Quark–Hadron Duality in Neutrino Reactions

Olga Lalakulich

Department of Subatomic and Radiation Physics
Ghent University, Belgium

+ E.Paschos (Dortmund Uni), W.Melnitchouk (JLab)
+ N. Jachowicz, Ch.Praet, J.Ryckebusch (Gent Uni)

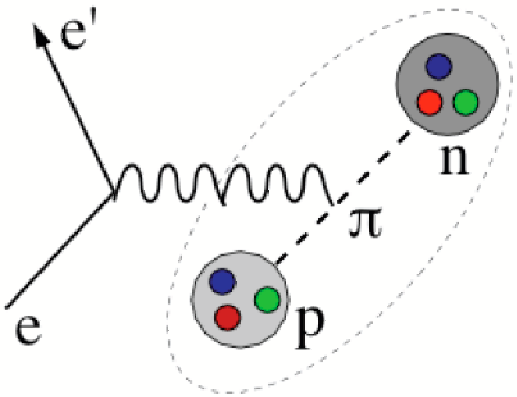
Neutrino-Nucleus Interactions in the Few-GeV Region, 2007

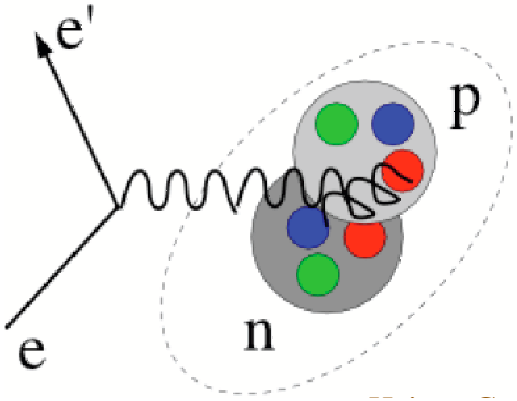
Outline

1. Quark-Hadron Duality: general conception
   - Relationships between meson–hadron and quark–gluon degrees of freedom
   - Quark–hadron duality is a general feature of strongly interacting landscape
2. Duality in neutrino–nucleon scattering compared to the electron–nucleon scattering
   - $F_2^{ep,\,en}$: Duality HOLDS in electron–nucleon scattering
   - Scaling variables for duality
   - $F_2^{\nu p,\,\nu n}$: In neutrino–nucleon scattering duality does NOT hold for proton and neutron
   - $F_2^{\nu p,\,\nu n}$: Duality HOLDS for the averaged structure functions
   - $2xF_1^{eN}$ in electron–nucleon scattering
   - $2xF_1^{\nu N}$ in neutrino–nucleon scattering
   - Duality for $xF_3^{\nu N}$ structure function
3. Duality in neutrino–nucleus scattering
   - Impulse approximation and nuclear shell model
   - Bound-state propagator for the struck nucleon
   - Duality for carbon nucleus
4. Applications of duality
   - Global fit of the neutrino–nucleus cross section ...
   - ... relies on duality
5. Summary

Quark–hadron duality

Relationships between meson–hadron and quark–gluon degrees of freedom





At low energies the effects of confinement impose a more efficient description in terms of collective degrees of freedom, the physical mesons and baryons or hadrons

At high energies, asymptotic freedom of QCD allows for an efficient description in terms of quarks and gluons or partons, weakly interacting at short distances

There exist instances where low-energy hadronic phenomena, averaged over appropriate energy intervals, closely resemble those at asymptotically high energies, calculated in terms of quark-gluon degrees of freedom

Duality is a general feature of strongly interacting landscape

Quark–hadron duality in different phenomena:

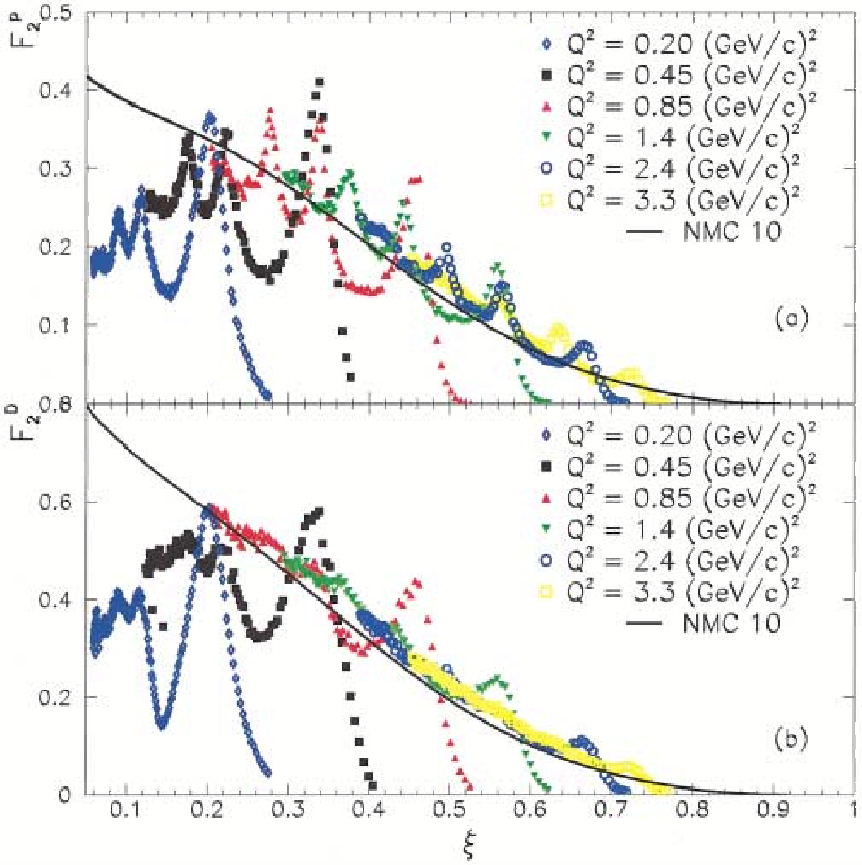
- in piN scattering establish relations between resonances and Regge pole exchange, background and Pomeron exchange
- in πp scattering helps to establish the Froissart bound on the x-section
- in hadronic decays of heavy mesons (extraction of V_{cb} and V_{ub} relies crucially on duality)
- in semileptonic decays of heavy mesons
- in electron-nucleon and neutrino-nucleon scattering

see W. Melnitchoul, R. Ent, C. Keppel, Phys Rept 406 for the review

$F_2^{ep,en}$: Duality HOLDS in electron–nucleon scattering

Duality holds for both proton and deuteriuim targets (=for neutron target)

Niculescu, PRL85



JLAB: recent experimental data on F_2 of the reactions $ep \to eX$, $eD \to DX$ in the resonance region

solid curve — global fit to the world's DIS data by NMC collaboration

The data at various values of Q^2 and W average to a smooth curve if expressed in terms of ξ.

