Present Status of the Phenomenology and Theory of Very Light Sterile Neutrinos

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Three Flavor Mixing Hypothesis Fits All* Data Really Well.

parameter	best fit $\pm 1\sigma$	2σ	3σ
Δm^2_{21} $[10^{-5}\text{eV}^2]$	7.62 ± 0.19	7.27–8.01	7.12–8.20
Δm^2_{31} $[10^{-3}\text{eV}^2]$	$2.53^{+0.08}_{-0.10}$ $-(2.40^{+0.10}_{-0.07})$	$2.34 - 2.69$ $-(2.25 - 2.59)$	$2.26 - 2.77$ $-(2.15 - 2.68)$
$\sin^2\theta_{12}$	$0.320^{+0.015}_{-0.017}$	0.29–0.35	0.27–0.37
$\sin^2\theta_{23}$	$0.49^{+0.08}_{-0.05}$ $0.53^{+0.05}_{-0.07}$	0.41–0.62 0.42–0.62	0.39–0.64
$\sin^2\theta_{13}$	$0.026^{+0.003}_{-0.004}$ $0.027^{+0.003}_{-0.004}$	0.019–0.033 0.020–0.034	0.015–0.036 0.016–0.037
δ	$\left(0.83^{+0.54}_{-0.64}\right)\pi$ 0.07π [a]	$0 - 2\pi$	$0 - 2\pi$

* Modulo short-baseline anomalies.

[Forero, Tórtola, Valle, 1205.4018]

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

What we have **really measured** (very roughly): [see, e.g., Antusch et al, hep-ph/0607020]

- Two mass-squared differences, at several percent level – many probes;
- $|U_{e2}|^2$ – solar data;
- $|U_{\mu 2}|^2 + |U_{\tau 2}|^2$ – solar data;
- $|U_{e2}|^2 |U_{e1}|^2$ – KamLAND;
- $|U_{\mu 3}|^2 (1 - |U_{\mu 3}|^2)$ – atmospheric data, K2K, MINOS;
- $|U_{e3}|^2 (1 - |U_{e3}|^2)$ – Double Chooz, Daya Bay, RENO;
- $|U_{e3}|^2 |U_{\mu 3}|^2$ (upper bound $\rightarrow$ evidence) – MINOS, T2K.

We still have a ways to go!

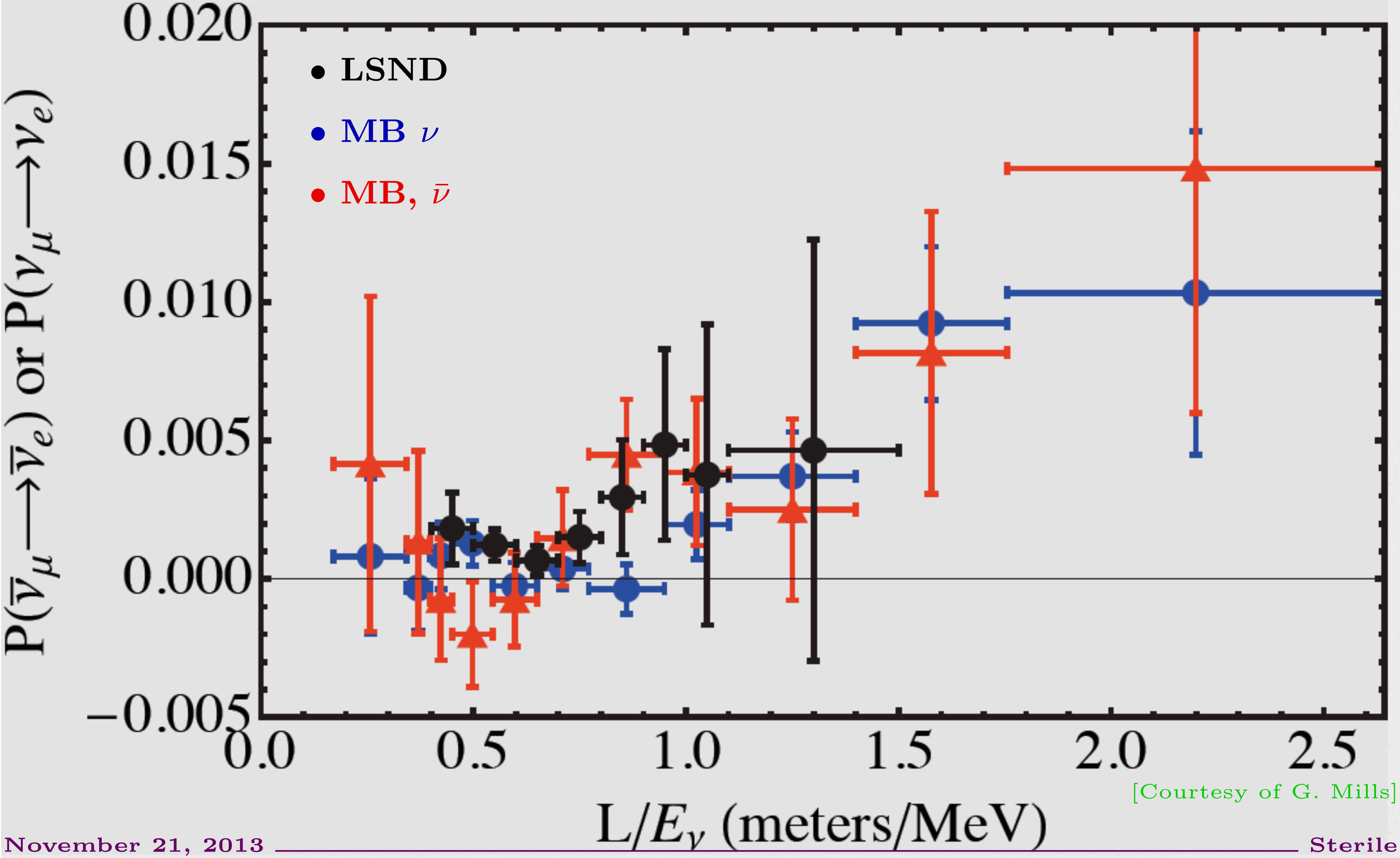
Not all is well: The Short Baseline Anomalies

Different data sets, sensitive to L/E values small enough that the known oscillation frequencies do not have "time" to operate, point to unexpected neutrino behavior. These include

- $\nu_\mu \to \nu_e$ appearance — LSND, MiniBooNE;
- $\nu_e \to \nu_{\rm other}$ disappearance — radioactive sources;
- $\bar{\nu}_e \to \bar{\nu}_{\rm other}$ disappearance — reactor experiments.

