EFFECTS OF BODY MULTIPOLES AND FRINGE-FILEDS IN THE HIGGS FACTORY

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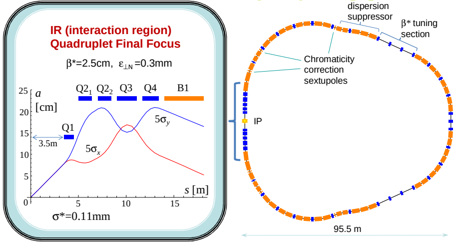
Abstract

The $\mu^+\mu^-$ collider lattice for the 125 GeV c.o.m Higgs Factory (HF) promising an average luminosity in excess of $2{\cdot}10^{31}$cm^{-2}s^{-1} with a 4 MW proton driver is considered. There are specific features for this lattice: relatively large emittance of muon beams $\varepsilon_{\perp N}=0.3\pi{\cdot}$mm·rad; small β-functions at the interaction point β*=2.5cm; very large apertures of magnets in the Interaction Region (IR) with diameters up to 50 cm; very large β-functions in IR (up to 3 km). In this lattice, the beam dynamics is very sensitive to magnetic field configurations in IR magnets. Below, the effects on beam stability of the **fringe fields (FF)** and **multipole field errors (ME)** in bodies of IR quadrupoles are presented.

HF Collider Ring Layout [1]



Final Focus Quads used in simulations

	Q1	Q2	Q3	Q4
Design aperture radius (mm)	135	225	225	225
length (m)	1.0	1.4	2.05	1.7
Aperture radius of simulated quads (mm)	160	250	250	250
Ref. radius of harmonic expansion (cm)	135	225	225	225

ME[2] converted into MAD-X multipolar kicks

Q1	R_{ref}=0.135m; L=1.0m; k1=0.35556			
name	b_n	name	k_{n-1}	$k_{n-1}l$
b_2	10000	k1	+3.5556E-01	+3.5556E-01
b_6	-0.5579	k5	-7.1667E+00	-7.1667E+00
b_{10}	-0.4727	k9	-5.5282E+07	-5.5282E+07
b_{14}	3.4501	k13	+2.0846E+16	+2.0846E+16
b_{18}	-4.6258	k17	-0.48068E+24	-0.48068E+24
b_{22}	2.4380	k21	-2.0786E+33	-2.0786E+33
b_{26}	-1.2205	k25	-5.0127E+41	-5.0127E+41
b_{30}	0.4151	k29	+2.9259E+50	+2.9259E+50

Q3	R_{ref}=0.225m; L=2.05m; k1=+0.2085945			
name	b_n	name	k_{n-1}	$k_{n-1}l$
b_2	10000	k1	+2.0859E-01	+4.2761E-01
b_6	-0.1840	k5	-1.7970E-01	-3.6838E-01
b_{10}	-0.5747	k9	-6.62270E+05	-1.3577E+06
b_{14}	-0.9377	k13	-7.2352E+12	-1.4832E+13
b_{18}	-10.4111	k17	-1.7903E+21	-3.6701E+21
b_{22}	7.6648	k21	+7.3874E+28	+1.5144E+29
b_{26}	-5.3676	k25	-6.1282E+36	-1.2563E+37
b_{30}	3.0646	k29	+7.7823E+44	+1.5954E+45

Q2	R_{ref}=0.225m; L=1.4m; k1=-0.1709481			
name	b_n	name	k_{n-1}	$k_{n-1}l$
b_2	10000	k1	-1.7095E-01	-2.3933E-01
b_6	-0.1840	k5	+1.4728E-01	+2.0619E-01
b_{10}	-0.5747	k9	+5.42770E+05	+7.5988E+05
b_{14}	-0.9377	k13	+5.9296E+12	+8.3014E+12
b_{18}	-10.4111	k17	+1.4673E+21	+2.0542E+21
b_{22}	7.6648	k21	-6.0544E+28	-8.4762E+28
b_{26}	-5.3676	k25	+5.0223E+36	+7.0312E+36
b_{30}	3.0646	k29	-6.3780E+44	-8.9292E+44

Q4	R_{ref}=0.225m; L=1.70m; k1=-0.1184142			
name	b_n	name	k_{n-1}	$k_{n-1}l$
b_2	10000	k1	-1.1841E-01	-2.0130E-01
b_6	-0.1840	k5	+1.0201E-01	+1.7342E-01
b_{10}	-0.5747	k9	+3.75950E+05	+6.39120E+05
b_{14}	-0.9377	k13	+4.1072E+12	+6.9822E+12
b_{18}	-10.4111	k17	+1.0163E+21	+1.7277E+21
b_{22}	7.6648	k21	-4.1936E+28	-7.1291E+28
b_{26}	-5.3676	k25	+3.4788E+36	+5.9140E+36
b_{30}	3.0646	k29	-4.4178E+44	-7.5103E+44

