Modeling the Transition Region from High to Low Q^2

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OUTLINE

I *Introduction*

- *Modeling the transition region from Deep Inelastic Scattering (DIS)*
- *High Twists in structure functions*
- *Overview of phenomenological observations*

II *Analysis of charged lepton scattering*

- *Determination of dynamical High Twist terms*
- *Structure function limit to $Q^2 \to 0$*
- *Extrapolation to resonance region*
- *Comparison with low Q data*

III *Neutrino (antineutrino) scattering*

- *Axial-vector current and structure function limit for $Q^2 \to 0$*
- *Comparison with low Q data*

MODELING THE TRANSITION REGION

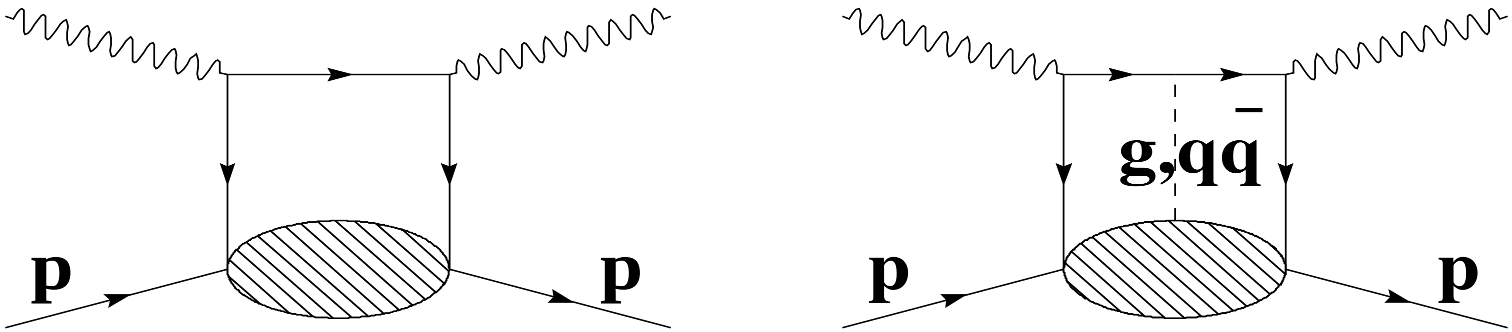
- *The large momentum transfer* (Q) *regime is well understood in terms of QCD parton model. Predictions have been verified with a variety of probes* $(e, \mu, \nu, \bar{\nu})$ *and targets*

  $\Longrightarrow$ *Universality of partonic description.*

- *By lowering progressively* Q *nonperturbative phenomena become more and more important for structure functions (SF):*
  - *What is the limit of applicability of perturbative calculation?*
  - *What is the magnitude of nonperturbative effects?*
  - *Typical scales relevant for different contributions?*

- *Natural to extrapolate DIS calculations to lower* Q *values or to the resonance region assuming a smooth transition. The range of validity of such approach would depend on different issues:*
  - *High Twists*
  - *Transition from dynamics to the regime dominated by current conservation*
  - *Axial and vector currents*
  - *Target description (e.g. nuclear effects)*

HIGH TWISTS IN STRUCTURE FUNCTIONS



- *The Operator Product Expansion (OPE) allows to express unpolarized SFs in terms of powers of $1/Q^2$, with coefficients related to local operators of a given "twist" τ:*

$$F_{T,2,3}(x, Q^2) = F^{\tau 2}_{T,2,3}(x, Q^2) + \frac{H^{\tau 4}_{T,2,3}}{Q^2} + \frac{H^{\tau 6}_{T,2,3}}{Q^4} + ...$$

- *The Leading Twist (LT) term $\tau = 2$ in the series corresponds to free quark scattering and is responsible for the scaling of structure functions via perturbative QCD $\alpha_s(Q^2)$ corrections. It dominates for $Q^2 \gtrsim 10$ GeV^2.*

- *The Higher Twist (HT) terms $\tau = 4, 6, ...$ reflect the strength of the multi-parton correlations (qq and qg). They are suppressed by powers of $1/Q^2$ (power corrections):*
  - *Twist-4 important for $Q^2 \lesssim 10$ GeV^2*
  - *Twist-6 relevant only for $Q^2 \lesssim 3$ GeV^2*

- *The kinematical High Twists associated with the finite mass of the target nucleon are mostly relevant at large x^2M^2/Q^2 values. The corresponding corrections involving powers of $1/Q^2$ are usually incorporated into the leading twist term ($\tau = 2$) following the prescription by Georgi and Politzer (1976):*

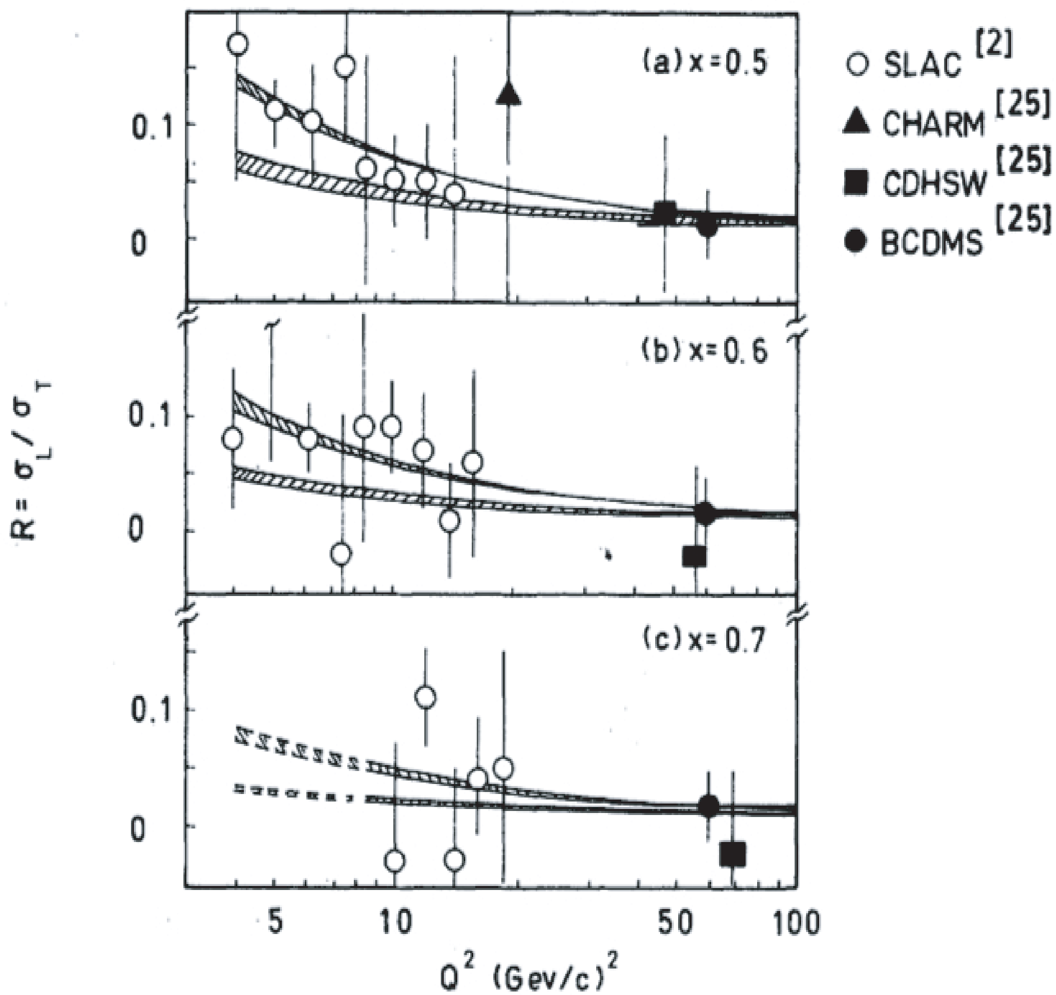
$$F^{\mathrm{LT}}_{T,2,3}(x, Q^2) \to F^{\mathrm{LT,TMC}}_{T,2,3}(x, Q^2)$$

where the calculation involves the Nachtman variable $\xi = 2x/(1+\sqrt{1+4M^2x^2/Q^2})$. Well established procedure (Kretzer and Reno 2004), but difficulty for $x \to 1$ due to wrong threshold behaviour ($F_i^{\mathrm{LT}}(\xi, Q^2) > 0$).

$\Longrightarrow$ *See talk by Wally Melnitchouk at this workshop.*

- *The dynamical High Twists ($\tau > 2$) related to multi-parton correlations can be determined phenomenologically from data by exploiting their specific Q^2 dependence which allows a separation from the LT term. Due to their nonperturbative nature, models (e.g. renormalons) provides only a qualitative description for such contributions.*

OVERVIEW OF PHENOMENOLOGICAL OBSERVATIONS

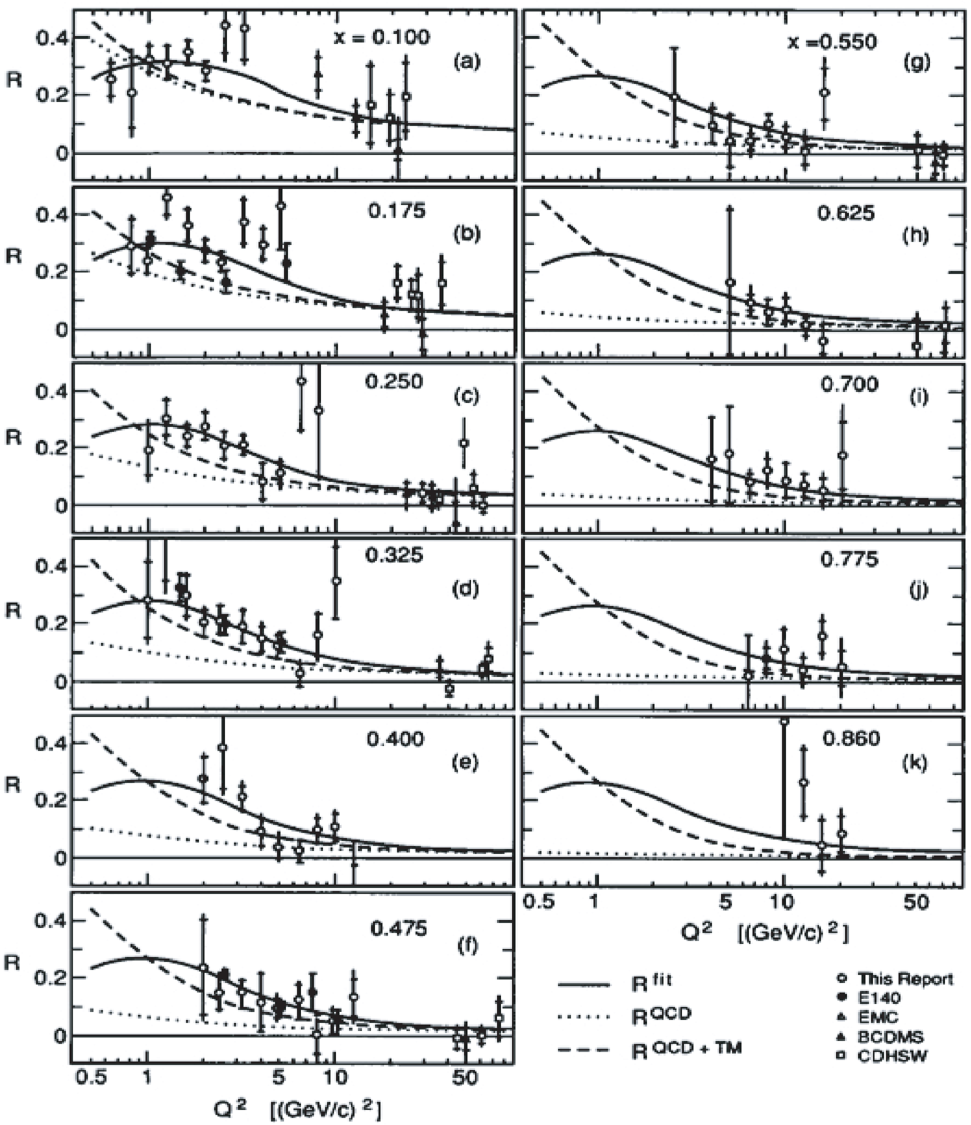


- *Miramontes, Miramontes and Sanchez (1989) analyzed world data on the ratio* $R = \sigma_L/\sigma_T$ *by introducing explicit high twist terms in* F_L

- *Parameterization of twist-4 in* F_L *as:*

$$F_L^{\tau 4}(x, Q^2) = \frac{8k^2}{Q^2} F_2^{\mathrm{LO}}$$

- *They concluded the excess of SLAC data on* R *was evidence for the presence of substantial high twist contributions and obtained* $k^2 = 0.05^{+0.02}_{-0.03}$ *at 99% CL*



