g-2 Experiments: From Brookhaven to Fermilab

Academic Lecture Series
10/2/2013

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Outline for today

- Recap experimental principles from Lee’s talk
- Statistical precision
- Data collection and precision fitting
- Controlling systematics in the ω_a analysis
- Conclusions

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Intersperse some lessons learned in BNL g-2 and contrast BNL with FNAL as we go

Goal for this talk

A little less of this…

Goal for this talk

Muon g-2

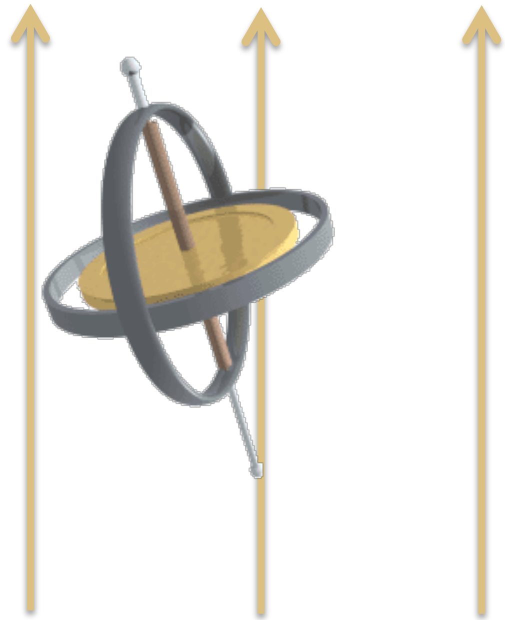
And more of this…

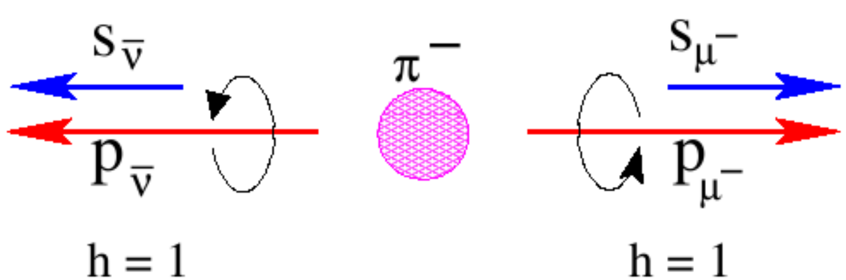
Principles from Lee's talk

- Place polarized muons in known magnetic field, measure precession
  - Muon mass 200x electron -> 40,000x more sensitive to higher mass exchanges
  - Makes up for incredible precision of a_e

$$\lambda_{sens} \propto \left(\frac{m_\mu}{m_e}\right)^2 \approx 40,000$$

- Can naturally get a nearly 100% polarized muon source by capturing highest (or lowest) energy muons





$$\omega_S = -g\frac{QeB}{2m}$$

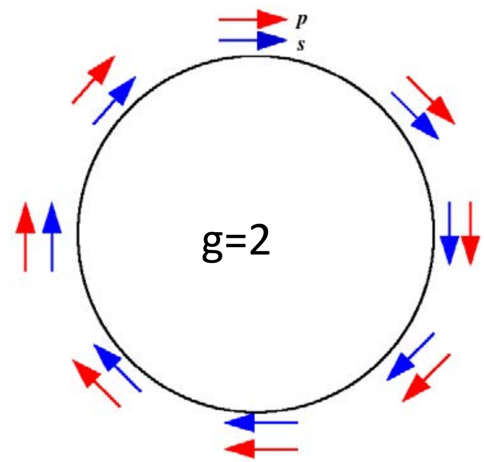
$$a_\mu = \frac{g-2}{2}$$

Principles from Lee's talk

- Inject beam into storage ring instead to measure a_μ directly

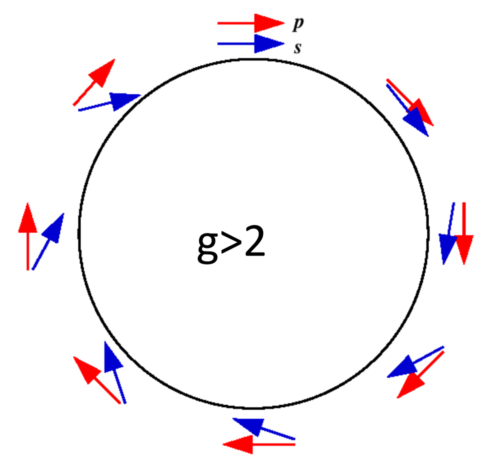
$$\omega_C = -\frac{QeB}{m\gamma}; \qquad \omega_S = -g\frac{QeB}{2m} - (1-\gamma)\frac{QeB}{\gamma m}$$

$$\omega_a = \omega_S - \omega_C = -\left(\frac{g-2}{2}\right)\frac{QeB}{m} = -a\frac{QeB}{m}$$



- Since g = 2.0023… gain factor of 800 for free in a_μ precision relative to at rest expts

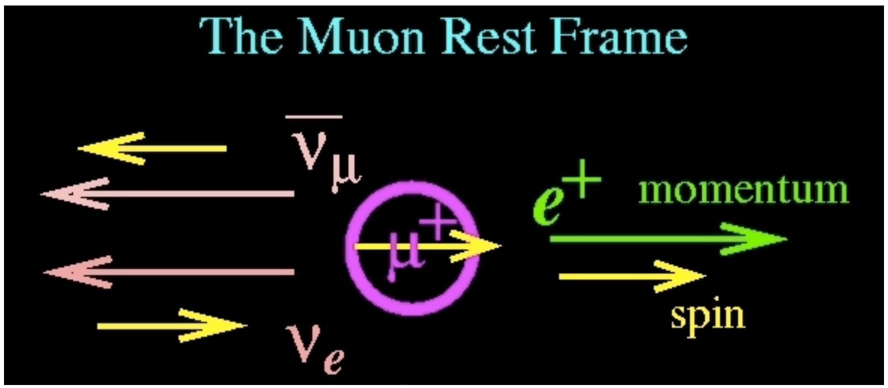
- Use magic momentum to allow vertical focus
  - ϒ=29.3, pμ = 3.094 GeV/c

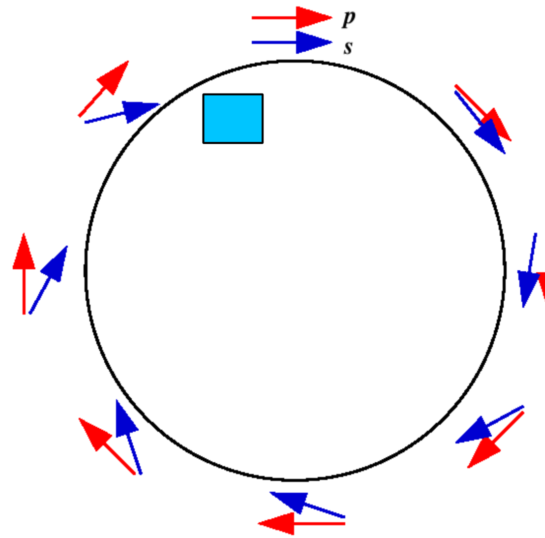


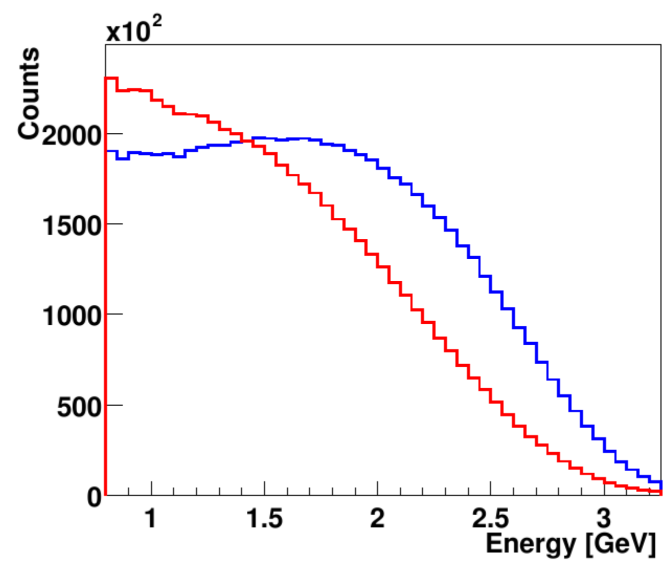
$$\vec{\omega}_a = -\frac{Qe}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]$$

Principles from Lee's talk

- Parity violation in muon decay results in highest energy decay positrons being emitted in direction of underlying muon spin



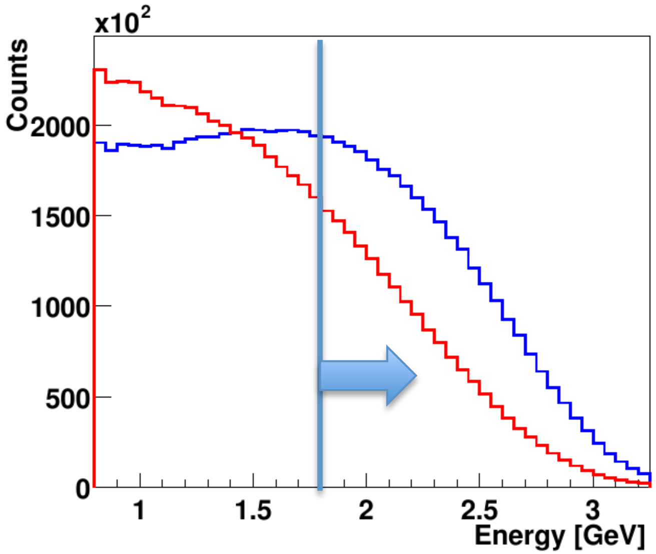


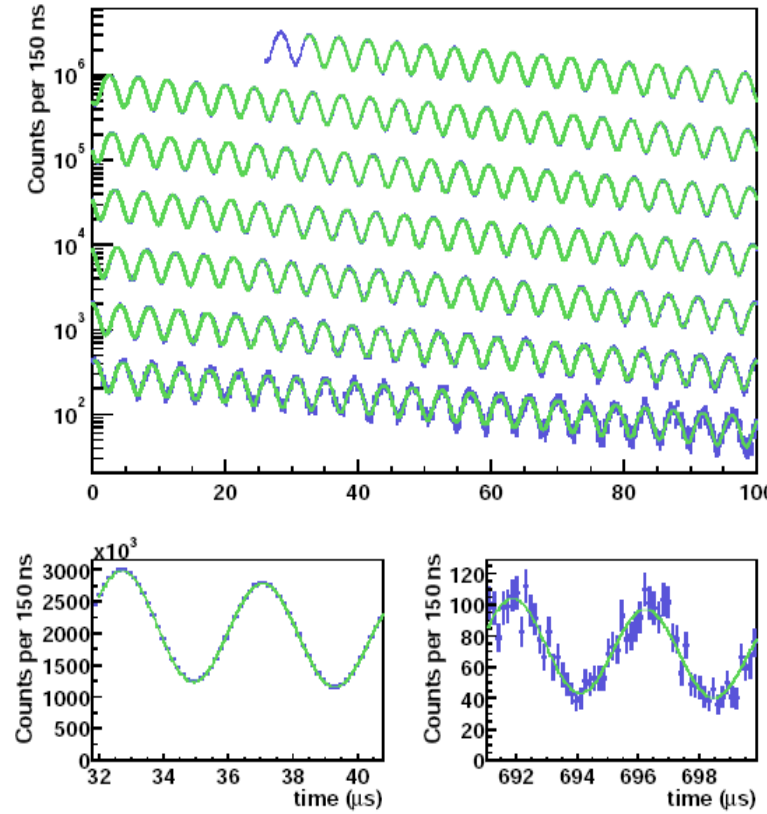


- No need to directly observe the muon spin, just look for a modulation in the energy spectrum of decay positrons

Principles from Lee's talk

Muon g-2





- Apply cut on energy, bin data in time, 'wiggle' plot emerges for ω_a

$$a_\mu = \frac{\frac{\omega_a}{\omega_p}}{\frac{\mu_\mu}{\mu_p} - \frac{\omega_a}{\omega_p}}$$

$$f(t) \simeq N_0 e^{-\lambda t}[1 + A\cos(\omega_a t + \phi)]$$

- Same principles used in CERN III, BNL, and FNAL
- Often referred to as 'textbook' due to all the underlying fundamental principles that conspired to give us this window into the quantum world

Interesting Aside: CERN muon g-2 experiments were initiated in 1958 by Leon Lederman to answer the question of whether the muon was really a 'heavy electron'.

"There he started the famous g-2 experiment and managed to confuse it so badly that it took 26 physicists nineteen years to finish."

Leon's Unauthorized Autobiography

http://history.fnal.gov/autobiography.html

Statistical Precision

Statistical precision

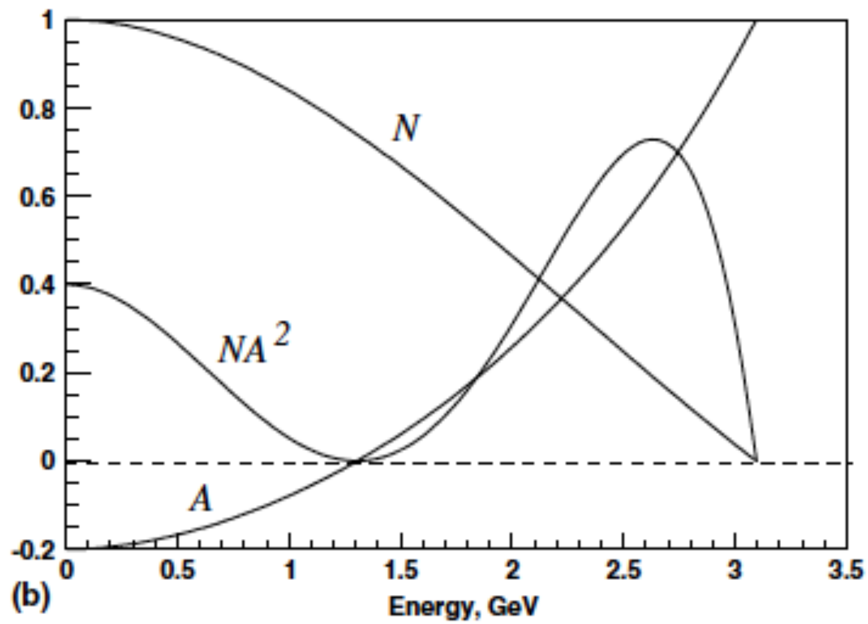
- Since g-2 is all about statistical precision and this is an 'academic lecture'…a quick aside for statistics nuts

$$f(t) \simeq N_0 e^{-\lambda t}[1 + A\cos(\omega_a t + \phi)]$$

$$\frac{\delta\omega_a}{\omega_a} = \frac{\sqrt{2}}{\omega_a A \gamma \tau \sqrt{N}}$$

$$f(y,t|\vec{p}) = \frac{n(y)}{\tau} e^{-t/\tau}[1 - A(y)\cos(\omega t + \phi)] \qquad y = \frac{E}{E_{\max}}$$

- Same equation as above but redefined to be a pdf and being careful to note that the number density and asymmetry are energy-dependent



Cramer-Rao Lower Bound

- Really cool theorem in parameter estimation called the Cramer-Rao lower bound (CRLB)
  - Basically says that for any unbiased estimator there exists a lower bound on the variance of an estimated parameter
  - Furthermore, that lower bound can be calculated from by inverting the Fisher discriminant

$$\sigma^2_{p_i} \geq \left[I^{-1}\right]_{ii}$$

$$\begin{aligned} I_{ij} &= -N_t \left\langle \frac{\partial^2 \ln f}{\partial p_i \partial p_j} \right\rangle, \\ &= N_t \left\langle \frac{1}{f^2} \frac{\partial f}{\partial p_i} \frac{\partial f}{\partial p_j} \right\rangle - N_t \left\langle \frac{1}{f} \frac{\partial^2 f}{\partial p_i \partial p_j} \right\rangle, \\ &= N_t \left\langle \frac{1}{f^2} \frac{\partial f}{\partial p_i} \frac{\partial f}{\partial p_j} \right\rangle. \end{aligned}$$

Compare CRLB with MLE for g-2 frequency

Muon g-2

- Simplify calculation since only correlated parameters matter and N, τ, and A are not correlated with ω

$$\begin{pmatrix} \sigma_\omega^2 & \sigma_{\omega\phi}^2 \\ \sigma_{\omega\phi}^2 & \sigma_\phi^2 \end{pmatrix} = \frac{1}{N_t} \begin{pmatrix} \left\langle \frac{1}{f^2}\frac{\partial f}{\partial\omega}\frac{\partial f}{\partial\omega} \right\rangle & \left\langle \frac{1}{f^2}\frac{\partial f}{\partial\omega}\frac{\partial f}{\partial\phi} \right\rangle \\ \left\langle \frac{1}{f^2}\frac{\partial f}{\partial\omega}\frac{\partial f}{\partial\phi} \right\rangle & \left\langle \frac{1}{f^2}\frac{\partial f}{\partial\phi}\frac{\partial f}{\partial\phi} \right\rangle \end{pmatrix}^{-1}$$

$$\begin{aligned}
\left\langle \frac{1}{f^2}\frac{\partial f}{\partial\omega}\frac{\partial f}{\partial\omega} \right\rangle &= \int_0^\infty \int_0^1 \left(\frac{1}{f^2}\frac{\partial f}{\partial\omega}\frac{\partial f}{\partial\omega} \right) f\, dy dt, \\
&= \int_0^\infty \int_0^1 \frac{1}{f}\frac{\partial f}{\partial\omega}\frac{\partial f}{\partial\omega} dy dt, \\
&= \int_0^\infty \int_0^1 \frac{n(y)}{\tau} e^{-t/\tau} \frac{A^2(y) t^2 \sin^2(\omega t + \phi)}{1 - A(y)\cos(\omega t + \phi)} dy dt, \\
&\approx \int_0^1 \tau^2 n(y) A^2(y), \\
&= \langle \tau^2 A^2 \rangle, \\
&= \tau^2 \langle A^2 \rangle.
\end{aligned}$$

$$\left\langle \frac{1}{f^2}\frac{\partial f}{\partial\omega}\frac{\partial f}{\partial\phi} \right\rangle \approx \frac{1}{2}\tau\langle A^2 \rangle,$$

$$\left\langle \frac{1}{f^2}\frac{\partial f}{\partial\phi}\frac{\partial f}{\partial\phi} \right\rangle \approx \frac{1}{2}\langle A^2 \rangle.$$

Extract CRLB for ω:

