Analysis of Cooling Lattices

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Fixed Point and Map

- Use ICOOL to find
  - Fixed point for one period
  - Linear map about that fixed point
- 4-D and 6-D
  - 4-D at fixed energy
    - As a function of energy
    - RF off, no absorbers
  - 6-D
    - RF on, absorbers in place
    - No stochastics
- Use 4-D results to do dynamic aperture scan vs. energy

Fixed Point and Map

- Information from 4-D
  - Energy-dependent closed orbit
  - Dispersion
  - Beta functions vs. energy
  - Tunes (transverse) vs. energy
  - Time of flight vs. energy
- Information from 6-D
  - Damping (growth) rates for modes
  - Energy of fixed point

Math

- From linear map M, find A such that $MA = AR$, R diagonal blocks like:

$$\begin{bmatrix} \lambda\cos\mu & \lambda\sin\mu \\ -\lambda\sin\mu & \lambda\cos\mu \end{bmatrix}$$

- μ phase advance, λ gives damping/growth
- A normalized to $\boldsymbol{a}_{2k-1}^T \boldsymbol{J} \boldsymbol{a}_{2k} = 1$
  - $\boldsymbol{a}_k$, $1 \leqslant k \leqslant 6$ are columns of A
- Dynamic aperture scan

$$\boldsymbol{z} = \sqrt{2J_1}\boldsymbol{a}_1 + \sqrt{2J_2}\boldsymbol{a}_3$$

- Beta functions

$$\beta_k = a_{1,2k-1}^2 + a_{1,2k}^2 + a_{3,2k-1}^2 + a_{3,2k}^2$$

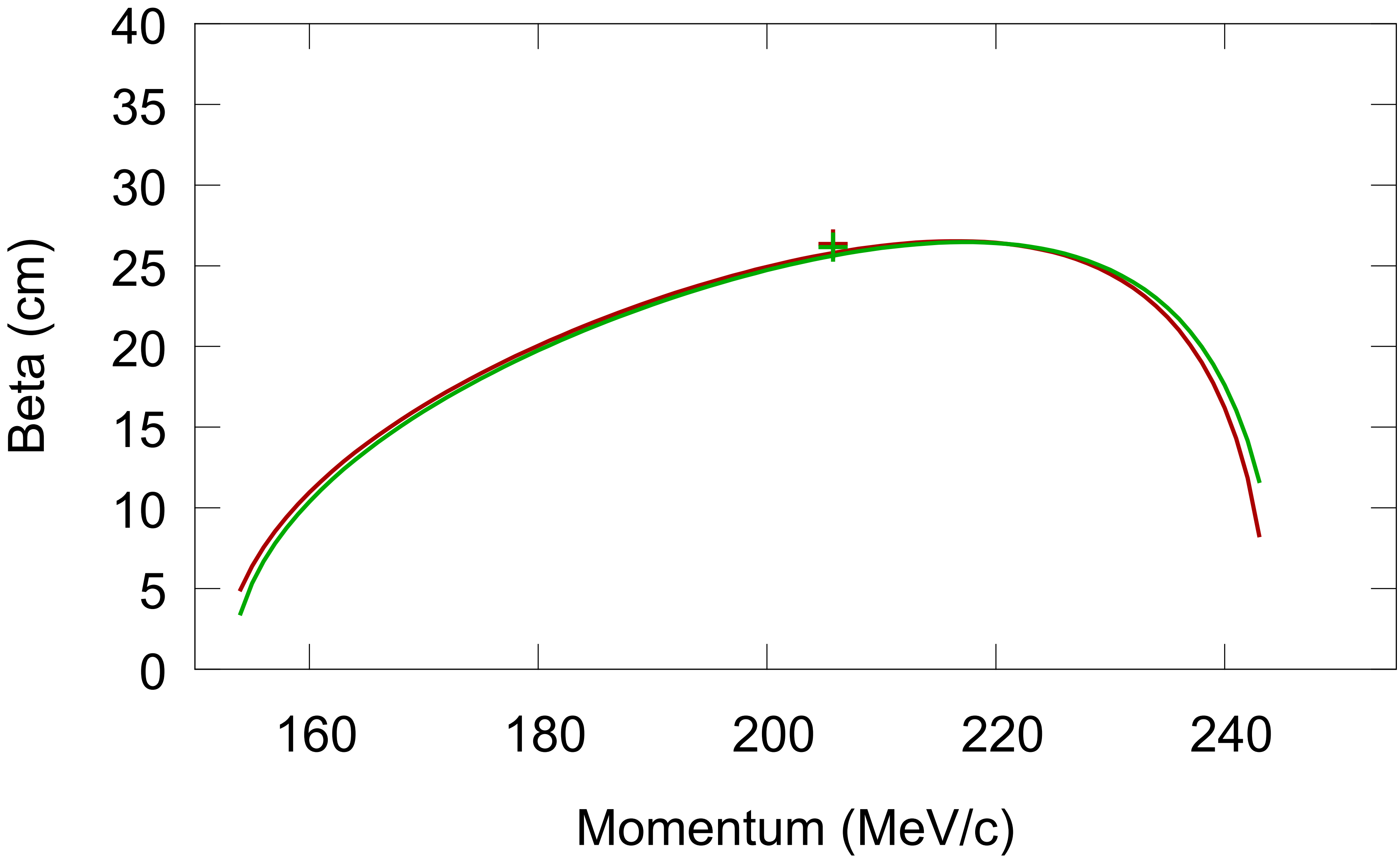
Lattices

- All operate between π and 2π resonances for single cell
- Guggenheim and rectilinear FOFO:
  - Single sinusoidal oscillation of solenoid field in cell
  - Solenoids tilted to give dipole field
  - Wedge absorbers couple to dispersion
  - Guggenheim bends, rectilinear FOFO straight
    - Closed orbit moves in Rectilinear FOFO, uses solenoid end fields to counter dipole
- Planar snake
  - Solenoid field opposite in adjacent cells
  - Two-cell period
  - Planar absorbers: momentum dispersion

