Predictions from High Scale Mixing Unification Hypothesis

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ABSTRACT

Starting with 'High Scale Mixing Unification' hypothesis, we investigate the renormalization group evolution of mixing parameters and masses for both Dirac and Majorana type neutrinos. Following this hypothesis, the PMNS mixing parameters are taken to be identical to the CKM ones at a unifying high scale. Then, they are evolved to a low scale using MSSM renormalization-group equations. For both type of neutrinos, the renormalization group evolution "naturally" results in a non-zero and small value of leptonic mixing angle θ_{13}. One of the important predictions of this analysis is the prediction for octant of the mixing angle θ_{23}. We also elaborate on the important differences between Dirac and Majorana neutrinos within our framework and how to experimentally distinguish between the two scenarios. Furthermore, for both cases, we also derive constraints on the allowed parameter range for the SUSY breaking and unification scales, for which this hypothesis works. The results are novel and can be tested by present and future experiments.

Motivation

- Neutrinos are probably the most mysterious and ill understood of all known particles. In past neutrinos have thrown up quite a few surprises. They still keep on surprising us !!
- Recent measurements have conclusively established neutrino mixing angle θ_{13} to be non-zero. As a result, several models are facing stringent constraints.
- Is there a "natural" way of understanding non-zero and "relatively large" θ_{13}?
- **High Scale Mixing Unification (HSMU)** of CKM and PMNS parameters provides one such explanation.
- The smallness of PMNS mixing angle θ_{13} is related to the smallness of the corresponding CKM mixing angle.

Current Experimental Scenario

- Global Fits for neutrino oscillation parameters **(M. C. Gonzalez-Garcia et.al. JHEP 1212 (2012) 123)** :

Quantity	Best Fit ± 1-σ	3-σ Range
$\theta_{12}/^\circ$	$33.36^{+0.81}_{-0.78}$	31.09 – 35.89
$\theta_{23}/^\circ$	$40.0^{+2.1}_{-1.5} \oplus 50.4^{+1.3}_{-1.3}$	35.8 – 54.8
$\theta_{13}/^\circ$	$8.66^{+0.44}_{-0.46}$	7.19 – 9.96
$\delta_{\rm CP}/^\circ$	300^{+66}_{-138}	0 – 360
Δm^2_{21} (10^{-5} eV^2)	$7.50^{+0.18}_{-0.19}$	7.00 – 8.09
Δm^2_{31} (10^{-3} eV^2) (N)	$2.473^{+0.070}_{-0.067}$	2.276 – 2.695
Δm^2_{23} (10^{-3} eV^2) (I)	$2.427^{+0.042}_{-0.065}$	2.242 – 2.649

- Averaged electron neutrino mass (m_β): Not yet measured ($m_\beta < 2$ eV, KATRIN sensitivity ≈ 0.2 eV).
- Neutrinoless Double Beta Decay: Not Observed (Effective Majorana mass $m_{\beta\beta} < 0.38$ eV).

High Scale Mixing Unification

- Unification of seemingly unrelated phenomenon is an old and quite fruitful notion. Has lead to much advancement in our understanding: Electro-Magnetism, Electro-Weak force etc.
- Current research: Unification of forces.
- Grand Unified Theories (GUTs): Unification of gauge couplings.
- Key Ingredient: Quarks and Leptons in same multiplet.
- Flavor structure of quarks and leptons: Not totally disconnected.
- Interesting possibility: "High Scale" Unification of CKM and PMNS mixing parameters. Large radiative magnification of PMNS angles is required to obtain them within current experimental 3-σ range at low scales **(R.N. Mohapatra et.al. Phys. Rev. D 69 (2004) 053007)**.

Majorana Neutrinos

- Model independent approach: Assume HSMU at some "High Scale". Details of the "High Scale" theory not needed.
- Below High Scale: MSSM + Type-I seesaw mechanism.
- Effective left handed neutrino mass matrix

$$m_\nu(\mu) = -\frac{v^2}{2} Y_\nu^T(\mu) M_R^{-1}(\mu) Y_\nu(\mu)$$

Right handed neutrinos integrated out below their mass threshold.