Scaling variables for duality

The most general scaling variable includes target mass correstion and finite quark mass

$$\xi_B = \frac{Q^2 + \sqrt{Q^4 + 4m_q^2 Q^2}}{2m_N\nu(1+\sqrt{1+Q^2/\nu^2})}$$

Barbieri, Ellis, Gaillard, Ross

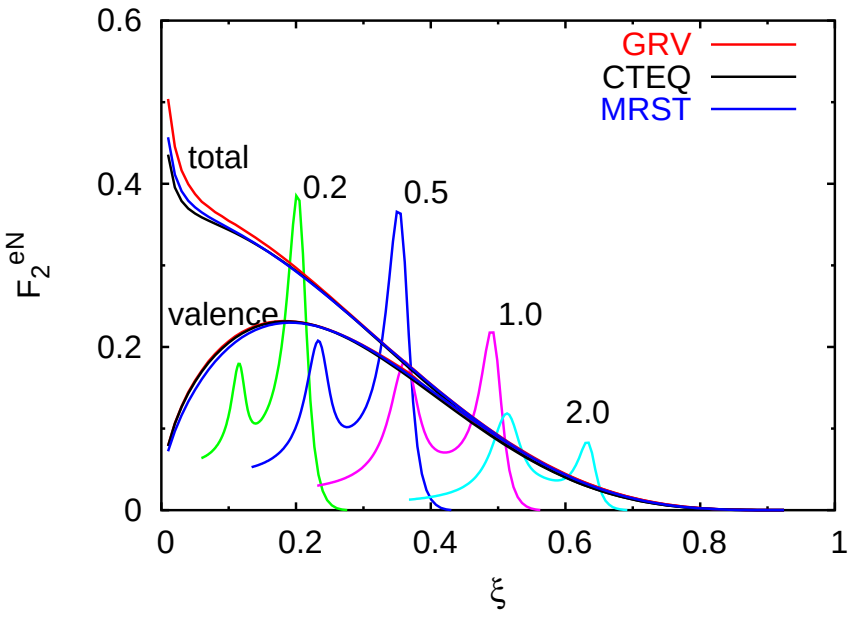
Nachmann scaling variable ξ

$$\xi = \lim_{m_q \to 0} \xi_B = \frac{2Q^2/2m_N\nu}{(1+\sqrt{1+Q^2/\nu^2})} = \frac{2x}{(1+\sqrt{1+4m_N^2x^2/Q^2})}$$

Expanding ξ in powers of $1/Q^2$ at high Q^2 gives the variable $\frac{2m_N\nu+m_N^2}{Q^2}$, found emperically in 1970 by Bloom and Gilman and used in their pioneer work on duality

$$\frac{1}{\xi} \approx \frac{1}{x}\left(1+\frac{m_N^2x^2}{Q^2}\right) = \frac{2m_N\nu+m_N^2}{Q^2}$$

At very high Q^2, neglecting m_N^2/Q^2, we get $\xi \approx \frac{2x}{1+1} = x$ - Bjorken variable (see Melnitchouk, Ent, Keppel, Phys.Rep. 406)



$Q^2 = 0.2,\ 0.5,\ 1.0,\ 2\ \mathrm{GeV}^2$

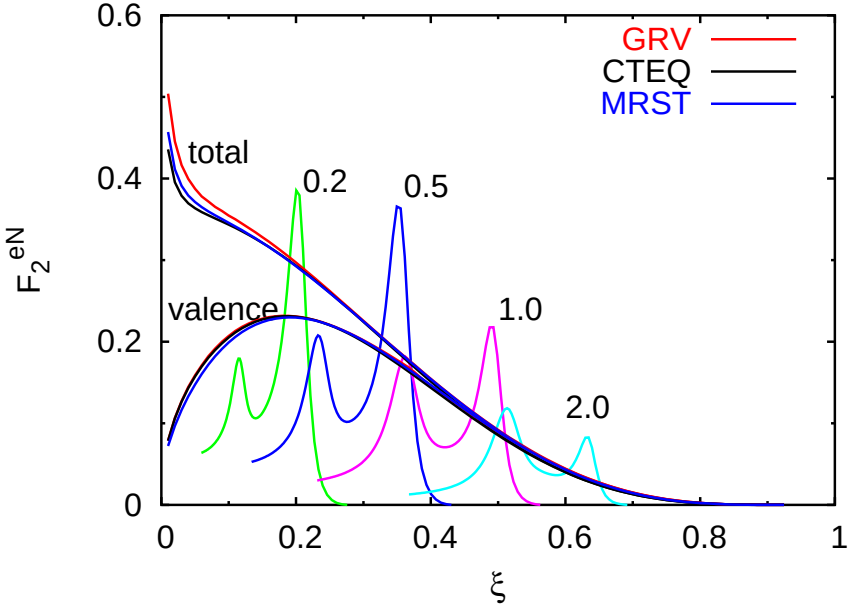
Resonance structure functions: isobar model with phenominological form factors OL,Paschos, PRD 71, 74 includes the first four low-lying baryon resonances $P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$

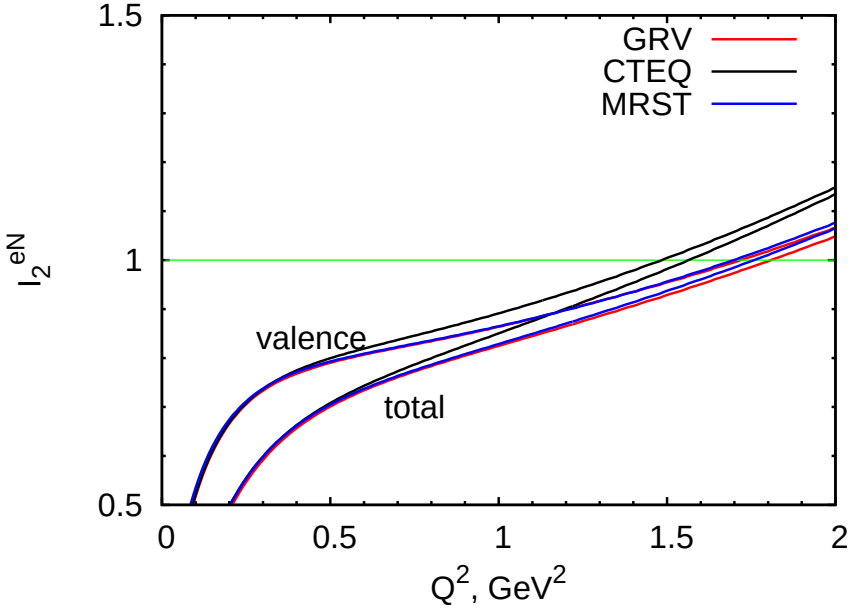
DIS structure functions: leading twist calculation with different parametrizations (GRV, CTEQ, MRST) of the parton distribution functions

