Transient Response of a pill box cavity to a short beam pulse

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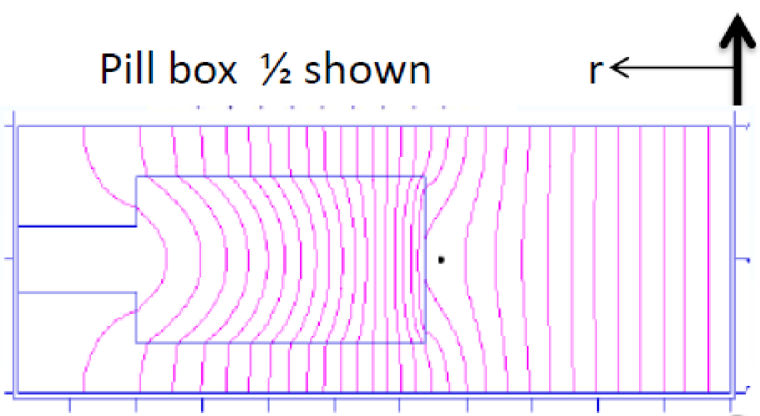
Data from simulation by Frank

- A dielectric loaded cavity with:

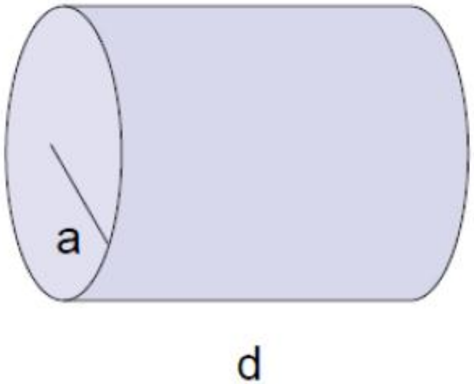
  f = 650 MHz

  Beam pulse: gaussian line charge 1 pC sigma = 1cm with v = c was sent thru the cavity on center. Cavity response was calculated for 6 cycles and Ez recorded on axis every 5 ps.



Cavity h = 2.73 cm r=12 cm ?

TM_{nmp} modes



$$E_z \sim (A\cos n\phi + B\sin n\phi) J_n(k_c\rho)\cos\left(\frac{p\pi}{d}z\right)$$

$$E_\rho \sim (A\cos n\phi + B\sin n\phi) J_n'(k_c\rho)\sin\left(\frac{p\pi}{d}z\right)$$

$$E_\phi \sim (A\cos n\phi - B\sin n\phi) J_n(k_c\rho)\sin\left(\frac{p\pi}{d}z\right)$$

$J_n(k_{nm}a) = 0$

$p = 0,1,2,3\ldots$

From boundary conditions.
p begins at 0.

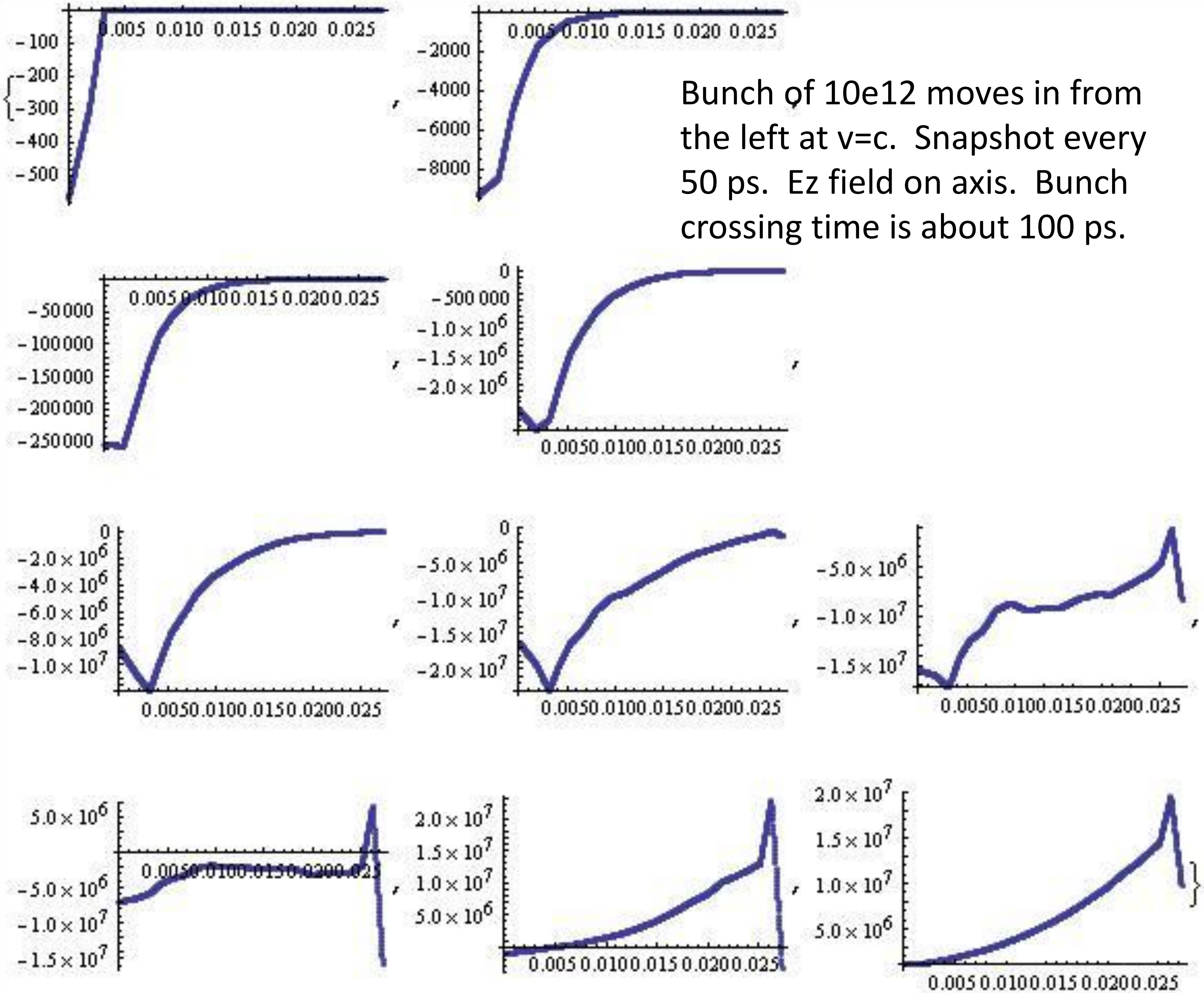
Eq. for freq of mode:

