Nuclear Structure, Double-Beta Decay, and Physics Beyond the Standard Model

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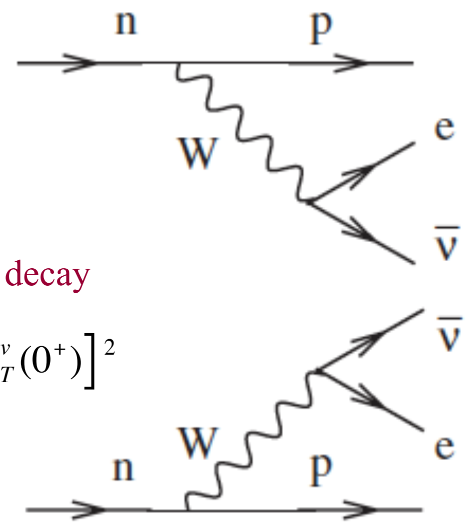
Classical Double Beta Decay Problem

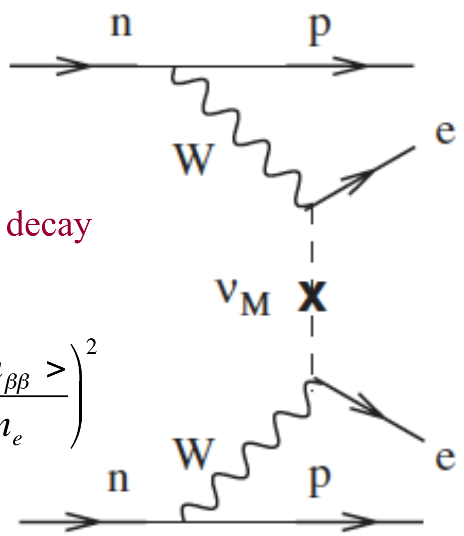
Isotope	$T_{1/2}(2\nu)$ (years)	$M^{2\nu}$
^{48}Ca	$4.4^{+0.6}_{-0.5} \times 10^{19}$	$0.0238^{+0.0015}_{-0.0017}$
^{76}Ge	$(1.5 \pm 0.1) \times 10^{21}$	$0.0716^{+0.0025}_{-0.0023}$
^{82}Se	$(0.92 \pm 0.07) \times 10^{20}$	$0.0503^{+0.0020}_{-0.0018}$
^{96}Zr	$(2.3 \pm 0.2) \times 10^{19}$	$0.0491^{+0.0023}_{-0.0020}$
^{100}Mo	$(7.1 \pm 0.4) \times 10^{18}$	$0.1258^{+0.0037}_{-0.0034}$
^{100}Mo-^{100}Ru(0^+_1)	$5.9^{+0.8}_{-0.6} \times 10^{20}$	$0.1017^{+0.0056}_{-0.0063}$
^{116}Cd	$(2.8 \pm 0.2) \times 10^{19}$	$0.0695^{+0.0025}_{-0.0024}$
^{128}Te	$(1.9 \pm 0.4) \times 10^{24}$	$0.0249^{+0.0031}_{-0.0023}$
^{130}Te	$(6.8^{+1.2}_{-1.1}) \times 10^{20}$	$0.0175^{+0.0016}_{-0.0014}$
^{150}Nd	$(8.2 \pm 0.9) \times 10^{18}$	$0.0320^{+0.0018}_{-0.0017}$
^{150}Nd-^{150}Sm(0^+_1)	$1.33^{+0.45}_{-0.26} \times 10^{20}$	$0.0250^{+0.0029}_{-0.0034}$
^{238}U	$(2.0 \pm 0.6) \times 10^{21}$	$0.0271^{+0.0053}_{-0.0033}$
^{136}Xe	2.23×10^{21}	0.020

A.S. Barabash, PRC 81 (2010)



2-neutrino double beta decay

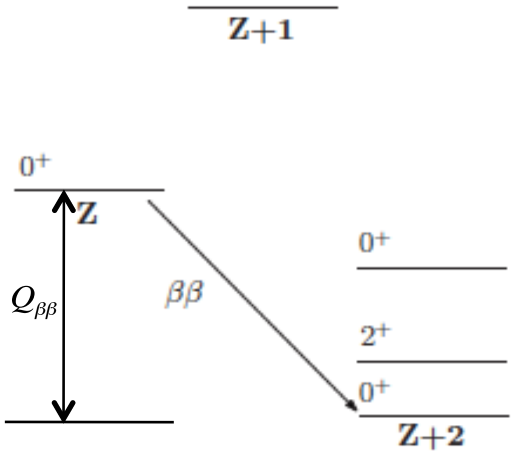
$$T^{-1}_{1/2}(2\nu) = G^{2\nu}(Q_{\beta\beta})\left[M^{2\nu}_{GT}(0^+)\right]^2$$



neutrinoless double beta decay

$$T^{-1}_{1/2}(0\nu) = G^{0\nu}(Q_{\beta\beta})\left[M^{0\nu}(0^+)\right]^2\left(\frac{< m_{\beta\beta} >}{m_e}\right)^2$$

$$\langle m_{\beta\beta}\rangle = \left|\sum_k m_k U^2_{ek}\right|$$



Adapted from Avignone, Elliot, Engel, Rev. Mod. Phys. 80, 481 (2008) -> RMP08

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}\,|\nu_i\rangle$$
$$|\nu_i\rangle = \sum_\alpha U_{\alpha i}\,|\nu_\alpha\rangle$$

Neutrino Masses

PMNS – matrix

$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{bmatrix} e^{i\alpha 1/2} & 0 & 0 \\ 0 & e^{i\alpha 2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$c_{12} \equiv \cos\theta_{12},\ s_{12} = \sin\theta_{12},\ etc$$

$$\Delta m^2_{21} \approx 7.5 \times 10^{-5}\ eV^2\,(solar)$$
$$\left|\Delta m^2_{32}\right| \approx 2.4 \times 10^{-3}\ eV^2\,(atmospheric)$$

- Tritium decay:

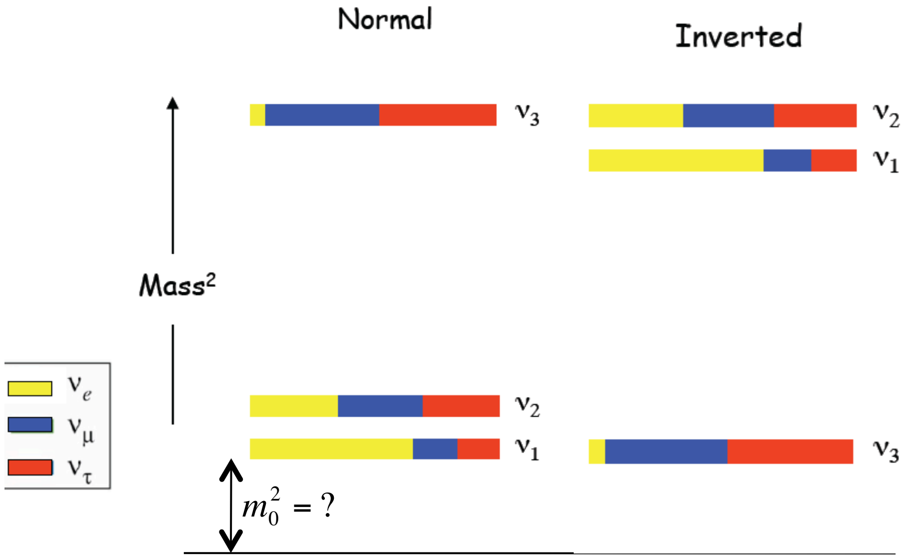
$$^3H \rightarrow {}^3He + e^- + \overline{\nu}_e$$

