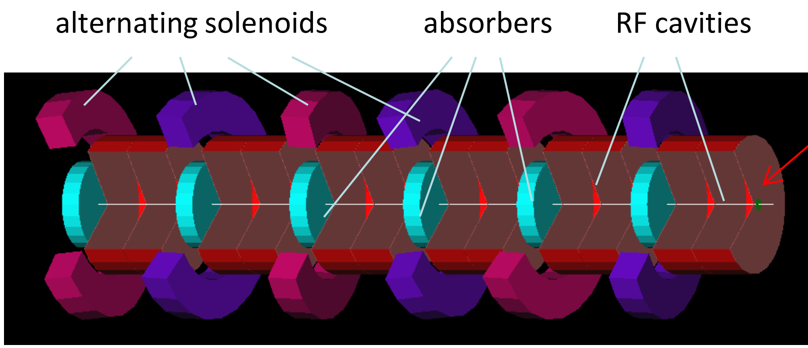
325 MHz Helical FOFO Snake for Initial Stage of 6D Ionization Cooling

Y. Alexahin
(FNAL APC)

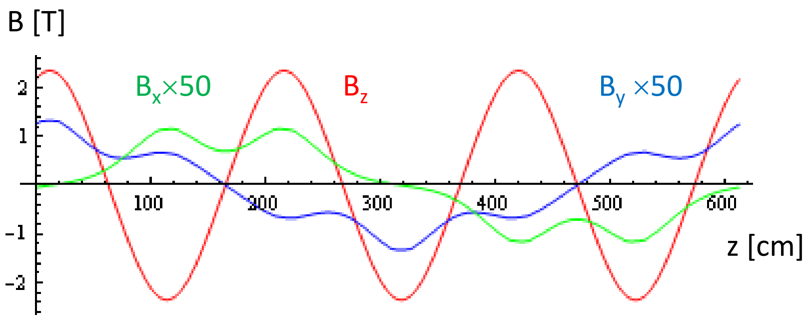
Disclaimer: the present design is probably far from the optimum, my goal was to develop and check principal solutions

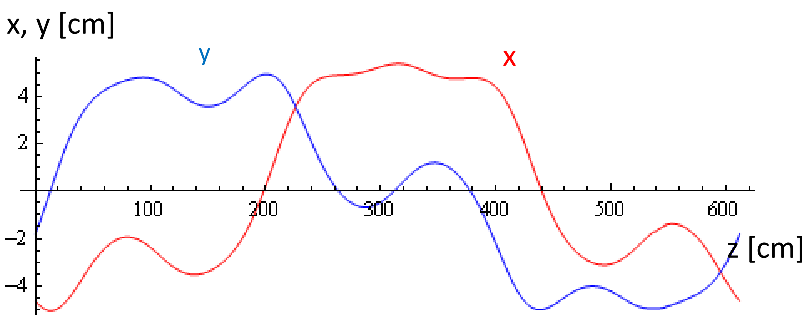
Basic Idea



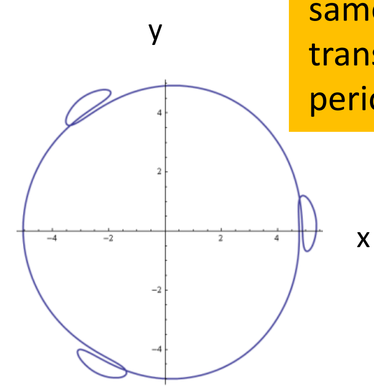
There was a hope that placing RF between solenoids will help to increase maximum gradient – not confirmed thus far

- The idea: create rotating $B_{\perp}$ field by periodically tilting solenoids, e.g. with 6-solenoid period.
- Periodic orbits for μ+ and μ- look exactly the same, just shifted by a half period (3 solenoids).
- With tune $Q_{\perp}$>1 (per period) $\mathbf{r}\cdot\mathbf{D}>0$ $\Rightarrow$ muons with higher momentum make a longer path $\Rightarrow$ longitudinal cooling achieved even with planar absorbers



