Hadronic Light-by-Light contribution to the muon g-2 from lattice QCD

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Collaborators

Work on g-2 done in collaboration with

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RBC/UKQCD Collaboration	Norikazu Yamada (KEK)
	Norman Christ (Columbia)
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Outline

The magnetic moment of the muon

Interaction of particle with static magnetic field

$$V(\vec{x}) \;=\; -\vec{\mu}\cdot\vec{B}_{\rm ext}$$

The magnetic moment $\vec{\mu}$ is proportional to its spin ($c = \hbar = 1$)

$$\vec{\mu} \;=\; g\left(\frac{e}{2m}\right)\vec{S}$$

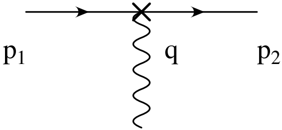
The Landé g-factor is predicted from the free Dirac eq. to be

$$g \;=\; 2$$

for elementary fermions

The magnetic moment of the muon

In interacting quantum (field) theory g gets corrections



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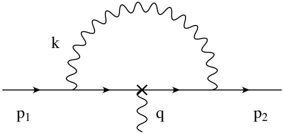


$+ \ldots$

$$\gamma^\mu \;\rightarrow\; \Gamma^\mu(q) = \left(\gamma^\mu F_1(q^2) + \frac{i\,\sigma^{\mu\nu}\,q_\nu}{2m} F_2(q^2)\right)$$

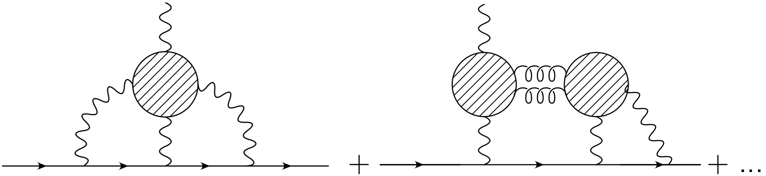
which results from Lorentz and gauge invariance when the muon is on-mass-shell.

$$F_2(0) \;=\; \frac{g-2}{2} \equiv a_\mu \qquad (F_1(0) = 1)$$

(the anomalous magnetic moment, or anomaly)

Outline

The hadronic light-by-light amplitude



Blobs: all possible hadronic states

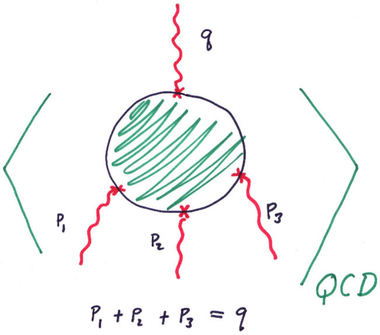
Model estimates put this $\mathcal{O}(\alpha^3)$ contribution at about $(10 - 12) \times 10^{-10}$ with a 25-40% uncertainty

No dispersion relation *a'la* vacuum polarization

Dominated by pion pole (models)

Lattice regulator: model independent, approximations systematically improvable

Lattice QCD: conventional approach



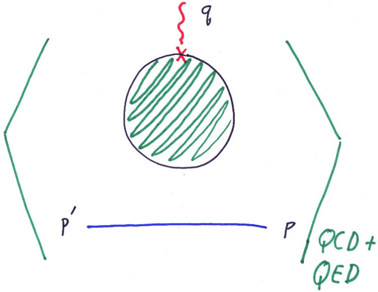
Correlation of 4 EM currents $\Pi^{\mu\nu\rho\sigma}(q, p_1, p_2)$

Two independent momenta +external mom q

Compute for all possible values of p_1 and p_2 ($O(V^2)$) four index tensor

several q (extrap $q \to 0$), fit, plug into perturbative QED two-loop integrals

Alternate approach: Lattice QCD+QED



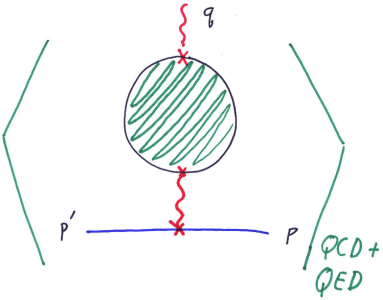
Average over combined gluon *and* photon gauge configurations

