Status of Low-β FOFO Snake for Final Stage of 6D Ionization Cooling

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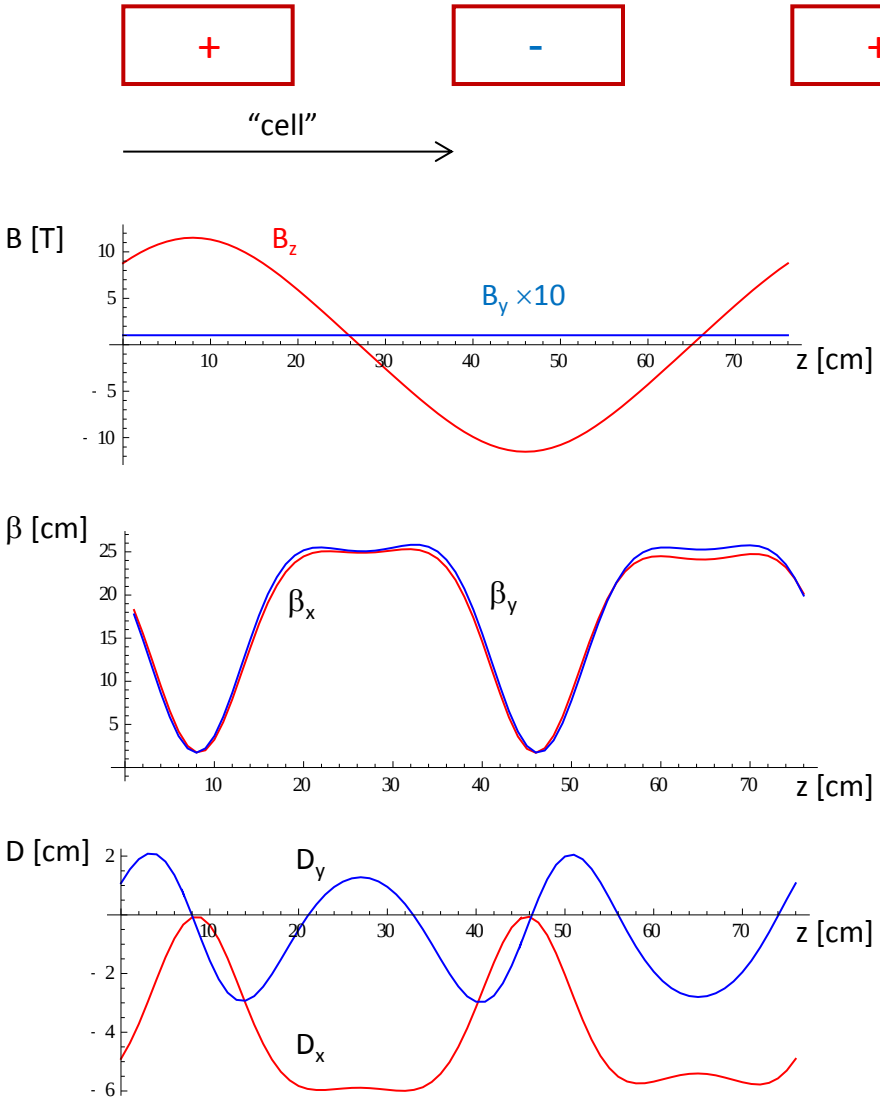
- This work is a continuation of effort reported at MCDW'09 (BNL) and NFMCC'10 (Miss. Univ.) and resumed now after a 3 year hiatus
- Is it worthwhile?

Motivation



FOFO snake with phase advance >180°/cell has a number of attractive features:

♦ Apparent technological simplicity (RF between solenoids, not inside)

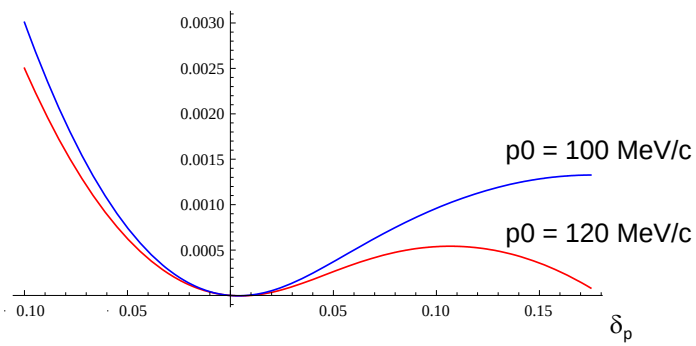
♦ Potentially higher compactness: phase advance / absorber (π+) is somewhat smaller than in RFOFO ($3\pi/2$-)

- However, phase advance / period (2π+) is higher creating problems with beam dynamics.