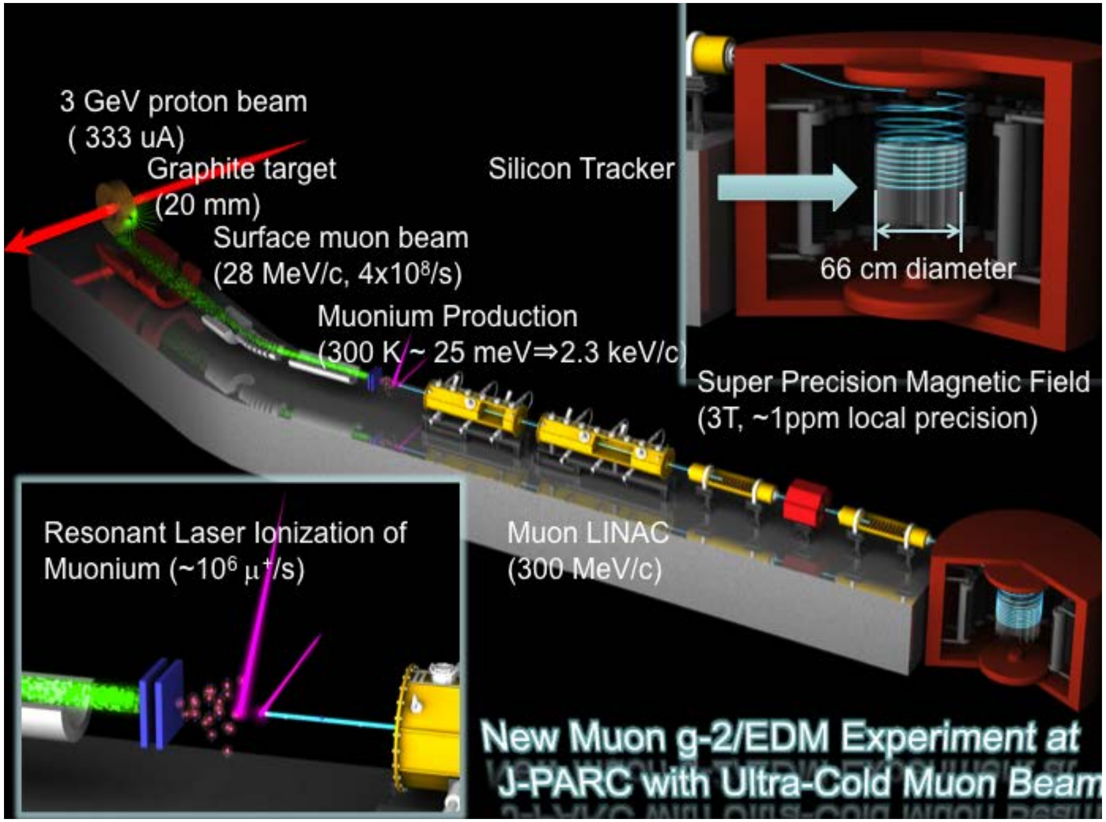
$$\sigma_\omega^2(\text{CRLB}) \geq \frac{2}{N_t \tau^2 \langle A^2 \rangle},$$

Comparison to prior expression:

$$\frac{\delta\omega_a}{\omega_a} = \frac{\sqrt{2}}{\omega_a A \gamma \tau \sqrt{N}}$$

Aside on J-PARC g-2

- There is a very clever proposal by our Japanese colleagues to do g-2 in way very different that the magic momentum technique
  - Run at a much lower momentum, use MRI-style magnet with better field
  - Eliminate vertical focusing by use a ultra-cold muon beam
  - Would be great to have a 2nd experiment with completely different systematics



$$\frac{\delta\omega_a}{\omega_a} = \frac{\sqrt{2}}{\omega_a A \gamma \tau \sqrt{N}}$$

- Relativistic gamma is 3 instead of 29.3
- A is reduced since reaccelerated muon start with 0% polarization, can throw away half to get to 50%
- The FNAL experiment plans to measure ~2e11 muons

Maximum Likelihood Fit Achieves the CRLB

Muon g-2

- By definition, the efficiency of a parameter estimation is defined relative to the CRLB

$$\varepsilon_{p_i}(\text{estimator}) = \frac{\sigma^2_{p_i}(\text{CRLB})}{\sigma^2_{p_i}(\text{estimator})}$$

- Can also derive that a maximum likelihood estimate (MLE) will achieve the CRLB

$$\mathcal{L}(\vec{p}) = \prod_{i=1}^{N_t} f(x_i|\vec{p}) \quad \Rightarrow \quad \varepsilon(\text{MLE}) = 1$$

Might conclude that MLE is the best way to fit the g-2 data

MLE and g-2

$$f(y,t|\vec{p}) = \frac{n(y)}{\tau} e^{-t/\tau}[1 - A(y)\cos(\omega t + \phi)]$$

$$\mathcal{L}(\vec{p}) = \prod_{i=1}^{N_t} f(x_i|\vec{p})$$

- Some practical difficulties with MLE
  - The functions n(y) and A(y) are not really known. Start out calculable from V-A in muon rest frame and boosting back to lab, but then perturbed by real world acceptance and resolution effects
  - With 2e11 samples expected at FNAL (1e10 at BNL) computationally intense to explore parameter space
  - No goodness-of-fit criteria comes directly with MLE

- Instead we bin the data and use least-squares estimation, a.k.a. χ^2 fits

$$\chi^2 = \sum_{k=1}^{N_{\rm bin}} \frac{(F_k - N_k)^2}{F_k}$$

Compare CRLB with LSE for g-2 frequency

Muon g-2

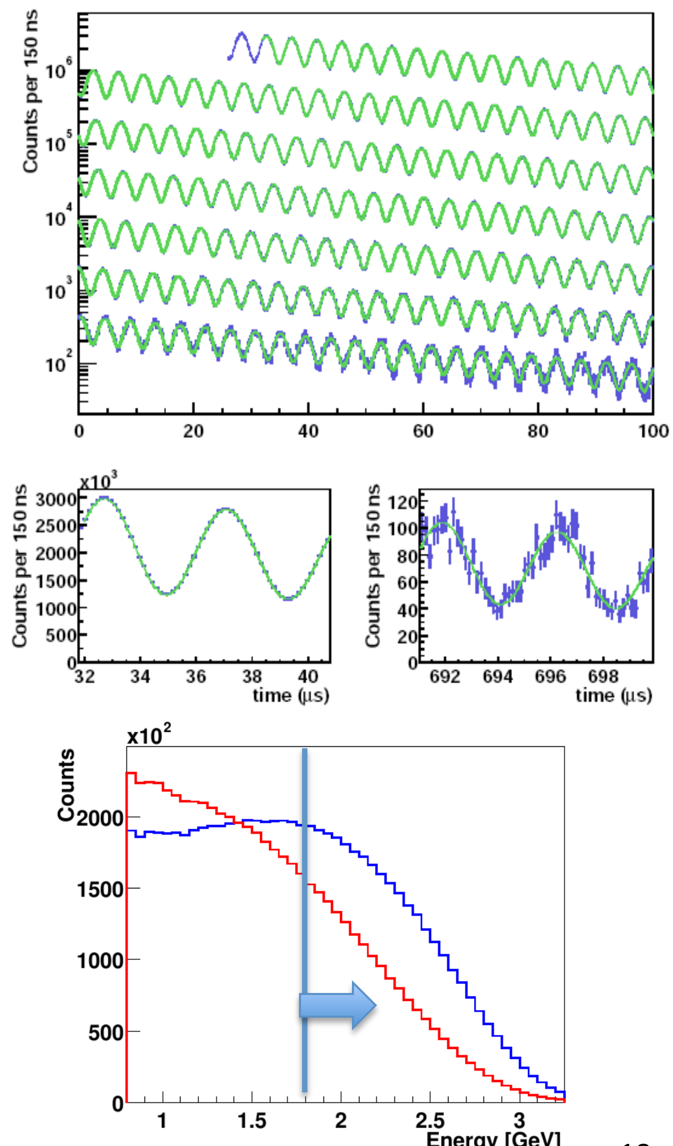
- Start with the same functional form as before, except not a pdf

$$F(y,t|\vec{p}) = \frac{N(y)}{\tau} e^{-t/\tau}\left[1 - A(y)\cos(\omega t + \phi)\right]$$

$$\begin{aligned}
F(t|\vec{p}) &= \int_{y_{\min}}^{1} N(y)\frac{e^{-t/\tau}}{\tau}\left[1 - A(y)\cos(\omega t+\phi)\right]dy, \\
&= \frac{\int_{y_{\min}}^{1} N(y)\frac{e^{-t/\tau}}{\tau}\left[1 - A(y)\cos(\omega t+\phi)\right]dy}{\int_{y_{\min}}^{1} N(y)dy}\int_{y_{\min}}^{1} N(y)dy, \\
&= N_t\left\langle \frac{e^{-t/\tau}}{\tau}\left[1 - A(y)\cos(\omega t+\phi)\right]\right\rangle, \\
&= N_t\frac{e^{-t/\tau}}{\tau}\left[1 - \langle A\rangle\cos(\omega t+\phi)\right].
\end{aligned}$$

$$\sigma^2_{p_i}(\mathrm{LSE}) = \left[S^{-1}\right]_{ii} \qquad S_{ij} = \frac{1}{2}\frac{\partial^2\chi^2}{\partial p_i \partial p_j}$$

$$\varepsilon(\mathrm{LSE}) = \frac{\langle A\rangle^2}{\langle A^2\rangle}$$



Statistics wrap-up

Muon g-2

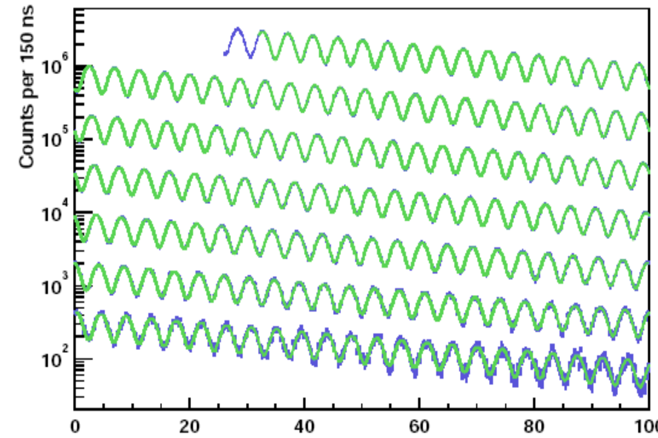
- In the end, just a mathematically rigorous way of saying something somewhat intuitively obvious
  - If you bin the data and integrate over A(y) then you lose some statistical precision
  - Error is about 10% larger

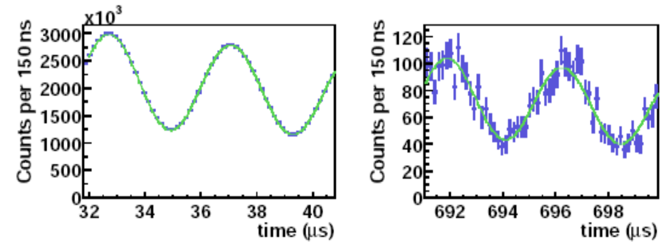
- Can now imagine at least 3 ways of fitting the data
  - One fit integrated over A(y) > threshold, turns out 1.8 GeV maximizes statistical power
  - Many fits in individual bins of energy, y
  - One fit with the data weighted by your best guess at A(y)
  - All have different sensitivities to systematic errors

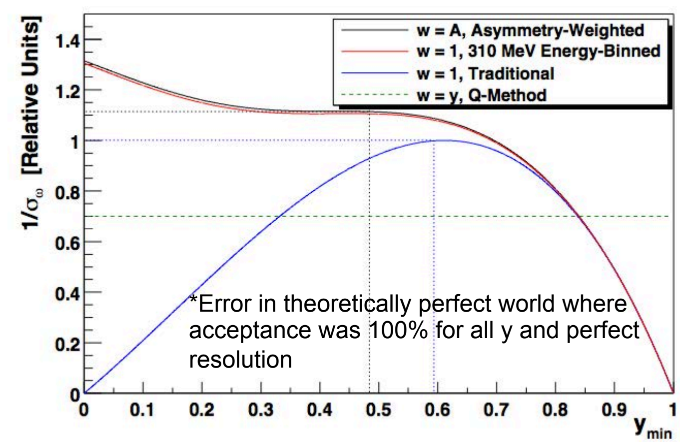
$$F_w(t|\vec{p}) = \int_{y_{\min}}^{1} N(y)w(y)\frac{e^{-t/\tau}}{\tau}\left[1 - A(y)\cos(\omega t + \phi)\right]dy,$$

$$= N_t \frac{e^{-t/\tau}}{\tau}\left[w - \langle wA\rangle \cos(\omega t + \phi)\right].$$

$$\sigma_\omega^2(wLSE) = \frac{2\langle w^2\rangle}{N_t\tau^2\langle Aw\rangle^2}$$







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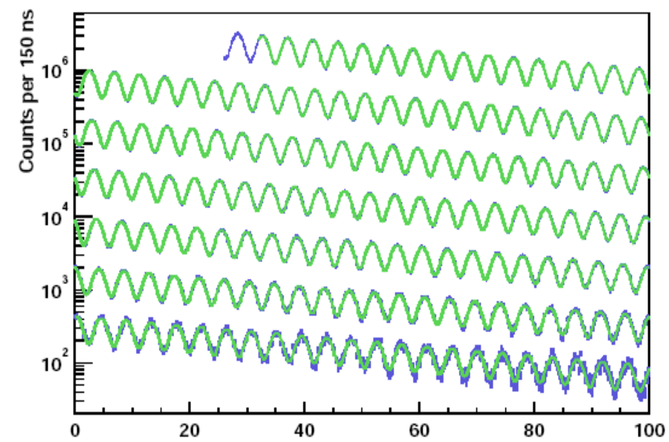
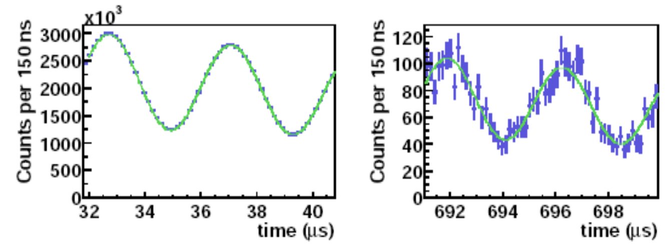
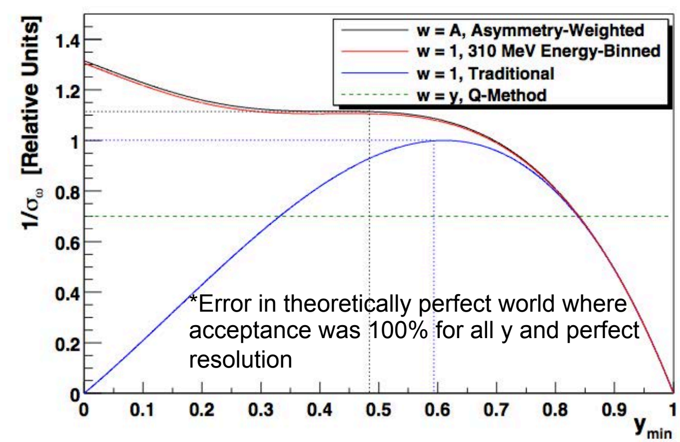




Why torture you with all of the math?

Because for many, this is part of the allure and challenge of precision experiments.

Not being able to take anything for granted leads to numerous intellectual challenges



Another example

Muon g-2

- How do we know this isn't a biased estimator?

  Turns out it is!

$$\chi^2 = \sum_{k=1}^{N_{\text{bin}}} \frac{(F_k - N_k)^2}{F_k}$$

- If it is biased, how much?

  Not enough to worry at BNL precision, still needs to be revisited for FNAL

- Should we use F_k or N_k in the denominator?

  Used N_k at BNL because it is simpler and was mathematically proven to be OK, but a linear combination ends up being the minimum bias

Pages 2 and 3 from Sergei Redin's 16 pg note on the matter

Equation (7), which explicitly relates statistical fluctuations of fit parameters to statistical fluctuations of number of counts in individual histogram channels, is extremely useful. For instance, it can be used to evaluate correlations of parameters:

$$\begin{aligned}
\langle \Delta x_i \, \Delta x_j \rangle &= \sum_{ab} \left(\mathcal{A}^{-1}\right)_{ia} \left(\mathcal{A}^{-1}\right)_{jb} \sum_{nm} \left(\frac{f'_a}{f_\circ}\right)_n \left(\frac{f'_b}{f_\circ}\right)_m \langle (\mathcal{N}_n - f_\circ)(\mathcal{N}_m - f_\circ) \rangle = \\
&= \sum_{ab} \left(\mathcal{A}^{-1}\right)_{ia} \left(\mathcal{A}^{-1}\right)_{jb} \sum_n \frac{f'_a f'_b}{f_\circ} = \sum_{ab} \left(\mathcal{A}^{-1}\right)_{ia} \left(\mathcal{A}^{-1}\right)_{jb} \mathcal{A}_{ab} = \left(\mathcal{A}^{-1}\right)_{ij} = \qquad (8) \\
&= \frac{A_{ij}}{det(\mathcal{A})} = (-1)^{i+j} \frac{M_{ij}}{det(\mathcal{A})} \qquad (9)
\end{aligned}$$

As a specific, but the most practical, case of eqs.(8) and (9), one can immediately obtain equation for statistical errors of fit parameters:

$$\begin{aligned}
\sigma_i^2 &\equiv \left\langle (\Delta x_i)^2 \right\rangle = \left(\mathcal{A}^{-1}\right)_{ii} = \qquad (10) \\
&= \frac{A_{ii}}{det(\mathcal{A})} = \frac{M_{ii}}{det(\mathcal{A})} \qquad (11)
\end{aligned}$$

where M_{ij} and A_{ij} are minors and cofactors of matrix elements of symmetrical matrix $\mathcal{A}$ and $det(\mathcal{A})$ is its determinant. Equation (7) also can be used for very simple evaluation of "Kawall band" formula, see Appendix I.

