow \nu_e$ — accelerator experiments.

The simplest and **only satisfactory** explanation of **all** this data is that neutrinos have distinct masses, and mix.

Summarizing:

Both the solar and atmospheric puzzles can be properly explained in terms of **two-flavor** neutrino oscillations:

- **solar:** $\nu_e \leftrightarrow \nu_a$ (linear combination of ν_μ and ν_τ): $\Delta m^2 \sim 10^{-4} \text{ eV}^2$, $\sin^2 \theta \sim 0.3$.
- **atmospheric:** $\nu_\mu \leftrightarrow \nu_\tau$: $\Delta m^2 \sim 10^{-3} \text{ eV}^2$, $\sin^2 \theta \sim 0.5$ (“maximal mixing”).
- **short-baseline reactors:** $\nu_e \leftrightarrow \nu_a$ (linear combination of ν_μ and ν_τ): $\Delta m^2 \sim 10^{-3} \text{ eV}^2$, $\sin^2 \theta \sim 0.02$.

A Really Reasonable, Simple Paradigm:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{e\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Definition of neutrino mass eigenstates (who are ν_1, ν_2, ν_3):

- $m_1^2 < m_2^2$ $\Delta m_{13}^2 < 0$ – Inverted Mass Hierarchy
- $m_2^2 - m_1^2 \ll |m_3^2 - m_{1,2}^2|$ $\Delta m_{13}^2 > 0$ – Normal Mass Hierarchy

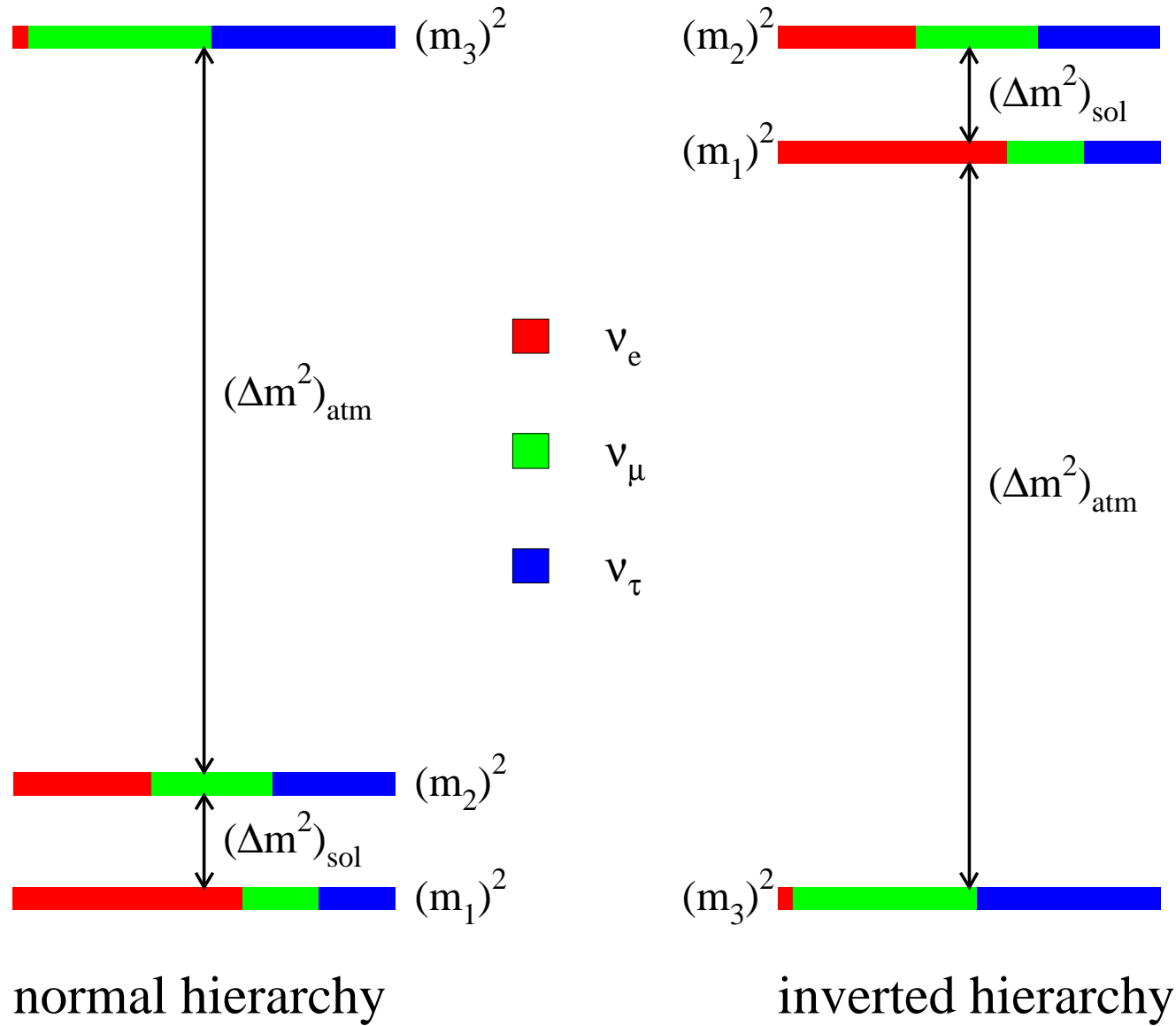
$$\tan^2 \theta_{12} \equiv \frac{|U_{e2}|^2}{|U_{e1}|^2}; \quad \tan^2 \theta_{23} \equiv \frac{|U_{\mu3}|^2}{|U_{\tau3}|^2}; \quad U_{e3} \equiv \sin \theta_{13} e^{-i\delta}$$

Three-Flavor Paradigm Fits All* Data Really Well (arXiv:1209.3023):

	Free Fluxes + RSBL		Huber Fluxes, no RSBL	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2 \theta_{12}$	0.30 ± 0.013	$0.27 \rightarrow 0.34$	0.31 ± 0.013	$0.27 \rightarrow 0.35$
$\theta_{12}/^\circ$	33.3 ± 0.8	$31 \rightarrow 36$	33.9 ± 0.8	$31 \rightarrow 36$
$\sin^2 \theta_{23}$	$0.41_{-0.025}^{+0.037} \oplus 0.59_{-0.022}^{+0.021}$	$0.34 \rightarrow 0.67$	$0.41_{-0.029}^{+0.030} \oplus 0.60_{-0.026}^{+0.020}$	$0.34 \rightarrow 0.67$
$\theta_{23}/^\circ$	$40.0_{-1.5}^{+2.1} \oplus 50.4_{-1.3}^{+1.2}$	$36 \rightarrow 55$	$40.1_{-1.7}^{+2.1} \oplus 50.7_{-1.5}^{+1.1}$	$36 \rightarrow 55$
$\sin^2 \theta_{13}$	0.023 ± 0.0023	$0.016 \rightarrow 0.030$	0.025 ± 0.0023	$0.018 \rightarrow 0.033$
$\theta_{13}/^\circ$	$8.6_{-0.46}^{+0.44}$	$7.2 \rightarrow 9.5$	$9.2_{-0.45}^{+0.42}$	$7.7 \rightarrow 10.$
$\delta_{CP}/^\circ$	240_{-74}^{+102}	$0 \rightarrow 360$	238_{-51}^{+95}	$0 \rightarrow 360$
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	7.50 ± 0.185	$7.00 \rightarrow 8.09$	$7.50_{-0.160}^{+0.205}$	$7.04 \rightarrow 8.12$
$\frac{\Delta m_{31}^2}{10^{-3} \text{ eV}^2}$ (N)	$2.47_{-0.067}^{+0.069}$	$2.27 \rightarrow 2.69$	$2.49_{-0.051}^{+0.055}$	$2.29 \rightarrow 2.71$
$\frac{\Delta m_{32}^2}{10^{-3} \text{ eV}^2}$ (I)	$-2.43_{-0.065}^{+0.042}$	$-2.65 \rightarrow -2.24$	$-2.47_{-0.064}^{+0.073}$	$-2.68 \rightarrow -2.25$

Table 1: Three-flavour oscillation parameters from our fit to global data after the Neutrino 2012 conference. For “Free Fluxes + RSBL” reactor fluxes have been left free in the fit and short baseline reactor data (RSBL) with $L \lesssim 100$ m are included; for “Huber Fluxes, no RSBL” the flux prediction from [42] are adopted and RSBL data are not used in the fit.

* **Modulo Short-Baseline Anomalies (David Schmitz lecture this Thursday)**



$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

What we have **really measured** (very roughly): [see, e.g., Antusch et al, hep-ph/0607020]

- Two mass-squared differences, at several percent level – many probes;
- $|U_{e2}|^2$ – solar data;
- $|U_{\mu2}|^2 + |U_{\tau2}|^2$ – solar data;
- $|U_{e2}|^2|U_{e1}|^2$ – KamLAND;
- $|U_{\mu3}|^2(1 - |U_{\mu3}|^2)$ – atmospheric data, K2K, MINOS;
- $|U_{e3}|^2(1 - |U_{e3}|^2)$ – Double Chooz, Daya Bay, RENO;
- $|U_{e3}|^2|U_{\mu3}|^2$ (upper bound \rightarrow evidence) – MINOS, T2K.

Lots of Room for Surprises!

More Neutrinos(?)

If there are more neutrinos with a well-defined mass, it is easy to extend the Paradigm:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_? \\ \vdots \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} & \cdots \\ U_{\mu1} & U_{\mu2} & U_{\mu3} & U_{\mu4} & \cdots \\ U_{\tau1} & U_{\tau2} & U_{\tau3} & U_{\tau4} & \cdots \\ U_{?1} & U_{?2} & U_{?3} & U_{?4} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \vdots \end{pmatrix}$$

- New mass eigenstates easy: ν_4 with mass m_4 , ν_5 with mass m_5 , etc.
- What are these new “flavor” (or weak) eigenstates $\nu_?$?

Number from e^+e^- Colliders

Number of Light ν Types

VALUE	DOCUMENT ID	TECN
2.9840 ± 0.0082	¹ LEP-SLC 06	RVUE
3.00 ± 0.05	² LEP 92	RVUE

• • • We do not use the following data for averages, fits, limits, etc. • • •

¹ Combined fit from ALEPH, DELPHI, L3 and OPAL Experiments.

² Simultaneous fits to all measured cross section data from all four LEP experimer

Number of Light ν Types from Direct Measurement of Invisible Z Width

In the following, the invisible Z width is obtained from studies of single-photon from the reaction $e^+e^- \rightarrow \nu\bar{\nu}\gamma$. All are obtained from LEP runs in the E_{cm}^{ee} 88–209 GeV.

VALUE	DOCUMENT ID	TECN	COMMENT
2.92 ± 0.05 OUR AVERAGE	Error includes scale factor of 1.2.		
$2.84 \pm 0.10 \pm 0.14$	ABDALLAH 05B	DLPH	$\sqrt{s} = 180\text{--}209$ G
$2.98 \pm 0.05 \pm 0.04$	ACHARD 04E	L3	1990–2000 LEP r
2.86 ± 0.09	HEISTER 03C	ALEP	$\sqrt{s} = 189\text{--}209$ G
$2.69 \pm 0.13 \pm 0.11$	ABBIENDI,G 00D	OPAL	1998 LEP run
$2.89 \pm 0.32 \pm 0.19$	ABREU 97J	DLPH	1993–1994 LEP r
$3.23 \pm 0.16 \pm 0.10$	AKERS 95C	OPAL	1990–1992 LEP r
$2.68 \pm 0.20 \pm 0.20$	BUSKULIC 93L	ALEP	1990–1991 LEP r

• • • We do not use the following data for averages, fits, limits, etc. • • •

$2.84 \pm 0.15 \pm 0.14$	ABREU 00Z	DLPH	1997–1998 LEP runs
3.01 ± 0.08	ACCIARRI 99R	L3	1991–1998 LEP runs
$3.1 \pm 0.6 \pm 0.1$	ADAM 96C	DLPH	$\sqrt{s} = 130, 136$ GeV

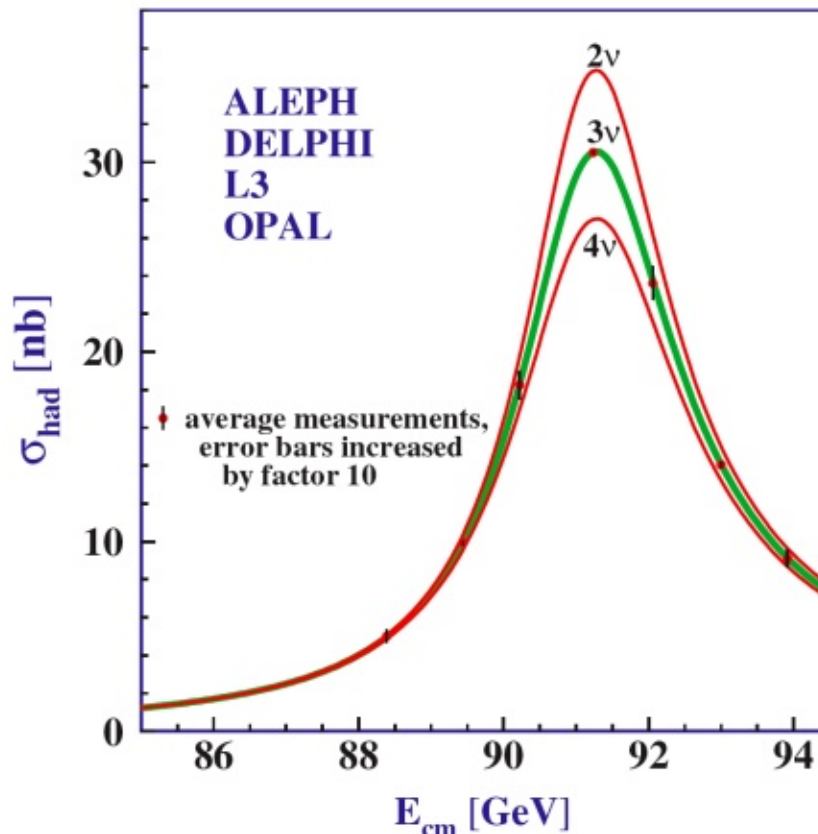
Limits from Astrophysics and Cosmology

Number of Light ν Types

("light" means $<$ about 1 MeV). See also OLIVE 81. For a review of limits based on Nucleosynthesis, Supernovae, and also on terrestrial experiments, see DENEGR I 90.

Also see "Big-Bang Nucleosynthesis" in this *Review*.