- Below seesaw scale: Effective dimension five neutrino mass operator

$$\mathcal{L}_{MSSM+\kappa} = \mathcal{L}_{MSSM} - \frac{1}{4}\kappa_{ij}\mathbf{l}^i_a \varepsilon^{ab}\mathbf{h}^{(u)}_b \mathbf{l}^j_c \varepsilon^{cd}\mathbf{h}^{(u)}_d \Big|_{\theta\theta}$$

- Testing HSMU: Need to run down the masses and mixing parameters from high scale to low scale (M_Z).
- RG running between High Scale and seesaw scale: Using standard MSSM RG equations within framework of Type-I seesaw mechanism.
- Below seesaw scale: RG running with dim-5 operator added to MSSM.
- Below SUSY breaking scale: RG running with dim-5 operator added to SM.

RG Equations

- The RG equation (at 1-loop) of the effective mass operator **(S. Antusch et.al. Nucl. Phys. B 674 (2003) 401)**:

$$16\pi^2 \frac{d\kappa}{dt} = C(Y_e^\dagger Y_e)^T \kappa + C\kappa(Y_e^\dagger Y_e) + \alpha\kappa$$

where $t = \ln(\mu/\mu_0)$, μ is the renormalization scale and $C = 1(\frac{-3}{2})$ in MSSM(SM).
- In MSSM and SM α reads

$$\begin{aligned}\alpha_{\rm MSSM} &= -\frac{6}{5}g_1^2 - 6g_2^2 + 6(y_t^2 + y_c^2 + y_u^2)\\ \alpha_{\rm SM} &= -3g_2^2 + 2(y_\tau^2 + y_\mu^2 + y_e^2) + 6(y_t^2 + y_b^2 + y_c^2\\ &+ y_s^2 + y_d^2 + y_u^2) + \lambda\end{aligned}$$

where y_f, $(f = \{e, d, u\})$ are the Yukawa couplings, g_i are gauge couplings and λ is SM Higg's quartic coupling.
- Parameters of interest: Masses, mixing angles and physical phases. Need to go to mass basis: diag(m_1, m_2, m_3).
- Parametrization of PMNS matrix:

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}\end{pmatrix} diag(e^{\frac{-i\phi_1}{2}}, e^{\frac{-i\phi_2}{2}}, 1)$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ $(i, j = 1, 2, 3)$
- RG running of masses

$$\begin{aligned}16\pi^2 \frac{dm_1}{dt} &= \left[\alpha + Cy_\tau^2\left(2s_{12}^2\, s_{23}^2 + F_1\right)\right] m_1\\ 16\pi^2 \frac{dm_2}{dt} &= \left[\alpha + Cy_\tau^2\left(2c_{12}^2\, s_{23}^2 + F_2\right)\right] m_2\\ 16\pi^2 \frac{dm_3}{dt} &= \left[\alpha + 2Cy_\tau^2\, c_{13}^2\, c_{23}^2\right] m_3\end{aligned}$$

- Where F_1 and F_2 are:

$$\begin{aligned}F_1 &= -s_{13}\,\sin 2\theta_{12}\,\sin 2\theta_{23}\,\cos\delta + 2s_{13}^2\,c_{12}^2\,c_{23}^2\,,\\ F_2 &= s_{13}\,\sin 2\theta_{12}\,\sin 2\theta_{23}\,\cos\delta + 2s_{13}^2\,s_{12}^2\,c_{23}^2\,.\end{aligned}$$

- RG running of angles

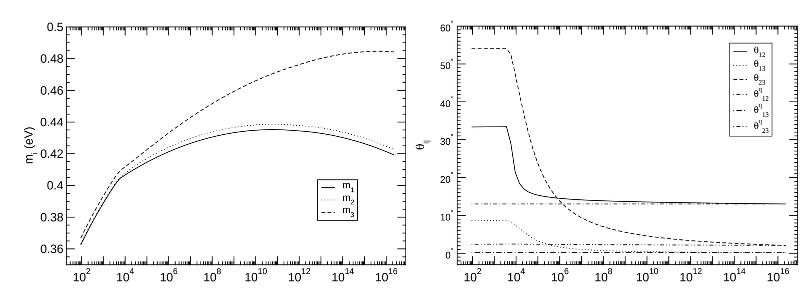
$$\begin{aligned}\frac{d\theta_{12}}{dt} &= -\frac{Cy_\tau^2}{32\pi^2}\,\sin 2\theta_{12}\,s_{23}^2\,\frac{|m_1\,e^{i\phi_1} + m_2\,e^{i\phi_2}|^2}{\Delta m_{21}^2} + \mathbf{O}(\theta_{13})\\ \frac{d\theta_{13}}{dt} &= \frac{Cy_\tau^2}{32\pi^2}\sin 2\theta_{12}\sin 2\theta_{23}\frac{m_3}{\Delta m_{32}^2\,(1+\xi)}\,[m_1\cos(\phi_1 - \delta)\\ &- (1+\xi)\,m_2\,\cos(\phi_2 - \delta) - \xi m_3\,\cos\delta] + \mathbf{O}(\theta_{13})\\ \frac{d\theta_{23}}{dt} &= -\frac{Cy_\tau^2}{32\pi^2}\,\sin 2\theta_{23}\,\frac{1}{\Delta m_{32}^2}\,\Big[c_{12}^2\,|m_2\,e^{i\phi_2} + m_3|^2\\ &+ s_{12}^2\,\frac{|m_1\,e^{i\phi_1} + m_3|^2}{1+\xi}\Big] + \mathbf{O}(\theta_{13})\end{aligned}$$