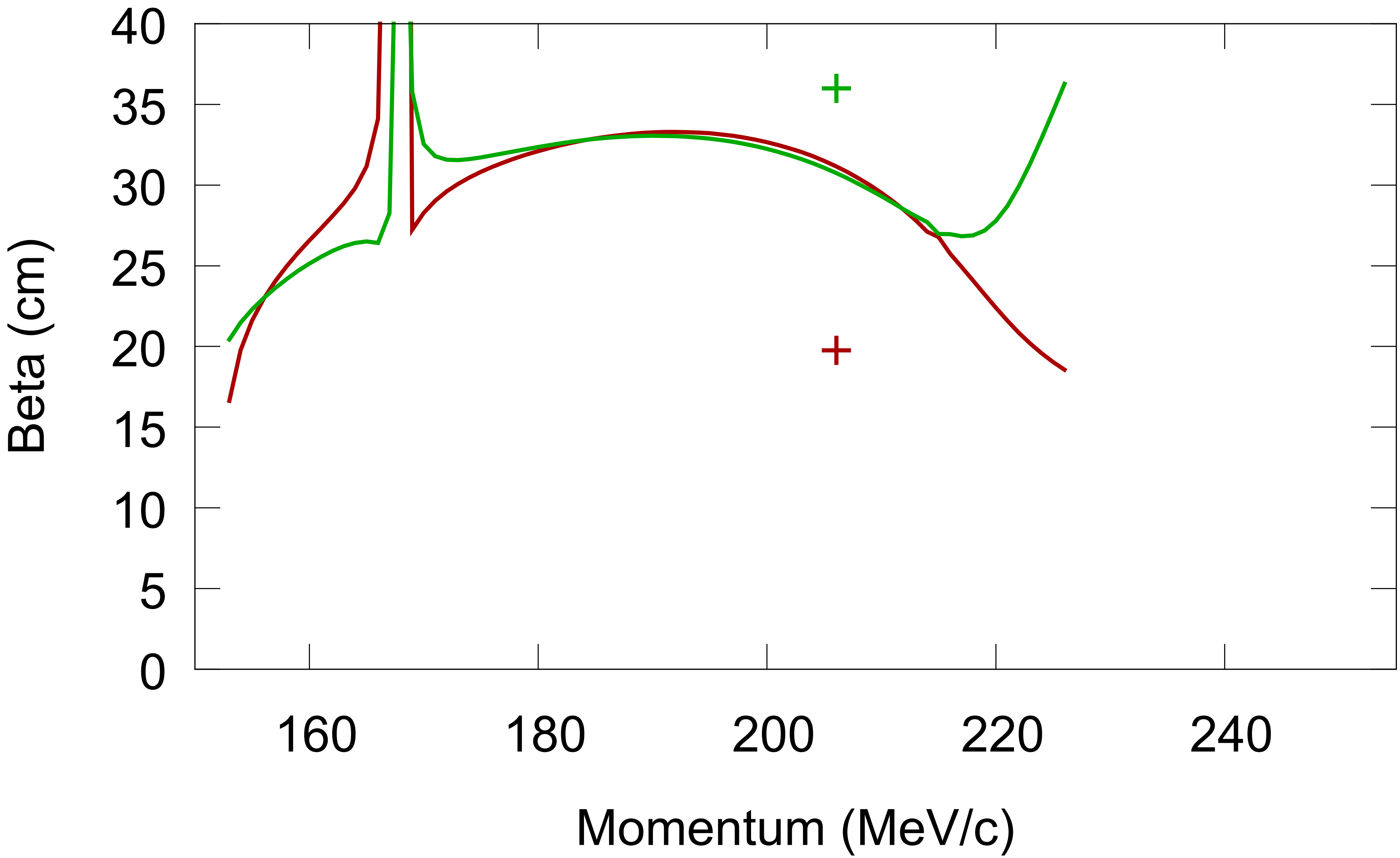
Beta Functions

- Find β vs. energy at fixed energy
- Also find 6-D fixed point, and beta functions
- Guggenheim and Rectilinear FOFO
  - Betas for 6-D close to fixed energy
- Planar snake
  - Resonance appears at 3π two-cell phase advance
  - Betas for 6-D very different from fixed energy
  - Appear to have coupling resonance from 212–215 MeV/c