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< 4.10	95	³ MORESCO 12	COSM	

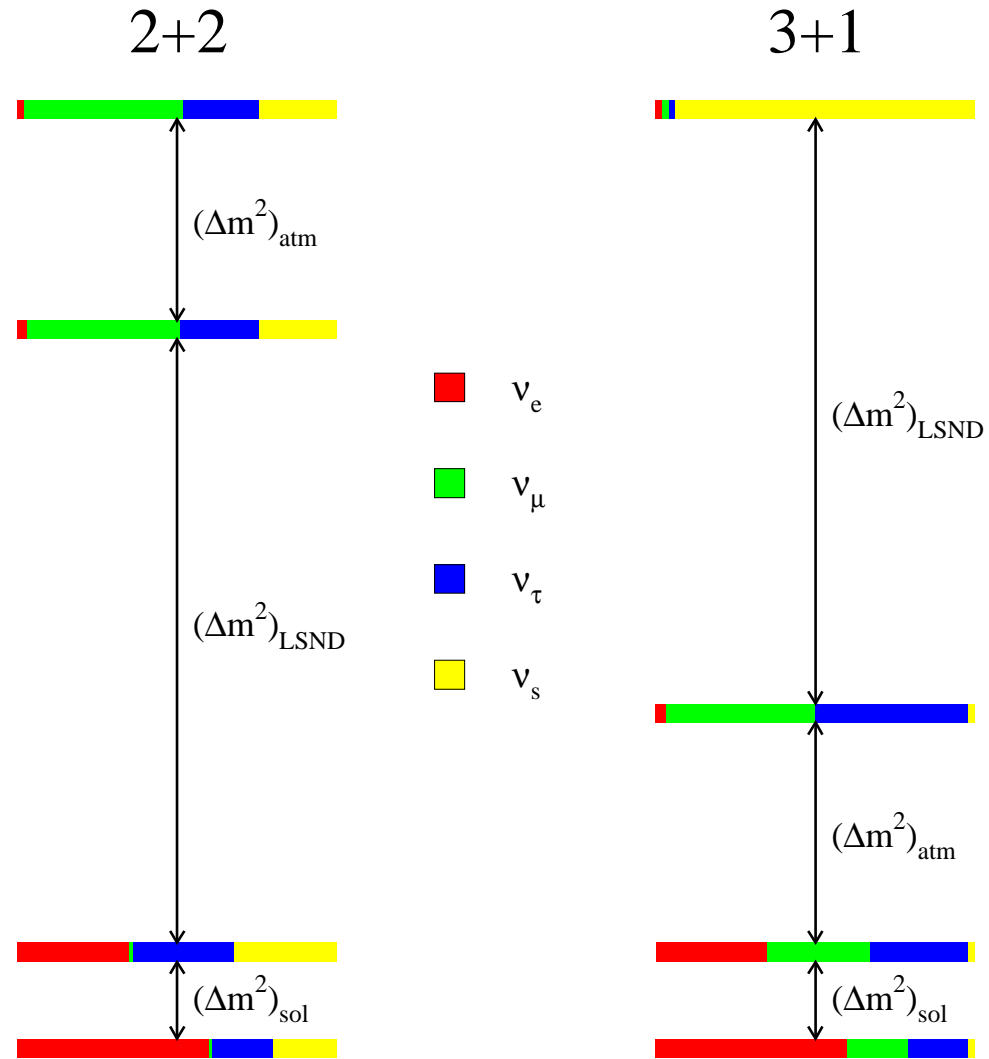


New neutrinos don't couple to the Z-boson if they are light (~ 45 GeV)

Hence STERILE neutrinos

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_{s_1} \\ \vdots \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} & \cdots \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} & \cdots \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} & \cdots \\ U_{s_1 1} & U_{s_1 2} & U_{s_1 3} & U_{s_1 4} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \vdots \end{pmatrix}$$

[Parameterizing the matrix is interesting. See AdG, Jenkins, PRD78, 053003 (2008)]



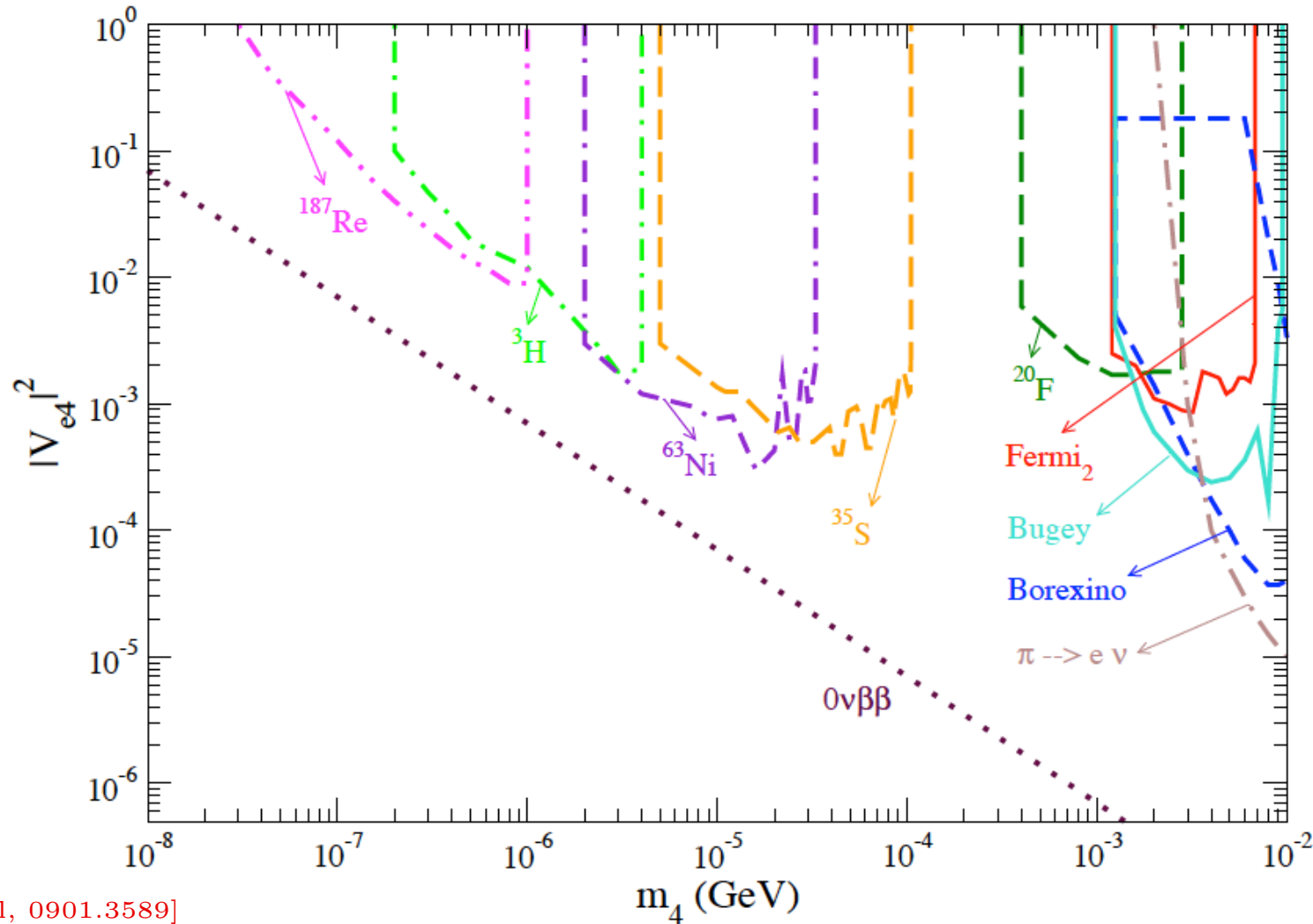
\Rightarrow 2+2 requires large sterile effects in either solar or atmospheric oscillations, not observed

I'll concentrate on “pure” sterile neutrinos (no other interactions with anyone). Such states only interact with the SM via weak mixing with the active neutrinos we know and love.

There are many theoretical complaints related to light sterile neutrinos:

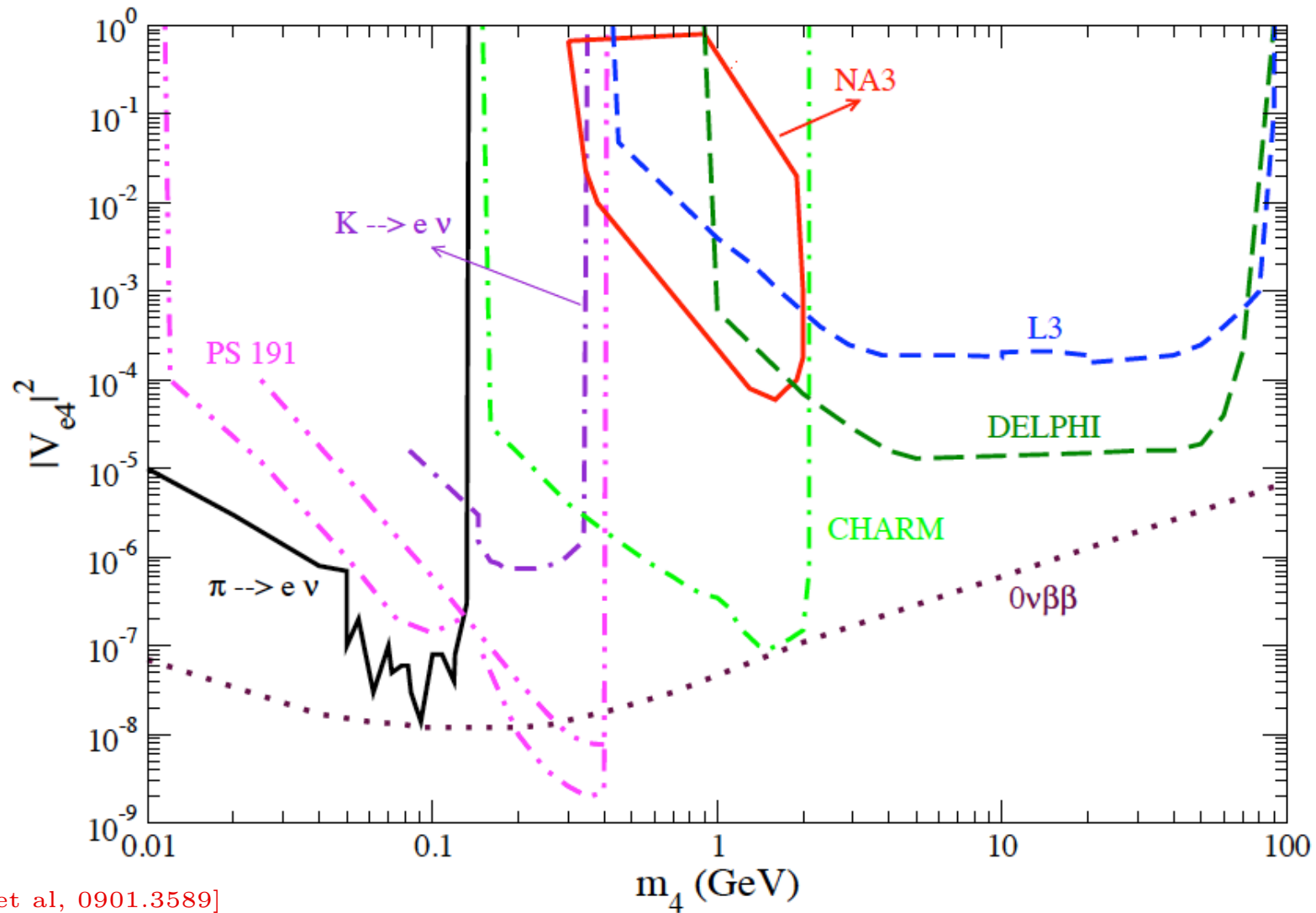
- Who ordered that? What are sterile neutrinos good for?
- Why would they be light? Sterile neutrinos are “theoretically expected” to be very heavy...
- If there are sterile neutrinos, can we say anything about their properties? Say, is the sterile–active neutrino mixing angle calculable? Are there preferred regions of the sterile neutrino parameter space?
- ...

BOTTOM LINE: In spite of theoretical complaints, sterile neutrinos are a viable logical possibility. They are experimentally constrained (more later), but are certainly allowed. They do not depend on whether neutrinos are Majorana or Dirac, do not imply the existence of more charged leptons (or quarks), do not lead to theoretical inconsistencies (anomalies), etc.



[A. Atre et al, 0901.3589]

Figure 2: Bounds on $|V_{e4}|^2$ versus m_4 in the mass range 10 eV–10 MeV. The excluded regions with contours labeled ^{187}Re [76], ^3H [77], ^{63}Ni [78], ^{35}S [79], ^{20}F and Fermi_2 [80] refer to the bounds from kink searches. All the limits are given at 95% C.L. except for the ones from Ref. [80] which are at 90% C.L.. The areas delimited by short dashed (blue) contour labeled Borexino and solid (cyan) contour labeled Bugey are excluded at 90% C.L. by searches of N_4 decays from the Borexino Counting Test facility [81] and Ref. [82] respectively. The region with long-dash-dotted (grey) contour, labelled $\pi \rightarrow e\nu$, is excluded by peak searches [83]. The dotted (maroon) line labelled $0\nu\beta\beta$ indicates the bound from searches of neutrinoless double beta-decay [84].



[A. Atre et al, 0901.3589]

Figure 3: Bounds on $|V_{e4}|^2$ versus m_4 in the mass range 10 MeV–100 GeV. The areas with solid (black) contour labeled $\pi \rightarrow e\nu$ and double dash dotted (purple) contour labeled $K \rightarrow e\nu$ are excluded by peak searches [83, 85]. Limits at 90% C.L. from beam-dump experiments are taken from Ref. [86] (PS191), Ref. [87] (NA3) and Ref. [88] (CHARM). The limits from contours labeled DELPHI and L3 are at 95% C.L. and are taken from Refs. [89] and [90] respectively. The excluded region with dotted (maroon) contour is derived from a reanalysis of neutrinoless double beta decay experimental data [84].