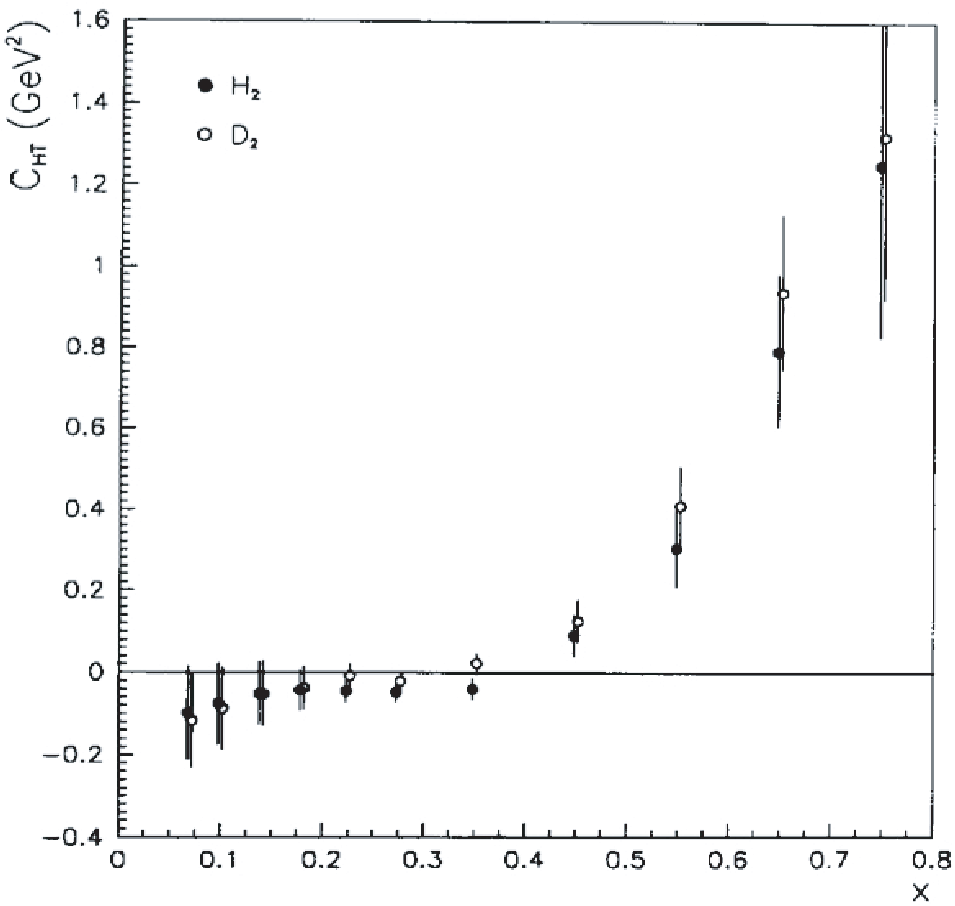
- *Whitlow et al. (1990) re-analyzed SLAC data and parameterized $R = \sigma_L/\sigma_T$ from SLAC,EMC,BCDMS and CDHSW*

- *Phenomenological fit to R data with:*

$$R^{\rm fit} = \frac{b_1}{\ln(Q^2/\Lambda^2)}\Theta(x,Q^2) + \frac{b_2}{Q^2} + \frac{b_3}{Q^4 + 0.3^2}$$

where $\Theta(x,Q^2) = 1 + 12\frac{Q^2}{Q^2+1}\frac{0.125^2}{0.125^2+x^2}$

- *They concluded $R(x,Q^2)$ is higher than QCD predictions including TMC and their phenomenological function contained HT-like terms in $1/Q^2$ and $1/Q^4$*

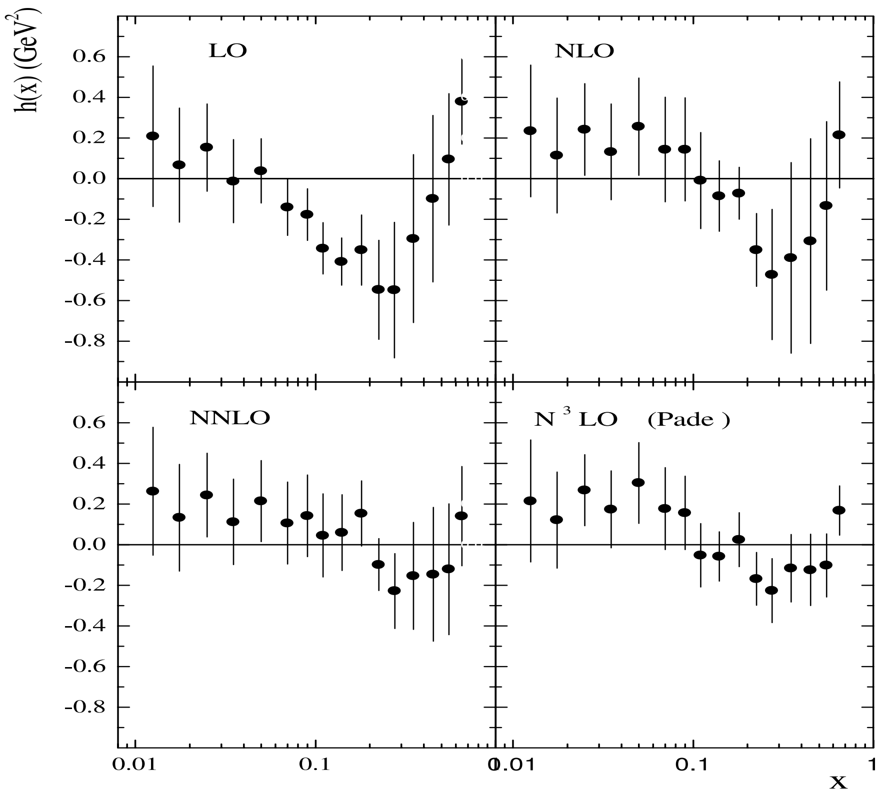


- *Virchaux and Milsztajn (1992) extracted high twists in F_2 from a NLO fit to SLAC and BCDMS data.*

- *F_2 parameterization in factorized form:*

$$F_2(x,Q^2) = F_2^{\mathrm{LT,TMC}}(x,Q^2)\left(1+\frac{C_{\mathrm{HT}}(x)}{Q^2}\right)$$

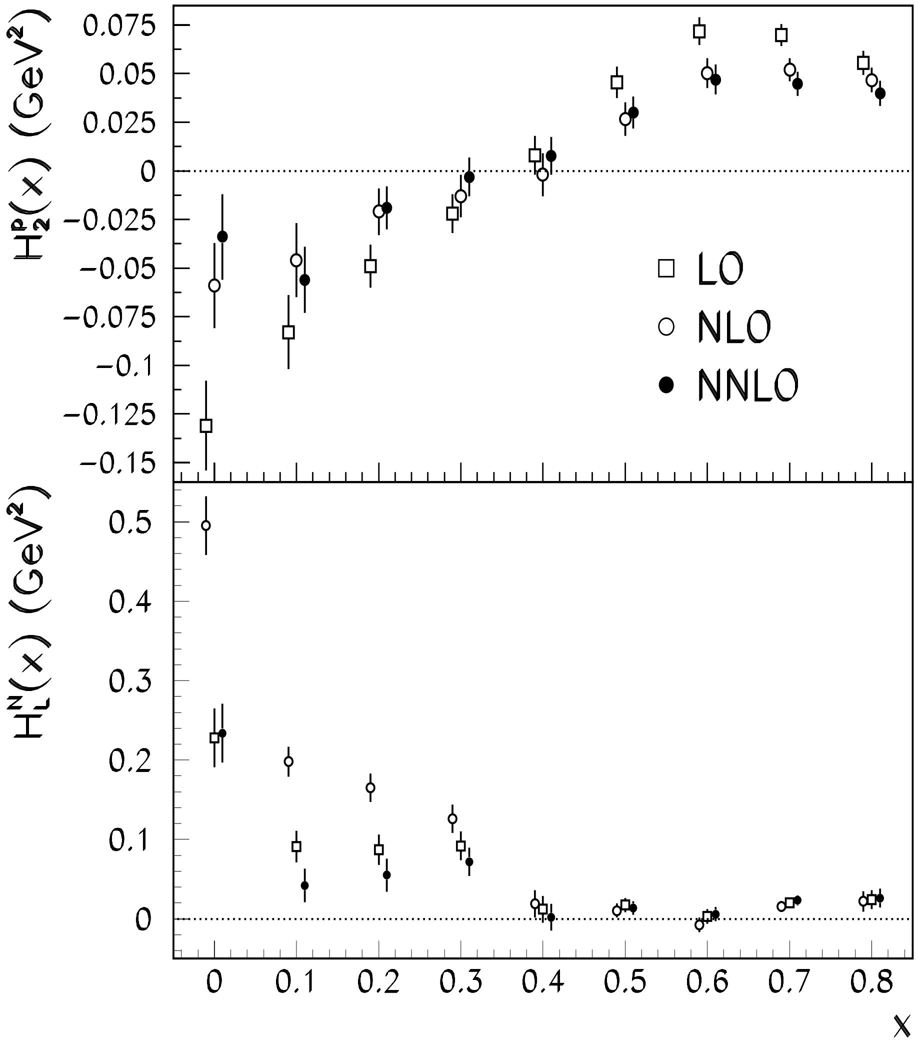
- *The inclusion of an additional term as $D_{\mathrm{HT}}(x)/Q^4$ provided only a marginal improvement in the fit and therefore it was discarded*



- *Kataev, Parente and Sidorov (1999) extracted HT in xF_3 from fits to CCFR ν data at different orders in pQCD.*
- *xF_3 parameterization in additive form:*

$$xF_3(x,Q^2) = xF_3^{\rm LT,TMC}(x,Q^2) + \frac{h(x)}{Q^2}$$

- *Although statistically limited, the analysis found a reduction of twist-4 terms by increasing the order of pQCD calculation from LO to N^{3}LO. Similar results obtained for F_2 and R from NLO to NNLO by Yang and Bodek (2000).*



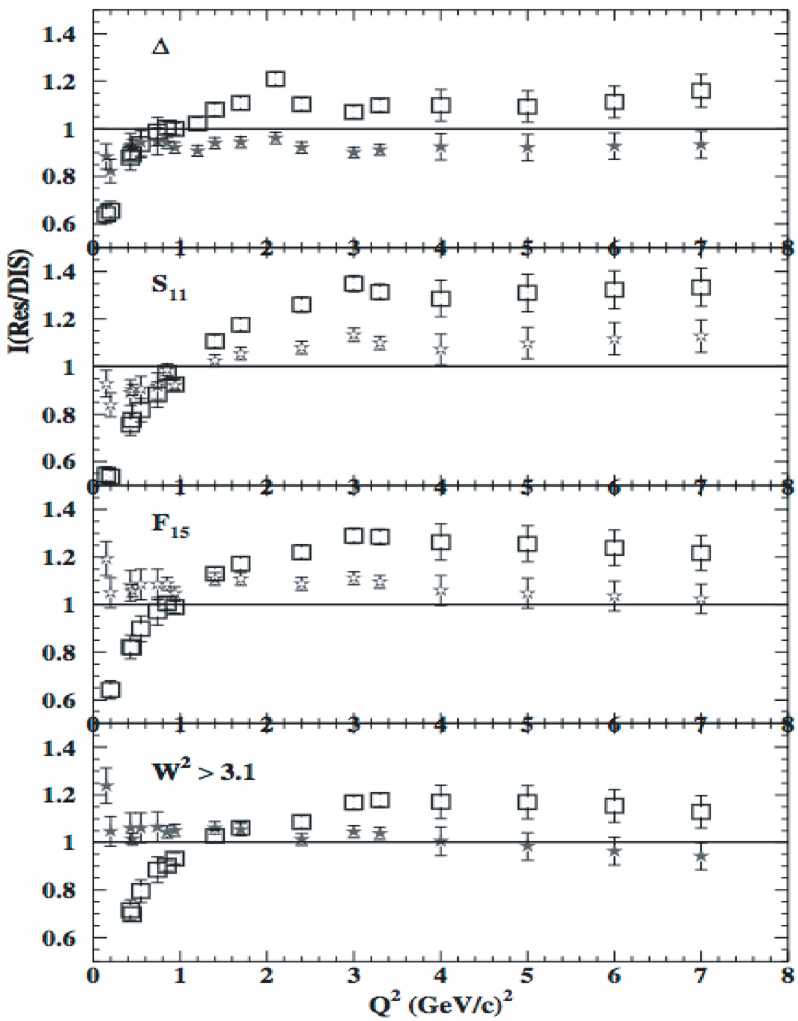
- *Alekhin (2002) extracted high twists in F_2 and F_L from fits to DIS data to SLAC,BCDMS,NMC,HERA with $Q^2 > 2.5$ GeV^2 at different pQCD orders.*

- *SF parameterization in additive form:*

$$F_{2,L}(x,Q^2) = F_{2,L}^{\mathrm{LT,TMC}}(x,Q^2) + \frac{H(x)_{2,L}}{Q^2}$$

- *Only marginal reduction of twist-4 terms was observed by increasing the order of pQCD calculation from NLO to NNLO*