Betas: Rectilinear FOFO



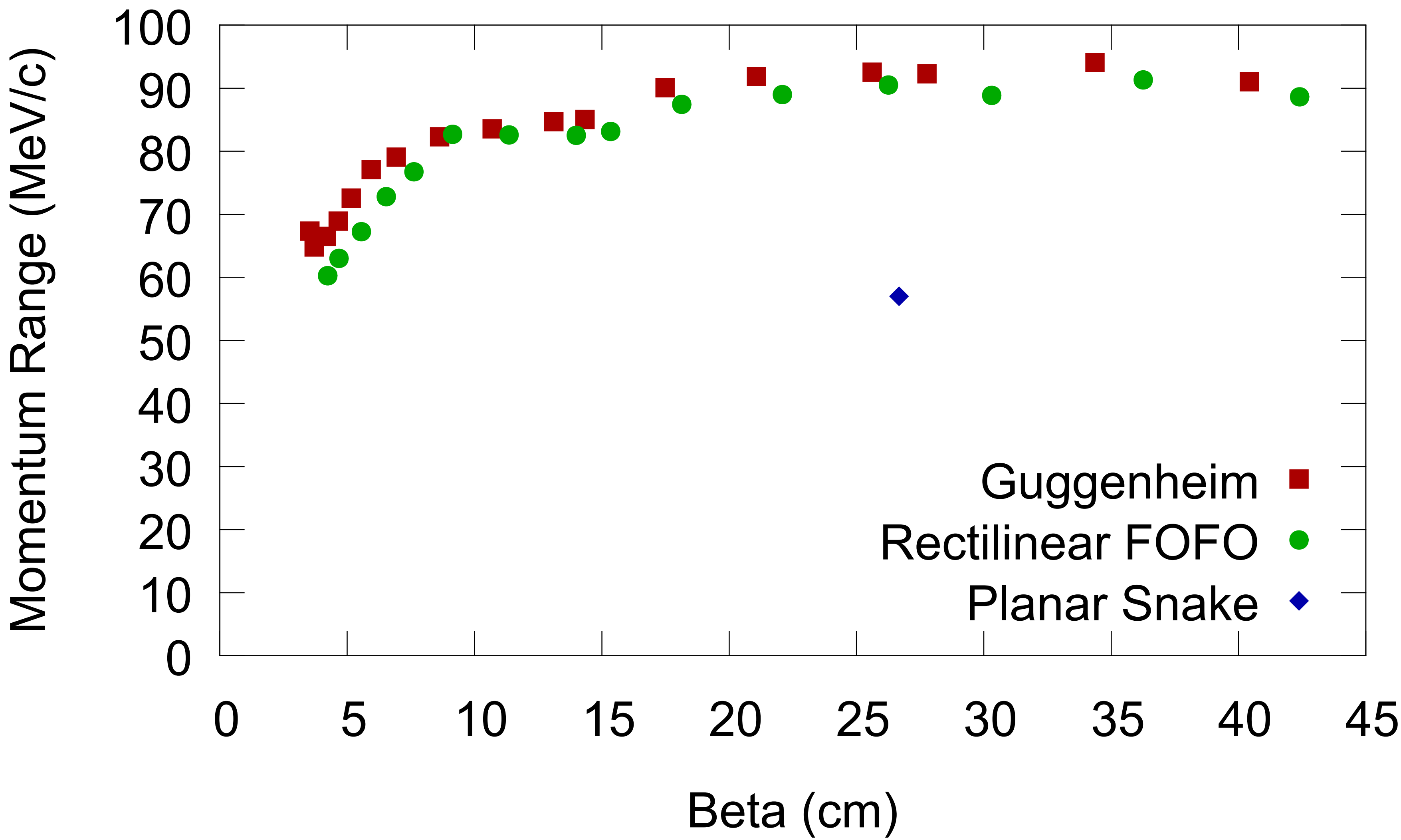
Betas: Planar Snake



Energy Acceptance

- Energy acceptance
  - Drops when lattices no longer scale
  - Worse for Rectilinear FOFO at that point
  - Poor for planar snake

Energy Acceptance



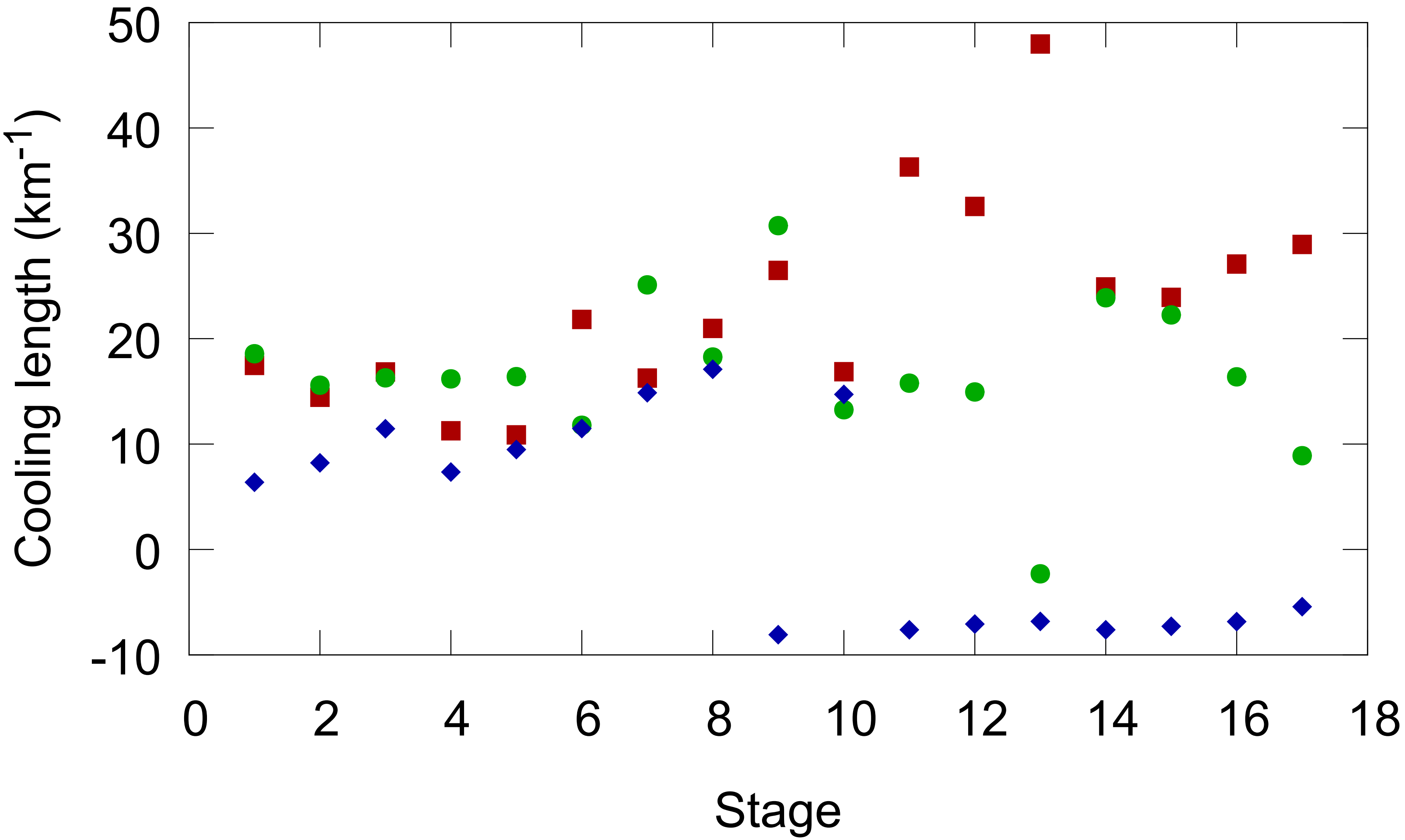
Planar Snake Eigensystem

- All tunes have fractional part of 0.25
- Two planes strongly coupled
  - Eigenellipses have nonzero area in horizontal and longitudinal
  - Areas about equal
  - Compare dispersion
    - Eigenellipses have projections into other plane
    - But projected as a line, not an ellipse
- Thus sitting on a synchro-betatron resonance
  - This is the source of the apparent dispersion Bob sees
  - Makes lattice performance sensitive to design
  - 3π resonance may not be limiting

Guggenheim 6-D Eigenvalues

- Look at eigenvalue magnitudes
- Some eigenvalues are unstable longitudinally
  - Closed orbit on side of “house” shaped wedge
  - Still see wedge on average, but smaller slope
  - Less nonlinear if use slope, modest length penalty
- Eigenvalue split for stage 13