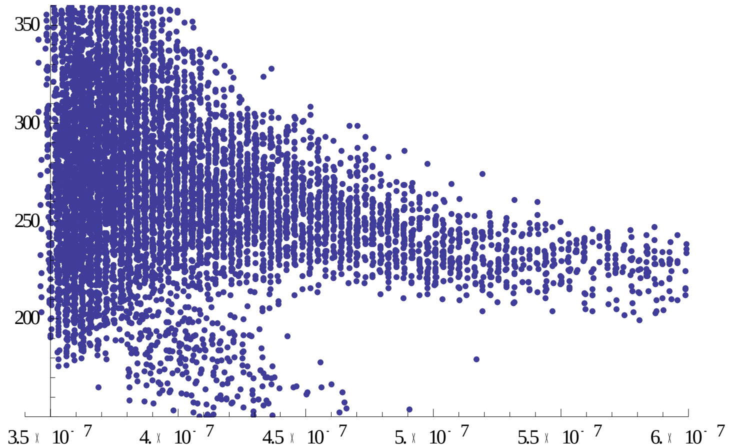


Periodic orbit for p=200MeV/c

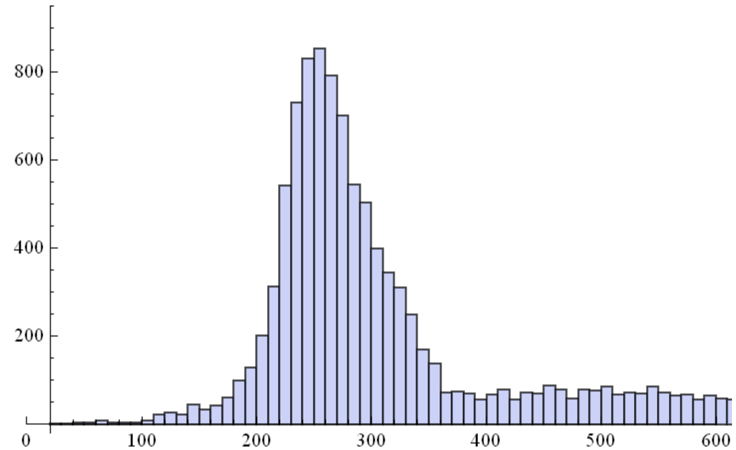


μ^+ and μ^- make the same trajectory with translation by half period (3 solenoids)

Beam from Dave's new rotator



μ^+ longitudinal phase space (number of strong bunches increased ~ f_{RF})



μ^+ momentum distribution, MeV/c (μ^- look the same)

Muon beam parameters obtained with Gaussian fit:

	$N^{(150<p<360)}$	$N^{(core)}$	$p^{(cnt)}$, MeV/c	$\sigma_\perp$, cm	σ_p, MeV/c	β_z, cm	ε_{mN}, cm			ε_{6D}, cm^3
μ^+	7998	7329	248.0	7.6	29.8	69.9	1.2	2.2	2.4	6.2
μ^-	9020	8248	248.8	7.4	28.2	71.9	1.2	2.1	2.2	5.6

$\beta_\perp$=82.8 cm for p=248.4 MeV/c Bz=2T

N.B.: There is a large imbalance in the transverse normal mode emittances, can it be used for better matching?

Challenges & Solutions for Initial 6D cooler

Challenges

- **Tight apertures with 325 MHz RF:**
  R_{window}=20cm must accommodate >2.5 $\sigma_\perp$ + helical orbits + dispersion contribution $\Rightarrow$
  for $\sigma_\perp = R_{window}/3$ and $\varepsilon_{\perp N}$=2cm $\beta_\perp$ =53cm $\Rightarrow$ very compact lattice !
- **Large momentum spread**
  – a lot of trouble e.g. in $\beta_\perp$ and orbit matching (for two signs simultaneously!)
- **High central momentum**

