Lattice QCD and the International Linear Collider

G. Peter Lepage

Cornell University (and DAMTP, Cambridge)

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Is an ILC useful?

- ILC ⇒ high-precision measurements of Higgs branching fractions, to study Beyond the Standard Model (BSM).
- BSM requires accurate SM parameters: e.g.,
  - $\Gamma(h \to b\bar{b})$ to 0.3% requires m_b to 0.3% and α_s to 0.5%.
  - $\Gamma(h \to c\bar{c})$ to 0.7% requires m_c to 0.7% and α_s to 1%.
  - $\Gamma(h \to gg)$ to 0.6% requires α_s to 0.6%.
- Need: m_b to 0.15%, m_c to 0.35%, α_s to 0.25%
- Is this possible over the next decade or so?
  - Most dramatic recent develops are from LQCD.

See: Lepage, Mackenzie and Peskin, soon.

Eg: current-current correlator in LQCD

Compute for heavy valence quark *h:*

$\overline{\psi}_h \gamma_5 \psi_h$

$$G(t) \equiv a^6 \sum_{\mathbf{x}} (am_{oh})^2 \langle 0 | j_{5h}(\mathbf{x}, t) j_{5h}(0,0) | 0 \rangle$$

- Mass factors imply UV finite (PCAC because HISQ).
- Implies:

$$G_{\text{cont}}(t) = G_{\text{lat}}(t) + \mathcal{O}(a^2) \qquad \text{for all } t \neq 0$$

Moments

Low-n moments perturbative ($E_{\text{threshold}} = 2m_h$):

$$G_n \equiv \sum_t (t/a)^n G(t)$$
$$\to \frac{\partial^n}{\partial E^n} G(E=0)$$

Implies:

from continuum 3rd-order perturbation theory — gives coupling

from lattice simulations (in place of experiment)

$$G_n = \frac{g_n(\alpha_{\overline{\text{MS}}}(m_h))}{(am_h(m_h))^{n-4}}$$

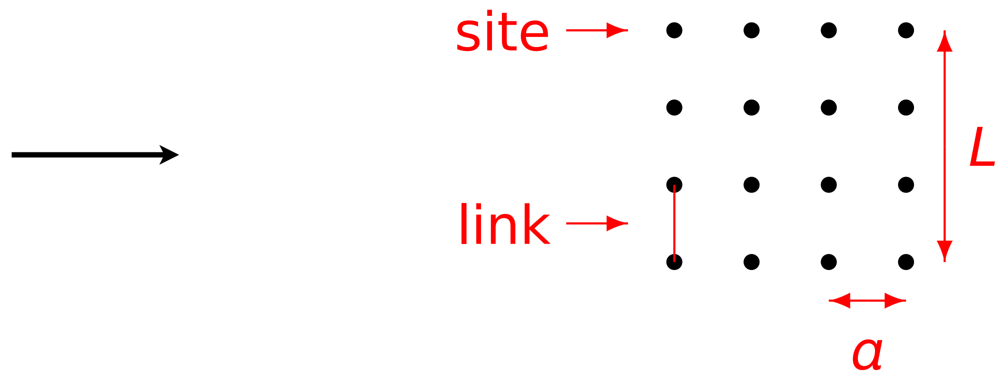
gives mass $m_h(m_h)$ (m_h only scale)

N.B. Fit n=4,6,8,10 for range of m_h to get high-precision m_c, m_b, and α_s!

What is lattice QCD (LQCD)?

Continuous

Space & Time



⇒ Fields $\psi(x)$, $A_\mu(x)$ specified only at grid sites (or links); interpolate for other points.

⇒ Solving QCD → multidimensional integration:

$$\int \mathcal{D}A_\mu \ldots \mathrm{e}^{-\int L dt} \longrightarrow \int \prod_{x_j \epsilon \mathrm{grid}} dA_\mu(x_j) \ldots \mathrm{e}^{-a \sum L_j}$$

LQCD simulations

1. Integrate path integral numerically — need Monte Carlo simulation methods.

2. Tune five free parameters — bare $m_u = m_d$, m_s, m_c, m_b, and α_s — using, *e.g.*, $m(\pi)$, $m(K)$, $m(\eta_c)$, $m(\eta_b)$ and f_π.