Problems with Low-Beta FOFO Snake

- Dispersion = 0 at focal points ⇒ stronger transverse field is required
- Large difference in cooling rates of the two transverse normal modes
- Momentum acceptance limited from above by sign change in the slippage factor
- Momentum acceptance limited from below by fast increase in the ionization loss rate

$$K(\delta_p) = \int^{\delta_p} [1 - \lambda(\xi) \frac{\beta_0}{\beta(\xi)}] \, d\xi$$



The major mechanism of losses is diffusion over the maximum of long. “kinetic energy” – change of the slip factor sign at higher values of momentum.

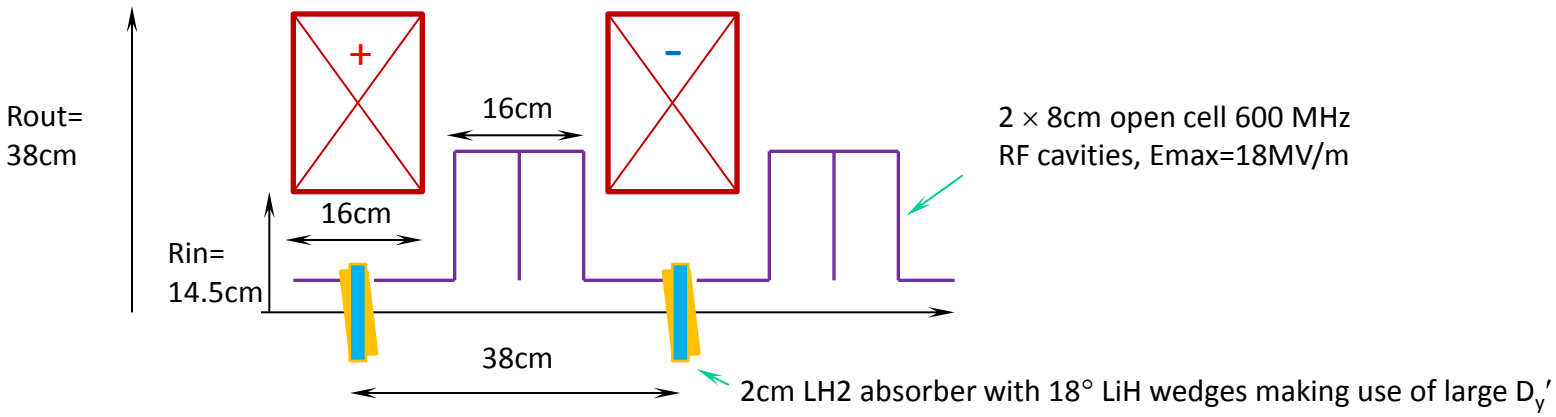
$\lambda = L(\delta_p)/L(0)$ – relative length of the periodic orbit

- Maximum $\beta_\perp$ is reached between solenoids where the field nonlinearity is also at maximum:

$$\frac{\partial^3}{\partial r^3} B_r \sim \frac{\partial^3}{\partial z^3} B_z$$

This creates difficulties with the transverse acceptance as well

Geometry & Parameters



Total length of 2-cell period 2 × 38cm = 76cm

Bz_axis=11.5T (Bz_coil=17.3T, j < 200A/mm^2) for p_0=100MeV/c, constant By=0.01T

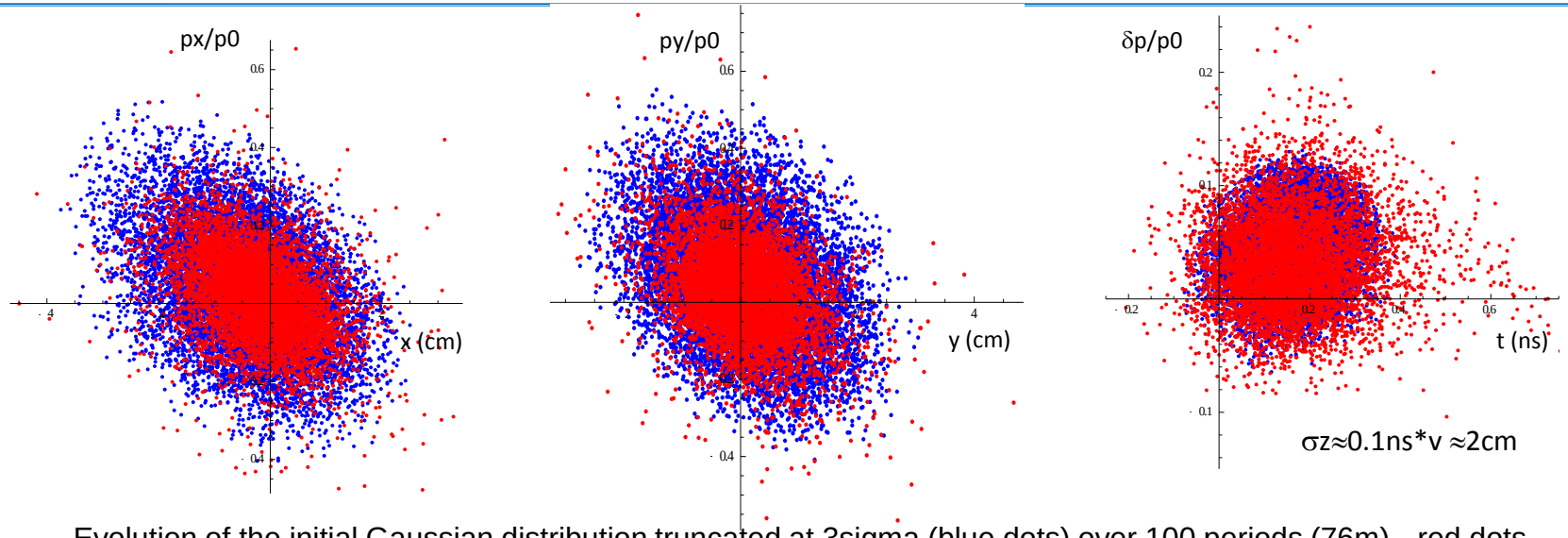
The transverse modes cooling rates can be equalized by 1-periodic **quadrupole** field with gradient **1.1T/m** between the solenoids (proposed by R.Palmer difference in solenoids also works but makes transition worse).

Normal mode tunes (including cooling rates) and normalized equilibrium emittances:

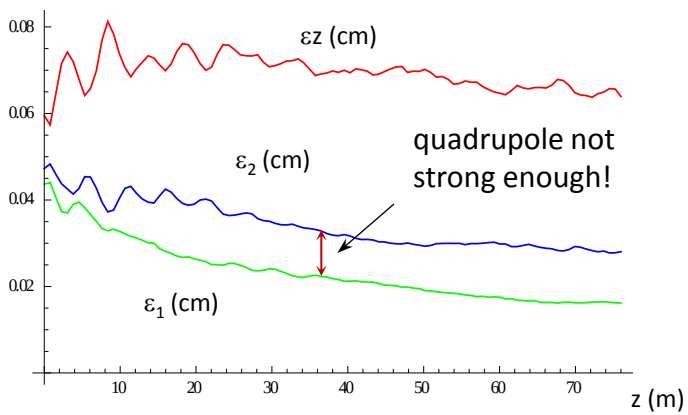
tune*	1.229 + 0.00149 i	1.245 + 0.00144 i	0.109 +0.00042 i
ε_N (mm)	0.183	0.201	1.03

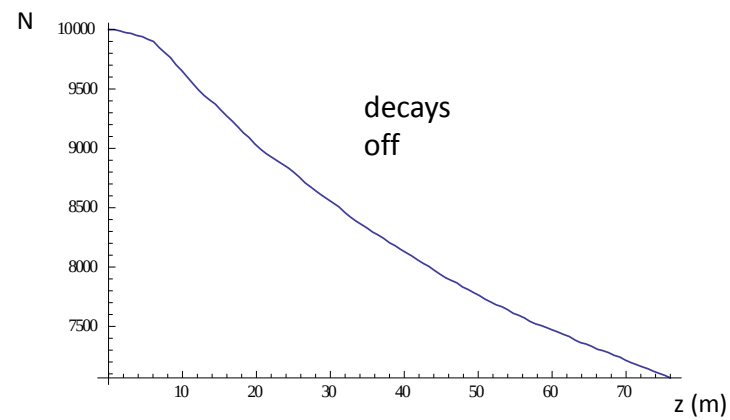
*) Transverse phase advance / period is (almost) 2.5π