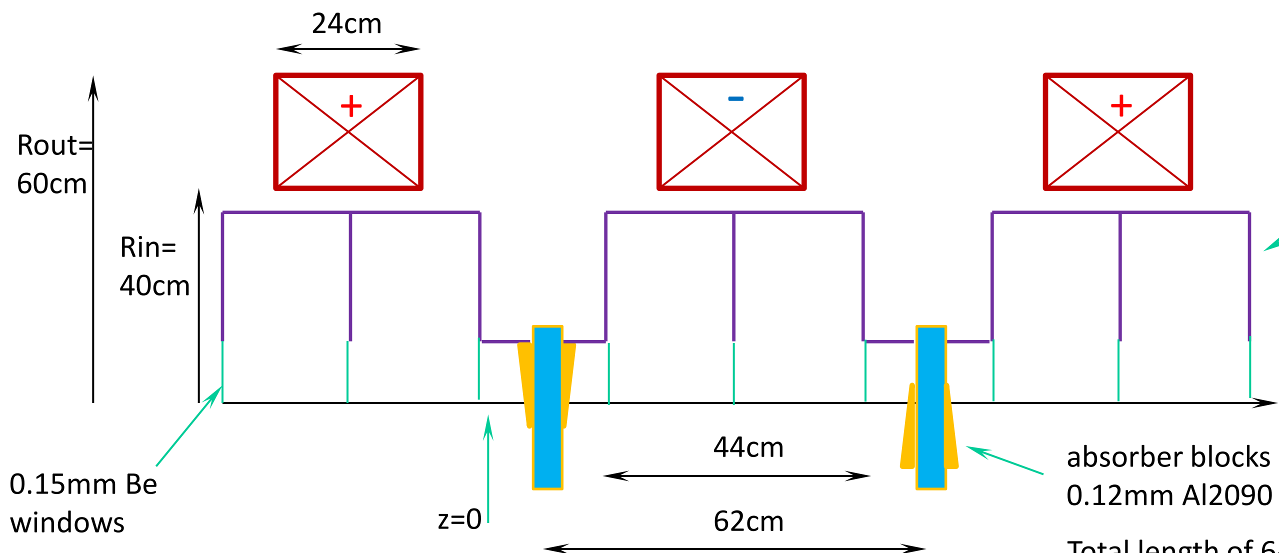
  – difficulty with momentum acceptance due to slippage factor crossing 0.
- **Strong momentum-betatron amplitude correlation due to low < $\beta_\perp$ >**

Solutions

- **Reduce helix amplitude**
  $\Rightarrow$ smaller dispersion and momentum compaction factor
- **Restore longitudinal cooling with wedges**
  (is it possible for two signs simultaneously?)
- **Lower momentum as fast as possible**
- **Use special shape absorbers to introduce required to reduce overshoot**

New Lattice Composition

Not to scale!



2 × 22cm 325 MHz RF cavities, Emax=21.7MV/m, (there is no indication so far that cavities between solenoids can achieve higher gradient)

absorber blocks of 11.5cm planar LH2 with 0.12mm Al2090 windows and rotating LiH wedges

Total length of 6-cell period = 372cm

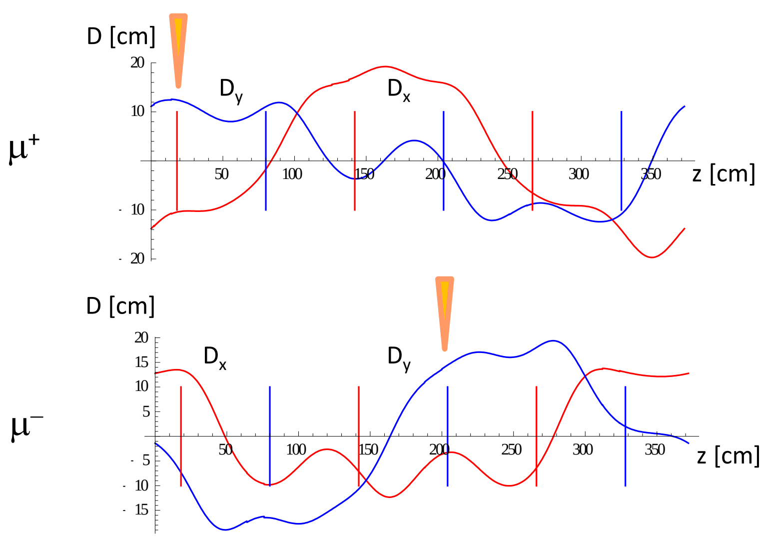
Bz_axis=4.6 T (j < 136A/mm²) for p_0=244MeV/c (p_{max}=248MeV/c), solenoid pitch angle 3mrad

The transverse modes cooling rates are equalized by constant **quadrupole** field with gradient **0.12T/m**

Direction of LiH wedges has periodicity 2, for now it is rotating by 120° from one block to another – just one parameter (initial orientation) to optimize (for given wedge angle).