However, ensemble average of $\Delta \boldsymbol{x}$ in eq.(7) vanishes:

$$\langle \Delta x_i \rangle = \left\langle \sum_j \left(\mathcal{A}^{-1}\right)_{ij} \sum_n \frac{f'_j}{f_\circ}(\mathcal{N}_n - f_\circ) \right\rangle = \sum_j \left(\mathcal{A}^{-1}\right)_{ij} \sum_n \frac{f'_j}{f_\circ} \langle \mathcal{N}_n - f_\circ \rangle = 0 \qquad (12)$$

since $\langle \mathcal{N}_n - f_\circ \rangle = 0$. Thus the approximation made in eq.(6) is not sufficient to calculate bias of fit parameters. In the next-to-leading approximation for the log.likelihood minimization we have:

$$\begin{aligned}
0 &= \frac{\partial \mathcal{L}}{\partial x_j} = \sum_n f'_j \left(-\frac{\mathcal{N}_n}{f} + 1\right) = \sum_n f'_j \frac{f - \mathcal{N}_n}{f} = \\
&= \sum_n \frac{f'_j + \sum_i f''_{ji} \Delta x_i + \ldots}{f_\circ + \sum_i f'_i \Delta x_i + \ldots} \times \left(f_\circ - \mathcal{N}_n + \sum_i f'_i \, \Delta x_i + \frac{1}{2} \sum_{ik} f''_{ik} \, \Delta x_i \, \Delta x_k + \ldots\right) \approx \qquad (13) \\
&\approx \sum_n \frac{f'_j}{f_\circ} \left(f_\circ - \mathcal{N}_n + \sum_i f'_i \, \Delta x_i + \frac{1}{2} \sum_{ik} f''_{ik} \, \Delta x_i \, \Delta x_k\right) \left(1 - \frac{\sum_i f'_i \Delta x_i}{f_\circ} + \frac{\sum_i f''_{ik} \Delta x_i \Delta x_i}{f_\circ}\right)
\end{aligned}$$

We search for solution for eq.(13) in form of successive approximations: $\Delta x_i = \Delta x_i^\circ + \Delta x_i^1 + \ldots$, where Δx_i° is the leading approximation, given in eq.(7):

$$\Delta x_i^\circ = \sum_j \left(\mathcal{A}^{-1}\right)_{ij} \sum_n \frac{f'_j}{f_\circ}(\mathcal{N}_n - f_\circ) \qquad (14)$$

and Δx_i^1 is the next-to-leading term. For Δx_i^1 we have equation:

$$\begin{aligned}
&\sum_n \frac{f'_i}{f_\circ} \left[\sum_p f'_p \, \Delta x_p^1 + \frac{1}{2} \sum_j \sum_k f''_{jk} \, \Delta x_j^\circ \, \Delta x_k^\circ + \right. \\
&\left. + \left(-\frac{\sum_j f'_j \, \Delta x_j^\circ}{f_\circ} + \frac{\sum_j f''_{ij} \, \Delta x_j^\circ}{f'_i}\right) \left((f_\circ - \mathcal{N}_n) + \sum_k f'_k \, \Delta x_k^\circ\right) \right] = 0 \qquad (15)
\end{aligned}$$

Equation (7), which explicitly relates statistical fluctuations of fit parameters to statistical fluctuations of number of counts in individual histogram channels, is extremely useful. For instance, it can be used to evaluate correlations of parameters:

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and Δx_i^1 is the next-to-leading term. For Δx_i^1 we have equation:

$$\begin{aligned}
&\sum_n \frac{f'_i}{f_\circ} \left[\sum_p f'_p \, \Delta x_p^1 + \frac{1}{2} \sum_j \sum_k f''_{jk} \, \Delta x_j^\circ \, \Delta x_k^\circ + \right. \\
&\left. + \left(-\frac{\sum_j f'_j \, \Delta x_j^\circ}{f_\circ} + \frac{\sum_j f''_{ij} \, \Delta x_j^\circ}{f'_i}\right) \left((f_\circ - \mathcal{N}_n) + \sum_k f'_k \, \Delta x_k^\circ\right) \right] = 0 \qquad (15)
\end{aligned}$$

Contrast BNL/FNAL: Statistics

- Bring E821 storage ring and associated equipment to Fermilab
- Modify anti-proton complex to provide intense, high-purity beam of 3.094 GeV/c muons
- Upgrade select subsystems to meet requirements for rates and systematics

- Scientific goal is 4-fold reduction in error relative to BNL
  - Increase stats x 21 to reduce stat error from 0.46 ppm to 0.1 ppm
  - Reduce systematics ω_a on from 0.2 ppm to 0.07 ppm
  - Reduce systematics ω_p on from 0.17 ppm to 0.07 ppm

Contrast BNL/FNAL: Rate Requirements

Item	Estimate
Protons per fill on target	10^{12} p
Positive-charged secondaries with $dp/p = \pm 2\%$	4.8×10^7
π^+ fraction of secondaries	0.48
π^+ flux entering FODO decay line	$> 2 \times 10^7$
Pion decay to muons in 220 m of M2/M3 line	0.72
Muon capture fraction with $dp/p < \pm 0.5\%$	0.0036
Muon survive decay 1800 m to storage ring	0.90
Muons flux at inflector entrance (per fill)	4.7×10^4
Transmission and storage using $(dp/p)_\mu = \pm 0.5\%$	0.10 ± 0.04
Stored muons per fill	$(4.7 \pm 1.9) \times 10^3$
Positrons accepted per fill (factors 0.15 x 0.63)	444 ± 180
Number of fills for 1.8 $\times 10^{11}$ events	$(4.1 \pm 1.7) \times 10^8$ fills
Time to collect statistics	(13 ± 5) months
Beam-on commissioning	2 months
Dedicated systematic studies periods	2 months
Net running time required	17 ± 5 months

Ratio of beam powers BNL/FNAL:

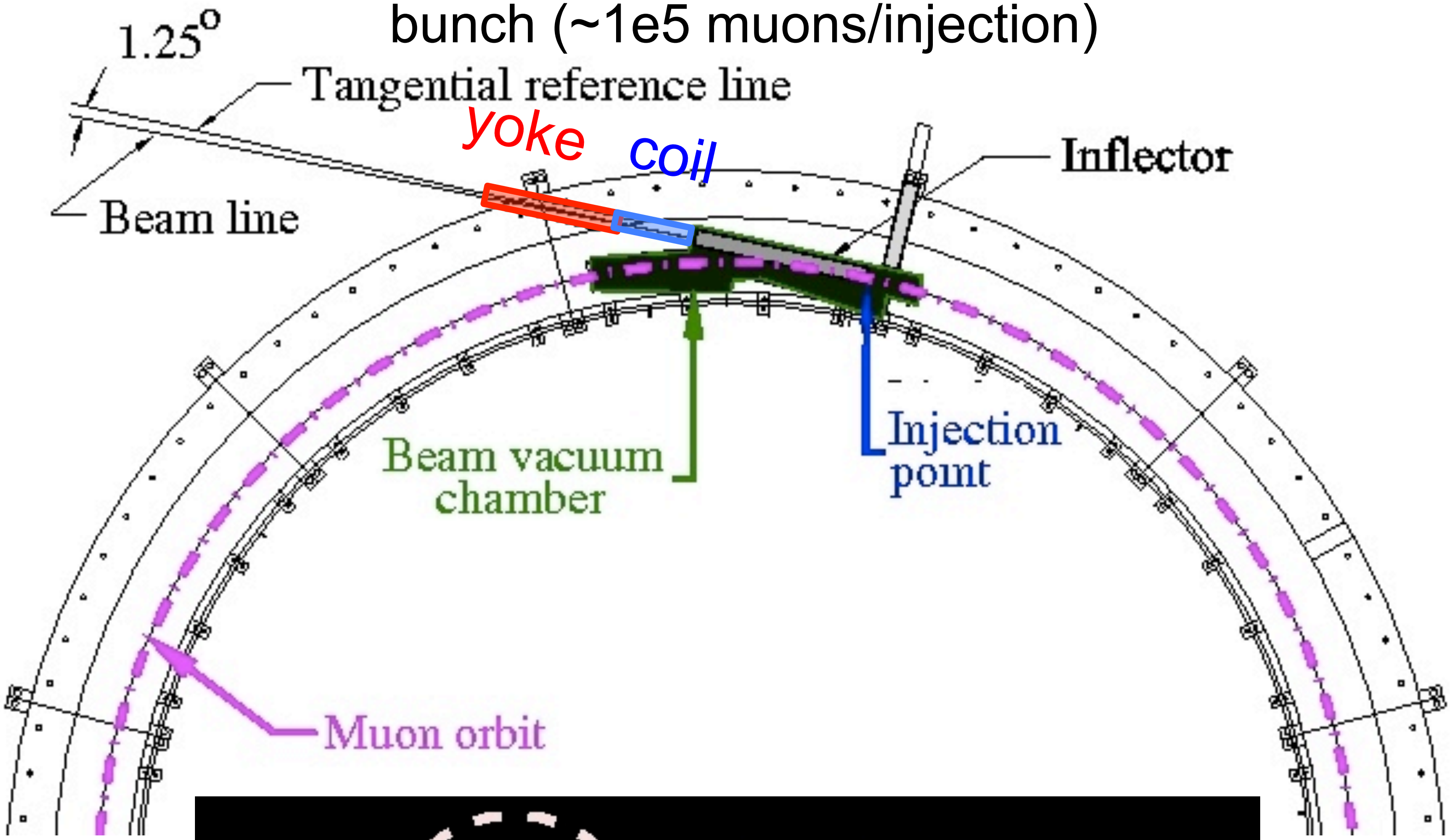
$$\frac{4e12 \text{ protons/fill} * (12 \text{ fills} / 2.7\text{s}) * 24 \text{ GeV}}{1e12 \text{ protons/fill} * (16 \text{ fills} / 1.3\text{s}) * 8 \text{ GeV}} = 4.3$$

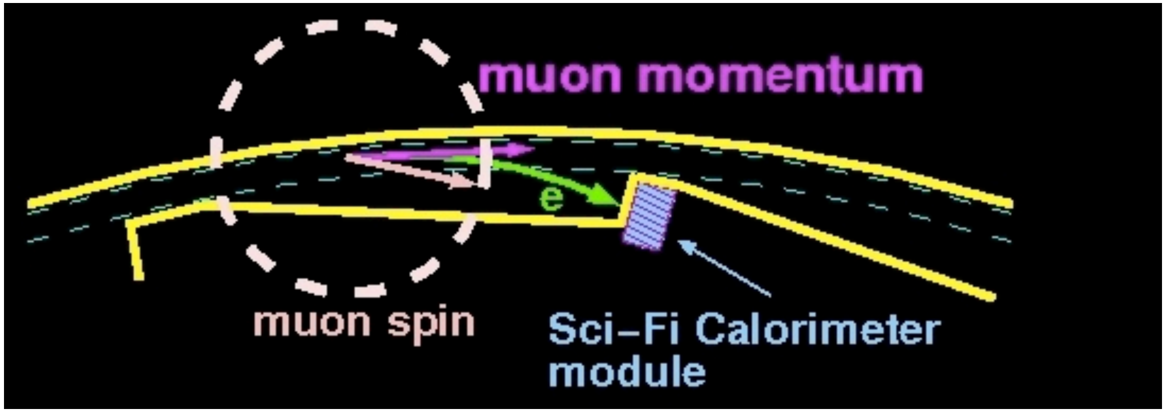
Achieving required statistics is a primary concern
- Need a factor 21 more statistics than BNL
- Beam power reduced by 4

Need a factor of 85 improvement in integrated beam coming from many other factors
- Collection of pions from lens
- Capture of decay muons in high density FODO channel
- p_π closer to magic momentum
- Longer decay channel
- Increased injection efficiency
- Earlier start time of fits
- Longer runtime

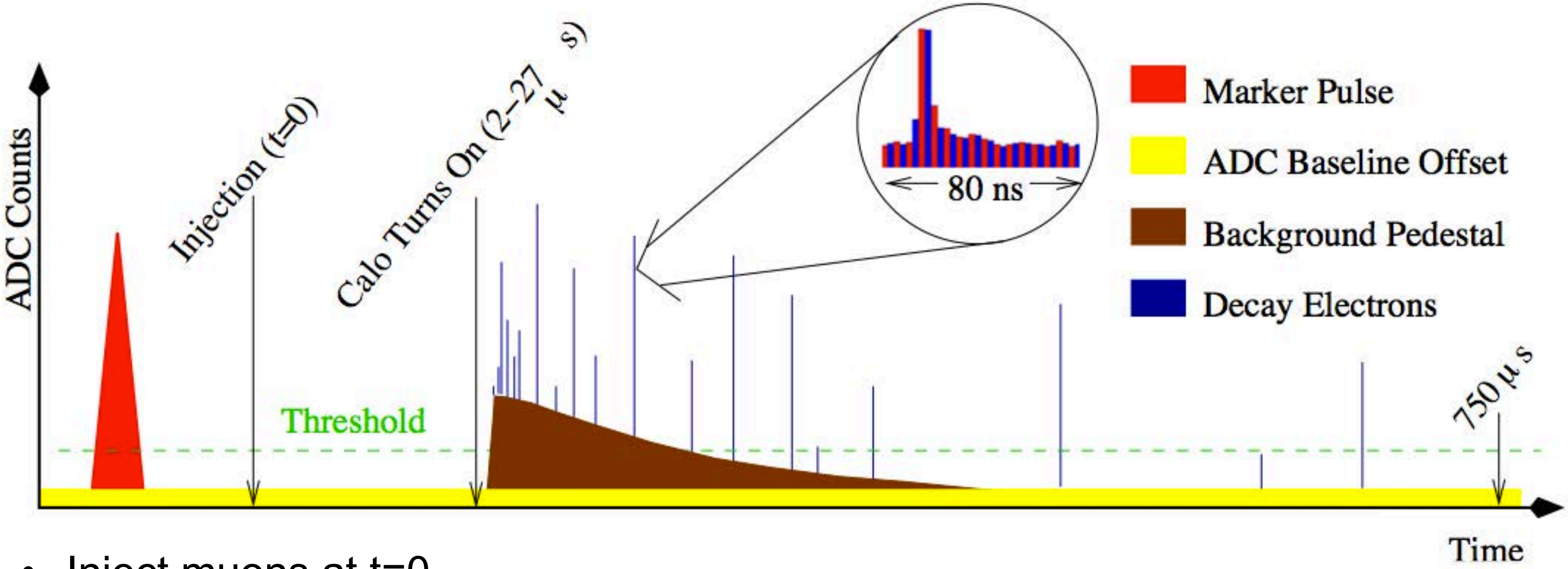
Collecting the data

Recall from Lee's talk we inject a single muon bunch (~1e5 muons/injection)



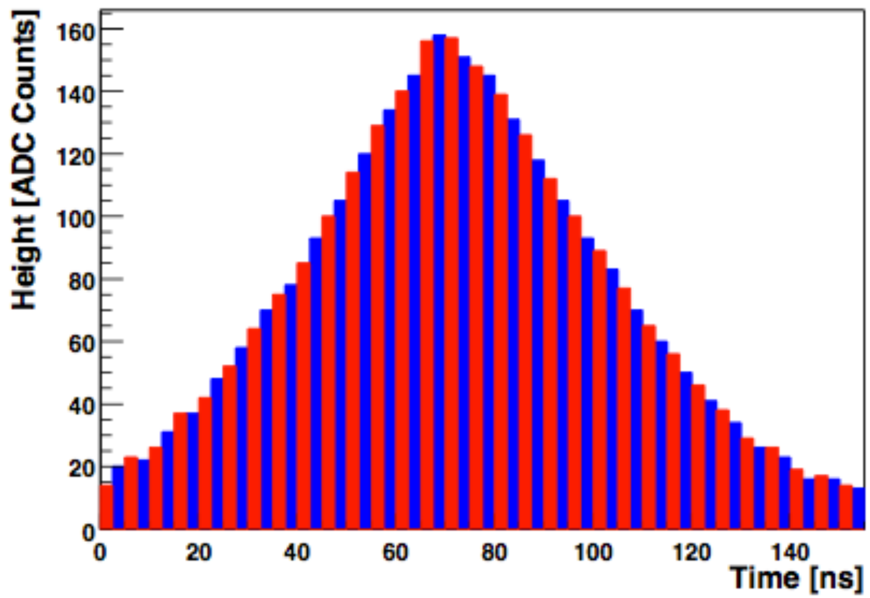


Data record in a single calorimeter at BNL

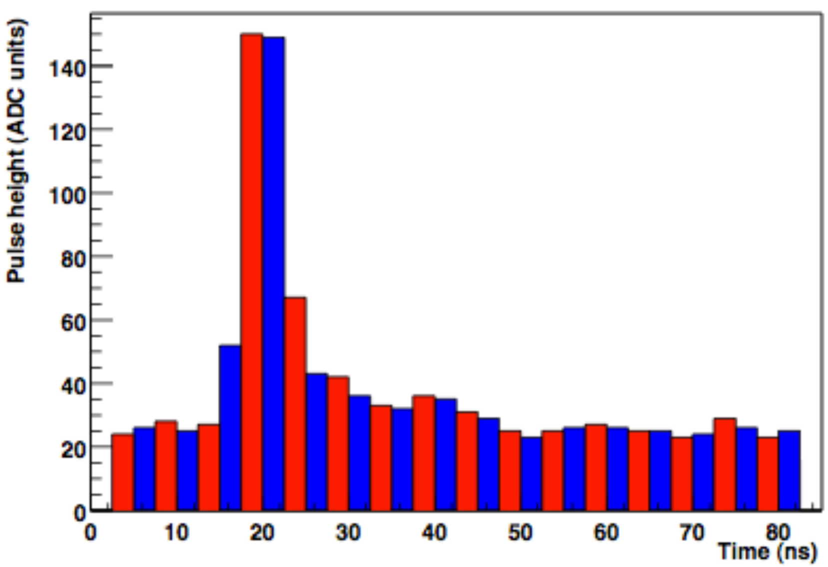


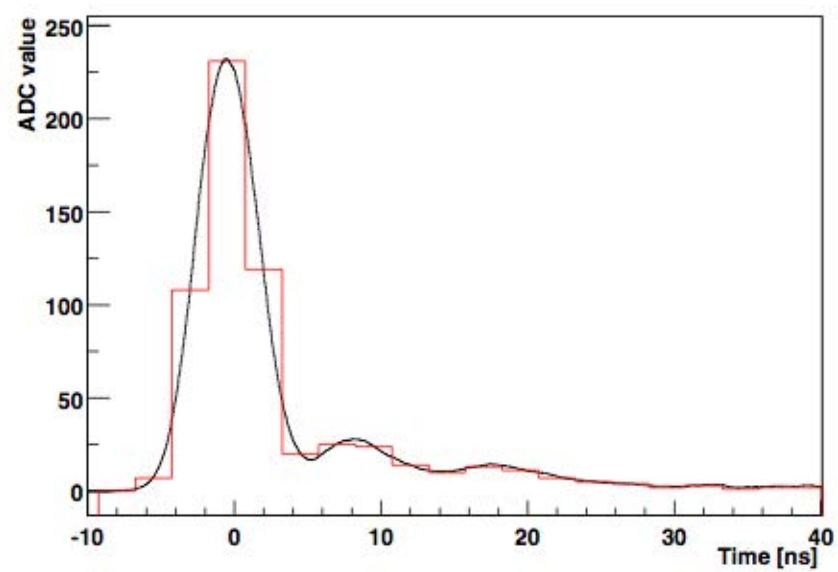
- Inject muons at t=0
- Calorimeter PMTs gated off due to hadronic flash
  - Pions/protons entering ring with muons create blinding flash of light at injection
- Fits cannot start until >20 μs in any case due to some beam manipulations going on in the ring and the time constant for kicker eddy currents to subside
- Large pedestal in detectors near injection point
- Decay electron signals riding on pedestal
- Not really any PID, other than wash of low energy stuff creating pedestal, only muons and decay electrons (few protons)
  - Muons are MIPs, well below threshold at BNL

First step…pulse-fitting

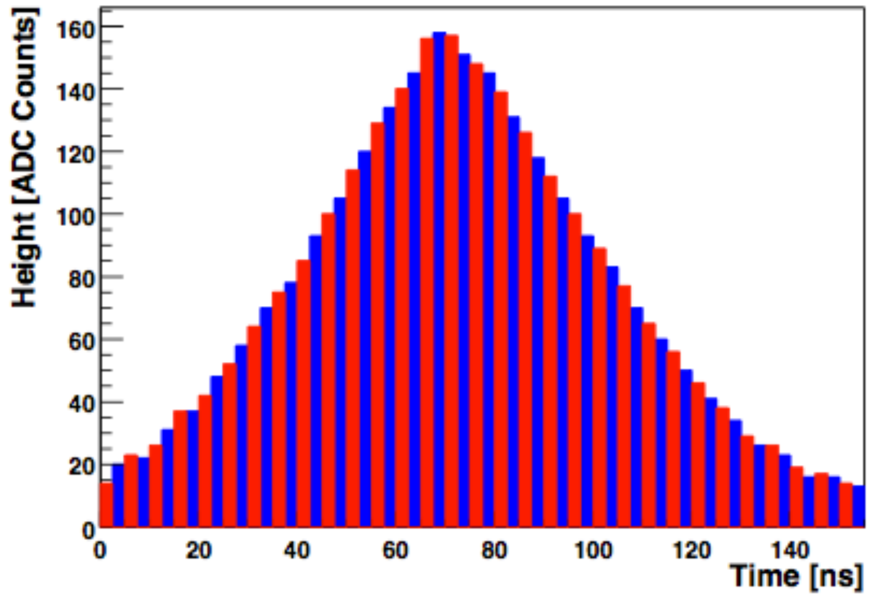


- BNL used two 200 MHz WFDs, sampling out of phase
  - Align with marker pulse
  - Calibrate relative gain of two WFDs
- Pulse fit to pulse-shape library to extract (E_i,t_i) of event accounting for the average electronic ringing in a particular calorimeter