$2\,\pi/\lambda = \text{Sqrt}[(\,b_m/a)^2 - (\pi p/d)^2\,]$

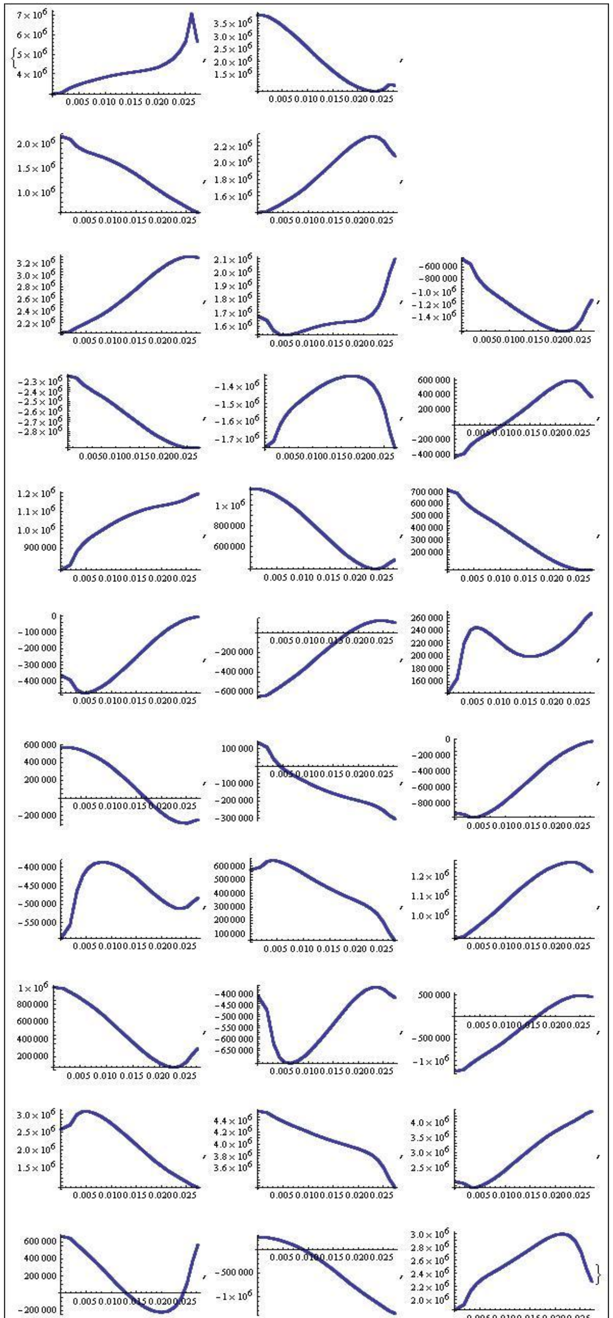
b_m Bessel roots

p = 0 means E_r and $E_\rho = 0$!!!
And cannot be excited with connector on the sides!

First TM cavity mode "usually" is TM_{011}.