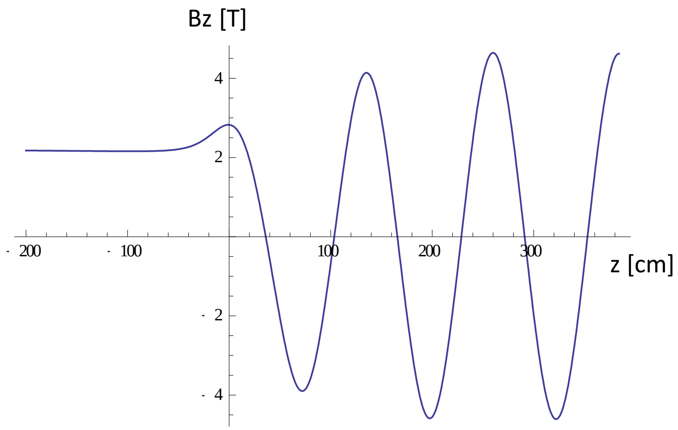
Normal mode tunes (including cooling rates) and normalized equilibrium emittances for wedge angle = 2×5.7°:

μ^+	tune	1.21 + 0.0058 i	1.24 + 0.0053 i	0.14 +0.0042 i
	ε_N (mm)	2.85	3.06	1.35
μ^-	tune	1.21 + 0.0051 i	1.24 + 0.0058 i	0.14 +0.0045 i
	ε_N (mm)	3.22	2.85	1.29

