Multistage rectilinear 6D cooling channel with 325 MHz RF and final LiH absorbers

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Considered 4-stages channel of length 450 m provides the cooling:

Transverse emittance – from 20 mm to 0.31 mm

Longitudinal emittance – from 20 mm to 1.5 mm

Transmission – 90% without decay, 62% with decay

Required magnetic field is accessible for NbTi - NbSn technology.

Bench-marks

Lithium hydride absorbers have to be applied in the last stages

because liquid hydrogen absorbers are incompatible with ultimately low beta-function

- Liquid hydrogen provides rather low decelerating gradient (~30 MeV/m at P = 200 MeV/c).
- Therefore, LH_2 wedge absorbers would be rather long (L = 0.3 − 0.4 m).
- It results in increase of average beta in the absorber which could be admissible only for earlier stages .

$$\beta_{ave} \approx \beta^* + \frac{L^2}{12\beta^*} \geq \frac{L}{\sqrt{3}}$$

- Additionally, LH_2 wedge absorber would have very large opening angle (150º or more).

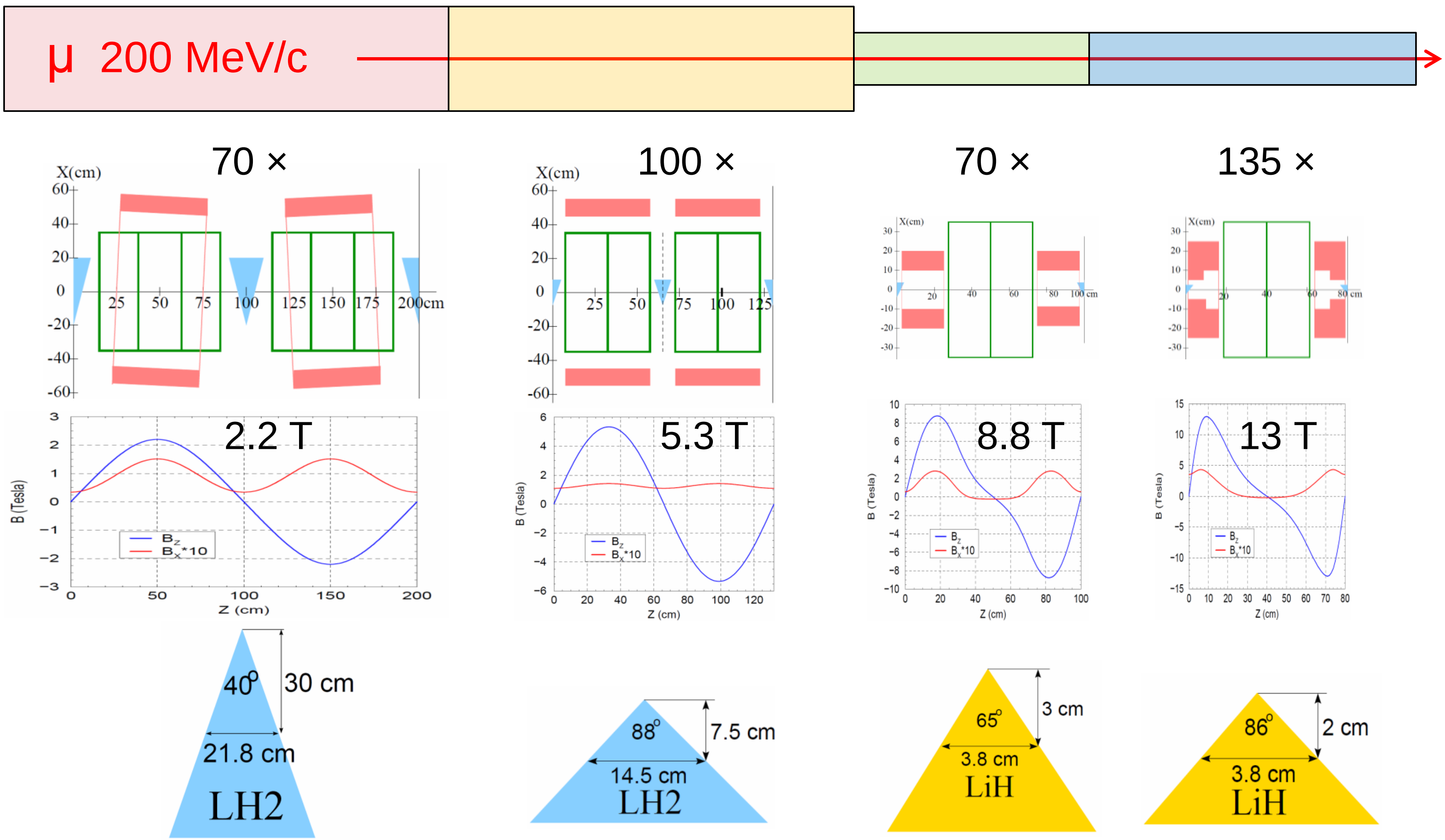
Unified RF (325 MHz througout) is preferable for rectilinear 6D cooling channel

- Less radius of higher RF cavities looks immaterially when cavities are placed out of solenoid coils which allocation is very characteristic for last stages of the rectilinear channel.
- Higher frequency → shorter separatrix → shorter bunch → higher momentum spread.
- The last point falls into a contradiction with a restricted momentum acceptance which is the main cause of the particle losses (it will be shown).