Global duality: on average the resonances appear to oscillate around and slide down the leading twist function

Similar results in Sato-Lee model Matsui,Sato,Lee, PRC 72 ($P_{33}(1232)$ resonance considered so far)

Local duality: ratio of the integrals over the finite interval of ξ





$$Q^2 = 0.2,\ 0.5,\ 1.0,\ 2\ \mathrm{GeV}^2$$

$$I_2(Q^2) = \frac{\int_{\xi_{\min}}^{\xi_{\max}} d\xi\ \mathcal{F}_2^{(\mathrm{res})}(\xi, Q^2)}{\int_{\xi_{\min}}^{\xi_{\max}} d\xi\ \mathcal{F}_2^{(\mathrm{LeadingTwist})}(\xi, Q^2)},$$

OL,Melnitchouk,Paschos, PRC 75

$$\xi_{\min} = \xi(Q^2, W = 1.6\ \mathrm{GeV}), \qquad \xi_{\max} = \xi(Q^2, W = 1.1\ \mathrm{GeV})$$

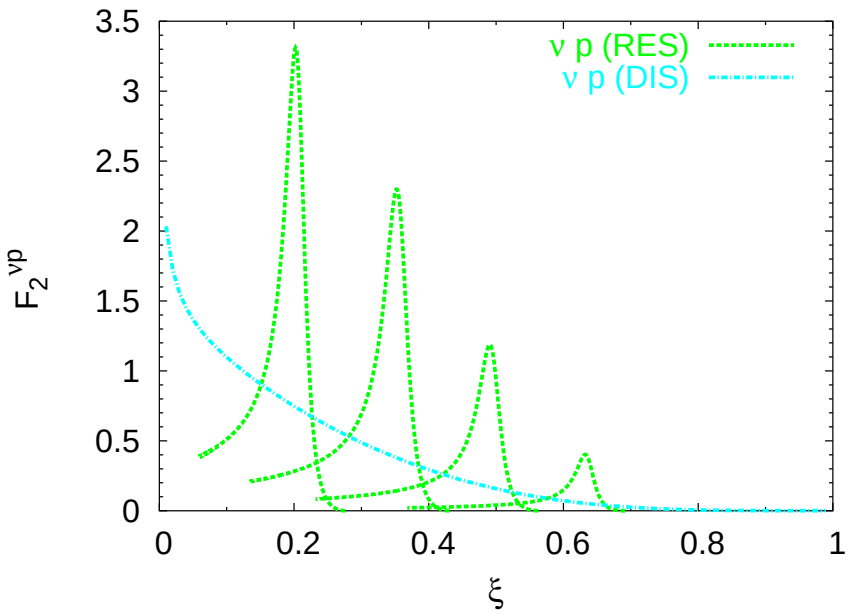
Two component duality: resonance curve agrees better with the valence–only structure function. The resonances are dual to the valence quarks, background (not shown here) to the sea quarks

$F_2^{\nu p,\nu n}$: In neutrino–nucleon scattering duality does NOT hold for proton and neutron targets separately

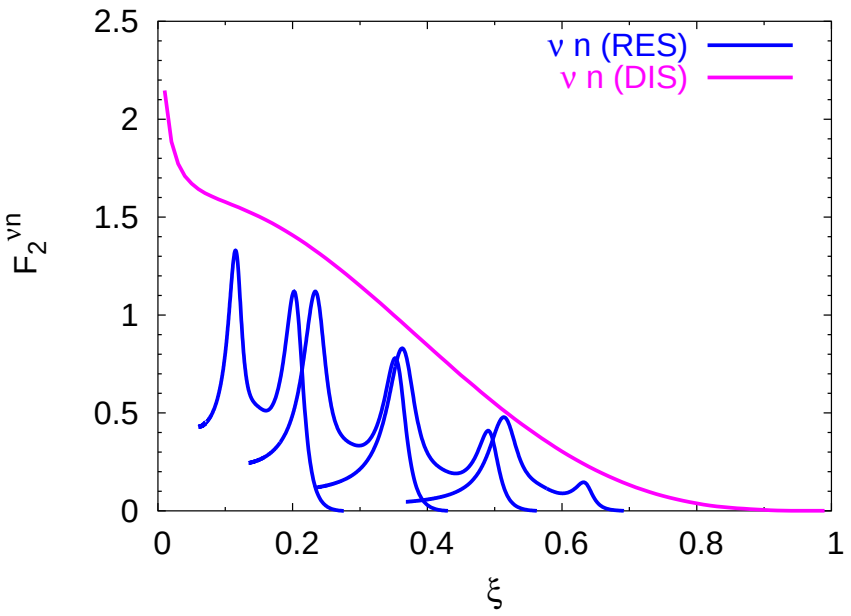
Low-lying resonances: $F_2^{\nu n(res)} < F_2^{\nu p(res)}$, DIS:$F_2^{\nu n(DIS)} > F_2^{\nu p(DIS)}$

$F_2^{\nu p(res-3/2)} = 3F_2^{\nu n(res-3/2)}$
$F_2^{\nu p(res-1/2)} \equiv 0$

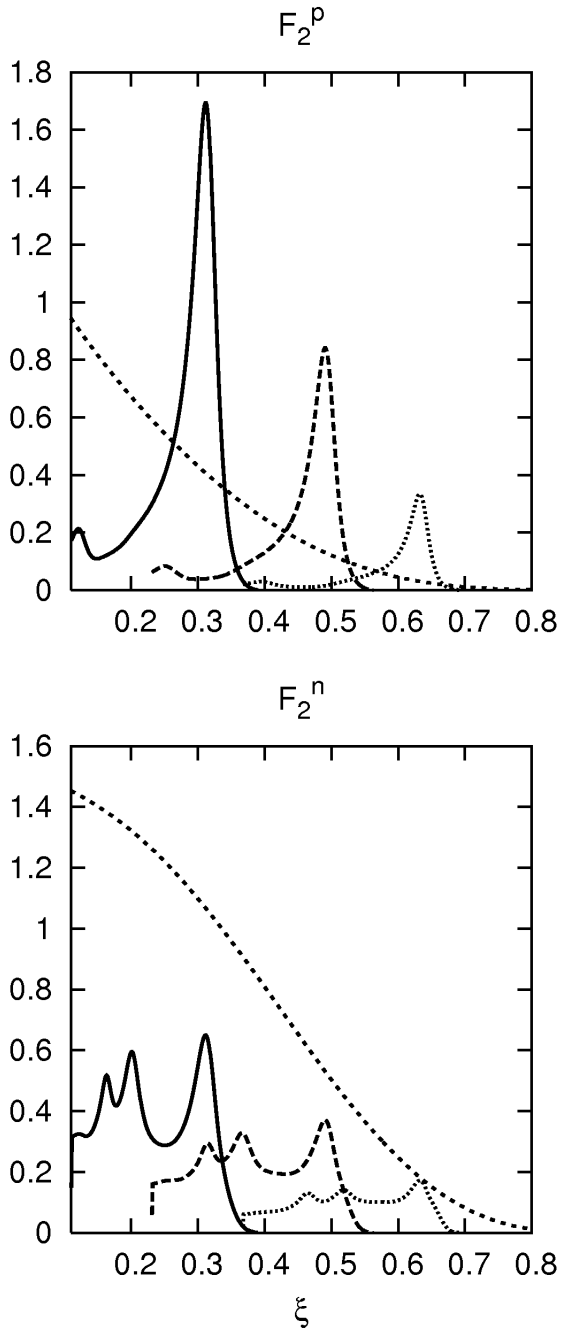
$F_2^{\nu n(res)}$: finite contributions from isispin-3/2 and -1/2 resonances