- *Significant high twist contribution to F_L observed for $x < 0.3$ in agreement with Miramontes et al.*



- *Niculescu et al. (2000) verified local quark-hadron duality at JLab*

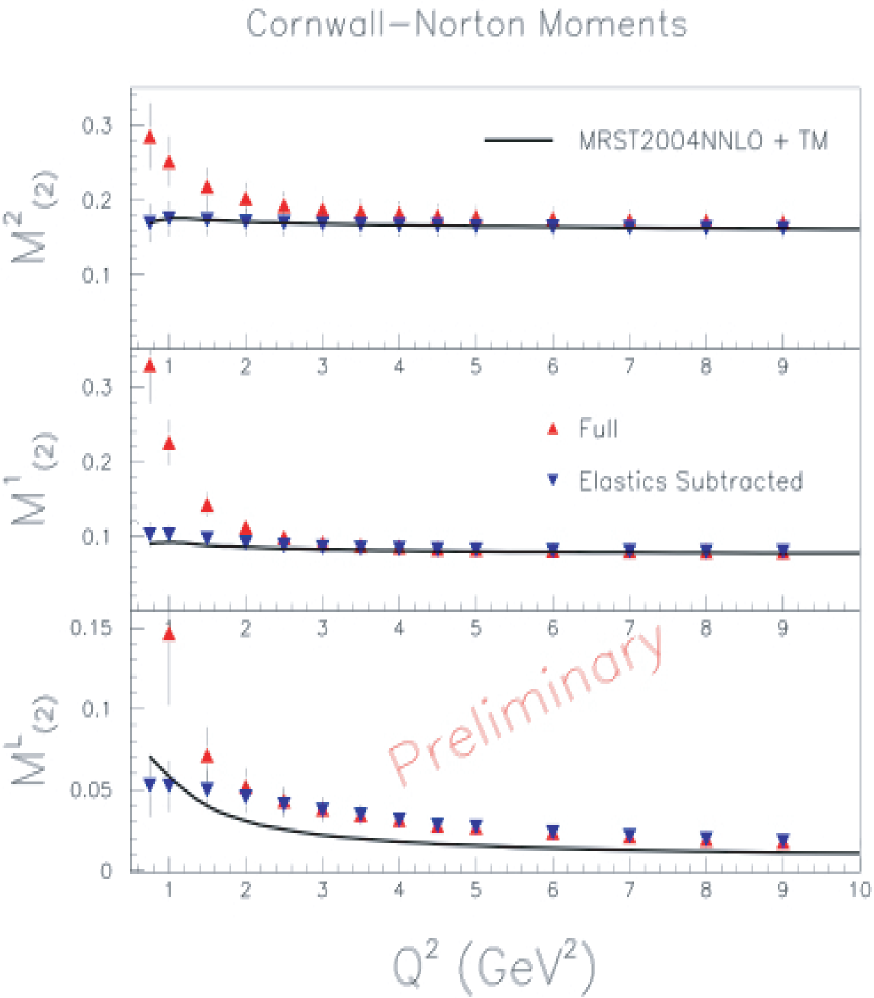
- *Analysis of integrated strength:*

$$I(Res/DIS) = \frac{\int_{\xi(W_1^2)}^{\xi(W_2^2)} F_2^{\rm data}(\xi, Q_0^2) d\xi}{\int_{\xi(a)}^{\xi(b)} F_2^{\rm scaling}(\xi, Q^2 = 10) d\xi}$$

where $\xi = 2x/(1 + \sqrt{1 + 4M^2x^2/Q^2})$ *and* $\xi(a, b)$ *corresponds to* $\xi(W_1^2, W_2^2)$

- *Results indicated a suppression of additional high twists in resonance region with respect to the average DIS behaviour, down to* $Q^2 \sim 1$ *GeV*^{2}.

- *Duality observed for local resonances as well as for the* $1 \leq W^2 \leq 4$ *GeV*^{2} *range*



- *Study of duality from structure function moments (Christy et al. 2005) at JLab*

- *The moments are defined as:*

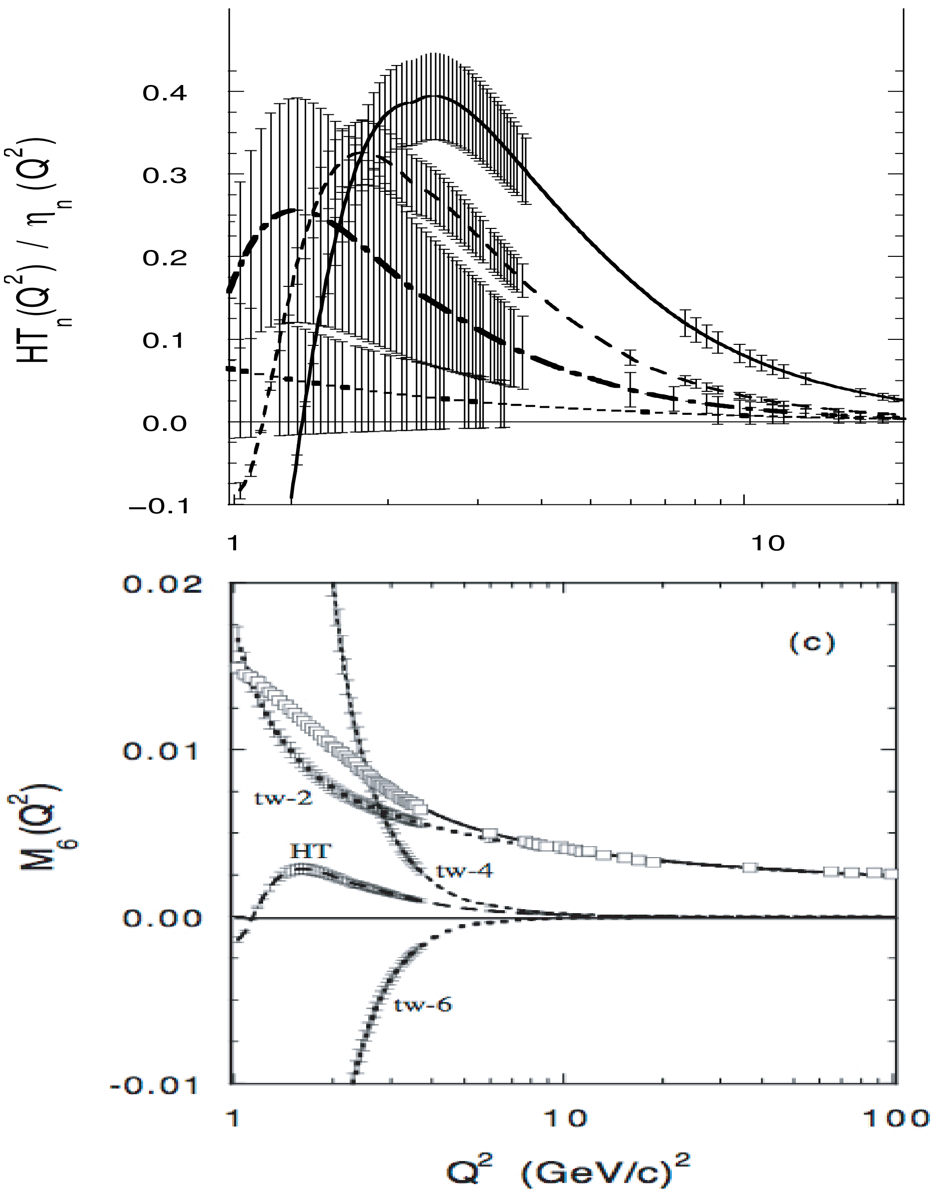
$$M_n^k(Q^2) = \int_0^1 dx\, x^{n-2} F_k(x, Q^2)$$

where $k = 1, 2, L$

- *High twist contributions visible from* Q^2 *dependence of moments beyond pQCD logarithmic scaling violations:*

$$M_n(Q^2) = A_n^{\tau 2} + \frac{A_n^{\tau 4}}{Q^2} + \frac{A_n^{\tau 6}}{Q^4} + ...$$

- *The* Q^2 *behaviour of* M_2 *in the resonance region indicates* F_2 *and* F_1 *are well described by pQCD down to* $Q^2 \sim 1$ *GeV*^{2}*.*



- *Extraction of moments for F_2^p in the resonance region by the CLAS collaboration at JLab (Osipenko et al. 2003)*

- *The leading twist dominates down to $2Q^2/n \sim 1$ GeV^2. The size of total high twist is comparable with that obtained from DIS and grows with x.*

- *The small HT contribution resulted from a cancellation between twist-4 and twist-6 terms, whose magnitude can even exceed the one of the leading twist (OPE convergence?). Liuti et al (2001) also observed the lower W region requires a negative $1/Q^4$ term.*

GENERAL INSIGHTS

- *There is a significant contribution to F_L from dynamical high twists. Most notably power corrections of the order of $1/Q^4$ may be needed to explain existing data on the ratio $R = \sigma_L/\sigma_T$*

- *To some extent dynamical high twists absorb missing orders in perturbation theory. Their size is therefore reduced by extending the order of pQCD calculation, becoming very small at NNLO.*

- *High twists affect only the region of $Q^2 \sim 1 \div 3$ GeV^2 and they are not important for the determination of leading twist*

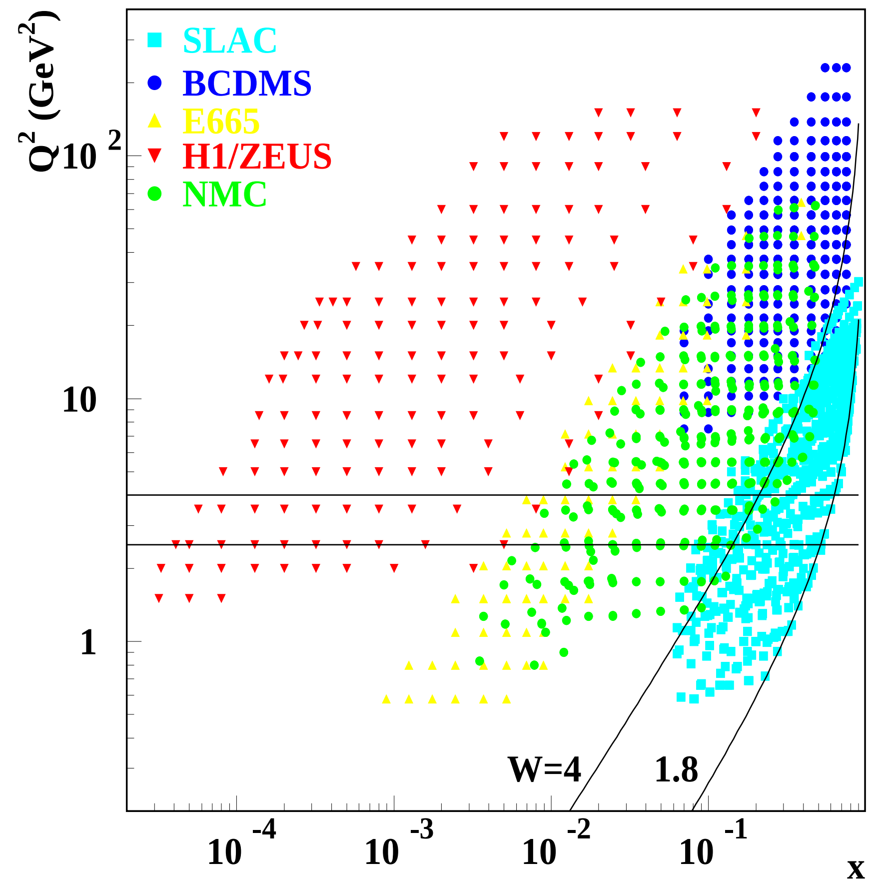
- *The quark-hadron duality implies a suppression of dynamical high twists in the resonance region.*