Required magnetic field should be reachable with NbSn superconductor

General view of the channel:

450 m long, ε_{trans} = 20→0.31 mm, ε_{long} = 20→1.5 mm, transmission 90%/62%

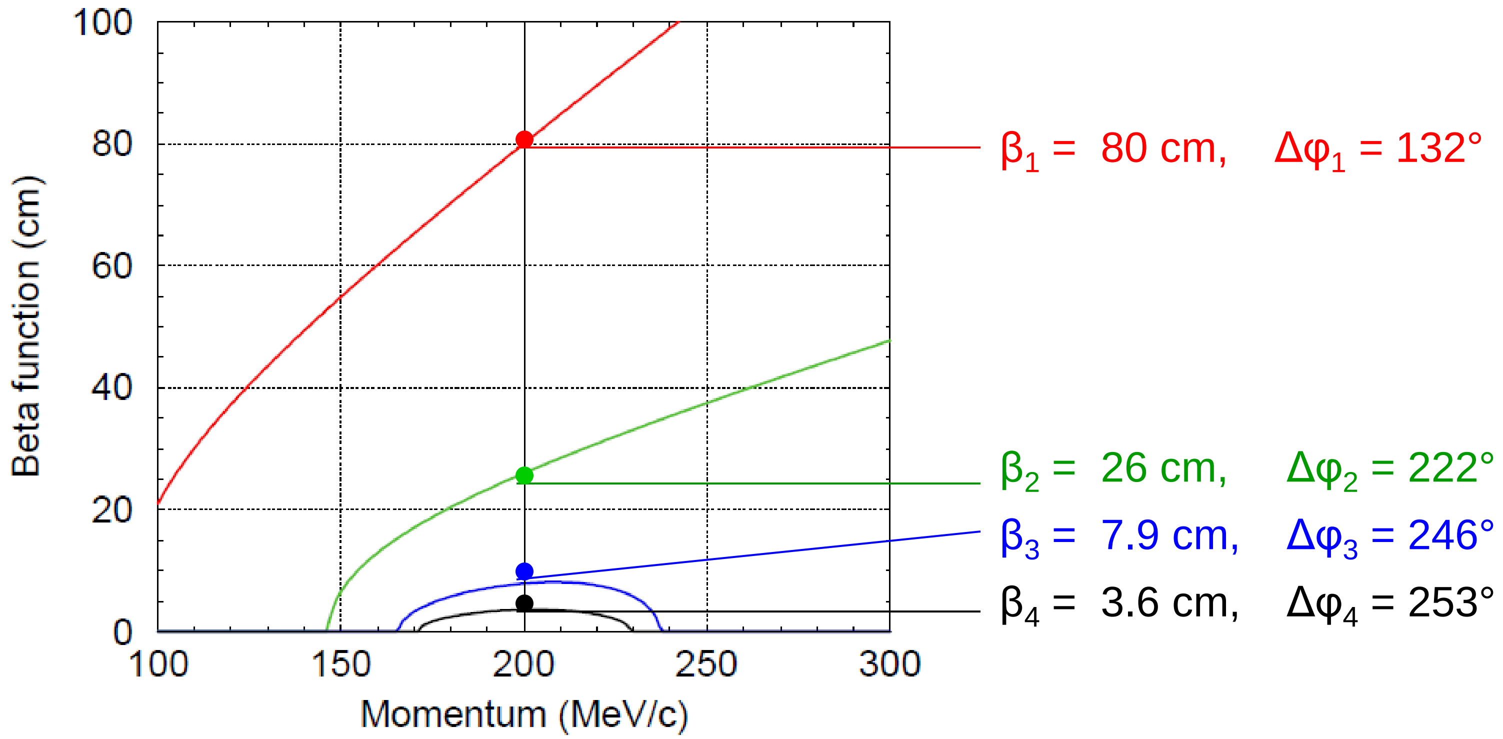


Cell parameters of the stages

Ref. momentum 200 MeV/c, RF 325 MHz -- 25 MV/m everywhere

Parameter	Units	Stage 1	Stage 2	Stage 3	Stage 4
Cell length	cm	200	132	100	80
Coil length	cm	50	50	24	16
Coil inner radius	cm	45	45	10	10&5
Coil thickness	cm	10	10	10	15&20
Coil tilt	mrad	±60	±15	±30	± 20
Current density	**A/mm^2**	**48.3**	**175**	**123**	**185**
Maximal field strength in coil	**T**	**3.73**	**12.3**	**10.1**	**15.6**
Synchronous phase	deg	23	23	44	44
LH_2 absorber center thickness	**cm**	**21.8**	**14.5**	~~21~~	~~21~~
Absorber opening angle	**deg**	**40**	**88**	~~148~~	~~158~~
LiH absorber center thickness	**cm**	3.9	2.6	**3.8**	**3.8**
Absorber opening angle	**deg**	7.4	20	**65**	**86**