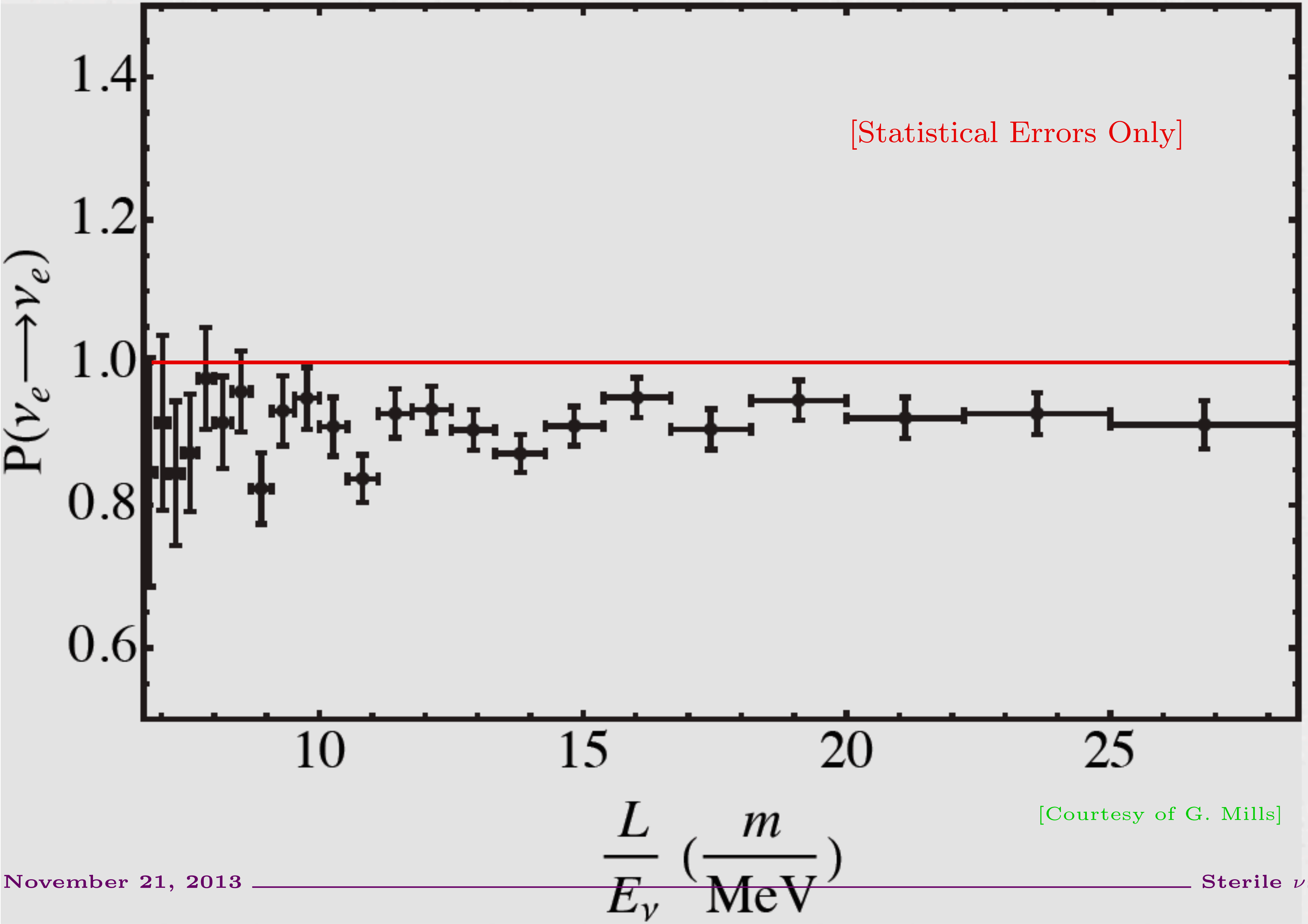
None are entirely convincing, either individually or combined. However, there may be something very very interesting going on here...

MiniBooNE & LSND



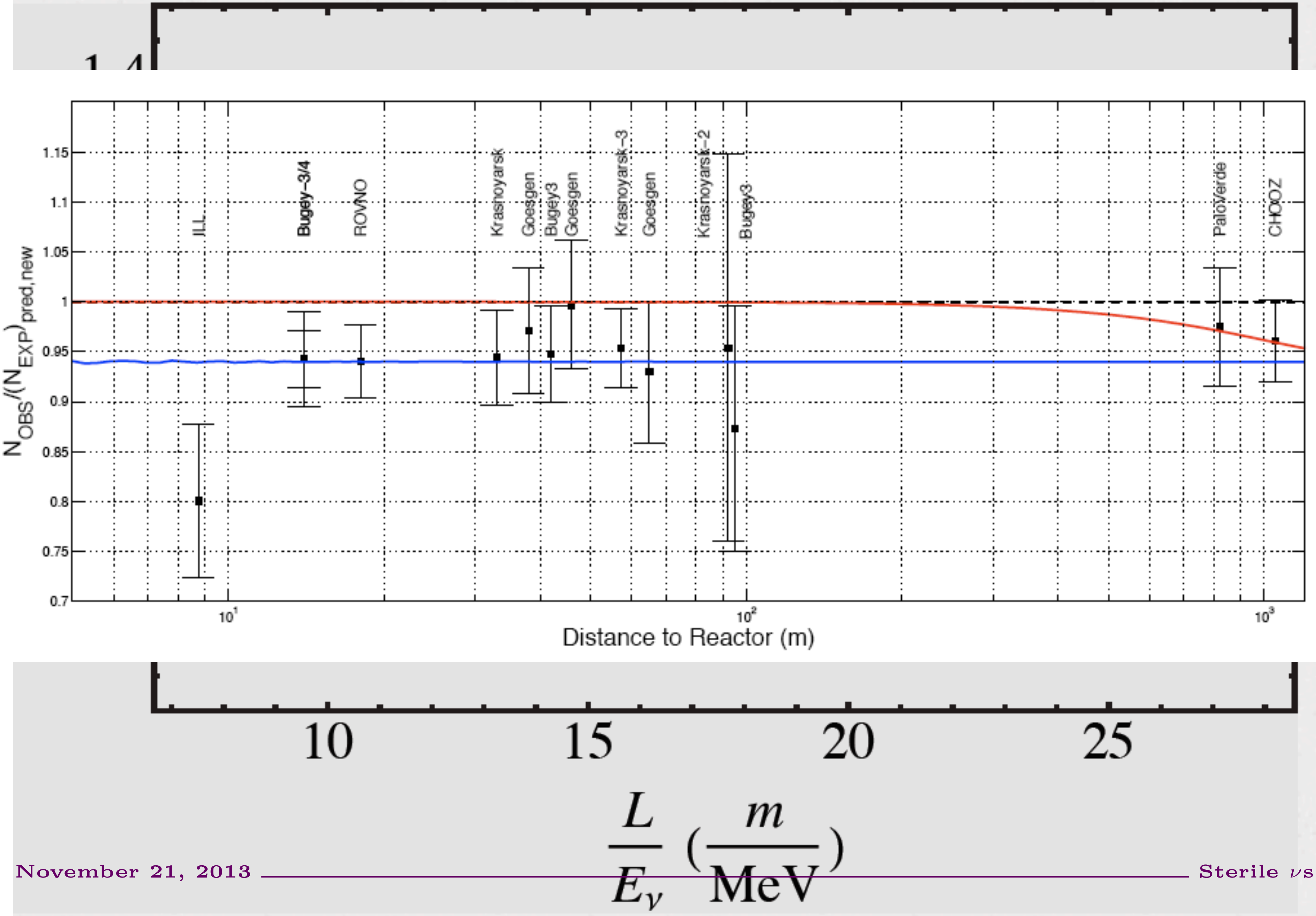
[Courtesy of G. Mills]

Bugey 40 m



[Courtesy of G. Mills]

Bugey 40 m



1.4
N_OBS/(N_EXP)_pred,new
1.15
1.1
1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
ILL
Bugey-3/4
ROVNO
Krasnoyarsk
Goesgen
Bugey3
Goesgen
Krasnoyarsk-3
Goesgen
Krasnoyarsk-2
Bugey3
PaloVerde
CHOOZ
10^1
10^2
10^3
Distance to Reactor (m)
10
15
20
25
L/E_ν (m/MeV)

What is Going on Here?

- Are these "anomalies" related?
- Is this neutrino oscillations, other new physics, or something else?
- Are these related to the origin of neutrino masses and lepton mixing?
- How do clear this up **definitively**?

Need new clever experiments, of the short-baseline type!

Observable wish list:

- ν_μ disappearance (and antineutrino);
- ν_e disappearance (and antineutrino);
- $\nu_\mu \leftrightarrow \nu_e$ appearance;
- $\nu_{\mu,e} \rightarrow \nu_\tau$ appearance.

[see talk by Jon Link]

A neutrino oscillation solution require new neutrino states ν_4, ν_5, etc with masses m_4, m_5, etc. Reason is simple: L/E too small (hence *Short Baseline Anomalies*).

The probability that ν_4 is measured as a ν_e is U_{e4}, the probability that ν_5 is measured as a ν_μ is $U_{\mu 5}$, and so on.

Bottom line: Fits to *all data* are mediocre – no "feel good" solution! On the other hand, I think it is not correct to say the hypothesis is safely ruled out ...

