Collective Neutrino Oscillations

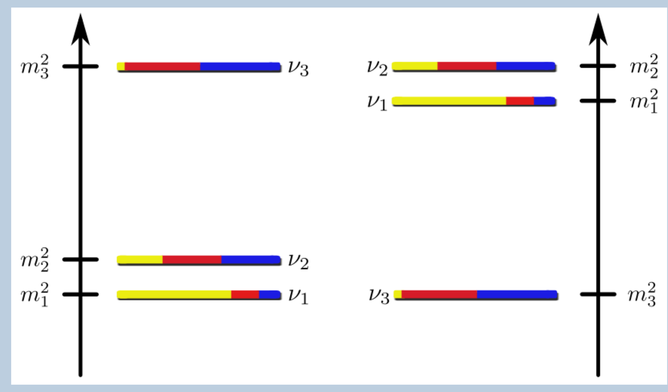
Shashank Shalgar, Huaiyu Duan
University of New Mexico, duan@unm.edu, shashankshalgar@unm.edu

Introduction

99% of the energy emitted by core collapse supernovae is in the form of neutrinos. Highly non-linear effects, due to neutrino self-interactions, that are present only in dense environments such as a supernova core, offers an unique opportunity to study neutrino properties. We demonstrate a new technique for calculating probability of collective neutrino oscillations.

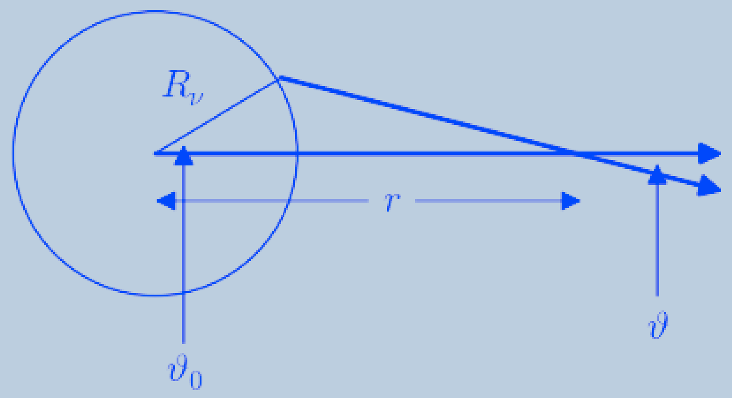
Neutrino Mass Hierarchy

The sign of the atmospheric mass squared difference ($\Delta m_{13}^2 \equiv m_3^2 - m_1^2$) has not been measured yet. It is known that the evolution of neutrinos in the interior of supernova strongly depends on the sign of this oscillation parameter. The collective neutrino oscillation probability is very different for two mass hierarchies.



Collective Neutrino Oscillations

Collective neutrino oscillations are a result of neutrino self-interactions mediated by Z-boson, along with the vacuum mixing. This is similar to the familiar MSW effect, but the ambient neutrinos are now relativistic and the effective potential is dependent on the relative angle between two neutrino trajectories.



$$i\frac{d}{dt}\rho(E,\cos\vartheta,t) = [H, \rho(E,\cos\vartheta)]$$

The Hamiltonian, H, has three components,

$$\begin{aligned} H &\equiv H_{vac} + H_{matt} + H_{self} \\ H_{self} &= \frac{\sqrt{2}G_F}{2\pi R_\nu^2}\int_0^\infty \int_0^1 (1-\cos\vartheta\cos\vartheta') \\ &\times \sum_{\alpha=e,x}\left(\frac{L_\alpha}{\langle E_\alpha\rangle}\rho_\alpha(E,\cos\vartheta') - \frac{L_{\bar\alpha}}{\langle E_{\bar\alpha}\rangle}\rho_{\bar\alpha}(E,\cos\vartheta')\right) dE d\cos\vartheta', \end{aligned}$$

where, G_F is the Fermi constant, L_α and $\langle E_\alpha\rangle$ are the luminosities and average energies of the initial spectra.

Angle Bin Method

The equations of motions are highly non-linear in nature because the Hamiltonian is a function of ρ. These equations of motion cannot be solved analytically. Even numerically it is not possible to straightaway solve the equations because of the dependence on continuous parameters, ϑ and E. However, it is possible to discretize the dependence on ϑ and E and convert the integrals in the Hamiltonian to sums.