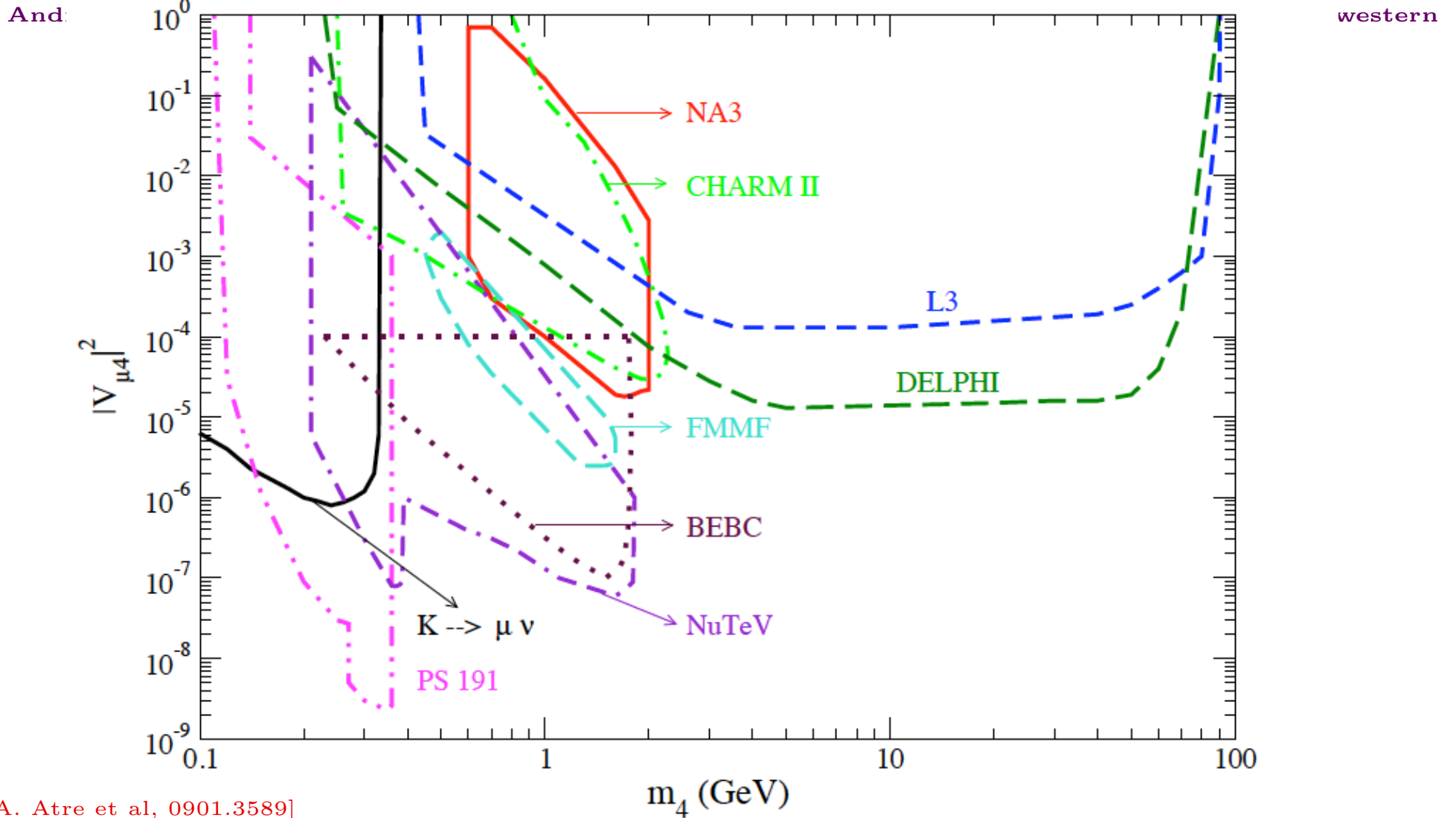
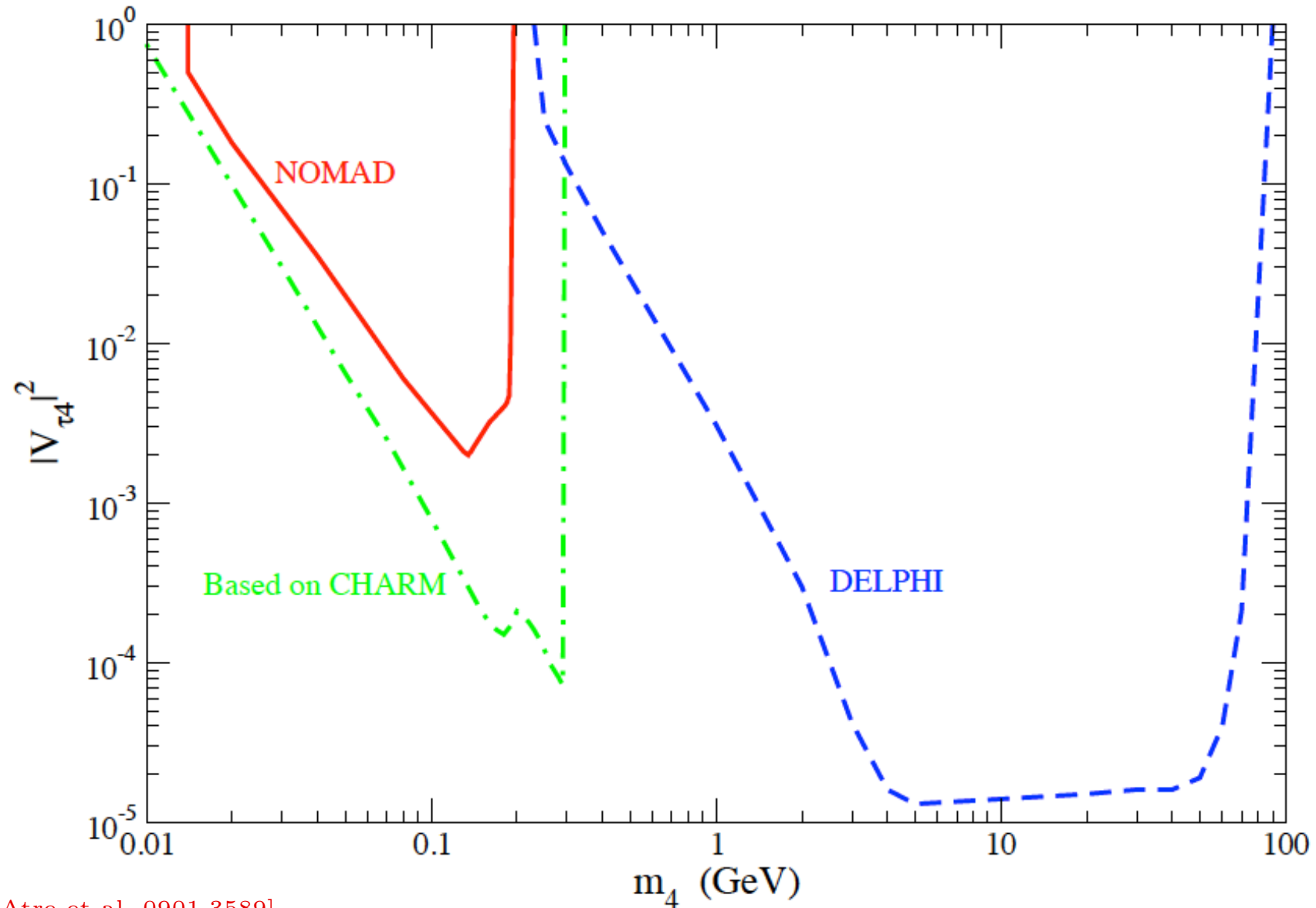
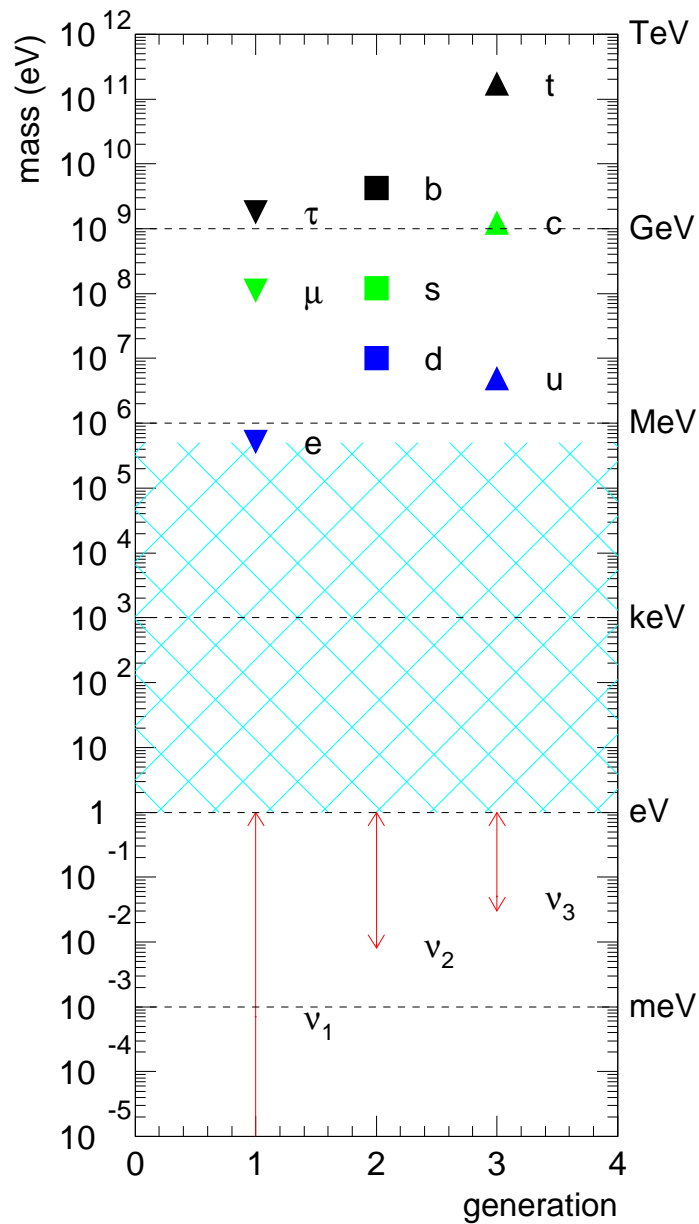


Figure 4: Limits on $|V_{\mu 4}|^2$ versus m_4 in the mass range 100 MeV–100 GeV come from peak searches and from N_4 decays. The area with solid (black) contour labeled $K \rightarrow \mu \nu$ [92] is excluded by peak searches. The bounds indicated by contours labeled by PS191 [86], NA3 [87], BEBC [93], FMMF [94], NuTeV [95] and CHARMII [96] are at 90% C.L., while DELPHI [89] and L3 [90] are at 95% C.L. and are deduced from searches of visible products in N_4 decays. For the beam dump experiments NA3, PS191, BEBC, FMMF and NuTeV we give an estimate of the upper limit for the excluded values of the mixing angle.



[A. Atre et al, 0901.3589]

Figure 5: Bounds on $|V_{\tau 4}|^2$ versus m_4 from searches of decays of heavy neutrinos, given in Ref. [97] (CHARM) and in Ref. [98] (NOMAD) at 90% C.L., and in Ref. [89] (DELPHI) at 95% C.L.



Why Neutrino Oscillations are a Big Deal:

⇐ **NEUTRINOS HAVE TINY MASSES**

What Does It Mean?

Who Cares About Neutrino Masses: “Palpable” Evidence of Physics Beyond the Standard Model*

The SM we all learned in school predicts that neutrinos are strictly massless. Massive neutrinos imply that the the SM is incomplete and needs to be replaced/modified.

Furthermore, the SM has to be replaced by something qualitatively different.

* There is only a handful of questions our understanding of fundamental physics is yet to explain properly. These are in order of palpability (these are personal. Feel free to complain)

- What is the physics behind electroweak symmetry breaking? (Higgs (✓?)).
- What is the dark matter? (not in SM).
- Why does the Universe appear to be accelerating? Why does it appear that the Universe underwent rapid acceleration in the past? (certainly not in SM!).

What is the New Standard Model? [ν SM]

The short answer is – WE DON'T KNOW. Not enough available info!



Equivalently, there are several completely different ways of addressing neutrino masses. The key issue is to understand what else the ν SM candidates can do. [are they falsifiable?, are they “simple”?, do they address other outstanding problems in physics?, etc]

Candidate ν SM: The One I'll Concentrate On

SM as an effective field theory – non-renormalizable operators

$$\mathcal{L}_{\nu\text{SM}} \supset -y_{ij} \frac{L^i H L^j H}{2\Lambda} + \mathcal{O}\left(\frac{1}{\Lambda^2}\right) + H.c.$$

There is only one dimension five operator [Weinberg, 1979]. If $\Lambda \gg 1$ TeV, it leads to only one observable consequence...

$$\text{after EWSB: } \mathcal{L}_{\nu\text{SM}} \supset \frac{m_{ij}}{2} \nu^i \nu^j; \quad m_{ij} = y_{ij} \frac{v^2}{\Lambda}.$$

- Neutrino masses are small: $\Lambda \gg v \rightarrow m_\nu \ll m_f$ ($f = e, \mu, u, d$, etc)
- Neutrinos are Majorana fermions – Lepton number is violated!
- ν SM effective theory – not valid for energies above *at most* Λ/y .
- Define $y_{\text{max}} \equiv 1 \Rightarrow$ data require $\Lambda \sim 10^{14}$ GeV.

What else is this “good for”? Depends on the ultraviolet completion!

The Seesaw Lagrangian

A simple^a, renormalizable Lagrangian that allows for neutrino masses is

$$\mathcal{L}_\nu = \mathcal{L}_{\text{old}} - \lambda_{\alpha i} L^\alpha H N^i - \sum_{i=1}^3 \frac{M_i}{2} N^i N^i + H.c.,$$

where N_i ($i = 1, 2, 3$, for concreteness) are SM gauge singlet fermions.

\mathcal{L}_ν is the most general, renormalizable Lagrangian consistent with the SM gauge group and particle content, plus the addition of the N_i fields.

After electroweak symmetry breaking, \mathcal{L}_ν describes, besides all other SM degrees of freedom, six Majorana fermions: **six neutrinos**.

^aOnly requires the introduction of three fermionic degrees of freedom, no new interactions or symmetries.

To be determined from data: λ and M .

The data can be summarized as follows: there is evidence for three neutrinos, mostly “active” (linear combinations of ν_e , ν_μ , and ν_τ). At least two of them are massive and, if there are other neutrinos, they have to be “sterile.”

This provides very little information concerning the magnitude of M_i (assume $M_1 \sim M_2 \sim M_3$).

Theoretically, there is prejudice in favor of very large M : $M \gg v$. Popular examples include $M \sim M_{\text{GUT}}$ (GUT scale), or $M \sim 1 \text{ TeV}$ (EWSB scale).

Furthermore, $\lambda \sim 1$ translates into $M \sim 10^{14} \text{ GeV}$, while thermal leptogenesis requires the lightest M_i to be around 10^{10} GeV .

we can impose very, very few experimental constraints on M

What We Know About M :

- $M = 0$: the six neutrinos “fuse” into three Dirac states. Neutrino mass matrix given by $\mu_{\alpha i} \equiv \lambda_{\alpha i} \nu$.

The symmetry of \mathcal{L}_ν is enhanced: $U(1)_{B-L}$ is an exact global symmetry of the Lagrangian if all M_i vanish. Small M_i values are 'tHooft natural.

- $M \gg \mu$: the six neutrinos split up into three mostly active, light ones, and three, mostly sterile, heavy ones. The light neutrino mass matrix is given by $m_{\alpha\beta} = \sum_i \mu_{\alpha i} M_i^{-1} \mu_{\beta i}$ [$m \propto 1/\Lambda \Rightarrow \Lambda = M/\mu^2$].