	$\chi^2_{\rm min}/{\rm dof}$	GOF	$\chi^2_{\rm PG}/{\rm dof}$	PG	$\chi^2_{\rm app,glob}$	$\Delta\chi^2_{\rm app}$	$\chi^2_{\rm dis,glob}$	$\Delta\chi^2_{\rm dis}$
3+1	712/(689 − 9)	19%	18.0/2	1.2×10^{-4}	95.8/68	7.9	616/621	10.1
3+2	701/(689 − 14)	23%	25.8/4	3.4×10^{-5}	92.4/68	19.7	609/621	6.1
1+3+1	694/(689 − 14)	30%	16.8/4	2.1×10^{-3}	82.4/68	7.8	611/621	9.0

Table 7. Global χ^2 minima, GOF values, and parameter goodness-of-fit (PG) test [125] for the consistency of appearance versus disappearance experiments in the 3+1, 3+2, and 1+3+1 schemes. The corresponding parameter values at the global best fit points are given in Tab. 8. The last four columns give the contributions of appearance and disappearance data to $\chi^2_{\rm PG}$, see Eq. (6.2).

J. Kopp *et al*, arXiv:1303.3011

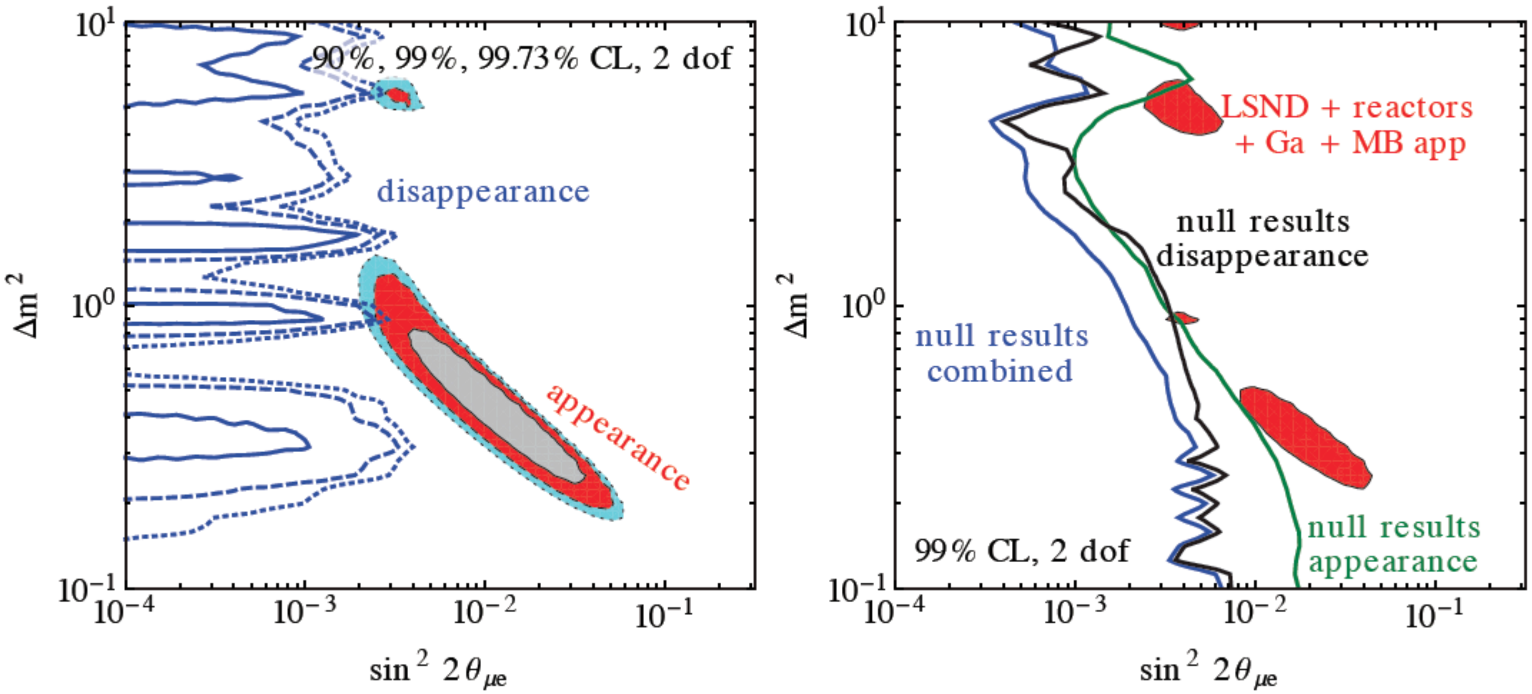
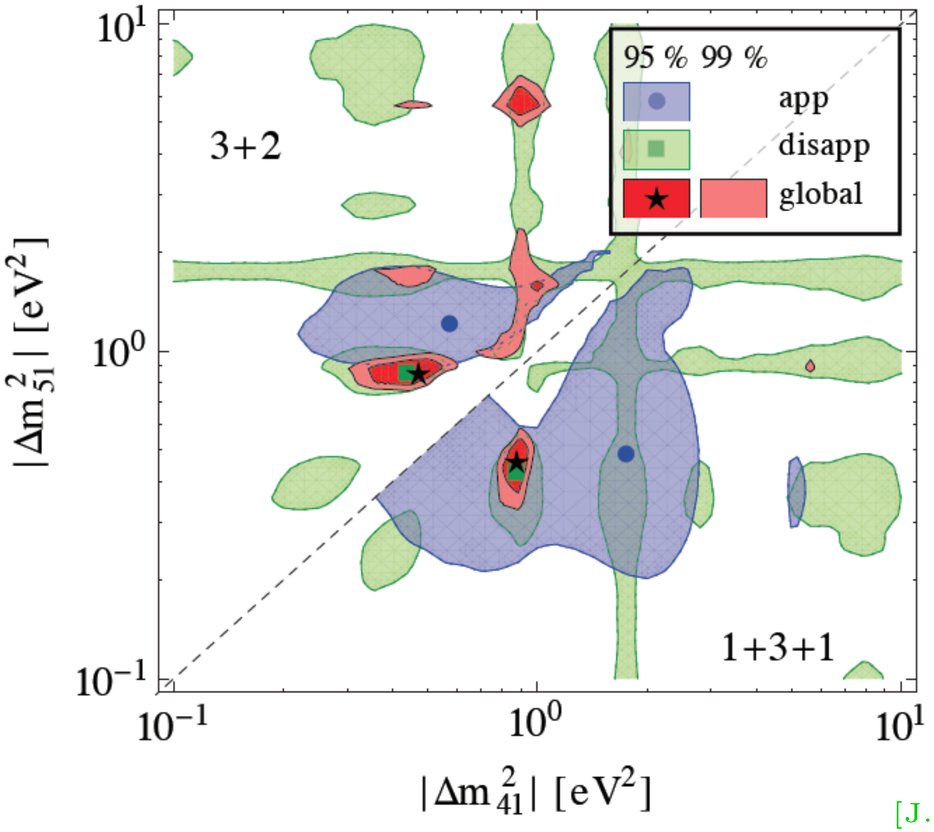


Figure 8. Results of the global fit in the 3+1 scenario, shown as exclusion limits and allowed regions for the effective mixing angle $\sin^2 2\theta_{\mu e} = 4|U_{e4}|^2|U_{\mu 4}|^2$ and the mass squared difference Δm^2_{41}. Left: Comparison of the parameter region preferred by appearance data (LSND, MiniBooNE appearance analysis, NOMAD, KARMEN, ICARUS, E776) to the exclusion limit from disappearance data (atmospheric, solar, reactors, Gallium, CDHS, MINOS, MiniBooNE disappearance, KARMEN and LSND ν_e–^{12}C scattering). Right: Regions preferred by experiments reporting a signal for sterile neutrinos (LSND, MiniBooNE, SBL reactors, Gallium) versus the constraints from all other data, shown separately for disappearance and appearance experiments, as well as their combination.