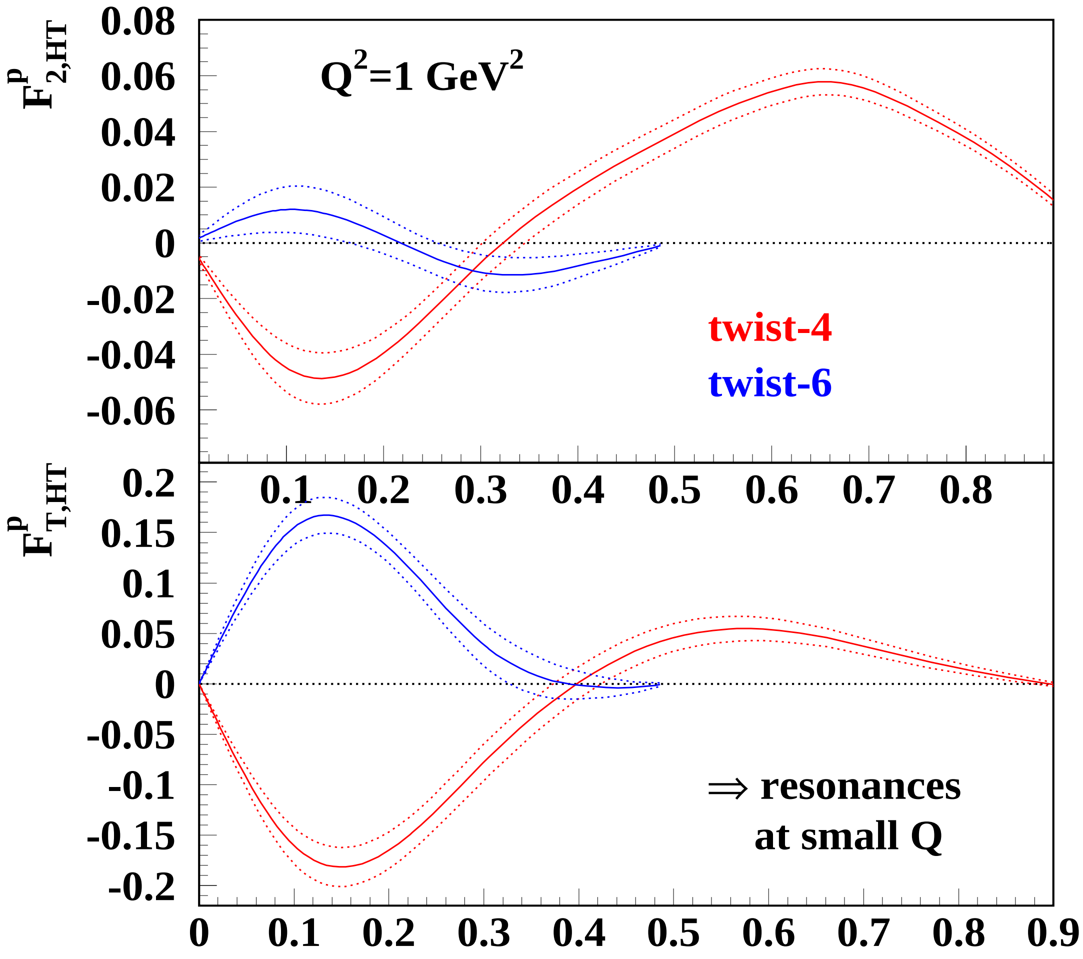
DETERMINATION OF DYNAMICAL HIGH TWISTS

- *Perform global fit to the charged lepton DIS and Drell-Yan data samples with* $\boxed{Q^2 > 1.0\ GeV^2}$ *and* $W > 1.8$ *GeV is used. The leading twist is calculated in the NNLO approximation, with parton distributions evolved from* $Q_0^2 = 9$ *GeV*^{2}*.*

- *The dynamical twist-4,6 terms,* $H_{2,T}^{\tau 4, \tau 6}(x)$*, are parameterized in a model-independent way by cubic splines with values at* $x = 0.1, 0.3, 0.5, 0.7, 0.9$ *which are fitted from data.*

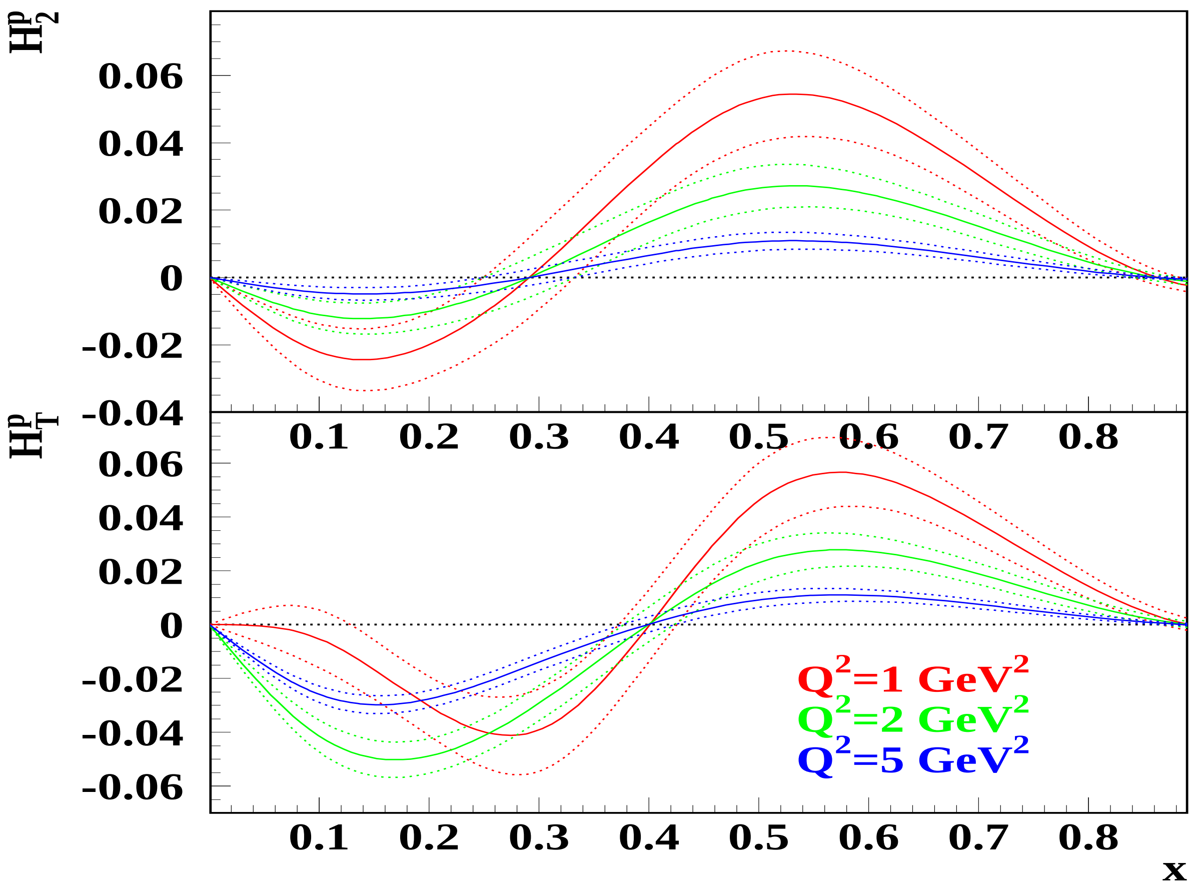
- *Few external constraints are imposed:*
  - $H_{2,T}^{\tau 4,\tau 6}(0) = 0$ *since no clear evidence for saturation effects is found at HERA;*
  - $H_{2,T}^{\tau 6} = 0$ *at* $x > 0.5$ *due to the impossibility to extract them out of the resonance region.*





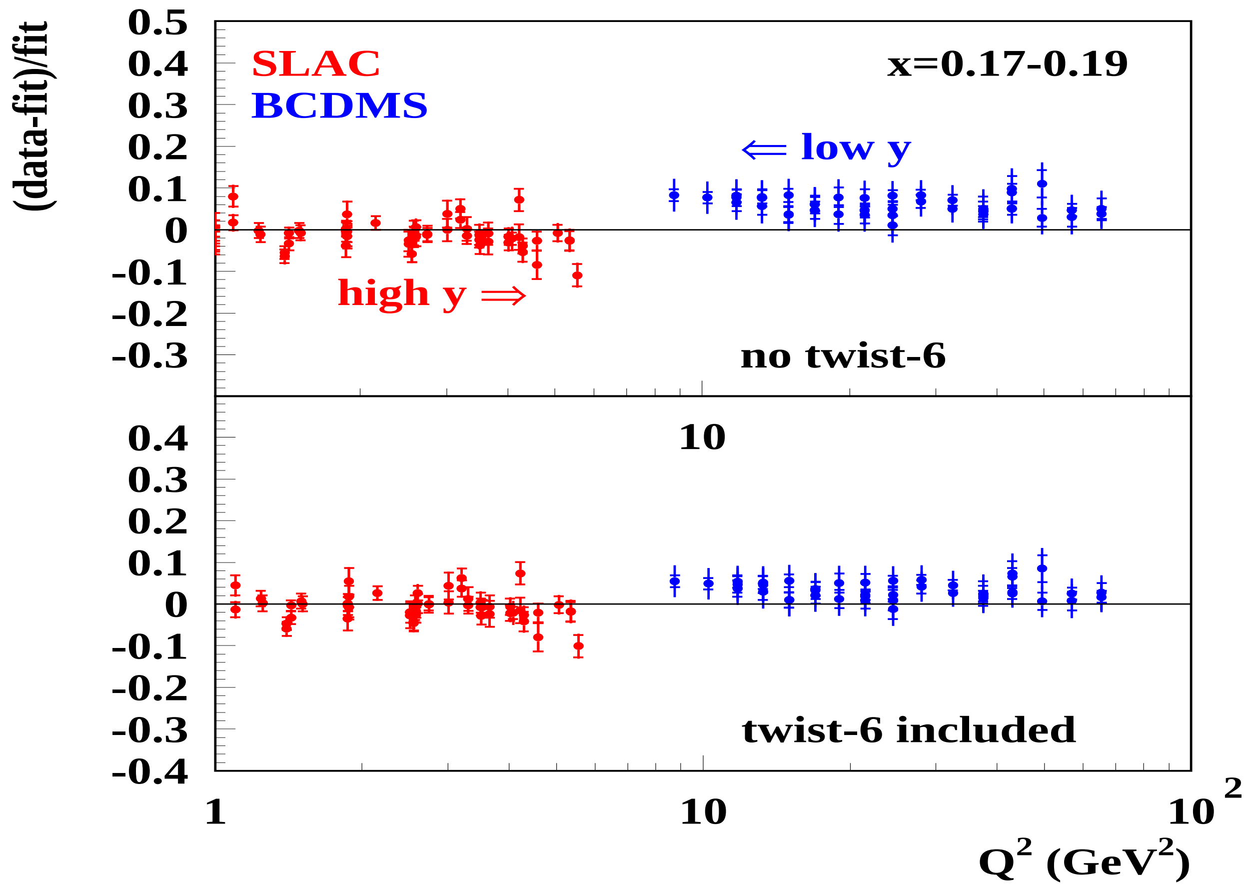
- *The high twist terms in F_2 demonstrate good convergence: $H_2^{\tau 6}$ is much smaller than $H_2^{\tau 4}$ and it is comparable to zero within the errors.*

- *For F_T the picture is different: the magnitudes of $H_T^{\tau 6}$ and $H_T^{\tau 4}$ terms are comparable and somehow compensate each other. Poor convergence of the OPE?*



The total high twist contribution to F_T indicates a weak dependence on Q^2 for $x < 0.3$

ORIGIN OF TWIST-6 TERMS



The twist-6 terms in F_T arise due to a mismatch between SLAC and BCDMS data at $Q^2 = 5 \div 10$ GeV^2 but different y

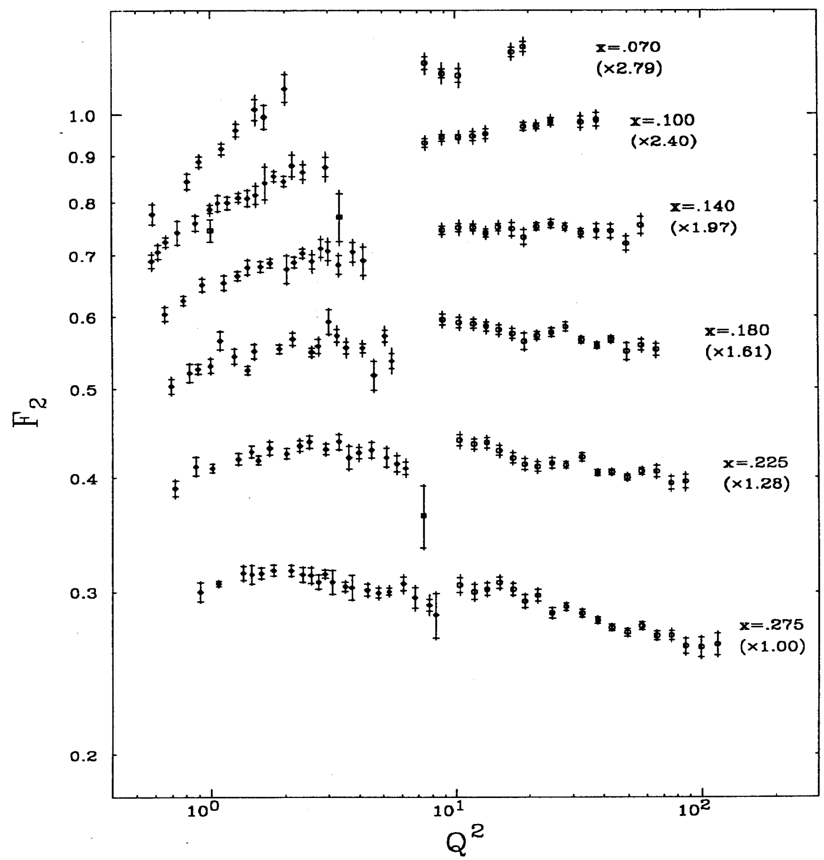


Figure 5.14. Shown is a comparison of the SLAC and BCDMS hydrogen F_2 results. A relative normalization of 1.000 is assumed. See also the caption to Figure 5.9. Figure continues on next page.