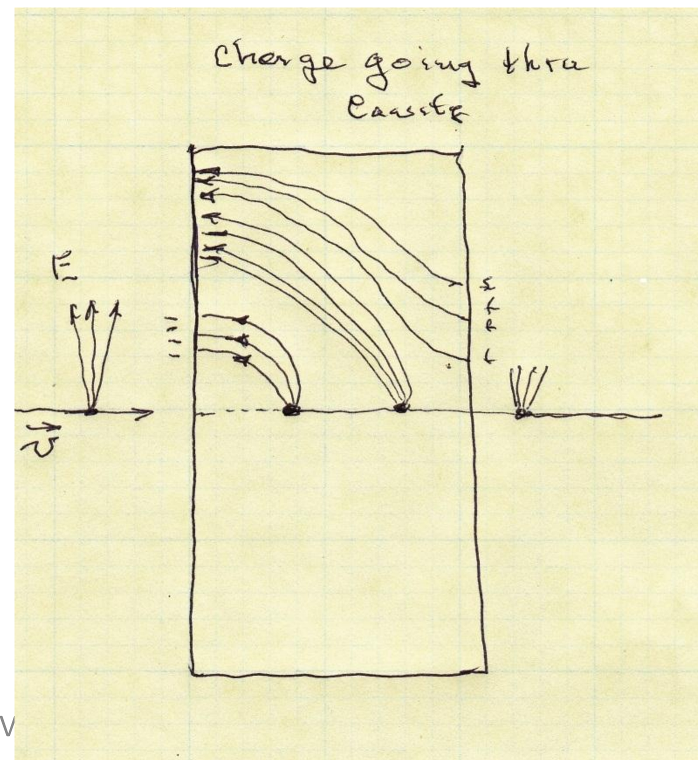


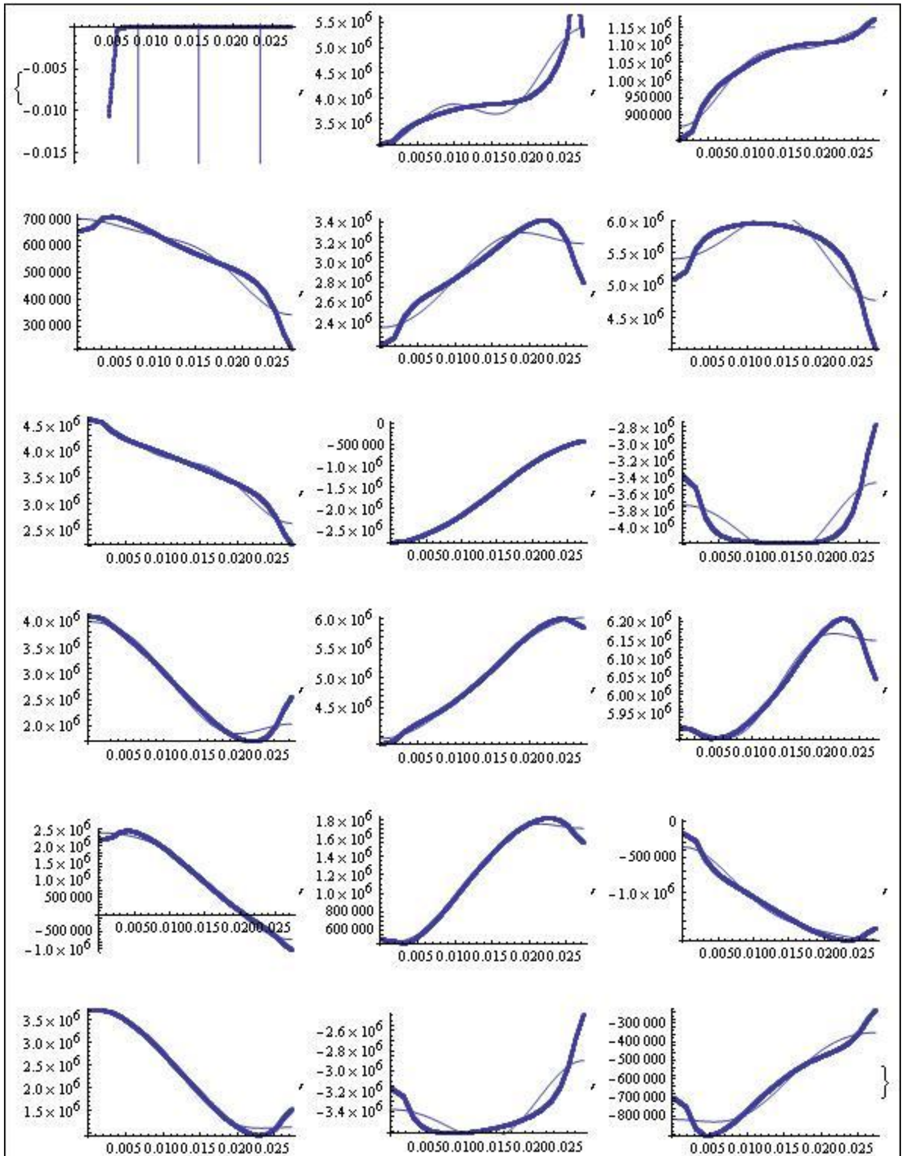
Bunch of 10e12 moves in from the left at v=c. Snapshot every 50 ps. Ez field on axis. Bunch crossing time is about 100 ps.



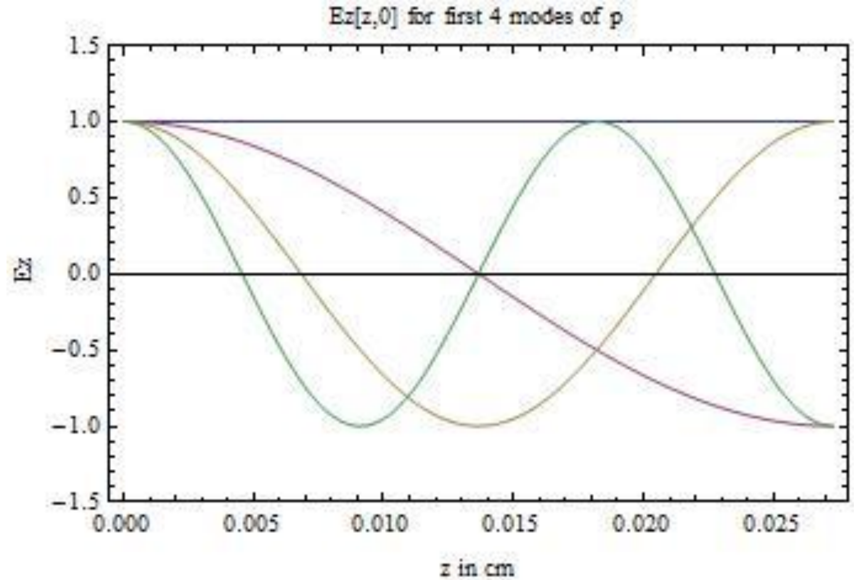
Ez[z] snapshots over 6 cycles of 650 MHz.

Ez[z,0] = Constant Cos[pπz/d]   p={0,1,2,3} which leads to a high frequency modes. p=0 is the normal mode we use for acceleration. The plots at right for the fundamental mode would be constant horizontal lines at heights given by Sin[ωt] if only the p=0 mode was excited. The z variation shows directly that the higher p modes were excited.