This the **seesaw mechanism**. Neutrinos are Majorana fermions.

Lepton number is not a good symmetry of \mathcal{L}_ν , even though L -violating effects are hard to come by.

- $M \sim \mu$: six states have similar masses. Active–sterile mixing is very large. This scenario is (generically) ruled out by active neutrino data (atmospheric, solar, KamLAND, K2K, etc).

Why are Neutrino Masses Small in the $M \neq 0$ Case?

If $\mu \ll M$, below the mass scale M ,

$$\mathcal{L}_5 = \frac{LHLH}{\Lambda}.$$

Neutrino masses are small if $\Lambda \gg \langle H \rangle$. Data require $\Lambda \sim 10^{14}$ GeV.

In the case of the seesaw,

$$\Lambda \sim \frac{M}{\lambda^2},$$

so neutrino masses are small if either

- they are generated by physics at a very high energy scale $M \gg v$ (high-energy seesaw); **or**
- they arise out of a very weak coupling between the SM and a new, hidden sector (low-energy seesaw); **or**
- cancellations among different contributions render neutrino masses accidentally small (“fine-tuning”).

High-Energy Seesaw: Brief Comments

- This is everyone's favorite scenario.
- Upper bound for M (e.g. Maltoni, Niczyporuk, Willenbrock, hep-ph/0006358):

$$M < 7.6 \times 10^{15} \text{ GeV} \times \left(\frac{0.1 \text{ eV}}{m_\nu} \right).$$

- Naturalness 'hint' (e.g., Casas, Espinosa, Hidalgo, hep-ph/0410298):

$$M < 10^7 \text{ GeV}.$$

- Physics “too” heavy! No observable consequence other than leptogenesis. From thermal leptogenesis $M > 10^9 \text{ GeV}$. Will we ever convince ourselves that this is correct? (e.g., Buckley, Murayama, hep-ph/0606088)

High-energy seesaw has no other observable consequences, except, perhaps, ...

Baryogenesis via Leptogenesis

One of the most basic questions we are allowed to ask (with any real hope of getting an answer) is whether the **observed baryon asymmetry** of the Universe can be obtained **from a baryon–antibaryon symmetric initial condition** plus well understood **dynamics**. [**Baryogenesis**]

This isn't just for aesthetic reasons. If the early Universe undergoes a period of **inflation**, baryogenesis is required, as inflation would wipe out any pre-existing baryon asymmetry.

It turns out that massive neutrinos can help solve this puzzle!

In the old SM, (electroweak) baryogenesis does not work – not enough CP-invariance violation, Higgs boson too light.

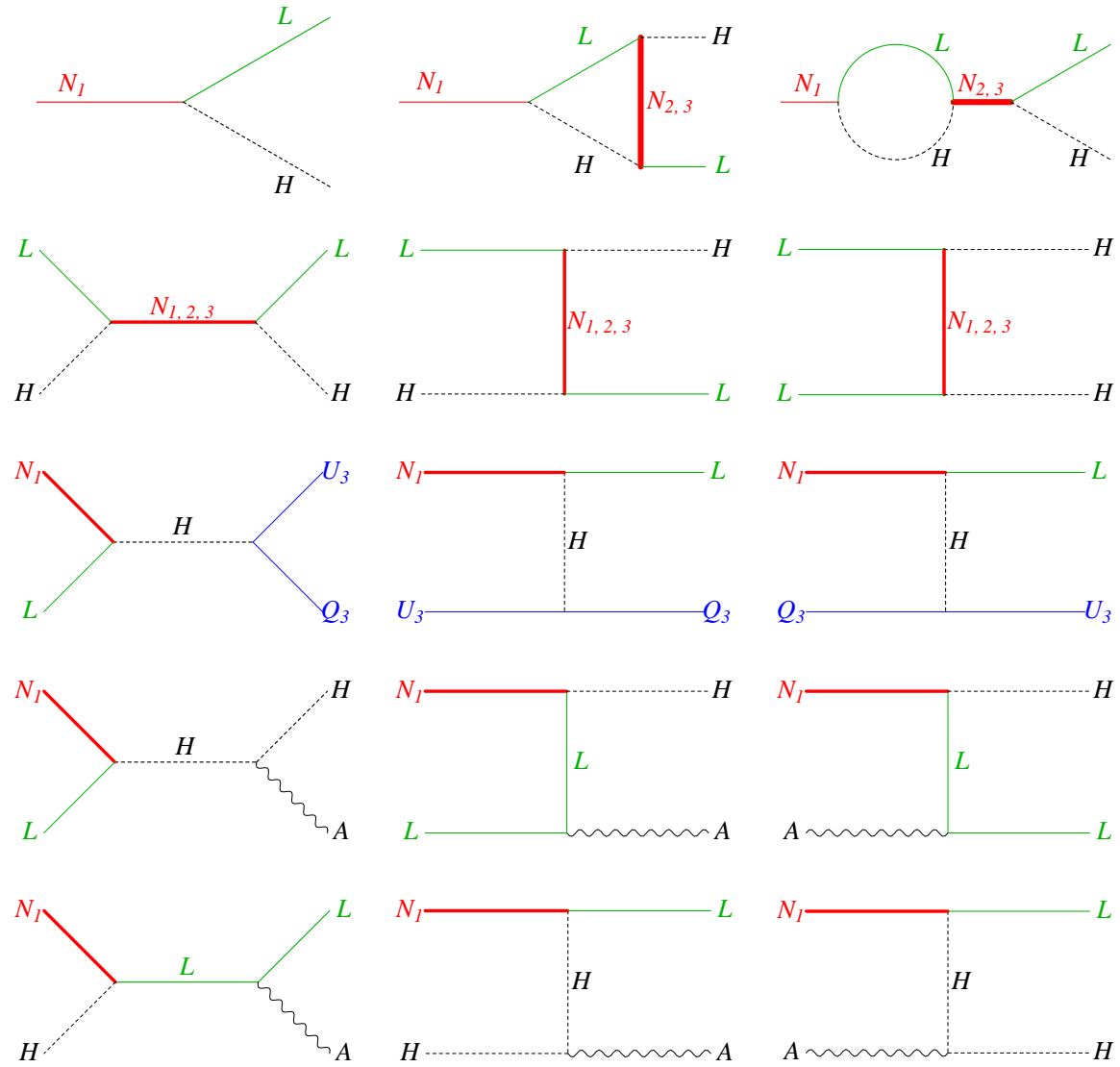
Neutrinos help by providing all the necessary ingredients for successful baryogenesis via leptogenesis.

- Violation of lepton number, which later on is transformed into baryon number by nonperturbative, finite temperature electroweak effects (in one version of the ν SM, lepton number is broken at a high energy scale M).
- Violation of C-invariance and CP-invariance (weak interactions, plus new CP-odd phases).
- Deviation from thermal equilibrium (depending on the strength of the relevant interactions).

E.g. – thermal, seesaw leptogenesis,

$$\mathcal{L} \supset -y_{i\alpha} L^i H N^\alpha - \frac{M_N^{\alpha\beta}}{2} N_\alpha N_\beta + H.c.$$

[Fukugita, Yanagida]



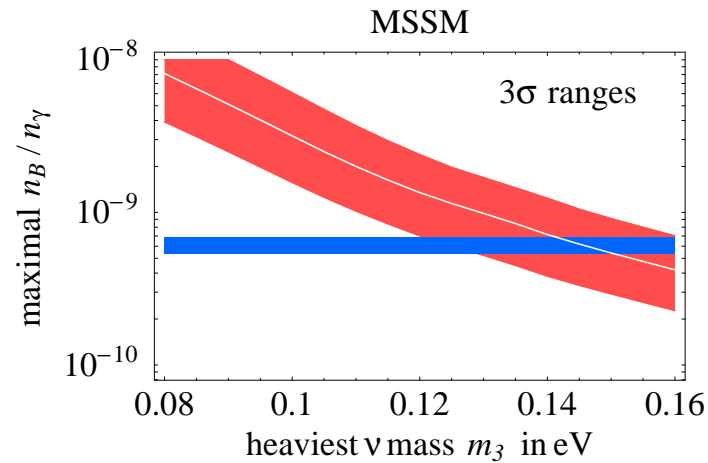
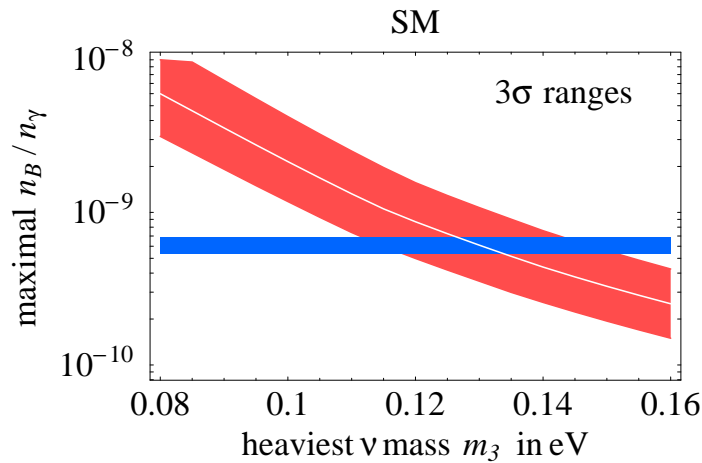
- L-violating processes
- $y \Rightarrow$ CP-violation
- deviation from thermal eq. constrains combinations of M_N and y .
- need to yield correct m_ν

not trivial!

[G. Giudice *et al*, hep-ph/0310123]

E.g. – thermal, seesaw leptogenesis,

$$\mathcal{L} \supset -y_{i\alpha} L^i H N^\alpha - \frac{M_N^{\alpha\beta}}{2} N_\alpha N_\beta + H.c.$$



[G. Giudice *et al.*, hep-ph/0310123]

It did not have to work – but it does

MSSM picture does not quite work – gravitino problem

(there are ways around it, of course...)

Relationship to Low Energy Observables?

In general ... no. This is very easy to understand. The baryon asymmetry depends on the (high energy) physics responsible for lepton-number violation. Neutrino masses are a (small) consequence of this physics, albeit the only observable one at the low-energy experiments we can perform nowadays.

see-saw: y, M_N have more physical parameters than $m_\nu = y^t M_N^{-1} y$.

There could be a relationship, but it requires that we know more about the high energy Lagrangian (model dependent).^a The day will come when we have enough evidence to refute leptogenesis (or strongly suspect that it is correct), but more information is really necessary (charged-lepton flavor violation, collider data on EWSB, lepton-number violation, etc).

^aBut listen to Boris's "plausibility argument." He will lecture on something else in the very near future, but I am sure he will be delighted to tell you more about it if you inquire!

Low-Energy Seesaw [AdG PRD72,033005]

The other end of the M spectrum ($M < 100$ GeV). What do we get?

- Neutrino masses are small because the Yukawa couplings are very small $\lambda \in [10^{-6}, 10^{-11}]$;
- No standard thermal leptogenesis – right-handed neutrinos way too light? [For a possible alternative see Canetti, Shaposhnikov, arXiv: 1006.0133 and reference therein.]
- No obvious connection with other energy scales (EWSB, GUTs, etc);
- Right-handed neutrinos are propagating degrees of freedom. They look like sterile neutrinos \Rightarrow sterile neutrinos associated with the fact that the active neutrinos have mass;
- sterile–active mixing can be predicted – hypothesis is falsifiable!
- Small values of M are natural (in the ‘tHooft sense). In fact, theoretically, no value of M should be discriminated against!

More Details, assuming three right-handed neutrinos N :

$$m_\nu = \begin{pmatrix} 0 & \lambda v \\ (\lambda v)^t & M \end{pmatrix},$$

M is diagonal, and all its eigenvalues are real and positive. The charged lepton mass matrix also diagonal, real, and positive.

To leading order in $(\lambda v)M^{-1}$, the three lightest neutrino mass eigenvalues are given by the eigenvalues of

$$m_a = \lambda v M^{-1} (\lambda v)^t,$$

where m_a is the mostly active neutrino mass matrix, while the heavy sterile neutrino masses coincide with the eigenvalues of M .

6×6 mixing matrix U [$U^t m_\nu U = \text{diag}(m_1, m_2, m_3, m_4, m_5, m_6)$] is

$$U = \begin{pmatrix} V & \Theta \\ -\Theta^\dagger V & 1_{n \times n} \end{pmatrix},$$

where V is the active neutrino mixing matrix (MNS matrix)

$$V^t m_a V = \text{diag}(m_1, m_2, m_3),$$

and the matrix that governs active–sterile mixing is

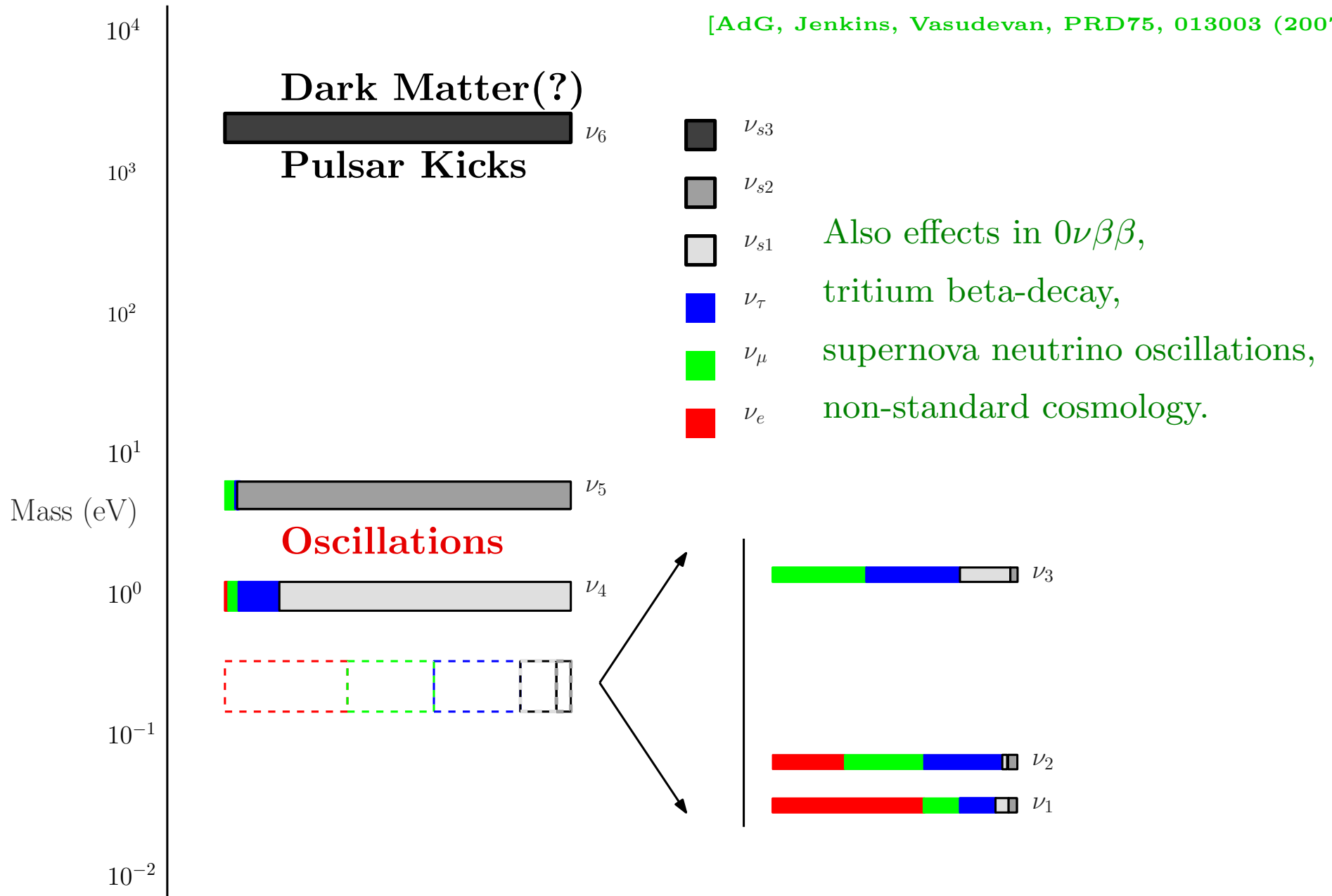
$$\Theta = (\lambda\nu)^* M^{-1}.$$

One can solve for the Yukawa couplings and re-express

$$\Theta = V \sqrt{\text{diag}(m_1, m_2, m_3)} R^\dagger M^{-1/2},$$

where R is a complex orthogonal matrix $RR^t = 1$.