$Q^2 = 0.2,\ 0.5,\ 1.0,\ 2\ \mathrm{GeV}^2$



OL, Melnitchouk, Paschos, PRC 75



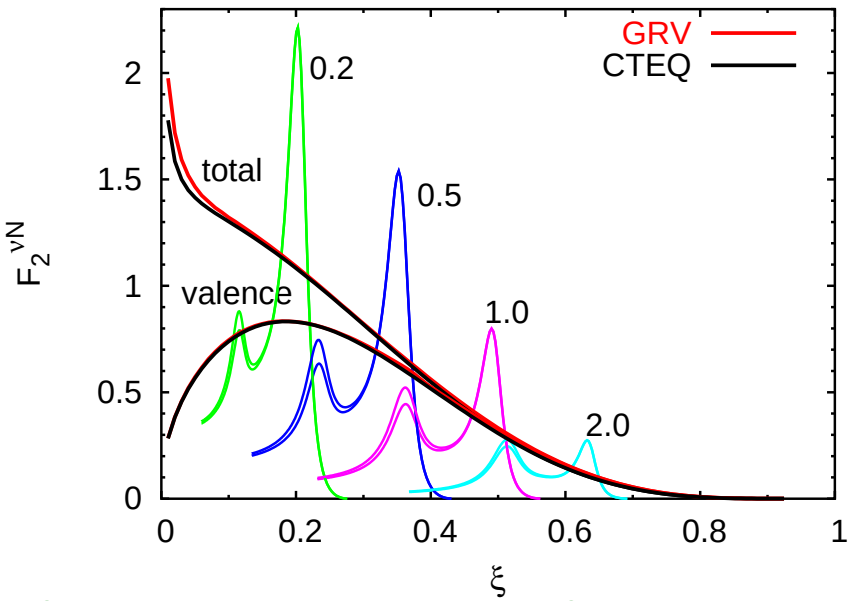
Similar results in the framework of Rein–Sehgal Model Graczyk, Juszczak, Sobczyk, Nucl Phys A781 (19 resonances included in the model)

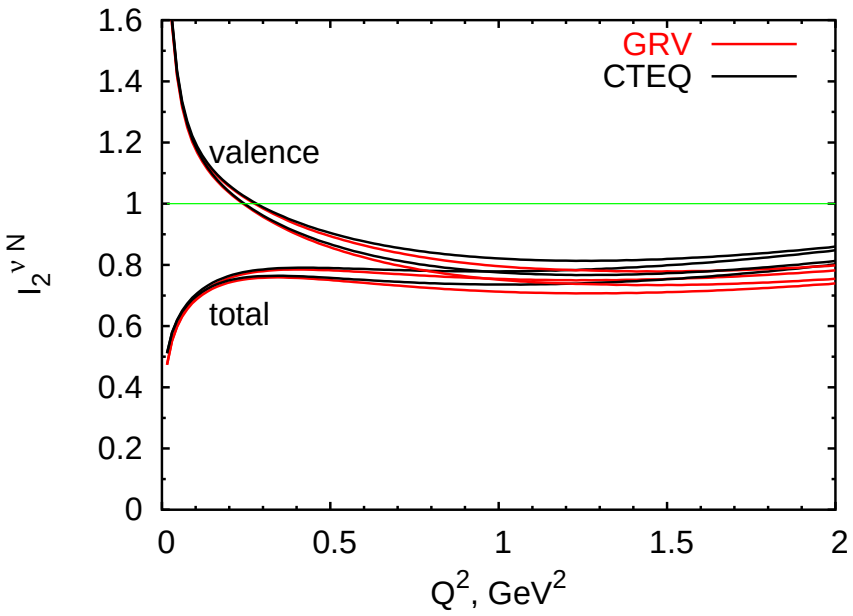
$P_{33}(1232)$,
$P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$,
$P_{33}(1600)$,
$S_{11}(1650)$, $D_{15}(1675)$, $F_{15}(1680)$

Interplay between the resonances with different isospins:

isospin-3/2 resonances give strength to the proton structure functions, while isospin-1/2 resonances contribute to the neutron structure function only

$F_2^{\nu p,\nu n}$: Duality HOLDS for the averaged structure functions





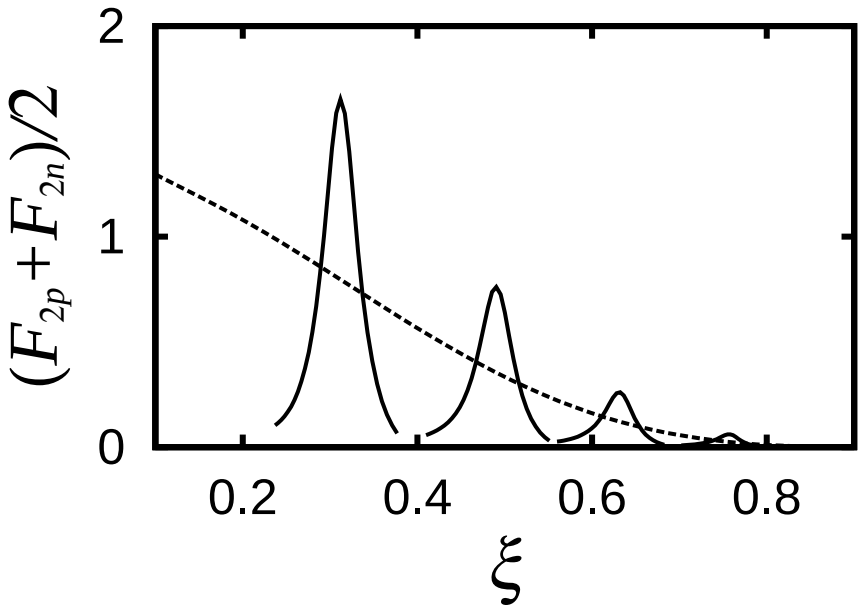
$Q^2 = 0.2,\ 0.5,\ 1.0,\ 2\ \mathrm{GeV}^2$

two curves in the second resonance region reflect the uncertainty in their axial form factors