where $\xi = \Delta m_{21}^2/\Delta m_{32}^2$.

Implementing HSMU: Two Step Process

- **Bottom - Up**
- Start from known values of gauge couplings, quark mixing angles, masses of quarks and charged leptons at low energies (M_Z).
- Use RG equations: Obtain the corresponding values at high energies.
- HSMU hypothesis: Take neutrino mixing angles same as the quark mixing angles at the high scale.
- **Top - Down**
- Neutrino masses at high scale: Unknown parameters.
- Determine these three parameters such that: Low energy values of the oscillation parameters lie within their 3-σ range.
- Natural "High Scale": Grand Unified Theory (GUT) Scale. Take scale of HSMU as GUT scale i.e. 2×10^{16} GeV.
- Choice of seesaw scale: HSMU realized for varied range of seesaw scale. For sake of definiteness: Choose typical Seesaw scale of 10^{12} GeV.
- HSMU depends weakly on choice of SUSY breaking scale ($M_{\rm SUSY}$). For sake of definiteness take $M_{\rm SUSY}$ = 5 TeV.
- Larger values of $\tan\beta$: Enhanced magnification.

RG Evolution of Masses and Mixing Angles



- Hierarchical quark masses: RG running of CKM mixing angles is almost negligible.
- RG running of leptonic mixing angles.

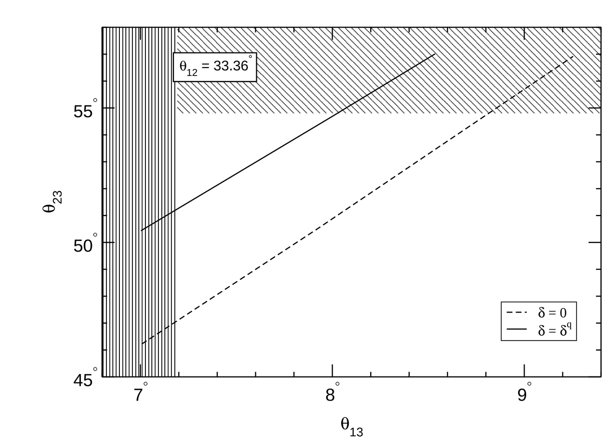
$$\frac{d\theta_{12}}{dt} \propto \frac{m^2}{\Delta m_{21}^2};\quad \frac{d\theta_{13}}{dt}, \frac{d\theta_{23}}{dt} \propto \frac{m^2}{\Delta m_{32}^2}\,.$$

- Quasi-degenerate neutrinos with normal hierarchy: Large radiative magnification of leptonic mixing angles.

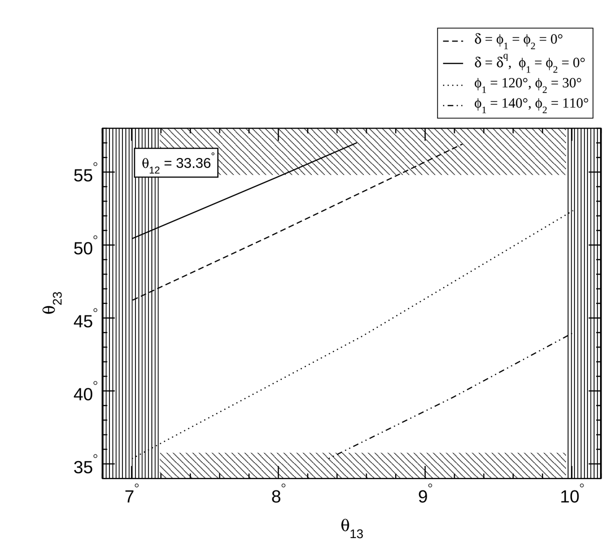
Results

Octant of θ_{23}

- RG evolution of θ_{13} and θ_{23} are correlated.
- For the case of no CP violation: θ_{23} always lies in second octant.