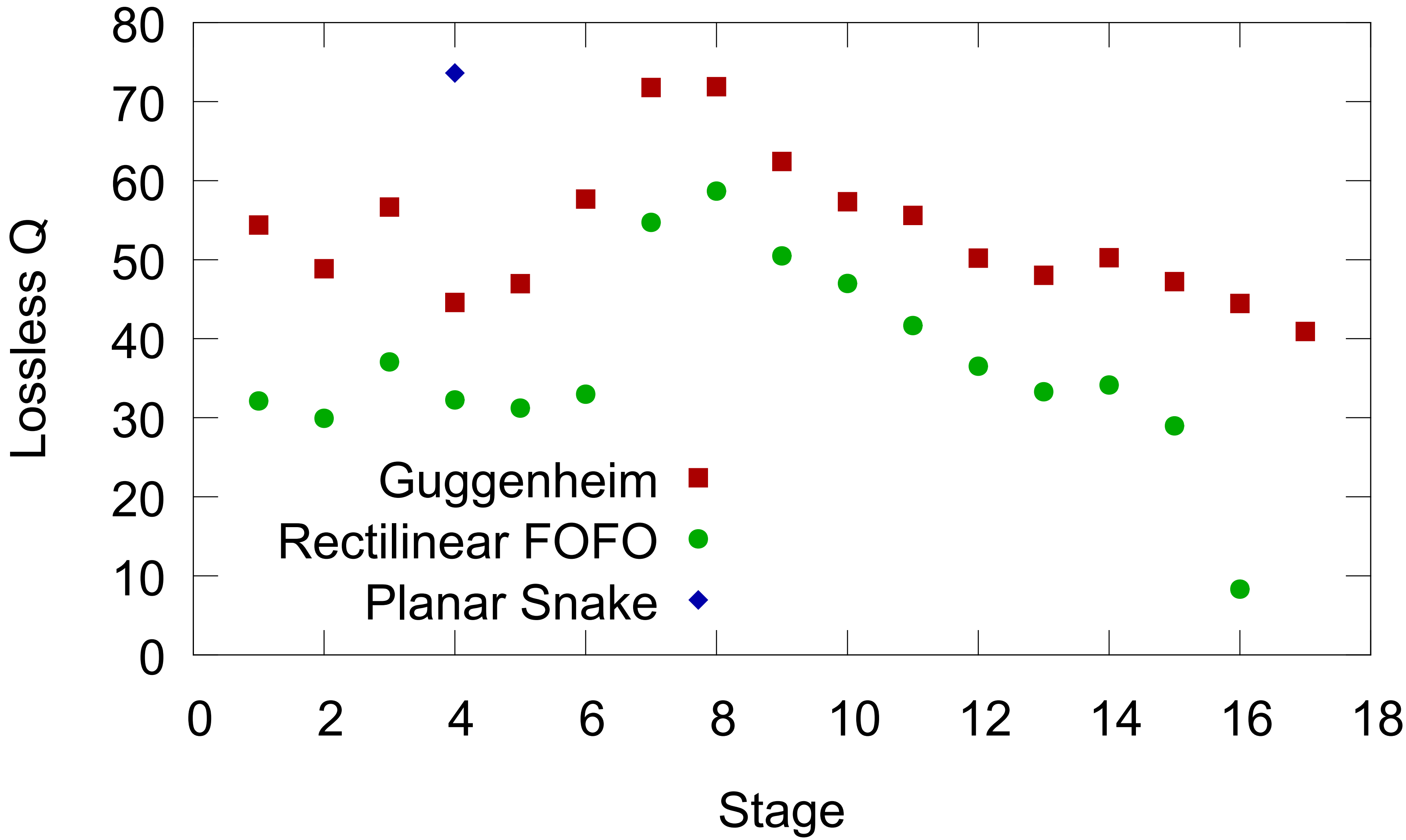
Guggenheim Eigenvalue Magnitude



Lossless Q

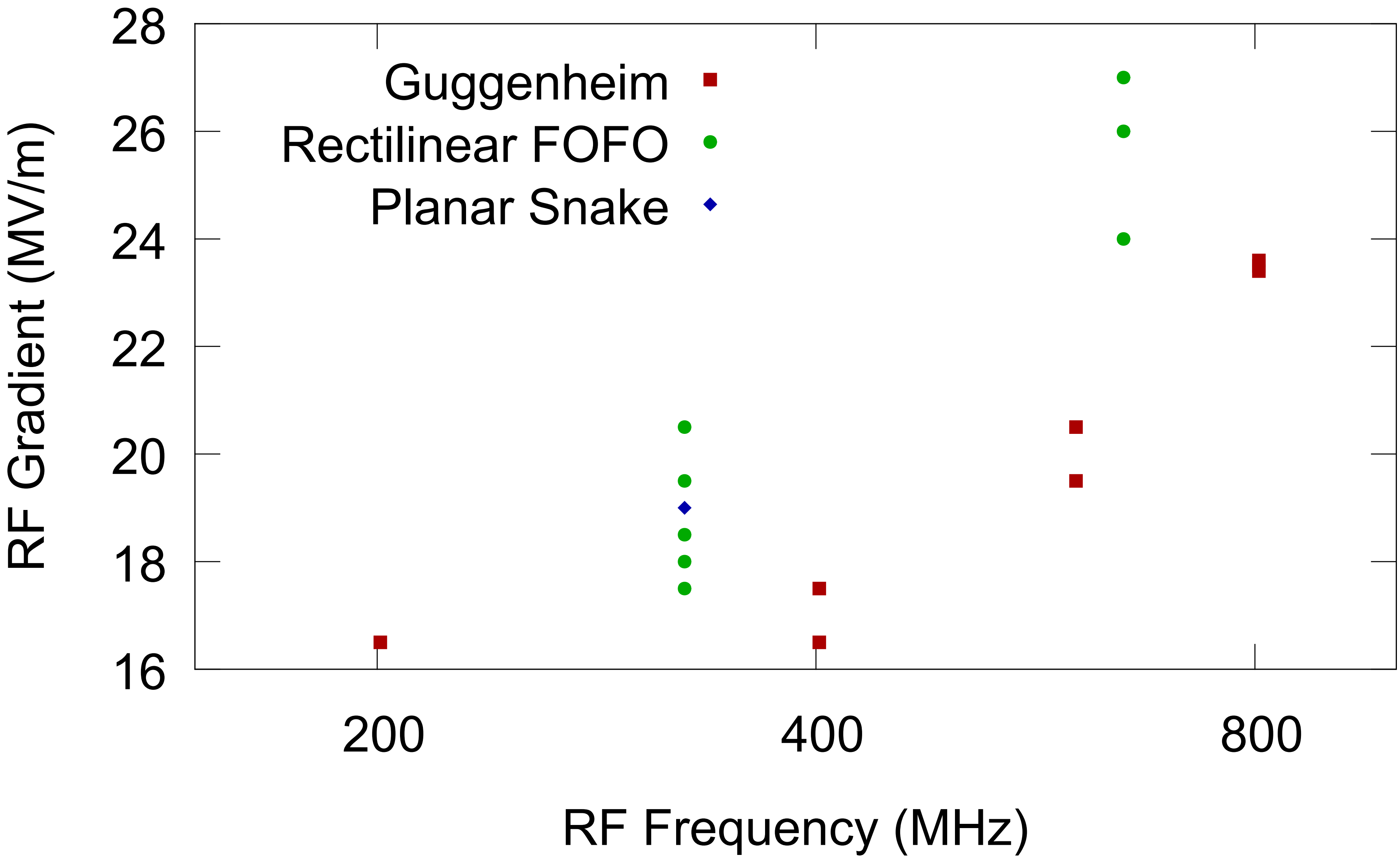
- Q definition: $(d\varepsilon_6/\varepsilon_6)/(dN/N)$
- Ignore performance reduction other then decays:
  - Scattering/straggling
    - Approach to equilibrium
    - Particles kicked outside dynamic aperture
  - Non-stochastic dynamic losses
- Only eigenvalues, energy, length matter
- Rectilinear FOFO significantly worse
  - Despite high gradients
  - Could explain worse performance
  - Should be fixable: more energy loss