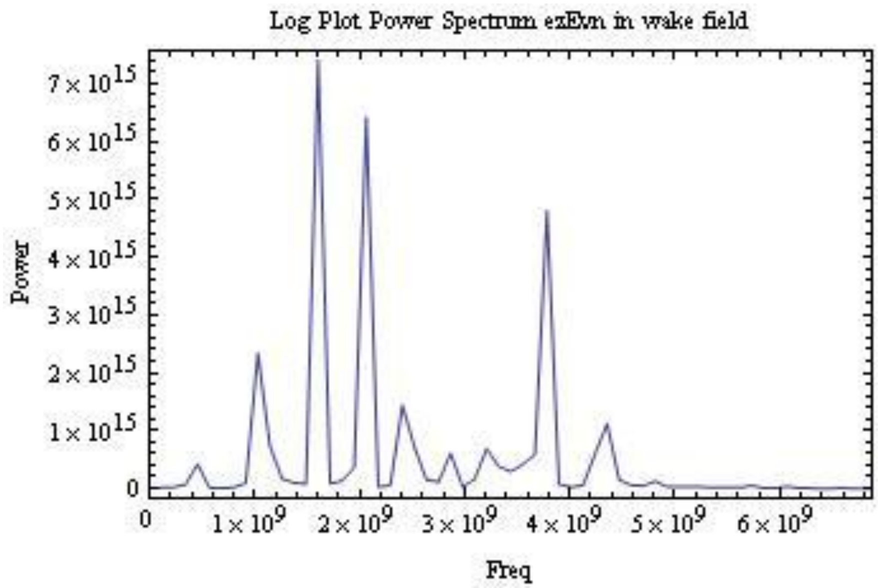


Ez vs z fits every 500 ns using
4 terms of Cos[p Pi z/d],{p= 0,1,2,3}



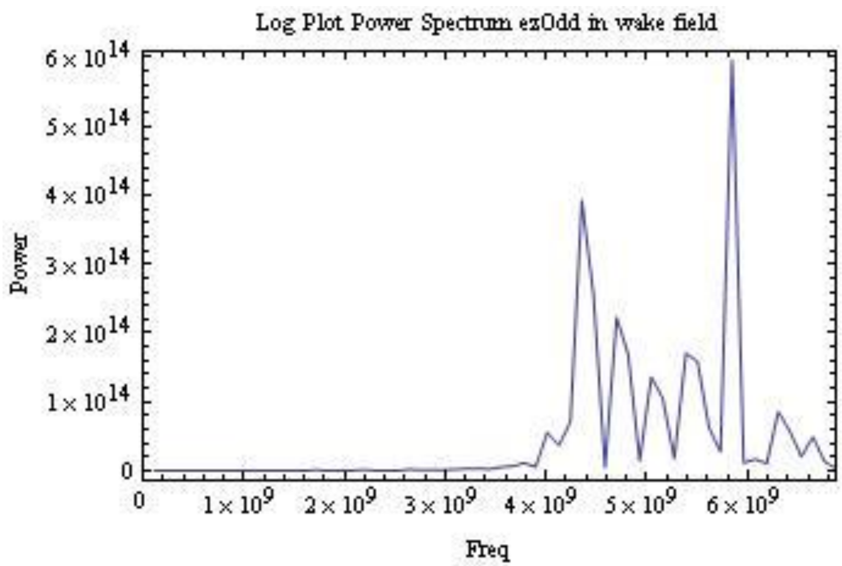
The light solid line shows the results of fitting with the above set of functions.

Note: the modes 1, 3 are odd and
modes 0, 2 re even.



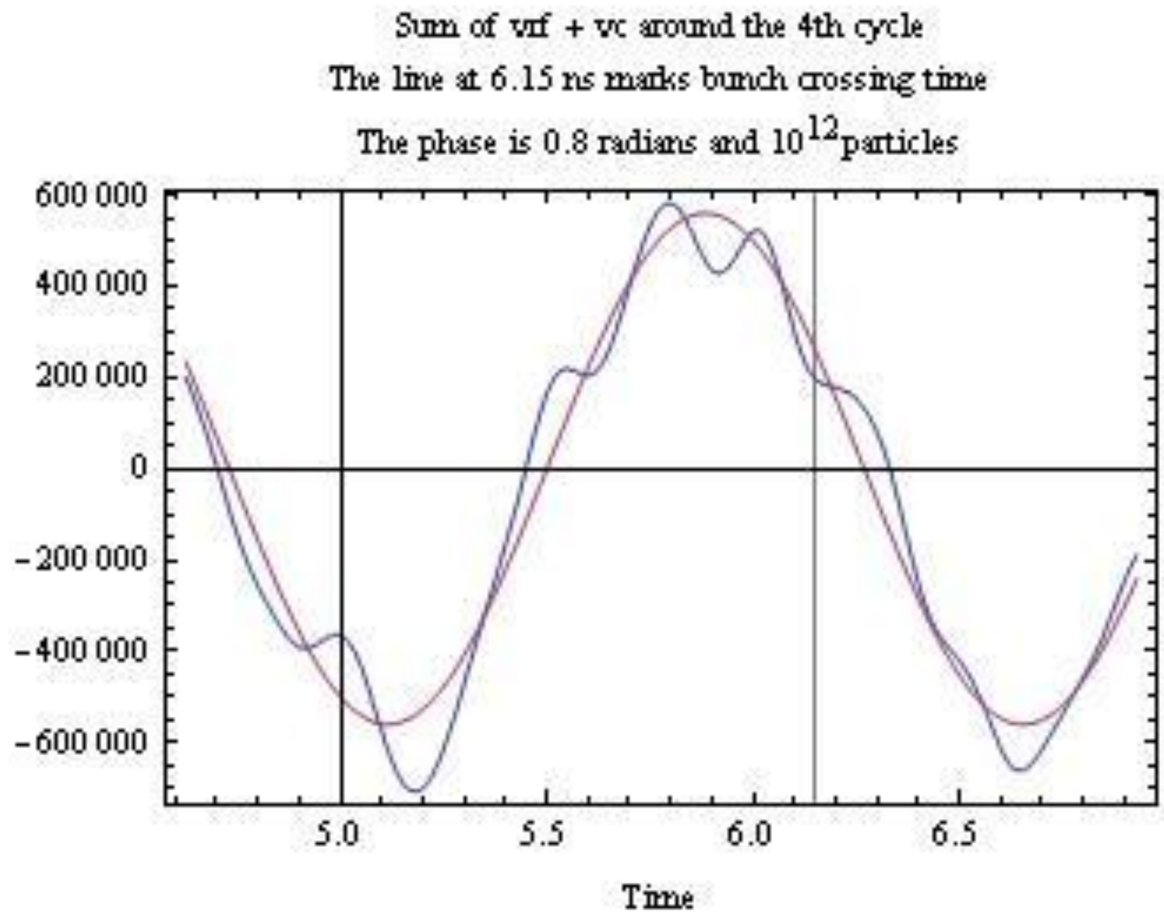
Results from analysis of simulation.