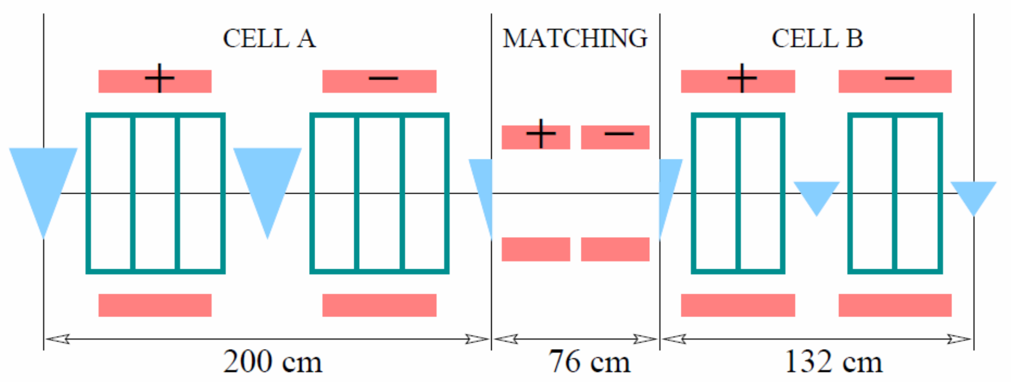
Beta-function against particle momentum in the stages



- Cells of 1st and 2nd stages have symmetrically arranged coils.
  They provide large momentum acceptance but should have coils of big radius for cavities.
- Coils of 3rd and 4th stages have small radius and asymmetrical arrangement providing ultimately small beta-function and a lot of room for cavities between the coils. However, its momentum acceptance is rather small being bounded by strong π and 2π resonances.

Matching of 1st and 2nd stages is effected by 90º FODO cell

Joining of the sub-stages

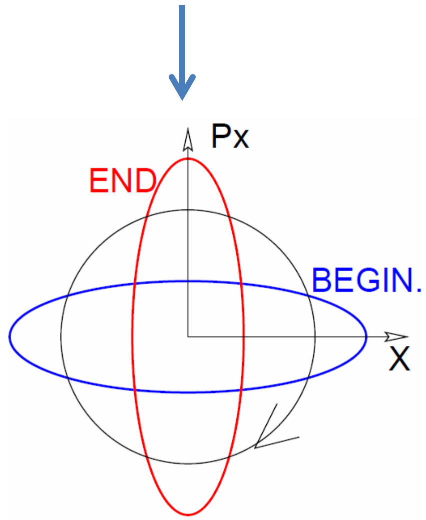


Matching section

Betatron phase advance 90º

Inherent beta 47 cm ≈ $(\beta_A\beta_B)^{1/2}$

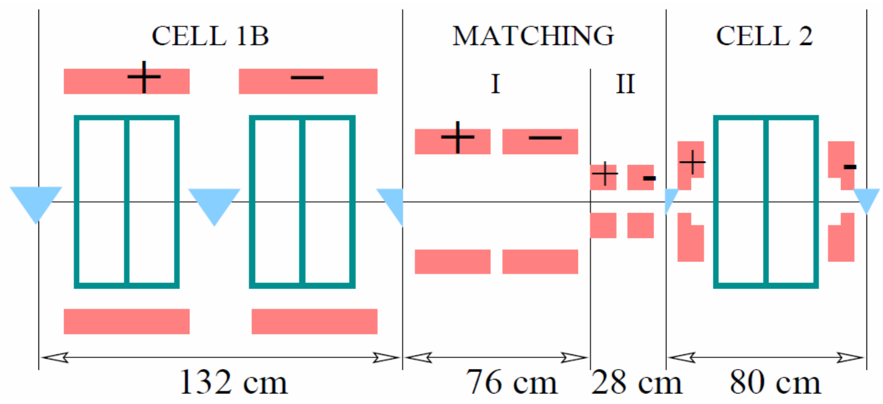
Phase space transformation



Parameters of the matching section

Section length	76 cm
Coil length	30 cm
Coil inner radius	20 cm
Coil thickness	10 cm
Current density	95.2 A/mm^2
Axial field	3.85 T
Coil field	6.47 T

Design of these matching section is in a progress. The schematic is supposed which has been successfully used for previous version of the channel.