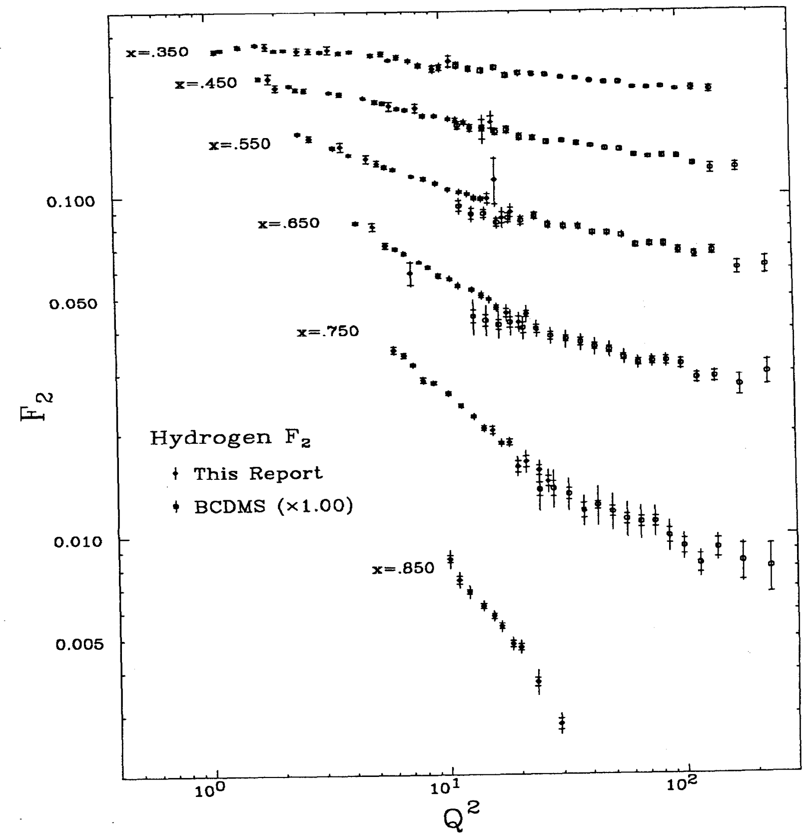
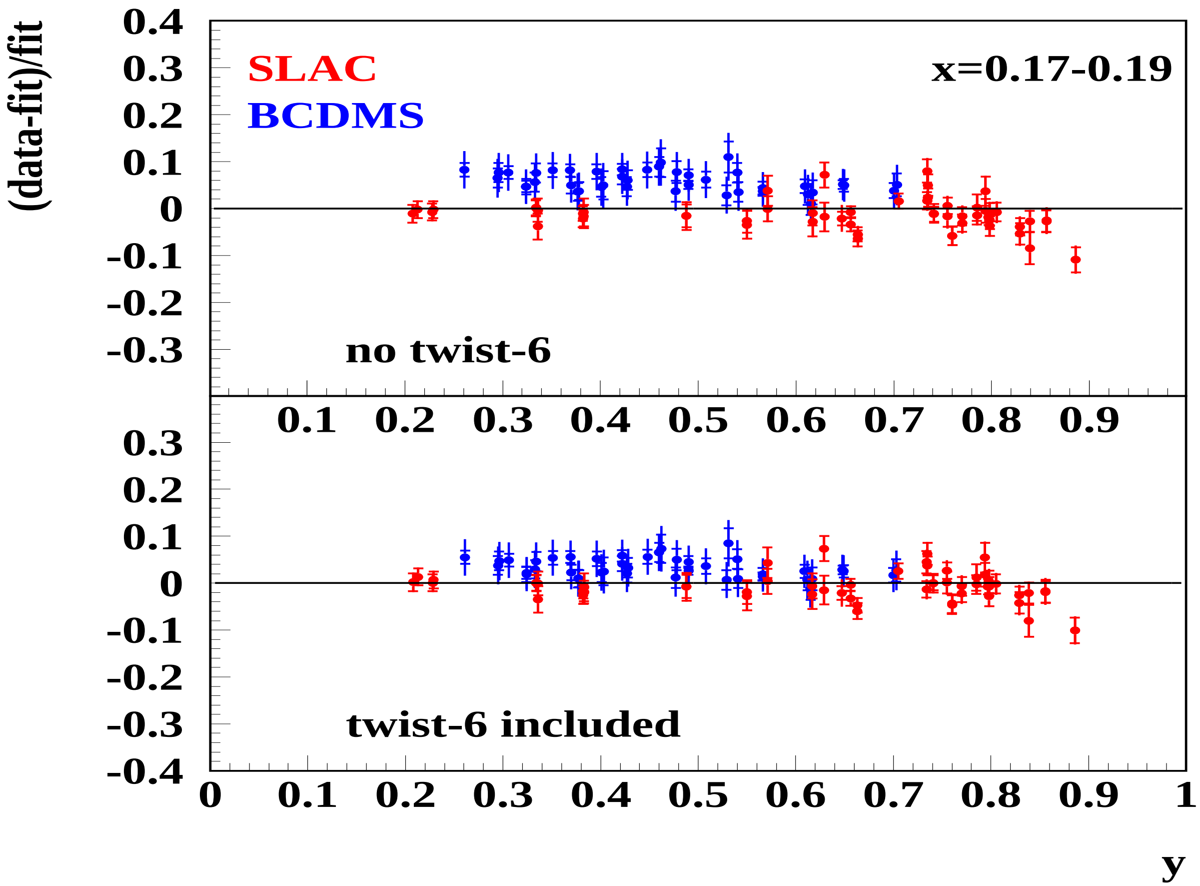
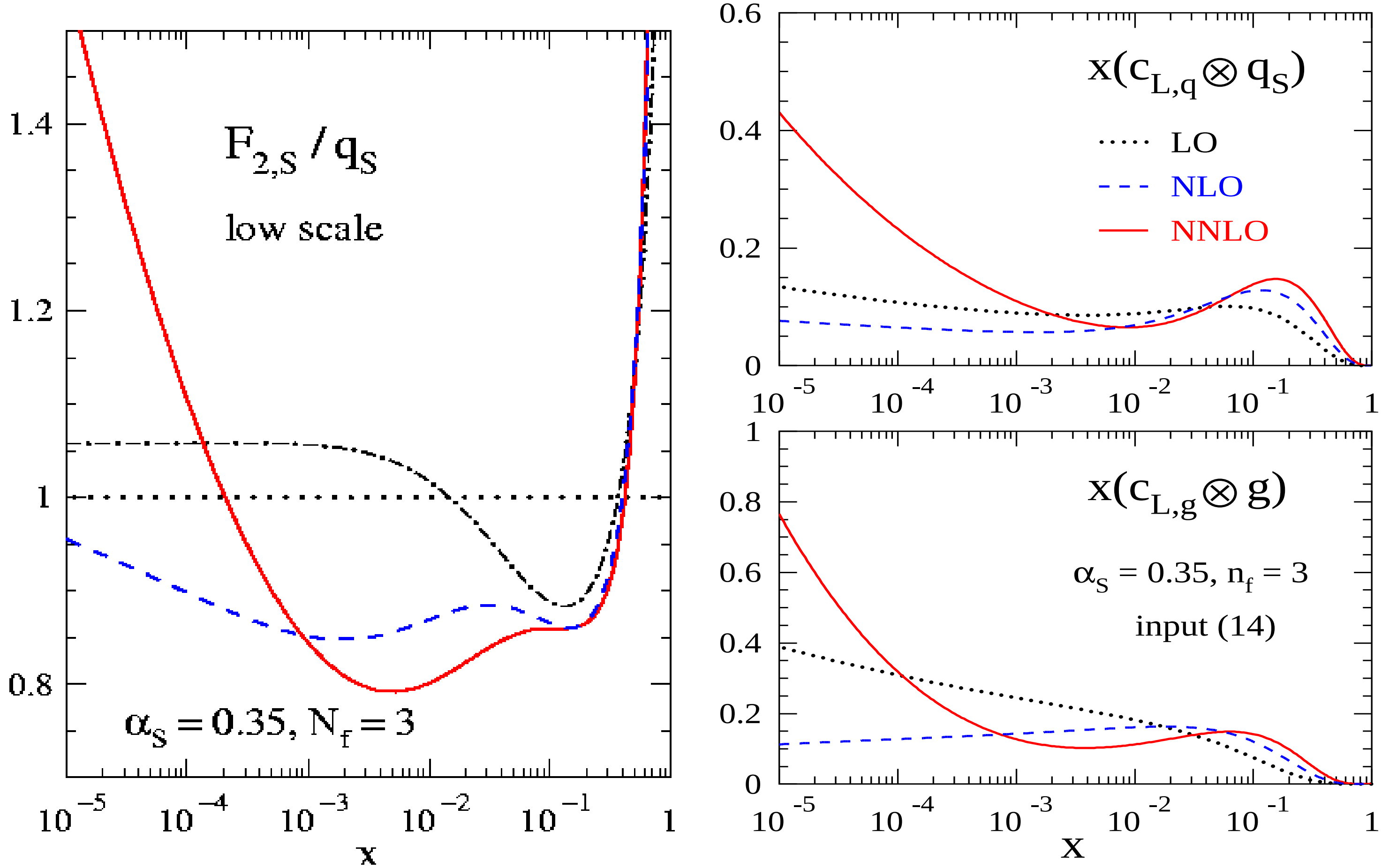


Figure 5.14/continued: Comparison of SLAC and BCDMS hydrogen F_2 results.