$$m_{\nu_e} = \sqrt{\sum_i \left|U_{ei}\right|^2 m_i^2} < 2.2eV\,(Mainz\ \exp.)$$

$$Katrin\ \exp.(in\ progress):\ goal\ m_{\nu_e} < 0.3eV$$

- Cosmology: CMB power spectrum, BAO, etc,

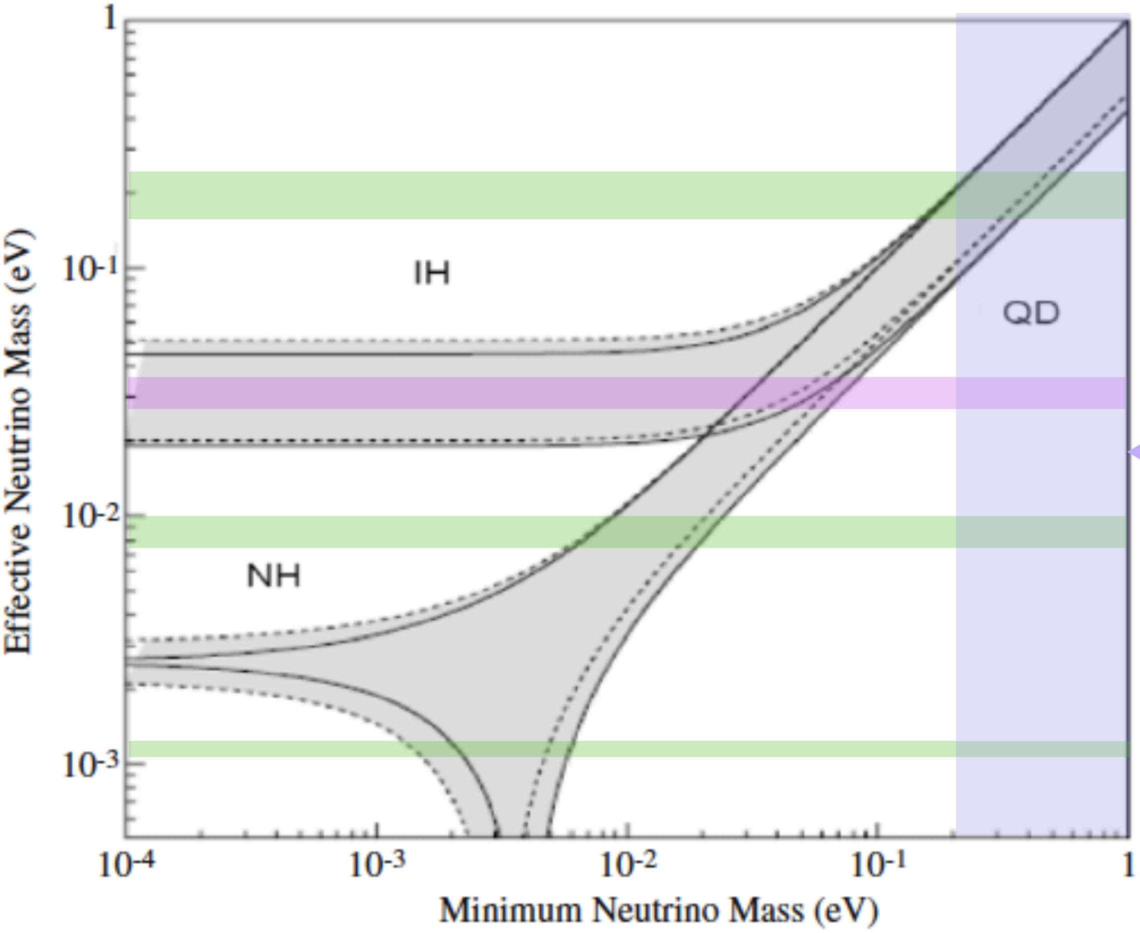
$$\sum_{i=1}^{3} m_i < 0.23eV$$



Two neutrino mass hierarchies

Neutrino ββ effective mass

H. Ejiri / Progress in Particle and Nuclear Physics 64 (2010) 249–257



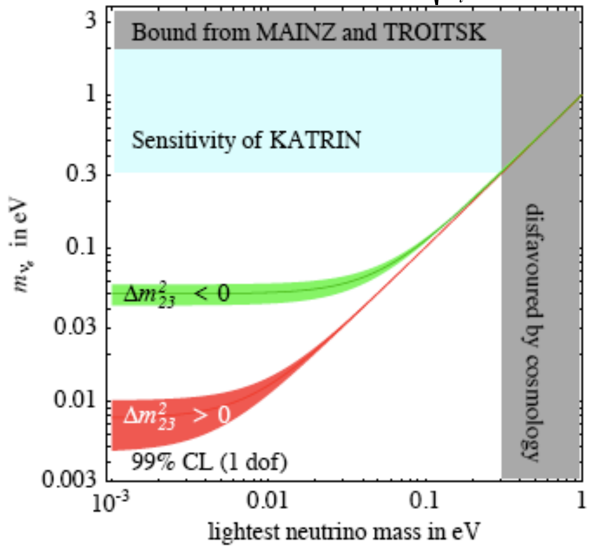
CMB constraint

$$\langle m_{\beta\beta}\rangle = \left|\sum_{k=1}^{3} m_k U_{ek}^2\right|$$
$$= \left|c_{12}^2 c_{13}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right|$$

$$\phi_2 = \alpha_2 - \alpha_1 \qquad \phi_3 = -\alpha_1 - 2\delta$$

$$T_{1/2}^{-1}(0\nu) = G^{0\nu}(Q_{\beta\beta})\left[M^{0\nu}(0^+)\right]^2\left(\frac{< m_{\beta\beta} >}{m_e}\right)^2$$

$$m_{\nu_e} = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$



The origin of Majorana neutrino masses

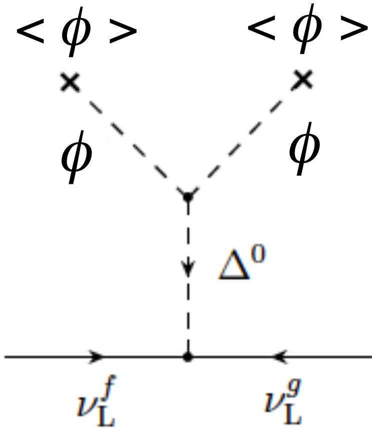
$$-\mathcal{L} \supset \frac{1}{2}\left(\overline{\nu'_L}\ \overline{\nu'^c_R}\right)\begin{pmatrix}0 & m_{LR}\\ m_{LR}^T & M_{RR}\end{pmatrix}\begin{pmatrix}\nu'^c_L\\ \nu'_R\end{pmatrix}+h.c.$$

$$\frac{1}{2}\left(\overline{\nu'_L}\ \overline{\nu'^c_R}\right)\begin{pmatrix}m_{LL} & m_{LR}\\ m_{LR}^T & M_{RR}\end{pmatrix}\begin{pmatrix}\nu'^c_L\\ \nu'_R\end{pmatrix}+h.c.$$