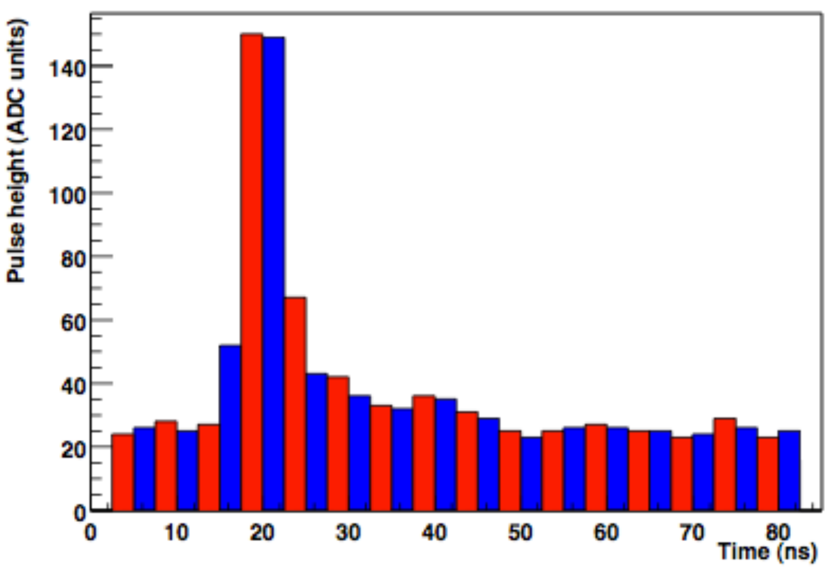
First step…pulse-fitting

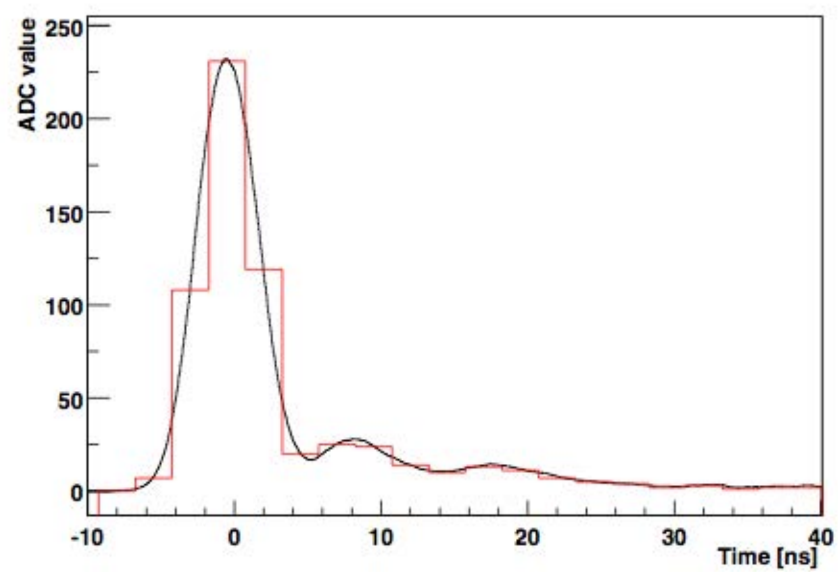


A lesson learned at BNL…

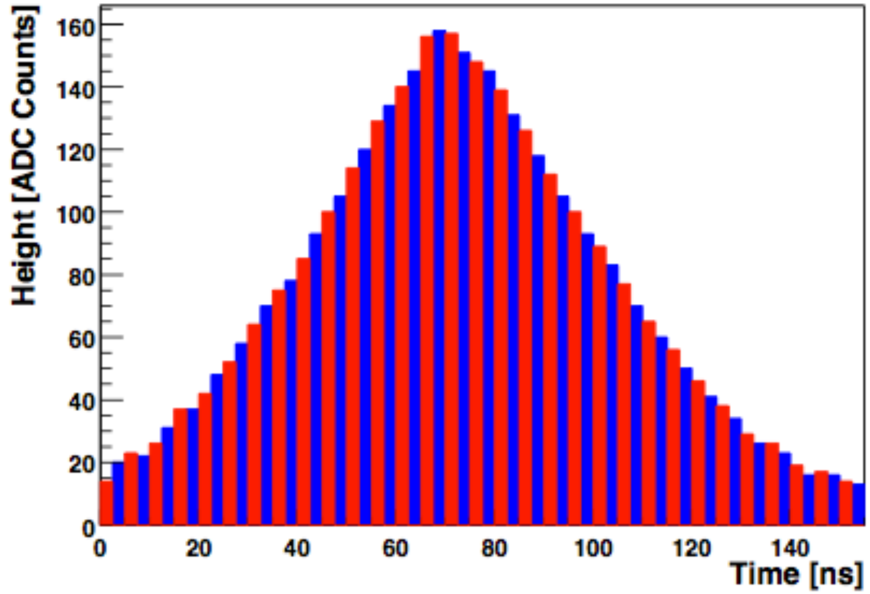
How would you fit for (E,t)?

- Assume uncertainty on bin height is +/-1 ADC count?
- Assume uncertainty scales with sqrt(ADC)?



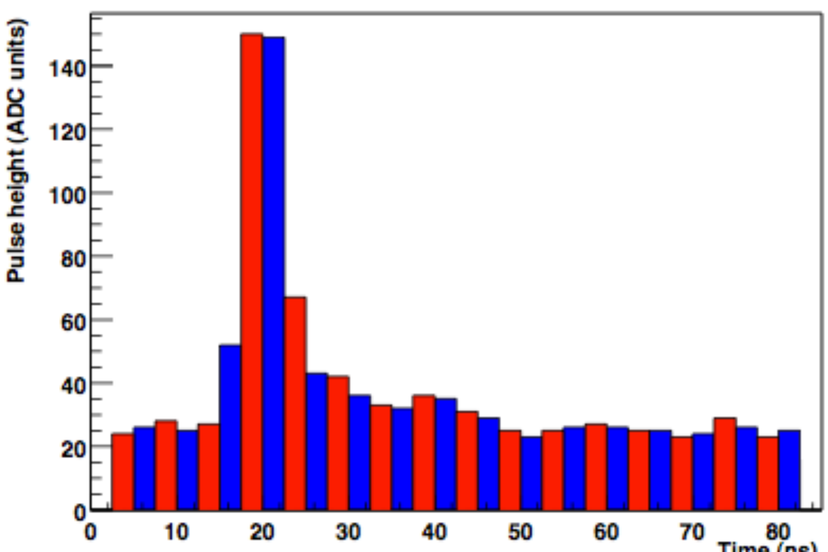


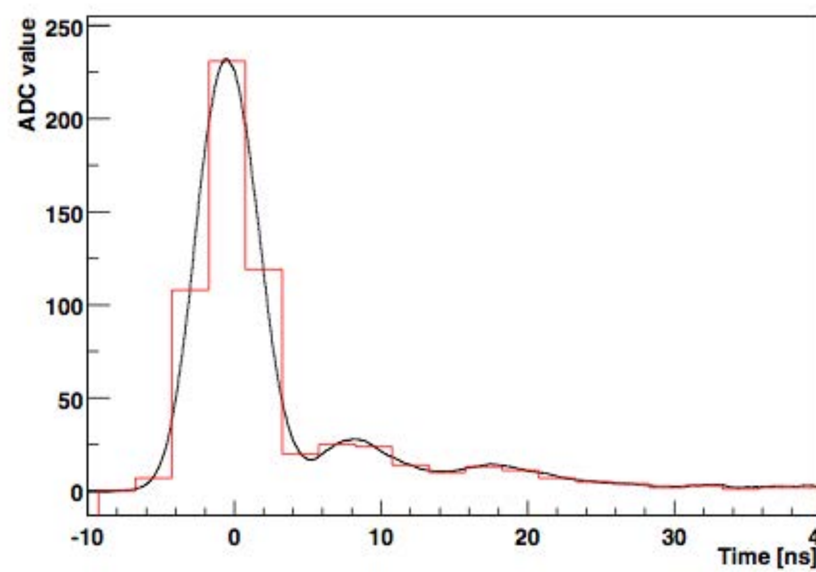
First step…pulse-fitting



Contrast BNL/FNAL

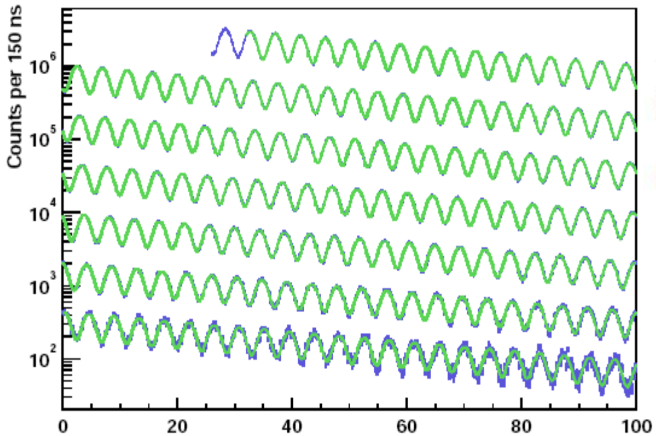
- FNAL using 500 MHz, single-phase WFDs
- SiPM readout of PbF_2 Cerenkov signals, fast and stable

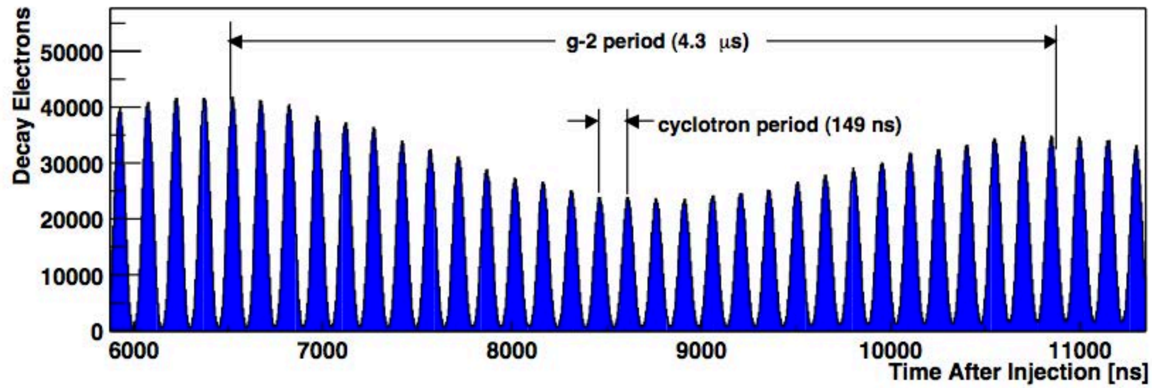


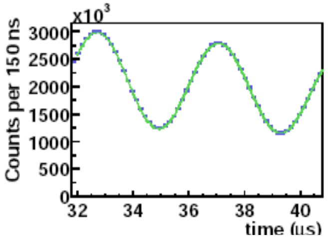


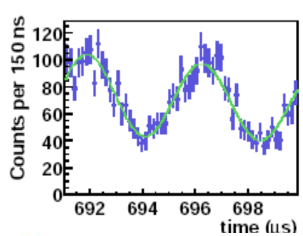
Controlling systematic errors on ω_a

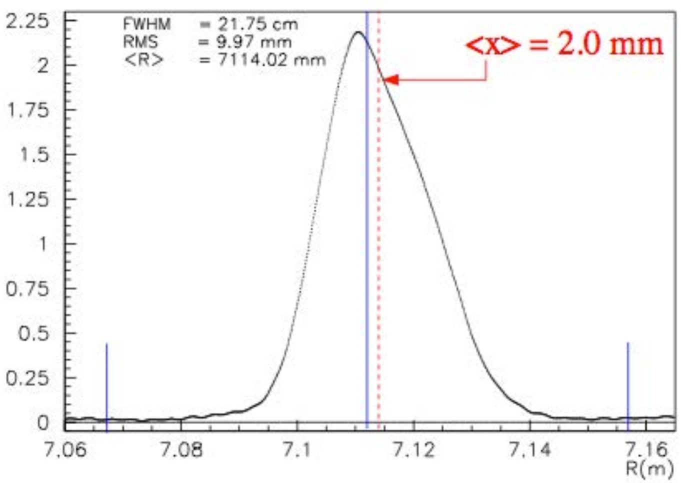
Fast Rotation





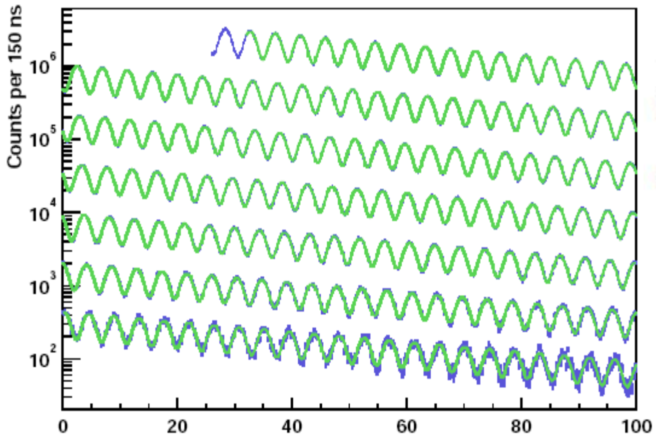


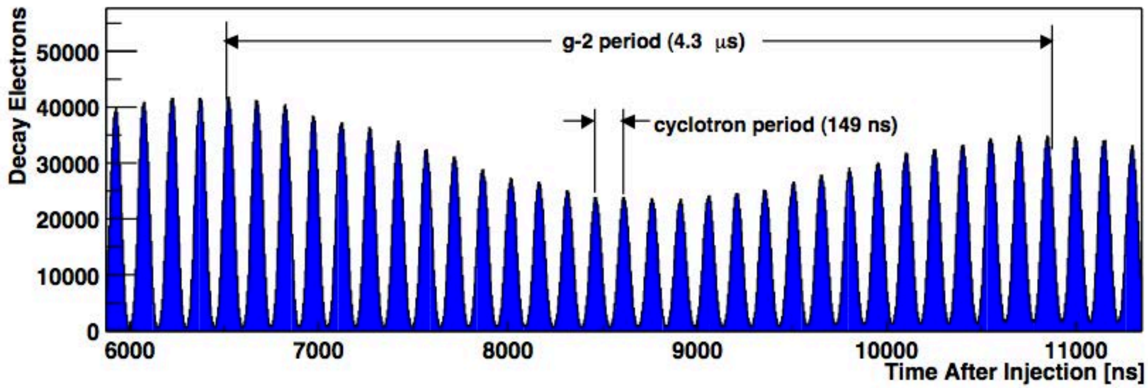


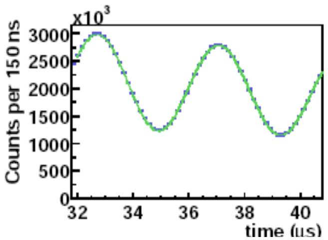


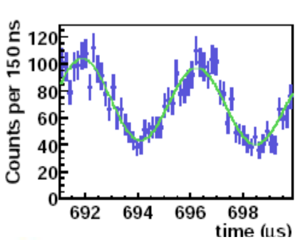
- Injected bunch width is <149 ns time for muons to go around the ring
  - Muons come in clumped and will slowly dephase due to dp/p
- Can see in plot above the fast rotation structure with the longer wavelength a_μ wiggle superimposed
- Actually a good feature, can extract the momentum spread of the stored muons

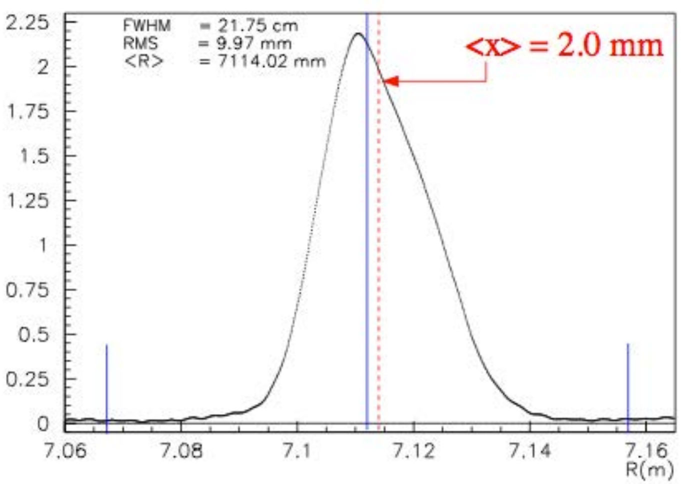
Fast Rotation







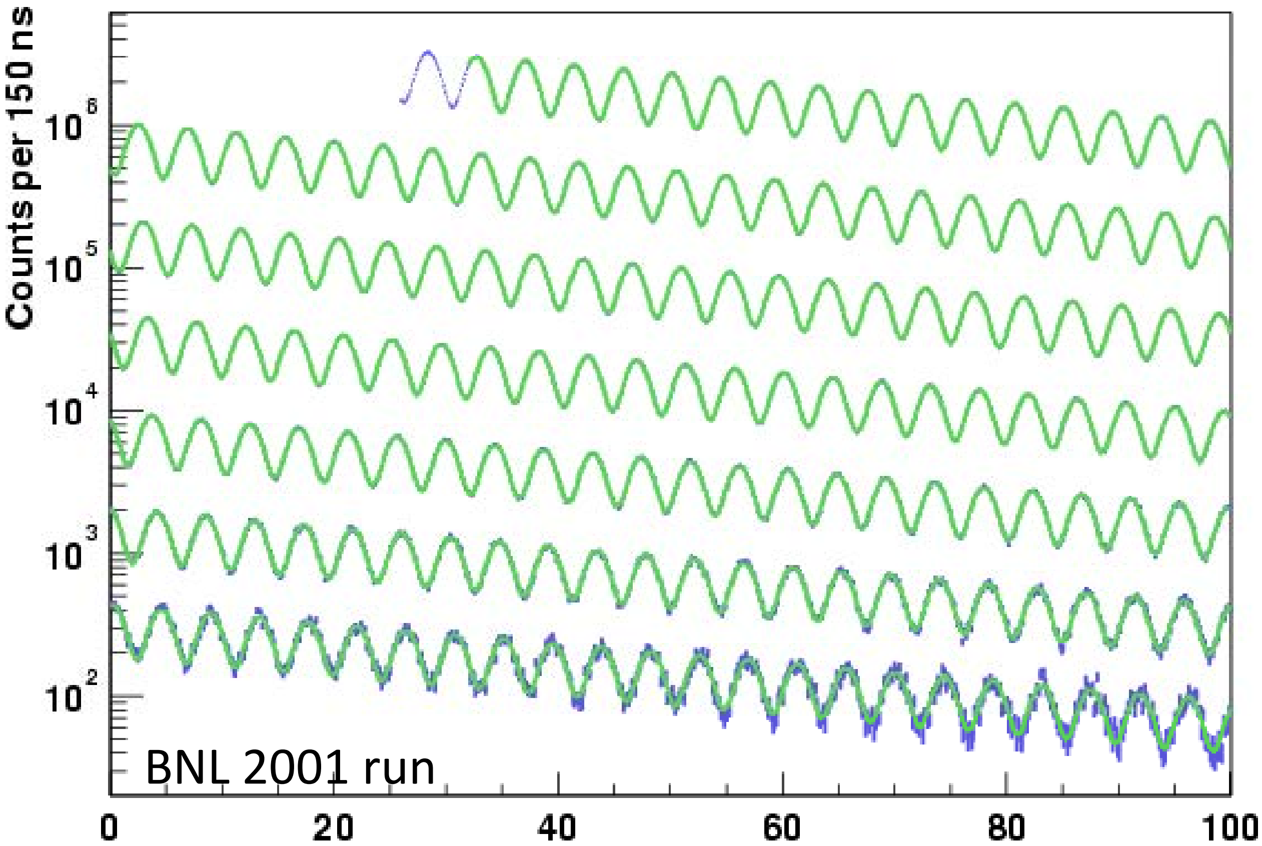




- Very hard to fit though
  - Not in 5 parameter function
  - Hard to pin down the envelope at the precision needed for getting a good χ^2.
- Two solutions employed
  - Randomize time of each fitted event by +/- 149 ns
  - Bin the data in 149ns bins

Early-to-late effects

- Experimental goal of 0.07 ppm systematic uncertainty
  - Must remove all biases from the fitting procedure



high energy positrons versus time

Dominant feature:

$$\cos(\omega_a t + \phi)$$

ϕ is the phase between the spin and momentum at the beginning of the fit.