Challenges

Solving equations of motion is very challenging due to the sheer number of equation that need to be solved simultaneously. Each density matrix contain four parameters for neutrino and anti-neutrino each. Typically, in order to get reliable results we need $\mathcal{O}(10^3)$ to $\mathcal{O}(10^4)$ angle bins for single split spectra and multiple split spectra respectively. In addition, we need at least a hundred energy bins to get reliable shape of the final flux.

$$\text{number of equations} > 8\times 100\times 1000 = 8\times 10^5$$

This number is true only for the simplest case. In most cases the number of equations that need to be solved is much greater.
On top of that, this is the case when we assume the spherical symmetry to hold. There have been recent studies that show that this assumption in not good in many cases. We suggest a better formalism that will reduce the number of equations that need to be solved.

Moments Method

We expand the density matrices and the Hamiltonian as a series in Legendre polynomials.

$$\begin{aligned} \rho &= \rho_0 P_0(\cos 2\vartheta_0) + \rho_1 P_1(\cos 2\vartheta_0) + \rho_2 P_2(\cos 2\vartheta_0) + \cdots \\ H &= H_0 P_0(\cos 2\vartheta_0) + H_1 P_1(\cos 2\vartheta_0) \end{aligned}$$

Here, ϑ_0 is related to ϑ as follows,

$$\cos\vartheta = \sqrt{1-\left(\frac{R_\nu}{r}\right)^2(1-\cos^2\vartheta_0)}.$$

The resulting equations in the moments are of the form,

$$i\frac{d}{dt}\rho_k = I(k,k)[H_0,\rho_k] + I(k,k+1)[H_1,\rho_{k+1}] + I(k,k-1)[H_1,\rho_{k-1}],$$

where, the coefficients $I(k,l)$ are constants that can be easily calculated.
Apart from the advantage of needing fewer equations, only ρ_0 is populated to begin with, and hence, using adaptive techniques is much easier with this method. Higher order terms need to be considered only when they become unstable.

