Academic Lecture: Evidence for Dark Matter from the Cosmos

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I. EVIDENCE

A. Rotation Curves

- Simple approximation is spherical distribution

$$a = \frac{v^2}{r} = \frac{GM(r)}{r^2} \tag{1}$$

  or $v \propto r^{-1/2}$ far from the gas and stars

- Flat rotation curves observed far from the central region where stars and gas are located

- Fit with 3 parameters (mass to light ratio of stars and 2 DM profile parameters); flat rotation curves if $\rho_{DM} \propto r^{-2}$

B. Lensing

- Tangential shear around a galaxy (far from the mass) satisfies

$$\gamma_t(R) = \frac{\langle\Sigma(< R)\rangle}{\Sigma_{\rm cr}} \tag{2}$$

  where

$$\langle\Sigma(< R)\rangle \equiv \frac{1}{\pi R^2}\int_{R'<R} d^2R'\Sigma(R') \to \frac{M}{\pi R^2} \tag{3}$$

  and $\Sigma_{\rm cr} \equiv \frac{D_s}{4\pi G D_L D_{LS}}$.

- Instead we observe tangential shear out to large radii as in Fig. 1. Falls off much more slowly than $1/R^2$

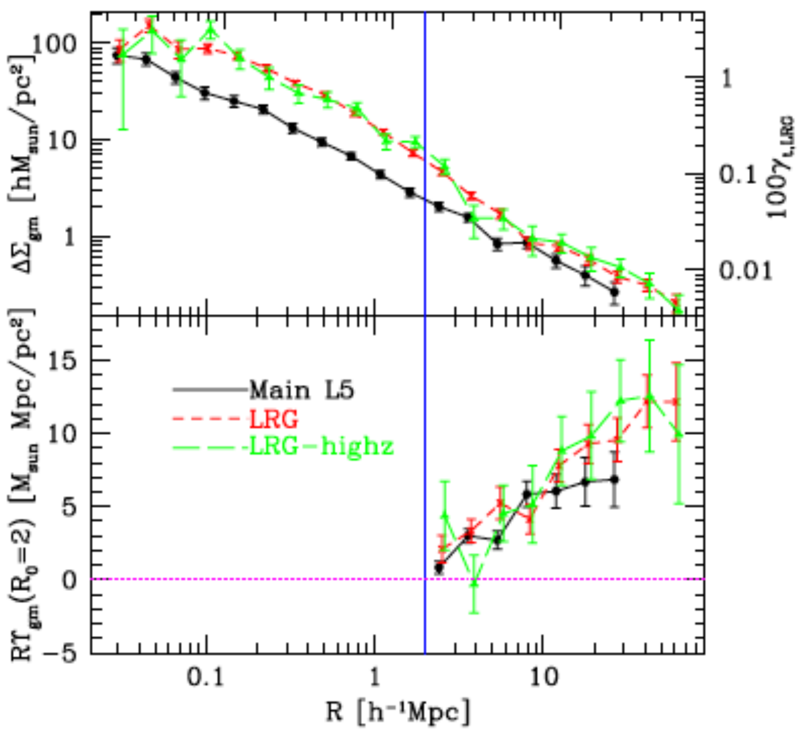

FIG. 1: Inferred average surface density ($\langle \int dz \rho(R,z) \rangle$) as a function of distance from galaxy centers. If the mass was concentrated in the center, this would fall off as R^{-2}. The much milder fall-off is evidence for dark matter. From 1207.1120.

C. Clusters

Our thinking of clusters is impossible to disentangle from dark matter.

- mass in galaxies much less than mass in gas much less than mass in dark matter
- formation in the context of dark matter "halos"; predictions for the number of these as a function of mass and redshift
- X-Ray observations relate temperature to total mass
- weak lensing measurements model independent measurements of mass, again require dark matter
- CMB observations of clusters (SZ effect) tightly correlated with mass (using simulations and lensing/X-ray calibrations)