See-saw mechanisms

$$m^{\nu}_{\rm LL} = -m^{\nu}_{\rm LR}M^{-1}_{\rm RR}m^{\nu T}_{\rm LR} \longrightarrow \begin{cases} m^{\nu}_{LL} \approx \dfrac{(100\ GeV)^2}{10^{14}\ GeV} = 0.1eV \\ m^{\nu}_{LL} \approx \dfrac{(300\ keV)^2}{1\ TeV} = 0.1eV \end{cases}$$

$$m^{\nu}_{\rm LL} \approx m^{\rm II}_{\rm LL} + m^{\rm I}_{\rm LL}$$



Left-Right Symmetric model

$$\begin{pmatrix}\nu'_L\\ \nu'^c_R\end{pmatrix} = W\begin{pmatrix}\nu_L\\ N^c_R\end{pmatrix} = \begin{pmatrix}U & S\\ T & V\end{pmatrix}\begin{pmatrix}\nu_L\\ N^c_R\end{pmatrix}$$

$$WW^+ = 1 \qquad UU^+ \approx 1 \qquad VV^+ \approx 1$$

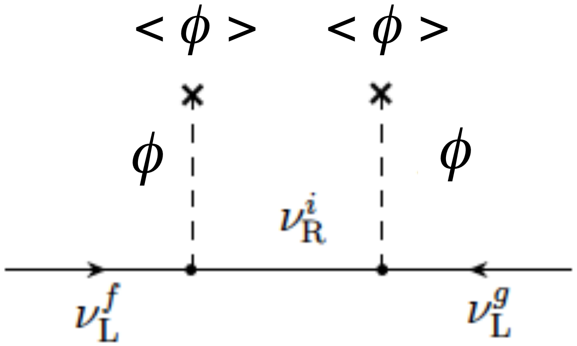
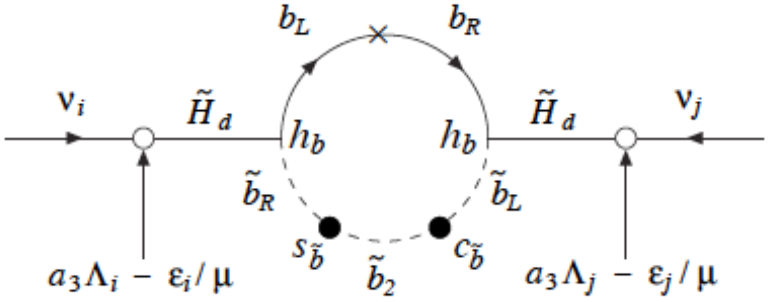


Diagram illustrating the type I see-saw mechanism

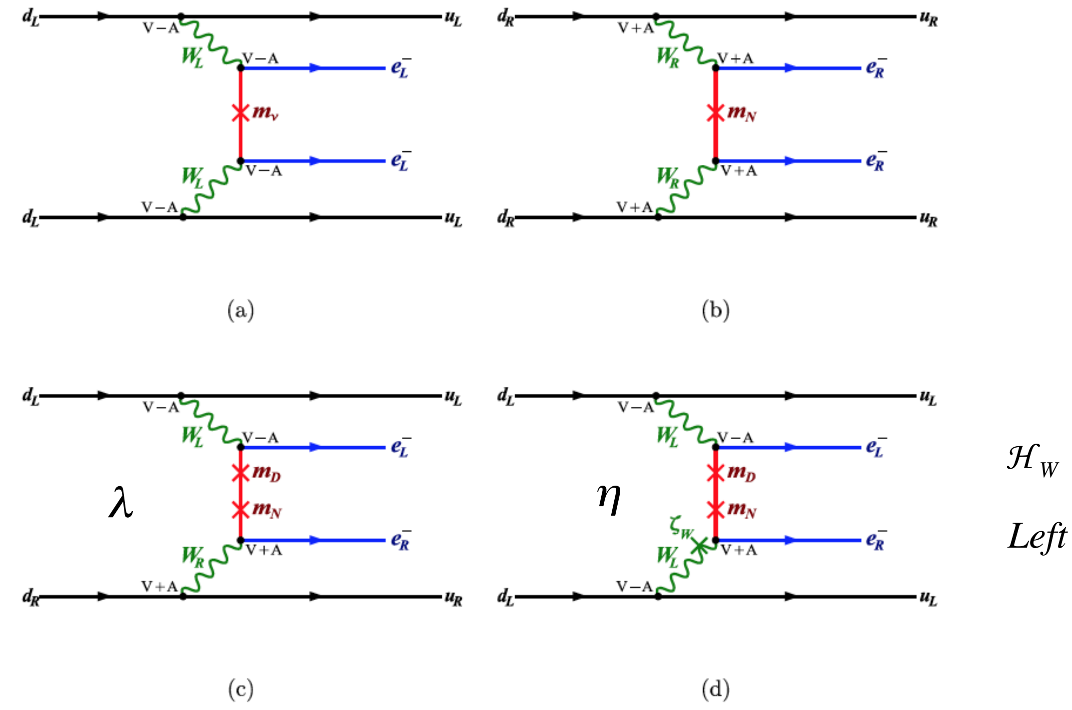
GUT/SUSY R-parity v. mechanism



arXiv:0710.4947v3

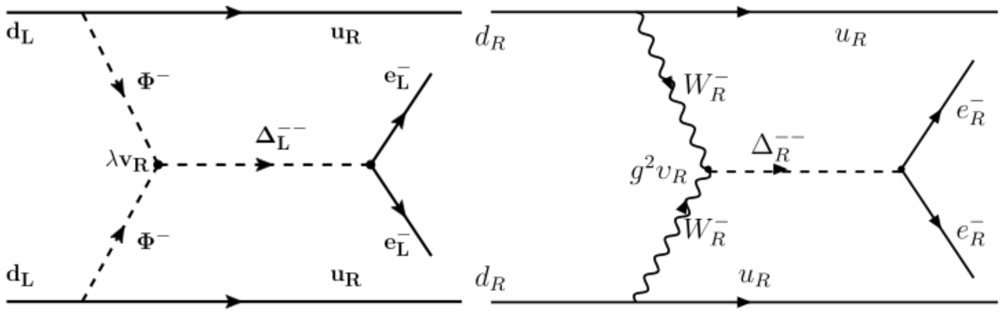
Possible contributions to 0ββ decay

DAS *et al.* PHYSICAL REVIEW D **86,** 055006 (2012)



(a) (b)

(c) (d)



(a) (b)

Low-energy effective Hamiltonian

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} j_L^\mu J_{L\mu}^+ + h.c.$$

$$j_{L/R}^\mu = \bar{e}\gamma^\mu\left(1\mp\gamma^5\right)\nu_e$$

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}}\left[j_L^\mu\left(J_{L\mu}^+ + \kappa J_{R\mu}^+\right) + j_R^\mu\left(\eta J_{L\mu}^+ + \lambda J_{R\mu}^+\right)\right] + h.c.$$

Left – right symmetric model

$$-\mathcal{L} \supset \frac{1}{2} h_{\alpha\beta}^T \left(\bar{\nu}_{\beta L} \;\; \bar{e}_{\alpha L}\right)\begin{pmatrix}\Delta^- & -\Delta^0 \\ \Delta^{--} & \Delta^-\end{pmatrix}\begin{pmatrix} e_R^c \\ -\nu_R^c\end{pmatrix} + hc$$

No neutrino exchange

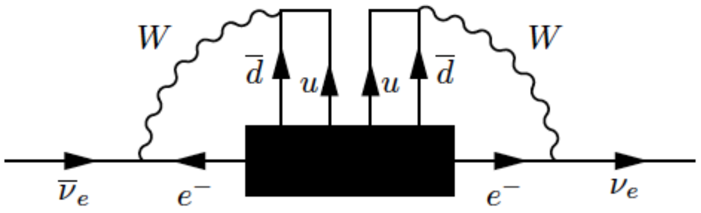
The Black Box Theorems

Black box I (electron neutrino)