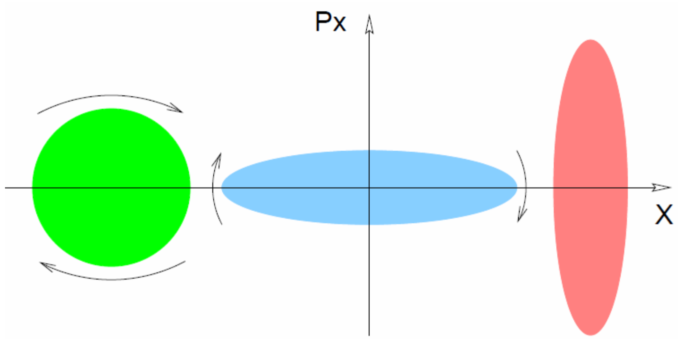
Matching section (200 MeV/c):

Two-parts system is used to decrease maximal field and chromatic effects

Inherent betas of the parts are 47 cm and 17 cm

Betatron phase advances are 90º

Phase space transformations as shown



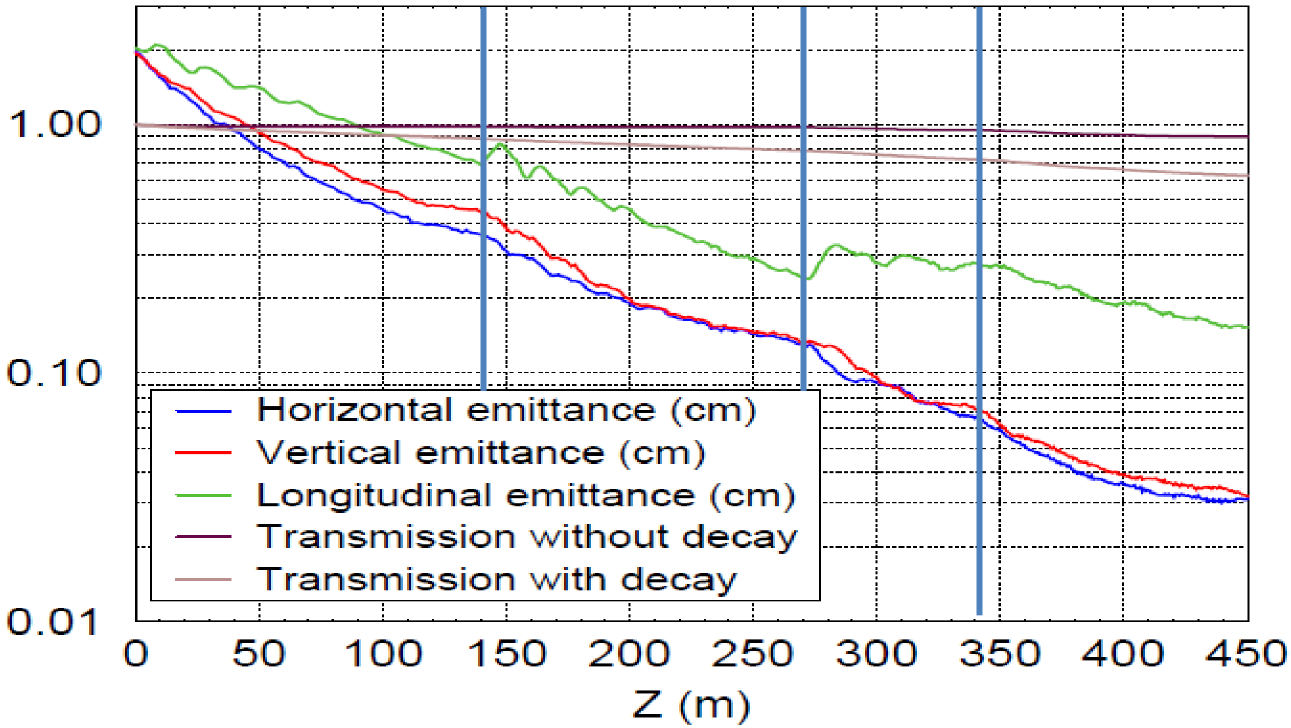
Single 90° cell would require magnetic field up to 20 T
Single 270° cell would require a modest field but creates unacceptable chromatiity.

Parameters of the matching section

	Unit	I	II
Section length	cm	76	28
Coil length	cm	30	13
Coil inner radius	cm	20	5
Coil thickness	cm	10	10
Current density	A/mm^2	95.2	254
Axial field	T	3.85	10.3
Coil field	T	6.47	13.1

Cooling with self-consistent initial distribution, and matching section or matrices between the stages