D. Large Scale Perturbations

- Any quantity, like $\delta, \delta T$, or Φ has zero mean so all the information is contained in the variance or power spectrum
- Best to think of Fourier space quantities, since small perturbations are linear so do not mix Fourier modes: $\langle \tilde{\Phi}(\vec{k})\tilde{\Phi}(\vec{k}')\rangle = (2\pi)^3\delta^3(\vec{k}+\vec{k}')P_\Phi(k)$
- Real space fluctuations were initially scale-invariant:

$$\langle \Phi^2(x)\rangle = \int \frac{d^3k}{(2\pi)^3} P_\Phi(k) = \int_0^\infty \frac{dk}{k}\,\frac{k^3 P_\Phi(k)}{2\pi^2} \tag{4}$$

  so $P_\Phi \propto k^{-3}$ initially

- Cartoon: large scale Φ remains constant in time; small scale Φ decays during radiation era and then remains constant
- Important to compare comoving wavelengths $1/k$ with comoving Horizon $1/(aH)$; modes with $k \gg aH$ are inside the horizon and can evolve causally; modes outside the horizon are acausal so do not evolve

E. Galaxy Power Spectrum

- δ does not grow until matter domination
- Then

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}\delta = 0 \tag{5}$$

  leads to $p(p-1) + (4/3)p - (3/2)(2/3)^2 = 0$ or $p^2 + p/3 - (2/3) = 0$ or $\delta \propto a$

- Poisson equation dictates that $k^2\Phi = 4\pi G\bar{\rho}a^2\delta \rightarrow \Phi =$constant (consistent with what we said above)
- Since $k^3 P_\Phi(k) =$ constant for all scales that entered the horizon after matter domination and decays on smaller scales, $P_\delta \propto k^4 P_\Phi \propto k$ on large scales, with the turn-over scale corresponding to that that entered the horizon at equality $k_{\rm EQ} = a_{\rm EQ}H(a_{\rm EQ})$, so turnover scale determines the matter density.

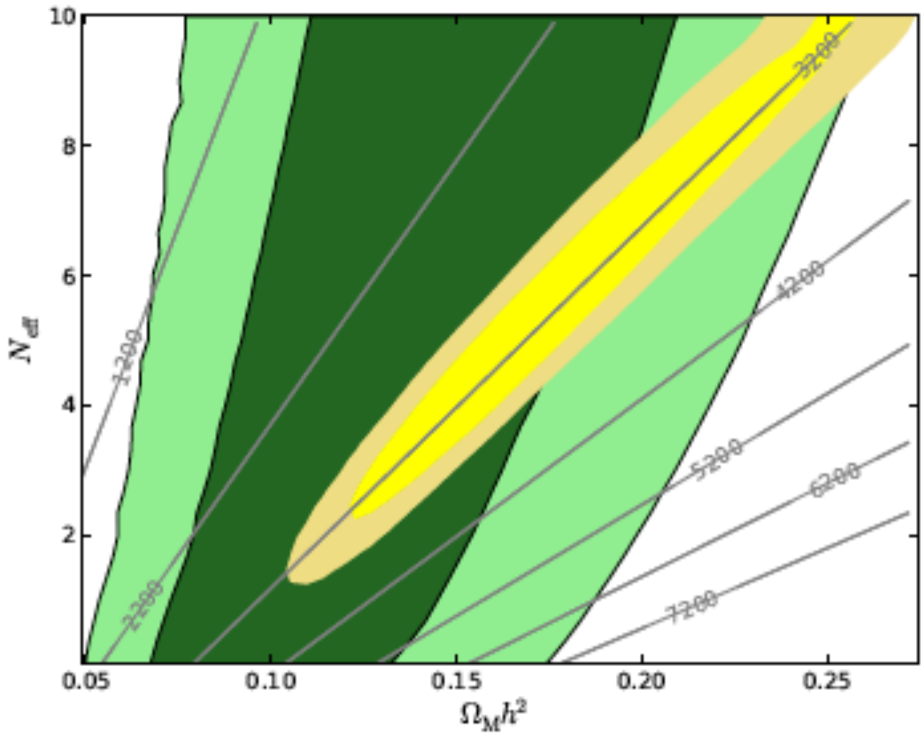

FIG. 2: Green region: Constraint on matter density and N_{eff}, the number of light neutrino species (equal to 3 in the standard model), from the turnover in the power spectrum (from `1211.5605`).

- Another important feature of power spectrum $k^3 P_\delta / 2\pi^2$, which quantifies fluctuations in the density is greater than one today on small scales. This ensures that we exist and would not have happened without DM, since we can pin down the amplitude of the baryon-photon perturbations at decoupling (of order 1 part in 10^4) and compute by how much they've grown (a factor of 1000). This is not enough to generate nonlinearities

- Both of these point to matter density 6 times larger than the baryonic content inferred, e.g., from nucleosynthesis

F. CMB

- Basic equation is the wave equation forced by gravity:

$$\ddot{T} + k^2 c_s^2 T = F[\Phi] \tag{6}$$

where the sounds speed is smaller than $c/\sqrt{3}$ because the photons are coupled to baryons.