J. Schechter and J.W.F Valle, PRD 25, 2951 (1982)

E. Takasugi, PLB 149, 372 (1984)

J.F. Nieves, PLB 145, 375 (1984)



$0\nu\beta\beta$ observed ⇔ (i) Electron neutrinos are Majorana fermions (with m > 0).

at some level (ii) Lepton number conservation is violated by 2 units

However:

M. Duerr et al, JHEP 06 (2011) 91

$$\left(\delta m_{\nu_e}\right)_{BB} \sim 10^{-24}\, eV << \sqrt{\left|\Delta m_{32}^2\right|} \approx 0.05\, eV$$

Black box II (all flavors + oscillations)

M. Hirsch, S. Kovalenko, I. Schmidt, PLB 646, 106 (2006)

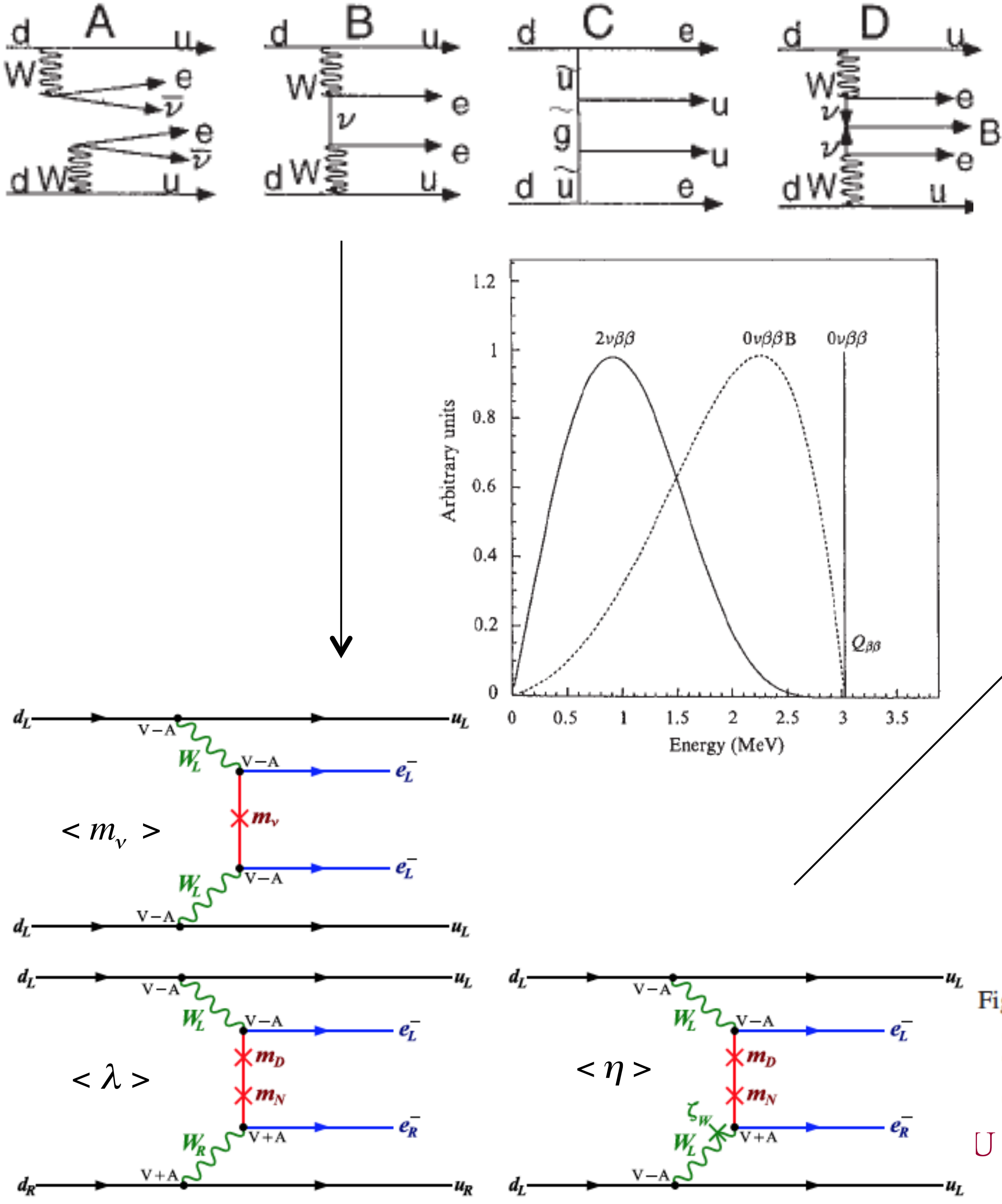
$0\nu\beta\beta$ observed ⇔ (i) Neutrinos are Majorana fermions.

at some level (ii) Lepton number conservation is violated by 2 units

Regardless of the dominant $0\nu\beta\beta$ mechanism!

$$\boxed{(iii)\ \ \left\langle m_{\beta\beta}\right\rangle = \left|\sum_{k=1}^{3} m_k U_{ek}^2\right| = \left|c_{12}^2 c_{13}^2 m_1 + c_{13}^2 s_{12}^2 m_2 e^{i\phi_2} + s_{13}^2 m_3 e^{i\phi_3}\right| > 0}$$

DBD signals from different mechanisms



J. Phys. Soc. Jpn., Vol. 74, No. 8, August, 2005

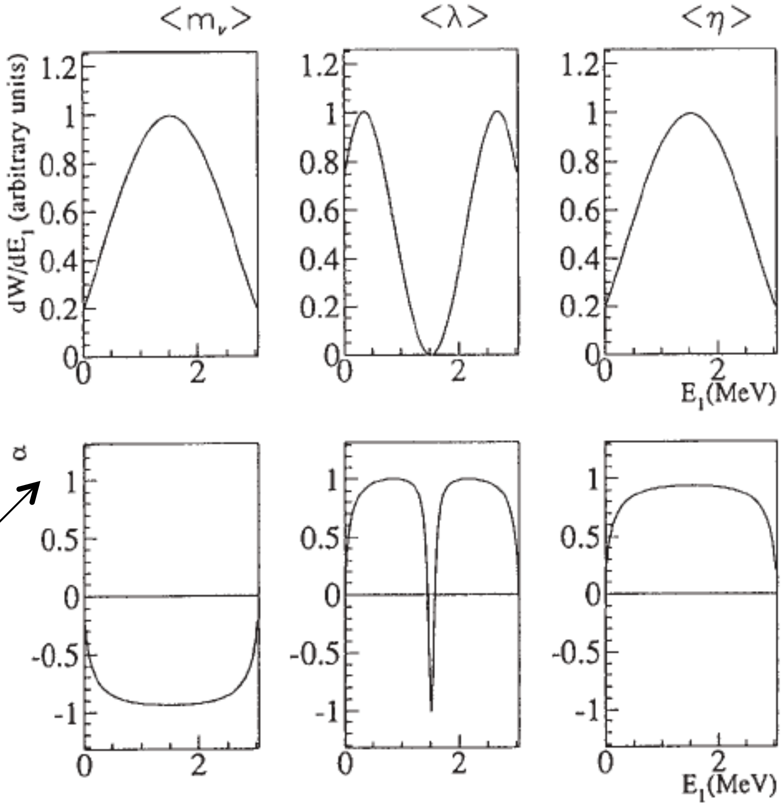
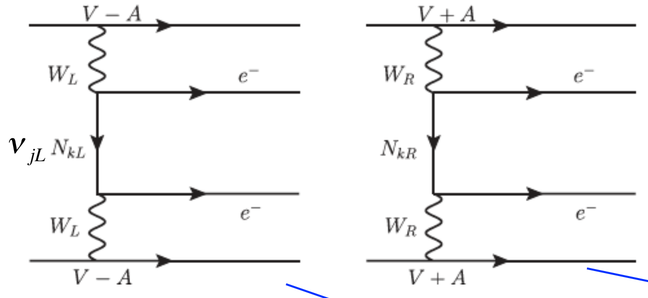