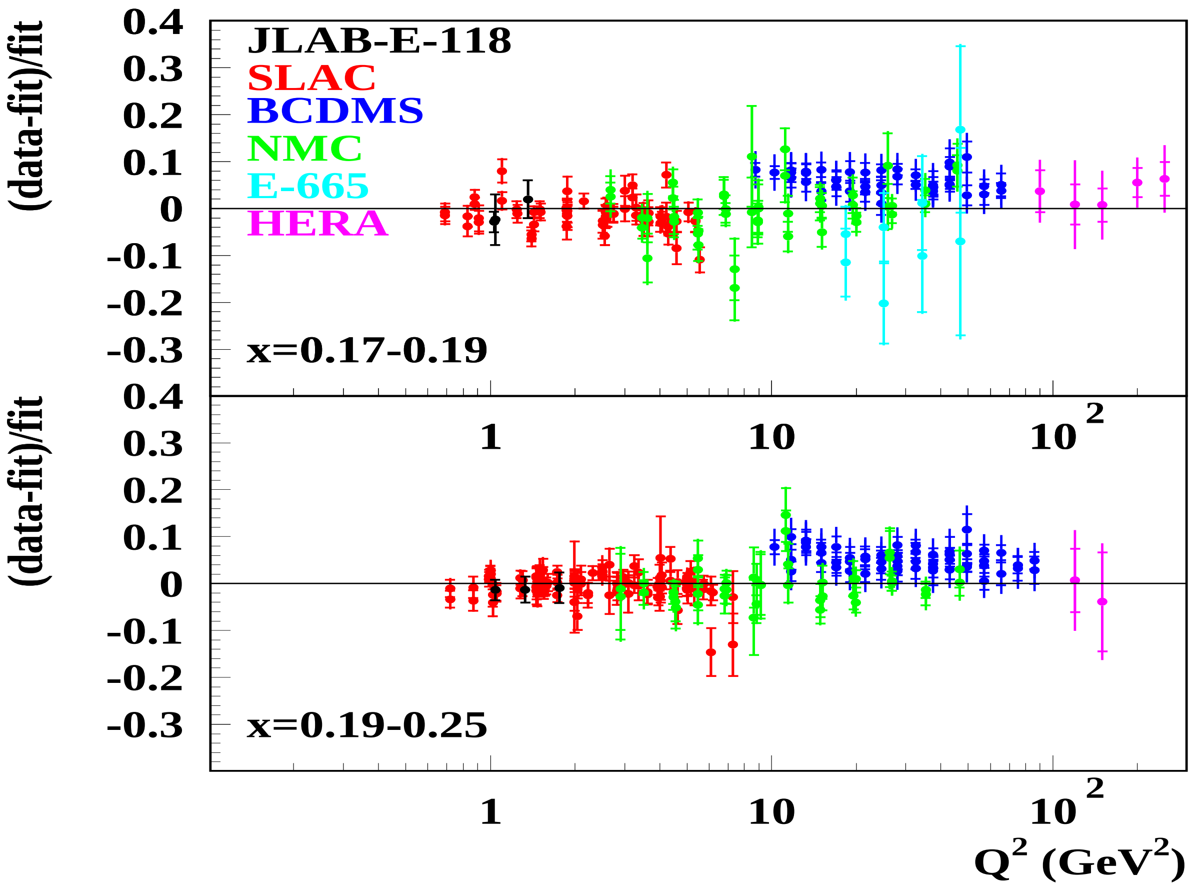
Whitlow et al. 90



Electro-weak corrections seem not responsible for the SLAC/BCDMS discrepancy

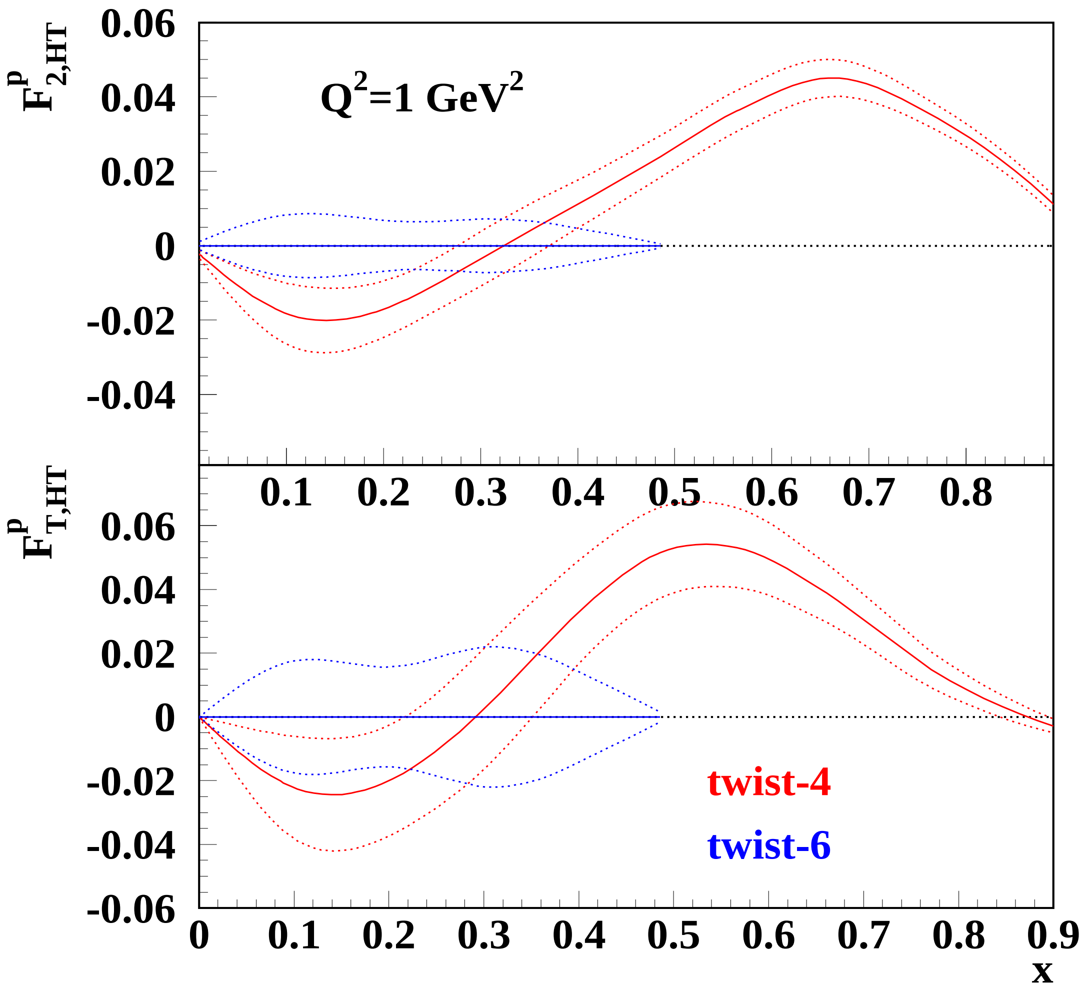


The $\mathcal{O}(\alpha_s^3)$ corrections (solid curves) to the DIS coefficient functions (Moch, Vermaseren, Vogt 04-05) do not change the fitted twist-6 terms



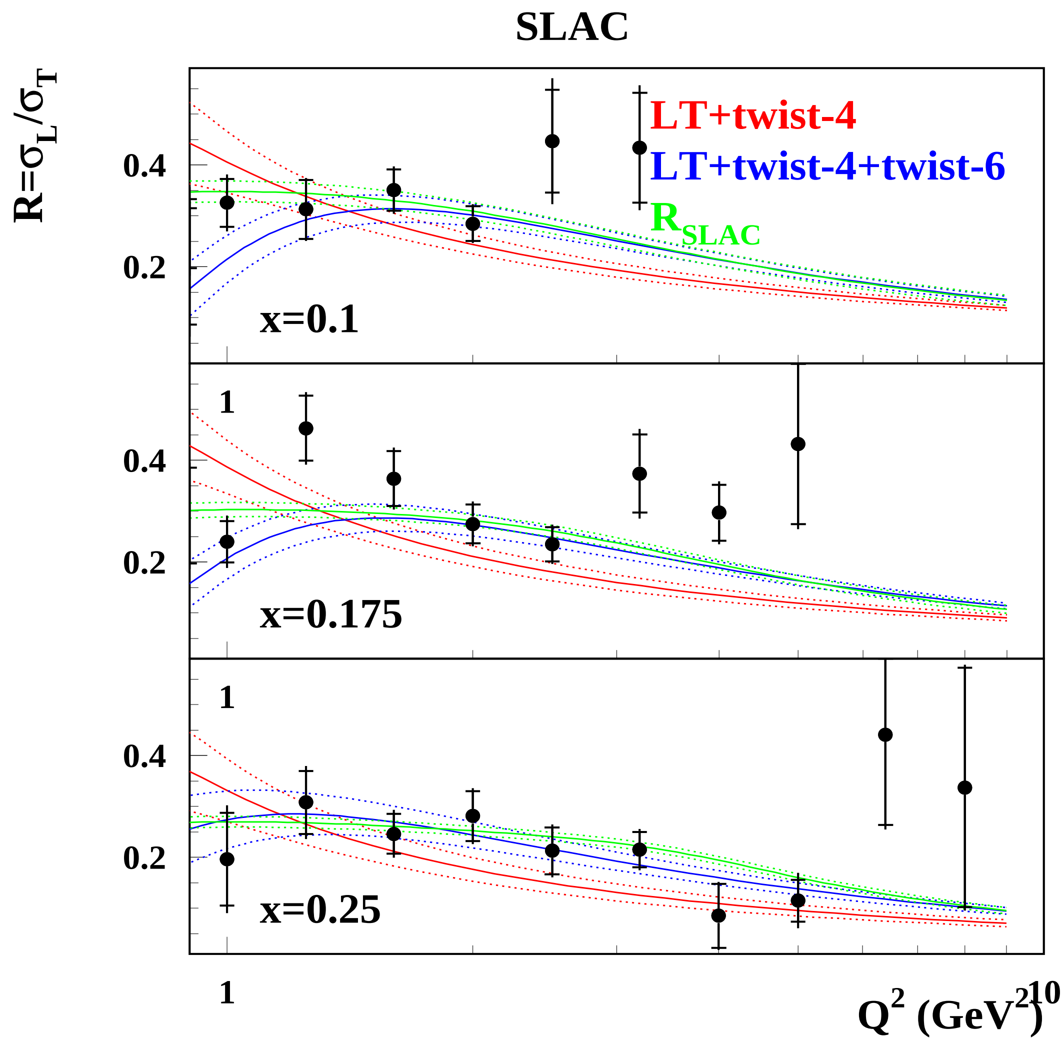
Preliminary data from JLab E-118 are in agreement with the low-Q part of SLAC data

NEW RESULTS ON HIGH TWISTS



- *Many indications twist-6 terms extracted from data are fake. Perform new fits with $H^{\tau 6}_{2,T} = 0$ in the full kinematic region.*
- *After dropping twist-6 terms the shape and magnitude of twist-4 contributions to F_2 and F_T are comparable*
- *The upper limit on the magnitude of twist-6 terms turns out to be ~ 0.02 GeV^2, well below twist-4 terms.*
- *Out of the resonance region the high twist contributions correspond to $\leq 10\%$ of the total structure functions, indicating a convergence of the OPE expansion.*

EVALUATION OF $R = \sigma_L/\sigma_T$



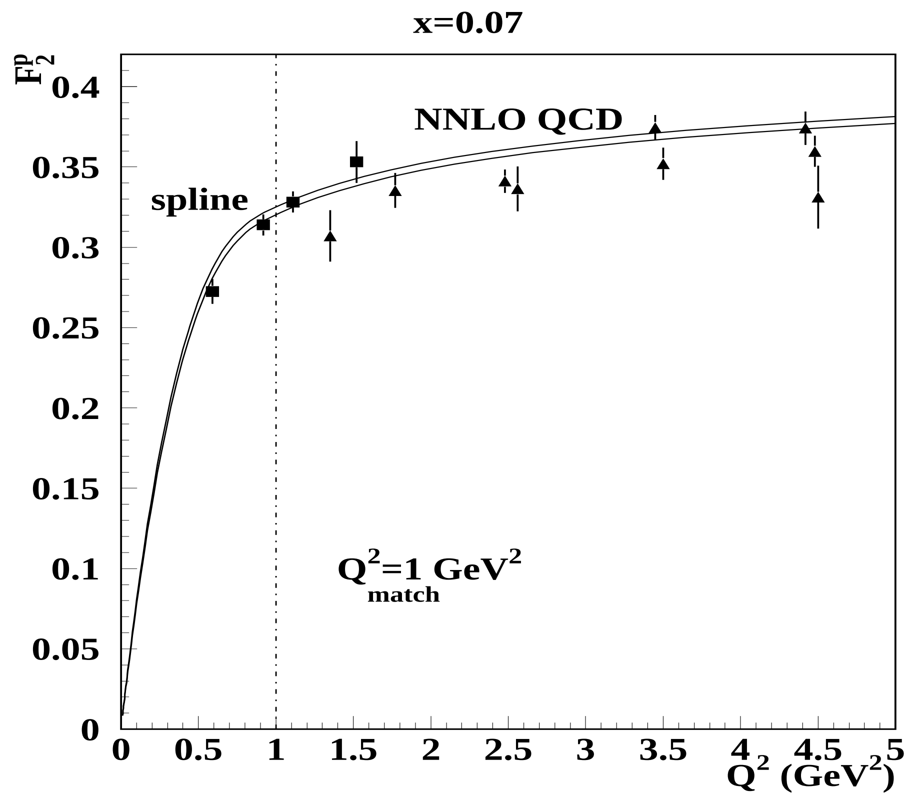
- The excess in SLAC data for $R = \sigma_L/\sigma_T$ at $x \sim 0.2$ with respect to the QCD predictions was considered as evidence of the large high twist contribution to R and F_L (Miramontes 92)

- Our results show instead such excess is connected with the discrepancy between SLAC and BCDMS and can be hardly attributed to the high twist contributions

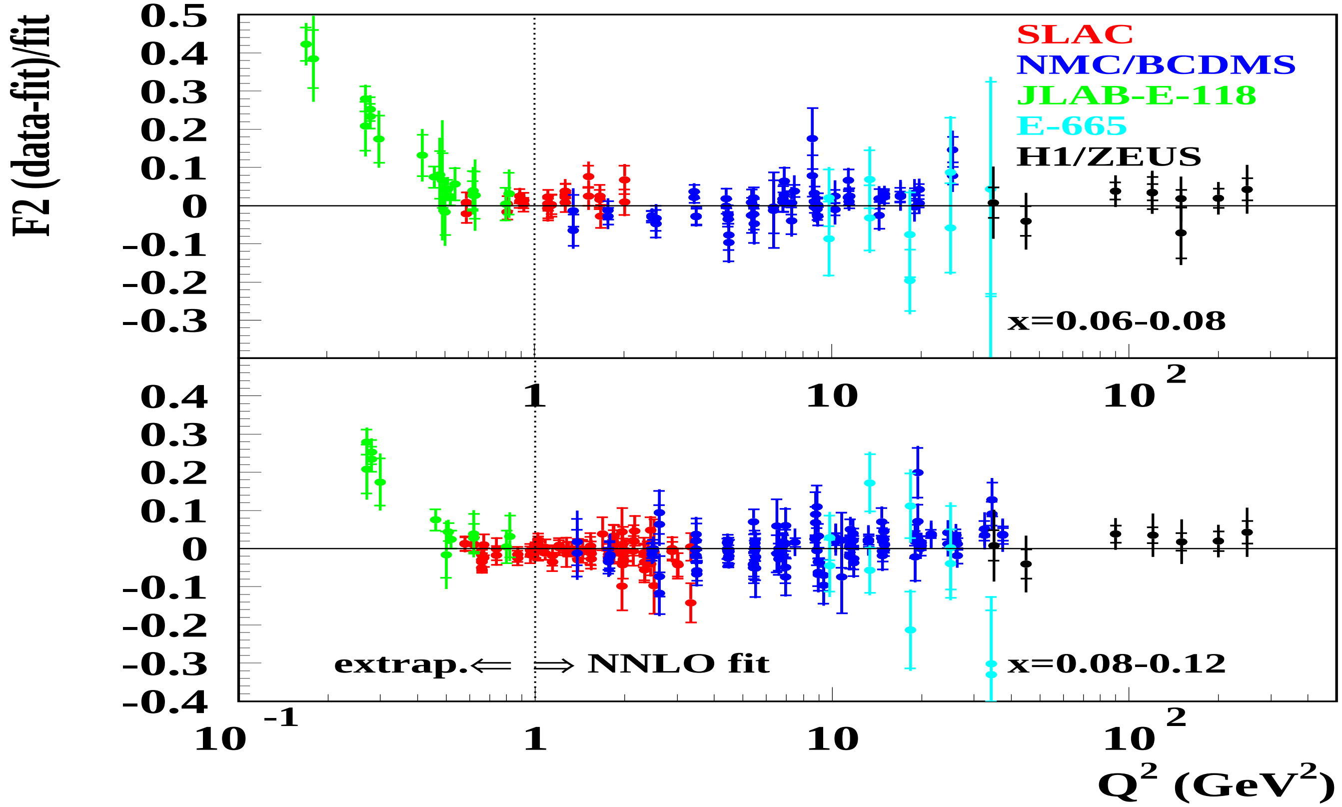
STRUCTURE FUNCTIONS DOWN TO $Q^2 = 0$

- **LOW Q** *SF are constructed as smooth interpolation between QCD approach (LT + HT) at high Q^2 and the limit for $Q^2 \to 0$ from current conservation arguments:*