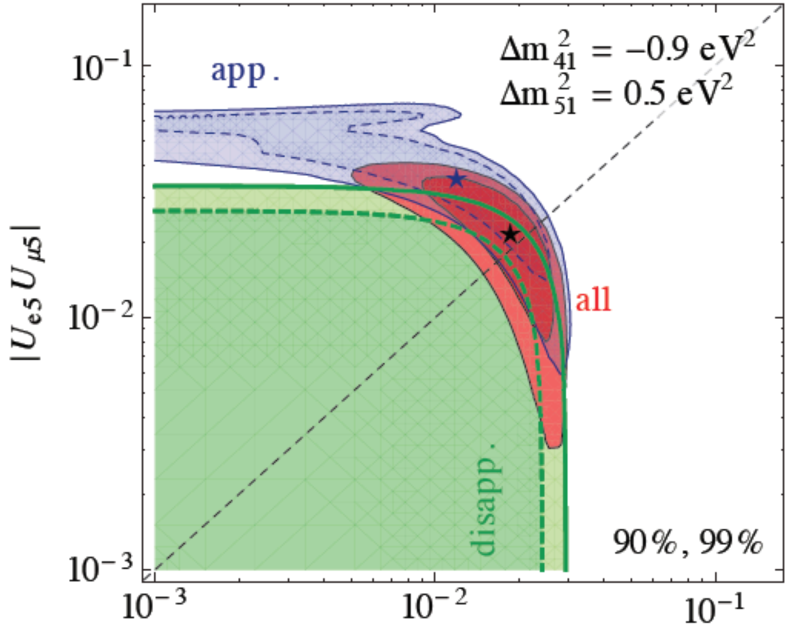
[J. Kopp *et al*, arXiv:1303.3011]

	Δm^2_{41} [eV^2]	$\lvert U_{e4}\rvert$	$\lvert U_{\mu 4}\rvert$	Δm^2_{51} [eV^2]	$\lvert U_{e5}\rvert$	$\lvert U_{\mu 5}\rvert$	$\gamma_{\mu e}$
3+1	0.93	0.15	0.17				
3+2	0.47	0.13	0.15	0.87	0.14	0.13	-0.15π
1+3+1	-0.87	0.15	0.13	0.47	0.13	0.17	0.06π

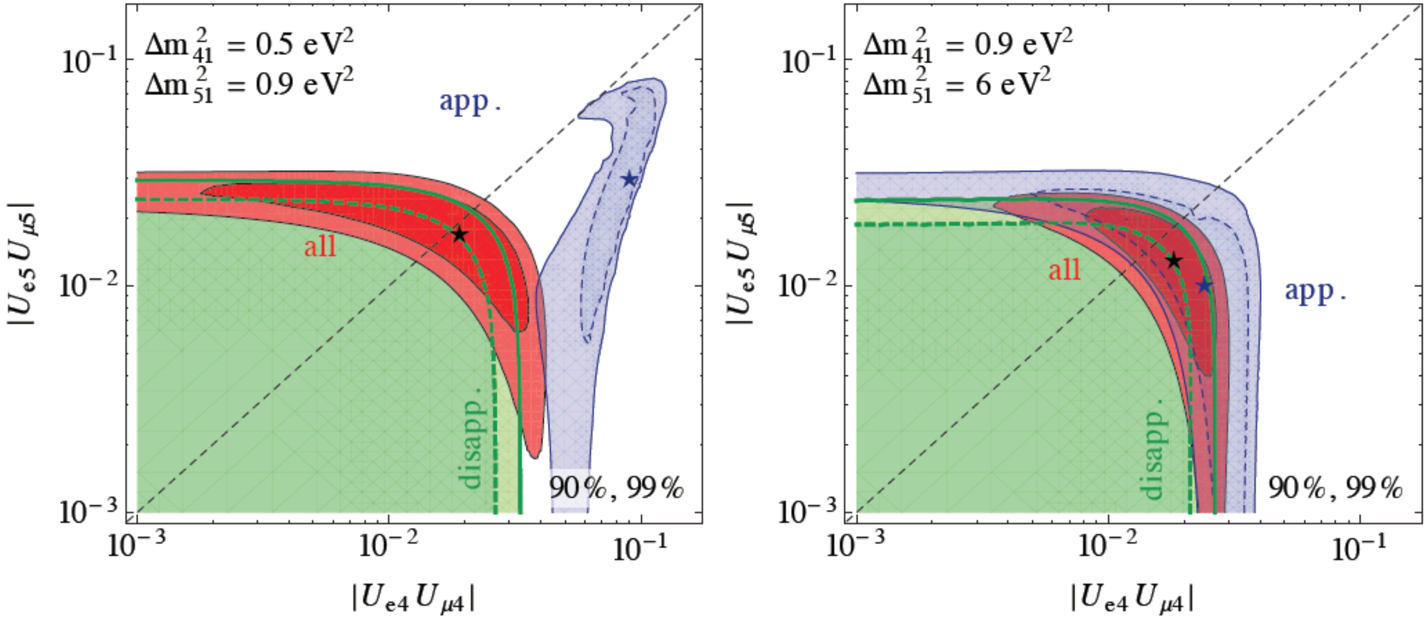
Table 8. Parameter values at the global best fit points for the 3+1, 3+2, and 1+3+1 mass schemes. $\gamma_{\mu e}$ is the complex phase relevant for SBL appearance experiments as defined in Eq. (2.2).



[J. Kopp *et al*, arXiv:1303.3011]



[J. Kopp *et al*, arXiv:1303.3011]



the 1+3+1 scheme.

Figure 10. Allowed regions for 3+2 in the plane of $|U_{e4}U_{\mu4}|$ vs. $|U_{e5}U_{\mu5}|$ for fixed values of Δm^2_{41} and Δm^2_{51}, at 90% and 99% CL (2 dof). We minimize over all undisplayed mixing parameters. We show the regions for appearance data (blue), disappearance data (green), and the global data (red).

Big Bang Neutrinos are Warm Dark Matter

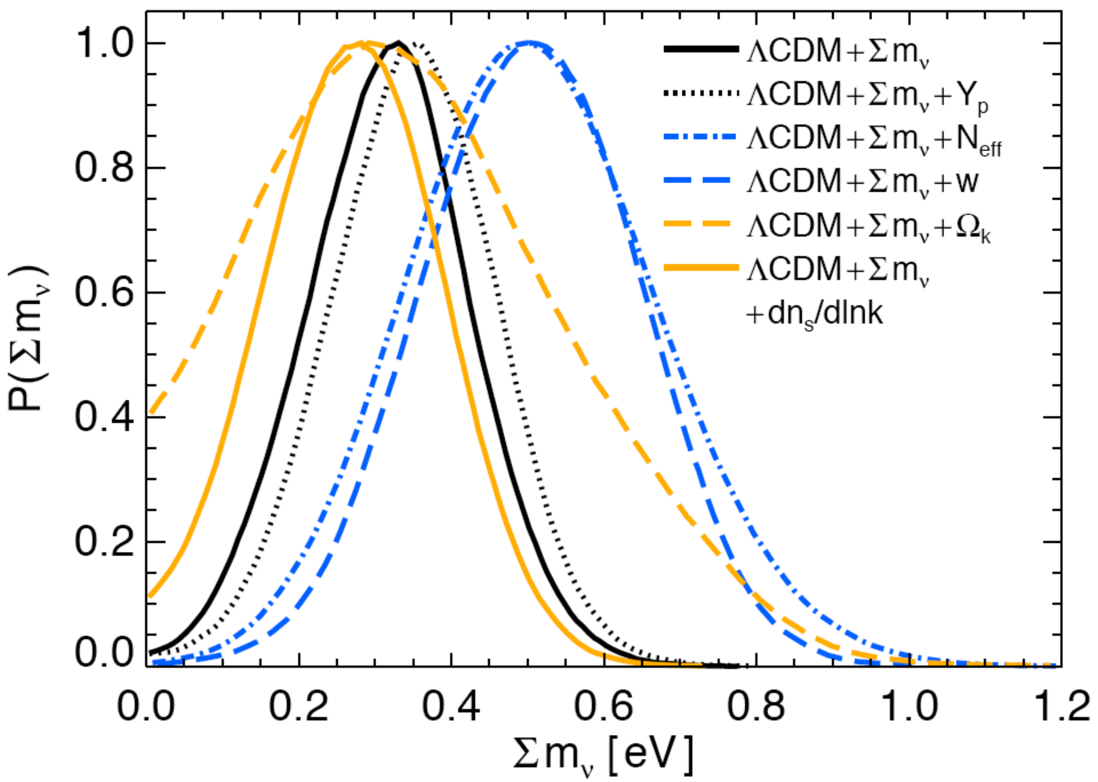


Fig. 10.— This figure illustrates the robustness of the neutrino mass detection to other parameter extensions. The marginalized one-dimensional posteriors for $\sum m_\nu$ are shown for two-parameter extensions to ΛCDM for the combined CMB+BAO+H_0+SPT$_{CL}$ data sets (for w, SNe are used instead of H_0). Allowing significant curvature or running can significantly reduce the preference for nonzero neutrino masses (to 1.7 and 2.4 σ respectively). Other extensions increase the preference for positive neutrino masses.

