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Phenomenology of μ-to-e conversion

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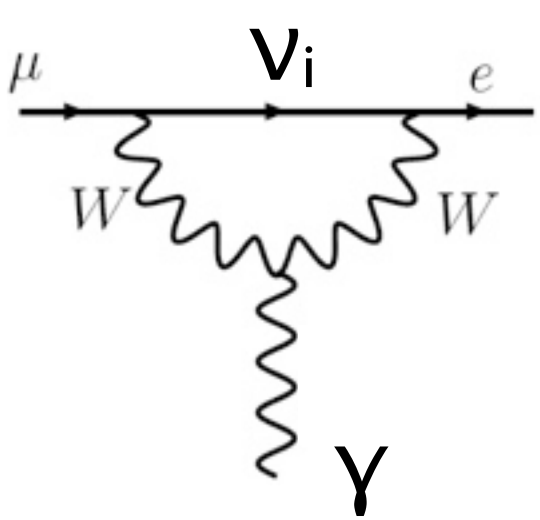
Outline

- Introduction: CLFV and new physics
- CLFV phenomenology: effective theory framework
- The model-discriminating power of mu-to-e conversion and needed theory input

LFV and BSM physics

- ν oscillations $\Rightarrow$ $L_{e,\mu,\tau}$ not conserved (accidental symmetries in SM)
- In SM + massive "active" ν, effective CLFV vertices are tiny (GIM)



$$\boxed{Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi} \left| \sum_{i=2,3} U^*_{\mu i} U_{ei} \frac{\Delta m^2_{1i}}{M_W^2} \right|^2 < 10^{-54}}$$

Petcov '77, Marciano-Sanda '77 ....

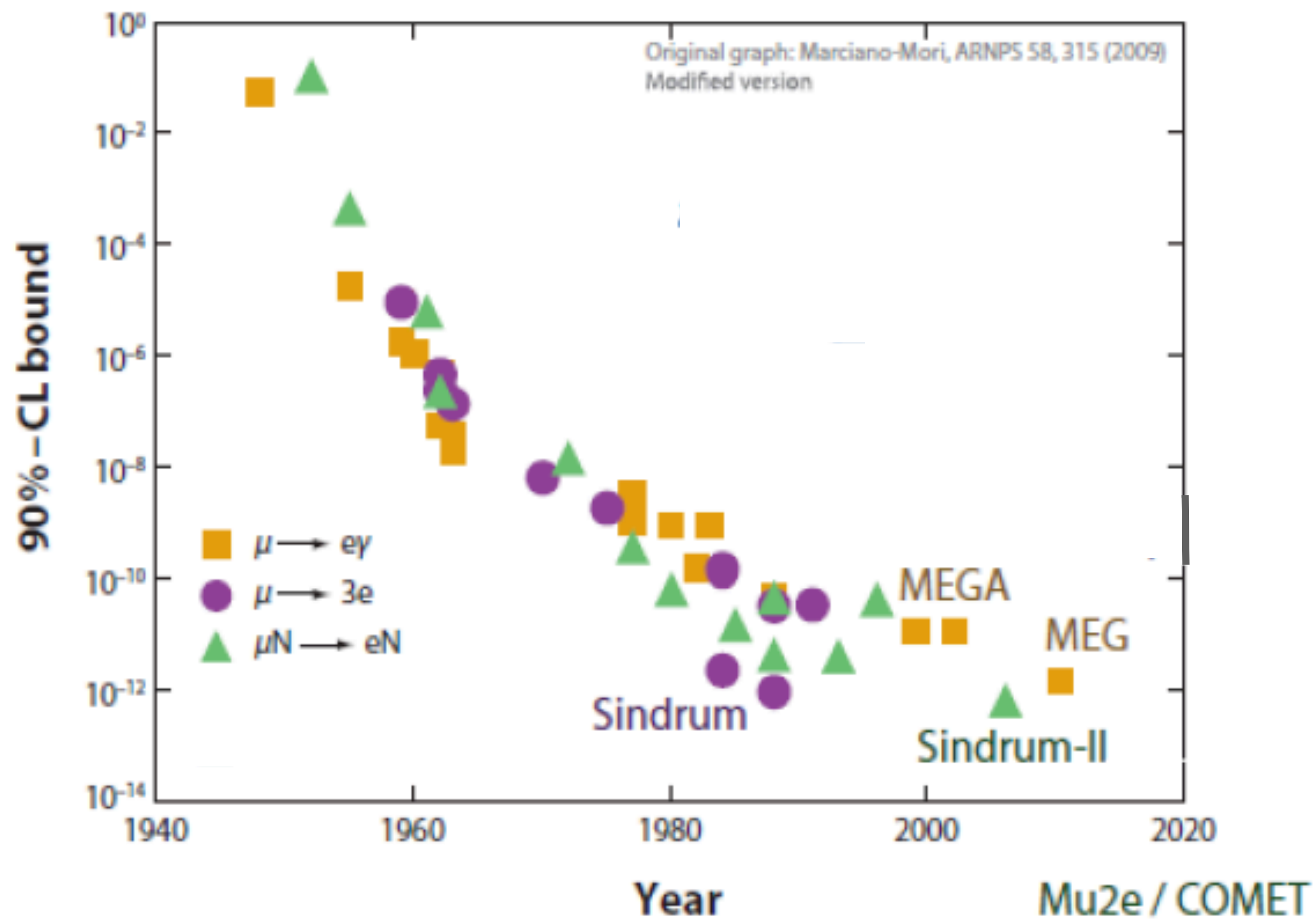
- Extremely clean probe of "BνSM" physics

dim-4 Dirac or
dim5 Majorana

$$\mathcal{L}_{\nu\mathrm{SM}} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_{\nu-\mathrm{mass}}$$

CLFV processes

- Muon processes :



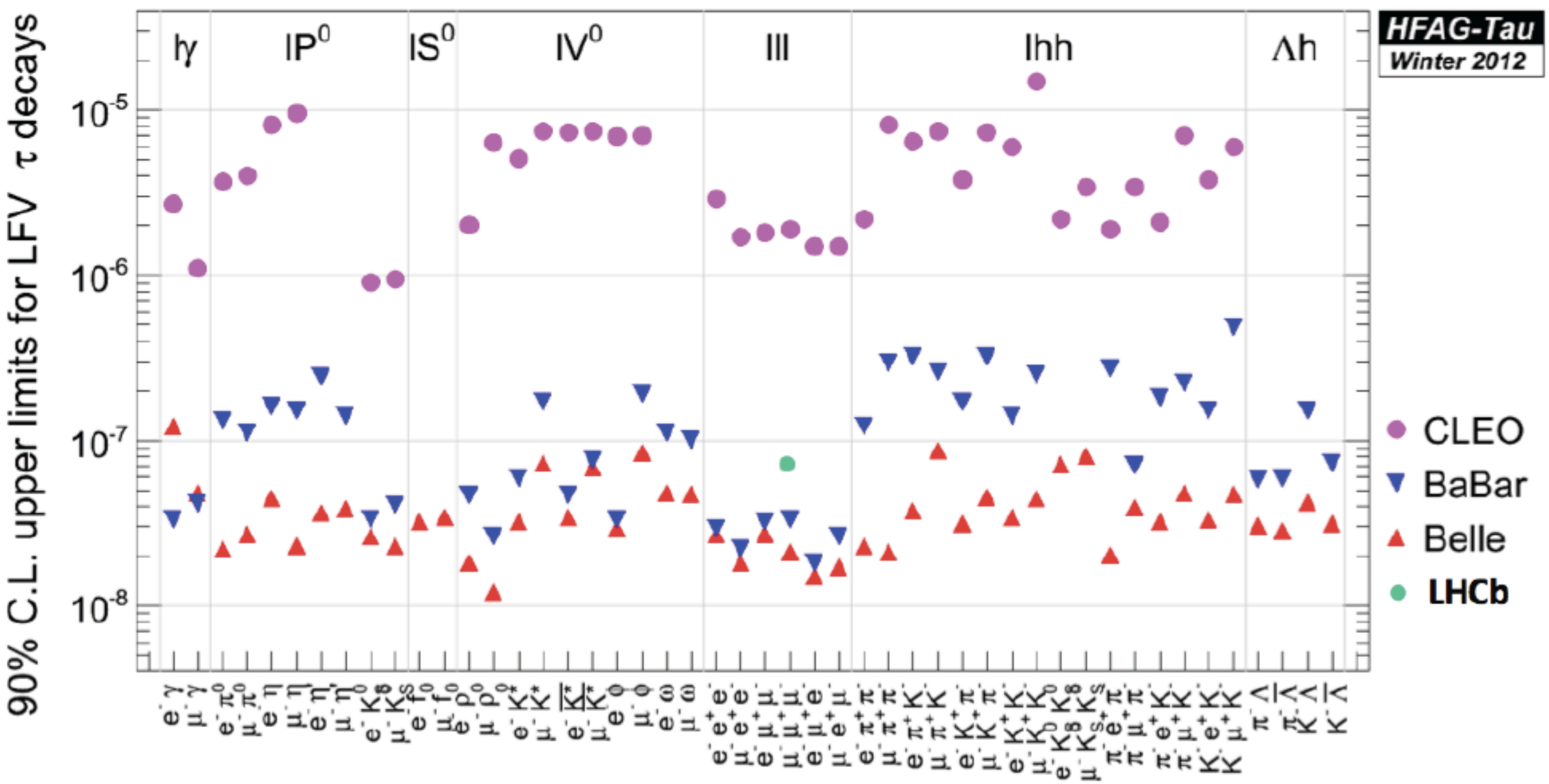
$$B_{\mu\to e\gamma} < 5.7\times 10^{-13}$$ ⟶ $10^{-/14}$ (MEG at PSI)

$$B_{\mu\to 3e} < 1.0\times 10^{-12}$$ ⟶ $10^{-15/16}$ (PSI)

$$B^{Ti}_{\mu-e} < 4.3\times 10^{-12}$$ ⟶ $10^{-16/17\to -18}$ (Mu2e, COMET)