Fig. 4. Energy and angular correlations for the ^{100}Mo $0\nu\beta\beta$ process caused by the mass and right-handed current terms of $\langle m \rangle$, $\langle \lambda \rangle$ and $\langle \eta \rangle$. Top: Calculated single-β spectra. Bottom: $\beta_1 - \beta_2$ angular correlation coefficients α defined by $W(\theta_{12}) = 1 + \alpha \cos\theta_{12}$.4)

U

The DBD half-life

PRD 83, 113003 (2011)



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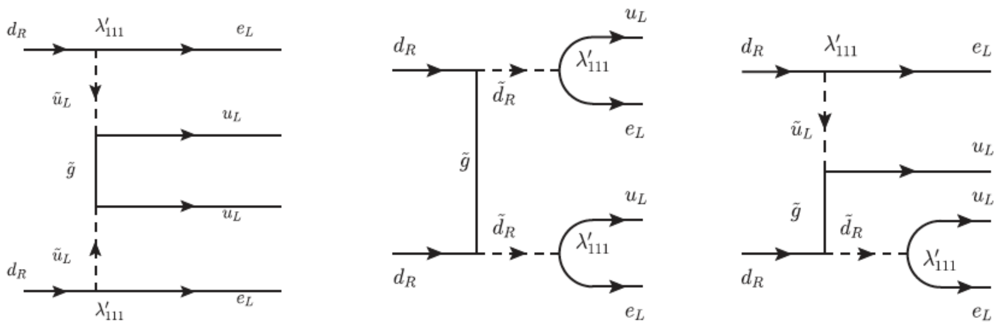


$$\begin{cases} \nu'_{eL} = \sum_k^{light} U_{ek}\nu_{kL} + \sum_k^{heavy} S_{ek} N_{kL} \\ \nu'_{eR} = \sum_k^{light} T^*_{ek}\nu_{kR} + \sum_k^{heavy} V^*_{ek} N_{kR} \end{cases}$$

$$\eta_{\nu L} = \frac{\langle m_{\beta\beta}\rangle}{m_e} \qquad \eta_{NL} = \sum_k^{heavy} S^2_{ek}\frac{m_p}{M_k} \qquad \eta_{NR} = \left(\frac{M_{WL}}{M_{WR}}\right)^4 \sum_k^{heavy} V^2_{ek}\frac{m_p}{M_k}$$

$$\langle\lambda\rangle = \left(\frac{M_{WL}}{M_{WR}}\right)^2 \sum_k^{light} U_{ek}T^*_{ek} \qquad \langle\eta\rangle = \zeta\sum_k^{light} U_{ek}T^*_{ek} \qquad W_R \approx \zeta W_1 + W_2$$

$$\left[T^{0\nu}_{1/2}\right]^{-1} = G^{0\nu}\left|\sum_j M_j\eta_j\right|^2 = G^{0\nu}\left|M^{(0\nu)}\eta_{\nu L} + M^{(0N)}\left(\eta_{NL}+\eta_{NR}\right) + \tilde{X}_\lambda < \lambda > + \tilde{X}_\eta < \eta > + M^{(0\lambda')}\eta_{\lambda'} + M^{(0\tilde{q})}\eta_{\tilde{q}} + \cdots\right|^2$$

(i) η_{NL} *neglijible in most models;* (ii) $\langle\eta\rangle$ & $\langle\lambda\rangle$ *ruled in/out by energy or angular distributions*

$$\left[T^{0\nu}_{1/2}\right]^{-1} \cong G^{0\nu}\left|M^{(0\nu)}\eta_{\nu L} + M^{(0N)}\eta_{NR}\right|^2 \approx G^{0\nu}\left[\left|M^{(0\nu)}\right|^2\left|\eta_{\nu L}\right|^2 + \left|M^{(0N)}\right|^2\left|\eta_{NR}\right|^2\right]$$

No interference terms!

2ν Double Beta Decay (DBD) of ^{48}Ca

$$T_{1/2}^{-1} = G_{2\nu}(Q_{\beta\beta})\left[M_{GT}^{2\nu}(0^+)\right]^2$$

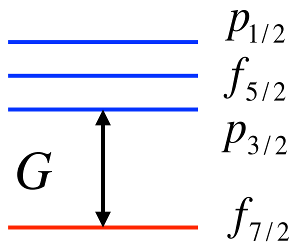
$$M_{\rm GT}^{2\nu}(0^+) = \sum_k \frac{\langle 0_f \| \sigma\tau^- \| 1_k^+\rangle\langle 1_k^+ \| \sigma\tau^- \| 0_i\rangle}{E_k + E_0}$$

$$^{48}Ca \xrightarrow{2\nu\,\beta\beta} {}^{48}Ti$$

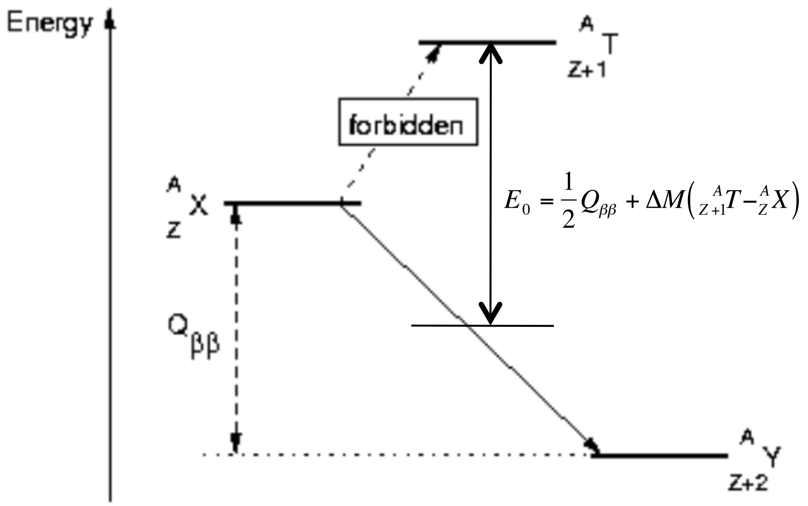
The choice of valence space is important!

ISR	48Ca	48Ti
pf	24.0	12.0
f7 p3	10.3	5.2

Ikeda satisfied in pf !