3. Tuned LQCD simulation = real QCD, with no free parameters. Compute vacuum expectation values of numerous operators for multiple lattice spacings a. Extrapolate to $a = 0$ to extract physics.

N.B. LQCD $\longleftrightarrow$ experiment.

Moments (again)

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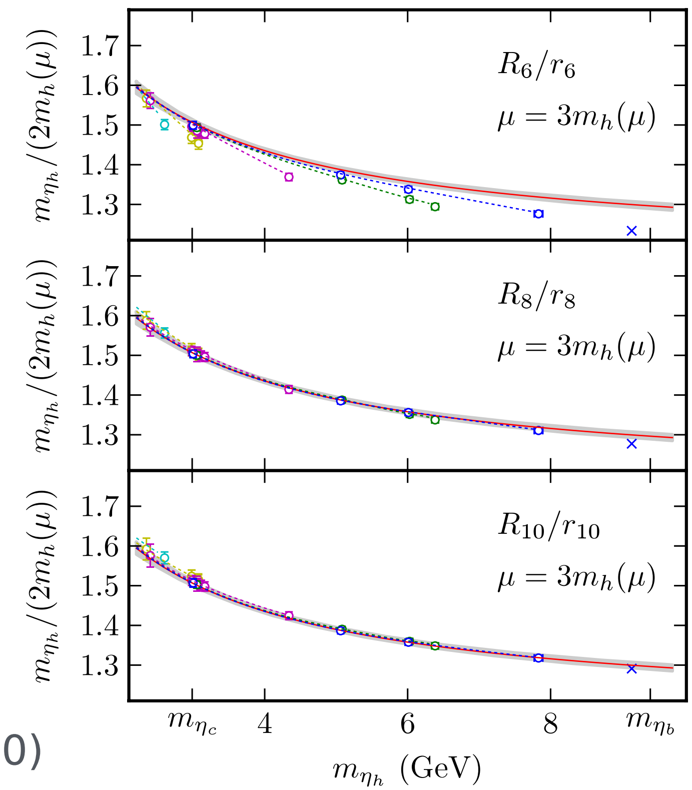
Current best results

$m_c(m_c, n_f=4)$ = 1.273(6) GeV (0.47%, need 0.35%)

$m_b(m_b, n_f=5)$ = 4.164(23) GeV (0.55%, need 0.15%)

$\alpha_{\overline{MS}}(M_Z, n_f=5)$ = 0.1183(7) (0.59%, need 0.25%)

	$m_c(3)$	$m_b(10)$	m_b/m_c	$\alpha_{\overline{MS}}(M_Z)$
a^2 extrapolation	0.2%	0.6%	0.5%	0.2%
Perturbation theory	0.5	0.1	0.5	0.4
Statistical errors	0.1	0.3	0.3	0.2
m_h extrapolation	0.1	0.1	0.2	0.0
Errors in r_1	0.2	0.1	0.1	0.1
Errors in r_1/a	0.1	0.3	0.2	0.1
Errors in m_{η_c}, m_{η_b}	0.2	0.1	0.2	0.0
α_0 prior	0.1	0.1	0.1	0.1
Gluon condensate	0.0	0.0	0.0	0.2
Total	0.6%	0.7%	0.8%	0.6%



McNeile et al (HPQCD), Phys. Rev. D82, 034512 (2010)

Simulate possible future results

Analysis of current data creates a fit function that describes the dependence of LQCD results on lattice spacing, perturbation theory, statistics, quark masses, *etc*:

$\Longrightarrow$ Predict what LQCD data at a smaller lattice spacing or a different quark mass or … will look like by evaluating fit function at smaller lattice spacing or …

$\Longrightarrow$ Add realistic noise to generate fake LQCD data with smaller lattice spacings and/or different mass and/or better statistics and/or …

- Rerun analysis but now on fake data plus existing LQCD data to extract new (fake) results for m_b, m_c, α_s.