1st and 2nd stages with LH_2 absorbers, 3rd and 4th − with LiH



Tr_1 = 90%

Tr_2 = 62%

E_{long} = 1.5 mm

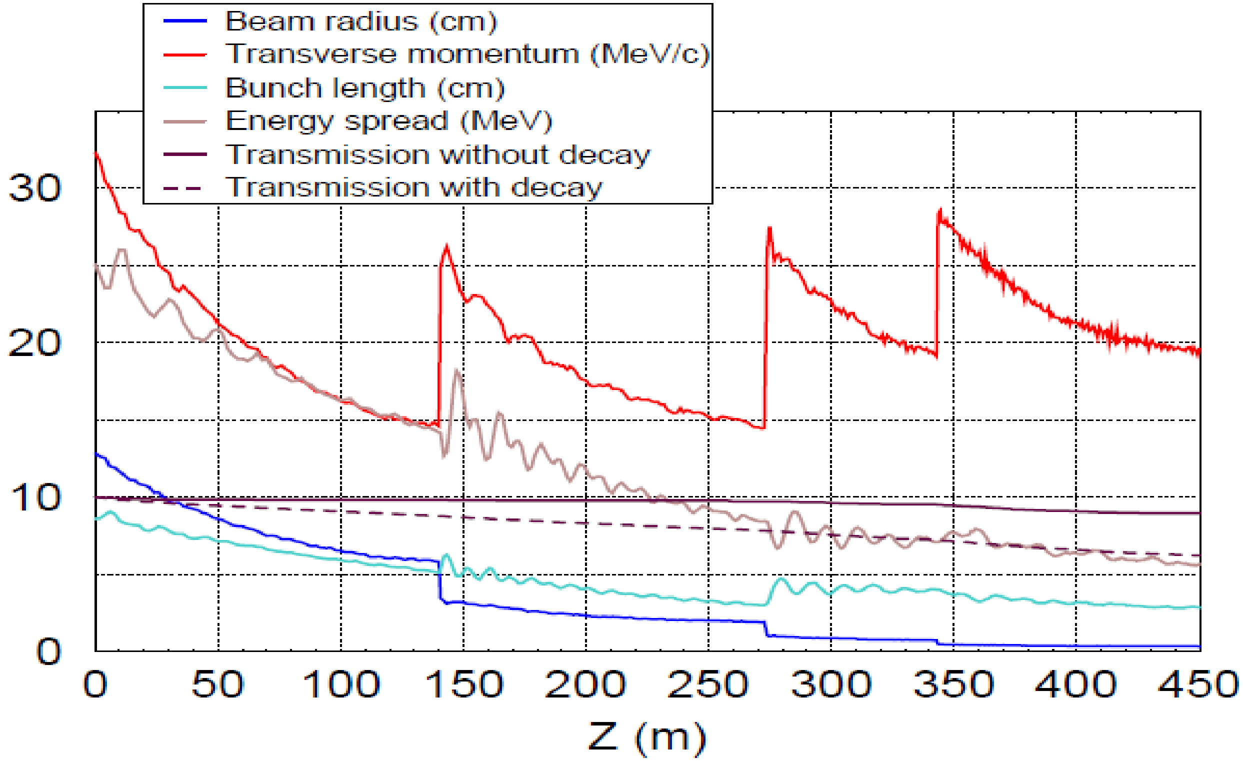
E_{trans}= 0.31 mm

Longitudinal emittance increases at the transition from each stage to next one.

The effect is caused by longitudinal − transverse correlations (nonlinear)

which cannot be controlled and corrected by the (linear) matching sections.

Beam size evolution at the cooling



The beam transverse momentum steeply rises between the stages.

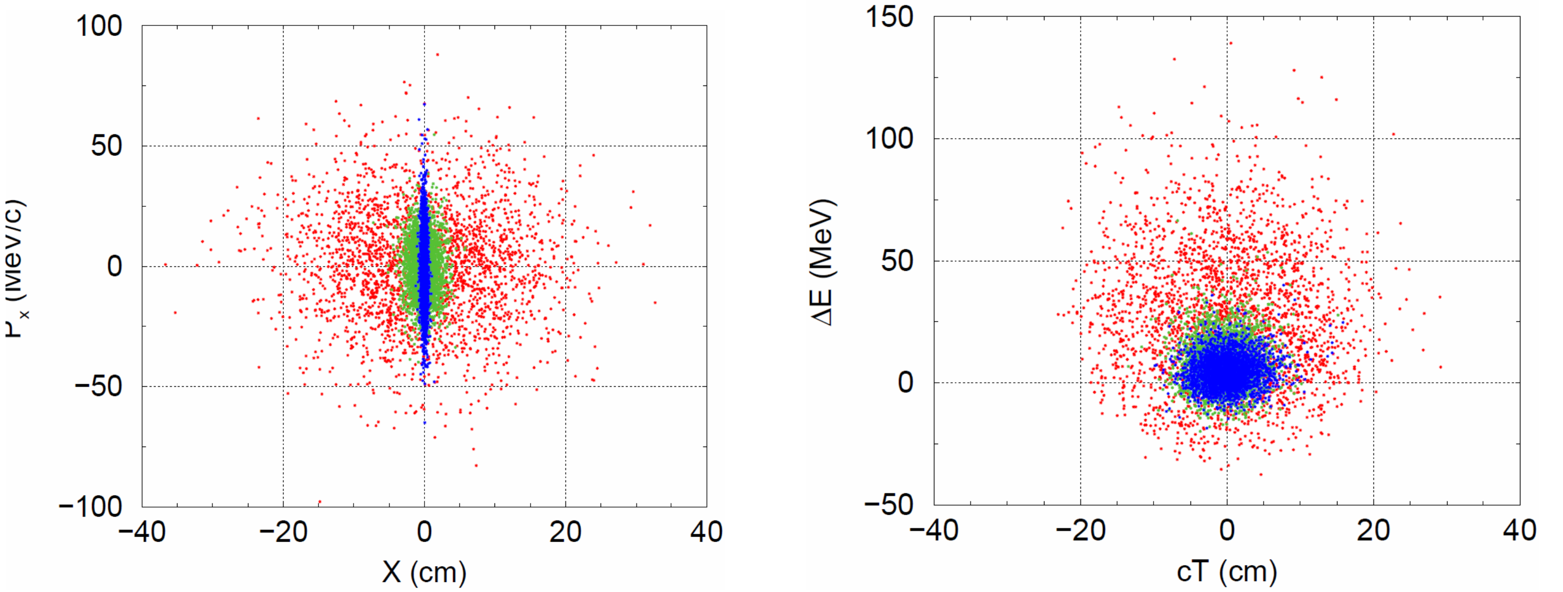
It causes a perturbation of the beam energy spread (correlations!)