CLFV processes

- Tau decays:



10^{-9} (or better?) sensitivities at Belle-II, LHCb

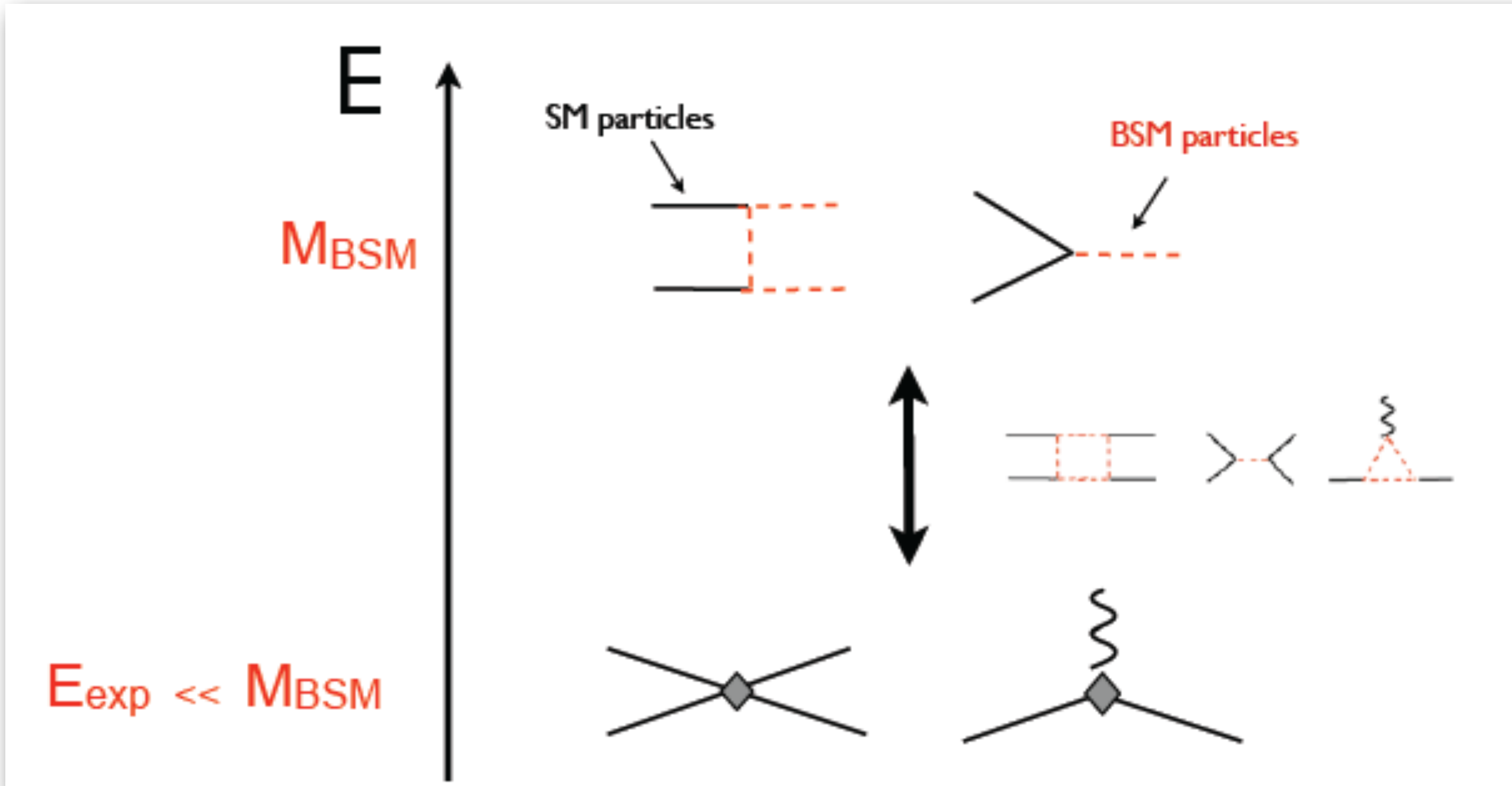
CLFV processes

- Great "discovery" tools
  - Observation near current limits $\Rightarrow$ BSM physics

- Great "model-discriminating" tools
  - What type of "mediator"?
    $\mu \to 3e$ vs $\mu \to e\gamma$ vs $\mu \to e$ conversion
    Z-dependence of $\mu \to e$ conversion
  - What sources of flavor breaking?
    $\mu \to e$ vs $\tau \to \mu$ vs $\tau \to e$

(Not discussed in this talk)

Effective theory framework



- At low energy, BSM dynamics described by local operators

$$\boxed{\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} O^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} O_i^{(6)} + \ldots}$$

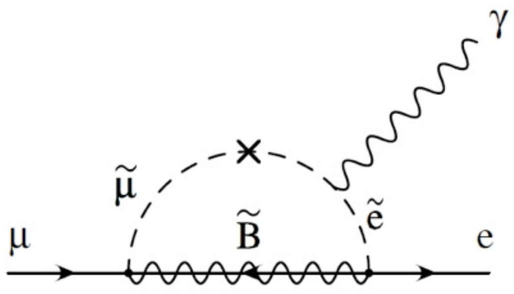
$\Lambda \leftrightarrow M_{\rm BSM}$ $\qquad$ $C_i \; [g_{\rm BSM}, \;\; M_a/M_b]$

- Each UV model generates a specific pattern of LFV operators

Rich phenomenology at dim=6

- Dipole

$$\boxed{\frac{[\alpha_D]^{ij}}{\Lambda^2}\, \varphi^\dagger\, \bar{e}_R^i \sigma_{\mu\nu} \ell_L^j\, F^{\mu\nu}}$$



Dominant in SUSY-GUT and SUSY see-saw scenarios

- Current limits on μ →eγ imply

$$\boxed{\Lambda/\sqrt{[\alpha_D]^{\mu e}} > 3.4 \times 10^4 \text{ TeV}}$$

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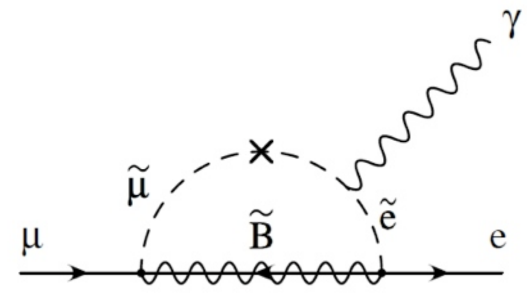
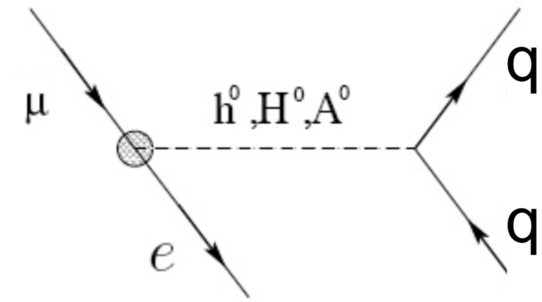


- Scalar

$$\frac{[\alpha_S]^{ij}}{\Lambda^2}\,\bar{e}^i_R\ell^j_L\,\bar{q}_L d_R$$

Dominant in RPV SUSY and RPC
SUSY for large tan(β) and low m_A



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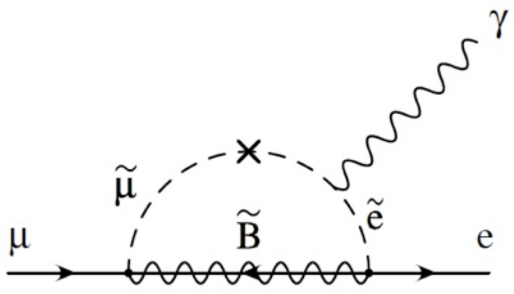

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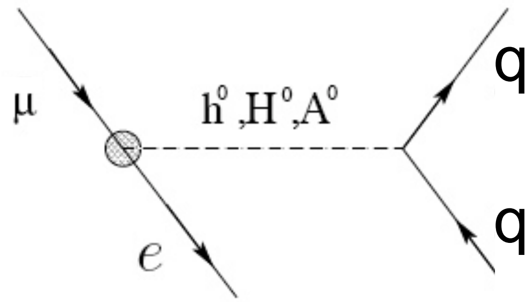
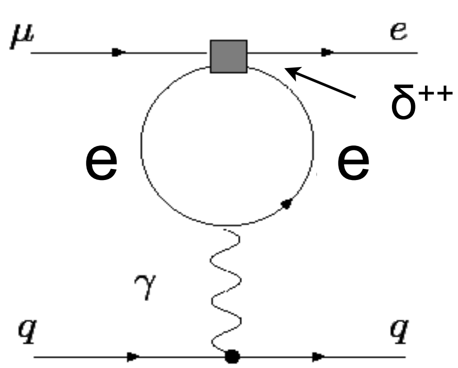


- Vector

$$\frac{[\alpha_{V(\gamma)}]^{ij}e_q}{\Lambda^2}\;\bar{\ell}^i_L\gamma_\mu\ell^j_L\;\bar{q}_L\gamma^\mu q_L$$