[AdG, Jenkins, Vasudevan, PRD75, 013003 (2007)]



Predictions: Neutrinoless Double-Beta Decay

The exchange of Majorana neutrinos mediates lepton-number violating neutrinoless double-beta decay, $0\nu\beta\beta$: $Z \rightarrow (Z + 2)e^-e^-$.

For light enough neutrinos, the amplitude for $0\nu\beta\beta$ is proportional to the effective neutrino mass

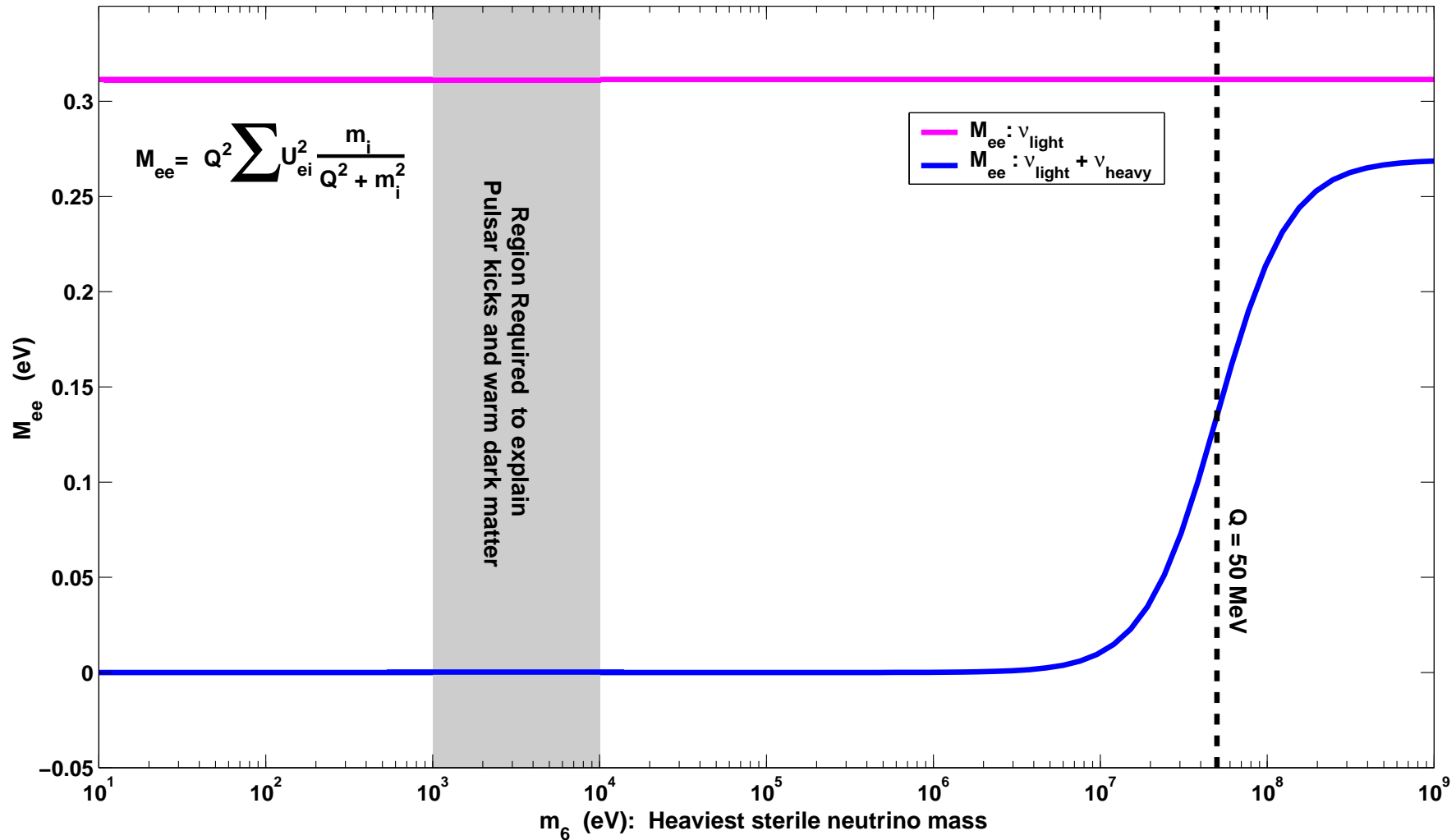
$$m_{ee} = \left| \sum_{i=1}^6 U_{ei}^2 m_i \right| \sim \left| \sum_{i=1}^3 U_{ei}^2 m_i + \sum_{i=1}^3 \vartheta_{ei}^2 M_i \right|.$$

However, upon further examination, $m_{ee} = 0$ in the eV-seesaw. **The contribution of light and heavy neutrinos exactly cancels!** This seems to remain true to a good approximation as long as $M_i \ll 1$ MeV.

$$\left[\mathcal{M} = \begin{pmatrix} 0 & \mu^T \\ \mu & M \end{pmatrix} \rightarrow m_{ee} \text{ is identically zero!} \right]$$

(lack of) sensitivity in $0\nu\beta\beta$ due to seesaw sterile neutrinos

[AdG, Jenkins, Vasudevan, hep-ph/0608147]



Predictions: **Tritium beta-decay**

Heavy neutrinos participate in tritium β -decay. Their contribution can be parameterized by

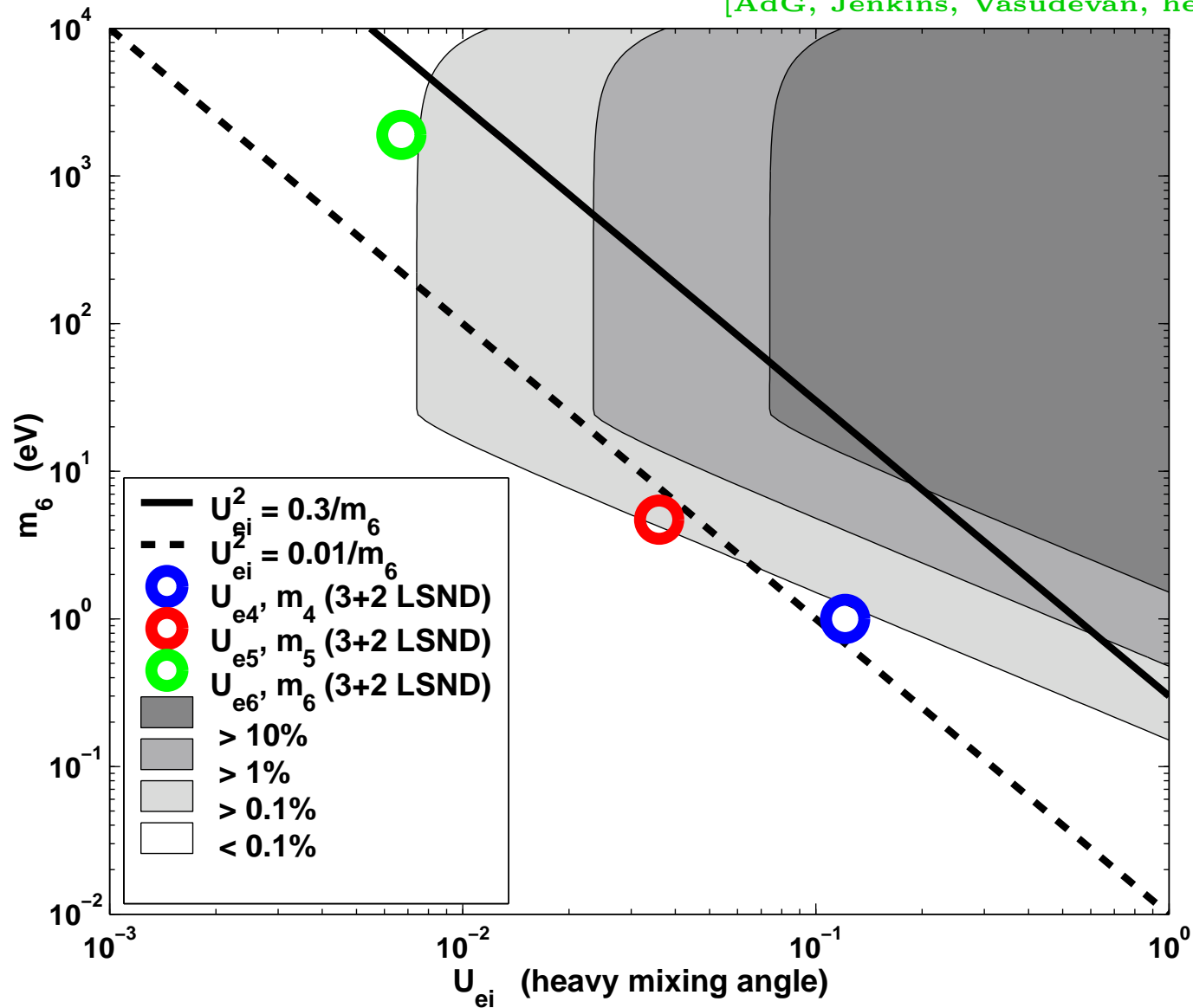
$$m_{\beta}^2 = \sum_{i=1}^6 |U_{ei}|^2 m_i^2 \simeq \sum_{i=1}^3 |U_{ei}|^2 m_i^2 + \sum_{i=1}^3 |U_{ei}|^2 m_i M_i,$$

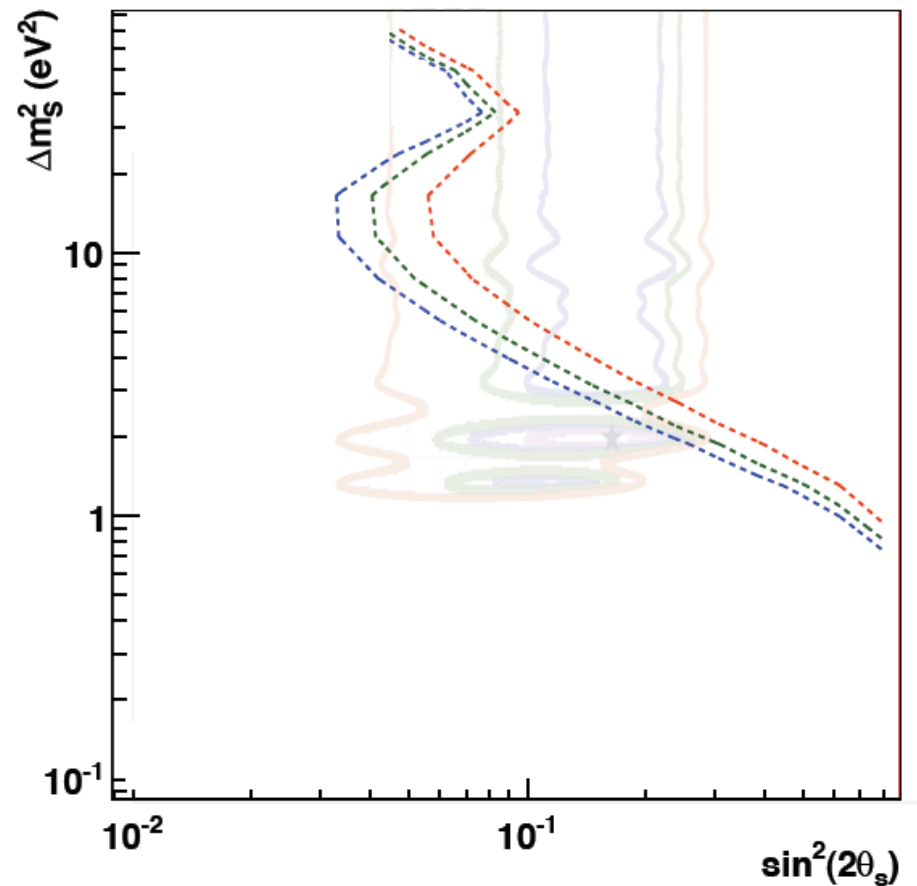
as long as M_i is not too heavy (above tens of eV). For example, in the case of a 3+2 solution to the LSND anomaly, the heaviest sterile state (with mass M_1) contributes the most: $m_{\beta}^2 \simeq 0.7 \text{ eV}^2 \left(\frac{|U_{e1}|^2}{0.7} \right) \left(\frac{m_1}{0.1 \text{ eV}} \right) \left(\frac{M_1}{10 \text{ eV}} \right)$.

NOTE: next generation experiment (KATRIN) will be sensitive to $O(10^{-1}) \text{ eV}^2$.

sensitivity of tritium beta decay to seesaw sterile neutrinos

[AdG, Jenkins, Vasudevan, hep-ph/0608147]





[Barrett, Formaggio, 1105.1326]

FIG. 2: Sensitivity of the KATRIN neutrino mass measurement for a sterile neutrino with relatively large mass splitting (dashed contours). Figure shows exclusion curves of mixing angle $\sin^2(2\theta_S)$ versus mass splitting $|\Delta m_S^2|^2$ for the 90% (blue), 95% (green), and 99% (red) C.L. after three years of data taking. Figure 7 from Ref. [2] show in solid curves in the background.