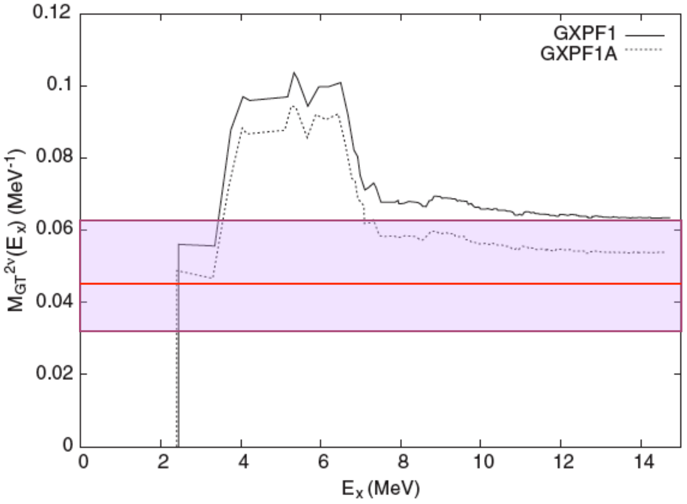


Neutrino 2014



$$B(GT) = \frac{\left|\langle f \| \sigma\cdot\tau \| i\rangle\right|^2}{(2J_i+1)}$$

$$Ikeda\ sum\ rule(ISR) = \sum B(GT;Z \to Z+1) - \sum B(GT;Z \to Z-1) = 3(N-Z)$$



$$\boxed{g_A\sigma\tau \xrightarrow{quenched} 0.77\,g_A\sigma\tau}$$

Horoi, Stoica, Brown,

PRC **75**, 034303 (2007)

The effect of larger model spaces for ^{48}Ca

M(0v)	SDPFU	SDPFMUP
$0\ \hbar\omega$	0.941	0.623
$0{+}2\,\hbar\omega$	1.182 (26%)	1.004 (61%)

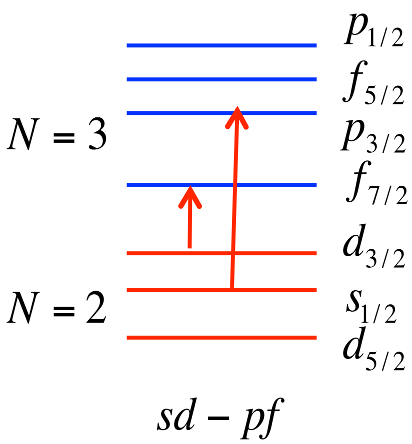
	M(0v)
$0\ \hbar\omega$ / GXPF1A	0.733
$0\ \hbar\omega + 2^{\text{nd}}$ ord./GXPF1A	1.301 (77%)

SDPFU: PRC 79, 014310 (2009)

SDPFMUP: PRC 86, 051301(R) (2012)

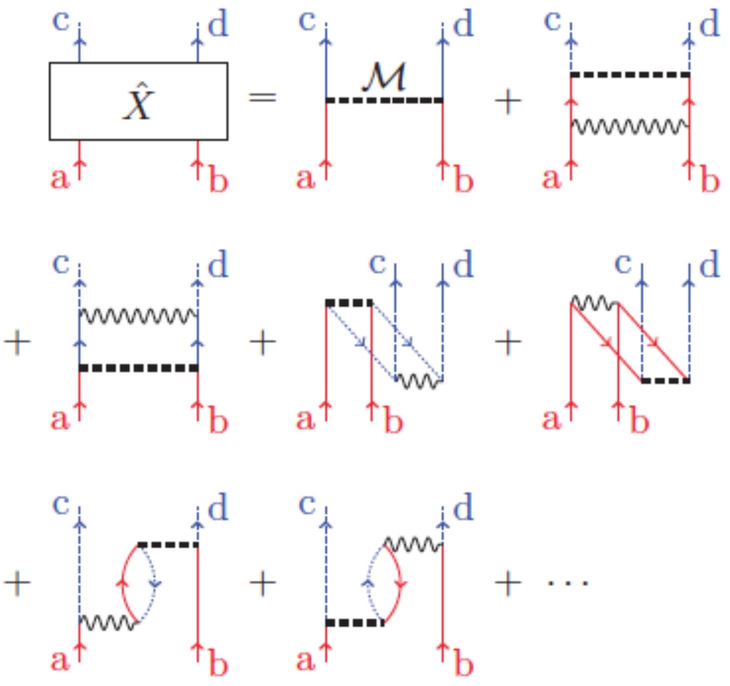
arXiv:1308.3815, PRC 89, 045502 (2014)

PRC 87, 064315 (2013)



See also M. Horoi,

PRC **87**, 014320 (2013)



Beyond Closure in Shell Model

$$M_S^{0\nu} = \sum_{\substack{p<p' \\ n<n' \\ p<n}} (\Gamma) \left\langle 0_f^+ \left\| \left[\left(a_p^+ a_{p'}^+ \right)^{\mathcal{J}} \left(\tilde{a}_{n'} \tilde{a}_n \right)^{\mathcal{J}} \right]^0 \right\| 0_i^+ \right\rangle \left\langle p\,p';\mathcal{J} \left| \int q^2 dq \left[\hat{S}\, \frac{h(q)\, j_\kappa(qr) G_{FS}^2 f_{SRC}^2}{q\left(q + \langle E \rangle\right)} \boldsymbol{\tau}_{1-} \boldsymbol{\tau}_{2-} \right] \right| n\,n';\mathcal{J} \right\rangle_{as} - closure$$

$$M_S^{0\nu} = \sum_{\substack{pp'nn' \\ J\,k\,\mathcal{J}}} (\tilde{\Gamma}) \left\langle 0_f^+ \left\| \left(a_p^+ \tilde{a}_n \right)^J \right\| J_k \right\rangle \left\langle J_k \left\| \left(a_{p'}^+ \tilde{a}_{n'} \right)^J \right\| 0_i^+ \right\rangle \left\langle p\,p';\mathcal{J} \left| \int q^2 dq \left[\hat{S}\, \frac{h(q)\, j_\kappa(qr) G_{FS}^2 f_{SRC}^2}{q\left(q + E_k^J\right)} \boldsymbol{\tau}_{1-} \boldsymbol{\tau}_{2-} \right] \right| n\,n';\mathcal{J} \right\rangle - exact$$

Challenge: there are about 100,000 J_k states in the sum for 48Ca

Much more intermediate states for heavier nuclei, such as ^{76}Ge!!!

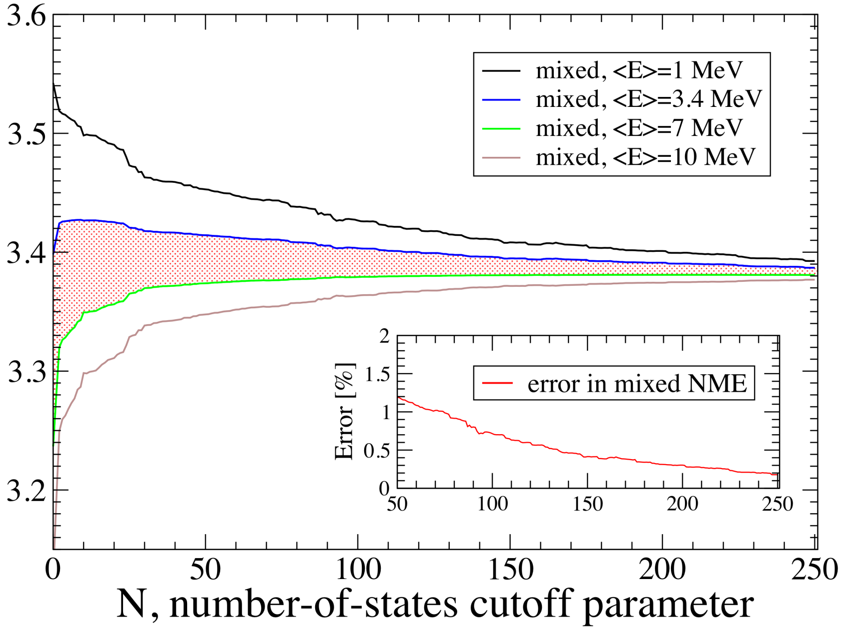
No-closure may need states out of the model space (not considered).