Quarks coupled to gluons and photons

muon coupled to photons

[Hayakawa, *et al.* hep-lat/0509016;
Chowdhury *et al.* (2008);
Chowdhury Ph. D. thesis (2009)]

Alternate approach: Lattice QCD+QED



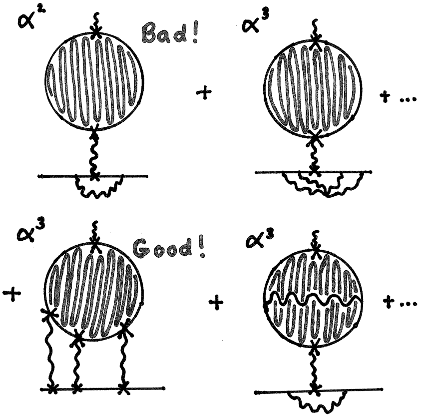
Attach one photon by hand (see why in a minute)

Correlation of hadronic loop and muon line

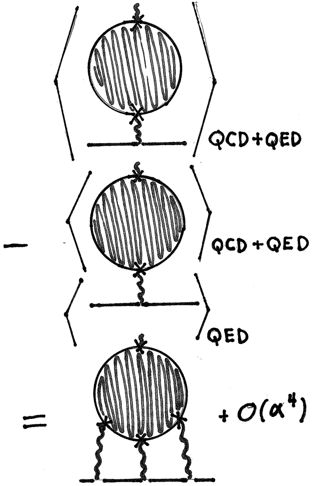
[Hayakawa, *et al.* hep-lat/0509016;
Chowdhury *et al.* (2008);
Chowdhury Ph. D. thesis (2009)]

Formally expand in α electromagnetic

The leading and next-to-leading contributions in α to magnetic part of correlation function come from



Subtraction of lowest order piece

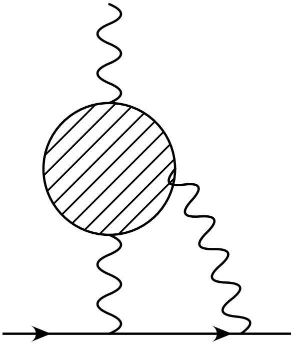


Subtraction term is product of separate averages of the loop and line

Gauge configurations identical in both, so two are highly correlated

In PT, correlation function and subtraction have same contributions except the light-by-light term which is absent in the subtraction

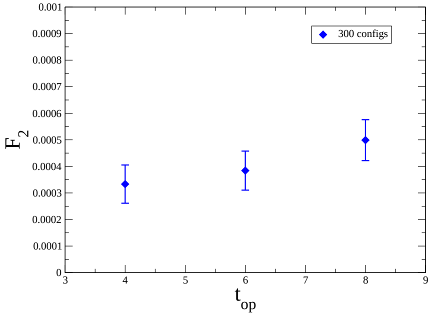
Subtraction of lowest order piece: two photons?



- absent in subtraction term, but vanishes due to Furry's theorem
- Only after averaging over gauge fields, potentially large error ($O(\alpha^2)$ compared to signal of $O(\alpha^3)$)
- Exact symmetry under $\mathbf{p} \to -\mathbf{p}$
  $e \to -e$ on muon line only
- If e unchanged, only effect is to flip the sign of all diagrams with two photons, so these cancel on each configuration.
- Observe large reductions in statistical errors after momentum averaging

QED test [Chowdhury Ph. D. thesis, UConn, 2009]

$F_2 = (3.96 \pm 0.70) \times 10^{-4}$



- $m_\mu / m_e = 40$
- $e = 1$
- $16^3 \times 32$ lattice size
- lowest non-zero momentum only ($|\mathbf{p}|/m_\mu \approx 1$)
- **stat error only**

- Expected size of enhancement (compared to $m_\mu / m_e = 1$)
- Continuum PT result: $\approx 10(\alpha/\pi)^3 = 1.63 \times 10^{-4}$ ($e = 1$)
- roughly consistent with PT result, large finite volume effect