ME ("magnet" notation) and MAD-X multipoles:

$$B_y + \mathrm{i}B_x = B_2\big|_{r=R_{\mathrm{ref}}} \times 10^{-4} \sum_{n=1}^{\infty} b_n (x+\mathrm{i}y)^{n-1} / R_{\mathrm{ref}}^{n-1}$$

$$B_y + iB_x = B\rho \cdot \sum_{m=0}^{\infty} k_m^{\mathrm{norm}} (x+iy)^m / m!$$

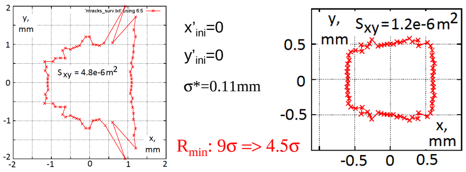
The indices are related as *m*=*n*-1, e.g. for quadrupole *m*=1 & *n*=2

MAD-X: "QUADRUPOLE, L=xxx, K1=xxx, **KNL={0,k1l,k2l,…}**;"

DA reduction with total 29 ME

Simulations done with MAD-X PTC-TRACK module[3]

DA in the planes of initial coordinate {x, y} at IP w/o ME (left) and with ME (right)



x'$_{\mathrm{ini}}$=0

y'$_{\mathrm{ini}}$=0

σ*=0.11mm

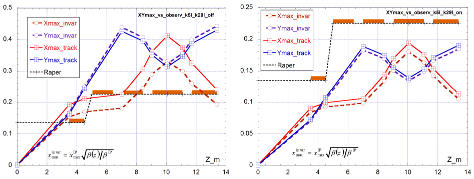
$R_{\min}$: 9σ => 4.5σ

FF & Phys. Apertures (PA) are OFF

ME in IR quads reduce DA ($R_{\min}\sim S_{xy}^{1/2}$) by a factor ~2

Beam envelopes relatively phys-aperture

without ME (left) and with all 29 ME (right):
linear optics (dotted-circle) & trajectories MADX (solid-square)



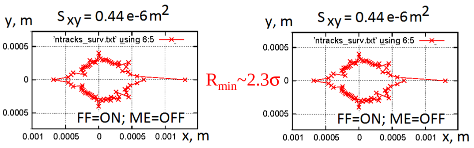
FRINGE FIELDS (FF)

- FF effects in accelerator magnets [4] can be important in rings with large beam emittances and short magnets.
- FF effects can be important for the MC ring as well (very large *β*-function; large magnet aperture R_{ap} ~40-80mm; short length of quadrupoles < 2 m).

Fringe Field Simulations with MADX

- MAD-X with PTC[5]: FF in "hard-edge"(HE) approach[5].
- FF region is "short" [4], if $|\beta'_{x,y}| l_{\mathrm{FF}} \ll \beta_{x,y}$, ($\beta'_{x,y}$ - derivatives of $\beta_{x,y}$-functions, l_{FF} – length of FF.
- This condition is fulfilled for IR magnets

FF with embedded in MAD-X PTC hard-edge approximation



FF effects ~ twice stronger than ME effects;
FF effects must be corrected in the first place
FF model must be adequate and reliable

Exporting Magnet Maps from COSY Infinity [6]

- to extend simulation capabilities beyond the HE approach
- usage of realistic FF falloffs generated by COSY[6,7]
- FF maps[7]: influence on both linear & non-linear motion.

"COSY INFINITY 9.0 Beam Physics Manual":

- Maps can be printed to ext. text file with command PM;
- FF for either built-in mode (FR) or general element (GE)
- Command "FR <m>" has been used for the map export to MADX
- Stand along maps are possible: (FR-1 & FR-2 => entry&exit FF)
- Maps for GE (from measurements) are also possible (in a future!).
- COSY maps can be symplectified with command "SY <M>"

We use the most accurate FF mode "FR 3" with FF falloffs based on the standard description by a six parameter Enge function

$$F(z) = \left\{1 + \exp\left[\sum_{i=1}^{6} a_i (z/D)^{i-1}\right]\right\}$$

Z – the distance along reference orbit; *D* – the full aperture.