G4BL Tracking



Evolution of the initial Gaussian distribution truncated at 3sigma (blue dots) over 100 periods (76m) - red dots

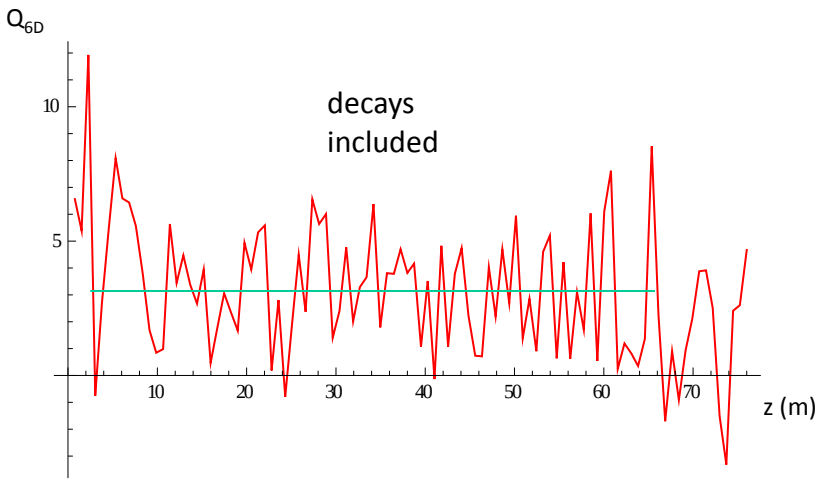




Normalized emittances (Gaussian fit) and intensity over 100 periods .

Final $\varepsilon_{\perp}$=0.21mm, total losses (with decay) = 40%.

Cooling Efficiency



R. Palmer's 6D quality factor

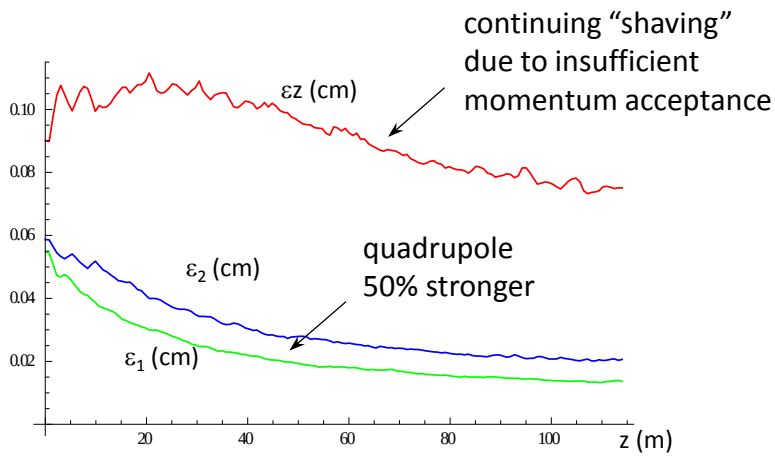
$$Q_{6D} = \frac{d \log \varepsilon_{6D}}{d \log N}$$

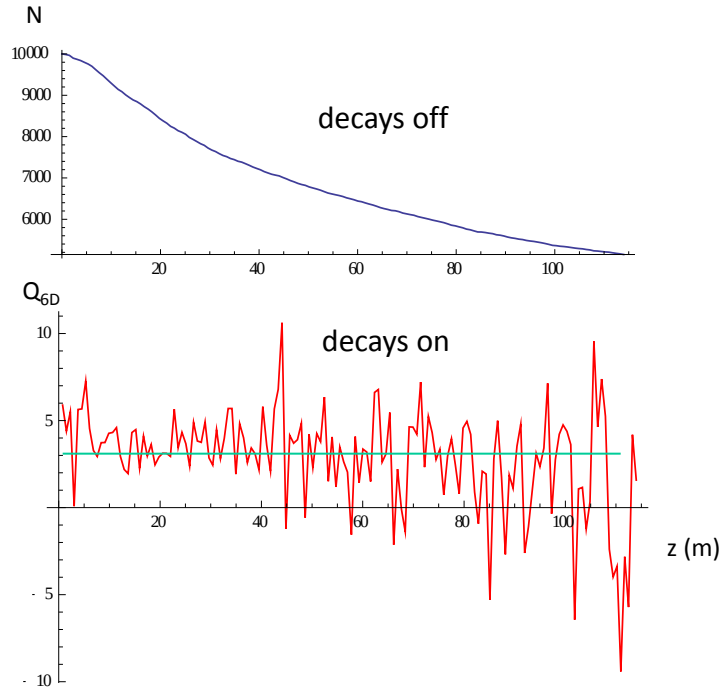
Average value $Q_{6D} \approx 3$

I tried to improve transmission by:

- Larger wedge angle (25°) – opposite result (!?)
- Changing tunes: ~ constant for $1.2<Q_\perp<1.25$, big drop for $Q_\perp<1.2$ ($Q_\perp$-$2Q_s$ SBR?) and $Q_\perp \approx 1.375$
- Rotation of LiH wedges to utilize both D_y' and D_x – no effect
- Lower momentum (90MeV/c) – no effect (this is actually good!)
- Deceleration from 100 to 90MeV/c over 50 periods – opposite result
- Higher RF frequency (650MHz) and higher voltage (20MV/m) – opposite result ($Q_\perp$-$2Q_s$ SBR?)
- Lower RF frequency (325MHz) and triangular pulse equivalent to 150MHz (this should also reduce space-charge effects)

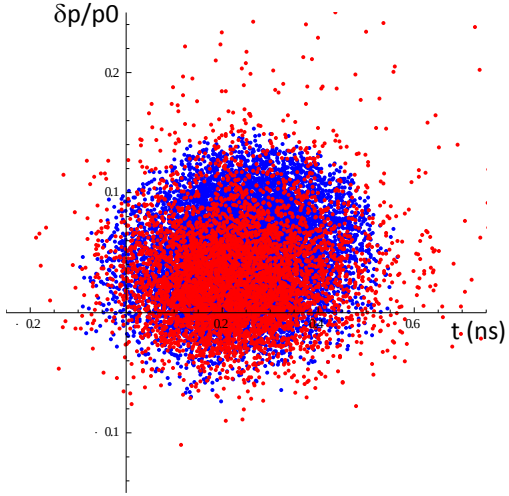
325 MHz RF





Normalized emittances (Gaussian fit), intensity and 6D quality factor over 150 periods (114m).

Final $\varepsilon_{\perp}$=0.17mm, total losses (with decay) = 60%.



bunch length increase is smaller than expected

Summary

- Low-beta FOFO-snake with LiH wedges does work allowing for normalized transverse emittance < 0.2mm
- Equalization of the transverse normal mode cooling rates can be achieved with either the solenoid current difference or a weak periodic quadrupole field (<2T/m)
- The major performance limitation is imposed by insufficient momentum acceptance
- There are still some possibilities to explore in order to improve transmission – may take a week more to exhaust them

325 MHz Helical FOFO Snake for Initial Stage of 6D Ionization Cooling

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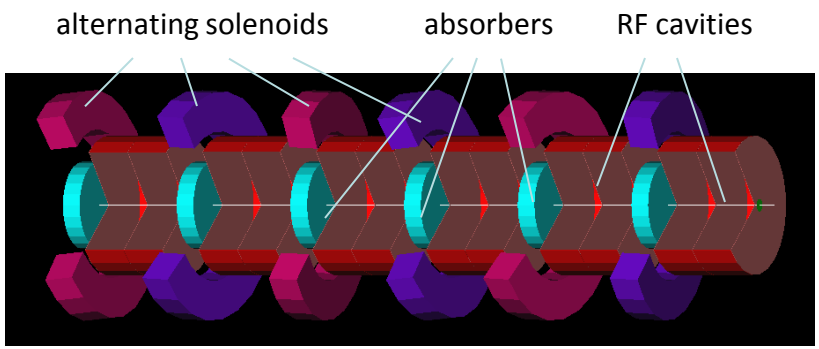
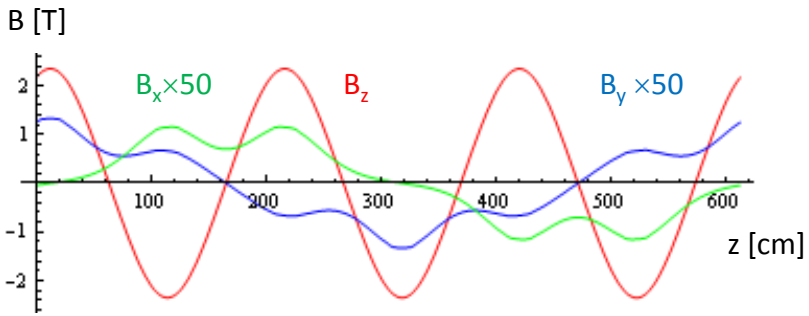
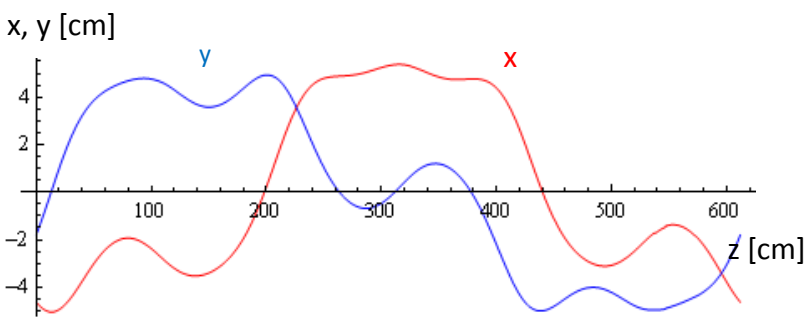
Basic Idea