Enhanced in triplet models (Type II seesaw), Left-Right symmetric models



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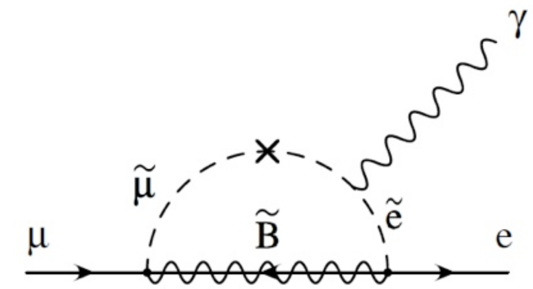

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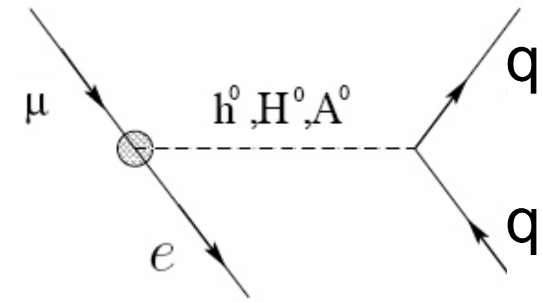

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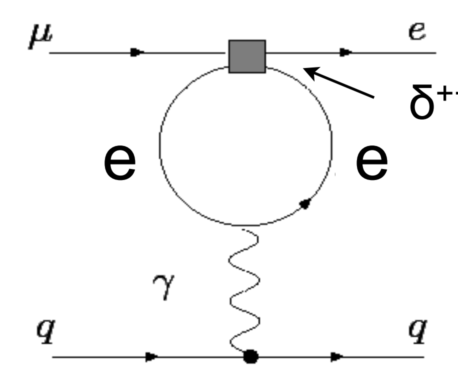

- Z-penguin

$$\frac{[\alpha_{V(Z)}]^{ij}}{\Lambda^2}\ \bar{\ell}^i_L \gamma_\mu \ell^j_L\ \varphi^\dagger D^\mu \varphi$$

Type III seesaw, ..

Rich phenomenology at dim=6

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$$\frac{[\alpha_D]^{ij}}{\Lambda^2}\ \varphi^\dagger\ \bar{e}^i_R \sigma_{\mu\nu} \ell^j_L\ F^{\mu\nu}$$

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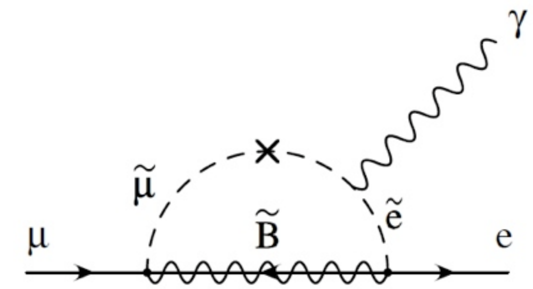

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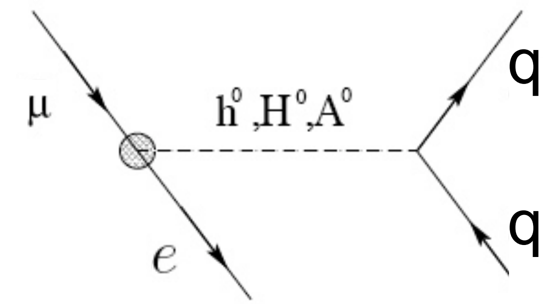

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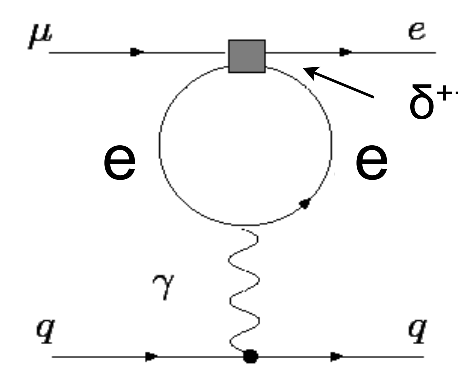

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Type III seesaw, ..

- 4 Leptons, ...

$$\frac{1}{\Lambda^2}\ \bar{L}^i \gamma_\mu L^j\ \bar{L} \gamma^\mu L$$

(LL)(RR), (RR)(RR)

Type II seesaw, RPV SUSY, LRSM

The μLFV matrix

	$\mu \to 3e$	$\mu \to e\gamma$	$\mu \to e$ conversion
$O^{4\ell}_{S,V}$	✓	–	–
O_D	✓	✓	✓
O^q_V	–	–	✓
O^q_S	–	–	✓

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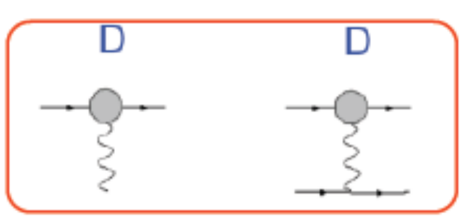
- μ → 3e vs μ →eγ: relative strength of dipole and 4L operators

$$\frac{\Gamma_{\mu\to 3e}}{\Gamma_{\mu\to e\gamma}} = \frac{\alpha}{4\pi} I_{\rm PS} \left(1 + \sum_i \frac{c_i^{(\rm contact)}}{c^{(\rm dipole)}} \right)$$



6 ×10^{-3}

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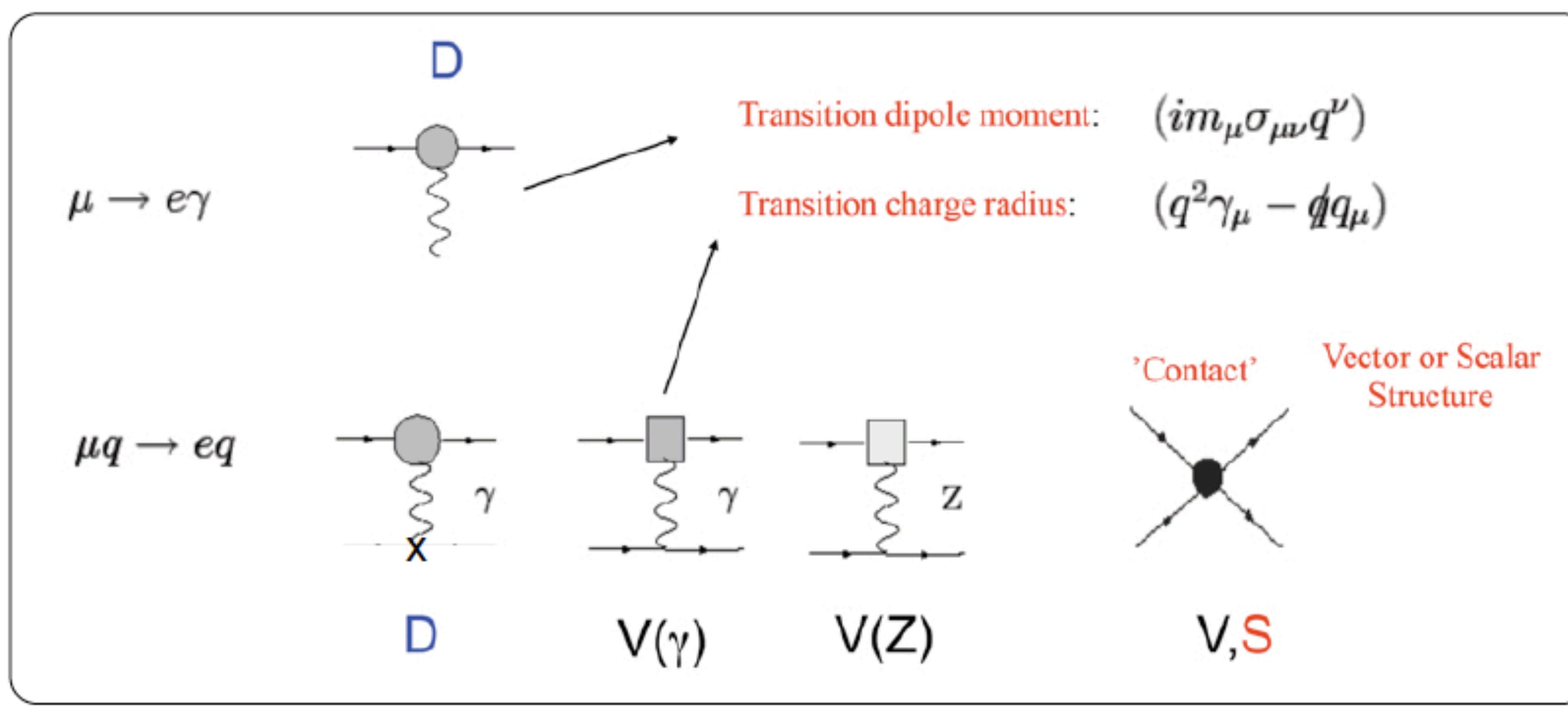
O^q_V	–	–	✓
O^q_S	–	–	✓

- μ →e vs μ →eγ and target-dependence of μ →e conversion: relative strength of dipole and quark operators



How does it work?