- For CP violating case: It can lie in either octant.



Predictions for $m_{\beta\beta}$ and m_β

- CP conserving case: $m_{\beta\beta} \approx$ 0.3-0.4 eV. In CP violating case, it can be as low as 0.1 eV.
- Value of $m_\beta \approx$ 0.3-0.4 eV. Can be probed by KATRIN.

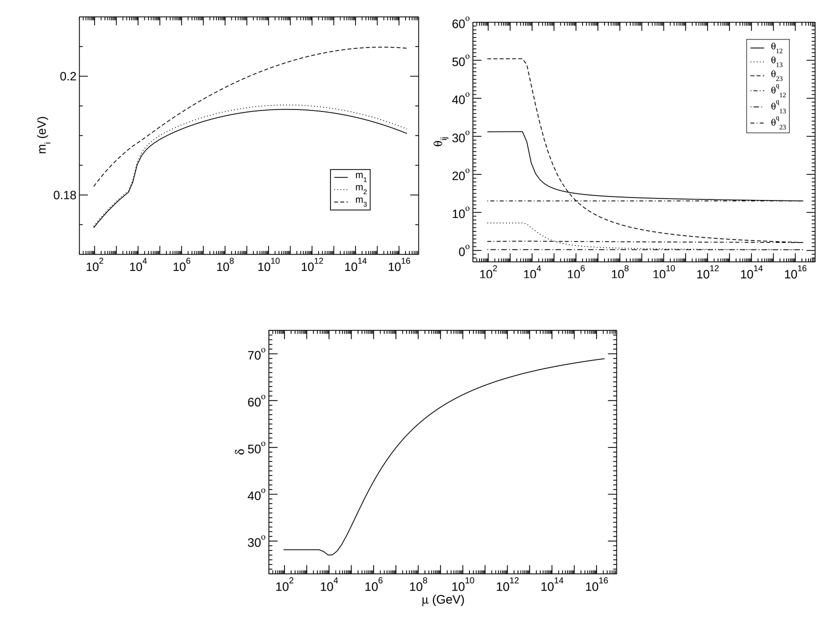
Dirac Neutrinos

- One of the most important open questions in neutrino physics: Whether neutrinos are Dirac or Majorana particles
- Answering this question: Essential to finding the underlying theory of neutrino masses and mixing.
- Current understanding: Dirac neutrinos as plausible as Majorana neutrinos.
- Instructive to see if HSMU can be implemented for Dirac Neutrinos as well.
- HSMU hypothesis: More natural for Dirac neutrinos than Majorana neutrinos.
- HSMU for Majorana Case: One has to treat the Majorana phases as free parameters. Majorana phases influence RG evolution of mixing angles. Conclusions subject to choice of Majorana phases.
- HSMU for Dirac Neutrinos: CKM and PMNS mixing parameters can be mapped in a one-to-one correspondence with each other at the unification scale.
- Clear and unambiguous predictions.

RG Evolution of Masses and Mixing Angles

- Implementing HSMU: Same as in Majorana case.
- **Bottom - Up**
- Start from known values of gauge couplings, quark mixing angles, masses of quarks and charged leptons at low energies (M_Z).
- Use RG equations: Obtain the corresponding values at high energies.
- HSMU hypothesis: Take neutrino mixing angles and phase same as the quark mixing angles and phase at the unification scale.
- **Top - Down**
- Neutrino masses at high scale: Unknown parameters.
- Determine these three parameters such that: Low energy values of the oscillation parameters lie within their 3-σ ranges.

RG Evolution

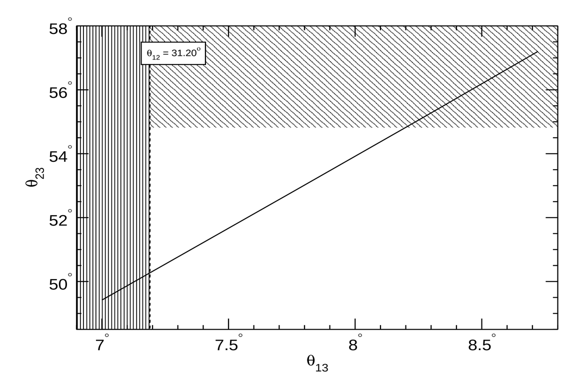


- Quasi-degenerate neutrinos: Large angle magnification in lepton sector.