  $\Longrightarrow$ Impact of fake data on errors.

- Also rerun analysis while pretending that 4^{th}-order perturbation theory is known (add fake 4^{th}-order coefficients).

$\Longrightarrow$ Map realistic scenarios for hardware/software/theory improvements onto improvements in the precision of m_b, m_c, α_s.

Scenarios for the next decade or so

Some combination of following likely doable:

- 4^{th}-order perturbation theory completed (continuum calculation).

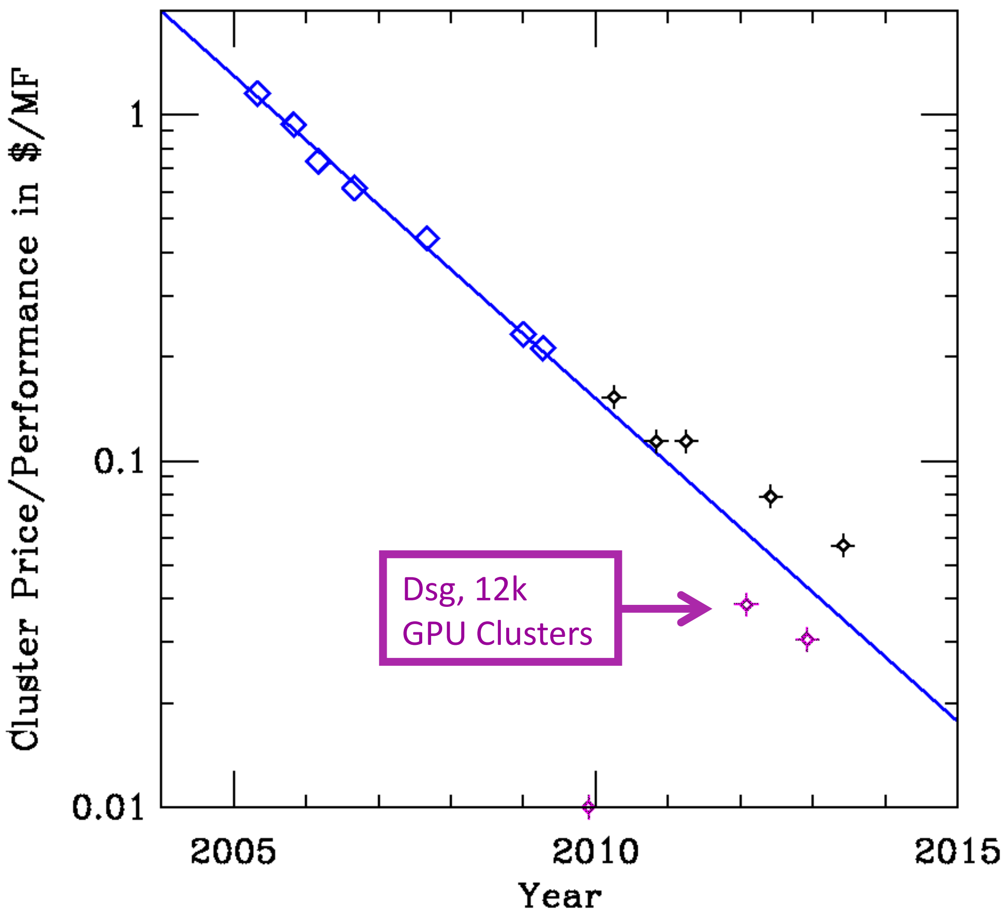
- 100x increase in computing power ⇒

  - Reduce lattice spacing to 0.03fm and 0.0225fm (from 0.045fm).

  - 100x increase in statistics.

How long for 100x in computing?