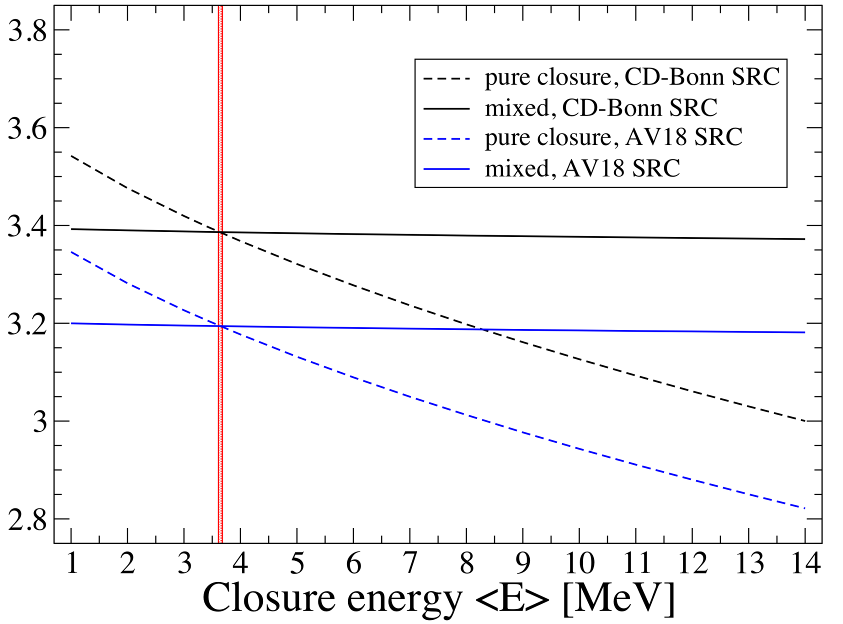
Minimal model spaces

^{82}Se : 6,146,681

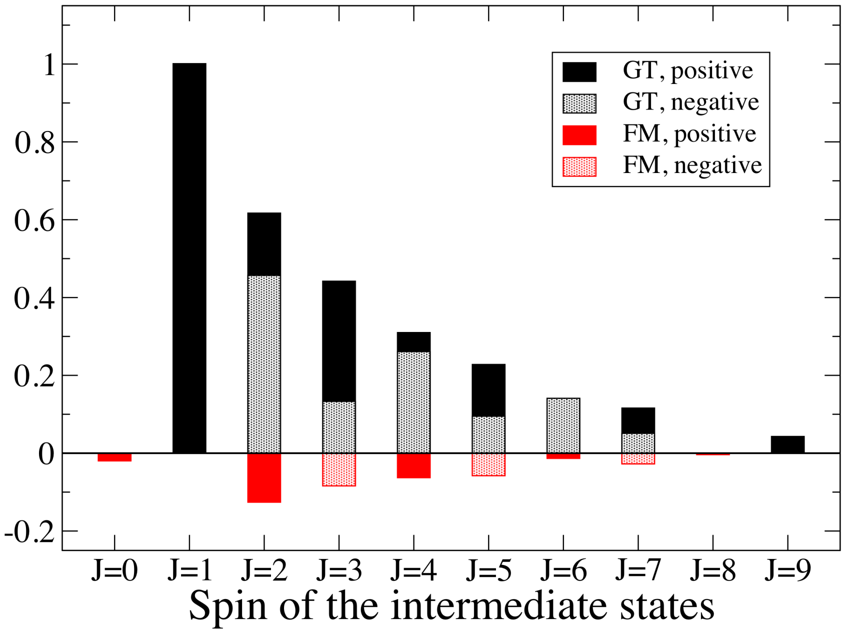
^{130}Te : 22,437,983

^{76}Ge : 89,472,767





$M^{0\nu}$ for ^{82}Se: PRC **89** 054304 (2014)



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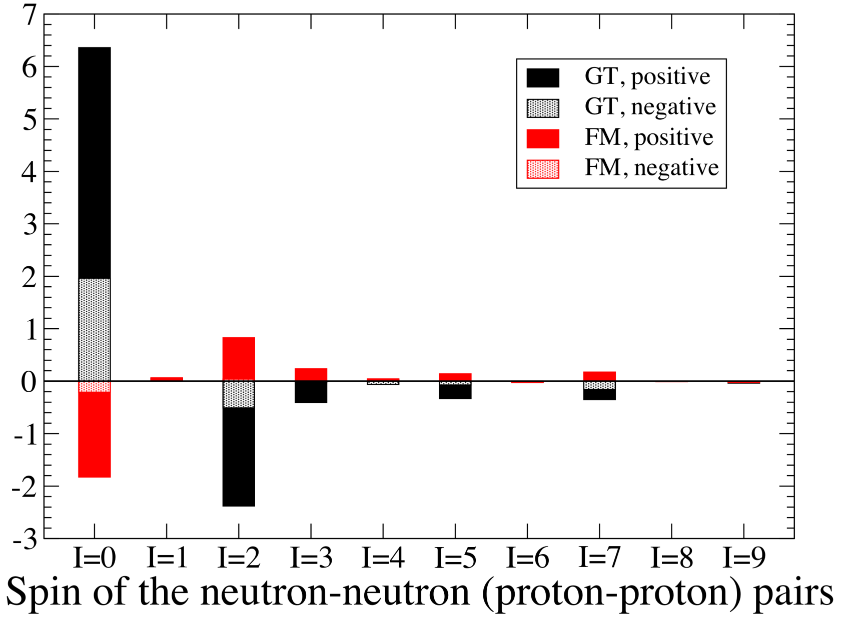


^{136}Xe 0νββ Results

$$M^{2\nu}_{\exp} = 0.019\ MeV^{-1}$$

M. Horoi and B.A. Brown, Phys. Rev. Lett. **110**, 222502 (2013)

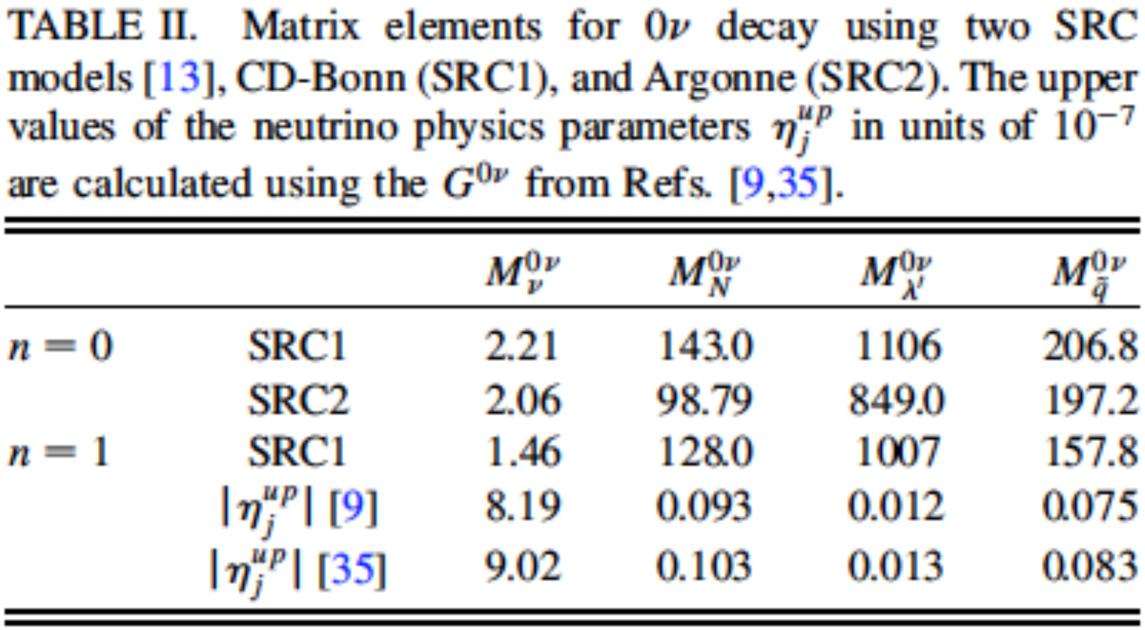


TABLE II. Matrix elements for 0ν decay using two SRC models [13], CD-Bonn (SRC1), and Argonne (SRC2). The upper values of the neutrino physics parameters η_j^{up} in units of 10^{-7} are calculated using the $G^{0\nu}$ from Refs. [9,35].