[Z. Hou *et al.* arXiv:1212.6267]

- Constrained by the Large Scale Structure of the Universe.

Constraints depend on

- Data set analysed;
- "Bias" on other parameters;
- ...

Bounds can be evaded with non-standard cosmology. Will we learn about neutrinos from cosmology or about cosmology from neutrinos?

Big Bang Neutrinos are Warm Dark Matter

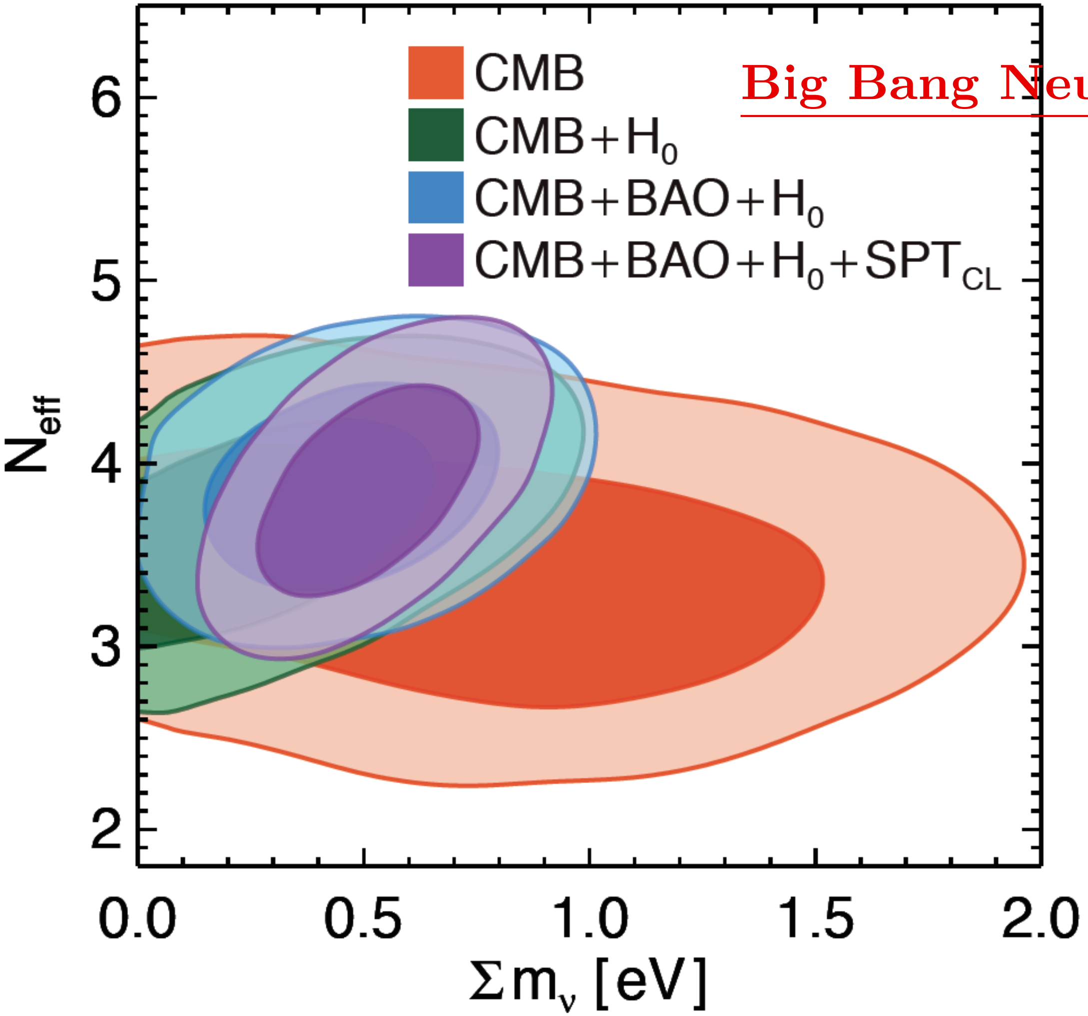


FIG. 18.— This figure demonstrates the impact of each combination of datasets on the constraints on $\sum m_\nu$ and $N_{\rm eff}$. The shaded contours are the 68% and 95% confidence intervals for the following data combinations: SPT+*WMAP7* (CMB; red), CMB+H_0 (green), CMB+H_0+BAO (blue), CMB+H_0+BAO+$\mathrm{SPT_{CL}}$ (purple). The combined data are in $>2\,\sigma$ tension with the ΛCDM assumption of three massless neutrino species.

[Z. Hou *et al.* arXiv:1212.6267]

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Big Bang Neutrinos are Warm Dark Matter

Planck Collaboration: Cosmological parameters

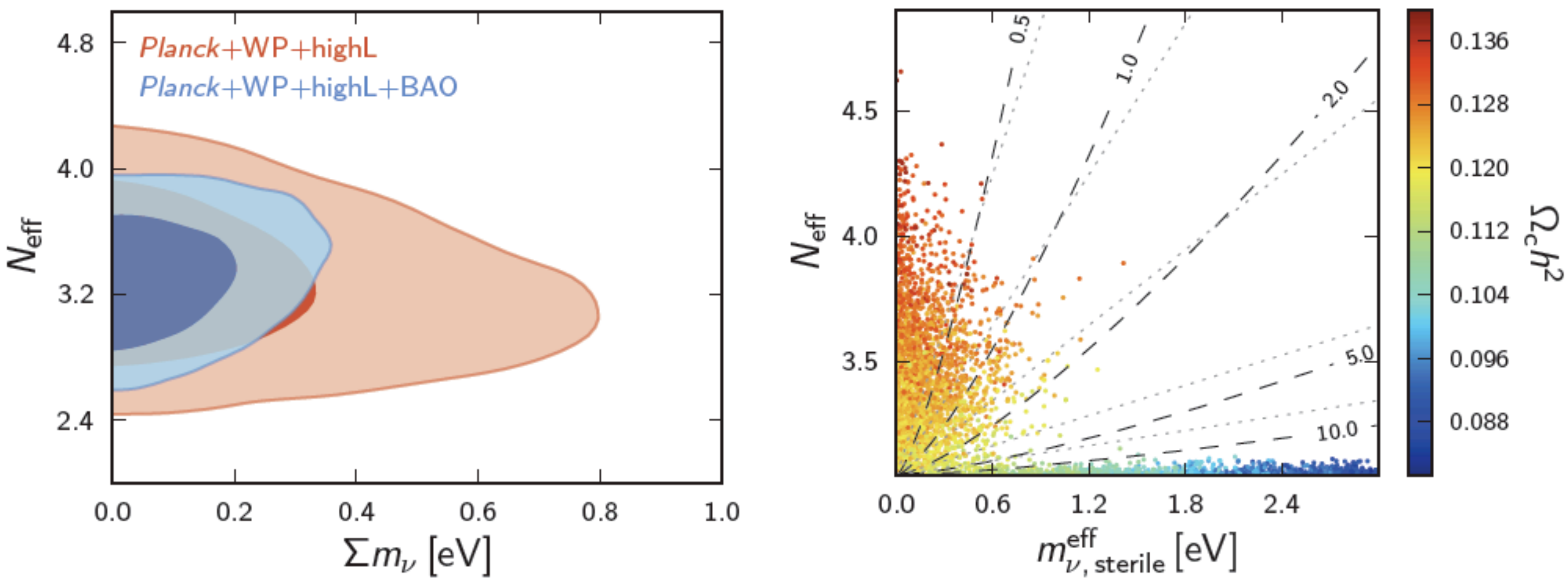


Fig. 28. *Left*: 2D joint posterior distribution between $N_{\rm eff}$ and $\sum m_\nu$ (the summed mass of the three active neutrinos) in models with extra massless neutrino-like species. *Right*: Samples in the $N_{\rm eff}$–$m^{\rm eff}_{\nu,\,\rm sterile}$ plane, colour-coded by $\Omega_c h^2$, in models with one massive sterile neutrino family, with effective mass $m^{\rm eff}_{\nu,\,\rm sterile}$, and the three active neutrinos as in the base ΛCDM model. The physical mass of the sterile neutrino in the thermal scenario, $m^{\rm thermal}_{\rm sterile}$, is constant along the grey dashed lines, with the indicated mass in eV. The physical mass in the Dodelson-Widrow scenario, $m^{\rm DW}_{\rm sterile}$, is constant along the dotted lines (with the value indicated on the adjacent dashed lines).