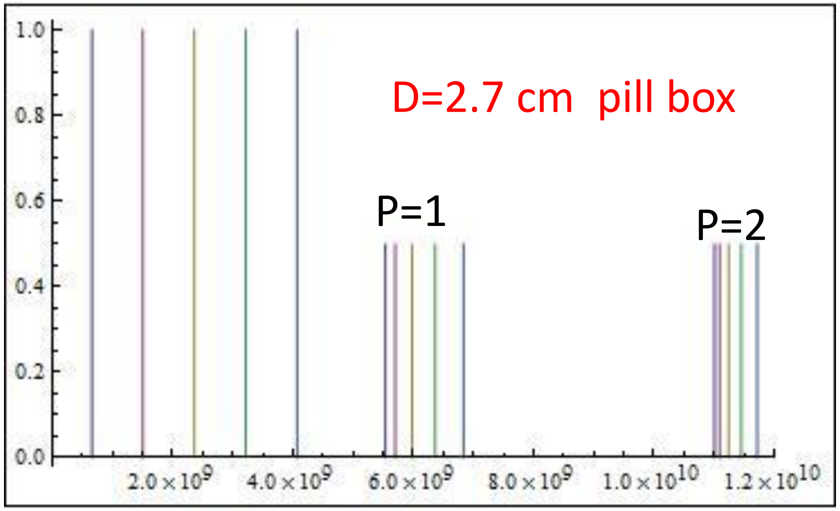
Even modes



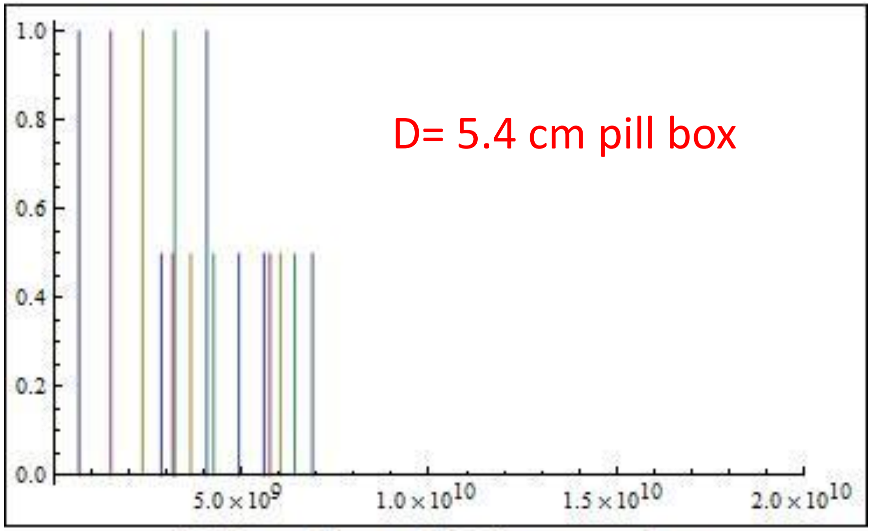
Odd modes

The plot below shows the total voltage across the cavity 4 cycles after a bunch of 10e12 particles pass thru. The voltage of the wake field was determined by integrating Ez[z,0] across the cavity. This is the voltage that the second bunch in the train would see. After 10 bunches pass thru the cavity the 11 bunch would see the sum of the previous 10 wake fields. They are not harmonically related and so one might guess that the answer would be about Sqrt[10] worse.

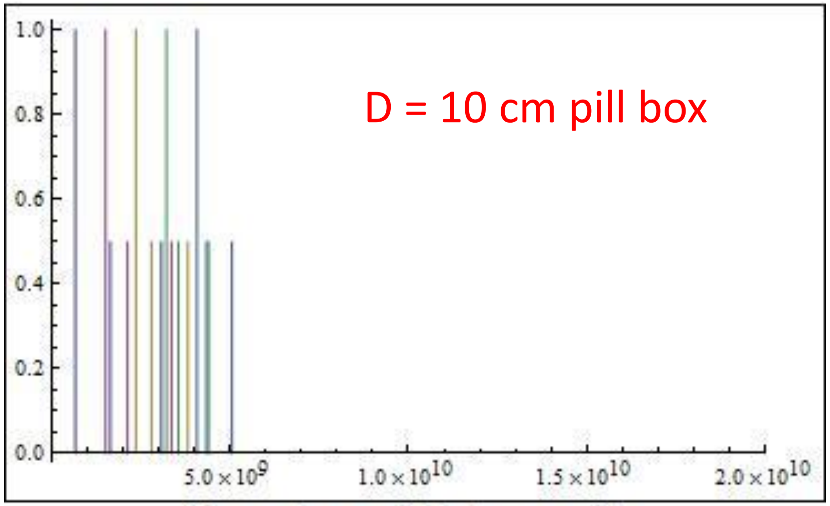




2.73 cm long pill box cavity
Showing n=0 with {k=1,2,3,4,5} and p= 0,1,2



5.4 cm long plii box cavity



10 cm long pill box cavity

Example spectrum of pill box cavities of different length. For longer cavities, the z dependent modes move down to lower frequencies.

Bunch-gas interaction: some thoughts

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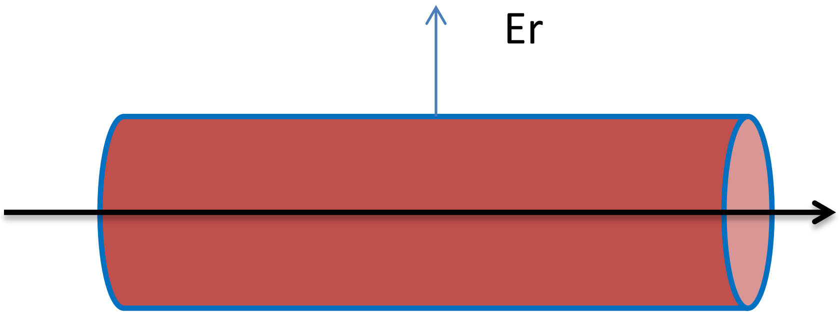
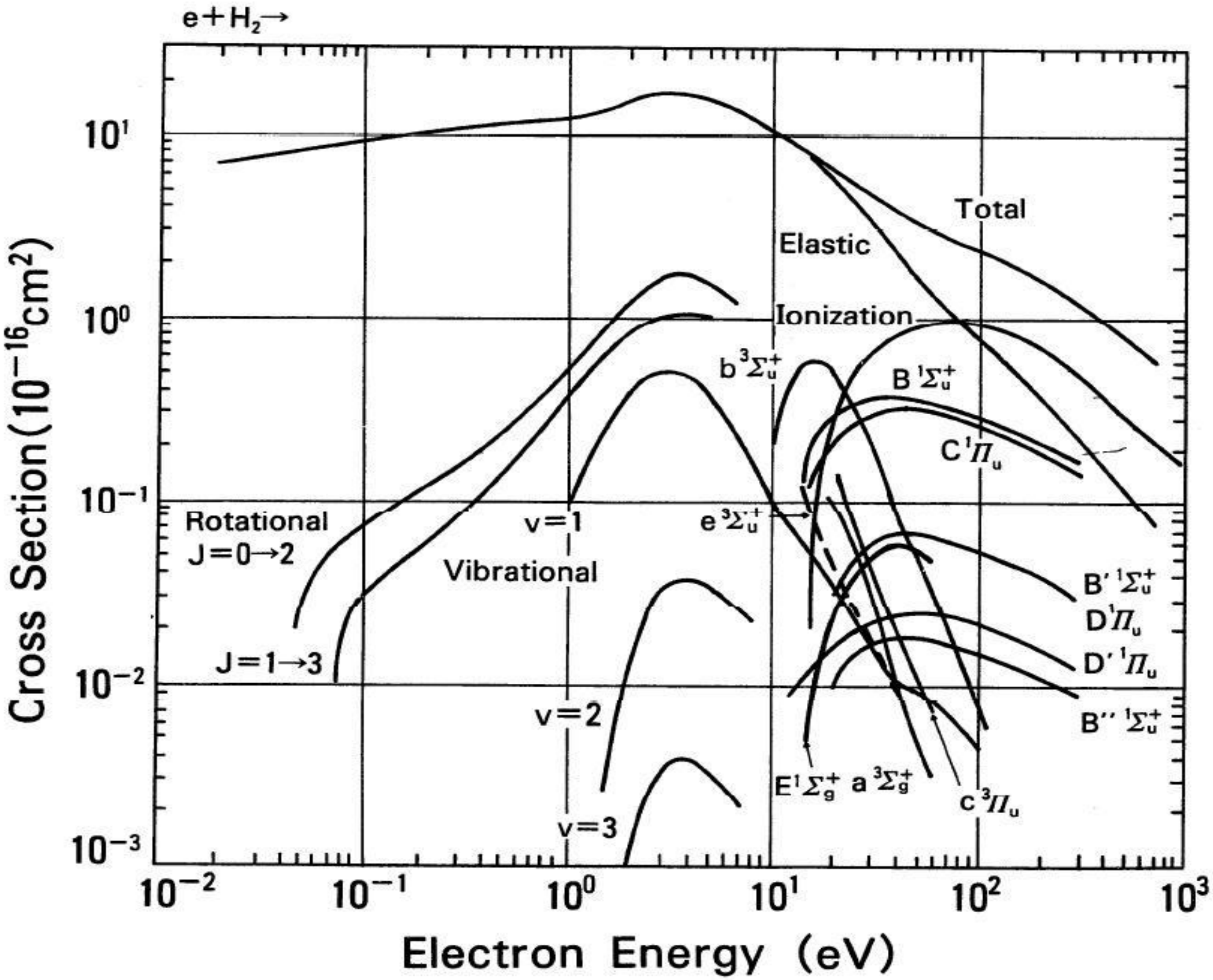
Model Beam Bunch-gas interaction