QED test: finite volume study

- Repeat calculation with 24^3 lattice volume
- Bigger box $F_2 = (1.19 \pm 0.32) \times 10^{-4}$
- Small box $F_2 = (3.96 \pm 0.70) \times 10^{-4}$
- finite volume effects manageable
- Continuum PT result: $\approx 10(\alpha/\pi)^3 = 1.63 \times 10^{-4}$ $(e = 1)$
- Roughly consistent with PT result

2+1f QCD+QED (PRELIMINARY)

- Same as before, but with $U = U(1) \times SU(3)$ [Duncan, *et al.*]
- QCD in the loop only (same in subtraction)
- QED in both loop and line
- 2+1 flavors of DWF (RBC/UKQCD)
- $a = 0.114$ fm, $16^3 \times 32\,(\times 16)$, $a^{-1} = 1.73$ GeV
- $m_q \approx 0.013$, $m_\pi \approx 420$ MeV
- $m_\mu \approx 692$ MeV ($m_\mu^{\mathrm{phys}} = 105.658367(4)$ MeV)
- 100 configurations (one QED conf. for each QCD conf.)
- $(N_s/4)^3 = 64$ (loop) propagator calculations/configuration

2+1f lattice QCD+QED (PRELIMINARY)

- $a_\mu(\mathrm{HLbL}) = (-11.6 \pm 1.2) \times 10^{-5} = -1.58 \pm 0.16 \times (\alpha/\pi)^3$ (lowest non-zero mom, $e = 1$)
- Magnitude 10 times bigger, sign opposite from models
- HLbL amplitude depends strongly on m_μ (m_μ^2 in models)
- Leading contribution is from pion pole
- Non-leading terms in models can give large, negative values (see arXiv:1309.2225 for summary and new results)
- Check subtraction is working by varying $e = 0.84$, 1.19
  - HLbL amplitude ($\sim e^4$) changes by ~ 0.5 and 2 ✓
  - while unsubtracted amplitude stays the same ✓

2+1f lattice QCD+QED (PRELIMINARY)

Easy to lower muon mass (muon line is cheap)

- Try $m_\mu \approx 190$ MeV
- $a_\mu(\mathrm{HLbL}) = (-0.96 \pm 0.36) \times 10^{-5} = -0.131 \pm 0.049 \times (\alpha/\pi)^3$ (lowest non-zero mom, $e = 1$).

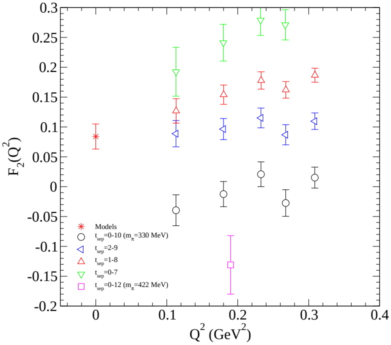
Right direction, right amount ...

2+1 flavor lattice QCD+QED (PRELIMINARY)

More realistic ensemble (RBC/UKQC DWF)

- Larger lattice size, 24^3 ($(2.7\text{ fm})^3$)
- Pion mass is smaller too, $m_\pi = 329$ MeV
- Same muon mass (190 MeV)
- $0.11 \lesssim Q^2 \lesssim 0.31\ \text{GeV}^2$
- Use All Mode Averaging (AMA)
  - 6^3 (5^3) point sources/configuration = 216 (125)
  - AMA approximation: "sloppy CG", $r_{\text{stop}} = 10^{-4}$

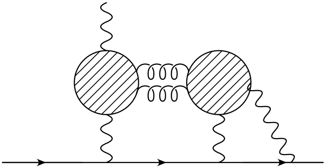
2+1f lattice QCD+QED (PRELIMINARY)



[Blum, Hayakawa, and Izubuchi (2014)]

- model value/error is "Glasgow Consensus"
  (arXiv:0901.0306 [hep-ph])
- Lattice: stat. error only
- Several source/sink separations for muon
- Possible significant excited state contamination
- $m_\pi = 329, 422$ MeV
- Pion pole may be emerging