- Conversion amplitude has non-trivial dependence on target atom, that distinguishes D, S, V underlying operators

$$M_{fi} \sim \langle e^-; A, Z| \int d^3x \, \hat{O}_\ell(x) \, \hat{O}_q(x) \; |\mu^-; A, Z\rangle$$

$$\sim \int d^3x \, \bar{\psi}_e O_\ell \psi_\mu \; \langle A, Z|\hat{O}_q|A, Z\rangle$$

Czarnecki-Marciano-Melnikov

Kitano-Koike-Okada

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Czarnecki-Marciano-Melnikov

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- Lepton wave-functions in EM field generated by nucleus

- Relativistic components of muon wave-function give different contributions to D,S,V overlap integrals. For example:

$$\bar{\psi}_e \gamma_0 \psi_\mu = \bar{\psi}_e \, \psi_\mu + O(v_\mu/c)$$

- Expect largest discrimination for heavy target nuclei

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- Sensitive to hadronic and nuclear properties

$$\langle A, Z | \; \bar{q} \Gamma q \; | A, Z \rangle$$

$$\downarrow$$

$$f_{\Gamma N}^{(q)} \langle A, Z | \; \bar{\psi}_N \Gamma \psi_N \; | A, Z \rangle$$

$$\downarrow$$

$$\begin{aligned}
\langle A, Z | \; \bar{\psi}_p (\gamma_0) \psi_p \; | A, Z \rangle &= Z \rho^{(p)} \\
\langle A, Z | \; \bar{\psi}_n (\gamma_0) \psi_n \; | A, Z \rangle &= (A - Z) \rho^{(n)}
\end{aligned}$$

- Dominant sources of uncertainty:

  - Scalar matrix elements $\langle i|\, m_q q\bar{q}\, |i\rangle = \sigma_q^{(i)} \bar{\psi}_i \psi_i$

$$\sigma_{\pi N} = \frac{m_u + m_d}{2} \langle p|\bar{u}u + \bar{d}d|p\rangle \rightarrow 53\ ^{+21}_{-10}\ \mathrm{MeV} \qquad \boxed{(45 \pm 15)\ \mathrm{MeV}}$$

ChPT JLQCD 2008 Lattice range 2012 (Kronfeld 1203.1204)

$$y = \frac{2\ \langle p|\bar{s}s|p\rangle}{\langle p|\bar{u}u + \bar{d}d|p\rangle} \in [0,\ 0.4] \rightarrow [0,\ 0.05] \qquad \boxed{[0.04,\ 0.12]}$$

  - Neutron density (heavy nuclei)

  - NLO chiral corrections in matching from quarks to nucleons?

$\mu \to e$ vs $\mu \to e\gamma$

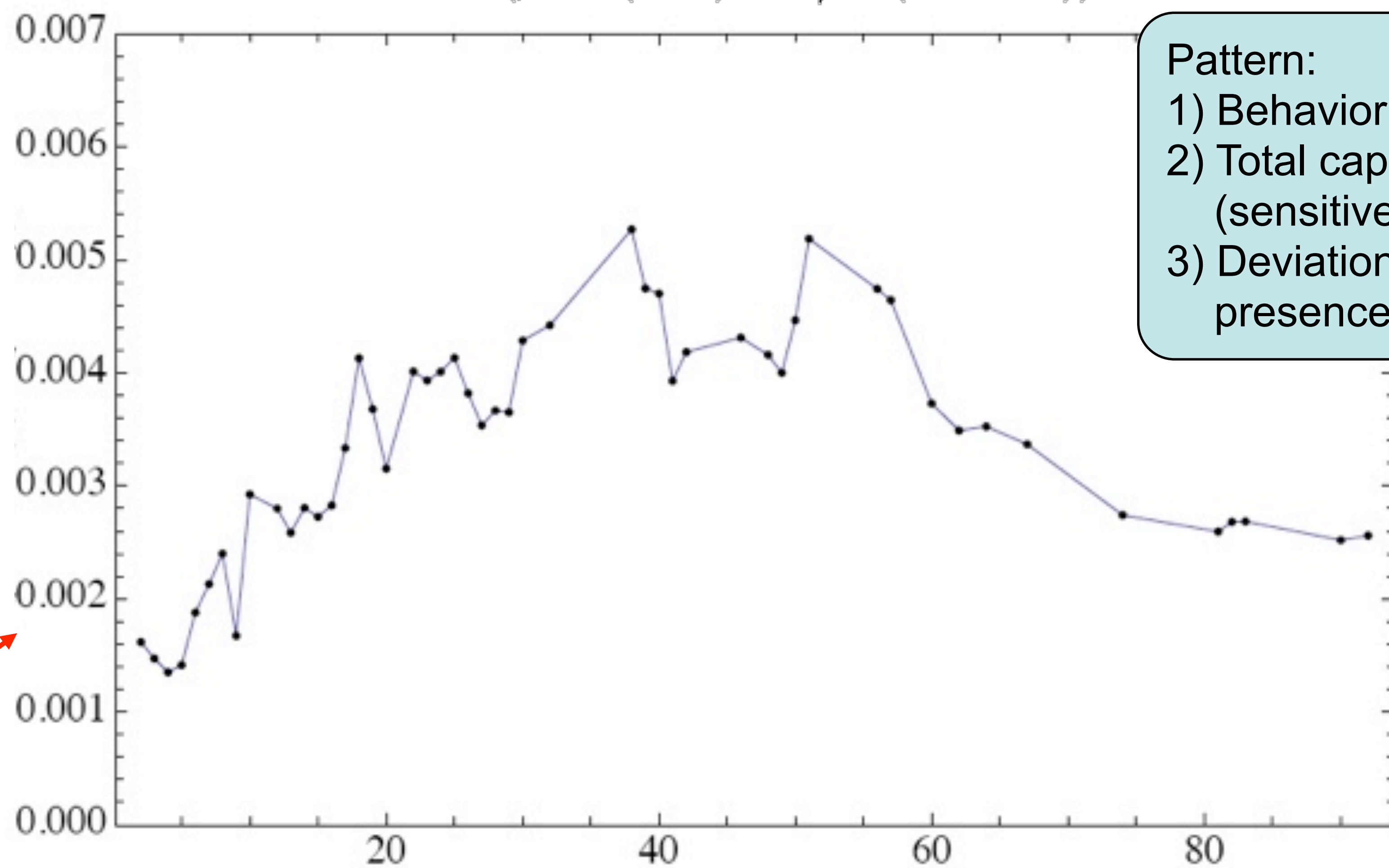
- By measuring $B(\mu \to e, Z)/B(\mu \to e\gamma)$ we can test the hypothesis of dipole dominance

$$B_{\mu\to e} = \frac{\Gamma(\mu^- + (Z,A) \to e^- + (Z,A))}{\Gamma(\mu^- + (Z,A) \to \nu_\mu + (Z-1,A))}$$

Kitano-Koike-Okada '02
VC-Kitano-Okada-Tuzon '09

Pattern:
1) Behavior of overlap integrals**
2) Total capture rate
   (sensitive to nuclear structure)
3) Deviations would indicate
   presence of scalar / vector terms

$$\frac{B(\mu \to e, Z)}{B(\mu \to e\gamma)}$$



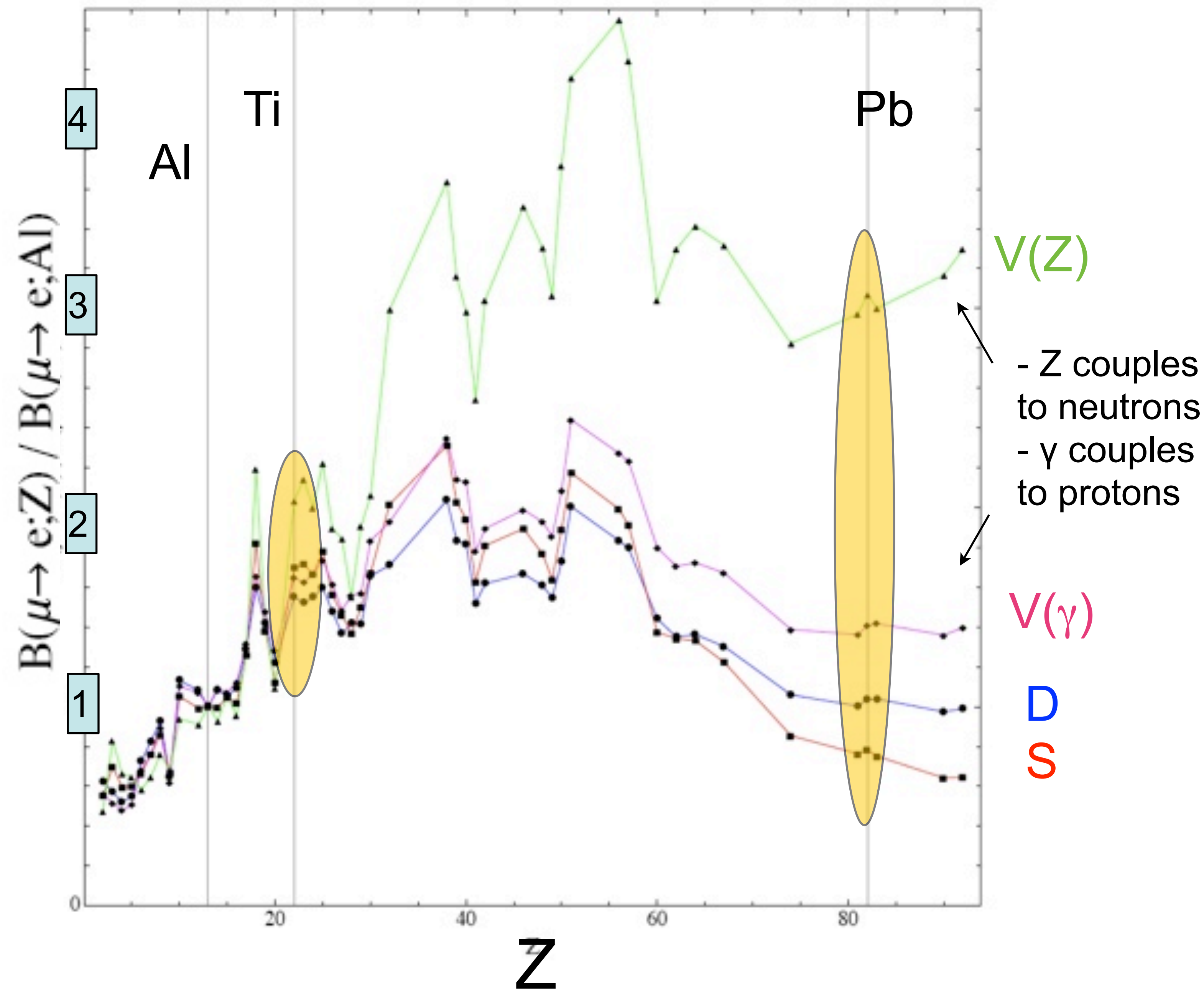
Z

$O(\alpha/\pi)$

VC-Kitano-Okada-Tuzon 2009

$\mu \to e$ vs $\mu \to e$

- $B(\mu \to e, Z_1)/B(\mu \to e, Z_2)$ tests any single-operator dominance model
- Essentially free of theory uncertainty (cancels in the ratio)