Sterile Neutrinos – Theory

It is easy to write down a theory for sterile neutrinos with any mass. They are gauge singlet fermions and can only couple to the SM via the *neutrino portal*, i.e., we can "only" see them because they mix with the neutrinos.

There are some technical issues one needs to deal with. We don't want the new mixing to lead to very large neutrino masses, and one has to get creative in order to add Dirac sterile neutrinos to the SM, but it is also doable [more often than not, we think about Majorana sterile neutrinos].

but...

- What are these sterile neutrinos? Who ordered that? Do they do anything?
- Why are they so light? Sterile neutrinos are "theoretically expected" to be very heavy...
- Can we say anything about the expected sterile–active neutrino mixing? Can short-baseline oscillations be predicted?
- ...

The Seesaw Lagrangian

A simple[a], renormalizable Lagrangian that allows for neutrino masses is

$$\mathcal{L}_\nu = \mathcal{L}_{\rm old} - \lambda_{\alpha i} L^\alpha H N^i - \sum_{i=1}^{3} \frac{M_i}{2} N^i N^i + H.c.,$$

where N_i ($i = 1, 2, 3$, for concreteness) are SM gauge singlet fermions.

$\mathcal{L}_\nu$ is the most general, renormalizable Lagrangian consistent with the SM gauge group and particle content, plus the addition of the N_i fields.

After electroweak symmetry breaking, $\mathcal{L}_\nu$ describes, besides all other SM degrees of freedom, six Majorana fermions: **six neutrinos.**

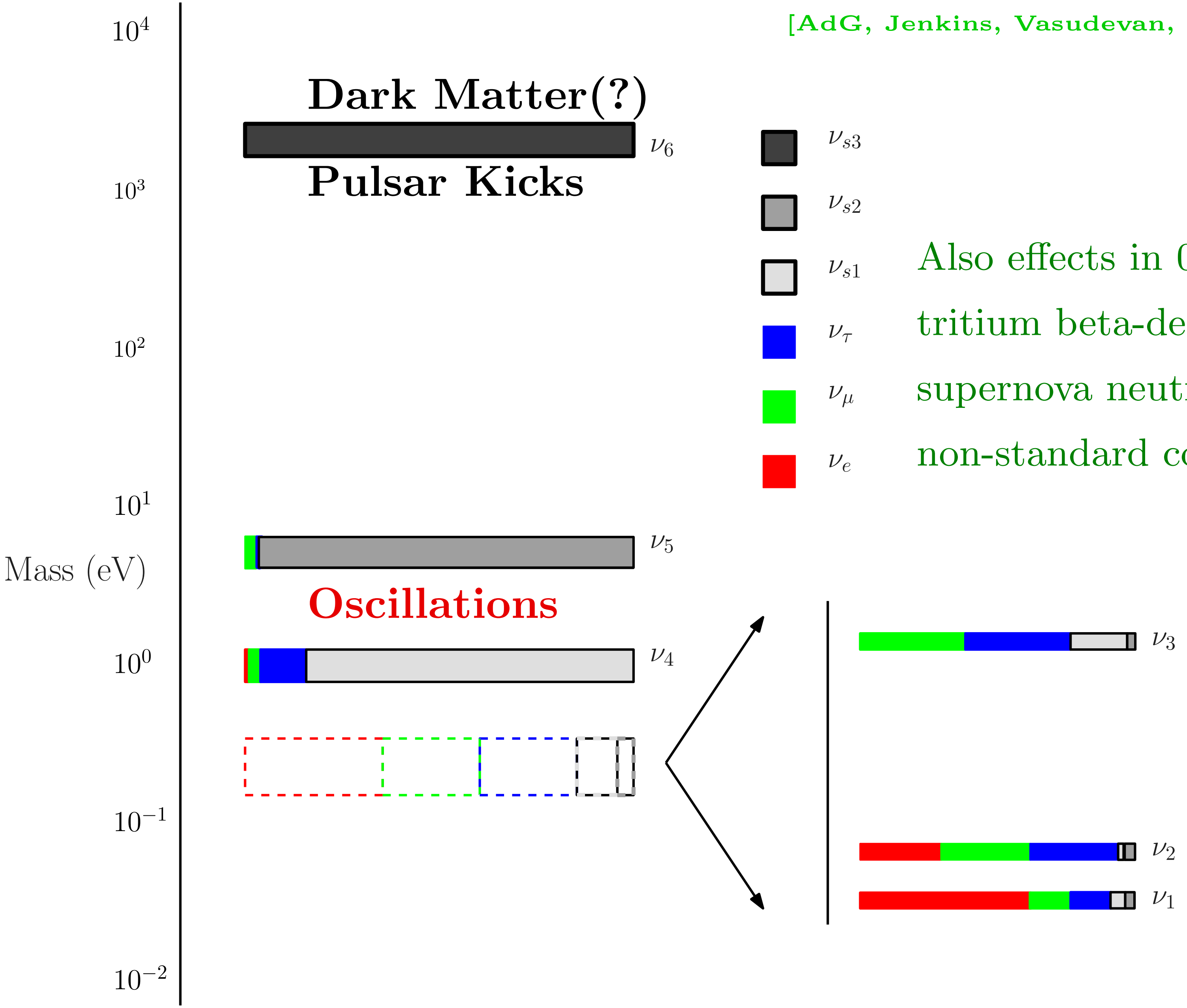
[a] Only requires the introduction of three fermionic degrees of freedom, no new interactions or symmetries.

Low-Energy Seesaw [AdG PRD72,033005)]

The other end of the M spectrum ($M \ll 100$ GeV). What do we get?

- Neutrino masses are small because the Yukawa couplings are very small $\lambda \in [10^{-6}, 10^{-11}]$;
- No standard thermal leptogenesis – right-handed neutrinos way too light? [For a possible alternative see Canetti, Shaposhnikov, arXiv: 1006.0133 and reference therein.]
- No obvious connection with other energy scales (EWSB, GUTs, etc);
- Right-handed neutrinos are propagating degrees of freedom. They look like sterile neutrinos $\Rightarrow$ sterile neutrinos associated with the fact that the active neutrinos have mass;
- sterile–active mixing can be predicted – hypothesis is falsifiable!
- Small values of M are natural (in the 'tHooft sense). In fact, theoretically, no value of M should be discriminated against!

[AdG, Jenkins, Vasudevan, PRD75, 013003 (2007)]



Also effects in $0\nu\beta\beta$,
tritium beta-decay,
supernova neutrino oscillations,
non-standard cosmology.

More Details, assuming three right-handed neutrinos N:

$$m_\nu = \left(\begin{array}{cc} 0 & \lambda v \\ (\lambda v)^t & M \end{array} \right),$$

M is diagonal, and all its eigenvalues are real and positive. The charged lepton mass matrix also diagonal, real, and positive.

To leading order in $(\lambda v)M^{-1}$, the three lightest neutrino mass eigenvalues are given by the eigenvalues of

$$m_a = \lambda v M^{-1} (\lambda v)^t,$$

where m_a is the mostly active neutrino mass matrix, while the heavy sterile neutrino masses coincide with the eigenvalues of M.