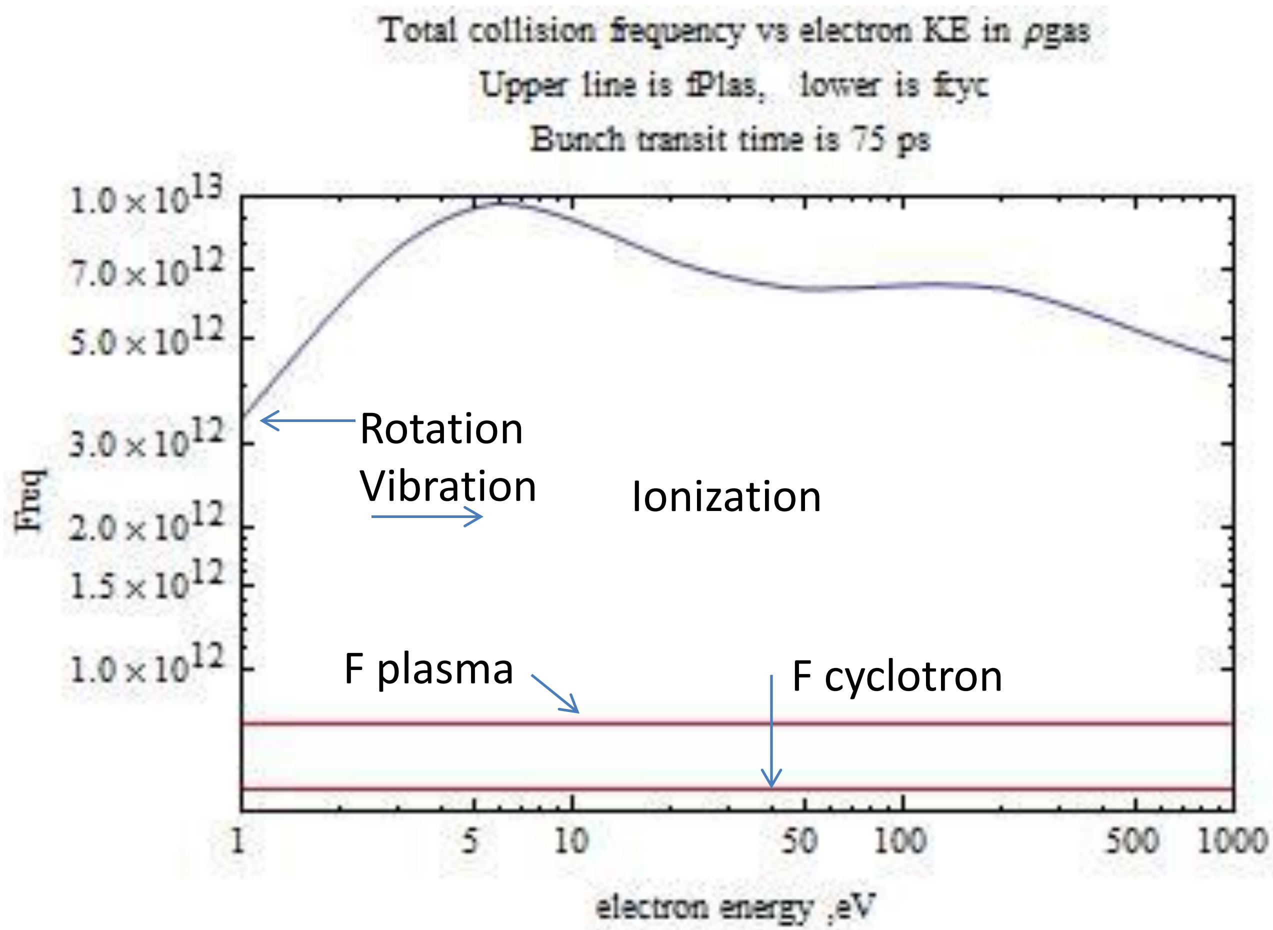
- 1. Bunch 2 cm long 2 mm radius 200 MeV . Transit time 75 ps
- 2. Bz = 20 T. pGas = 2600 psi
- 3. density H2 mol = 4.75 10 e21
- 4. density ions = 7.12 10 e15
- 5. Cyclotron F = 5.6 10 e11
- 6. Plasma F = 7.57 10e11
- 7. Er , z=0, r=2mm = 150 MV/m
- 8. Bϕ, z=0,r=2mm = .45 T