*Pilfered from Brendan Casey

Early-to-late effects

$$\cos(\omega_a t + \phi)$$

Leading systematics come from time dependence in the phase

Taylor expansion: $$\phi(t) = \phi_0 + \alpha t + \beta t^2 \cdots \approx \phi_0 + \alpha t$$

$$\cos(\omega_a t + \phi(t)) \approx \cos((\omega_a + \alpha)t + \phi_0)$$

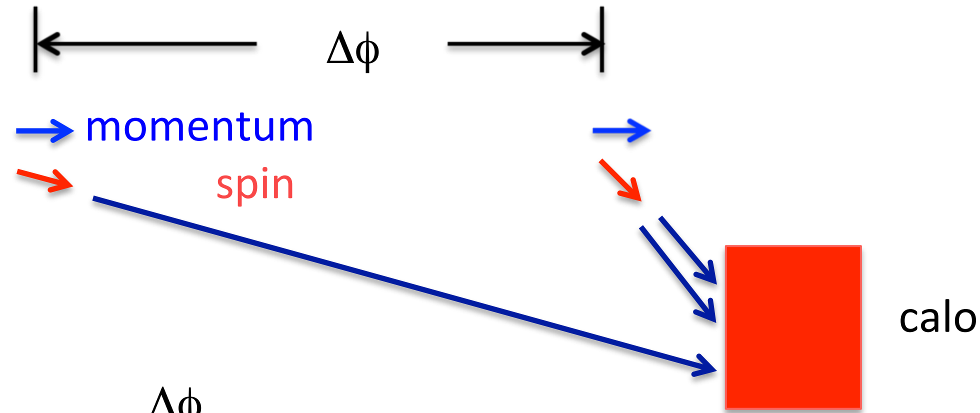
Things that change "early to late" in the fill typically lead to a time dependence in the phase of the accepted sample that directly biases the extracted value of ω_a

*Pilfered from Brendan Casey

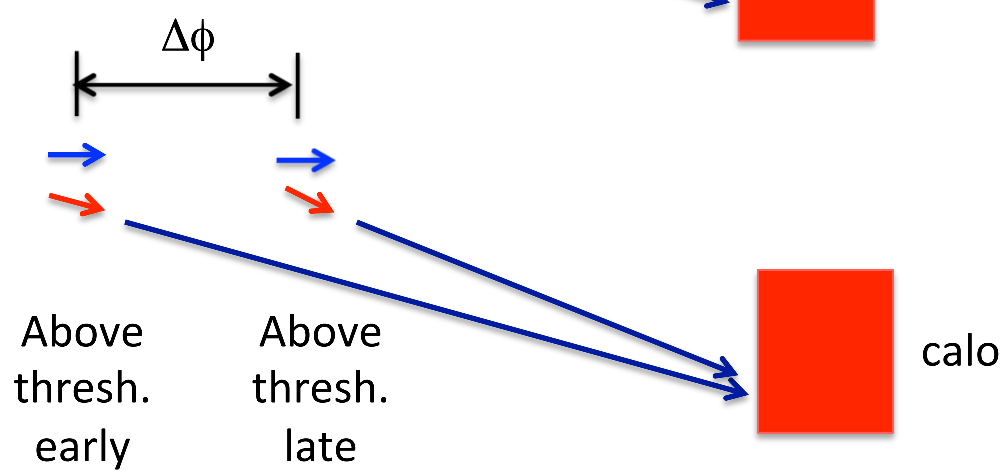
Two examples of early-to-late errors

Muon g-2

Pileup: two low energy positrons fake a high energy positron (happens early, not late)



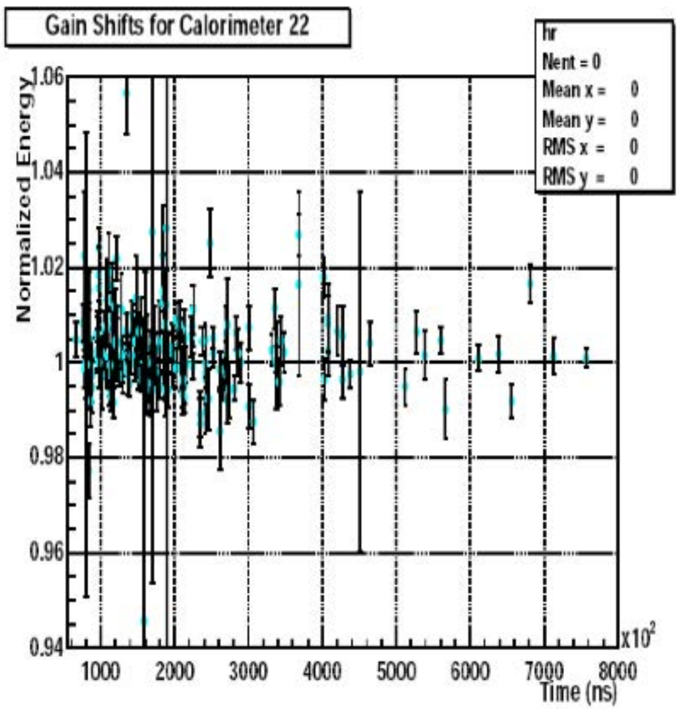
Gain change:
example: saturation
(happens early, not late)



Design not driven by absolute performance, but relative stability early to late

*Pilfered from Brendan Casey

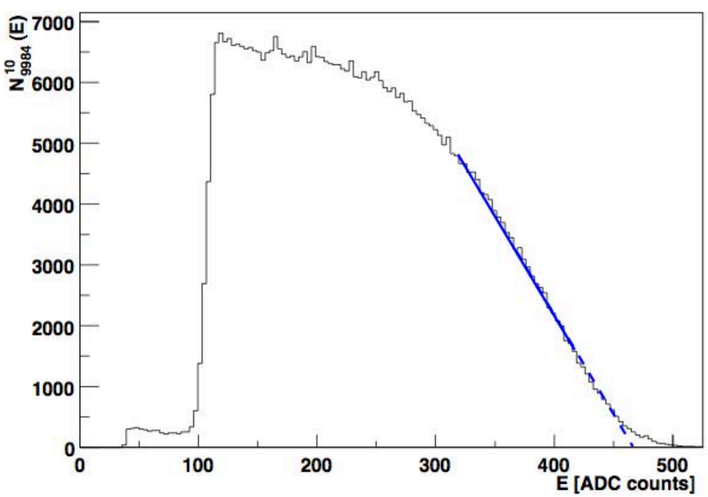
Correcting for gain changes



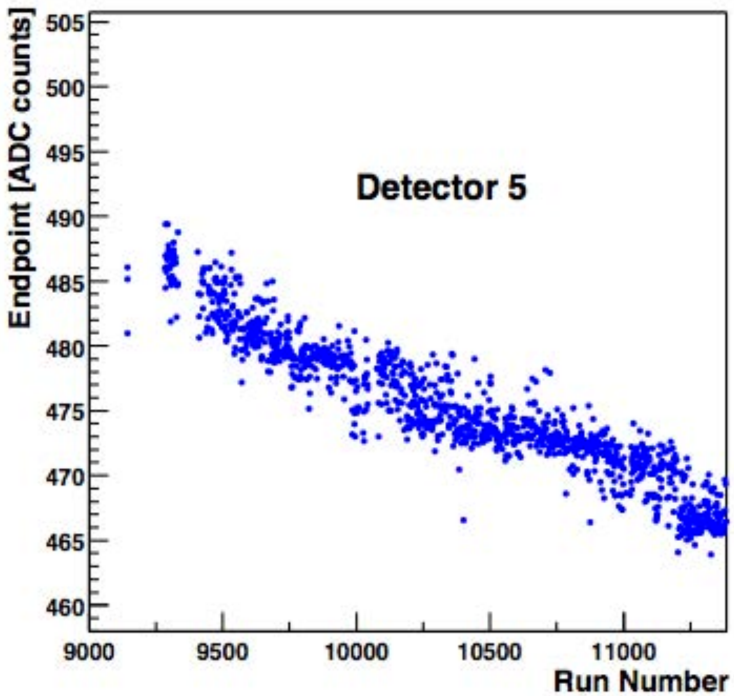
- BNL experiment had a laser calibration system, but could not reached required stability

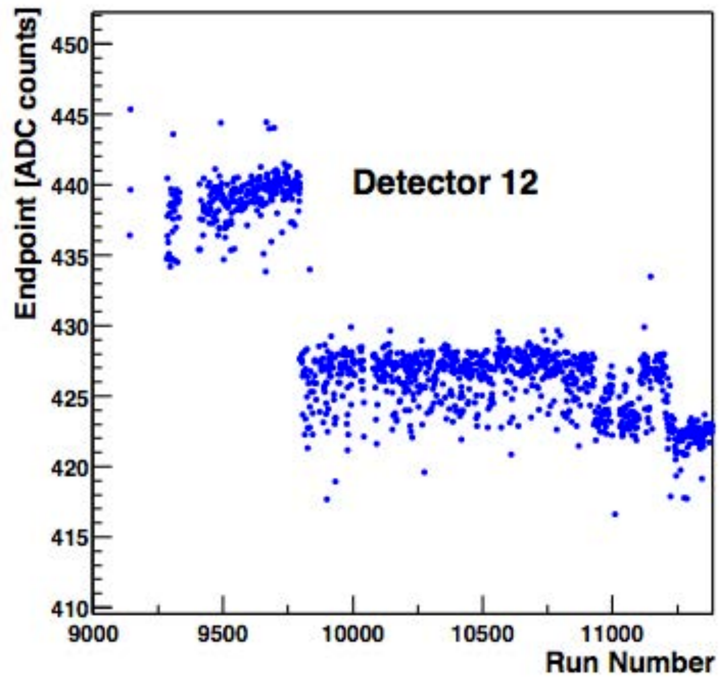
Contrast BNL/FNAL:
- Much more stable laser system being developed

- Ended up using endpoint of decay electron spectrum
  - Good because it scales with stats
  - Can only be binned in g-2 periods
  - Endpoint sensitive to pileup



Gain changes



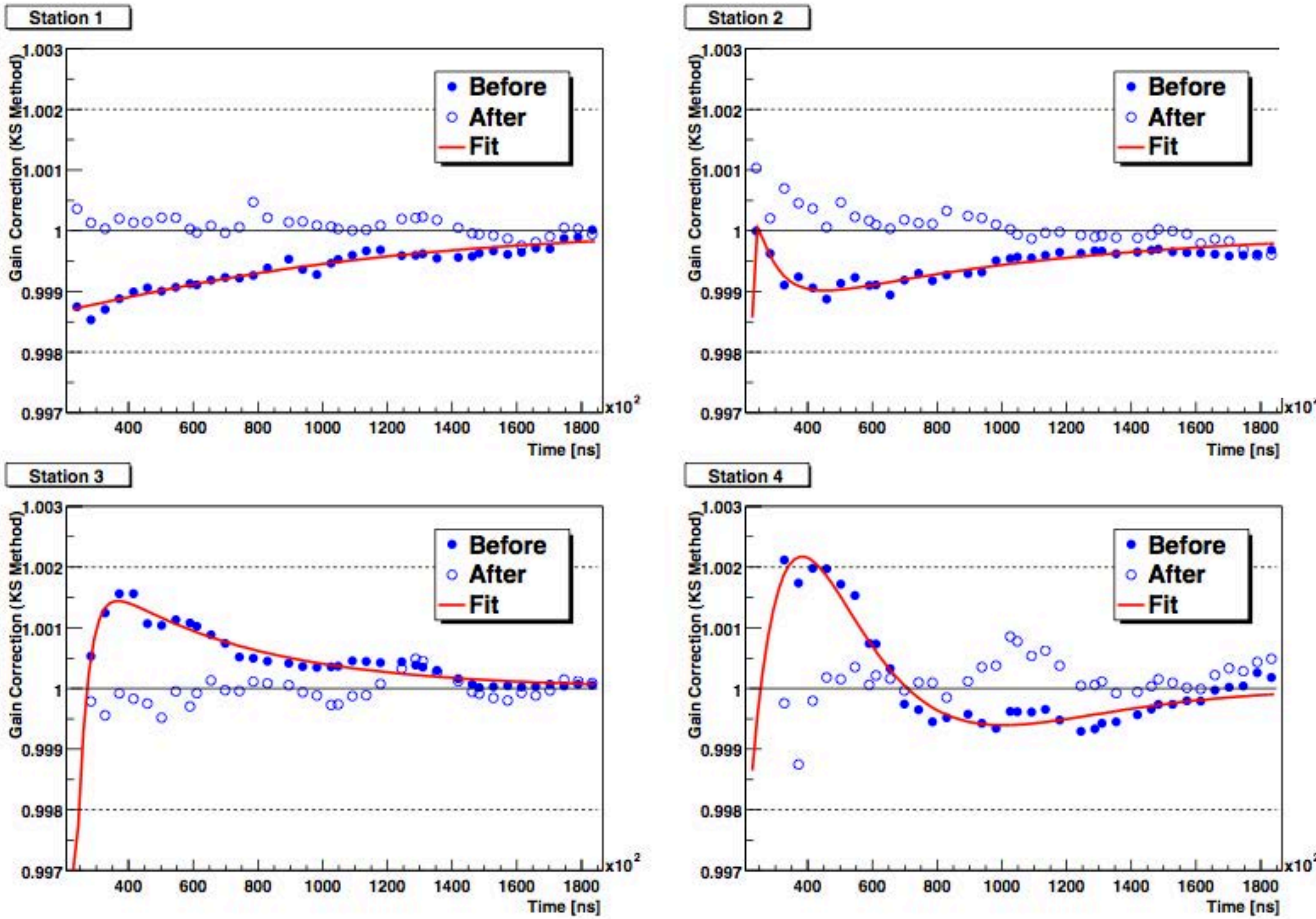


- Corrections over months of running
  - Can see degradation of detector 5 in hadronic flash region
  - Jumps in detector 12 are due to relative calibration of 4 PMTS

Contrast BNL/FNAL:
- No hadronic flash at FNAL
- Pions decay in >1 km beamline (compared to 80m at BNL)
- Protons removed by circulating in Debuncher long enough to kick out

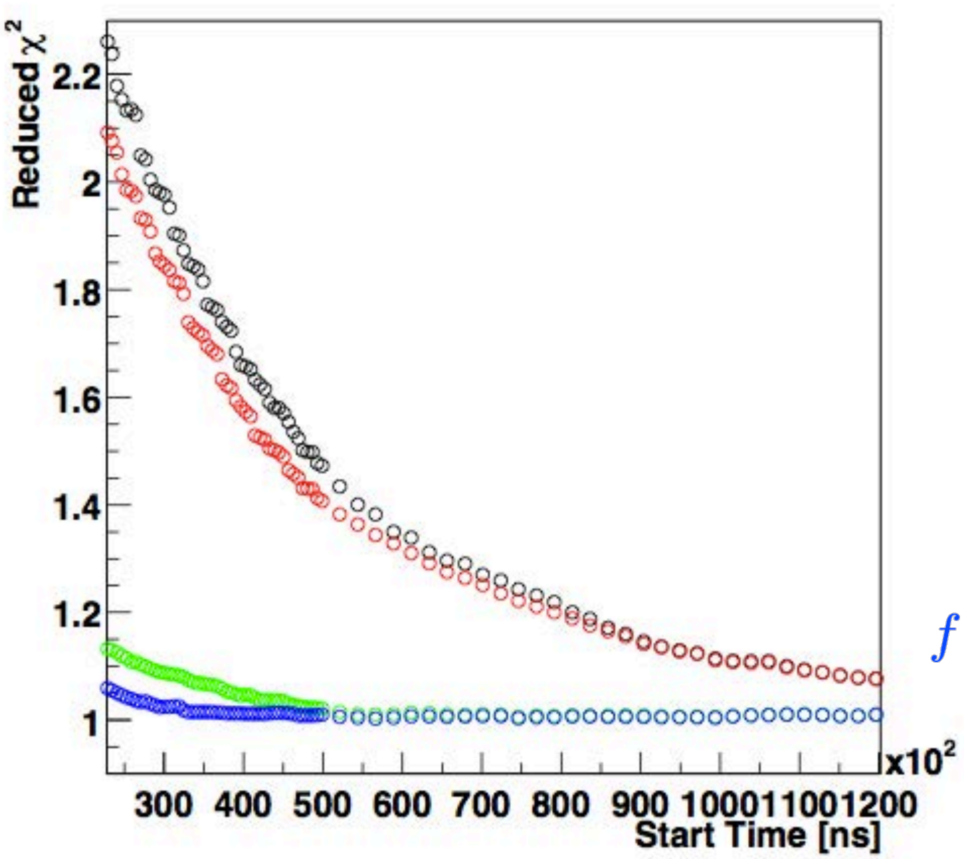
Important part is 'early-to-late' correction

Muon g-2



- Corrections for gain applied within measurement window

With all these corrections, must be ready to start fitting?