- *QCD matching point* $\boxed{Q^2_{\rm match} = 1.0\ {\rm GeV}^2}$
- *Conservation of Electromagnetic Current implies $F_2 \sim Q^2$ and $F_L \sim Q^4$ as $Q^2 \to 0$.*
- *A cubic spline interpolation at fixed x is applied from $Q^2 = 1.0$ GeV^2 to the $Q^2 = 0$ limit predicted by current conservation.*
- *Functions and derivatives match at $Q^2_{\rm match}$*
- *The region $0 \leq Q^2 \leq 1$ GeV^2 is determined by the asymptotics at $Q^2 \to 0$ and by the matching conditions at $Q^2 = Q^2_{\rm match}$.*

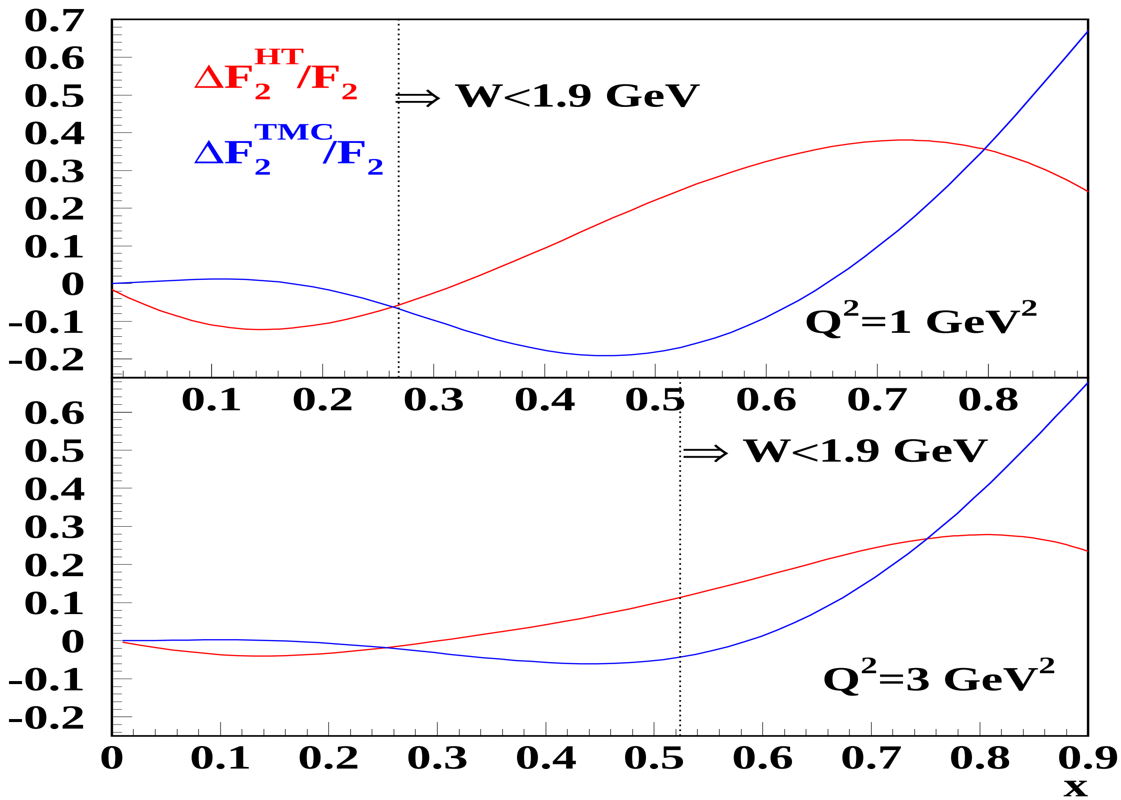


COMPARISON WITH LOW Q DATA



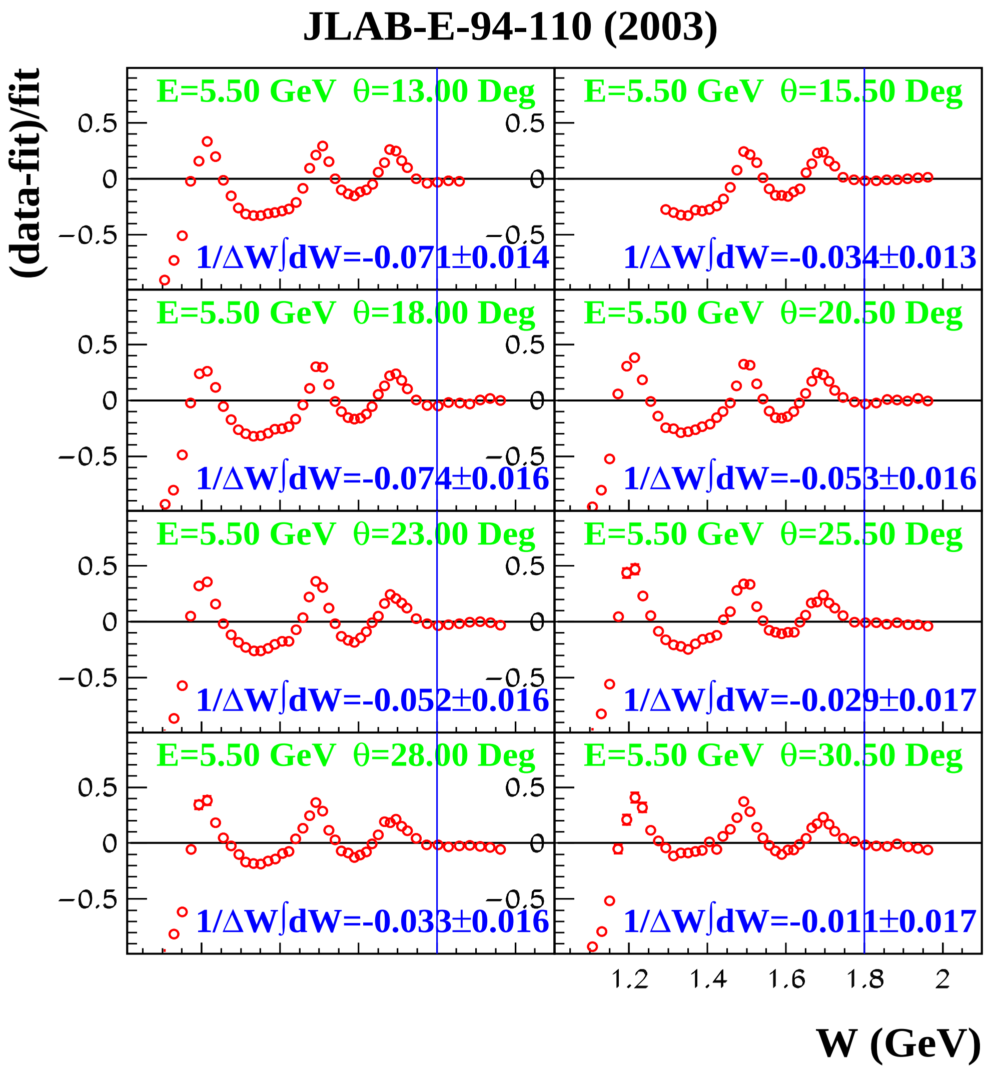
- *Data points with $Q^2 < 1.0$ GeV² (JLab, SLAC) are not included in fits*
- *The phenomenological extrapolation of the QCD phenomenology (LT + HT) provides a good description of charged lepton data down to $Q^2 \sim 0.5$ GeV²*

IMPACT OF HIGH TWISTS



- *The total High Twist contribution is a small correction to SFs at* $Q^2 \gtrsim 1$ *GeV*^{2}
- *The extrapolation of phenomenological HTs provides a sizeable correction in the resonance region* ($W < 1.9$ *GeV*)

EXTRAPOLATION TO RESONANCE REGION



- *Use data on p and D targets within the region $W > 1.9$ GeV in fits.*
- *Extrapolation of DIS calculations to the resonance region with $\boxed{W < 1.9 \text{ GeV}}$ (not used in fits) is consistent with Bloom-Gilman duality at percent level.*
- *Indirect indication the Twist-6 contributions are small at large x values.*

APPLICATION TO NEUTRINO SCATTERING

- *Neutrino scattering is characterized by the* AXIAL-VECTOR CURRENT *(V-A):*

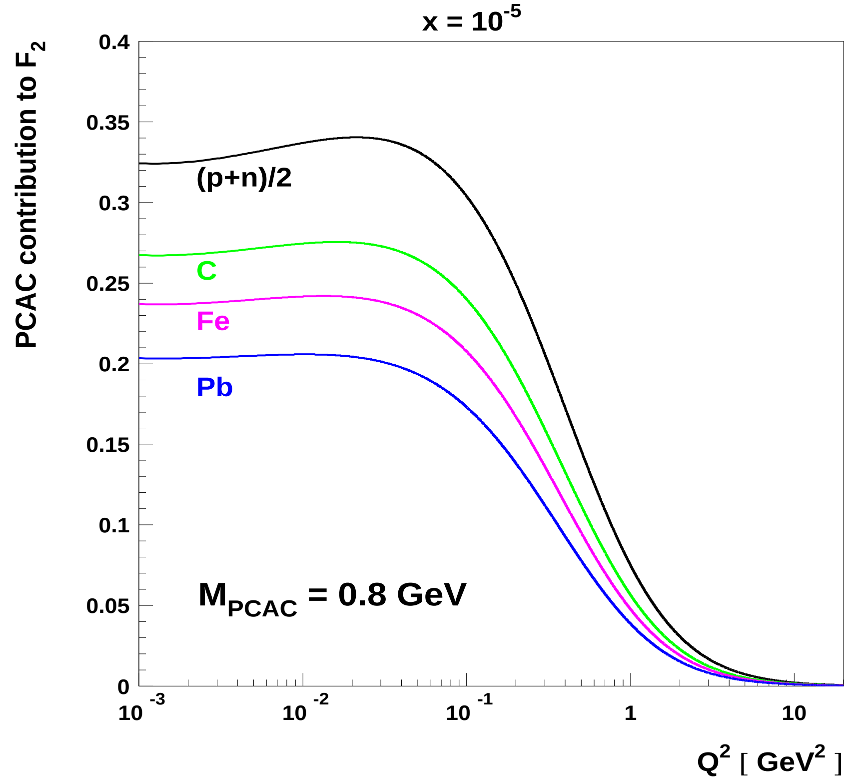
$$VV, AA \implies F_{1,2} \quad (or\ F_L,\ F_T)$$
$$VA \implies F_3$$

- *Conservation of Vector Current (CVC), in analogy to the charged leptons, implies:*

$$F_2, F_T \sim Q^2 \qquad F_L \sim Q^4 \qquad for \quad Q^2 \to 0$$

- *Axial Current is only Partially Conserved (PCAC) and dominates F_L at low Q^2:*

$$\partial A = f_\pi m_\pi^2 \varphi \quad \implies \quad F_L = \frac{f_\pi^2 \sigma_\pi}{\pi}$$



- *Transition scale between low and high Q^2 is* $\boxed{\mathrm{NOT}\ m_\pi^2}$ *but rather $M_{\mathrm{PCAC}} \sim 1\ GeV^2$, since direct pion contribution $\partial_\mu \varphi$ cancels out. We use the following model:*

$$F_L = \frac{f_\pi^2 \sigma_\pi}{\pi}(1 + Q^2/M^2_{\mathrm{PCAC}})^{-2} + \widetilde{F}_L \qquad with \quad \widetilde{F}_L \propto Q^4 \quad for \quad Q^2 \to 0$$

$\implies$ *Non vanishing contribution to $F_2 = (F_L + F_T)/(1 + 4x^2M^2/Q^2)$ for $Q \to 0$.*