Lossless Q



Lossless Q
80
70
60
50
40
30
20
10
0
0
2
4
6
8
10
12
14
16
18
Stage
Guggenheim
Rectilinear FOFO
Planar Snake

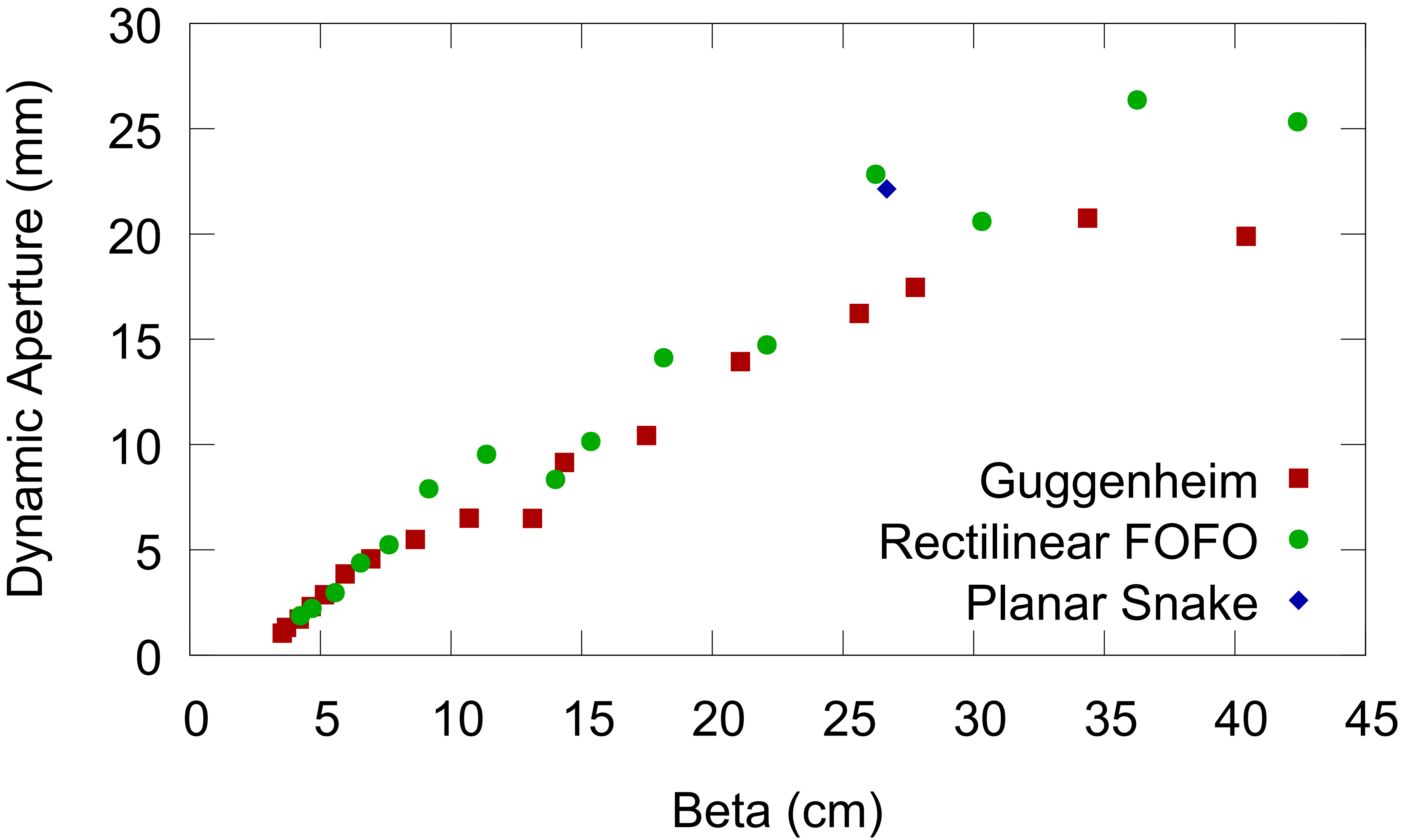
RF Gradients



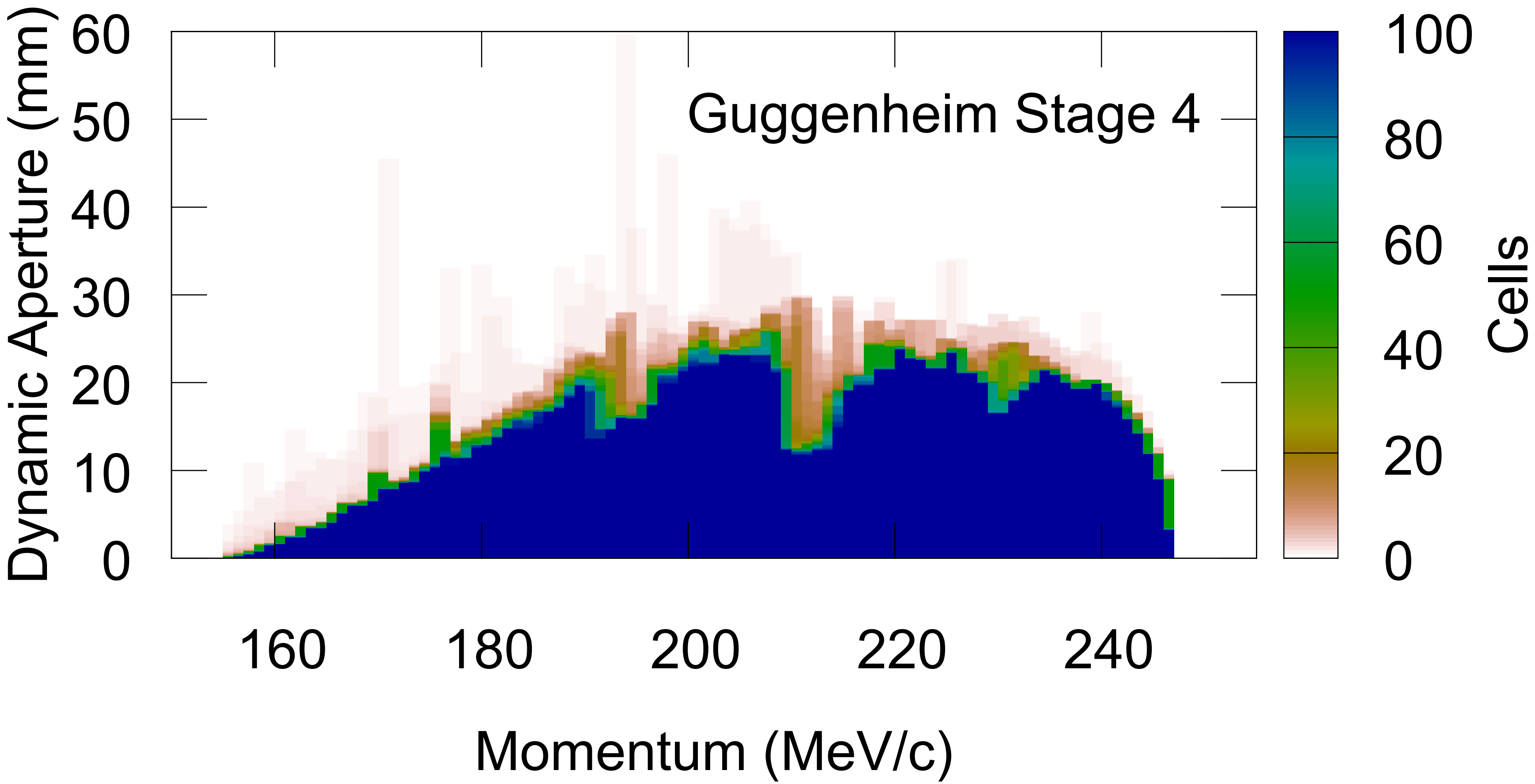
Dynamic Aperture

- Once lattices no longer scale, dynamic aperture drops drops faster than proportional to beta
- Much stronger effect than mometum acceptance reduction
- Every lattice exhibits resonance
  - Near tune of 0.75
  - May not have major impact
- Rectilinear FOFO has “softer” dynamic aperture
- Planar snake: dynamic aperture not approaching zero at high energy
- Some of this looks decidedly non-symplectic