6×6 mixing matrix U $[U^t m_\nu U = \text{diag}(m_1, m_2, m_3, m_4, m_5, m_6)]$ is

$$U = \begin{pmatrix} V & \Theta \\ -\Theta^\dagger V & 1_{n\times n} \end{pmatrix},$$

where V is the active neutrino mixing matrix (MNS matrix)

$$V^t m_a V = \text{diag}(m_1, m_2, m_3),$$

and the matrix that governs active–sterile mixing is

$$\Theta = (\lambda v)^* M^{-1}.$$

One can solve for the Yukawa couplings and re-express

$$\Theta = V\sqrt{\text{diag}(m_1, m_2, m_3)}R^\dagger M^{-1/2},$$

where R is a complex orthogonal matrix $RR^t = 1$.

Model independent constraints

Constraints depend, unfortunately, on m_i and M_i and R. E.g.,

$$U_{e4} = U_{e1}A\sqrt{\frac{m_1}{m_4}} + U_{e2}B\sqrt{\frac{m_2}{m_4}} + U_{e3}C\sqrt{\frac{m_3}{m_4}},$$

$$U_{\mu 4} = U_{\mu 1}A\sqrt{\frac{m_1}{m_4}} + U_{\mu 2}B\sqrt{\frac{m_2}{m_4}} + U_{\mu 3}C\sqrt{\frac{m_3}{m_4}},$$

$$U_{\tau 4} = U_{\tau 1}A\sqrt{\frac{m_1}{m_4}} + U_{\tau 2}B\sqrt{\frac{m_2}{m_4}} + U_{\tau 3}C\sqrt{\frac{m_3}{m_4}},$$

where

$$A^2 + B^2 + C^2 = 1.$$

One can pick A, B, C such that two of these vanish. But the other one is maximized, along with $U_{\alpha 5}$ and $U_{\alpha 6}$.

Can we (a) constrain the seesaw scale with combined bounds on $U_{\alpha 4}$ or (b) test the low energy seesaw if nonzero $U_{\alpha 4}$ are discovered?

AdG, Huang arXiv:1110.6122

Concrete Example: 2 right-handed neutrinos

$$X_{\rm normal} = \begin{pmatrix} 0.23e^{i\phi} & 0.1e^{i\delta} \\ (0.25-0.02e^{-i\delta})e^{i\phi} & 0.70 \\ -(0.25+0.02e^{-i\delta})e^{i\phi} & 0.70 \end{pmatrix} \begin{pmatrix} \cos\zeta & \sin\zeta \\ -\sin\zeta & \cos\zeta \end{pmatrix}$$

$$X_{\rm inverted} = \begin{pmatrix} 0.83e^{i\psi} & 0.55 \\ -(0.39+0.06e^{-i\delta})e^{i\psi} & 0.59-0.04e^{-i\delta} \\ (0.39-0.06e^{-i\delta})e^{i\psi} & -0.59-0.04e^{-i\delta} \end{pmatrix} \begin{pmatrix} \cos\zeta & \sin\zeta \\ -\sin\zeta & \cos\zeta \end{pmatrix}$$

$$\zeta \in C$$

where

$$X_{\rm normal\ (inverted)} = \Theta\sqrt{\frac{m_{\rm heavy}}{m_3\ \ (m_2)}}$$

Some Relevant Examples: [AdG, W-C Huang, arXiv:1110.6122]

$\zeta = 3/4\pi + i$, $\delta = 6/5\pi$, $\phi = \pi/2$ and a normal mass hierarchy,

$$X_{\rm normal} = \left(\begin{array}{cc} 0.41e^{-0.66i} & 0.45e^{1.03i} \\ 0.62e^{2.67i} & 0.61e^{-2.62i} \\ 1.27e^{2.44i} & 1.26e^{-2.41i} \end{array} \right).$$

$\zeta = 2/3\pi + 0.3i$, $\delta = 0$, $\psi = \pi/2$, and an inverted mass hierarchy,

$$X_{\rm inverted} = \left(\begin{array}{cc} 0.44e^{-2.24i} & 0.62e^{1.83i} \\ 0.69e^{2.66i} & 0.66e^{-2.14i} \\ 0.71e^{-0.39i} & 0.60e^{0.89i} \end{array} \right).$$

both accommodate 3+2 fit for $m_4^2 = 0.5$ eV^2 and $m_5^2 = 0.9$ eV^2. Furthermore, $|U_{\tau 4}|$ and $|U_{\tau 5}|$ are completely fixed. No more free parameters. They are also both larger than (or at least as large as $|U_{\mu 4}|$ and $|U_{\mu 5}|$).

$\nu_\mu \to \nu_\tau$ MUST be observed if this is the origin of the two mostly sterile neutrinos.

Making Predictions, for an inverted mass hierarchy, $m_4 = 1$ eV($\ll m_5$)

[AdG, Huang, 1110.6122]

- ν_e disappearance with an associated effective mixing angle $\sin^2 2\vartheta_{ee} > 0.02$. An interesting new proposal to closely expose the Daya Bay detectors to a strong β-emitting source would be sensitive to $\sin^2 2\vartheta_{ee} > 0.04$;
- ν_μ disappearance with an associated effective mixing angle $\sin^2 2\vartheta_{\mu\mu} > 0.07$, very close to the most recent MINOS lower bound;
- $\nu_\mu \leftrightarrow \nu_e$ transitions with an associated effective mixing angle $\sin^2 \vartheta_{e\mu} > 0.0004$;
- $\nu_\mu \leftrightarrow \nu_\tau$ transitions with an associated effective mixing angle $\sin^2 \vartheta_{\mu\tau} > 0.001$. A $\nu_\mu \to \nu_\tau$ appearance search sensitive to probabilities larger than 0.1% for a mass-squared difference of 1 eV^2 would definitively rule out $m_4 = 1$ eV if the neutrino mass hierarchy is inverted.