Historical processor-cluster price / performance data from measurements at USQCD (Fermilab):



- 10–15 years with cluster technology.
- ??? years with GPUs or ….

Simulation of simulations (% errors)

couplings $\propto \Gamma^{1/2}$

	$m_b(10)$	$\alpha_{\overline{\mathrm{MS}}}(M_Z)$	$m_c(3)$	$h \to b\bar{b}$	$h \to c\bar{c}$	$h \to gg$
current results (prelim.)	0.65	0.49	0.50	0.82	0.74	0.64
4^{th}-order	0.63	0.32	0.30	0.74	0.49	0.42
$a = 0.03$ fm	0.25	0.36	0.44	0.38	0.60	0.47
$a = 0.03$ and 0.023 fm	0.15	0.28	0.43	0.27	0.54	0.36
4^{th}-order, $a = 0.03$ fm	0.24	0.21	0.22	0.30	0.32	0.27
4^{th}-order, $a = 0.03$ and 0.023 fm	0.14	0.16	0.22	0.19	0.29	0.21
4^{th}-order, $a = 0.03$ and 0.023 fm, $100\times$stats	0.10	0.11	0.21	0.14	0.26	0.14
ILC goal	0.15	0.25	0.35	0.30	0.70	0.60

Are LQCD errors believable?

Lattice allows checks that are impossible in the continuum:

- Simultaneously fit data for a range of quark masses m_h between m_c and m_b $\Rightarrow$ treat coefficients of $\alpha_s(m_h)^n$ for $n>3$ as fit parameters:

$$g_n(\alpha_{\overline{\rm MS}}(m_h)) = c_{n0} + c_{n1}\alpha_{\overline{\rm MS}} + c_{n2}\alpha^2_{\overline{\rm MS}} + c_{n3}\alpha^3_{\overline{\rm MS}} \quad \longleftarrow \text{Pert'n th.}$$
$$+ c_{n4}\alpha^4_{\overline{\rm MS}} + c_{n5}\alpha^5_{\overline{\rm MS}} + \cdots$$

Fit parameters. (c_{n4}, c_{n5})

Much more reliable estimate of perturbative errors than, *e.g.*, variation as $\mu\to\mu/2$ and $\mu\to 2\mu$.

- Nonperturbative ratios of quark masses easy to measure in LQCD. Compare nonperturbative m_b/m_c (4.49(4)) with perturbative result (4.51(4)). Highly non-trivial check.

- Vary valence quark mass to vary/fit nonperturbative effects. O.P.E. implies:

$$G_n = G_n^{\mathrm{short-distance}} \left\{ 1 + d_n(\alpha_{\overline{\mathrm{MS}}}) \frac{\langle \alpha_s G^2/\pi \rangle}{(2m_h)^4} + \cdots \right\}$$

perturbation theory, small instantons, *etc* (no IR renormalons)

nonperturbative, very small (<0.1%)

N.B. Continuum results using $R(e\bar{e})$ data (instead of LQCD) to compute vector correlators give same masses, with similar errors, to within 1σ. (c.*f.*, Karlsruhe Group.)

Other LQCD technologies

- Higher moments (n=12, 14…) have smaller $(am_b)^2$ errors.

- Avoid $(am_b)^2$ errors by using NRQCD/Fermilab *b*-quark action for m_b current-current correlators. May need better perturbation theory for high-order moments (n=20). Cancel Z factors (nonperturbative). (HPQCD analysis soon)

- α_s from Wilson loops remains competitive at smaller lattice spacings (needs HISQ perturbation theory). Also light-quark vacuum polarization at large Euclidean q^2 (Adler function). Also …

- m_b and m_c from off-shell $m_h \langle q|\overline{\psi}_h\gamma_5\psi_h|q'\rangle$.

- …

Conclusions

- LQCD likely to deliver adequate precision for ILC needs.
- Likely to have multiple approaches for each parameter.
- Improved continuum perturbation theory would help.
- Highly-improved actions (like HISQ) help keep $(am_b)^2$ errors under control.