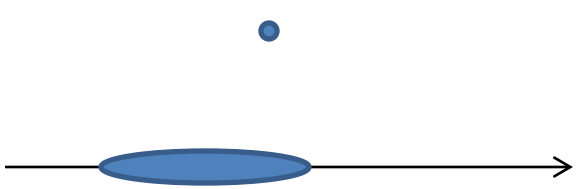


Moving with beta = .88

Electron – H2 cross sections

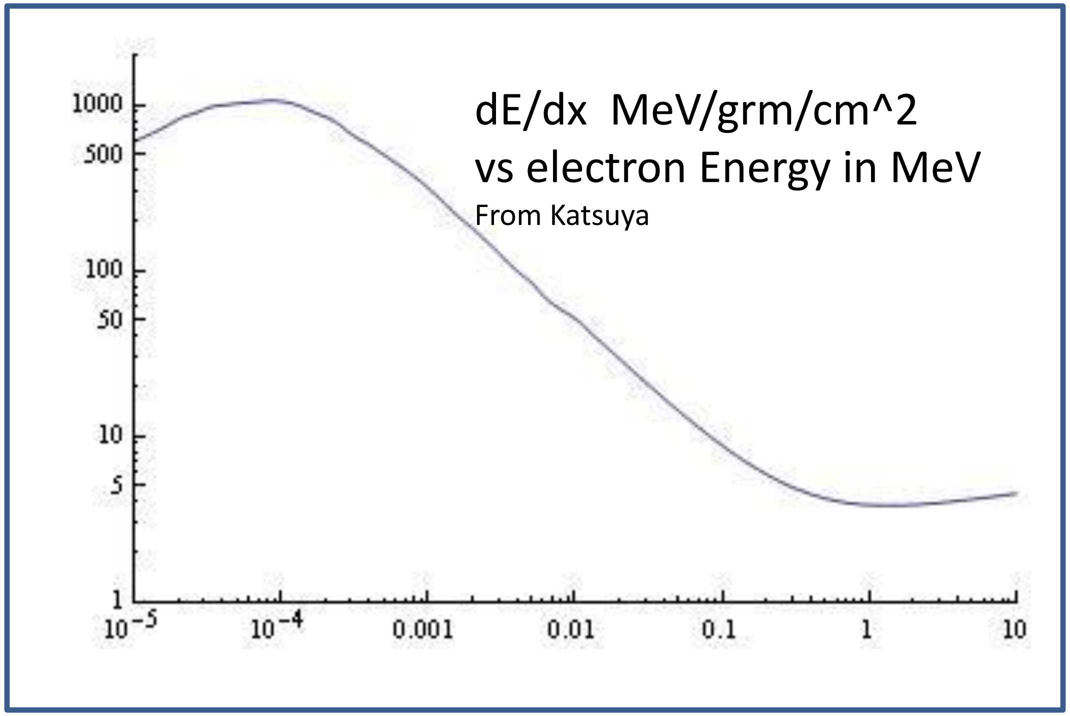






Ala Bethe-Bloch calculation for dE/dx
Trouble: 150 MV/m and 75 ps gives large Number. Impulse approx NG

$$\Delta p = \int e\, E_r[\chi]\, dt$$



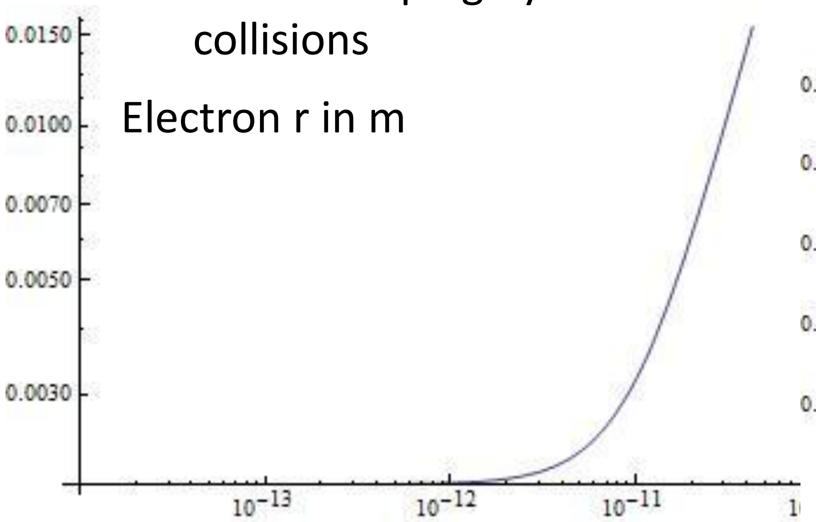
Need to make model for 0 < Te < 10 eV that matches onto the above curve. Use collision frequency and energy loss / collision for model shown on next slide. We should find velocities consistent with $v = \mu E$ where μ the classic mobility and is a function of E/pGas =11 V/cm/mmHg giving v 10 e7 cm/sec in a field of 150 Mv/m.

Wrong model but try anyway!

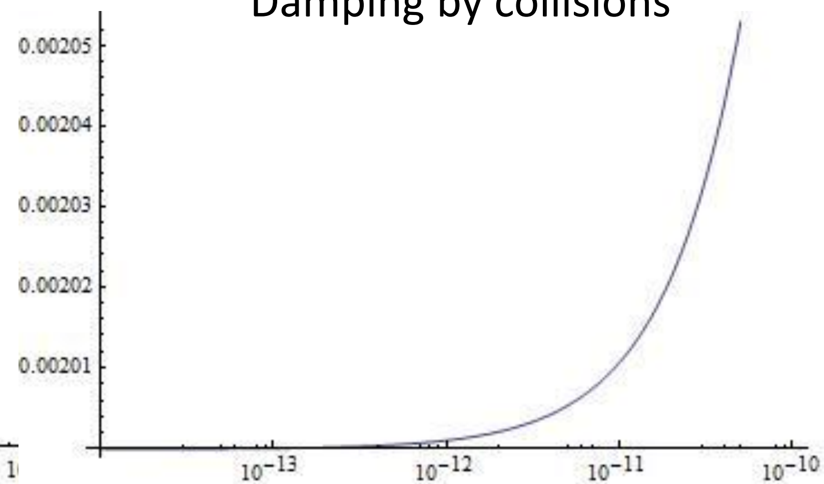
2^{nd} try: integrate $m\, d^2x/dt^2 = eE$ - Damping Term