and results in increase the longitudinal emittance.

However, the effect is not very strong in the four-stages channel,

so division of the stages on parts or adding of new stages seems to be not needed.

Transverse $X-P_x$ space Longitudinal $cT-\Delta E$ space



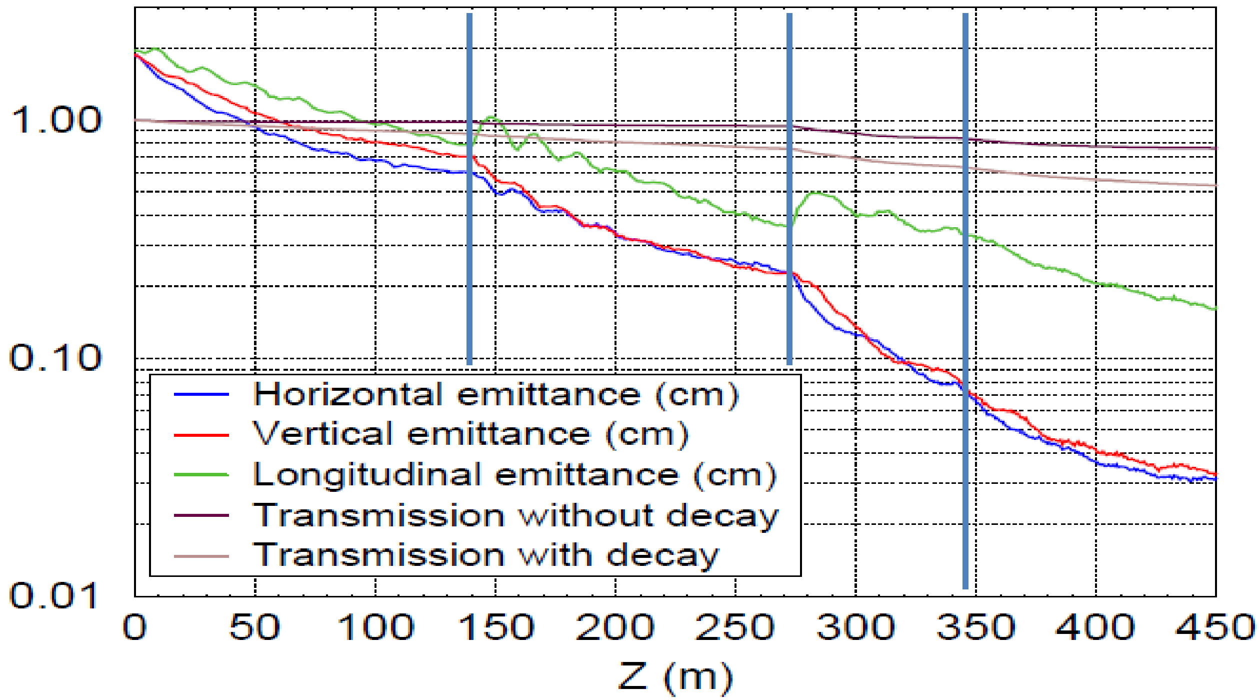
Red – injected beam, green – after 2nd stage, blue – at the end of the channel.

Longitudinal phase space is non-Gaussian because synchronous energy depends on betatron amplitude. The dependence is too complicated to control it.

Cooling by the same channel with LiH absorbers throughout

LH_2 absorbers are replaced by LiH ones in 1st and 2nd stages

to get the same energy loss. The wedge angles are decreased correspondingly.