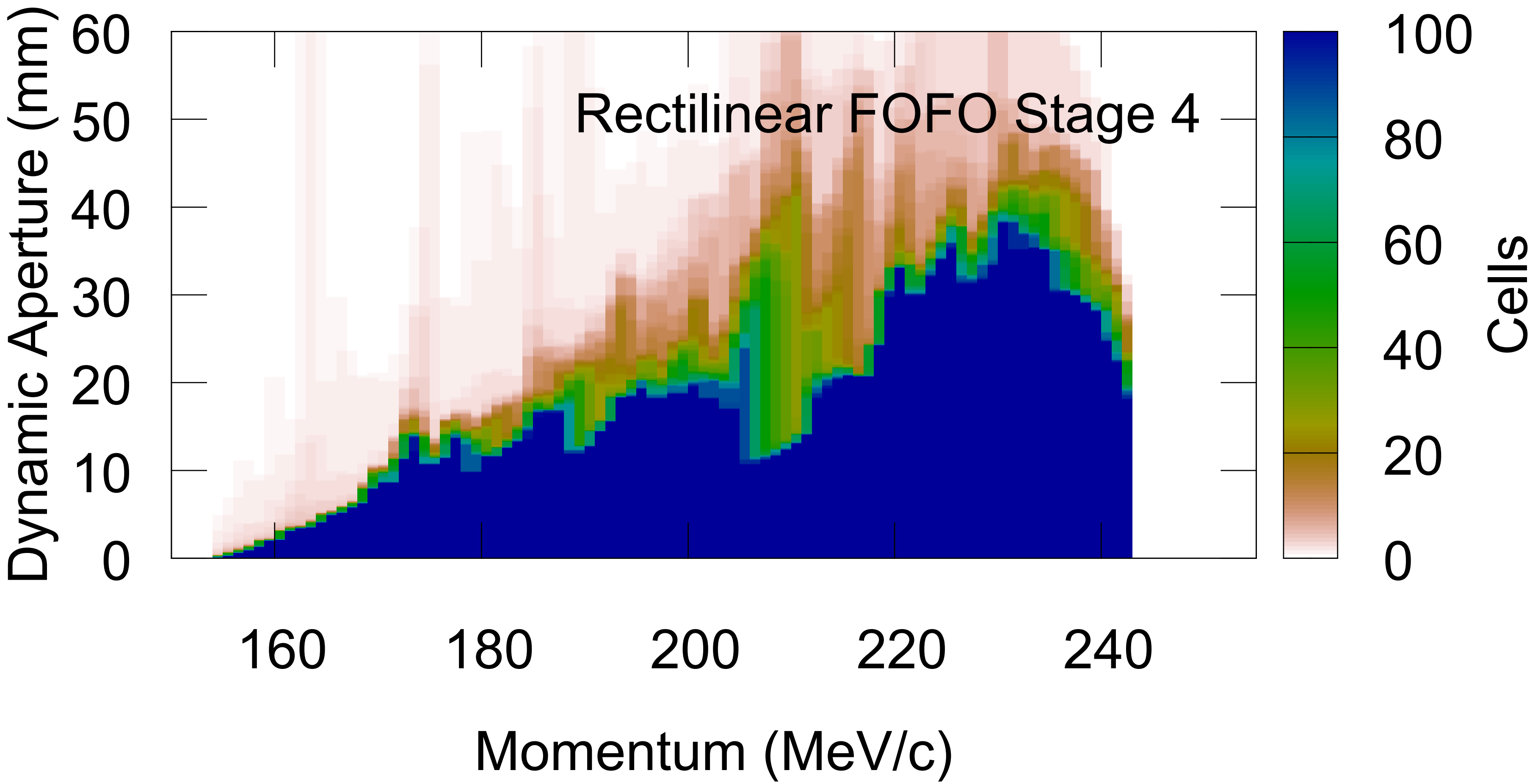
Dynamic Aperture



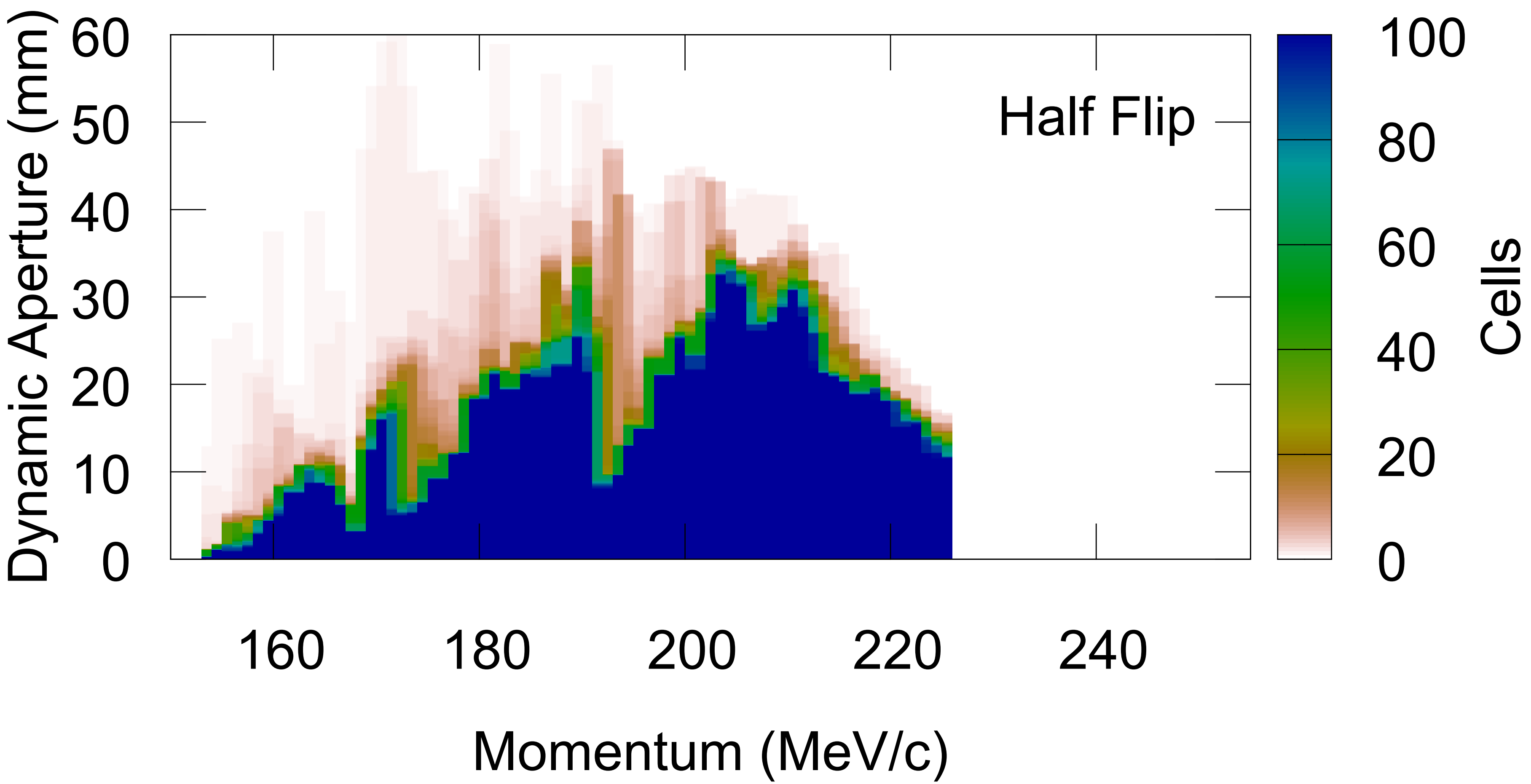
Dynamic Aperture