- Not even close!
- Black points show reduced χ^2 as a function of when fit is started using 5 parameter fit
- Allowed deviation in is sqrt(2/d) and because of 149ns binning, d numbers in the thousands
- Black points are unacceptable 50σ
- 5-parameter fitting function is pitifully inadequate

$$f(t) \simeq N_0 e^{-\lambda t}[1 + A\cos(\omega_a t + \phi)]$$

- What is still missing
  - Obtain red after correcting for pileup
  - Obtain green after including coherent betatron oscillations
  - Obtain blue after including muon losses

Actual fitting function

$$N(t) = \frac{N_0}{\tau}\Lambda(t)V(t)B(t)C(t)[1 - A'(t)\cos(\omega_a t + \phi'(t))], \text{ where}$$

$$B(t) = 1 - A_{br}e^{-t/\tau_{br}} \text{ with } \tau_{\rm br} = 5\mu\text{s}.$$ Beam relaxation

$$V(t) = (1 - e^{-t/\tau_{vw}}[1 - A_{vw}\cos(\omega_{vw}t + \phi_{vw})]),$$ Vertical breathing

$$A'(t) = A(1 - e^{-t/\tau_{cbo}}[1 - A_2\cos(\omega_{cbo}t + \phi_2)]), \text{ and}$$ 3 CBO terms

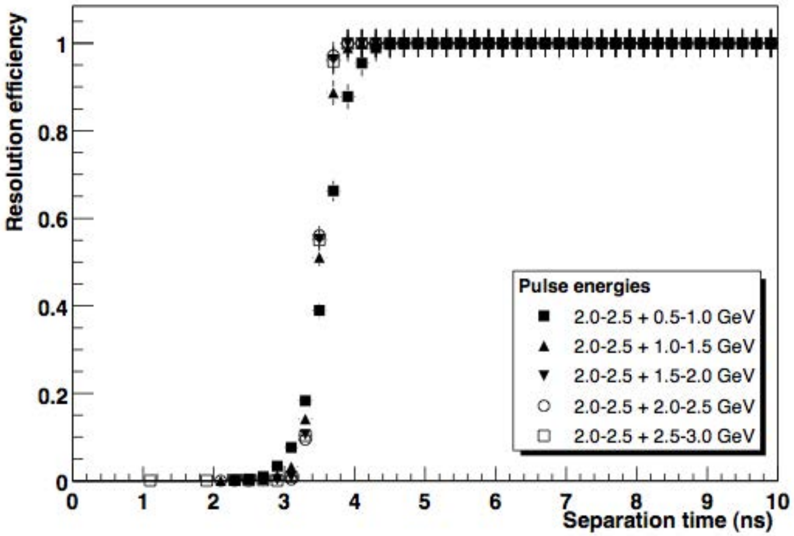
$$\phi'(t) = \phi(1 - e^{-t/\tau_{cbo}}[1 - A_3\cos(\omega_{cbo}t + \phi_3)]).$$

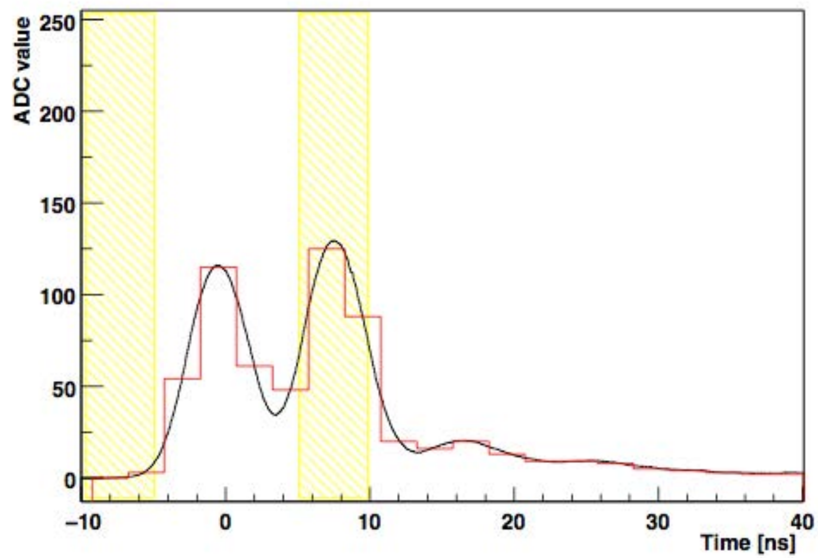
$$C(t) = 1 - e^{-t/\tau_{cbo}}[1 - A_1 cos(\omega_{cbo}t + \phi_1)].$$

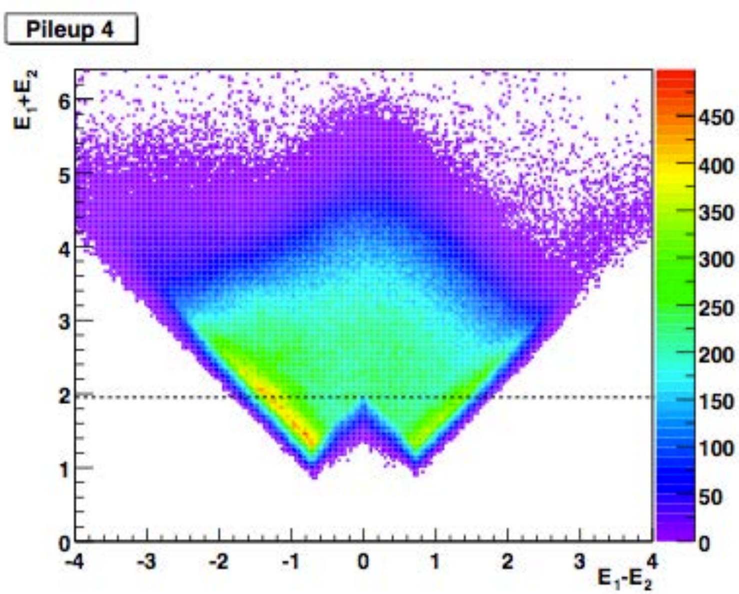
$$\Lambda(t) = 1 - Ce^{-t_0/\tau}\int_{t_0}^{t} L(t')e^{t'/\tau}dt',$$ Muons lost from ring

- Note, no pileup term since correction is constructed (like gain)
- Many more terms for Sergei to understand analytically ☺
- Every effect is intellectually challenging and every factor of 2 increase in statistics results in sensitivity to higher-order effects
- Luckily, these terms only weakly not couple to ω_a, but we consider an acceptable χ2 as a necessary condition

Pileup correction



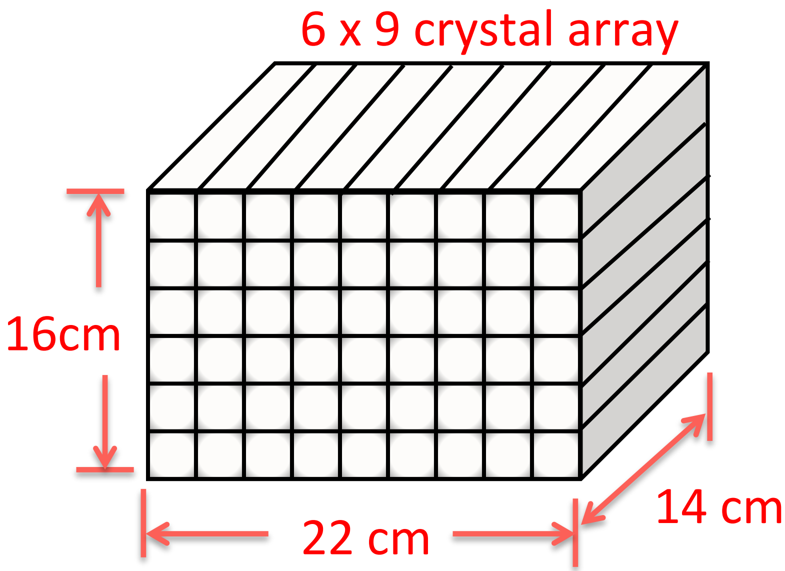


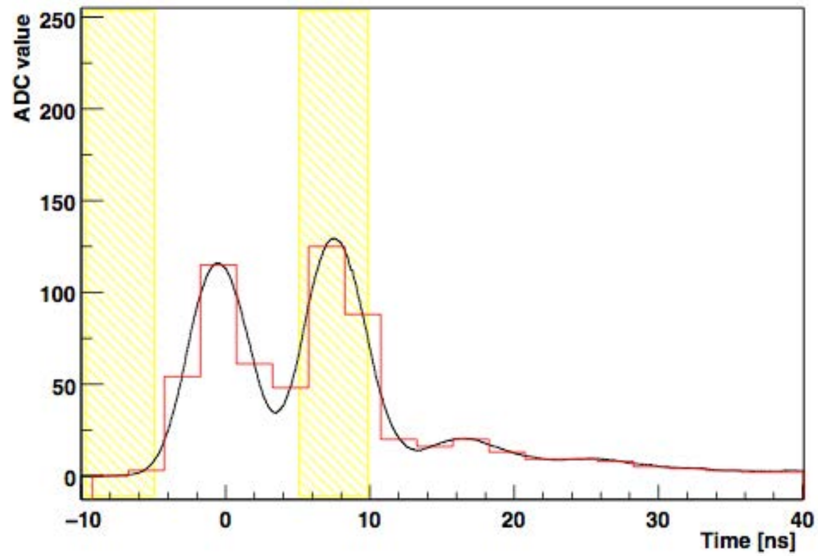


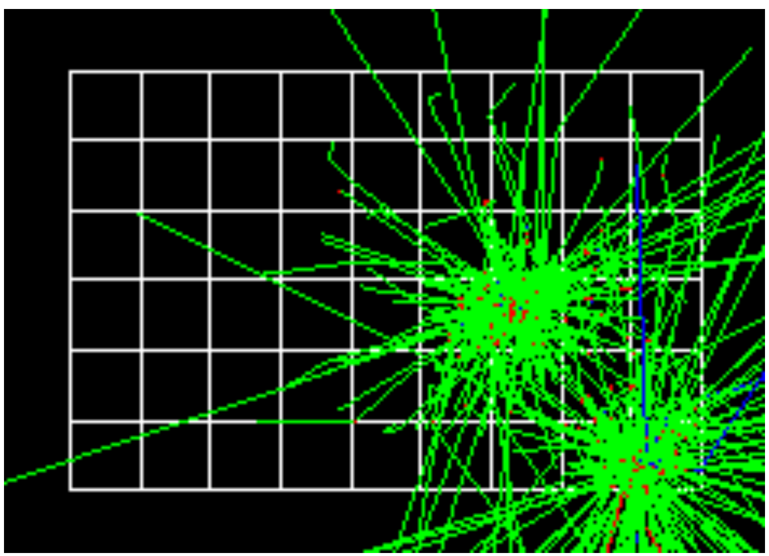
(a) Detector 4; Most Missing Pileup

- Pileup can be resolved in if dt<5ns
- Reconstructed by looking in side window around main pulse
- Complicated due to hardware and software thresholds
  - Example in asymmetric wings
  - Would really rather look in windows further out but BNL trigger only kept a few samples to either side of pulse

Pileup correction

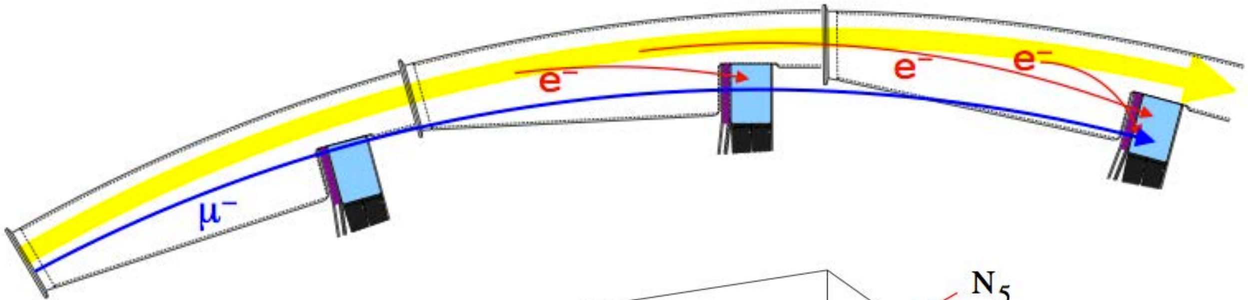


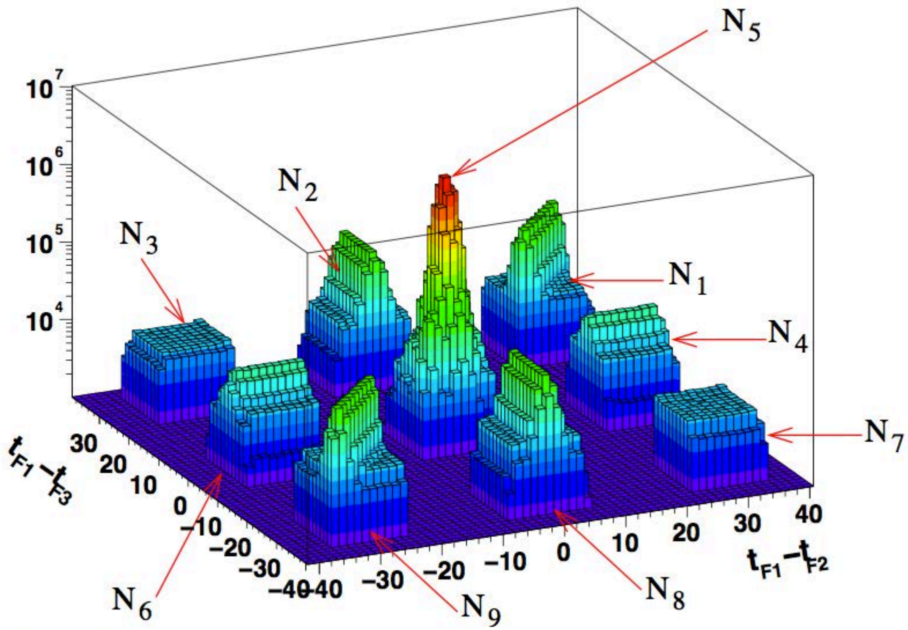




Contrast BNL/FNAL:
- Segmented detectors
- Faster WFDs
- Full data record kept

Muon losses

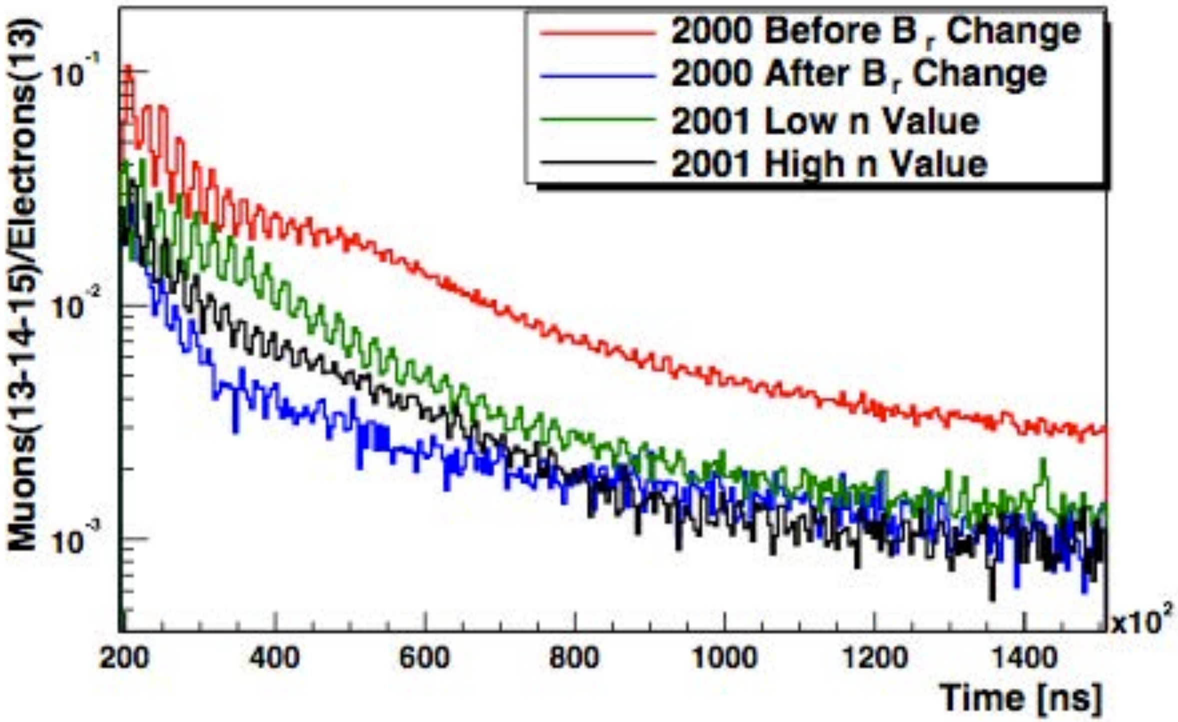




- Muons can hit collimators or other material, lose energy, and spiral out of ring
- Able to reconstruct at BNL by looking for triple coincidence on hodoscopes attached to front of calorimeters
- Double coincidence no good due to upstream calorimeter spraying downstream hodoscope

$$
\begin{aligned}
L_{\text{true}}(t) &= [S_1 \odot S_2 \odot S_3] \\
&= N_5(t) - \frac{1}{2}(N_2(t) + N_8(t)) - \frac{1}{2}(N_1(t) + N_9(t)) - \frac{1}{2}(N_4(t) + N_6(t)) \\
&\quad + 2 \cdot \frac{1}{2}(N_3(t) + N_7(t))
\end{aligned}
$$

Muon losses



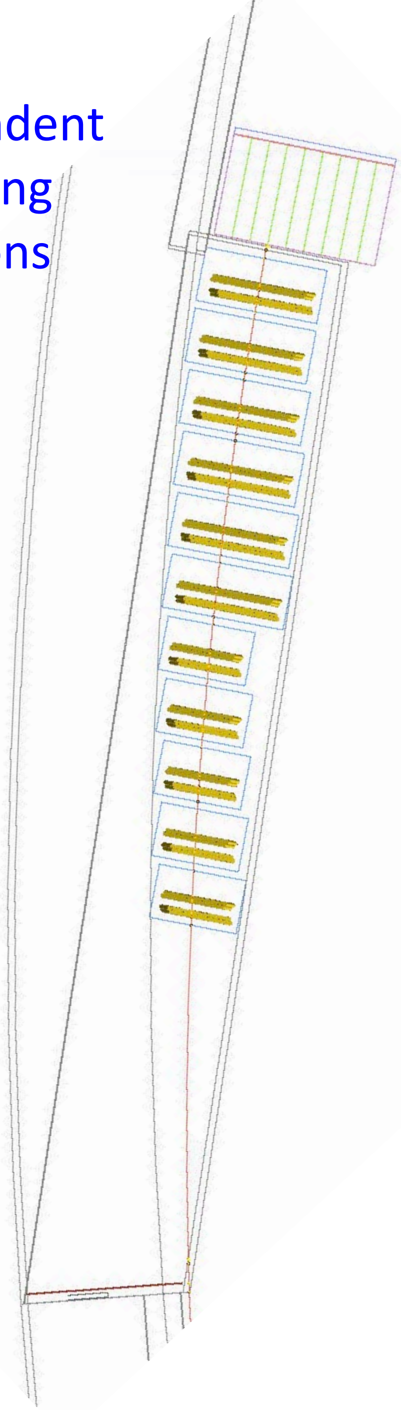
A lesson learned at BNL…

In part of the 2000 run at BNL the radial field in the magnet was set incorrectly…beam too high in aperture…losses large

$$
\begin{aligned}
L_{\text{true}}(t) &= [S_1 \odot S_2 \odot S_3] \\
&= N_5(t) - \frac{1}{2}(N_2(t) + N_8(t)) - \frac{1}{2}(N_1(t) + N_9(t)) - \frac{1}{2}(N_4(t) + N_6(t)) \\
&\quad + 2 \cdot \frac{1}{2}(N_3(t) + N_7(t))
\end{aligned}
$$