- Discrimination: need ~5% measure of Ti/Al or ~20% measure of Pb/Al
- Ideal world: use Al and a large Z-target (D,V,S have largest separation)

Beyond single operator dominance

- If "single-operator" dominance hypothesis fails, consider next simplest case: two-operator dominance (DV, DS, SV)

- Unknown parameters: $[\alpha_1]^{e\mu}/\Lambda^2$, $[\alpha_2]^{e\mu}/\Lambda^2$

- Hypothesis can be tested with two double ratios (three LFV measurements!!). For example:

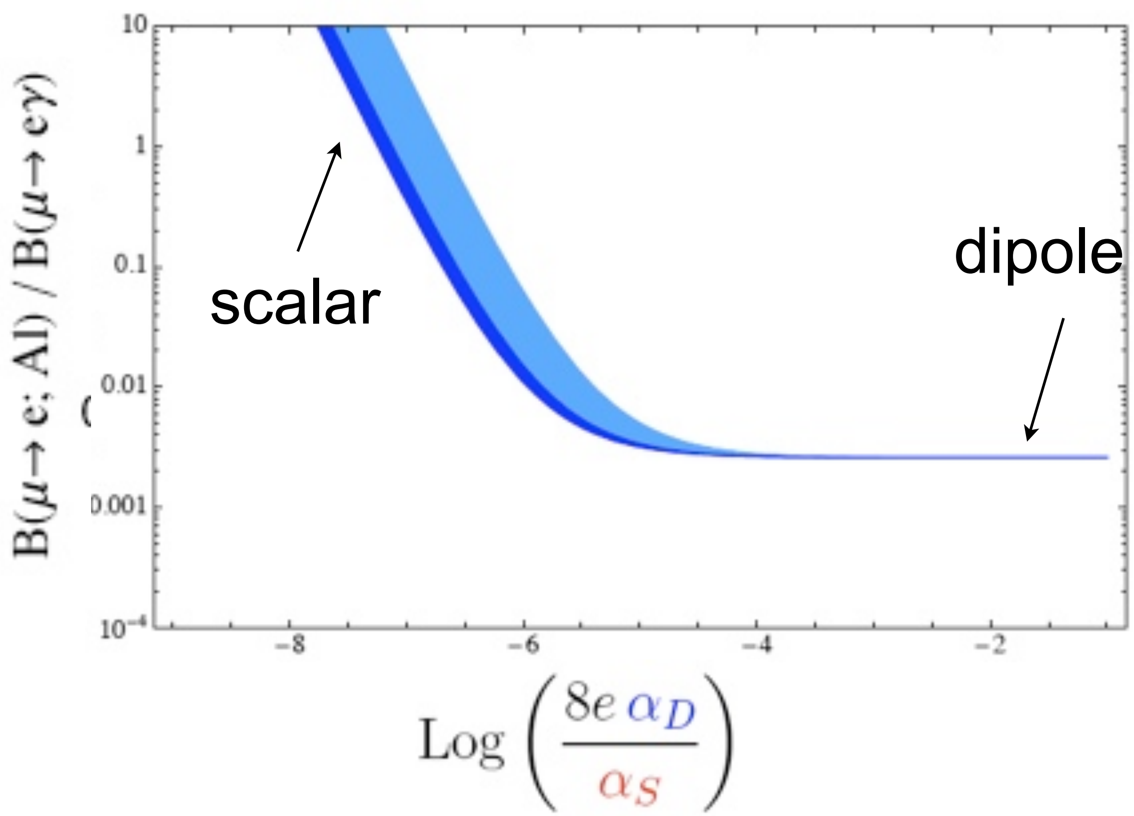
DV, DS $\longleftrightarrow$ $\dfrac{B(\mu \to e, Al)}{B(\mu \to e\gamma)}$ $\dfrac{B(\mu \to e, Pb)}{B(\mu \to e, Al)}$

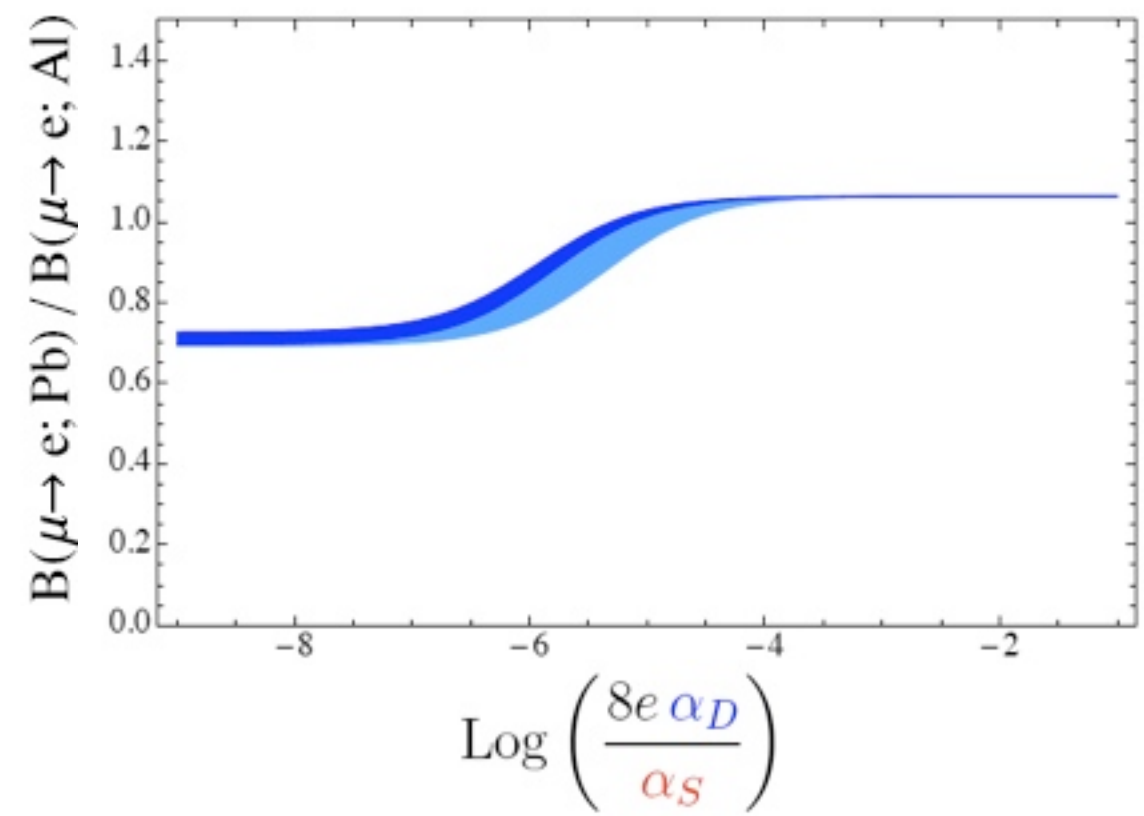
SV $\longleftrightarrow$ $\dfrac{B(\mu \to e, Ti)}{B(\mu \to e, Al)}$ $\dfrac{B(\mu \to e, Pb)}{B(\mu \to e, Al)}$

- Consider S and D: realized in SUSY via competition between dipole and scalar operator (mediated by Higgs exchange)

Relative sign: +

VC-Kitano-Okada-Tuzon 2009





- Uncertainty from strange form factor largely reduced by lattice QCD

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JLQCD 2008

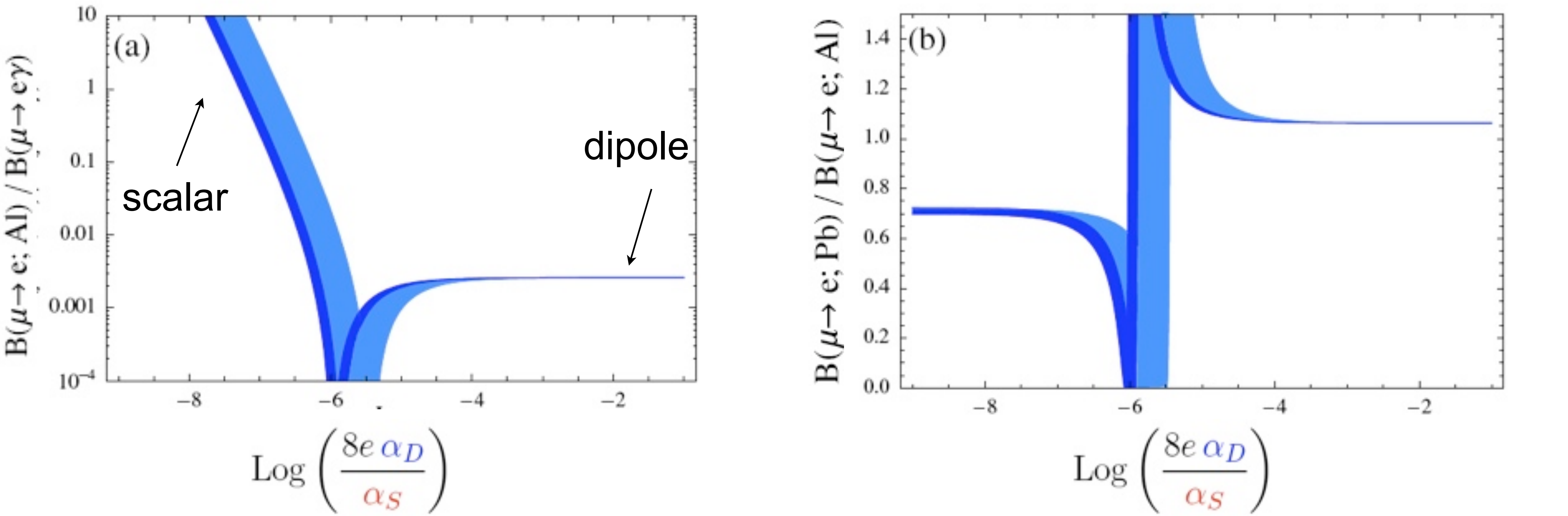
fat error band

thin error band → realistic discrimination

- Consider S and D: realized in SUSY via competition between dipole and scalar operator (mediated by Higgs exchange)