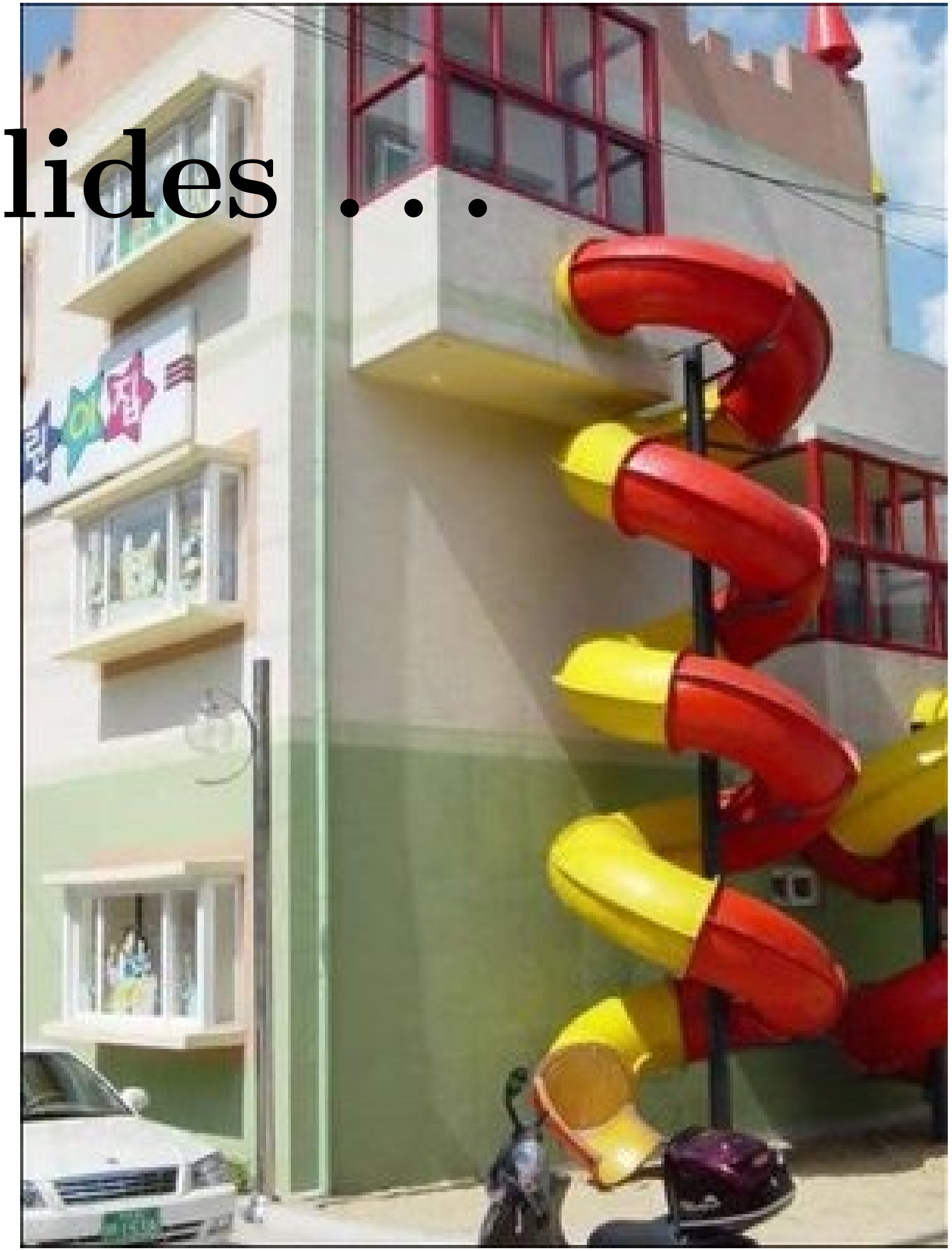
Concluding Statements

1. We know very little about the new physics uncovered by neutrino oscillations.

   - It could be renormalizable → “boring” Dirac neutrinos
   - It could be due to Physics at absurdly high energy scales $M \gg 1$ TeV → high energy seesaw. How can we ever convince ourselves that this is correct?
   - It could be due to very light new physics → low energy seesaw. Prediction: new light propagating degrees of freedom – sterile neutrinos
   - It could be due to new physics at the TeV scale → either weakly coupled, or via a more subtle lepton number breaking sector. Predictions: charged lepton flavor violation, collider signatures!

2. We need more experimental input!

Backup Slides . . .



What We Know About M:

- $M=0$: the six neutrinos "fuse" into three Dirac states. Neutrino mass matrix given by $\mu_{\alpha i}\equiv\lambda_{\alpha i}v$.

  The symmetry of $\mathcal{L}_\nu$ is enhanced: $U(1)_{B-L}$ is an exact global symmetry of the Lagrangian if all M_i vanish. Small M_i values are 'tHooft natural.

- $M\gg\mu$: the six neutrinos split up into three mostly active, light ones, and three, mostly sterile, heavy ones. The light neutrino mass matrix is given by $m_{\alpha\beta}=\sum_i\mu_{\alpha i}M_i^{-1}\mu_{\beta i}$ $[m\propto 1/\Lambda \ \Rightarrow \ \Lambda=M/\mu^2]$.

  This the **seesaw mechanism.** Neutrinos are Majorana fermions. Lepton number is not a good symmetry of $\mathcal{L}_\nu$, even though L-violating effects are hard to come by.

- $M\sim\mu$: six states have similar masses. Active–sterile mixing is very large. This scenario is (generically) ruled out by active neutrino data (atmospheric, solar, KamLAND, K2K, etc).

- $M\ll\mu$: neutrinos are quasi-Dirac fermions. Active–sterile mixing is maximal, but new oscillation lengths are very long (*cf.* 1 A.U.).

(

Why are Neutrino Masses Small in the $M \neq 0$ Case?

If $\mu \ll M$, below the mass scale M,

$$\mathcal{L}_5 = \frac{LHLH}{\Lambda}.$$

Neutrino masses are small if $\Lambda \gg \langle H \rangle$. Data require $\Lambda \sim 10^{14}$ GeV.

In the case of the seesaw,

$$\Lambda \sim \frac{M}{\lambda^2},$$

so neutrino masses are small if either

- they are generated by physics at a very high energy scale $M \gg v$ (high-energy seesaw); or
- they arise out of a very weak coupling between the SM and a new, hidden sector (low-energy seesaw); or
- cancellations among different contributions render neutrino masses accidentally small ("fine-tuning").

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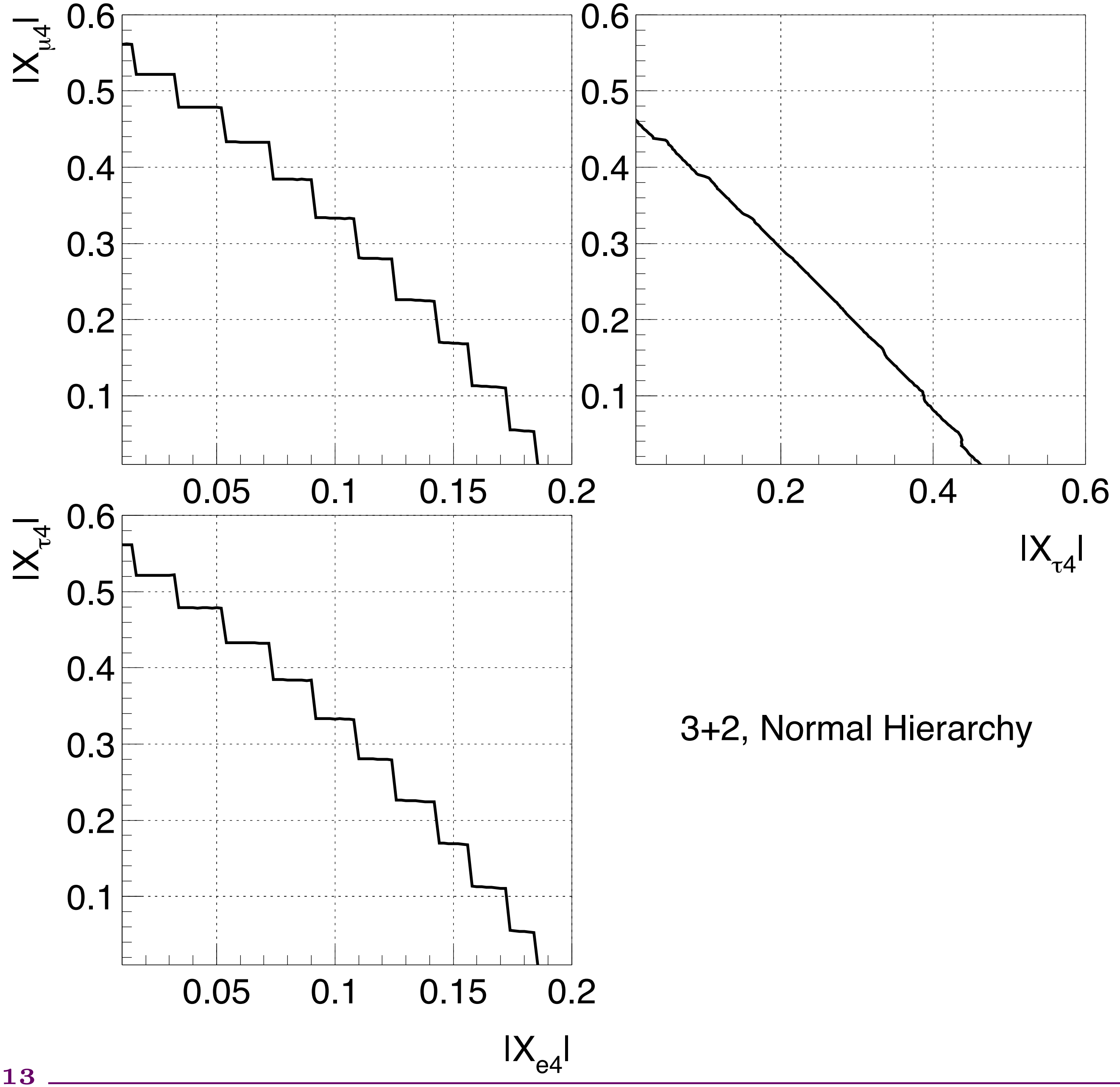

|X_μ4|
0.6
0.5
0.4
0.3
0.2
0.1
0.05
0.1
0.15
0.2
0.2
0.4
0.6
|X_τ4|
|X_τ4|
|X_e4|
3+2, Normal Hierarchy