Results

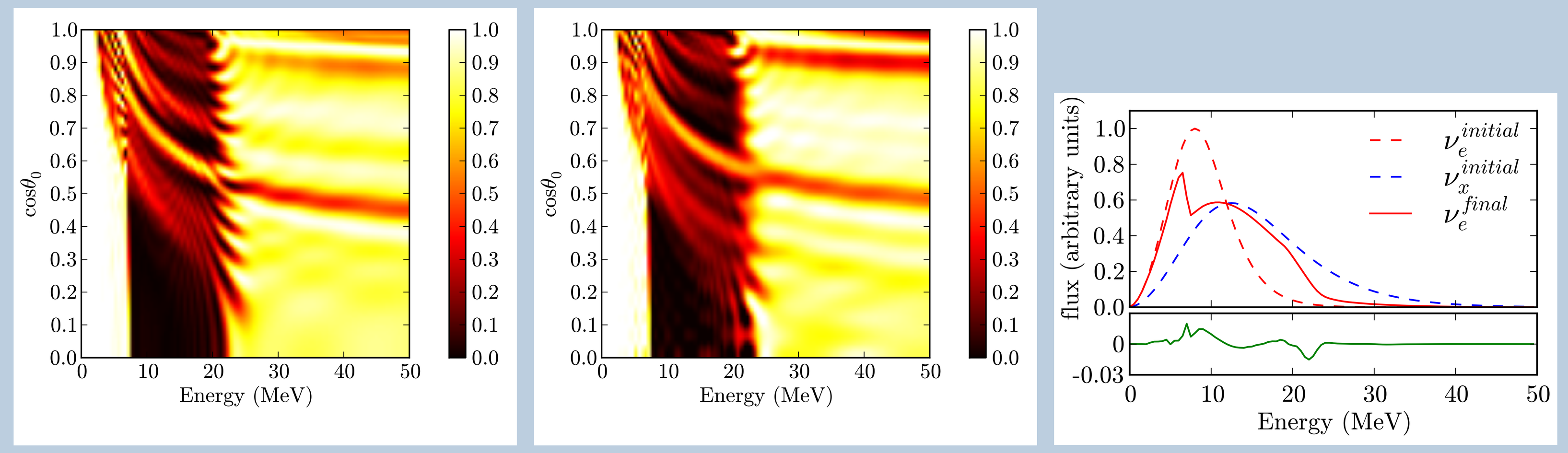


Fig. 1: Survival probability of electron neutrinos for inverted hierarchy as a function of emission angle and energy, using the angle bin method with 50,000 angle bins(left) and 300 moments(center). The figure on the right hand side shows the integrated flux at 400 km. The panel at the bottom shows the difference in the flux for the two methods. The initial flux used is a multi-split spectrum.

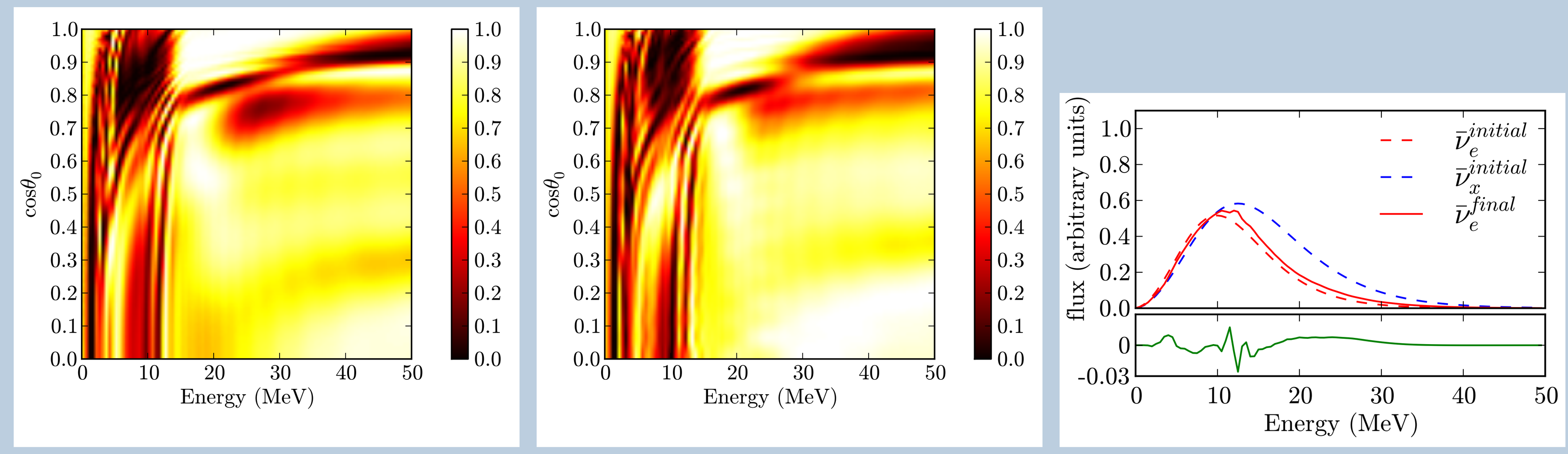


Fig. 2: Same as Fig. 1 but for anti-electron neutrinos.