On Early Universe Cosmology / Astrophysics

A combination of the SM of particle physics plus the “concordance cosmological model” severely constrain light, sterile neutrinos with significant active-sterile mixing. Taken at face value, not only is the eV-seesaw ruled out, but so are all oscillation solutions to the LSND anomaly.

Hence, eV-seesaw \rightarrow nonstandard particle physics and cosmology.

On the other hand...

- Right-handed neutrinos may make good warm dark matter particles.

Asaka, Blanchet, Shaposhnikov, hep-ph/0503065.

- Sterile neutrinos are known to help out with r-process nucleosynthesis in supernovae, ...
- ...and may help explain the peculiar peculiar velocities of pulsars.

Big Bang Neutrinos are Warm Dark Matter

Planck Collaboration: Cosmological parameters

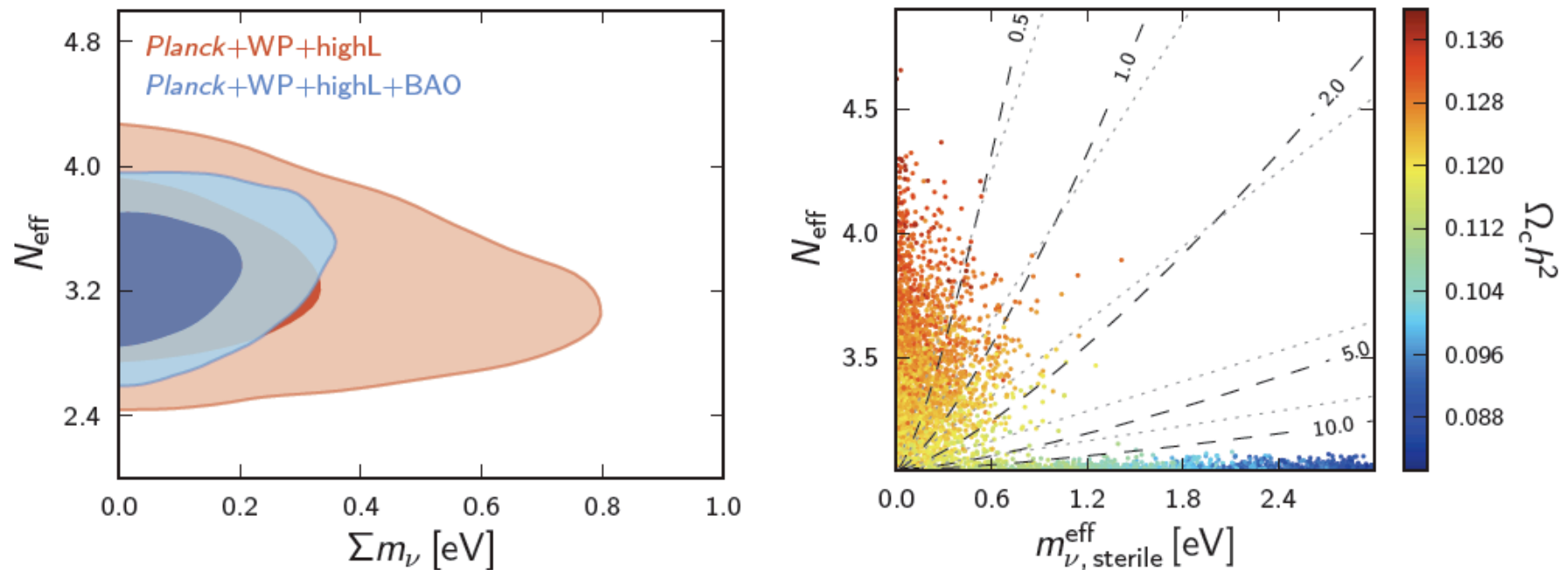


Fig. 28. *Left:* 2D joint posterior distribution between N_{eff} and $\sum m_\nu$ (the summed mass of the three active neutrinos) in models with extra massless neutrino-like species. *Right:* Samples in the $N_{\text{eff}}-m_{\nu, \text{sterile}}^{\text{eff}}$ plane, colour-coded by $\Omega_c h^2$, in models with one massive sterile neutrino family, with effective mass $m_{\nu, \text{sterile}}^{\text{eff}}$, and the three active neutrinos as in the base Λ CDM model. The physical mass of the sterile neutrino in the thermal scenario, $m_{\text{sterile}}^{\text{thermal}}$, is constant along the grey dashed lines, with the indicated mass in eV. The physical mass in the Dodelson-Widrow scenario, $m_{\text{sterile}}^{\text{DW}}$, is constant along the dotted lines (with the value indicated on the adjacent dashed lines).

What if $1 \text{ GeV} < M < 1 \text{ TeV}$?

Naively, one expects

$$\Theta \sim \sqrt{\frac{m_a}{M}} < 10^{-5} \sqrt{\frac{1 \text{ GeV}}{M}},$$

such that, for $M = 1 \text{ GeV}$ and above, sterile neutrino effects are mostly negligible.

However,

$$\Theta = V \sqrt{\text{diag}(m_1, m_2, m_3)} R^\dagger M^{-1/2},$$

and the magnitude of the entries of R can be arbitrarily large [$\cos(ix) = \cosh x \gg 1$ if $x > 1$].

This is true as long as

- $\lambda v \ll M$ (seesaw approximation holds)
- $\lambda < 4\pi$ (theory is “well-defined”)

This implies that, in principle, Θ is a quasi-free parameter – independent from light neutrino masses and mixing – as long as $\Theta \ll 1$ and $M < 1 \text{ TeV}$.

What Does $R \gg 1$ Mean?

It is illustrative to consider the case of one active neutrino of mass m_3 and two sterile ones, and further assume that $M_1 = M_2 = M$. In this case,

$$\Theta = \sqrt{\frac{m_3}{M}} \begin{pmatrix} \cos \zeta & \sin \zeta \end{pmatrix},$$

$$\lambda v = \sqrt{m_3 M} \begin{pmatrix} \cos \zeta^* & \sin \zeta^* \end{pmatrix} \equiv \begin{pmatrix} \lambda_1 & \lambda_2 \end{pmatrix}.$$

If ζ has a large imaginary part $\Rightarrow \Theta$ is (exponentially) larger than $(m_3/M)^{1/2}$, λ_i neutrino Yukawa couplings are much larger than $\sqrt{m_3 M}/v$

The reason for this is a strong cancellation between the contribution of the two different Yukawa couplings to the active neutrino mass

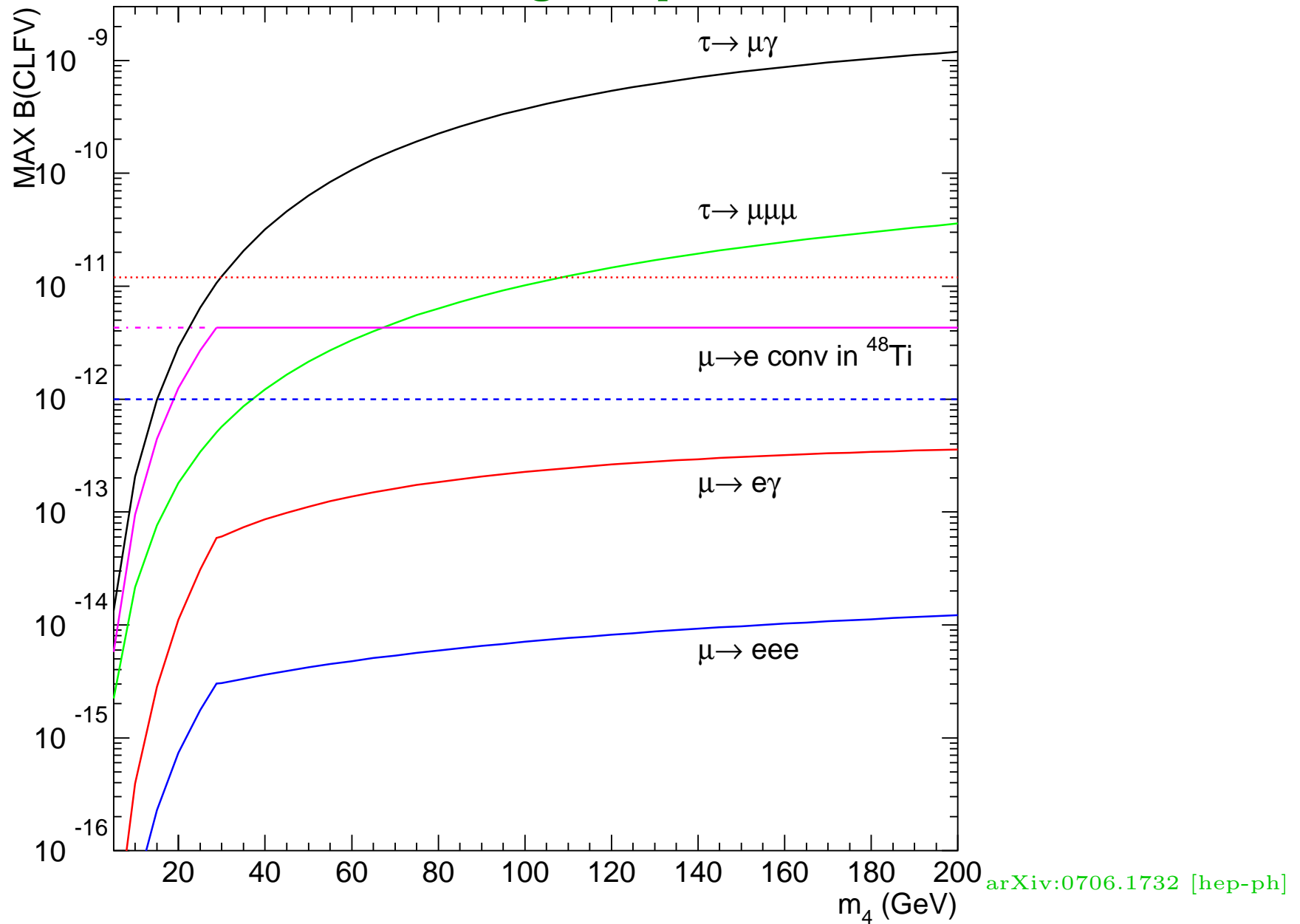
$$\Rightarrow m_3 = \lambda_1^2 v^2 / M + \lambda_2^2 v^2 / M.$$

For example: $m_3 = 0.1$ eV, $M = 100$ GeV, $\zeta = 14i \Rightarrow \lambda_1 \sim 0.244$, $\lambda_2 \sim -0.244i$, while $|y_1| - |y_2| \sim 3.38 \times 10^{-13}$.

NOTE: cancellation may be consequence of a symmetry (say, lepton number).

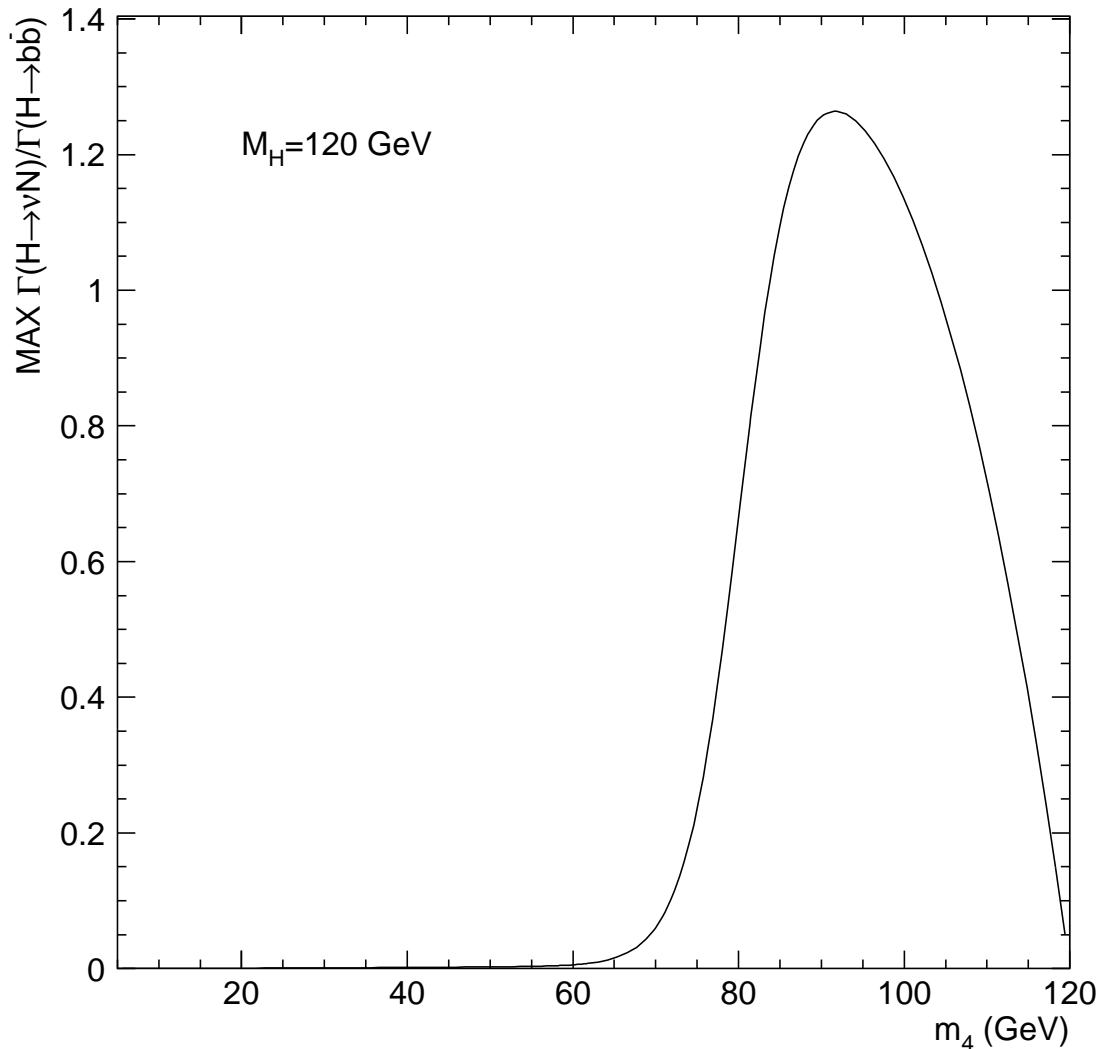
See, for example, the “inverse seesaw” Mohapatra and Valle, PRD34, 1642 (1986).

Constraints From Charged Lepton Flavor Violation



Weak Scale Seesaw, and Accidentally Light Neutrino Masses

[AdG arXiv:0706.1732 [hep-ph]]



What does the seesaw Lagrangian predict for the LHC?