Results

Octant of θ_{23}

- RG evolution of θ_{13} and θ_{23} correlated.



- For present 3-σ range of θ_{13}: Mixing angle θ_{23} always lies in second octant.

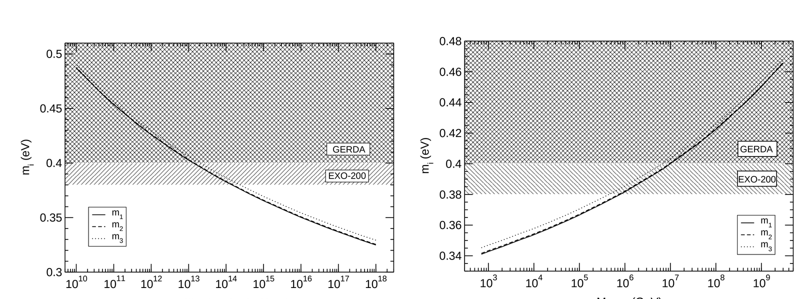
Predictions for m_β and $\delta_{\rm CP}$

- "Averaged electron neutrino mass" $m_\beta \approx$ 0.15-0.20 eV: Slightly below KATRIN's proposed sensitivity.
- Small CP violation: $\delta_{CP} \approx 15^\circ - 35^\circ$, $J_{\rm CP} \approx 0.1$.

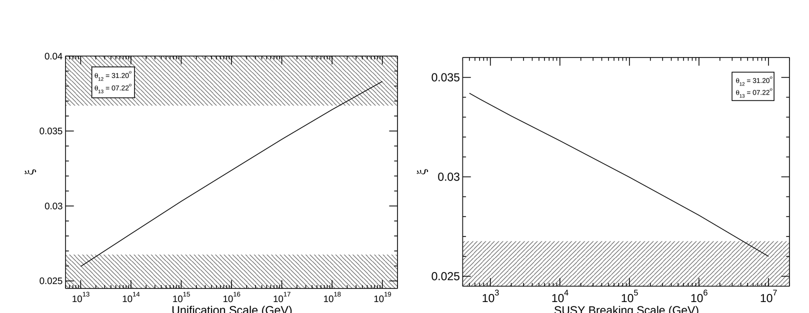
Variation of HSMU and SUSY Breaking Scale

- So far we assumed HSMU to be realized at GUT scale.
- HSMU does not depend on "details" of GUT scale theory.
- Instructive to analyze the effect of variation of HSMU scale.
- Similarly $M_{\rm SUSY}$ was taken as 5 TeV.
- In light of ongoing SUSY search: Important to analyze the dependence of HSMU on $M_{\rm SUSY}$.

Majorana Neutrinos



Dirac Neutrinos



- In both cases, HSMU scale should be above 10^{13} GeV. For Majorana neutrinos with CP violation, even lower scales are possible.
- In both cases, HSMU is consistent with experimental constraints, for SUSY breaking scales up to or more than 1000 TeV. Even higher scales are possible in Majorana case with CP violation.

Testing HSMU Hypothesis

- HSMU is quite predictive. Several experiments can test its predictions.

Experiment	Majorana	Dirac
$m_{\beta\beta}$ (observed)	✓	×
$m_{\beta\beta} < 0.1$ eV	×	-
KATRIN m_β (observed)	✓	✓
KATRIN m_β (not observed)	×	✓
$\theta_{23} > 45^\circ$	✓	✓
$\theta_{23} < 45^\circ$	✓	×
Mass Hierarchy (Normal)	✓	✓
Mass Hierarchy (Inverted)	×	×

Conclusions and Future Work

- High Scale Mixing Unification (HSMU) of PMNS and CKM parameters is an interesting possibility.
- It can be realized with both Dirac and Majorana type neutrinos.
- It naturally leads to non zero and "relatively large" values of θ_{13} consistent with present global fits.
- It leads to several predictions which can be tested by present and near future experiments.
- The scale of HSMU is roughly same as that of Grand Unified theories.
- This opens up the possibility of realizing HSMU through a GUT.
- Construction of such a GUT theory will put HSMU on a firmer footing.

G. Abbas, S. Gupta, G. Rajasekaran and R. Srivastava, Phys.Rev. D, 89 (2014) 093009, arXiv:1401.3399 [hep-ph] & arXiv:1312.7384 [hep-ph].