Relative sign: -

VC-Kitano-Okada-Tuzon 2009



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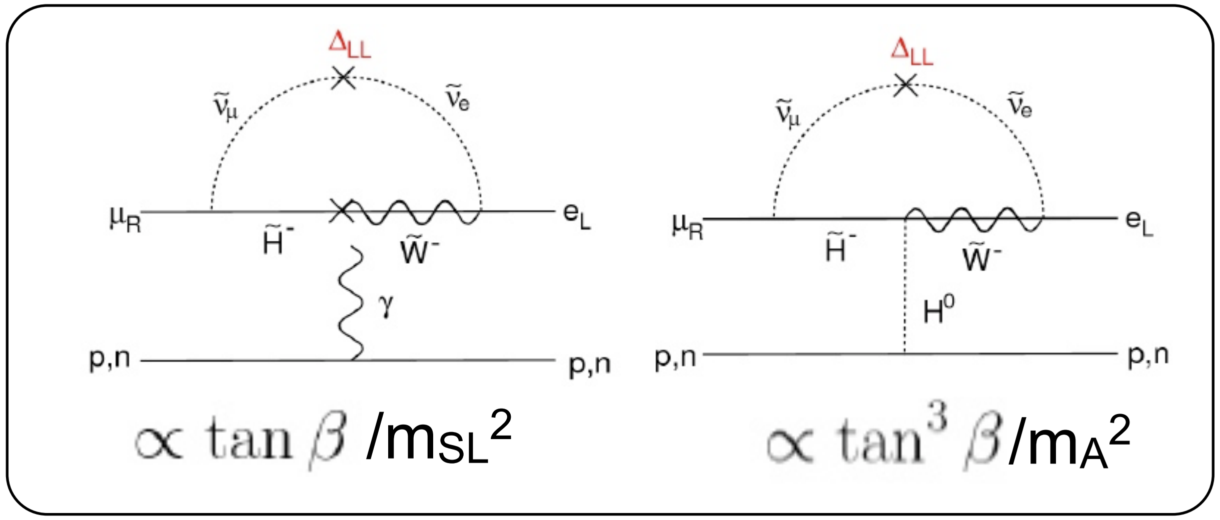
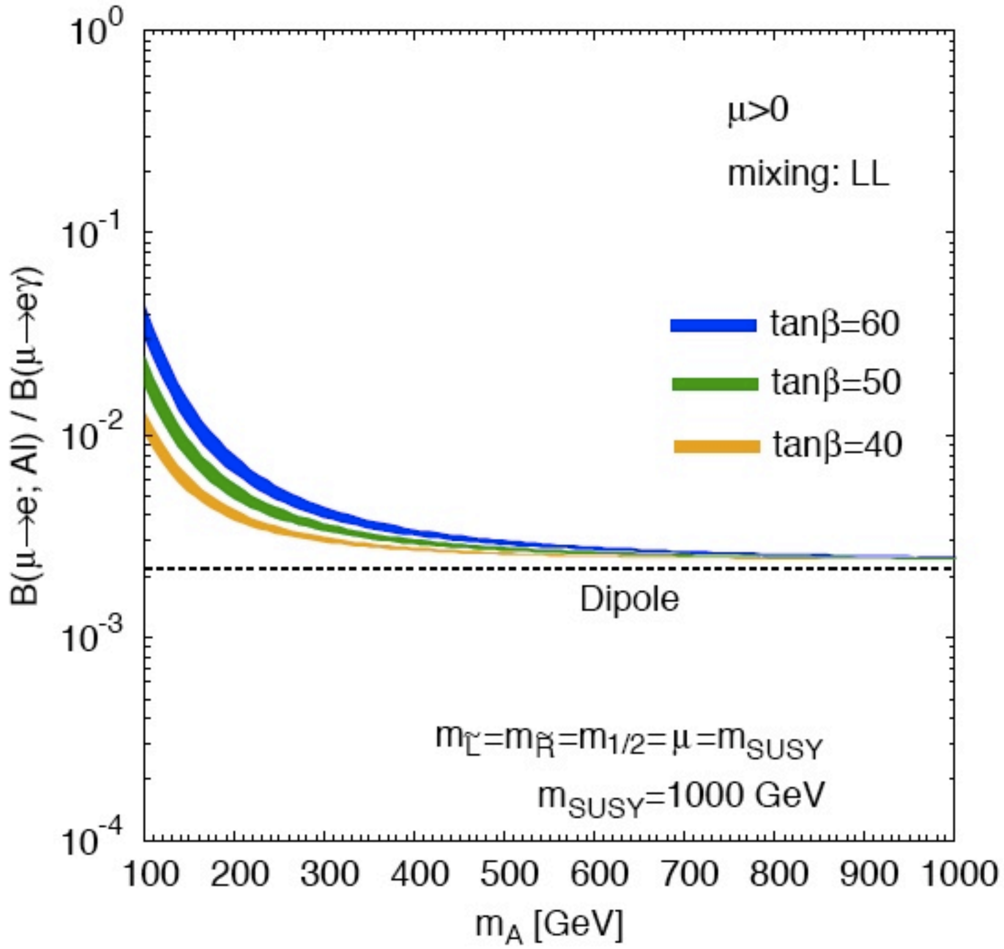
VC-Kitano-Okada-Tuzon '09

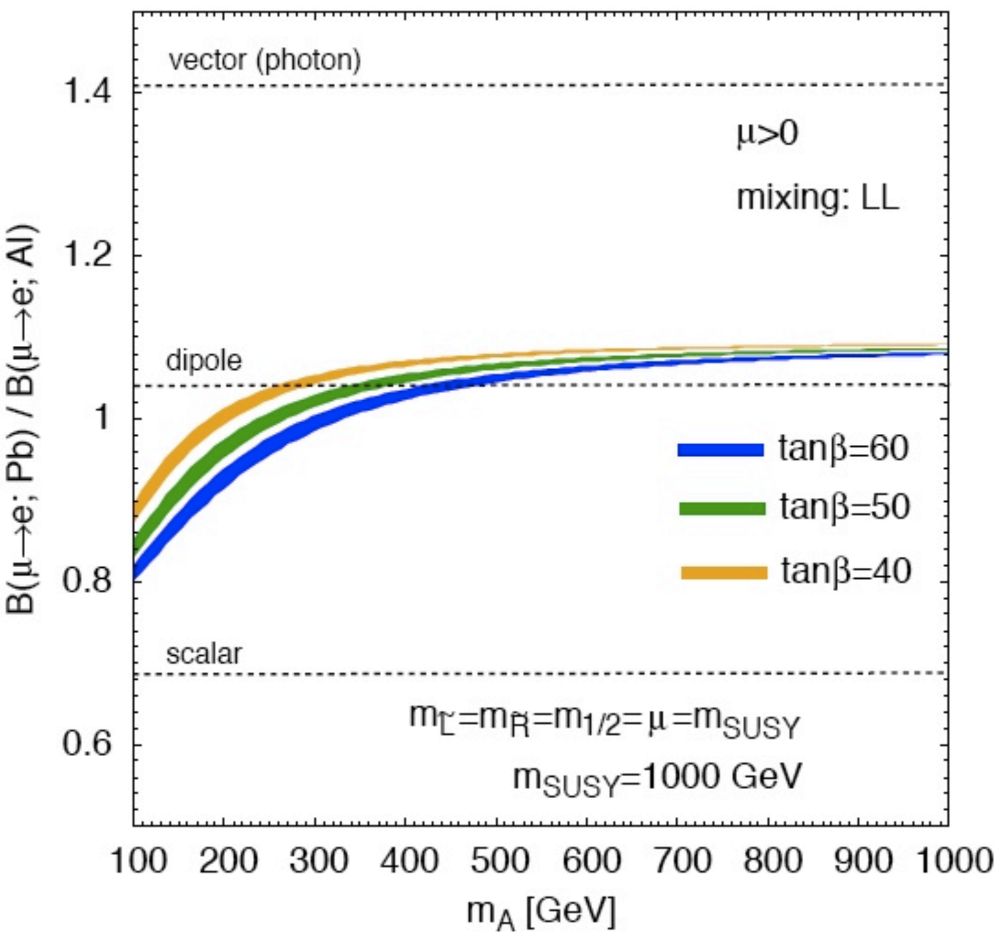
- Explicit realization in a SUSY scenario