Nothing much, unless...

- $M_N \sim 1 - 100$ GeV,
- Yukawa couplings larger than naive expectations.

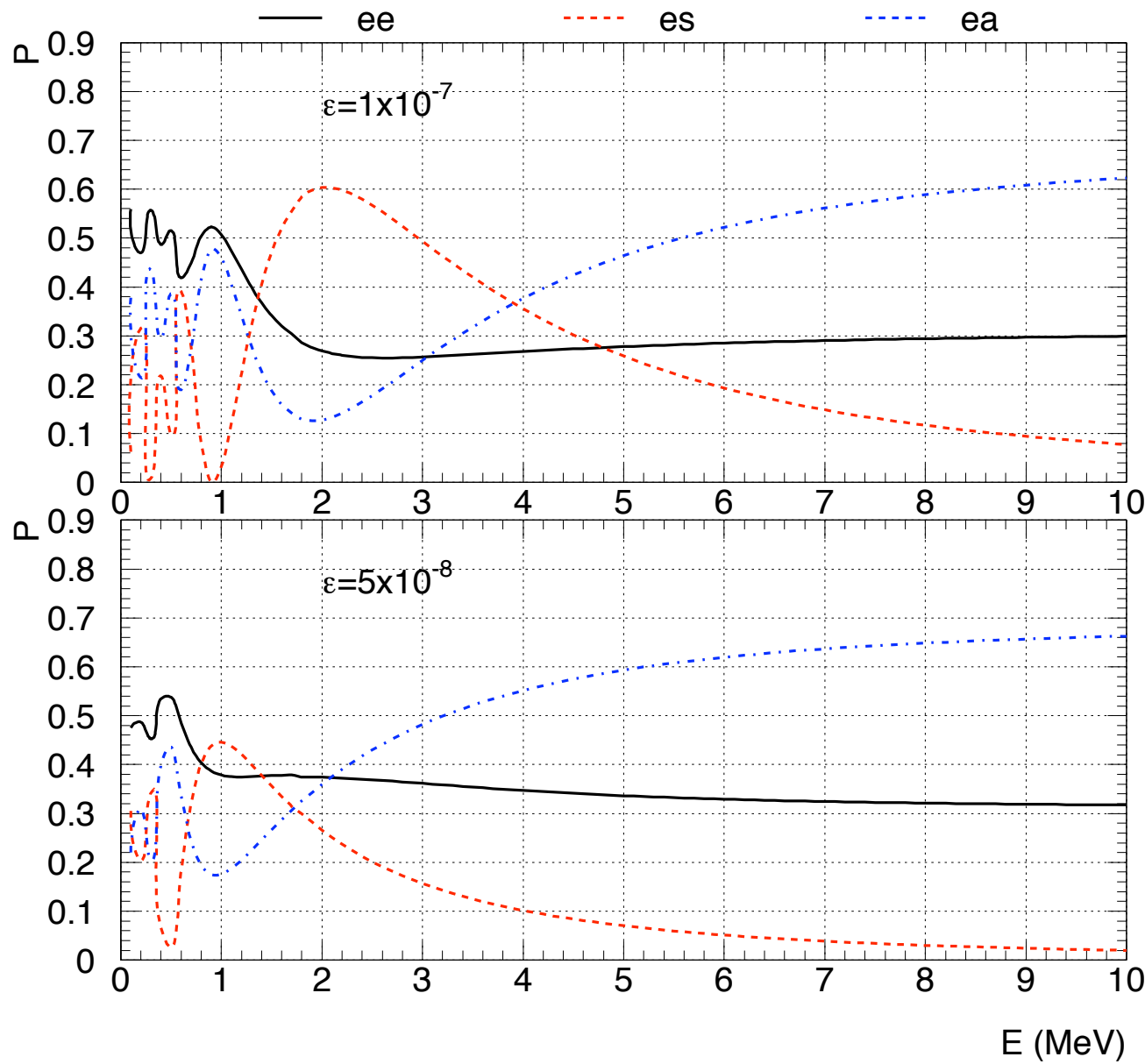
$\Leftarrow H \rightarrow \nu N$ as likely as $H \rightarrow b\bar{b}$!

(NOTE: $N \rightarrow \ell q' \bar{q}$ or $\ell \ell' \nu$ (prompt)
 “Weird” Higgs decay signature!)

Going All the Way: What Happens When $M \ll \mu$?

In this case, the six Weyl fermions pair up into three quasi-degenerate states (“quasi-Dirac fermions”).

These states are fifty–fifty active–sterile mixtures. In the limit $M \rightarrow 0$, we end up with Dirac neutrinos, which are clearly allowed by all the data.

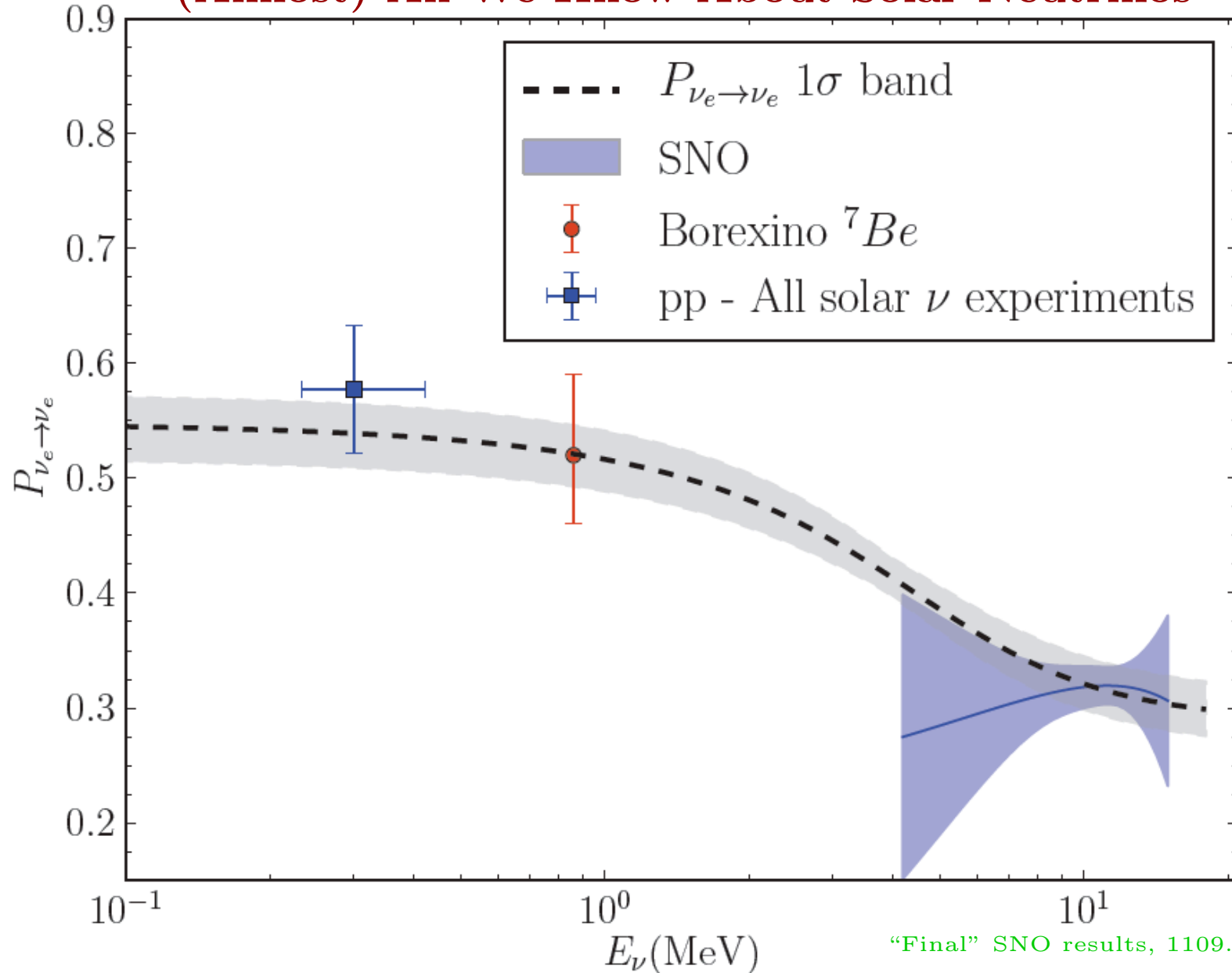


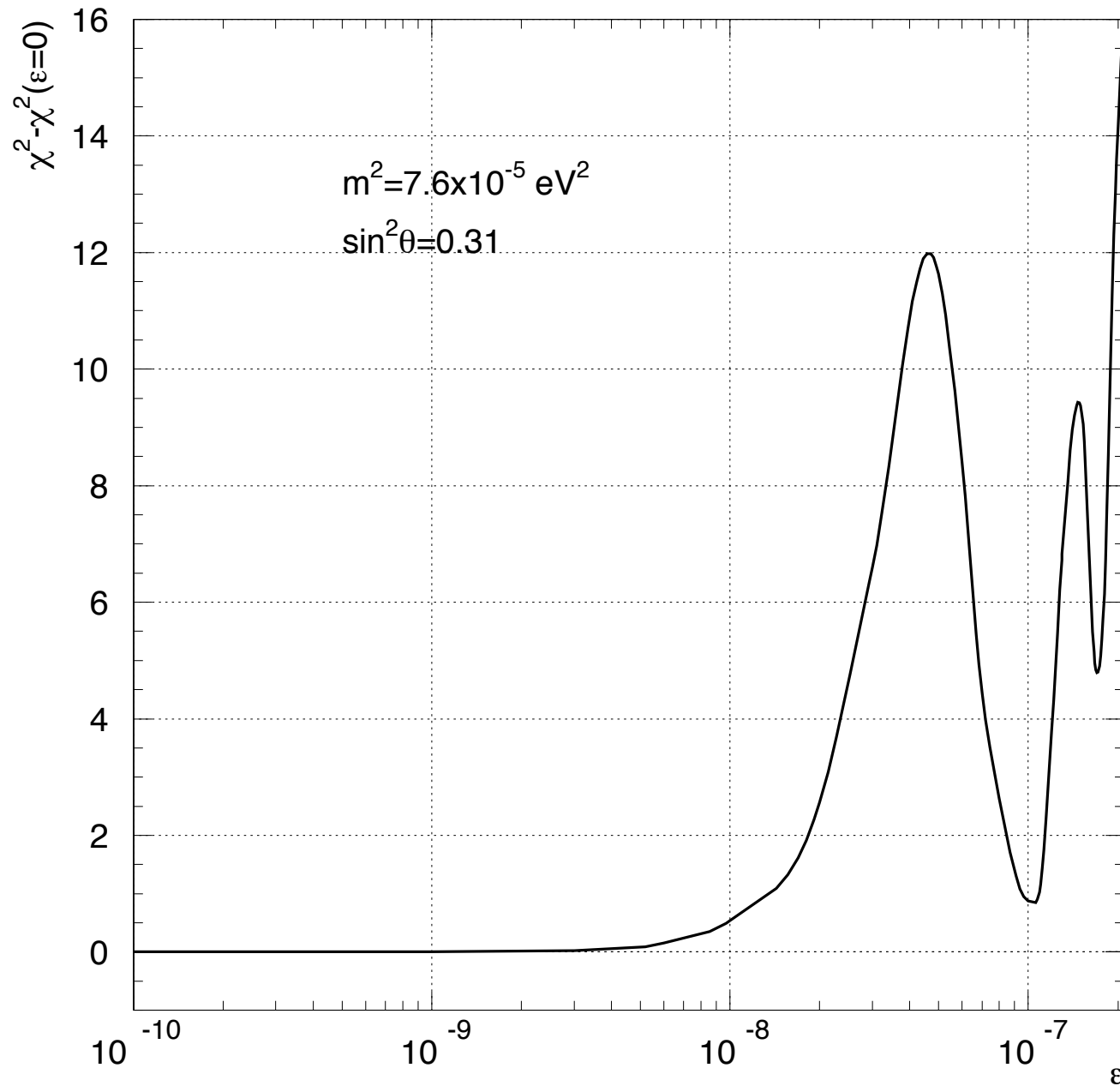
Quasi-Sterile Neutrinos

- tiny new $\Delta m^2 = \epsilon \Delta m_{12}^2$,
- maximal mixing!
- Effects in Solar ν_s

[AdG, Huang, Jenkins, arXiv:0906.1611]