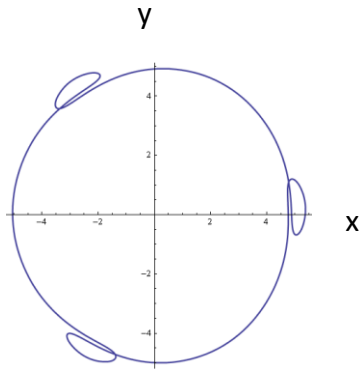




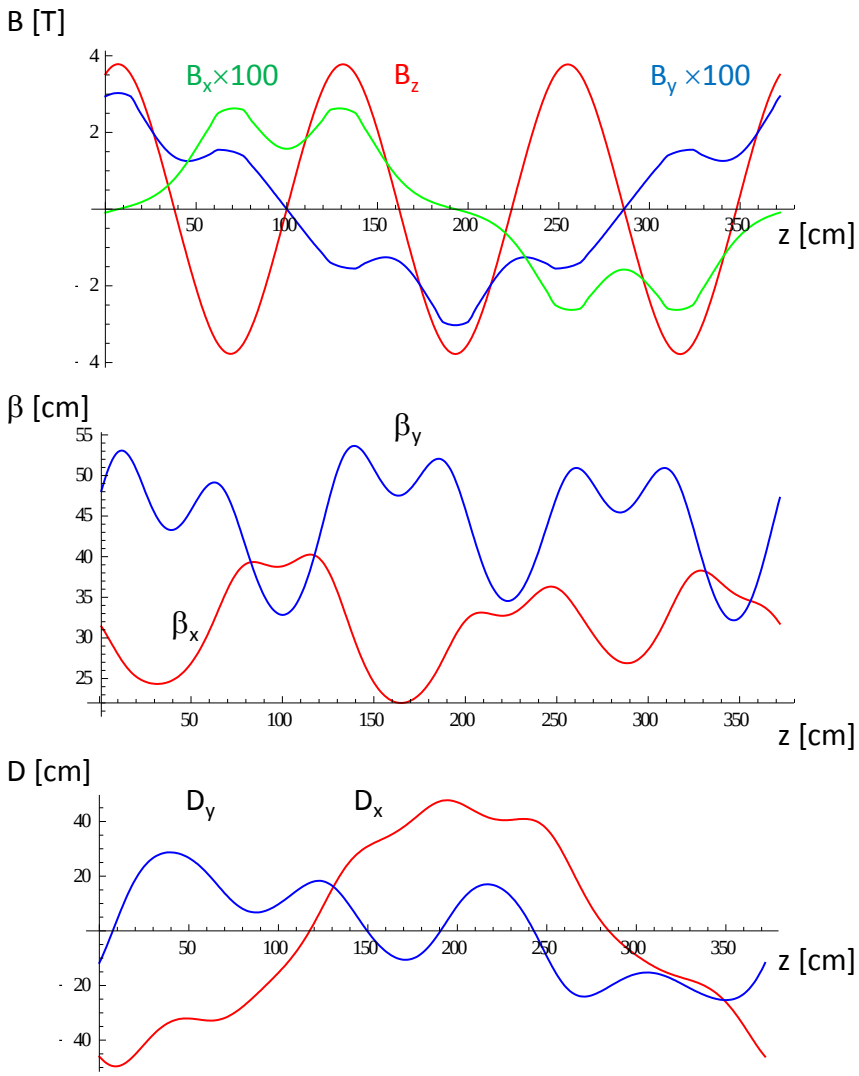


Periodic orbit for p=200MeV/c

- The idea: create rotating $B_\perp$ field by periodically tilting solenoids, e.g. with 6-solenoid period.
- Periodic orbits for μ+ and μ- look exactly the same, just shifted by a half period (3 solenoids).
- With tune $Q_\perp$>1 (per period) **r·D**>0 ⇒ muons with higher momentum make a longer path ⇒ longitudinal cooling achieved even with planar absorbers



Optics Functions