Small Damping by collisions

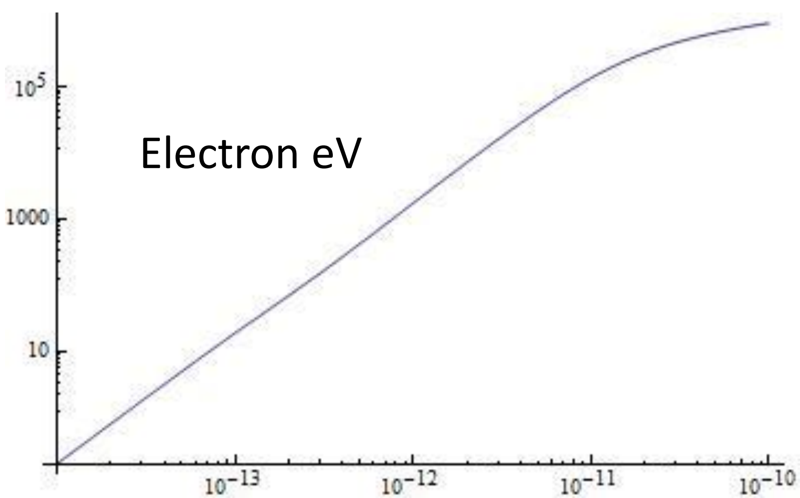
Electron r in m

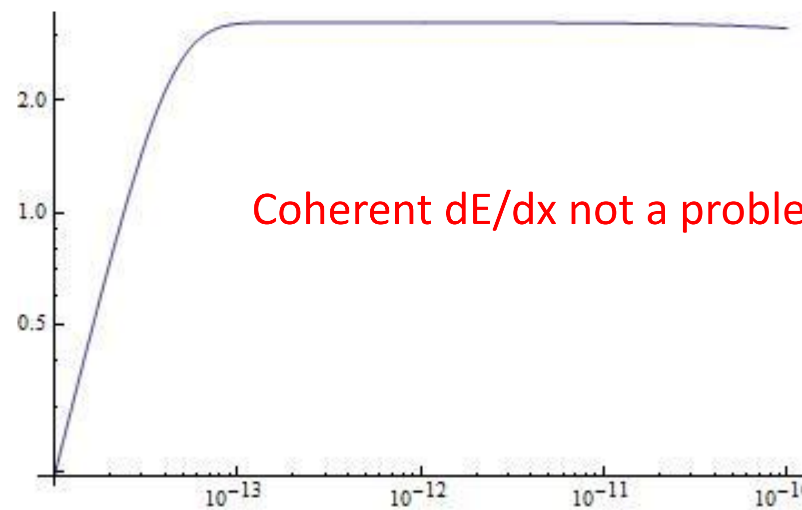


Damping by collisions



Electron eV





Coherent dE/dx not a problem

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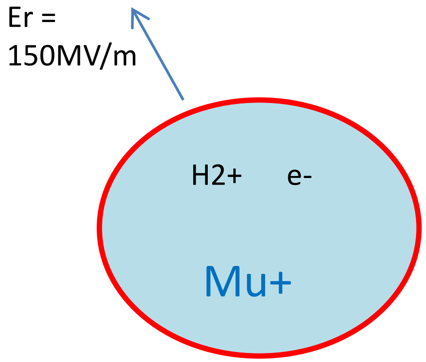
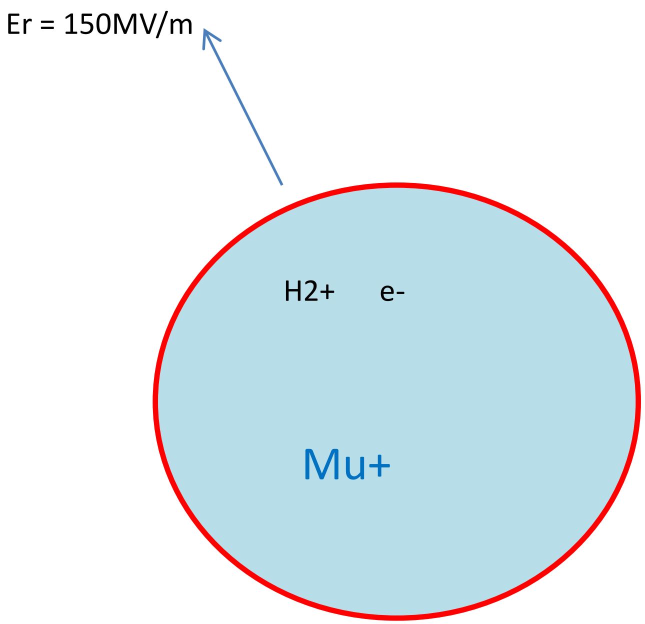
How about space charge neutralization?

1. Like electron cloud in accelerators. The beam makes a plasma e- and H2+ around the beam. Consider mu+ beam. The muons pull in the electrons, neutralize the electric field and the remaining B field focuses the beam. Or else there is an interaction between the cloud plasma and the bunch that causes blow up of the emittance. Is such a thing possible with an intense beam pulse in H2?
2. Two facts:
   1. Each muon makes 1000 ion pairs/cm of path. There are hence 1000 times as many + ions and 1000 times as many electrons / cm as there are beam charges.
   2. The resulting plasma frequency is very high … of the order of 10 e12 Hz



In the 2 mm circle there are 1000 e and H2+ for each muon. Plasma Neutral
The muons alone generate an Er = 150 MV/m.
1 part per 1000 unbalance of density of electrons will generate a field = to the beam particles!
So the beam E field can be easily neutralized in plasma period…….~ few pico seconds.



What is equilibrium state?

Inside the electrons move slightly to counter the muon field and make E=0.

This makes a ring around the outside of H2+ . The field from this ring is =0 on the inside and 150 MV/M on the outside.

Thickness of ring: $1/1000\ \pi r^2 = 2\pi r \Delta r$
$\Delta r = 1/2000\ r$. We can move an electron this far in 100 ps. So bunch can be neutralized but there is a linear difference in the plasma density between the front and back end of the bunch. Note sync osc will average this out! The plasma is very stiff, fPlas = 10 e12.

Suppose Er of bunch neutralized by plasma. Does the bunch $B\phi$ field result in strong self focus? $B\phi$ 0.45 T $R = m\,\beta/Bc$ = 3330 m. Very weak lens!

Some Questions to answer with good simulation

1. Is there a coherent energy loss from the bunch that adds on to the dEdx loss by ionization?
2. Are there plasma modes that can interact with the bunch phase space distribution?
3. Our measurement of the electron capture time indicates the time will be less than 1 ns, even as short as 0.1 ns. This time depends on the plasma temperature. We think this time is very short because of the high collision frequency. Are we missing something?

4. The arguments here would say the effects are small. However the simulation must be sufficiently detailed, including high collision frequency to answer some of these detailed questions. The first two questions are of beam dynamics, the last concerns ion chemistry.