Tr_1 = 76 %

Tr_2 = 52%

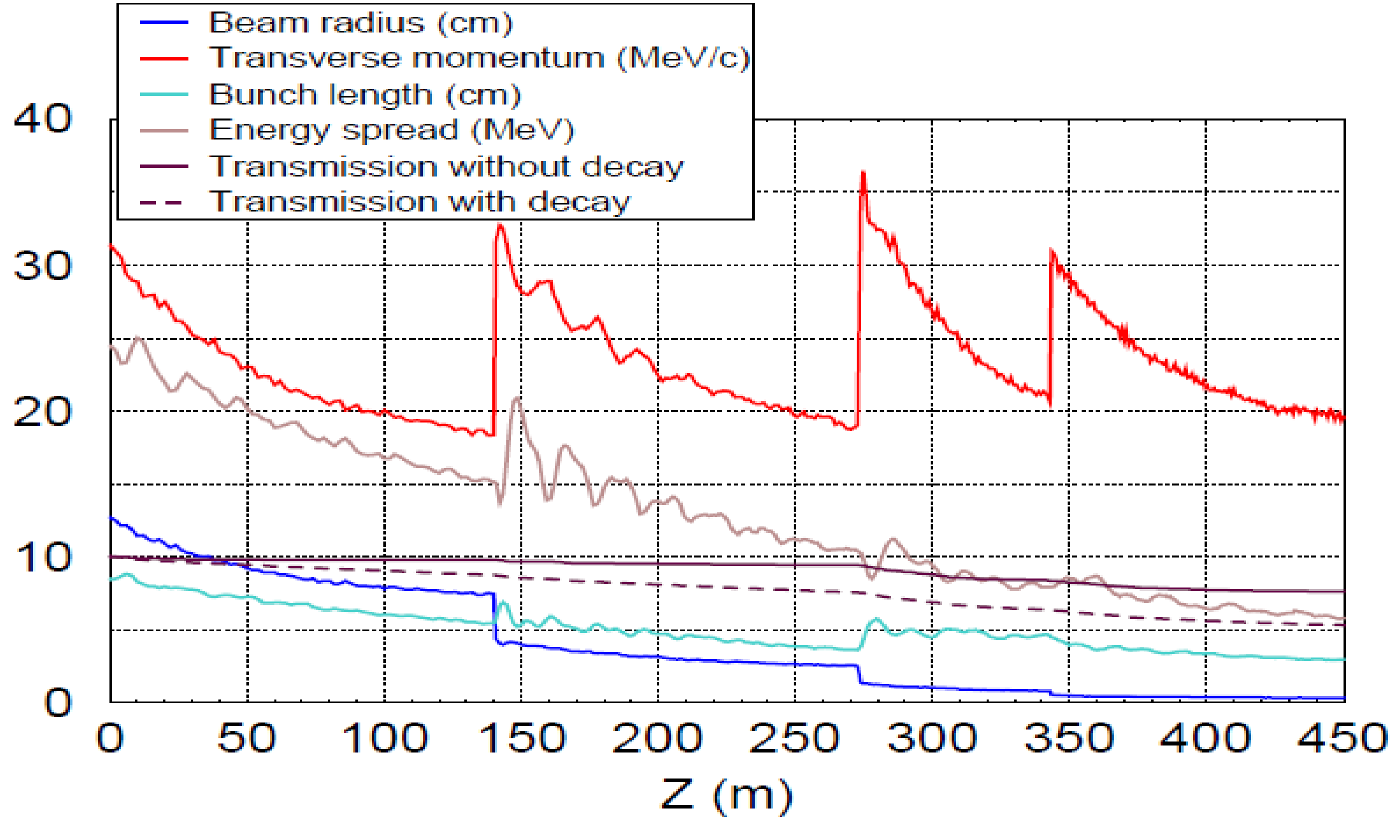
E_{long} = 1.6 mm

E_{trans}= 0.31 mm

Emittance of the cooled beam is actually the same as before the replacement

but transmission has fallen: 90% → 76% without decay, and 62% → 52% with decay.

Cooling by LiH absorbers – beam size evolution



- Transverse momentum increases in each matching but especially in the beginning of 3rd stage.
- Probably, that is the reason of higher particle loss starting from 270 m.
- Possibly, a lengthening of 2nd stage will help because an equilibrium is not reached there (though it increase decay loss).

How much collider luminosity increases through the cooling?