Dynamic Aperture



Dynamic Aperture



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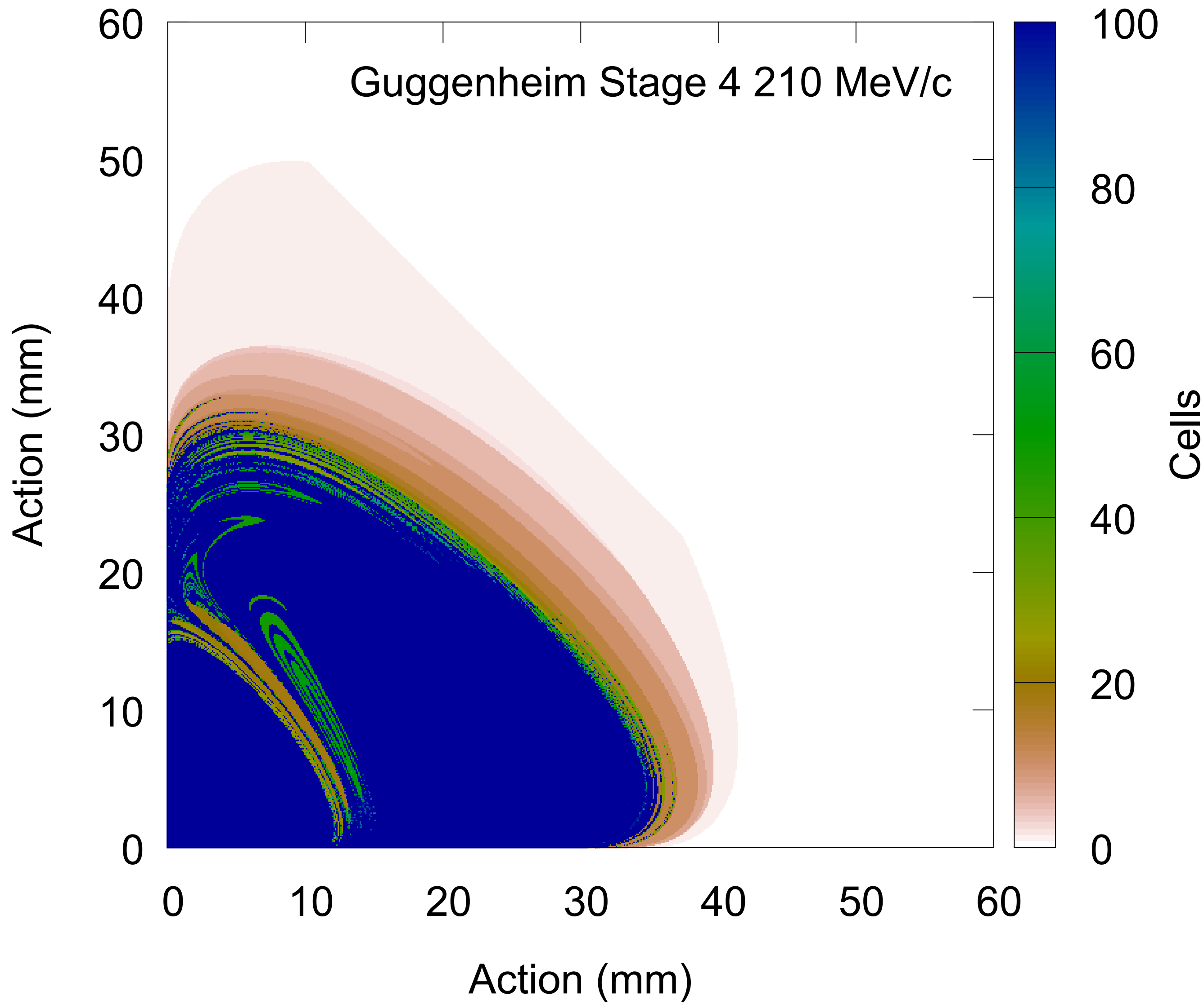