Total length of 6-cell period = 372cm vs 612cm @200MHz – I tried to reduce $\beta_\perp$ as much as reasonably possible

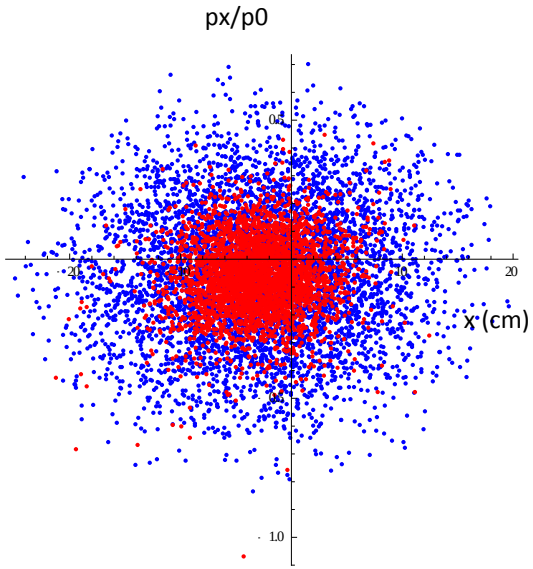
Bz_axis=3.8T (j < 200A/mm^2) for p_0=200MeV/c, solenoid pitch angle 5mrad

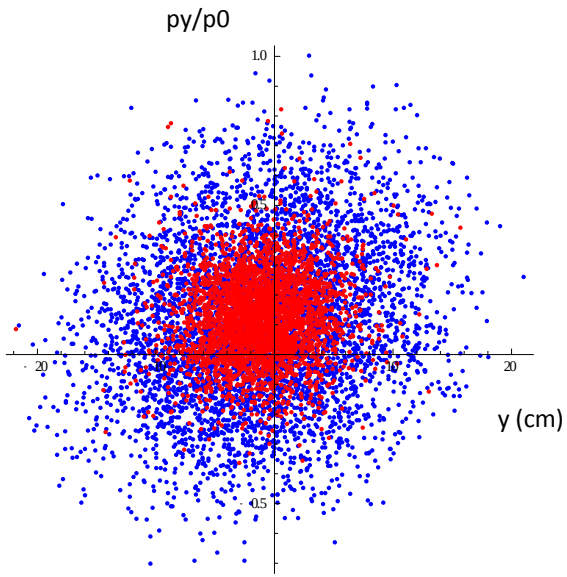
The transverse modes cooling rates are equalized by costant **quadrupole** field with gradient **0.12T/m**

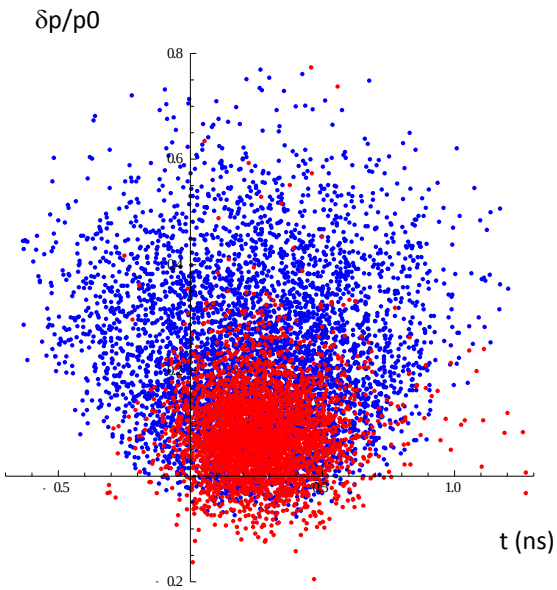
Normal mode tunes (including cooling rates) and normalized equilibrium emittances:

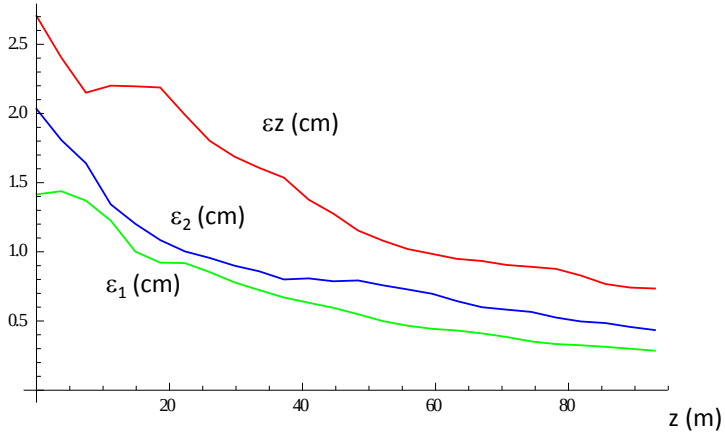
tune	1.21 + 0.0069 i	1.24 + 0.0069 i	0.16 +0.0031 i
ε_N (mm)	2.47	2.39	3.48

G4BL Tracking









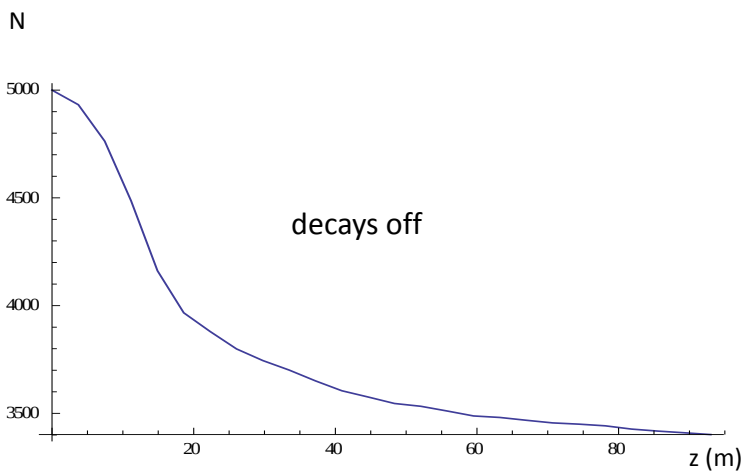
Initial Gaussian distribution includes all correlations up to 2nd order (including energy-transverse amplitude^2)

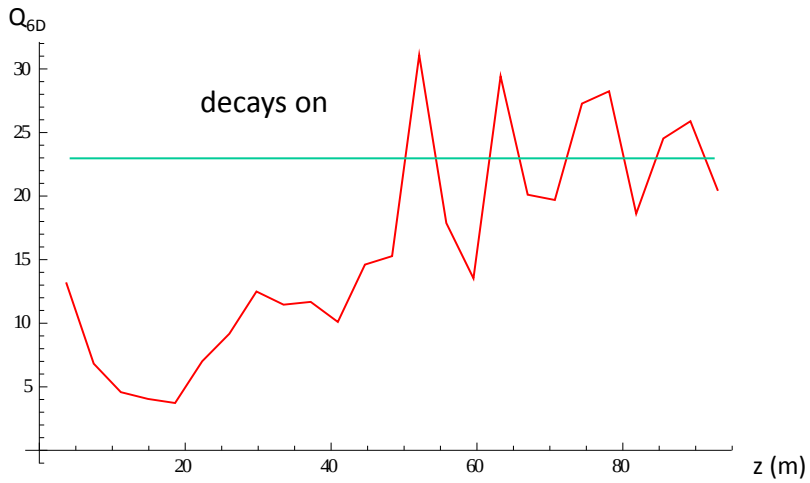
Horizontally beam extends over 20cm, transverse momentum exceeds p0=200MeV/c! - inevitably high losses in the beginning (next slide)

Normalized emittances (Gaussian fit) over 25 periods (93m) .

Final $\varepsilon_{\perp}$=3.5mm, $\varepsilon_{||}$ ~ twice larger

Cooling Efficiency





Final value of Q6D exceeds 20 – cooling can be continued.

Now I should try Dave's rotator output.