Local duality in neutrino scattering looks better than in electron scattering:

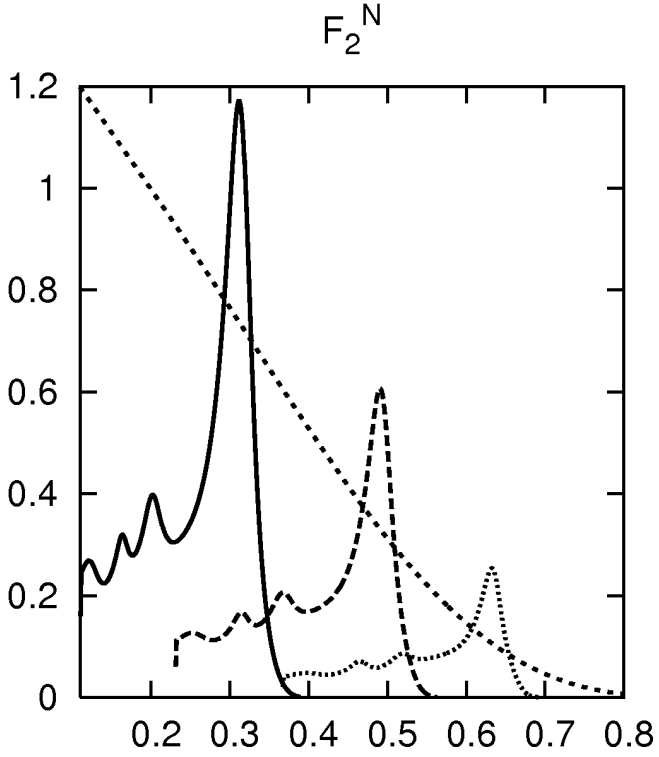
the ratio does not grow appreciably with Q^2

Similar results in Sato-Lee model
Matsui,Sato,Lee, PRC 72
($P_{33}(1232)$ resonance considered so far)



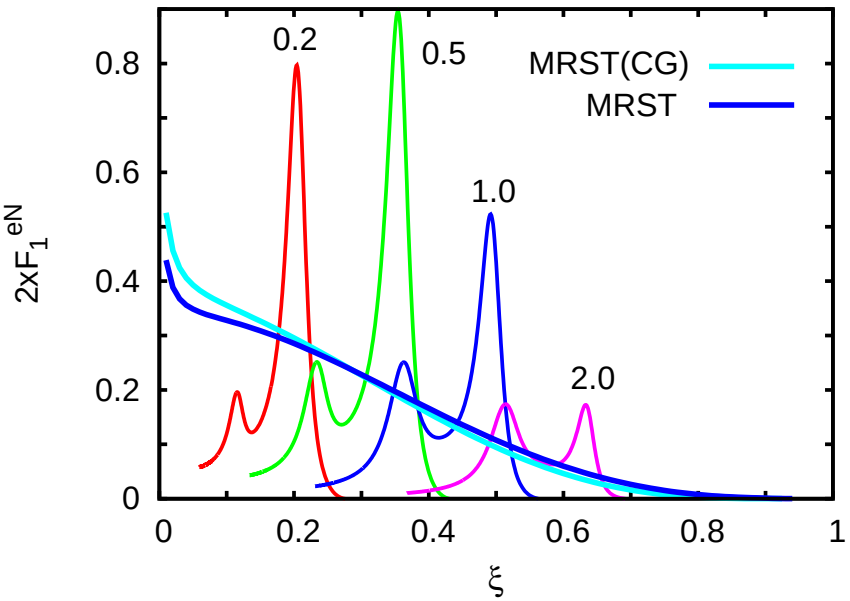
$Q^2 = 0.4,\ 1.0,\ 2,\ 4\ \mathrm{GeV}^2$

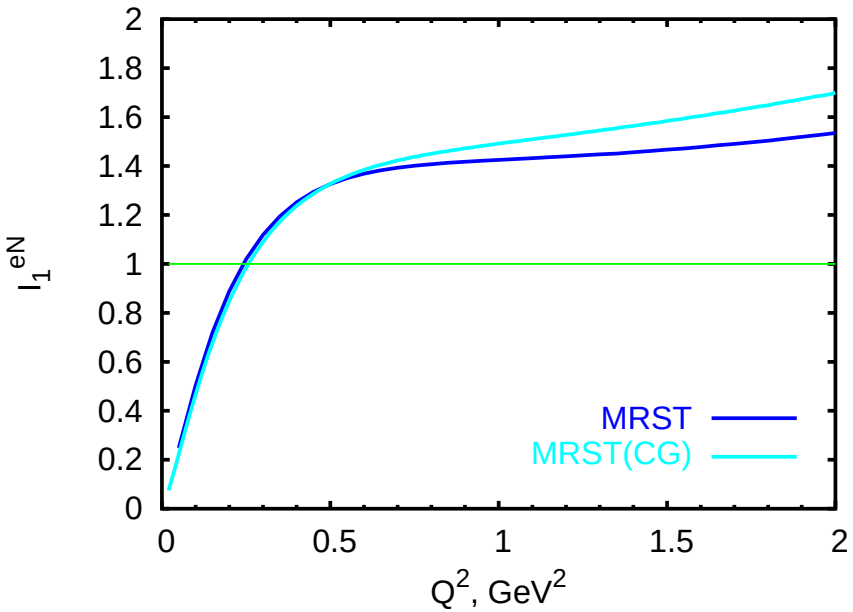
and Rein–Sehgal model
Graczyk, Juszczak, Sobczyk, Nucl Phys A781
(19 resonances included in the model)



$Q^2 = 0.4,\ 1.0,\ 2\ \mathrm{GeV}^2$

$2xF_1^{eN}$ in electron–nucleon scattering





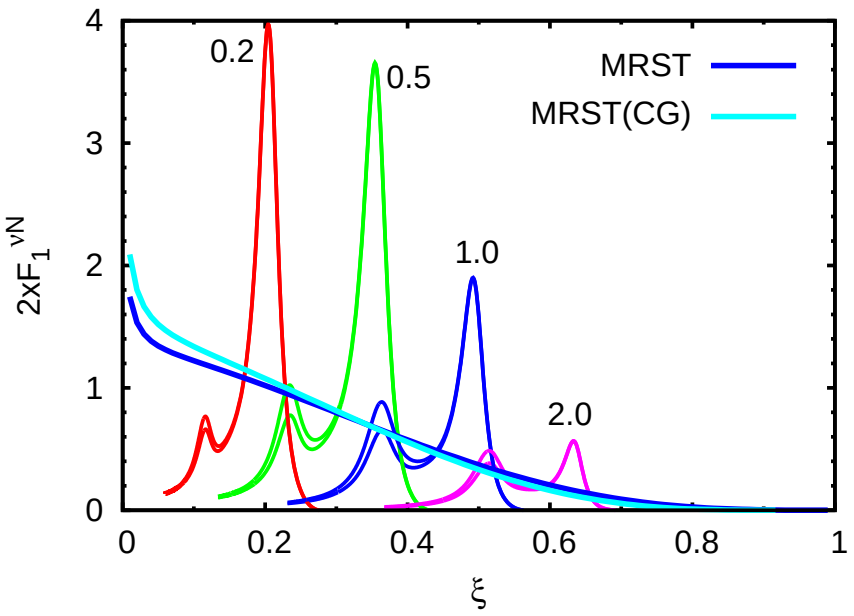
$$2xF_1^{DIS} = \left(1 + \frac{4m_N^2 x^2}{Q^2}\right) F_2 - F_L$$