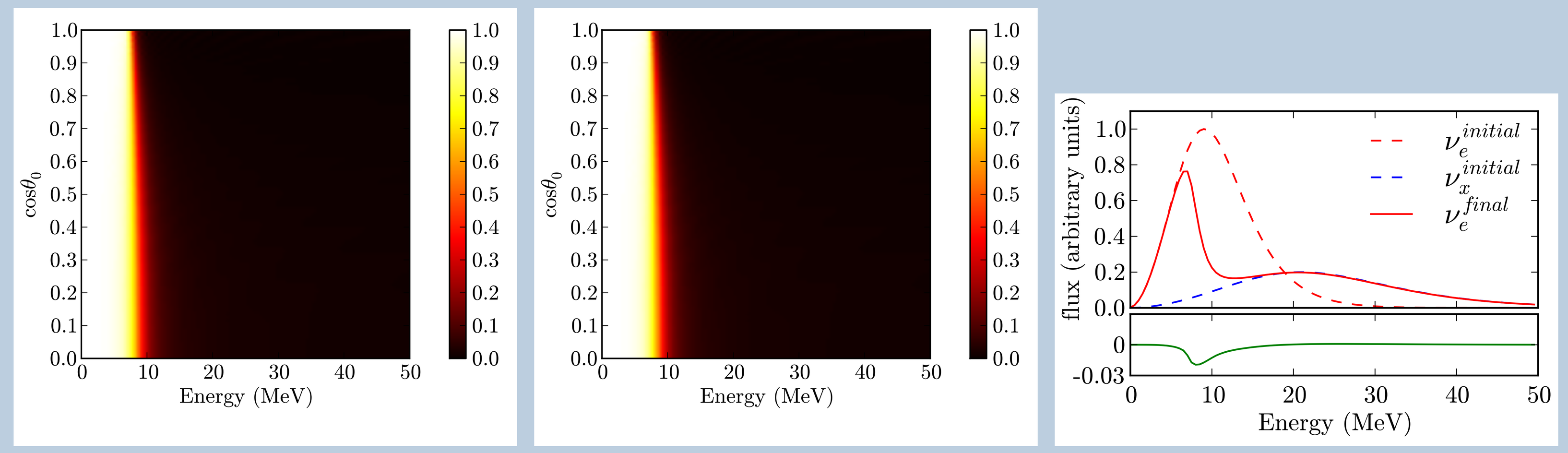


Fig. 3: Survival probability of electron neutrinos for inverted hierarchy as a function of emission angle and energy, using the angle bin method with 1,200 angle bins(left) and 200 moments(center). The figure on the right hand side shows the integrated flux at 200 km. The panel at the bottom shows the difference in the flux for the two methods. The initial flux used is a single-split spectrum.

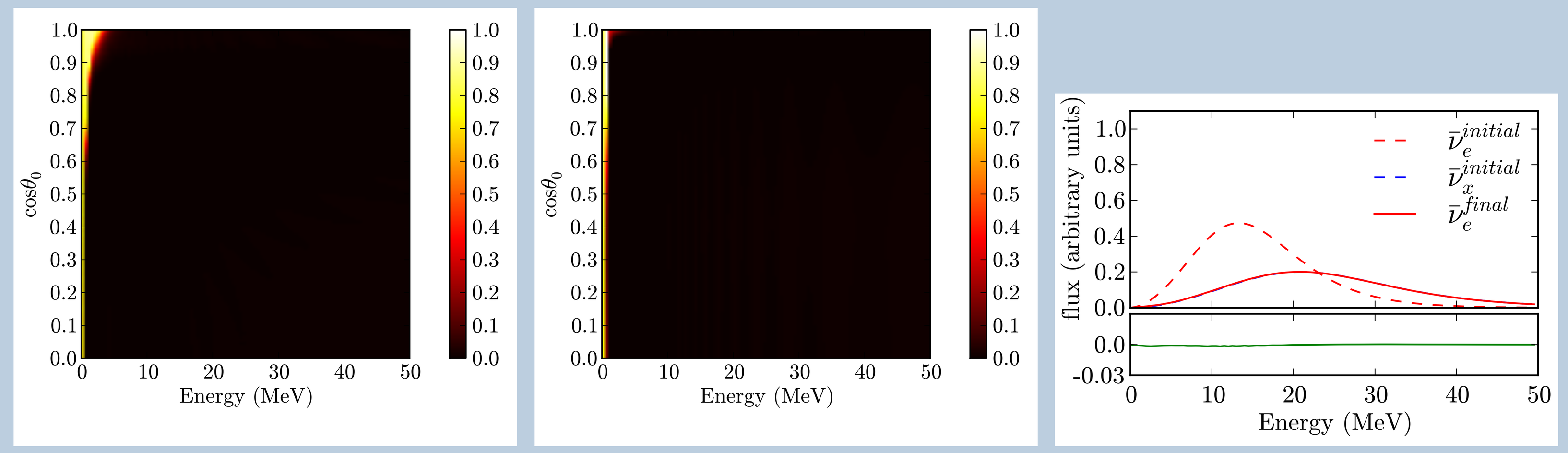


Fig. 4: Same as Fig. 3 but for anti-electron neutrinos.