Kitano-Koike-Komine-Okada 2003

- Dipole vs scalar operator (mediated by Higgs exchange) in SUSY see-saw models







VC-Kitano-Okada-Tuzon ‘09

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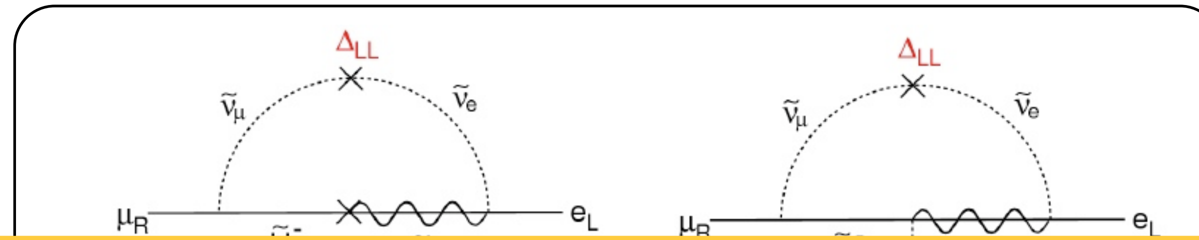


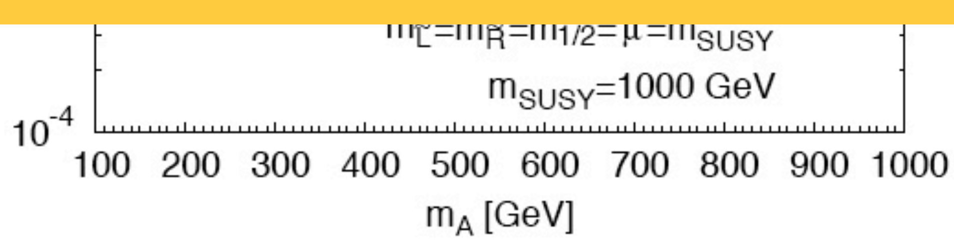
- Dipole vs scalar operator
  (mediated by Higgs exchange)

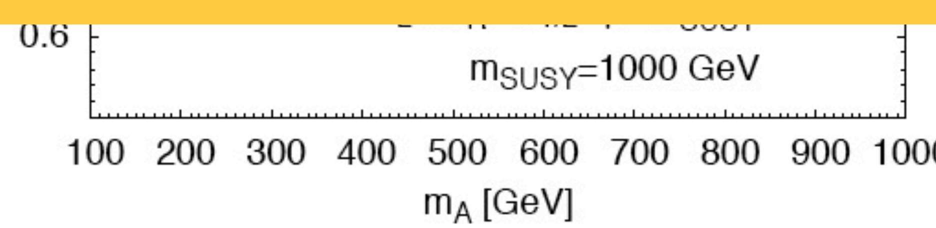
In summary:

- Theoretical hadronic uncertainties under control for 1-operator dominance

- need Lattice QCD for 2-operator models

- Realistic model discrimination requires measuring Ti/Al at <5% or Pb/Al at <20%





Conclusions

- Charged LFV: deep probes of physics BSM
- "Discovery" tools: clean, high scale reach
- "Model-discriminating" tools:

  - Operator structure $\rightarrow$ mediators
  - μe vs $\tau\mu$ vs τe $\rightarrow$ sources of flavor breaking

Exciting prospects in the next 5-10 years:

★ 3-4 orders of magnitude improvement in μ processes

★ 1-2 orders of magnitude improvement in τ processes

Extra Slides

** Qualitative behavior of overlap integrals

$\phi_e(x)$ → free outgoing electron wf

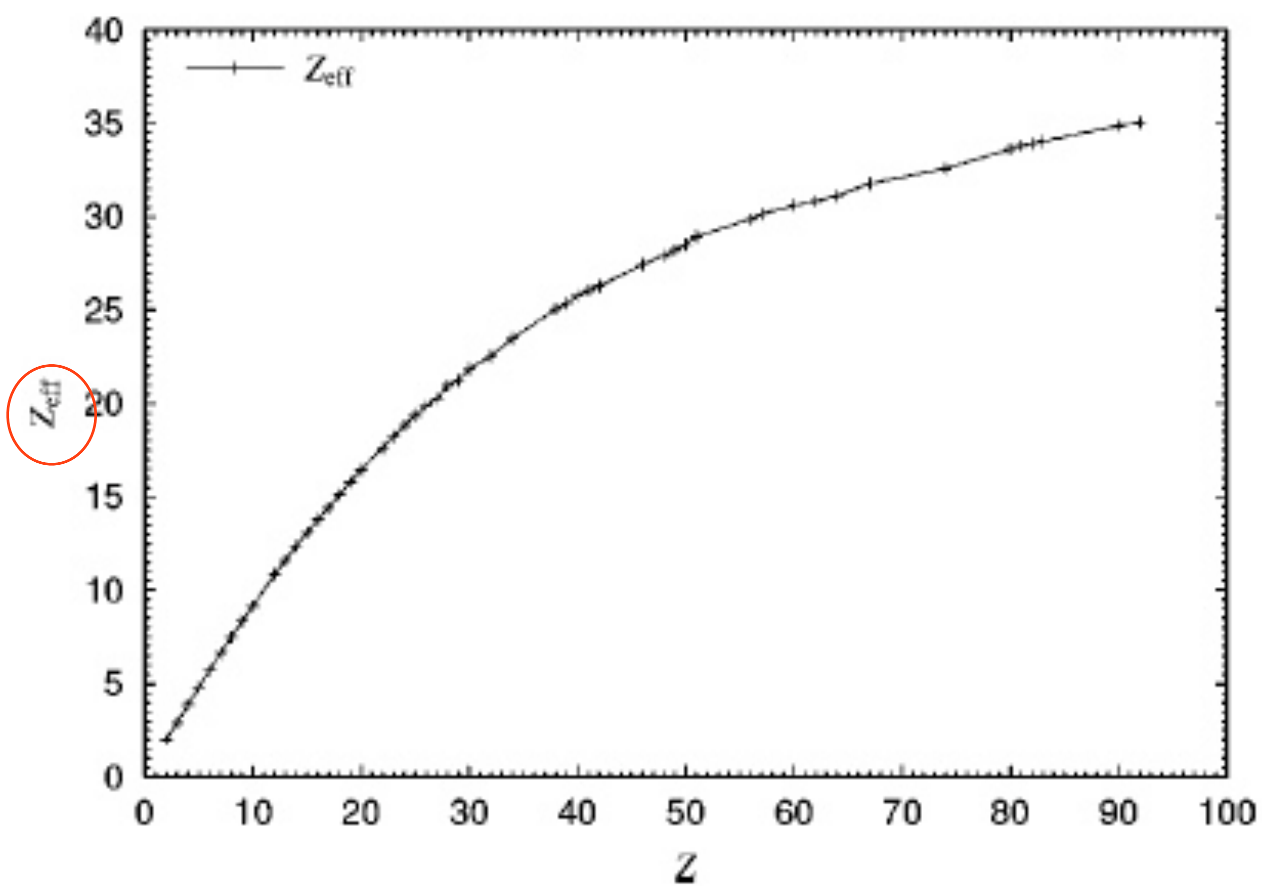
$\phi_\mu(x) \to \langle \phi_\mu(x) \rangle$ (average value)

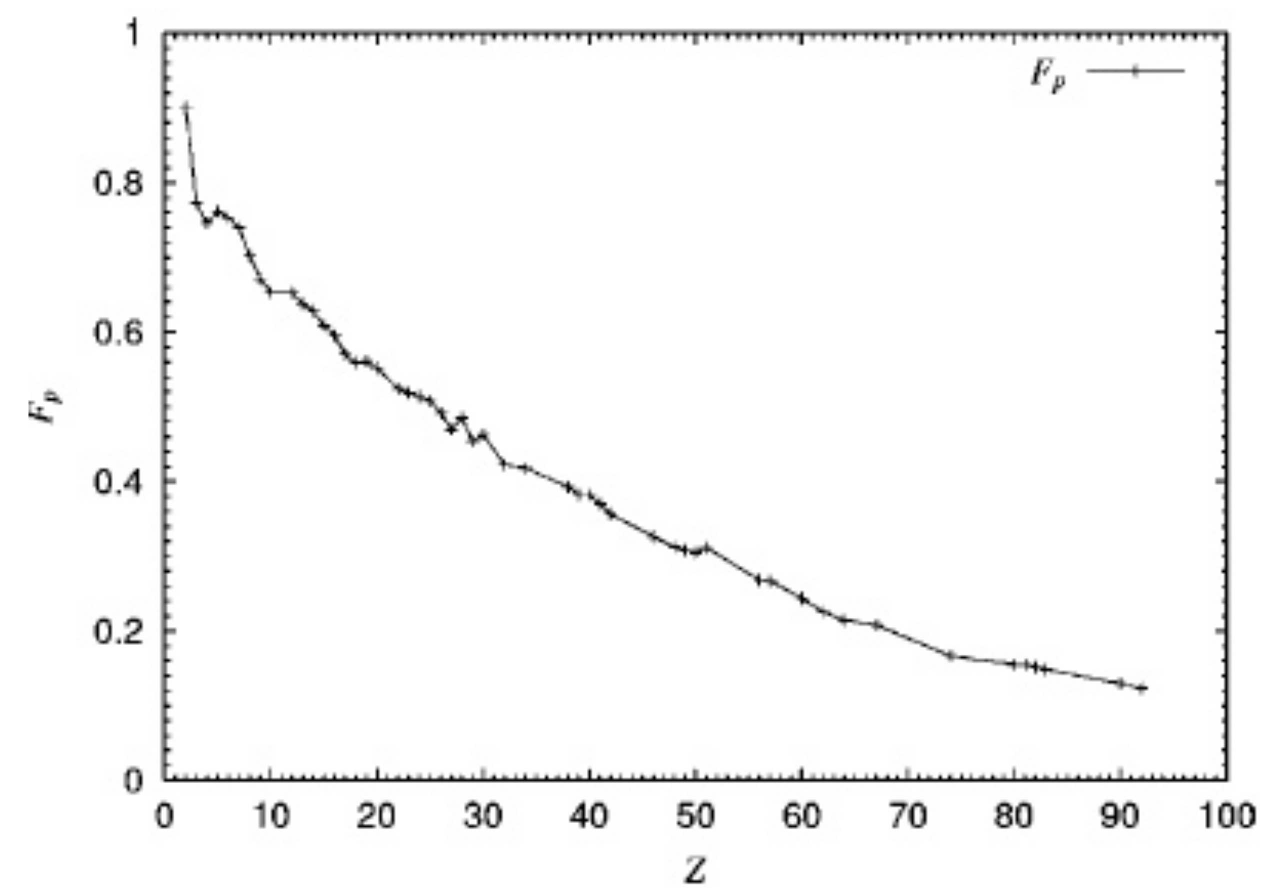
$$\boxed{I \sim \int d^3x \, \phi_e^*(x) \phi_\mu(x) \, \rho_p(x) \;\; \to \;\; \langle \phi_\mu \rangle \, F_p}$$