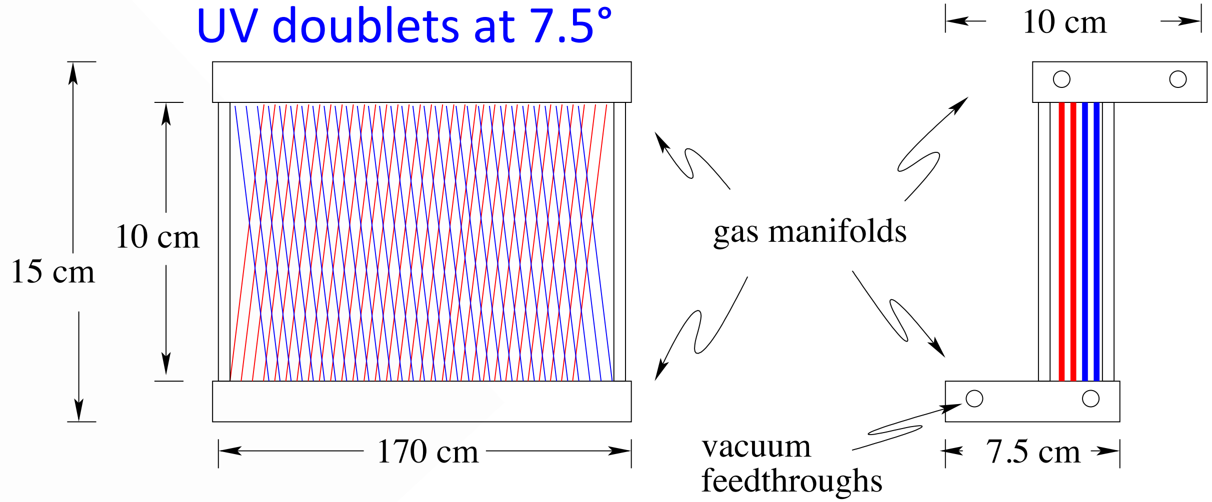
Muon losses

11 independent tracking stations



5 mm straws

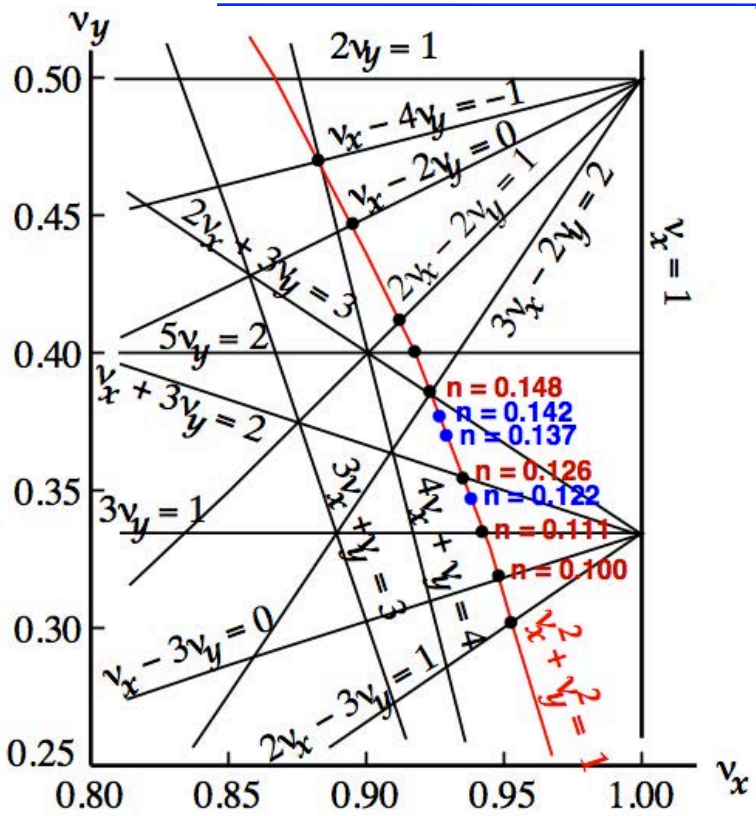
UV doublets at 7.5°



Contrast BNL/FNAL:

- In vacuo straw trackers being developed (B. Casey Early Career)
- Will give much better beam diagnostics, much more timely
- Also, calorimeters will be able to reconstruct muons without need for triple coincidence

Coherent Betatron Oscillations



- Electrostatic focusing causes beam to ‘swim’ and ‘breathe’ horizontally and vertically
- Can calculate expected frequencies based on strength of electric field…n value
- Creates time-dependent detector acceptance effects
  - Big impact on $\chi 2$, but little impact on ω_a

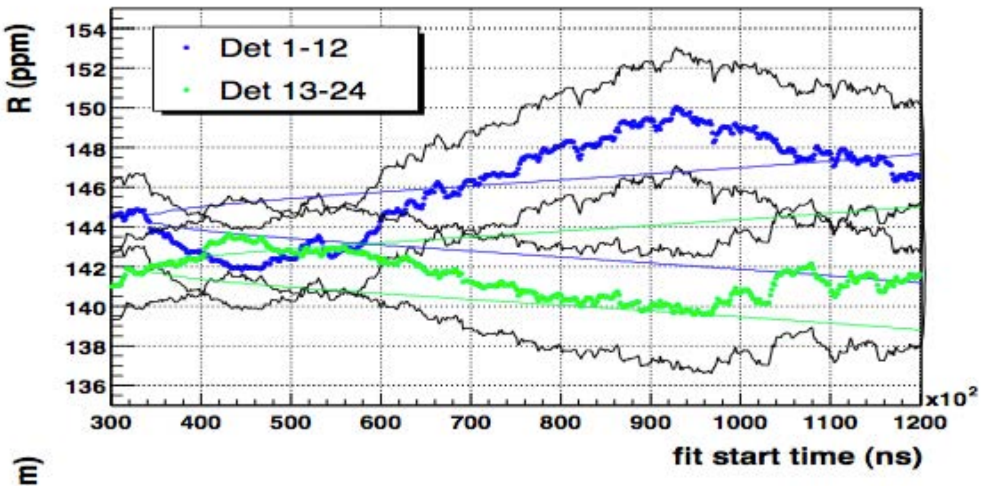
$$x(s) = x_e + A_x\sqrt{\beta_x(s)}\cos\left[\nu_x\frac{s}{R_0} + \phi_x(s)\right]$$

$$y(s) = A_y\sqrt{\beta_y(s)}\cos\left[\nu_y\frac{s}{R_0} + \phi_y(s)\right].$$

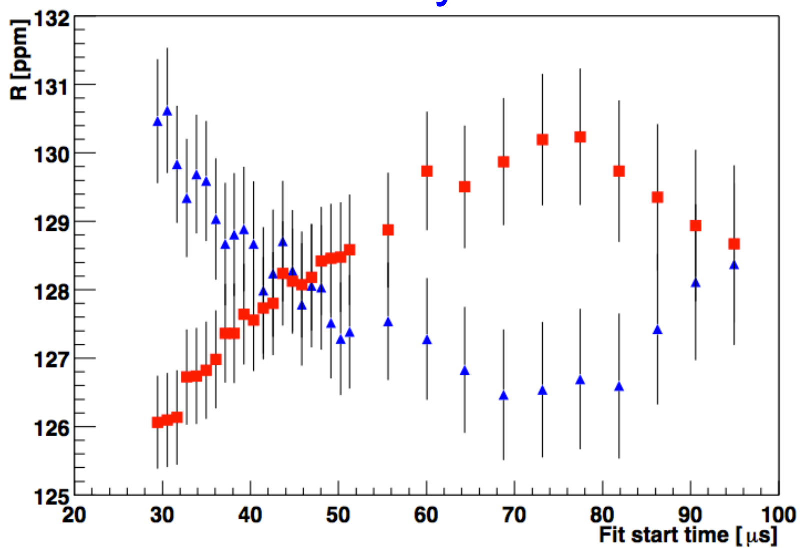
Coherent Betatron Oscillations

Muon g-2

1999 L. Duong Thesis



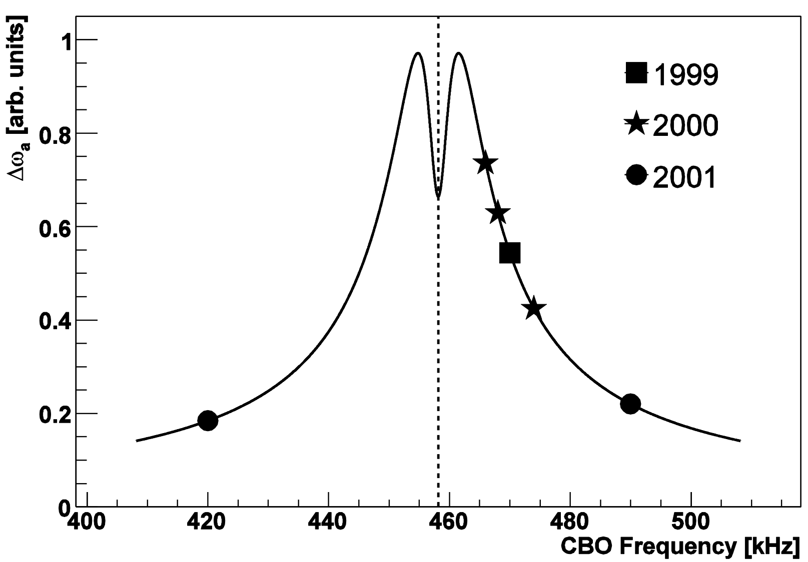
2000 F. Grey Thesis



Another lesson learned at BNL
- In 1999, statistics first significantly surpassed CERN III, but still 1/10th or so of final stats
- Started to see effect in data where detectors on two halves of ring got different results
- By 2000 run, data was practically screaming there was some kind of problem

Coherent Betatron Oscillations

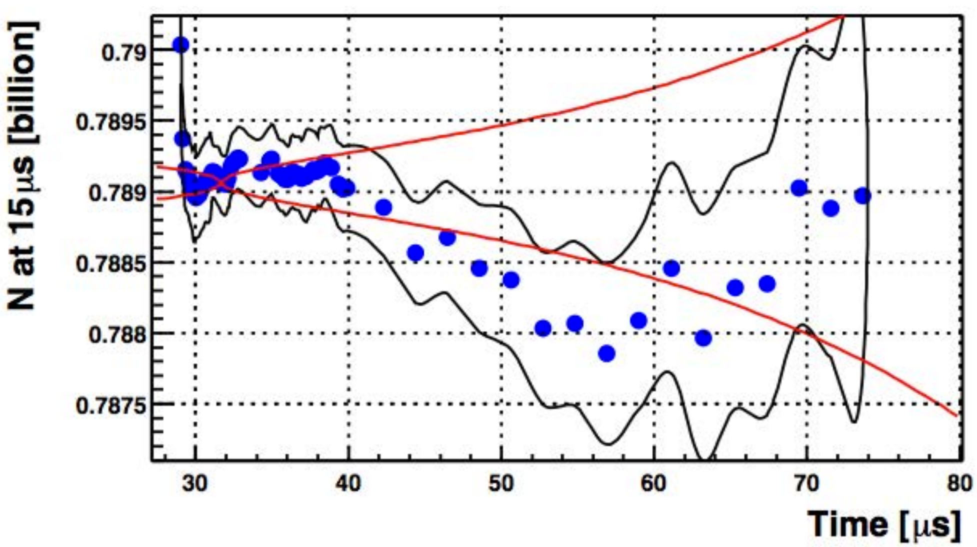
Quantity	Expression	Frequency	Period
f_a	$\frac{e}{2\pi mc} a_\mu B$	0.23 MHz	4.37 μs
f_c	$\frac{v}{2\pi R_0}$	6.7 MHz	149 ns
f_x	$\sqrt{1-n} f_c$	6.23 MHz	160 ns
f_y	$\sqrt{n} f_c$	2.48 MHz	402 ns
f_{CBO}	$f_c - f_x$	0.477 MHz	2.10 μs
f_{VW}	$f_c - 2f_y$	1.74 MHz	0.574 μs

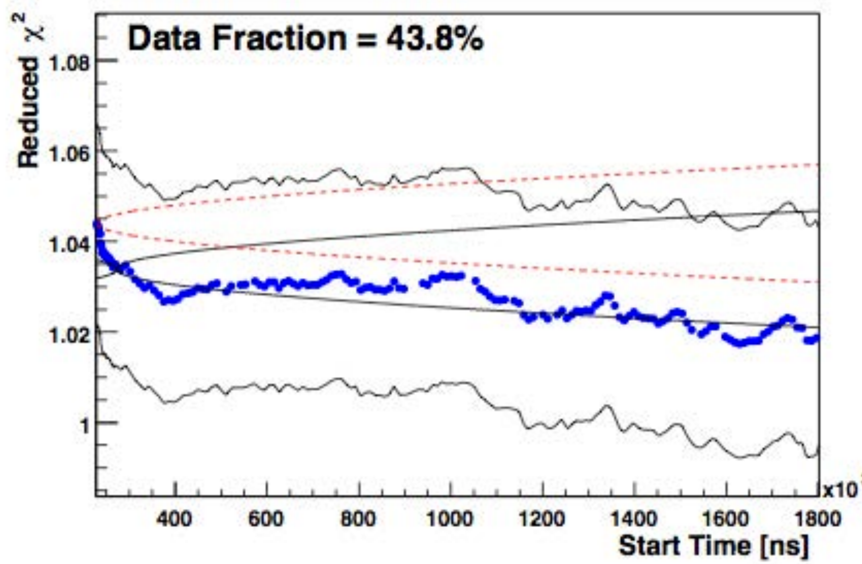


- Turns out that the difference frequency between the horizontal CBO and the cyclotron frequency were almost exactly at the 2nd harmonic of ω_a
  - Effect cancels when summing all detectors, but it is a clear example of how higher statistics helps one discover new effects
    - Part of what make a higher stat version of g-2 so critical

Many systematics have a characteristic time constant

Effect	Lifetime [μs]	Effect	Lifetime [μs]
Beam relaxation	5	Fast rotation	30
Vertical CBO	15	Pileup	32
Triple pileup	21	Detector gain	50
Muon losses	25	Radial CBO	100





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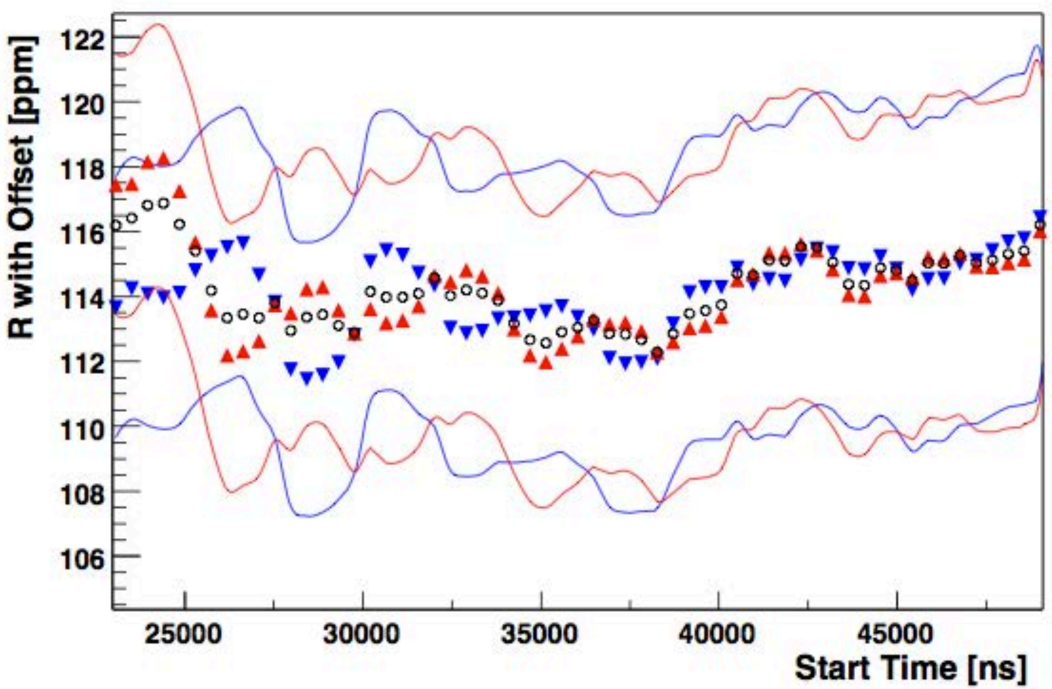


- Many data-driven consistency checks
- Can see here what happens in one detector if gain is uncorrected vs over-corrected by a factor of two

ω_a Systematic Requirements

E821 Error	Size [ppm]	Plan for the E989 $g-2$ Experiment	Goal [ppm]
Gain changes	0.12	Better laser calibration; low-energy threshold; temperature stability; segmentation to lower rates; no hadronic flash	0.02
Lost muons	0.09	Running at higher n-value to reduce losses; less scattering due to material at injection; muons reconstructed by calorimeters; tracking simulation	0.02
Pileup	0.08	Low-energy samples recorded; calorimeter segmentation; Cherenkov; improved analysis techniques; straw trackers cross-calibrate pileup efficiency	0.04
CBO	0.07	Higher n-value; straw trackers determine parameters	0.03
E-Field/Pitch	0.06	Straw trackers reconstruct muon distribution; better collimator alignment; tracking simulation; better kick	0.03
Diff. Decay	0.05[1]	better kicker; tracking simulation; apply correction	0.02
Total	0.20		0.07

Overall, ω_a systematics need to be reduced by a factor of 3

- Some errors were data-driven, precision of corrections scales with statistics
- Environmental improvements by changing run conditions, e.g. no hadronic flash
- Many hardware and analysis-driven improvements detailed in parallel sessions

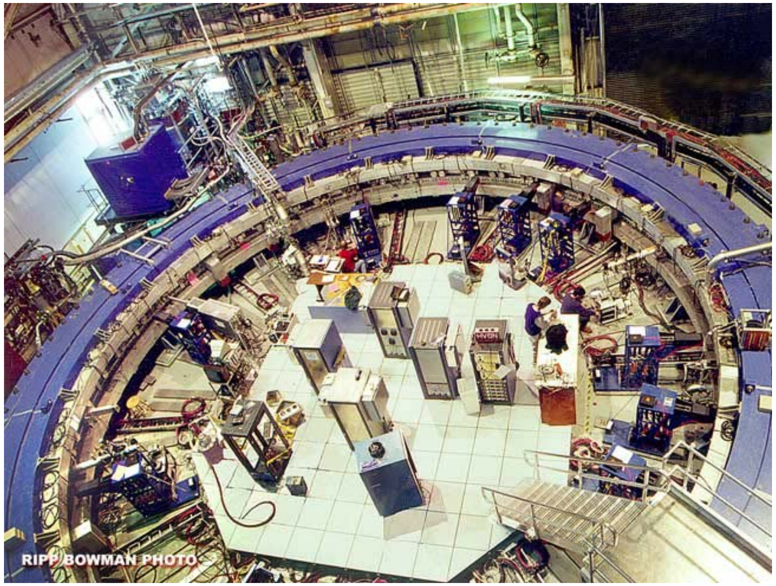
ω_p worthy of a whole extra lecture

E821 Error	Size [ppm]	Plan for the E989 $g-2$ Experiment	Goal [ppm]
Absolute field calibrations	0.05	Special 1.45 T calibration magnet with thermal enclosure; additional probes; better electronics	0.035
Trolley probe calibrations	0.09	Absolute cal probes that can calibrate off-central probes; better position accuracy by physical stops and/or optical survey; more frequent calibrations	0.03
Trolley measurements of B_0	0.05	Reduced rail irregularities; reduced position uncertainty by factor of 2; stabilized magnet field during measurements; smaller field gradients	0.03
Fixed probe interpolation	0.07	More frequent trolley runs; more fixed probes; better temperature stability of the magnet	0.03
Muon distribution	0.03	Additional probes at larger radii; improved field uniformity; improved muon tracking	0.01
Time-dependent external B fields	–	Direct measurement of external fields; simulations of impact; active feedback	0.005
Others	0.10	Improved trolley power supply; trolley probes extended to larger radii; reduced temperature effects on trolley; measure kicker field transients	0.05
Total	0.17		0.07