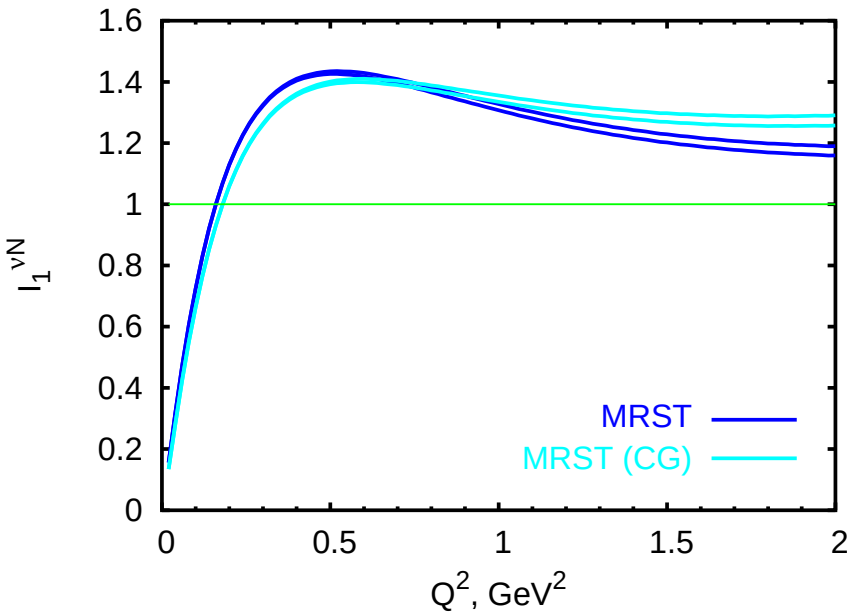
$$I_1(Q^2) = \frac{\int_{\xi_{\min}}^{\xi_{\max}} d\xi \; 2xF_1^{(\mathrm{res})}(\xi, Q^2)}{\int_{\xi_{\min}}^{\xi_{\max}} d\xi \; 2xF_1^{(\mathrm{LeadingTwist})}(\xi, Q^2)} \; ,$$

$2xF_1^{DIS(CG)} = F_2$ (Callan-Gross relation)

Target mass correction have a large effect on $2xF_1$ and would tend to increase the DIS leading twist function and hence reduce the ratio (see Steffens, Melnitchouk, PRC73)

$2xF_1^{\nu N}$ in neutrino–nucleon scattering





$$2xF_1^{DIS} = \left(1 + \frac{4m_N^2 x^2}{Q^2}\right) F_2 - F_L$$

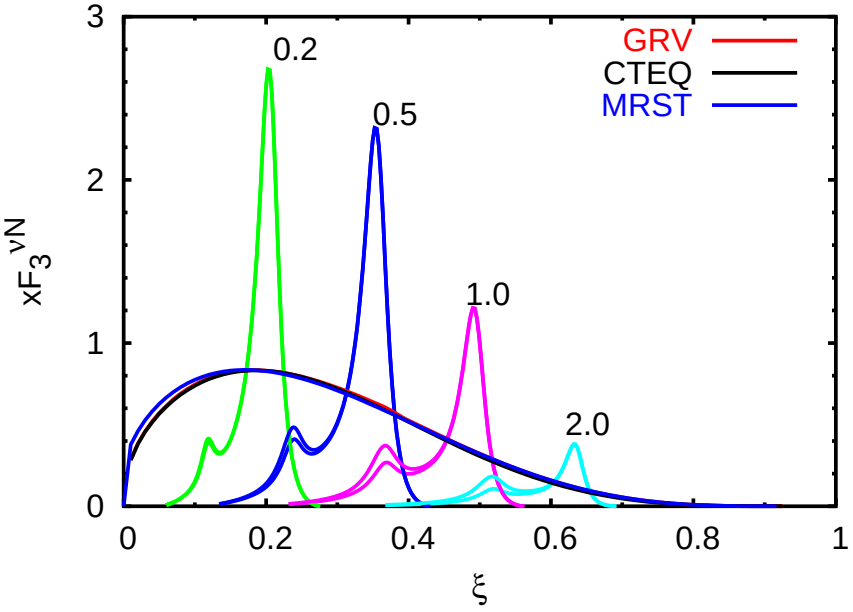
$$I_1^{\nu N}(Q^2) = \frac{\int_{\xi_{\min}}^{\xi_{\max}} d\xi \; 2xF_1^{(\mathrm{res})}(\xi, Q^2)}{\int_{\xi_{\min}}^{\xi_{\max}} d\xi \; 2xF_1^{(\mathrm{LeadingTwist})}(\xi, Q^2)} \, ,$$

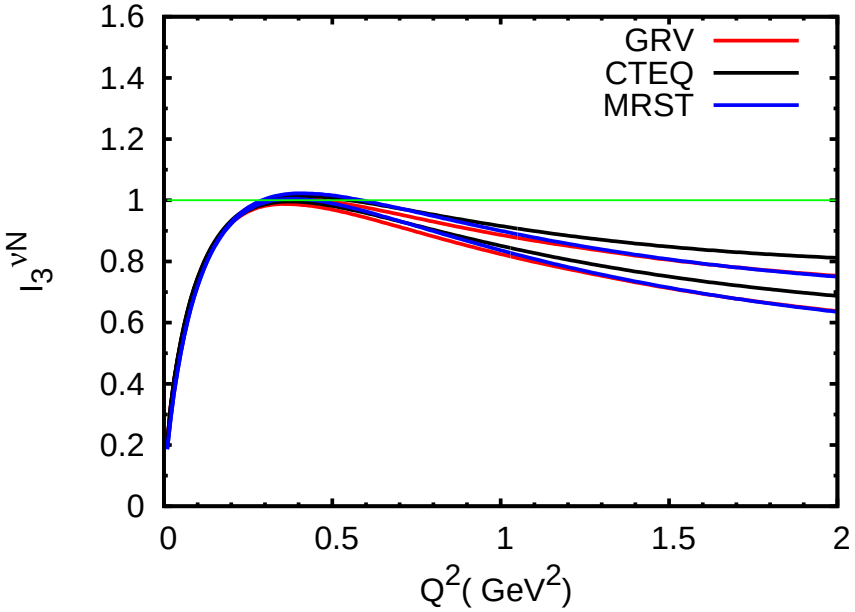
$2xF_1^{DIS(CG)} = F_2$ (Callan-Gross relation)