Collider luminosity

$$L \propto \frac{N^2}{(\sigma_x \sigma_y)_{ave}} \propto \frac{N^2}{\sqrt{\varepsilon_x \varepsilon_y (\beta_x \beta_y)_{ave}}}$$

Averaged β depends on β*, σ_z, and distribution
but in optimal case $\beta_{ave} \propto \sigma_z$ that is

$$L \propto \frac{N^2}{\sqrt{\varepsilon_x \varepsilon_y \sigma_z}}$$

With given longitudinal beta-function,

$$\sigma_z = \sqrt{\beta_z \varepsilon_z} \propto \sqrt{\varepsilon_{long}}$$ (case A)

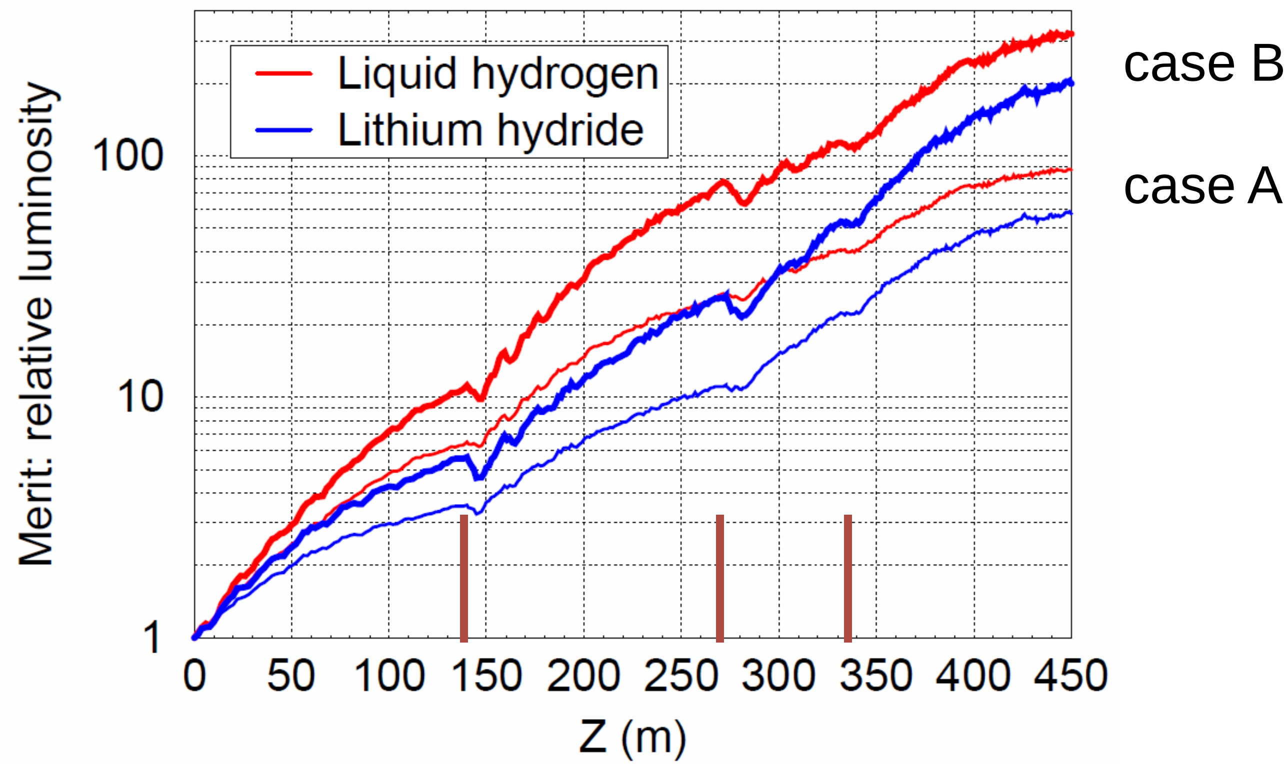
$$L_A \propto \frac{N^2}{\sqrt{\varepsilon_x \varepsilon_y \varepsilon_z}} \propto \frac{(Transmission)^2}{\sqrt{\varepsilon_{6D}}}$$

With given momentum spread,

$$\sigma_z = \varepsilon_z / \sigma_p \propto \varepsilon_{long}$$ (case B)

$$L_B \propto \frac{N^2}{\sqrt{\varepsilon_x \varepsilon_y \varepsilon_z}} \propto \frac{(Transmission)^2}{\varepsilon_{trans} \varepsilon_{long}}$$

Luminosity merit factors



Higher merit in case B does not mean a higher luminosity
but it means higher sensitivity of the luminosity to the beam emittance.

Conversely, the luminosity is, typically, lower in this case B
because used bunch size is larger than it would be possible technically.

Summary and Conclusion

- Multistage rectilinear channel can be applied for ultimate 6D cooling
- Using of a unified RF system is possible and expedient
- Lithium hydride absorbers have to be applied in the last stages whereas LH_2 absorbers are incompatible with ultimately low beta-function
- Based on these principles, 4-stages channel is designed having the parameters: length 450 m, RF 325 MHz, absorbers LH2 and LiH, and provides the cooling:

  transverse emittance – from 20 mm to 0.31 mm

  longitudinal emittance – from 20 mm to 1.5 mm

  transmission – 90% without decay, 62% with decay

- The channel merely with LiH absorbers provides the same emittances but less transmission: 76% without decay, 52% with decay.
- Required magnetic field is accessible for NbTi - NbSn technology.
- Luminosity merit factor is discussible.