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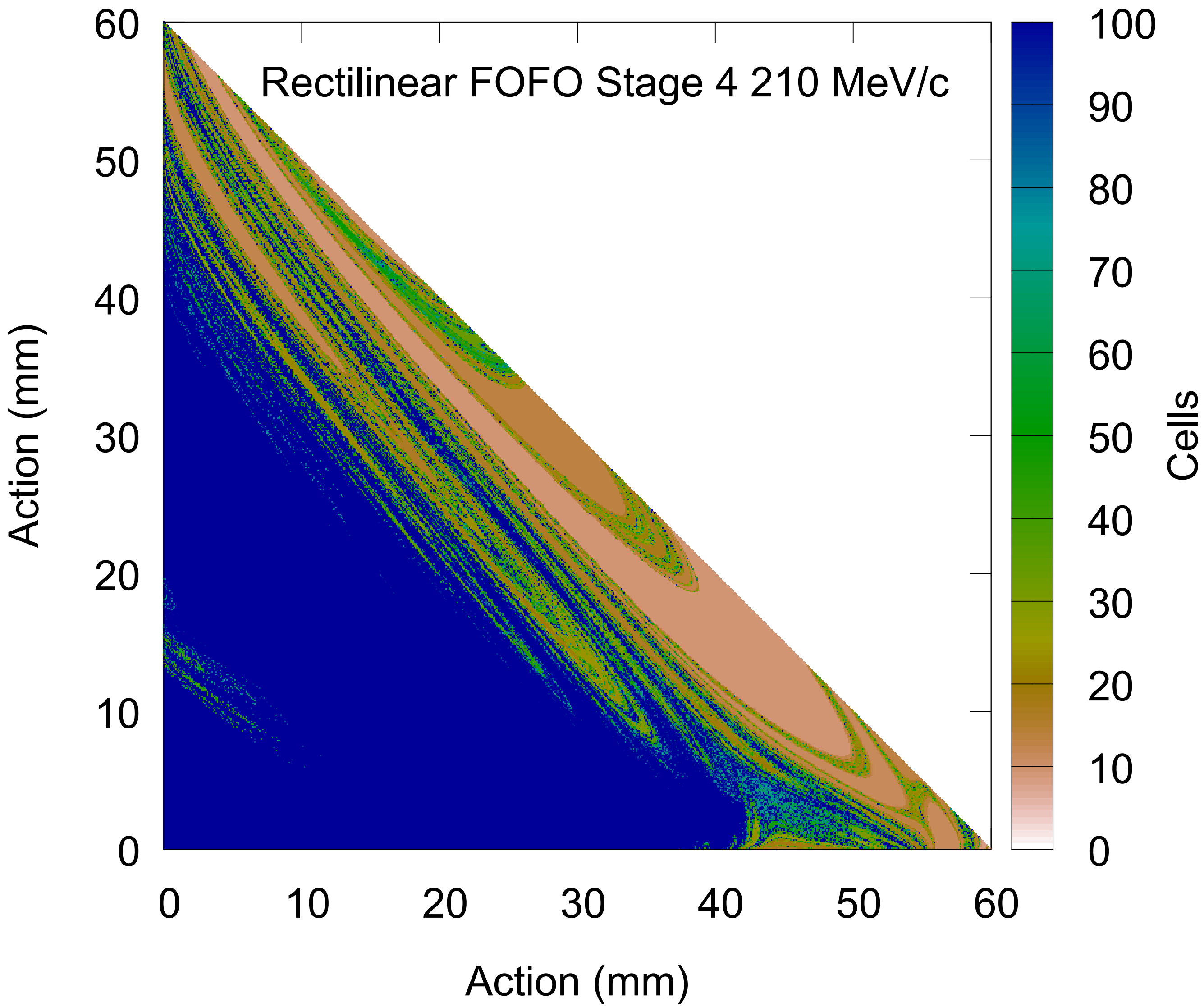

Practical Results

- Guggenheim might improve by using slope of wedge, not side
- Rectilinear FOFO should do better by going through more absorber
- Planar snake using synchro-betatron resonance
  - Probably difficult to achieve over wide parameter range
- Energy acceptance of planar snake very limited
- Transverse dynamic aperture probably the primary performance limitation for low β (as lattices no longer scaled down with β)

Additional Analysis

- Can look at the RF bucket and how it relates to the transverse passband
- Look at dynamic aperture in 6-D
- Study what gives some of the structures we see
  - Resonance near 0.75
  - Transverse coupling resonance in half-flip
  - Details of synchro-betatron resonance in half-flip
- Understand eigenvalue split in stage 13 Guggenheim
- Identify if non-symplectic behavior important

RF Gradients

- Need consistent value for comparison
- Cavity lengths also matter
- Propose consistent values
  - consistent with 17 MV/m at 201.25 MHz

Freq. MHz	Length cm	Grad MV/m	ΔE $v = c$ MeV	ΔE 200 MeV/c MeV
325	30	22	5.51	5.23
650	15	31	3.88	3.68
975	10	38	3.17	3.01