- Acoustic oscillations: for fixed k, amplitude oscillate with time. For fixed time, amplitudes oscillate with k.
- Nature of oscillations determined by characteristics of fluid. E.g. higher baryon density leads to smaller frequency, which leads to zero-point of oscillations more displaced: more disparity between first and second peaks
- Effects of gravitational forcing sensitive to the matter density and these too leave a distinctive imprint on the oscillations.
- CMB alone measures very robustly that the matter density is 6 times larger than the baryon density

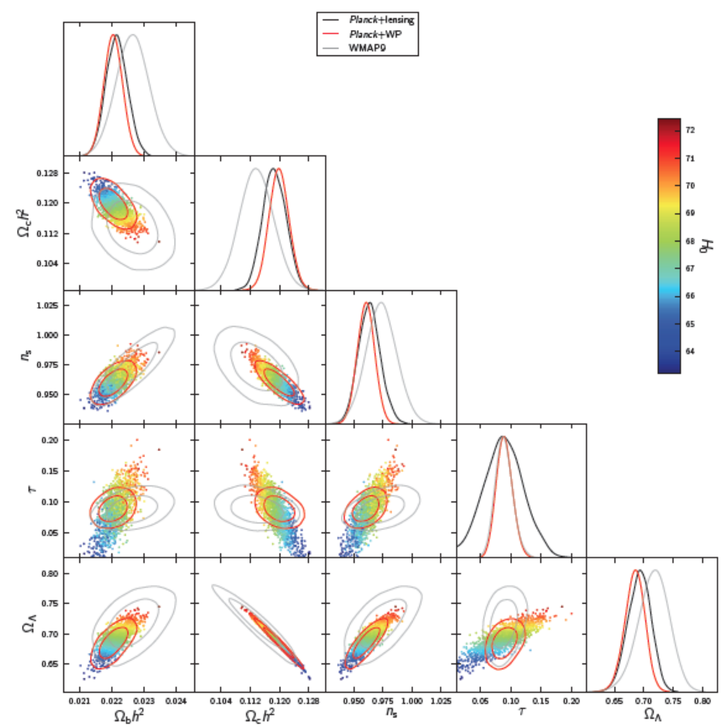

FIG. 3: Constraints from Planck CMB measurements. Notice in the second row, first column that the baryon density is constrained to be 6 times smaller than the cold dark matter density.

II. PROBLEMS

A. Satellites

- Simulations predict many low mass sub-halos
- Roughly the correct amount of massive sub-halos are seen in the MW, but the number of lighter sub-halos is too small

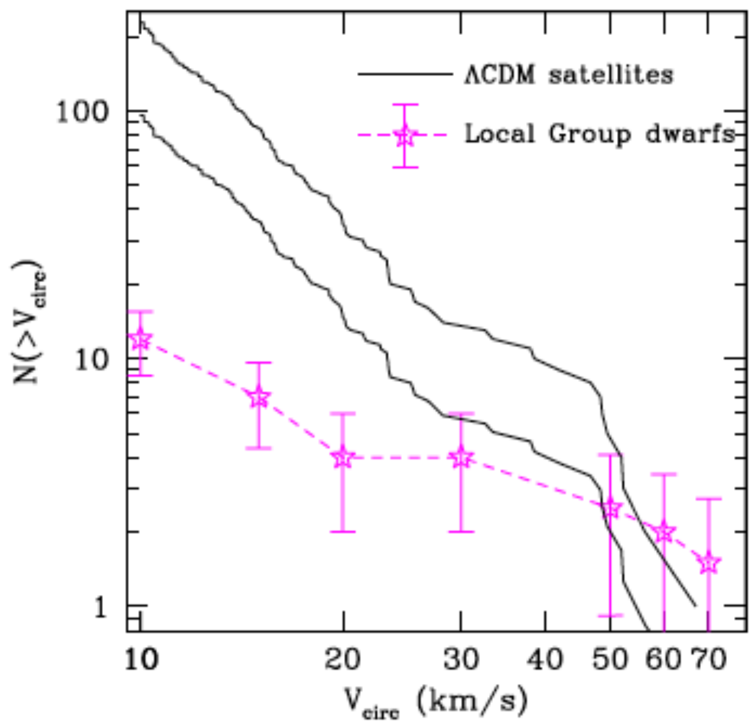

FIG. 4: Predicted vs. observed sub-halos (`0906.3295`).