(Almost) All We Know About Solar Neutrinos





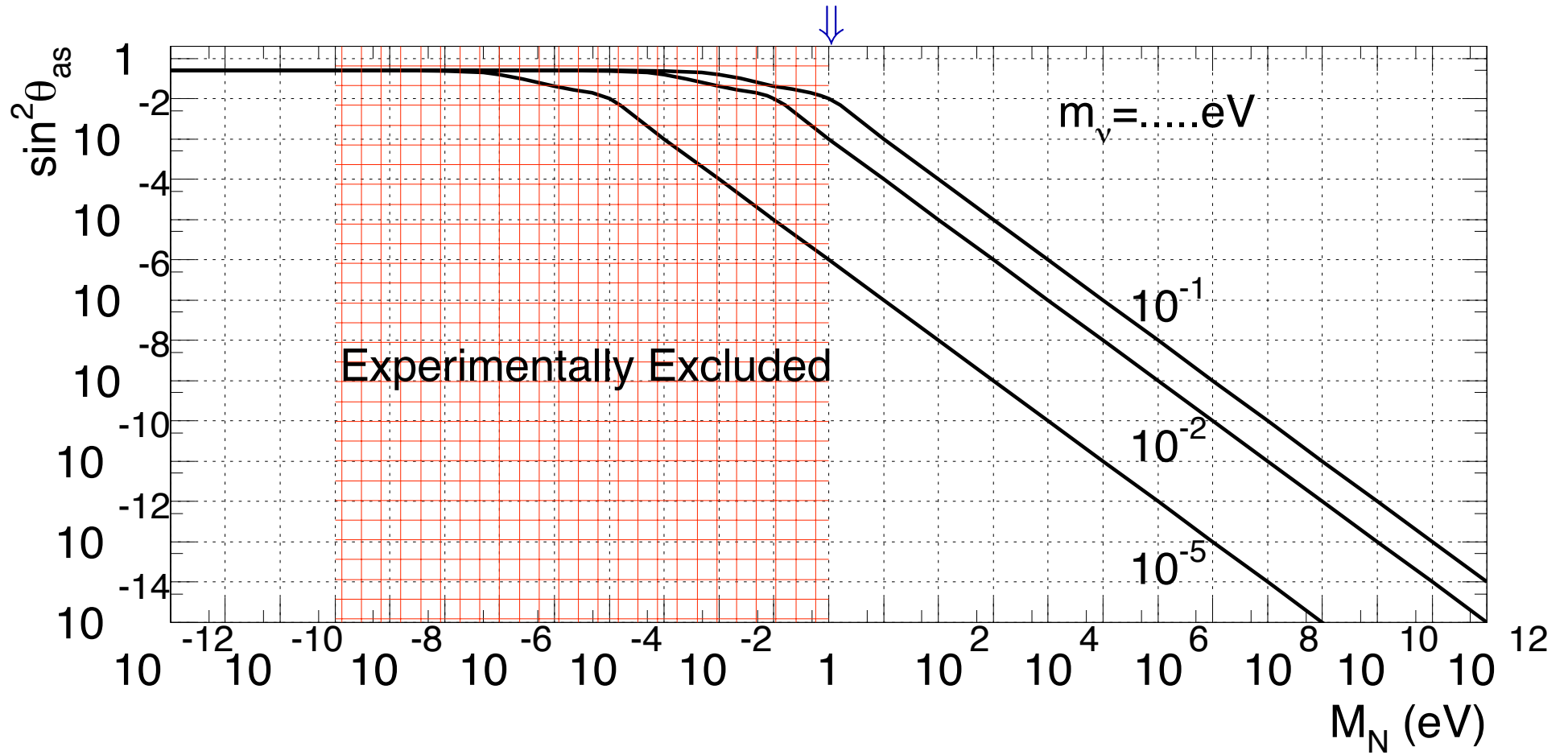
Quasi-Sterile Neutrinos

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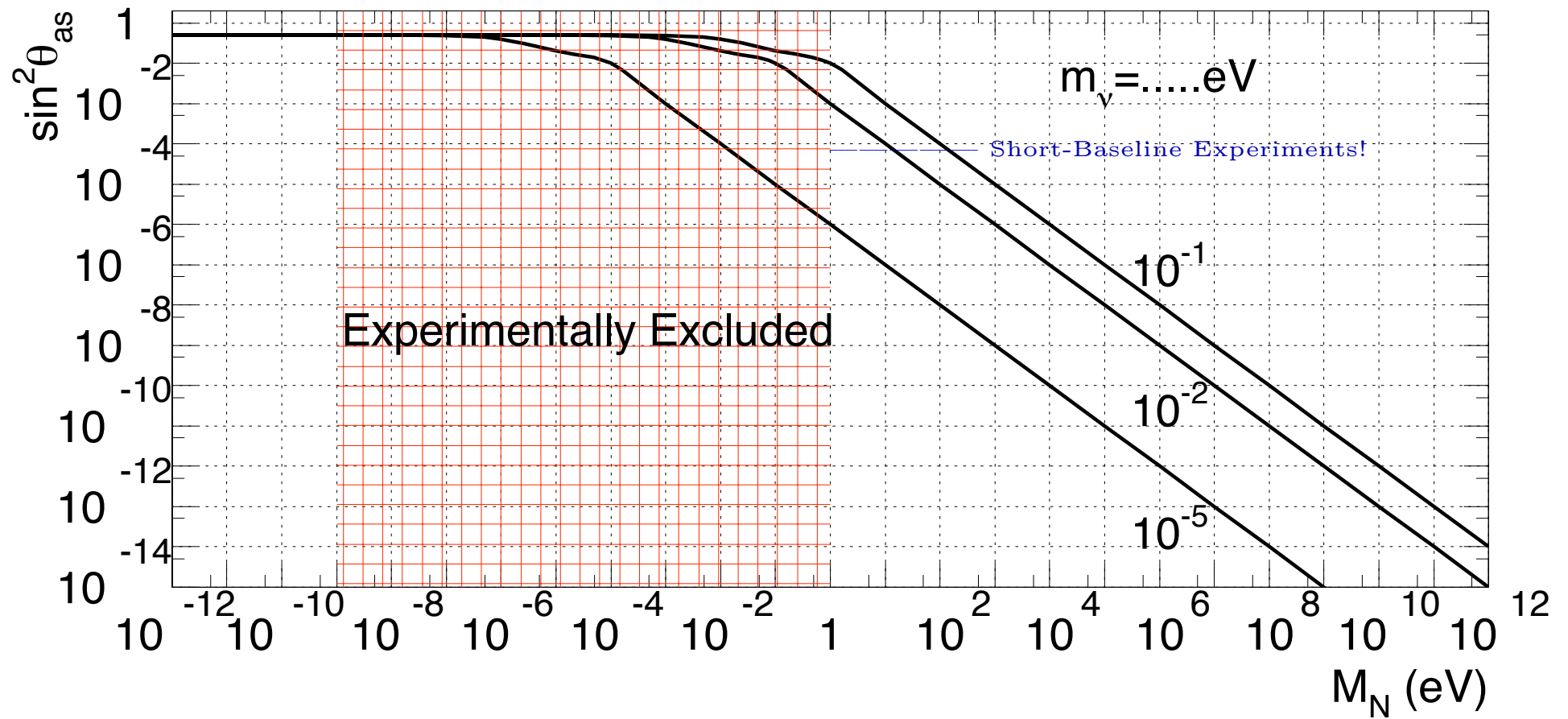
Constraining the Seesaw Lagrangian

[rough upper bound, see Donini et al, arXiv:1106.0064]



[AdG, Huang, Jenkins, arXiv:0906.1611]

Can we improve our sensitivity?



[AdG, Huang, Jenkins, arXiv:0906.1611]

Model independent constraints

Constraints depend, unfortunately, on m_i and M_i and R . E.g.,

$$U_{e4} = U_{e1}A\sqrt{\frac{m_1}{m_4}} + U_{e2}B\sqrt{\frac{m_2}{m_4}} + U_{e3}C\sqrt{\frac{m_3}{m_4}},$$

$$U_{\mu4} = U_{\mu1}A\sqrt{\frac{m_1}{m_4}} + U_{\mu2}B\sqrt{\frac{m_2}{m_4}} + U_{\mu3}C\sqrt{\frac{m_3}{m_4}},$$

$$U_{\tau4} = U_{\tau1}A\sqrt{\frac{m_1}{m_4}} + U_{\tau2}B\sqrt{\frac{m_2}{m_4}} + U_{\tau3}C\sqrt{\frac{m_3}{m_4}},$$

where

$$A^2 + B^2 + C^2 = 1.$$

One can pick A, B, C such that two of these vanish. But the other one is maximized, along with $U_{\alpha5}$ and $U_{\alpha6}$.

Can we (a) constrain the seesaw scale with combined bounds on $U_{\alpha4}$ or (b) testing the low energy seesaw if nonzero $U_{\alpha4}$ are discovered?

Concrete Example: 2 right-handed neutrinos

$$X_{\text{normal}} = \begin{pmatrix} 0.23e^{i\phi} & 0.1e^{i\delta} \\ (0.25 - 0.02e^{-i\delta})e^{i\phi} & 0.70 \\ -(0.25 + 0.02e^{-i\delta})e^{i\phi} & 0.70 \end{pmatrix} \begin{pmatrix} \cos \zeta & \sin \zeta \\ -\sin \zeta & \cos \zeta \end{pmatrix}$$

$$X_{\text{inverted}} = \begin{pmatrix} 0.83e^{i\psi} & 0.55 \\ -(0.39 + 0.06e^{-i\delta})e^{i\psi} & 0.59 - 0.04e^{-i\delta} \\ (0.39 - 0.06e^{-i\delta})e^{i\psi} & -0.59 - 0.04e^{-i\delta} \end{pmatrix} \begin{pmatrix} \cos \zeta & \sin \zeta \\ -\sin \zeta & \cos \zeta \end{pmatrix}$$

$$\zeta \in \mathcal{C}$$

where

$$X_{\text{normal (inverted)}} = \Theta \sqrt{\frac{m_{\text{heavy}}}{m_3 (m_2)}}$$

Some Relevant Examples: [AdG, W-C Huang, arXiv:1110.6122]

$\zeta = 3/4\pi + i$, $\delta = 6/5\pi$, $\phi = \pi/2$ and a normal mass hierarchy,

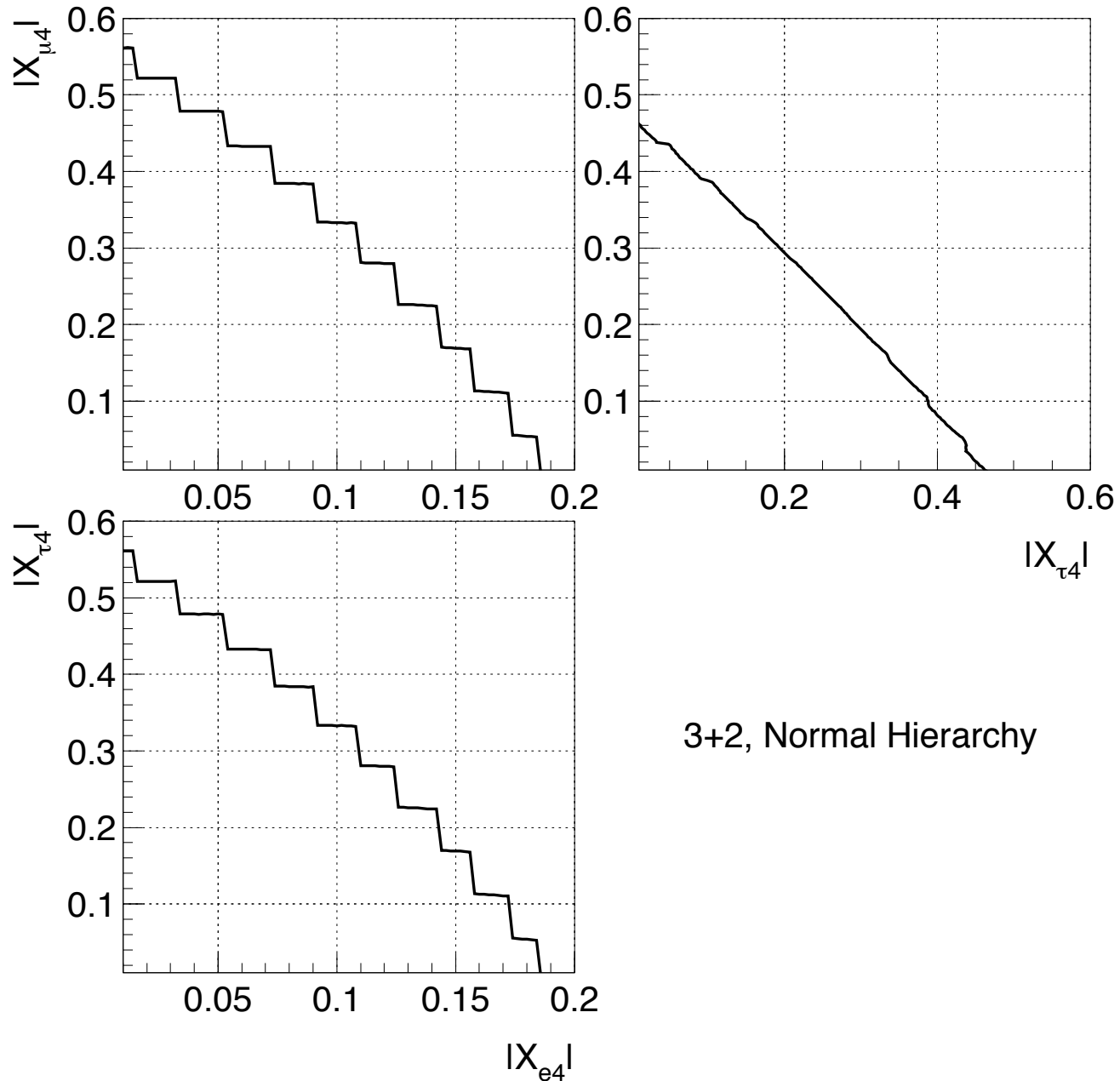
$$X_{\text{normal}} = \begin{pmatrix} 0.41e^{-0.66i} & 0.45e^{1.03i} \\ 0.62e^{2.67i} & 0.61e^{-2.62i} \\ 1.27e^{2.44i} & 1.26e^{-2.41i} \end{pmatrix}.$$

$\zeta = 2/3\pi + 0.3i$, $\delta = 0$, $\psi = \pi/2$, and an inverted mass hierarchy,

$$X_{\text{inverted}} = \begin{pmatrix} 0.44e^{-2.24i} & 0.62e^{1.83i} \\ 0.69e^{2.66i} & 0.66e^{-2.14i} \\ 0.71e^{-0.39i} & 0.60e^{0.89i} \end{pmatrix}.$$

both accommodate 3+2 fit for $m_4^2 = 0.5 \text{ eV}^2$ and $m_5^2 = 0.9 \text{ eV}^2$. Furthermore, $|U_{\tau 4}|$ and $|U_{\tau 5}|$ are completely fixed. No more free parameters. They are also both larger than (or at least as large as $|U_{\mu 4}|$ and $|U_{\mu 5}|$).

$\nu_\mu \rightarrow \nu_\tau$ MUST be observed if this is the origin of the two mostly sterile neutrinos.



Making Predictions, for an inverted mass hierarchy, $m_4 = 1 \text{ eV} (\ll m_5)$

- ν_e disappearance with an associated effective mixing angle $\sin^2 2\vartheta_{ee} > 0.02$. An interesting new proposal to closely expose the Daya Bay detectors to a strong β -emitting source would be sensitive to $\sin^2 2\vartheta_{ee} > 0.04$;
- ν_μ disappearance with an associated effective mixing angle $\sin^2 2\vartheta_{\mu\mu} > 0.07$, very close to the most recent MINOS lower bound;
- $\nu_\mu \leftrightarrow \nu_e$ transitions with an associated effective mixing angle $\sin^2 \vartheta_{e\mu} > 0.0004$;
- $\nu_\mu \leftrightarrow \nu_\tau$ transitions with an associated effective mixing angle $\sin^2 \vartheta_{\mu\tau} > 0.001$. A $\nu_\mu \rightarrow \nu_\tau$ appearance search sensitive to probabilities larger than 0.1% for a mass-squared difference of 1 eV^2 would definitively rule out $m_4 = 1 \text{ eV}$ if the neutrino mass hierarchy is inverted.

CONCLUSION

1. Sterile neutrinos are a very benign extension of the standard model. They are allowed by all experimental data as long as they are very heavy or very weakly coupled to the Standard Model.
2. If they exist, sterile neutrinos will only manifest themselves through mixing with the active neutrinos [neutrino portal]. We don't "see" the sterile neutrinos. We can only hope to determine that the three active states are made up of more than three massive states.
3. Not just a good idea, sterile neutrinos may be a "side effect" of the physics responsible for nonzero neutrinos masses. If they are light enough (mass below 10 eV?) they may be discovered in neutrino oscillation experiments. And we may get lucky in non-oscillation experiments for masses below 100 GeV!
4. Have we run into sterile neutrinos already? David Schmitz will tell you all about it on Thursday!