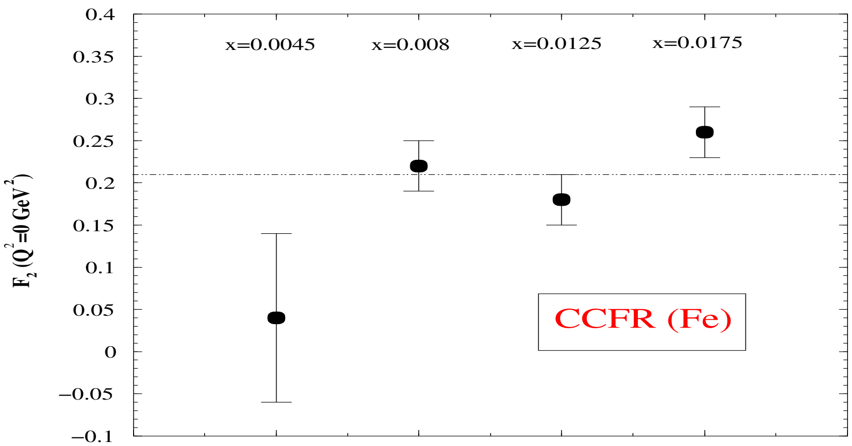
- *Predictions for the asymptotic value $F_2(Q^2 = 0)$ in neutrino CC scattering seem in agreement with the CCFR determination on Fe target 0.210 ± 0.02.*

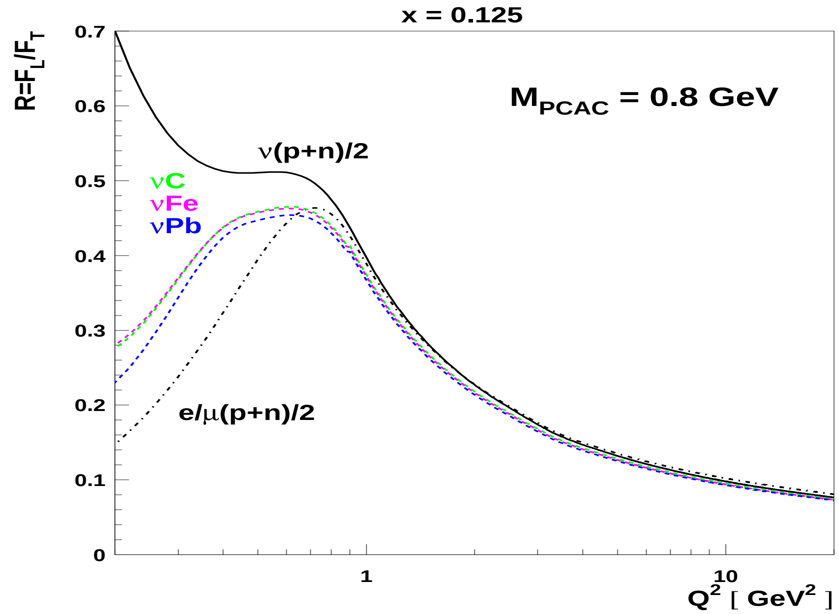
- *The finite PCAC contribution to F_L strongly affects the asymptotic behaviour of $\boxed{R = \sigma_L/\sigma_T}$ for $Q^2 \rightarrow 0$:*

$$F_T \sim Q^2$$
$$F_L \sim \frac{f_\pi^2 \sigma_\pi}{\pi} > 0$$

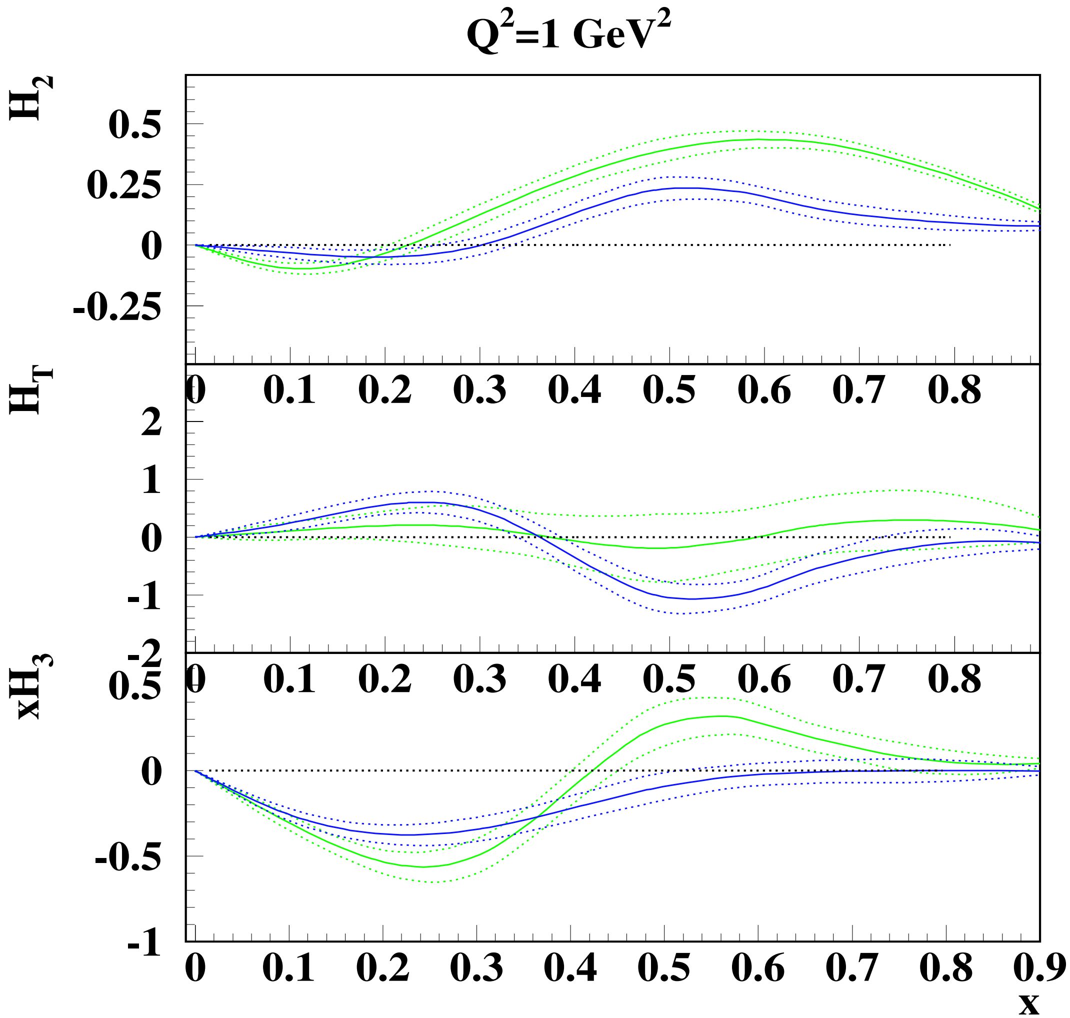
so that R is divergent for vanishing Q^2

$\Longrightarrow$ *Substantial difference with respect to charged lepton scattering.*



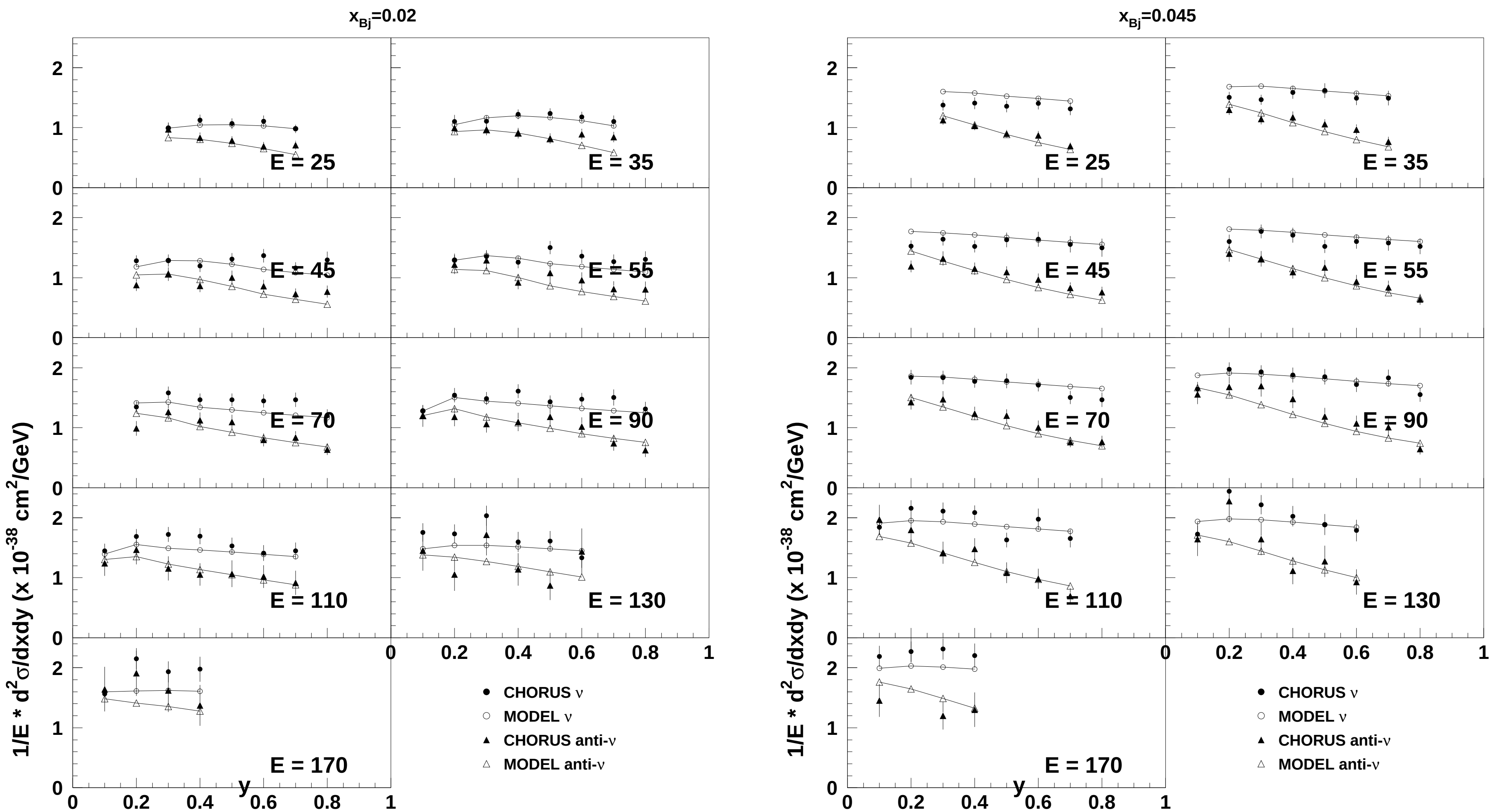


HIGH TWISTS IN NEUTRINO SCATTERING



- *Independent determination of the twist-4 terms from $\nu(\bar{\nu})$ cross-section data:*
  - *Global fit with charged lepton DIS and DY;*
  - *Nuclear corrections by Kulagin-Petti;*
  - *Electro-weak corrections by Arbuzov-Bardin.*
- *Discrepancies between CHORUS (blue curves) and NuTeV (green curves) do not allow unambiguous results*
- *Possible best strategy in global fits with charged lepton DIS and DY data:*
  - *Define $H_{2,T}^{\tau 4}$ for (anti)neutrinos as the one for charged leptons, rescaled according to the ratio of the corresponding leading twist terms;*
  - *Extract $H_3^{\tau 4}$ from (anti)neutrino data.*

COMPARISON WITH LOW Q DATA



Low x (< 0.05) CHORUS data on Pb target (PLB 632 (2006) 65.) reach $Q^2 \sim 0.25$ GeV^2 and can be used for tests of PCAC contribution (suggest $M_{\rm PCAC} \sim 0.8$ GeV)

SUMMARY

- *A detailed analysis of dynamical High Twists from world data on charged lepton DIS has been performed. Results provide new insights on High Twist contributions to SFs:*
  - *Part of the excess observed in SLAC data on $R = \sigma_L/\sigma_T$ with respect to pQCD predictions is related to the tension with BCDMS data. No evidence for actual twist-6 terms.*
  - *The magnitudes of twist-4 terms in F_2 and F_T are comparable and they do not vanish in the NNLO approximation for the leading twist.*
  - *Dynamical Twist-4 terms contribute to structure functions at $Q^2 \lesssim 10$ GeV^2 and are crucial for a precise modeling of the transition region.*
  - *Convergence of the OPE expansion has been verified with the extracted HT down to 1 GeV^2.*

- *A smooth interpolation between QCD phenomenology (Leading Twist + Twist-4) at large Q^2 and limits for $Q^2 \to 0$ from current conservation arguments has been used to parameterize low Q^2 structure functions.*

  $\Longrightarrow$ *Good description of data from charged lepton DIS down to $Q^2 \sim 0.5$ GeV^2.*

- *Once the effect of the axial-vector current is taken into account, (anti)neutrino charged-current data are, in general, consistent with charged lepton scattering data. Some discrepancies among different experiments*