- Some observational issues (measuring v_{max}; anisotropy; completeness), but the major difficulty is relating observed galaxies to predicted dark matter sub-halos

B. Cusp or Core

- simulations predict a *cusp*: $\rho \propto r^{-1}$ or $v \propto r^{1/2}$ on small scales. Variety of results over the last 15 years, but this general result still holds at least down to $r = 0.1$ kpc. Slope is much closer to -1 than to 0.

- data from dwarf galaxies well-fit assuming dark matter distribution is cored (e.g. $\rho \propto C$ as $r \to 0$), so that $v \propto r$ (at least the contribution from DM) on small scales
- could reconcile with "baryon blow-out" but require a lot of baryons to be expelled
- can check this idea in late-type Low Surface Brightness galaxies, which are much larger, therefore have deeper potential wells from which it would be much more difficult to expel baryons. Also, those found seem to be pretty quiet. Problem seems to persist, may require high resolution hydro sims

III. ALTERNATIVES: MOND

A. Rotation Curves; H_0

- Poisson usually says $\nabla^2\Phi \propto M$The gravitational force per mass in a circular orbit in Newton's theory is $a_N = v^2/r$. Suppose this changes to

$$a_N \to a_N \mu(a_N/a_0) \tag{7}$$

where

$$\mu(x) = \begin{cases} 1 & x \gg 1 \\ x & x \ll 1 \end{cases}. \tag{8}$$

So as long acceleration is greater than a_0, we get the standard result. In the small acceleration limit though,

$$\frac{a_N^2}{a_0} = \frac{MG}{r^2} \tag{9}$$

so that $v^4 = a_0 MG$, a flat rotation curve.

- This also predicts that $L \propto v^4$, a phenomenological law that is observed and difficult to explain in dark matter models.
- You can guess the typical value of a_0 by using $v \sim 200$km/sec and $r \sim 5$ kpc. Roughly a_0/c is not too different than $H_0 = 70$km/sec/Mpc.
- This should impress you: there are also modified gravity models that explain the acceleration of the universe. All of these have a fundamental parameter in the Lagrangian of order $H_0 \sim 10^{-33}$eV. This could be coincidence or a hint that there is new physics at this IR scale.

B. Scalar Tensor Models

The first few pages of Bekenstein's paper (0403694) give an excellent review of twenty years of work in this field.

- MOND is not a fully relativistic theory, from which predictions can be made.
- GR implicitly equates the metric that characterizes the curvature of space-time (the one that appears in the Einstein-Hilbert action) and the one that dictates how particles move. Scalar-tensor models differentiate between the two metrics:

$$\begin{aligned} S_{EH} &= \frac{1}{16\pi G}\int d^4x\sqrt{-g}R(g) \\ S_m &= \int d^4x\sqrt{-\tilde{g}}L_m \end{aligned} \tag{10}$$

  In scalar tensor models, the two are related via $g_{\mu\nu} = e^{2\phi/m_{Planck}}\tilde{g}_{\mu\nu}$.

- A scalar-tensor model is defined by the action for the scalar field. The simplest way to implement MOND is to choose the Lagrangian for ϕ to be

$$L_\phi = \frac{a_0^2}{8\pi G} f\left(\frac{8\pi G}{a_0^2}\partial_\mu\phi\partial^\mu\phi\right) \tag{11}$$

  A suitably chosen f reduces to MOND.

C. Lensing

- Photons behave the same in scalar-tensor models as they do in GR. Proof:

$$0 = ds^2 = g_{\mu\nu}dx^\mu dx^\nu \rightarrow_{S.T.} e^{2\phi/m_{\text{Planck}}} g_{\mu\nu}dx^\mu dx^\nu \tag{12}$$

- All lensing is done by baryons and there are not enough baryons in galaxies or clusters to account for observed lensing.

D. TeVeS

Introduce another new field, a vector field, to break the theorem by setting

$$\tilde{g}_{\mu\nu} = e^{-2\phi/m_{\text{Planck}}}\left[g_{\mu\nu}dx^\mu dx^\nu + 8\pi G A_\mu A_\nu\right] + 8\pi G A_\mu A_\nu e^{2\phi/m_{\text{Planck}}} \tag{13}$$

The vector field gives extra lensing.