The accuracy of local duality is about the same as for electron scattering

Target mass correction have a large effect on $2xF_1$ and would tend to increase the DIS leading twist function and hence reduce the ratio

Duality for $xF_3^{\nu N}$ structure function





$$I_3^{\nu N}(Q^2) = \frac{\int_{\xi_{\rm min}}^{\xi_{\rm max}} d\xi \; xF_3^{(\rm res)}(\xi, Q^2)}{\int_{\xi_{\rm min}}^{\xi_{\rm max}} d\xi \; xF_3^{(\rm LeadingTwist)}(\xi, Q^2)} \; ,$$

$F_3^{\nu N}$ is generally more sensitive to the choice of the axial form factors

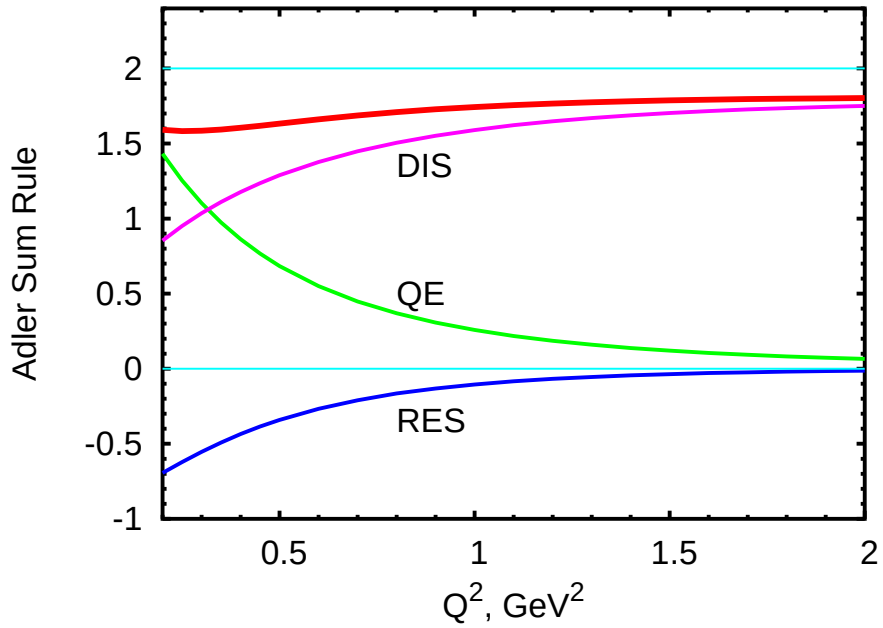
The accuracy of local duality about 30% is consistent with the estimated uncertainly of the axial form factors

Adler sum rule

$$\underbrace{\left[g_{1V}^{(QE)}\right]^2 + \left[g_{1A}^{(QE)}\right]^2 + \left[g_{2V}^{(QE)}\right]^2 \frac{Q^2}{2M^2}}_{\text{QE contribution}} + \int d\nu \underbrace{\left[W_2^{\nu n}(Q^2,\nu) - W_2^{\nu p}(Q^2,\nu)\right]}_{\text{RES+DIS}} = 2$$

RES: integration of resonance W_2 over ν region, corresponding to the $\xi_{min}(Q^2) < \xi < \xi_{max}(Q^2)$

DIS: integration of the leading twist F_2 over the remaining ξ interval $0 < \xi < \xi_{min}$, corresponding to $W > 1.6$ GeV

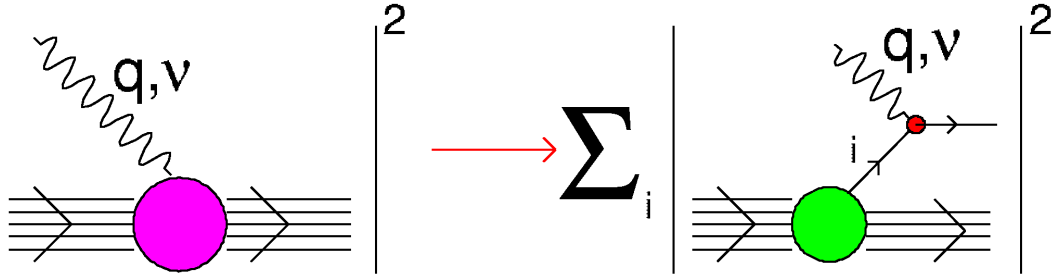


High-Q^2 limit (DIS)

$$\int d\nu \left[W_2^{\nu n}(Q^2,\nu) - W_2^{\nu p}(Q^2,\nu)\right] \equiv \int dx \frac{F_2^{\nu n}(Q^2,\nu) - F_2^{\nu p}(Q^2,\nu)}{x} = 2$$

Impulse approximation and nuclear shell model

picture from Benhar et al, PR D72



— target nucleus is seen by the probe lepton as a collection of individual nucleons

— the particles produced at the interaction vertex and the recoiling (A-1) system evolve independently on

— struck protons and neutrons in the nucleus are off-mass-shell with the binding (removal) energy of the corresponding nuclear shell

$$\frac{d\sigma^{C_6^{12}}}{dQ^2dW} = \int d^3p \left[2\frac{d\sigma_{\nu p}\,|_{1s^{1/2}}}{dQ^2dW} + 4\frac{d\sigma_{\nu p}\,\Big|_{1p^{3/2}}}{dQ^2dW} + 2\frac{d\sigma_{\nu n}\,|_{1s^{1/2}}}{dQ^2dW} + 4\frac{d\sigma_{\nu n}\,\Big|_{1p^{3/2}}}{dQ^2dW} \right]$$

Off-mass-shell effects of the struck nucleon are revealed in the kinematics and in the bound state propagator

Bound-state propagator for the struck nucleon

single nucleon bound-state relativistic wave functions calculated within the Walecka model in the Hartree approximation