Code (in COSYScript) is written for exporting 4 types of maps:
1)total map (FR3); 2)entry FF; 3)body (FR 0); 4)exit FF

Map symplectification has been added to our exporting script:
Symplectification errors can be controlled with command "SE<M>", which evaluates error for symplectic condition given as a norm of the matrix SE=||$M{\cdot}J{\cdot}M^{t}- J$ ||

Example: Map Symplectification for IR magnets

SE values for the 5th order maps before/after symplectification (by SY)

magnet name	magnet type	Map type			
		Total	Entry	Body	Exit
QLB1	QUAD	7E-16/1E-18	1E-16/3E-18	3E-17/3E-18	3E-16/3E-18
QLB2	**RBEND+Q**	3E-15/1E-17	**9E-6/7E-23**	1E-15/1E-19	**9E-6/1E-21**
QLB3	Quad	8E-16/2E-17	1E-16/2E-23	7E-18/1E-17	2E-16/1E-24
QLB4	RBEND+Q	2E-15/2E-17	**6E-6/3E-18**	2E-15/3E-19	**6E-6/3E-23**
BIR	RBEND	7E-14/1E-17	5E-16/3E-18	6E-16/2E-20	2E-15/2E-21
SLB1	SEXT	6E-17/9E-23	1E-17/4E-23	1E-21/9E-23	5E-17/3E-23

Importing COSY maps into MADX PTC-TRACK module

Modifications of MADX PTC TRACK done by VK:

- coordinate transformations COSY->MADX
- MADX subroutines for maps reading and particle tracking
- Testing runs and usage for 1.5 TeV MC lattice [8]

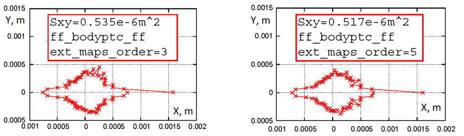
Linear part of COSY maps strongly perturbs linear optics.
Preserving linear optics, the PTC and COSY maps have been combined:

$$M_{\mathrm{FR\text{-}1}}^{[1]} \circ M_{\mathrm{FF=OFF}}^{\mathrm{PTC}} \circ M_{\mathrm{FR\text{-}2}}^{[1]}$$

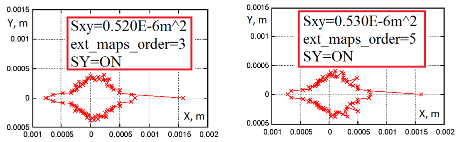
$M_{\mathrm{FF=OFF}}^{\mathrm{PTC}}, M_{\mathrm{FR\text{-}1}}^{[1]}, M_{\mathrm{FR\text{-}2}}^{[1]}$ are PTC map for magnet body, and two COSY maps of FF at magnet entrance and exit with the linear part replaced by unit matrices, resp.

DA with COSY maps

Before symplectification

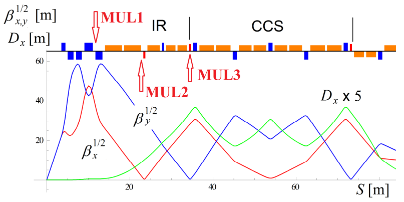


After symplectification



- ➢ DA reduction for realistic FF-falloffs (COSY) agrees with HE (MADX-PTC)
- ➢ Stable area S_{xy} for COSY > S_{xy} for HE ~ 15%
- ➢ Difference for Sxy between maps of orders 3 & 5 is less 3%
- ➢ Symplectification of maps: Sxy changes slightly within ~3%

DA correction with octupoles (against FF)



DA corrections are under the way:

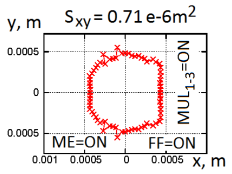
optimal strength of correcting octupoles in MUL1-3 were found first analytically (via detuning coeffs) and now are being sought numerically (K3 scanning).

K3%MUL1=+0.35

K3%MUL2=-5.2

K3%MUL3=-2.5

$R_{\min}$~3.9σ



Final remarks

- HF lattice requires simulations with adequate treatments of **systematic multipolar errors and fringe fields**
- **MADX modifications** in more detail described in [9].
- **FF effects** on DA are **more stronger** than ME effect.
- Results with **realistic FF falloffs** by COSY maps can be verified by comparison with results by PTC HE model
- DA for **realistic FF** is slightly **higher ~10%** than HE FF
- **Symplectified COSY maps** are now used
- DA values for the current HF lattice are practically the **same for symplectified and non-symplectified maps**
- DA reduction due to ME & FF has been corrected with octupoles and **the corrected DA~3.9σ**

References

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[3] V.Kapin & F.Schmidt, "PTC-TRACK Module", in MADX User's guide (http://mad.home.cern.ch/mad/)
[4] Y.Papaphilippou et al., "Deflections in magnet fringe fields", P.R. E-67, 046502, 2003.
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[6] M. Berz, "Computational Aspects of Design and Simulation: COSY INFINITY",NIM A-298, p.473, (1990): http://bt.pa.msu.edu/index_cosy.htm
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[8] Y. Alexahin, E.Gianfelice-Wendt, V. Kapin, "Effect of Field Errors in MC IR magnets on Beam Dynamics", Proc. IPAC-2012, pp.1257.
[9]V. Kapin, Y. Alexahin, " Simulations of Field Errors in IR magnets of a Muon Collider using MADX and COSY codes", Beam-docs, FNAL, 2012.