"Disconnected" diagrams



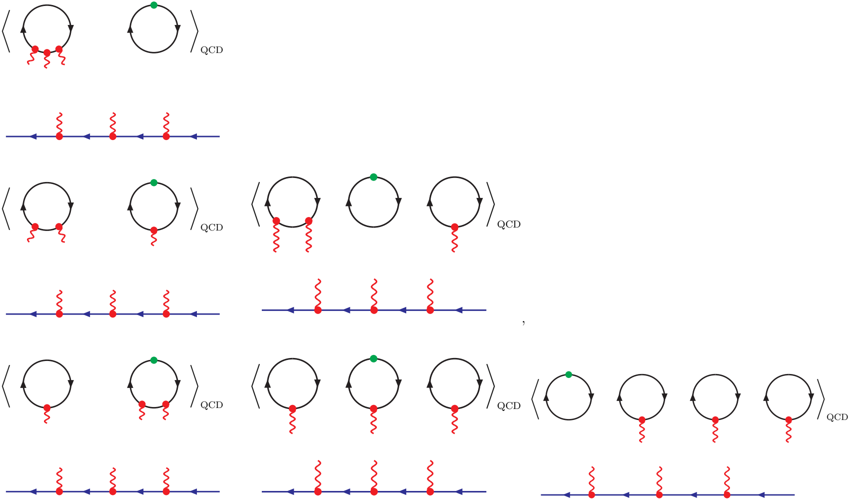
not calculated yet

Omission due to use of quenched QED, i.e., sea quarks not electrically charged. Two possibilities,

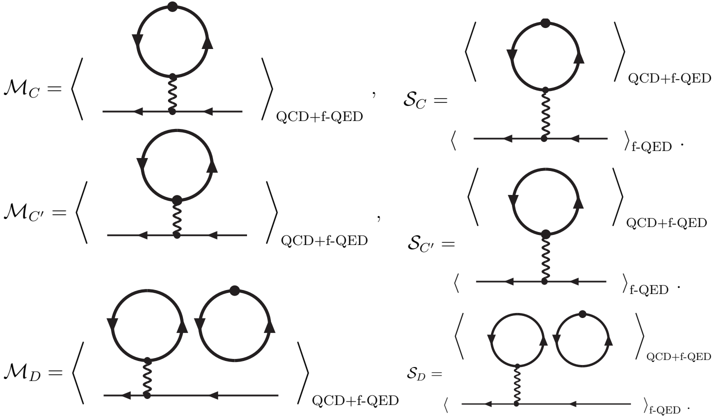
1. Re-weight in α (T. Ishikawa, *et al.*, Phys.Rev.Lett. 109 (2012) 072002) or
2. dynamical QED(+QCD) in HMC

Use same non-perturbative method as for quenched QED

Disconnected quark loop diagrams



Disconnected quark loops in non-perturbative method



Disconnected quark loops in non-perturbative method

Diagrams in non-perturbative method have various “multiplicities”

	$\mathcal{M}_C + \mathcal{M}_{C'}$	$\mathcal{M}_D$
LBL(4)	3	0
LBL(1,3)	0	3
LBL(2,2)	1	2
LBL(3,1)	2	1
LBL(1,1,2)	0	3
LBL(2,1,1)	1	2
LBL(1,1,1,1)	0	3

But, physical linear combination,
$\mathcal{M}_C + \mathcal{M}_{C'} + \mathcal{M}_D$
has overall factor of 3

more systematic errors

- quark mass
- sea-quark charge
- Finite volume
- $q^2 \to 0$ extrapolation
- $m_\mu \to m_{\mu,\,\mathrm{phys}}$
- excited states/"around the world" effects
- $a \to 0$
- QED renormalization
- ...

Timeline for calculation (very rough)

1. Connected part, near physical pion mass: end of 2014
2. Connected part, physical pion mass: mid 2015
3. First disconnected parts / dynamical QED: end of 2015
4. Refined calculation addressing all systematics: 201-?