E. Growth of Structure

Vector field, which was introduced to solve the problem of lensing, drives enhanced growth of structure (0608602). This is necessary in order to get from inhomogeneities observed at the time of the CMB to the nonlinear level.

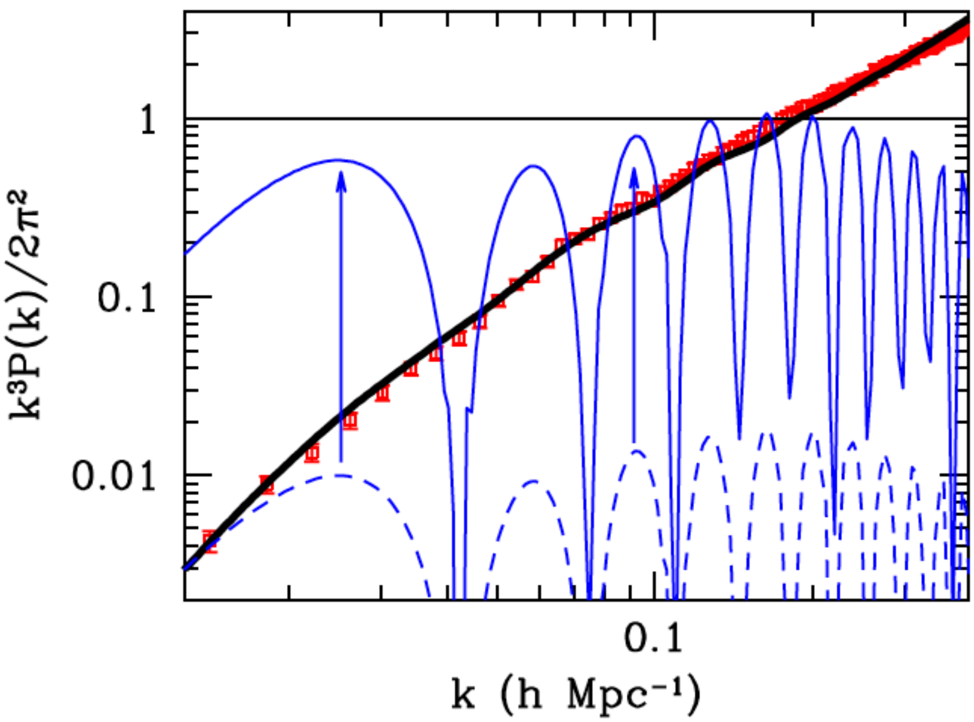

FIG. 5: Dimensionless measure of clumpiness. Data points shown in red. No-dark-matter model with normal growth of structure is dashed blue line. The universe remains linear and no interesting structures (including us) can form. The solid blue line shows that TeVeS, the model with an additional vector field, enhances the growth of structure so that the Universe can go nonlinear. From 1112.1320, which is short and readable.

F. Baryon Acoustic Oscillations

Note in Fig. 5 that, although the amplitude of the power spectrum is ok, the shape is decidedly not. There are very large acoustic oscillations. These are present in the baryons in dark matter models but only at the few percent level (because dark matter dominates the gravitational potentials). This prediction of huge order one oscillations in the spectrum is a big problem for any no-dark-matter model, including TeVeS.

Appendix A: Some cosmological facts

- Mpc $= 3 \times 10^{24}$ cm
- FRW metric: $ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2$
- Hubble expansion rate: $H = \dot{a}/a$
- Hubble rate today is called the *Hubble constant* and is measured to be about 70 km/sec/Mpc, sometimes written as $H_0 = 100h$ km/sec/Mpc
- Friedmann equation: $H^2 = 8\pi G\rho/3$ where ρ counts all the contributions to the energy density
- Energy census in units of critical density: Species i today contributes a fraction $\Omega_i \equiv \rho_i/\rho_{\rm cr}$, with $\rho_{\rm cr} \equiv 3H_0^2/(8\pi G) = 4 \times 10^{-11}\,{\rm eV}^4$
- Photon density: $\Omega_\gamma = 1.2 \times 10^{-5}$
- Baryon density: $\Omega_b = 0.05$
- Matter density: $\Omega_m = 0.26$
- Total density: $\Omega = 1$
- $a_{\rm EQ} = 4.15 \times 10^{-5}/(\Omega_m h^2) = 2.91 \times 10^{-4}$