		$M_\nu^{0\nu}$	$M_N^{0\nu}$	$M_{\lambda'}^{0\nu}$	$M_{\tilde{q}}^{0\nu}$
$n = 0$	SRC1	2.21	143.0	1106	206.8
	SRC2	2.06	98.79	849.0	197.2
$n = 1$	SRC1	1.46	128.0	1007	157.8
	$\vert\eta_j^{up}\vert$ [9]	8.19	0.093	0.012	0.075
	$\vert\eta_j^{up}\vert$ [35]	9.02	0.103	0.013	0.083

$$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu}|\eta_{\nu L}M^{0\nu}_{\nu} + \eta_N M^{0\nu}_N + \eta_{\lambda'}M^{0\nu}_{\lambda'} + \eta_{\tilde{q}}M^{0\nu}_{\tilde{q}}|^2,$$

n (0+)	n (1+)	M(2v)
0	0	0.062
0	1	0.091
1	1	0.037
1	2	0.020

$0g_{9/2}$

$0g_{7/2} 1d_{5/2}\ 1d_{3/2}$

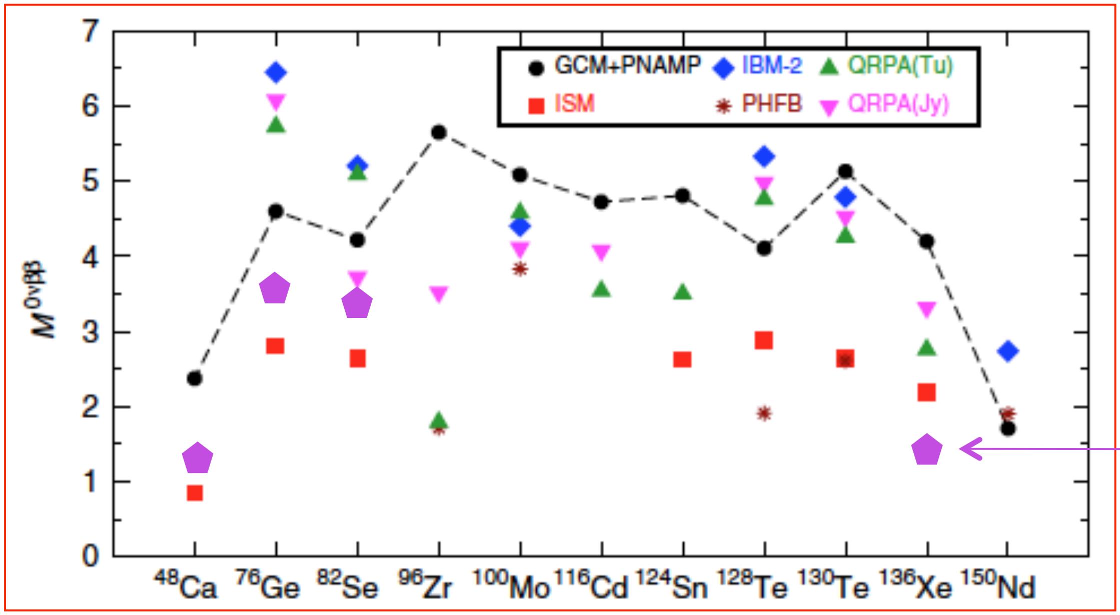
$2s_{5/2}\ 0h_{11/2}$

$0h_{9/2}$

$$\sum B(GT;Z\rightarrow Z+1) - \sum B(GT;Z\rightarrow Z-1) = 84$$

$$Ikeda: \quad 3(N-Z) = 84$$

Comparisons of $M^{0\nu}$ 0νββ Results



From T. Rodriguez, G. Martinez-Pinedo,

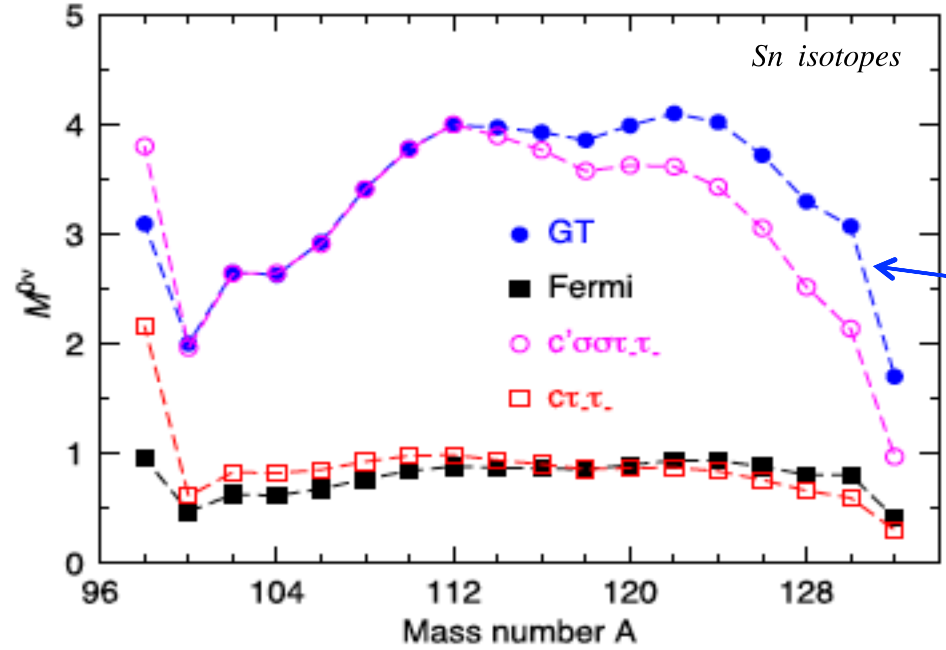
Phys. Rev. Lett. **105**, 252503 (2010)

⬟ Present Shell Model results:

Phys. Rev. Lett. **110**, 222502 (2013)

PRC **89**, 045502 & **88**, 064312 (2013)

PRC **89**, 054304 (2014)



T. Rodriguez, G. Martinez-Pinedo,

Phys. Lett. B **719**, 174 (2013)

Large jump down for magic no of neutrons !!!

Summary and Outlook

- Observation of neutrinoless double beta decay would signal physics beyond the Standard Model: **massive Majorana neutrinos, right-handed currents, SUSY LNV, etc**
- ^{48}Ca case suggests that 2ν double-beta decay can be described reasonably within the shell model with standard quenching, provided that all spin-orbit partners are included.
- Higher order effects for 0ν NME included: range 1.0 − 1.4
- Reliable 0νββ nuclear matrix elements could be used to identify the dominant mechanism if energy/angular correlations and data for several isotopes become available.
- The effects of the quenching and the missing spin-orbit partners are important (see the ^{136}Xe case), and they need to be further investigated for ^{76}Ge, ^{82}Se and ^{130}Te.