Outline

Summary/Outlook

- First HLbL lattice calculation encouraging
- Current/next steps
  1. 170 MeV pion, connected: pion pole?
  2. 420 MeV pion, connected: excited state contamination
  3. dynamical QED

Acknowledgments

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- Computational resources provided by the RIKEN BNL Research Center and USQCD Collaboration
- Lattice computations done on
  - QCDOC at BNL
  - Ds cluster at FNAL

Outline

Introduction

The hadronic light-by-light (HLbL) contribution ($O(\alpha^3)$)

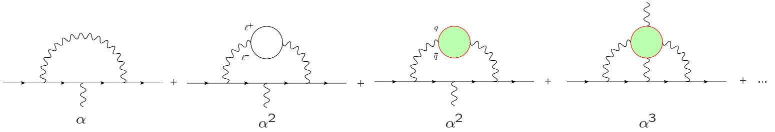
Summary/Outlook

Backup slides

The magnetic moment of the muon

Compute these corrections order-by-order in perturbation theory by expanding $\Gamma^\mu(q^2)$ in QED coupling constant

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137} + \ldots$$

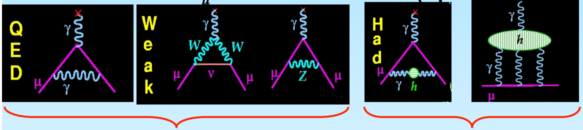


Corrections begin at $\mathcal{O}(\alpha)$; Schwinger term $= \frac{\alpha}{2\pi} = 0.0011614\ldots$

hadronic contributions $\sim 6 \times 10^{-5}$ times smaller (leading error).

Value in the standard model

The SM Value for a_μ from $e^+e^- \to$ *hadrons* (Updated 9/10)



CONTRIBUTION	RESULT ($\times 10^{-11}$) UNITS
QED (leptons)	$116\ 584\ 718.09 \pm 0.14 \pm 0.04_\alpha$
HVP(lo)	$6\ 914 \pm 42_{\text{exp}} \pm 14_{\text{rad}} \pm 7_{\text{pQCD}}$
HVP(ho)	$-98 \pm 1_{\text{exp}} \pm 0.3_{\text{rad}}$
HLxL	105 ± 26
EW	$152 \pm 2 \pm 1$
Total SM	$116\ 591\ 793 \pm 51$

A. Höcker Tau 2010, U. Manchester September 2010

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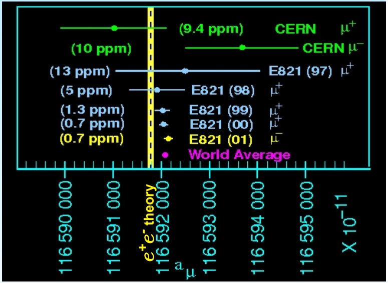
- new: QED thru $O(\alpha^5)$ [Aoyama, *et al.*, 2012]

Experimental value (dominated by BNL E821)

E821 achieved ± 0.54 ppm. The e^+e^- based theory is at the ~0.4 ppm level. Difference is ~3.6 σ



$$a_\mu^{exp} = 116\,592\,089(63) \times 10^{-11} \ (0.54\ \text{ppm})$$
$$\Delta a_\mu \equiv a_\mu^{exp} - a_\mu^{SM} = (287 \pm 80) \times 10^{-11}$$

Theory: arXiv:1010.4180v1 [hep-ph] Davier, Hoecker, Malaescu, and Zhang, Tau2010

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New experiments + new theory = (?) new physics

muon anomaly a_μ provides important test of the SM

- Fermilab E989, $\sim 2-3$ years away, 0.14 ppm
- J-PARC E34 ? (recently, lower priority than $\mu \to e$)
- $a_\mu(\mathrm{Expt})$-$a_\mu(\mathrm{SM}) = 287(63)(51)$ $(\times 10^{-11})$, or $\sim$ 3.6σ
  to 249(87) $(\times 10^{-11})$, or $\sim$ 2.9σ
- If both central values stay the same,
  - E989 ($\sim 4\times$ smaller error) $\to\sim$ 5σ
  - E989+new HLBL theory (models+lattice, 10%) $\to\sim$ 6σ
  - E989+new HLBL +new HVP (50% reduction) $\to\sim$ 8σ
- Big discrepancy! (New Physics $\sim 2\times$ Electroweak)
- Lattice calculations crucial