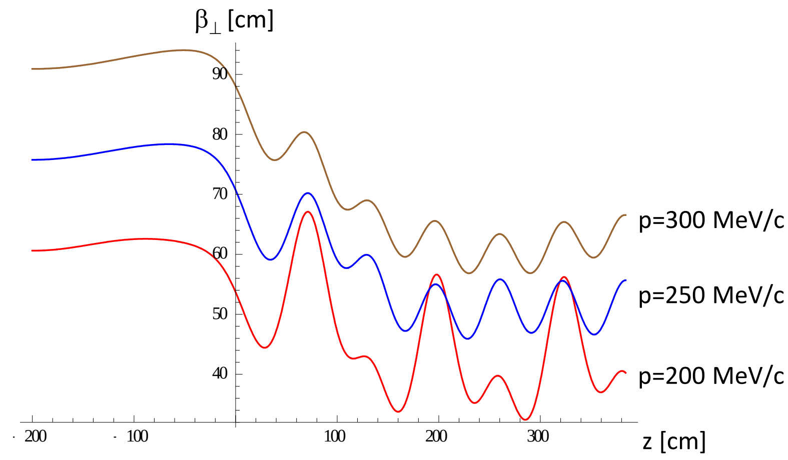


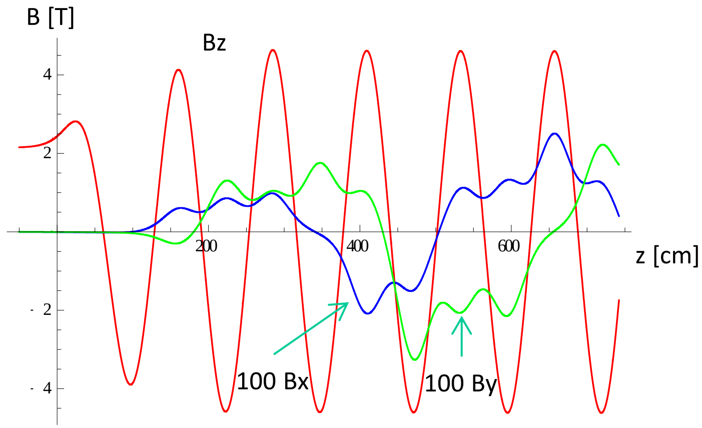
Dispersion and absorber blocks position

β-function and orbit matching



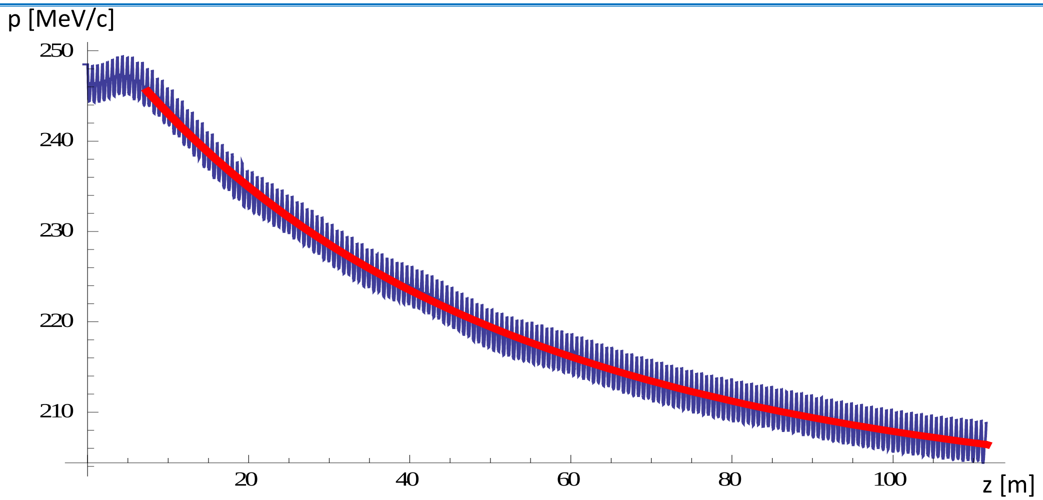
current in the first 4 solenoids (1st @ z=0 here) is used for β-matching to the rotator solenoid (increased to 2.2T)



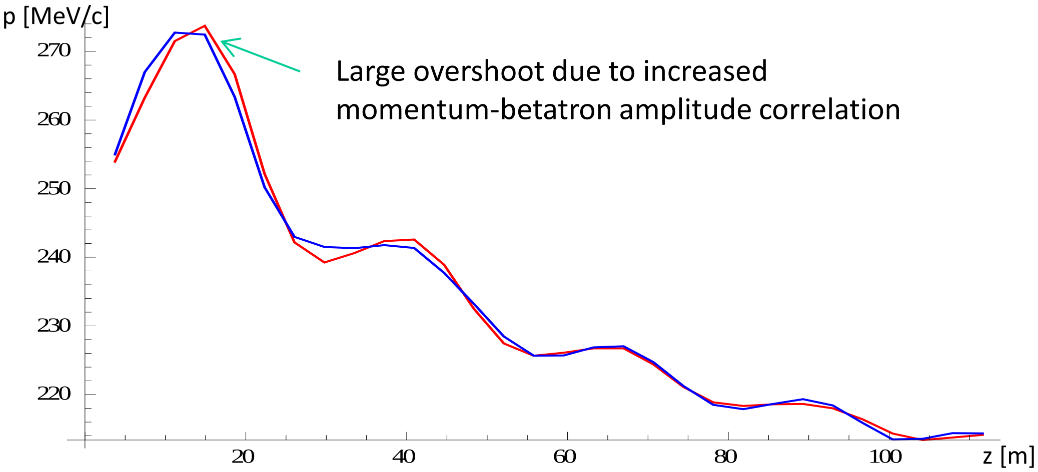


inclination of solenoids 3-9 is used for placing μ^+ and μ^- on their periodic orbits

Deceleration (G4BL tracking)

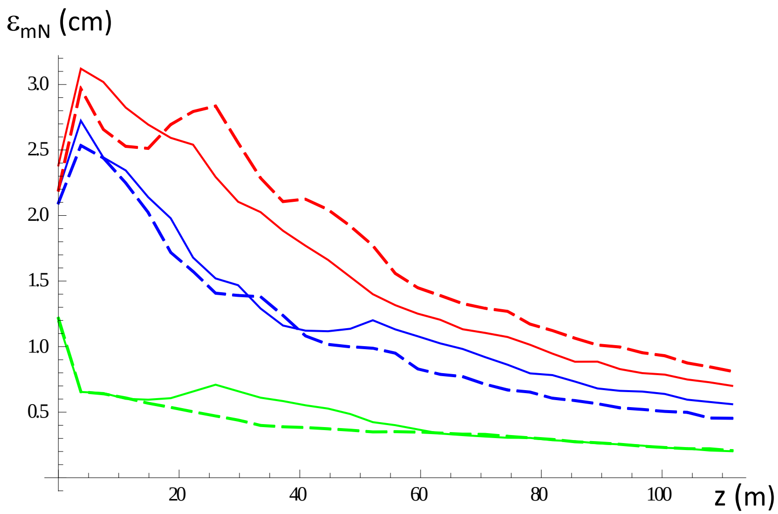


Momentum of reference particle (blue) and target value (red). Adjustment made by RF timing with fixed gradient. Large deviations are due to abrupt wedge angle changes at 235 and 225 MeV/c.