$p \sim m_\mu$

$$\langle \phi_\mu \rangle^2 = \int_0^\infty dr 4\pi r^2 (g_\mu^2 + f_\mu^2) \rho^{(p)} = \frac{4 m_\mu^3 \alpha^3 Z_{\mathrm{eff}}^4}{Z}$$

$$F_p = \int_0^\infty dr 4\pi r^2 \rho^{(p)} \frac{\sin m_\mu r}{m_\mu r}$$





Kitano-Koike-Okada

Benchmark models: D, S, V$_{(Z)}$, V$_{(\gamma)}$

$$\mathcal{L}_{\rm eff}^{(q)} = -\frac{1}{\Lambda^2}\Big[(C_{DR} m_\mu \bar{e}\sigma^{\rho\nu} P_L \mu + C_{DL} m_\mu \bar{e}\sigma^{\rho\nu} P_R \mu) F_{\rho\nu}$$
$$+ \sum_q (C_{VR}^{(q)} \bar{e}\gamma^\rho P_R \mu + C_{VL}^{(q)} \bar{e}\gamma^\rho P_L \mu)\bar{q}\gamma_\rho q$$
$$+ \sum_q (C_{SR}^{(q)} m_\mu m_q G_F \bar{e} P_L \mu + C_{SL}^{(q)} m_\mu m_q G_F \bar{e} P_R \mu)\bar{q}q \; + \text{H.c.}\Big]$$

Dipole model

$$C_D \equiv C_{DR} \neq 0, \qquad C_{\rm else} = 0.$$

Vector model: V(γ)

$$C_V \equiv C_{VR}^{(u)} = -2C_{VR}^{(d)} \neq 0, \qquad C_{\rm else} = 0,$$

Scalar model

$$C_S \equiv C_{SR}^{(d)} = C_{SR}^{(s)} = C_{SR}^{(b)} \neq 0,$$

$$C_{\rm else} = 0.$$

Vector model: V(Z)

$$C_V \equiv C_{VR}^{(u)} = \frac{C_{VR}^{(d)}}{a} \neq 0, \qquad C_{\rm else} = 0$$

$$a = \frac{T^3_{d_L} + T^3_{d_R} - (Q_{d_L} + Q_{d_R})\sin^2\theta_W}{T^3_{u_L} + T^3_{u_R} - (Q_{u_L} + Q_{u_R})\sin^2\theta_W} = -1.73. \qquad \tilde{C}_{VR}^{(n)}/\tilde{C}_{VR}^{(p)} = -9.26.$$

- Details on the uncertainties

	S	D	$V^{(\gamma)}$	$V^{(Z)}$
$\frac{B(\mu\to e,\mathrm{Ti})}{B(\mu\to e,\mathrm{Al})}$	$1.70 \pm 0.005_y$	1.55	1.65	2.0
$\frac{B(\mu\to e,\mathrm{Pb})}{B(\mu\to e,\mathrm{Al})}$	$0.69 \pm 0.02_{\rho_n}$	1.04	1.41	$2.67 \pm 0.06_{\rho_n}$

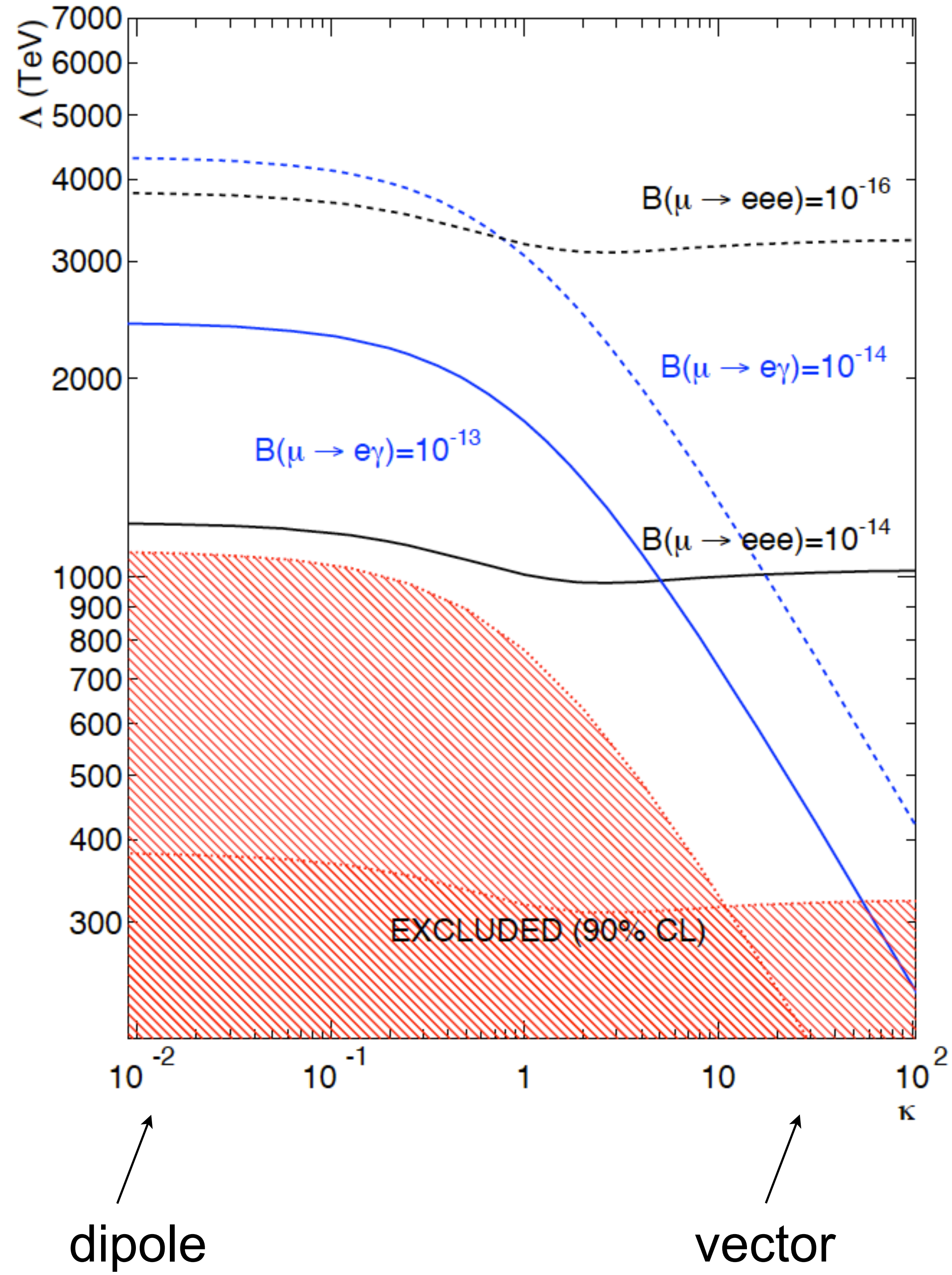
$\mu \to e\gamma$ vs $\mu \to 3e$

- A simple example with two operators

De Gouvea, Vogel 1303.4097

$$\mathcal{L}_{\rm CLFV} = \frac{m_\mu}{(\kappa+1)\Lambda^2}\bar{\mu}_R\sigma_{\mu\nu}e_L F^{\mu\nu} + h.c. \\ \frac{\kappa}{(1+\kappa)\Lambda^2}\bar{\mu}_L\gamma_\mu e_L\left(\bar{e}\gamma^\mu e\right) + h.c.\,.$$

- κ controls relative strength of dipole vs vector operator



$\mu \to e\gamma$ vs $\mu \to e$ conversion

- A simple example with two operators

De Gouvea, Vogel 1303.4097

$$\mathcal{L}_{\rm CLFV} = \frac{m_\mu}{(\kappa+1)\Lambda^2}\bar{\mu}_R\sigma_{\mu\nu}e_L F^{\mu\nu} + h.c.$$
$$\frac{\kappa}{(1+\kappa)\Lambda^2}\bar{\mu}_L\gamma_\mu e_L\left(\bar{u}_L\gamma^\mu u_L + \bar{d}_L\gamma^\mu d_L\right) + h.c.\,.$$

- κ controls relative strength of dipole vs vector operator