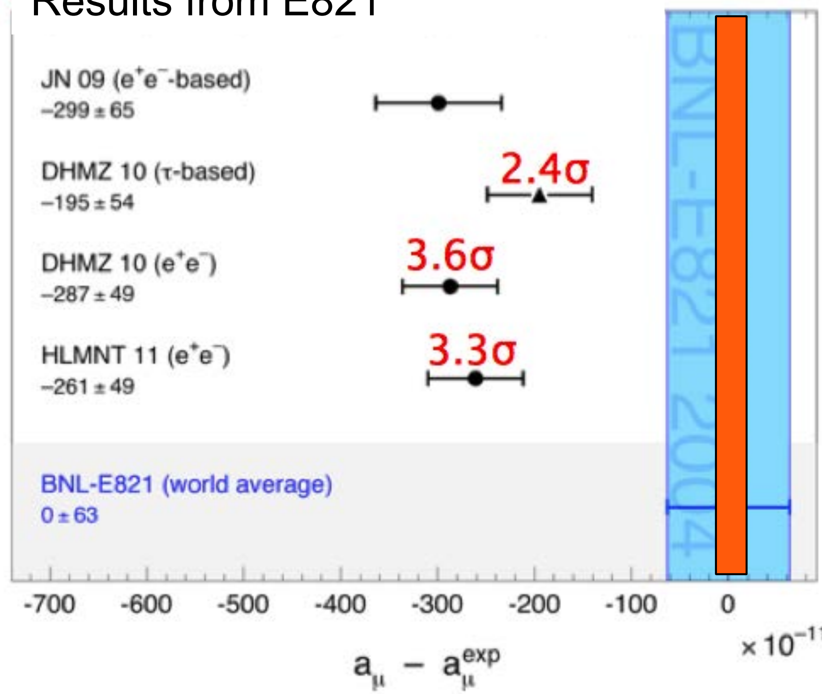
Overall, ω_p systematics need to be reduced by a factor of 2.5

- Better run conditions, e.g. temperature stability of experimental hall, more time to shim magnetic field to high uniformity, smaller stored muon distribution
- Also many hardware and simulation driven improvements detailed in parallel sessions

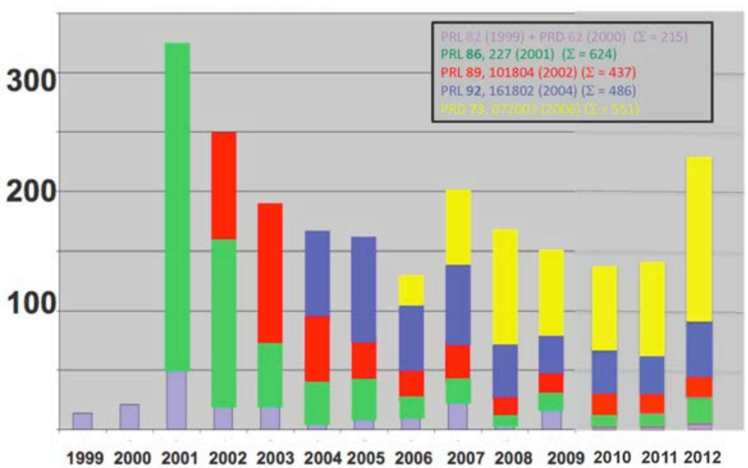
Outlook



Results from E821



Citations to E821 remain high



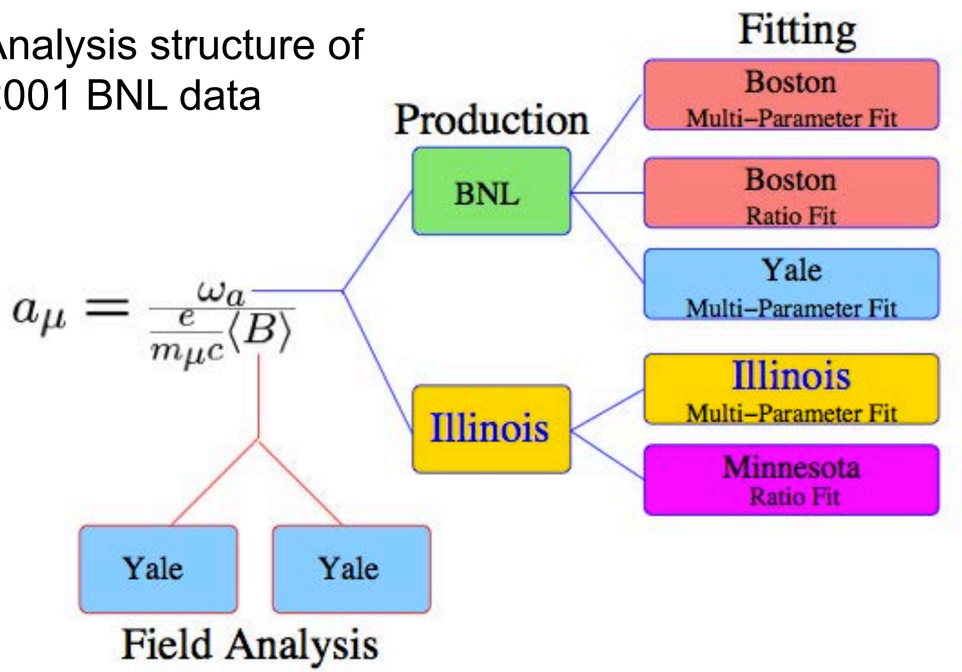
Primary scientific goal of FNAL experiment

- Reduce experimental error on a_μ by factor of 4
- If current discrepancy persists, significance will be pushed beyond 5σ discovery threshold
- Motivates further theoretical improvement

How does a single number experiment support so many dissertations?

Muon g-2

Analysis structure of 2001 BNL data



- Absolutely critical to have independent analyses from the very bottom up
  - Were it not for the separate BNL and Illinois productions it is not clear how long it would have taken to discover the pulse-fitting lesson
- Every factor of 2 in statistics brings new challenges

- Independent analyses of ω_a and ω_p are crucial
  - Analyzers allowed to make own decisions, consistency amongst all required at end of day to gain confidence
  - Analyses can be structured to have very different systematic sensitivities

Pre-1999 engineering runs and work supporting g-2	**Alex Peter Grossmann:** *Magnetic Field Determination in a Super-ferric Storage Ring for a Precise Measurement of the Muon Magnetic Anomaly*, University of Heidelberg, (July 1998). **Douglas Hodson Brown:** *Measurement of Three Pion Production in Electron Positron Annihilations for the Hadronic Contribution to the Anomalous Magnetic Moment of the Muon*, Boston University, (1998). **Joel Matthew Kindem:** *The Anomalous Magnetic Moment of the Positive Muon*, University of Minnesota, (September 1998). **Sergei Ivanovich Redin:** *Preparation and First Result of BNL Experiment E821 "A New Precision Measurement of the Muon (g-2) Value"*, Yale University (May 1999). **William James Deninger:** *A measurement of the magnetic field systematic correction to the muon anomalous magnetic moment associated with muon phase space in experiment BNL E821*, University of Illinois at Urbana-Champaign, (1999).
1999 run: $a_{\mu+}$ to 1.3 ppm	**Long Hoang Duong:** *A Precise Measurement of the Anomalous Magnetic Moment of the Positive Muon*, University of Minnesota, (December 2001). **Alexei Vitalyevich Trofimov:** *A New Precision Measurement of the Anomalous Magnetic Moment of the Positive Muon*, Boston University, (December 2001).
2000 run: $a_{\mu+}$ to 0.7 ppm	**Huaizhang Deng:** *Precise Measurement of the Positive Muon Anomalous Magnetic Moment*, Yale University, (September 2002). **Frederick Earl Gray Jr.:** *A Measurement of the Anomalous Magnetic Moment of the Positive Muon with a Precision of 0.7 Parts Per Million*, University of Illinois at Urbana-Champaign, (February 2003). **Benjamin Bousquet:** *A measurement of the anomalous magnetic moment of the positive muon to 0.7 ppm*, University of Minnesota, (2003).
2001 run: $a_{\mu-}$ to 0.7 ppm	**Jonathan M. Paley:** *Measurement of the Anomalous Magnetic Moment of the Negative Muon to 0.7 Parts Per Million*, Boston University, (April 2004). **Charles C. Polly:** *A measurement of the anomalous magnetic moment of the negative muon to 0.7 ppm*, University of Illinois at Urbana-Champaign, (2005)
Muon EDM	**Ronald Steven McNabb Jr.:** *An Improved Limit on the Electric Dipole Moment of the Muon*, University of Minnesota, (December 2003) **Steven Giron:** *Measuring the electric-dipole moment of the muon at BNL E821*, University of Minnesota, (2004). **Michael J. Sossong:** *A search for an electric dipole moment of the positive muon*, University of Illinois at Urbana-Champaign, (2005).
CPT, LV, & Relativity Tests	**Tao Qian:** *A precise measurement of muon lifetime at Brookhaven National Laboratory muon storage ring*, University of Minnesota, (2006) **Xiaobo Huang:** *CPT and Lorentz violation test in the BNL muon g-2 data*, Boston University, (2008).

Muon g-2

- 17 PhDs produced at BNL
- Just as many postdoc analyses
- Fermilab experiment will require even more
  - Increased precision
  - Longer run time
  - More sophisticated analyses
  - Trackers will open up whole new realm of analyses
  - Field requires more effort than BNL

Why does one more decimal place appeal to you?

$$g_\mu = 2.002\,331\,841\,78\,(126)$$

Why does one more decimal place appeal to you?

$$\left(\frac{1}{2m}(\vec{P}+e\vec{A})^2+\frac{e}{2m}\vec{\sigma}\cdot\vec{B}-eA^0\right)\psi_A=(E-m)\psi_A$$

$$g_\mu^{exp} = 2.002\,331\,841\,78\,(126)$$

Why does one more decimal place appeal to you?

Muon g-2

$$g_{\mu}^{exp} = 2.002\ 331\ 841\ 78\ (126)$$

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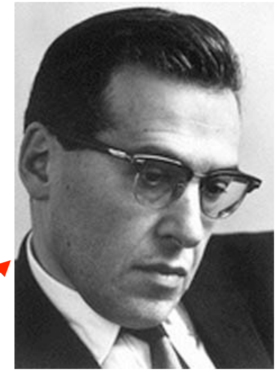
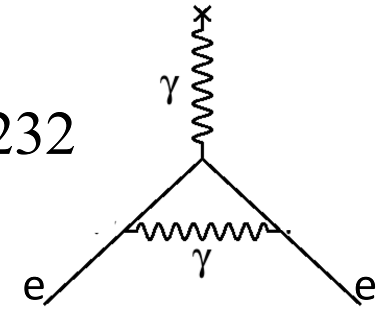


$$\frac{\alpha}{2\pi} = 0.00232$$



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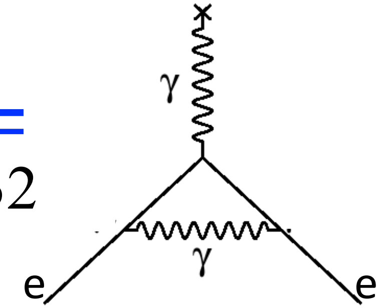
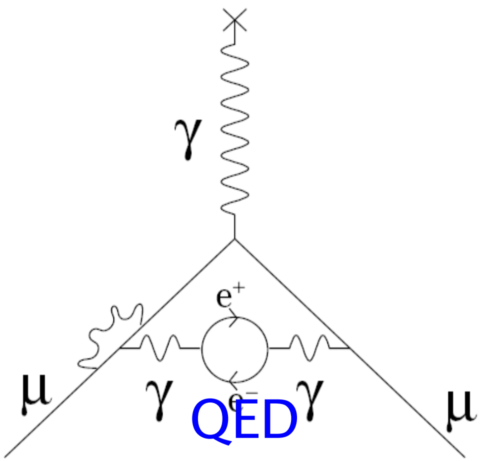


$$g_\mu^{exp} = 2.002\ 331\ 841\ 78\ (126)$$

$$2\ 331\ 694\ 36\ (0)$$



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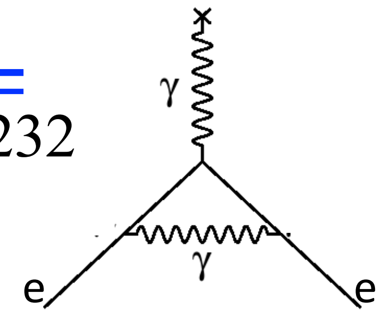
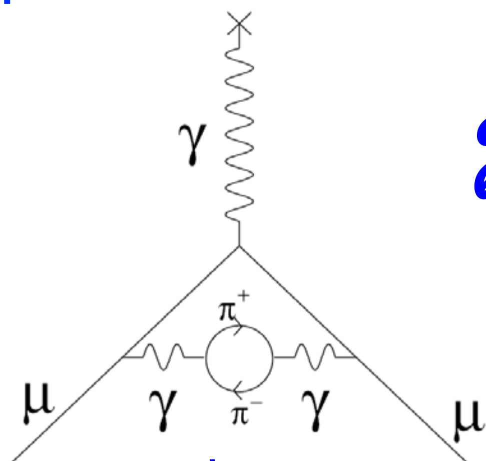


$$g_\mu^{exp} = 2.002\ 331\ 841\ 78\ (126)$$

$$2\ 331\ 694\ 36\ (0)$$

$$138\ 60\ (98)$$

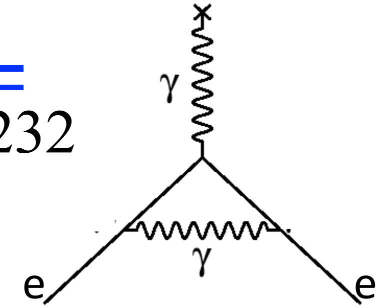


Hadronic

$$\lambda_{sens} \propto \left(\frac{m_\mu}{m_e}\right)^2 \approx 40,000$$

* Hadronic corrections for the electron g-2 don't show up until the 12th decimal

$$\frac{\alpha}{2\pi} = 0.00232$$



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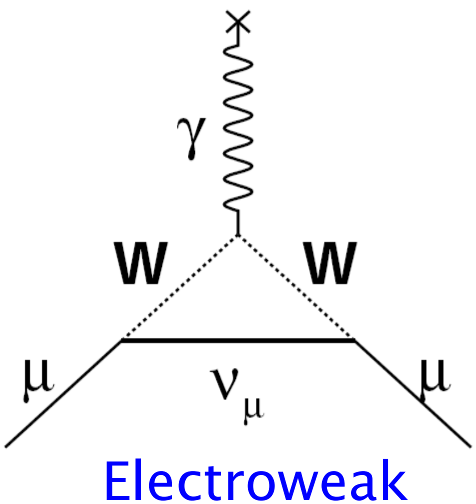
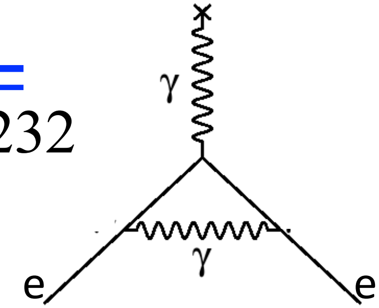


2 331 694 36 (0)

138 60 (98)

3 08 (4)

$$\frac{\alpha}{2\pi} = 0.00232$$



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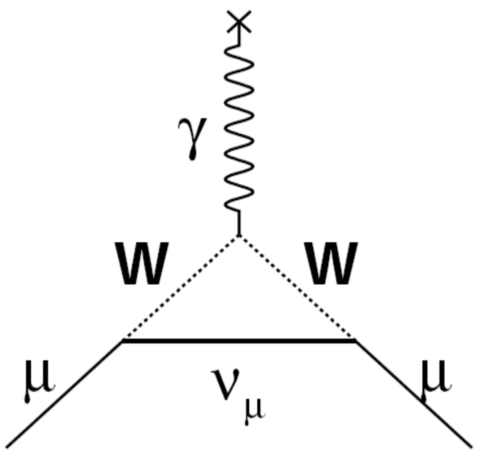
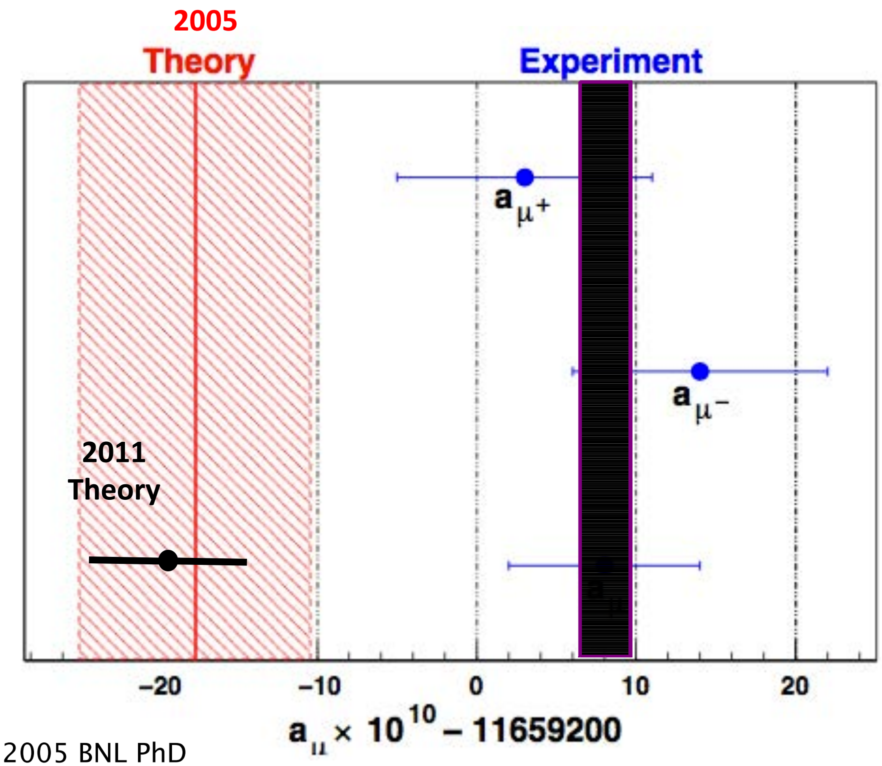


Electroweak

$$a_\mu^{exp} - a_\mu^{thy} = 287\ (80) \times 10^{-11}$$



2005 BNL PhD

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