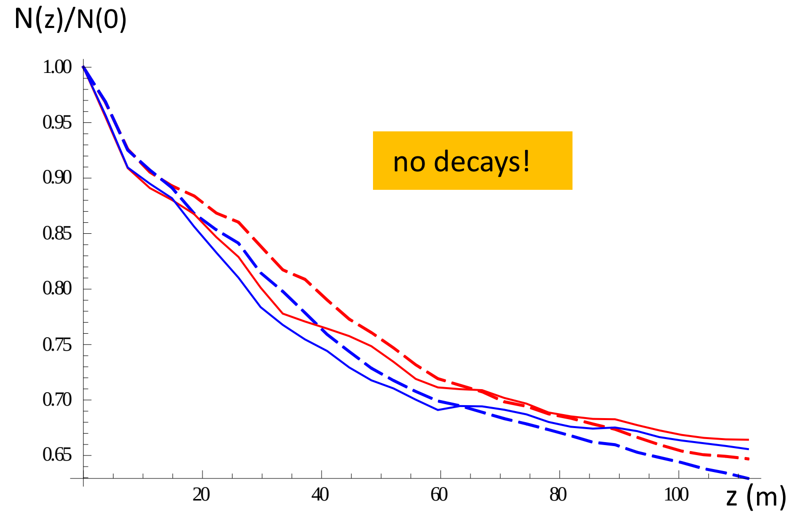


Central momentum from Gaussian fit: red – initial, blue – after an attempt to introduce momentum-betatron amplitude correlation with convex shape of the first 12 LH2 absorbers. No big effect on momentum, but transmission improved by ~3%

Cooling & Transmission (G4BL)



Normalized emittances from Gaussian fit: μ^+ - solid lines, μ^- - dashed lines.

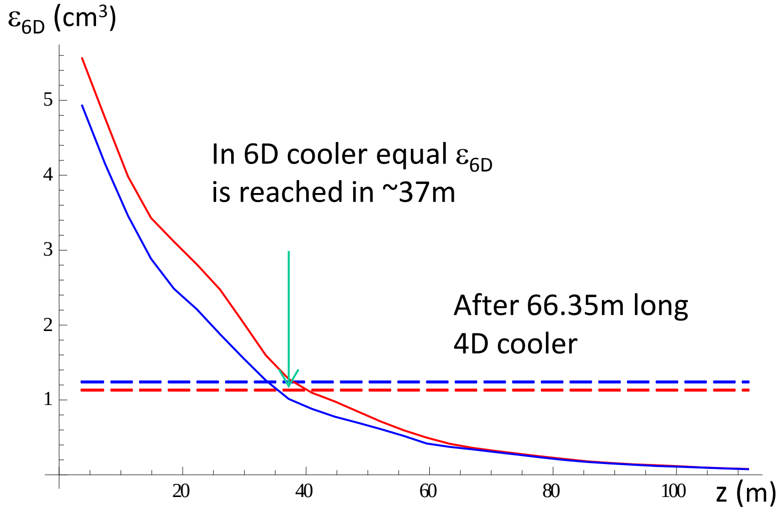


Transmission as a ratio of the number of muons in the core (Gaussian fit) - solid lines, and in the 150MeV/c <p< 360MeV/c range - dashed lines. Red lines - μ^+, blue lines - μ^-.

Final values (Gaussian fit):

	$N^{(150<p<360)}$	$N^{(core)}$	$p^{(cnt)}$, MeV/c	ε_{mN}, mm			ε_{6D}, mm^3
μ^+	5175	4868	214.3	2.0	5.6	7.0	80
μ^-	5677	5409	215.0	2.1	4.5	8.1	76

Comparison with Dave's 4D cooler & Summary



Normalized 6D emittances from Gaussian fit: μ^+ - red lines, μ^- - blue lines.

Transmission of core muons at equivalent ε_{6D} points in 4D and 6D coolers:

	μ^+	μ^-
4D	0.83	0.83
6D (no decays)	0.77	0.75

Decays will decrease 6D transmission by another couple of %%.

- HFOFO snake performs just a bit worse than 4D cooler transmission-wise.
- HFOFO performance can be improved with:
  - better choice of the wedge roll angles
  - better and smoother distribution of cooling rates
  - optimization of p(z) (and of course all other parameters)
- The main concern is RF performance in strong B (which is now II E)
  – HFOFO snake can be easily adapted to HPRF