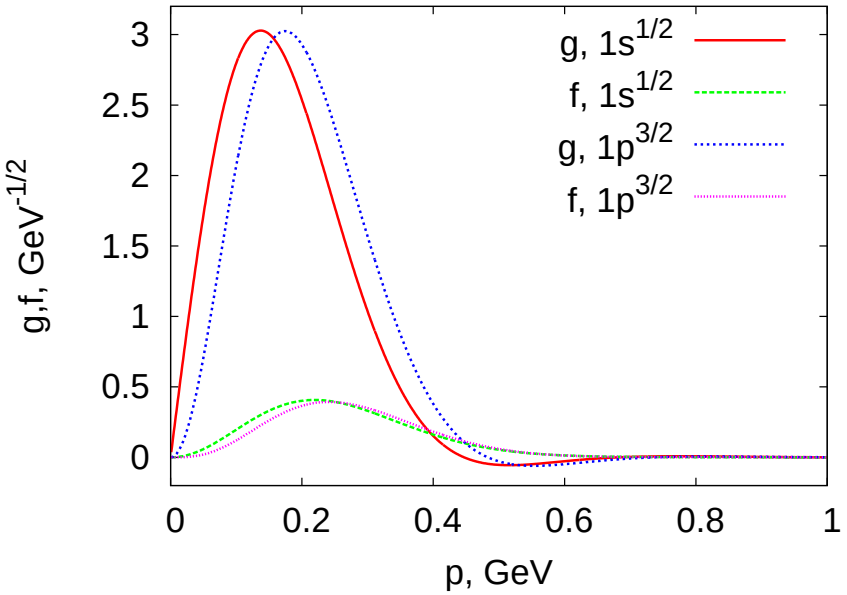
$$u_\alpha \equiv u_{n\kappa m}(\vec{p}) = \begin{bmatrix} g(|\vec{p}|)\mathcal{Y}_{\kappa m} \\ -f(|\vec{p}|)\mathcal{Y}_{-\kappa m} \end{bmatrix}$$

$$\frac{1}{2j+1}\sum_m u_{\alpha,m}(p)\bar{u}_{\alpha,m}(p) = (\not{p}_\alpha + m_\alpha)$$

$$m_\alpha = \frac{\pi}{|\vec{p}|^2}\left[g_\alpha^2(|\vec{p}|) - f_\alpha^2(|\vec{p}|)\right]$$

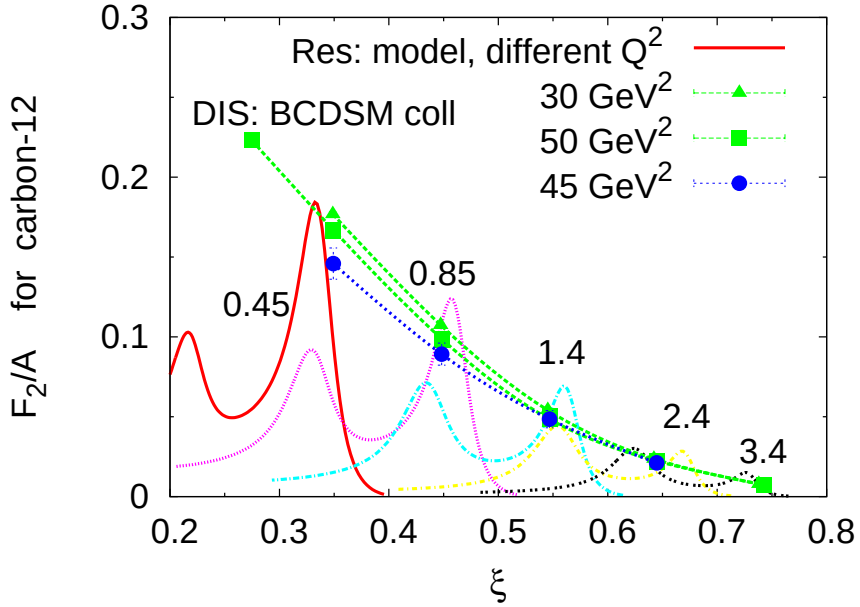
$$E_\alpha = \frac{\pi}{|\vec{p}|^2}\left[g_\alpha^2(|\vec{p}|) + f_\alpha^2(|\vec{p}|)\right]$$

$$\vec{p}_\alpha = \frac{\pi}{|\vec{p}|^2}\left[2g_\alpha^2(|\vec{p}|)f_\alpha^2(|\vec{p}|)\frac{\vec{p}}{|\vec{p}|}\right]$$



Duality for carbon nucleus

For nuclei, the Fermi motion and other medium effects broaden resonances, thus performing averaging

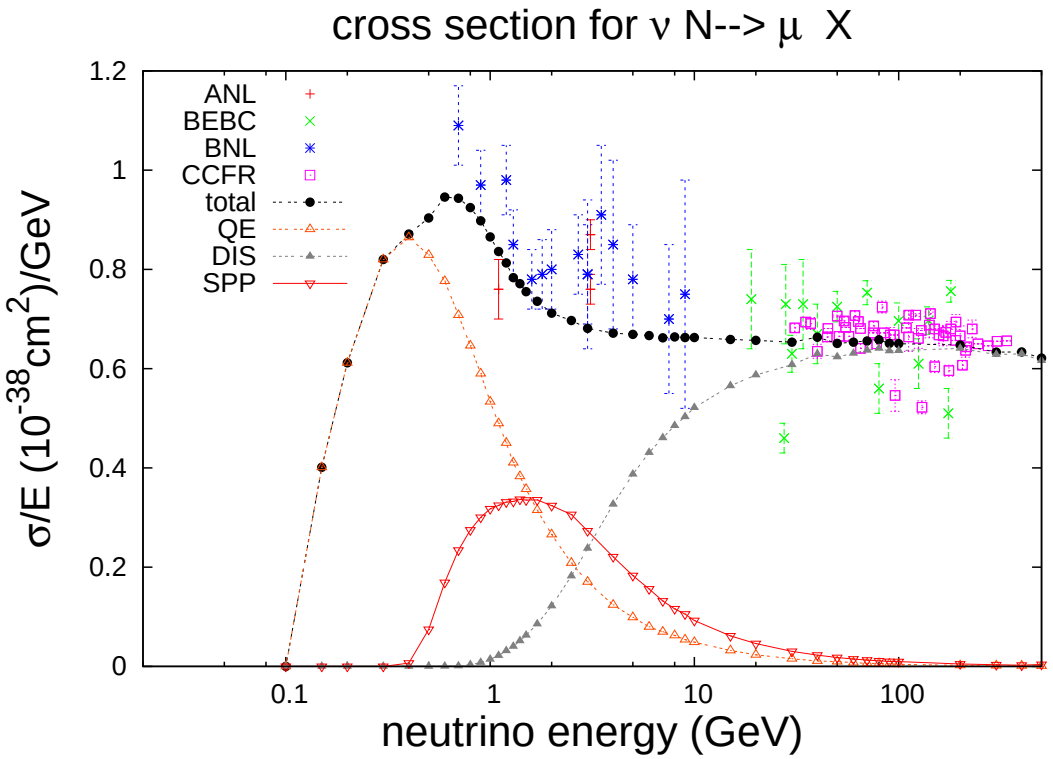


Resonance structure functions: isobar model with phenominological form factors OL,Paschos, PRD 71, 74 includes the first four low-lying baryon resonances $P_{33}(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$

Preliminary!

Global fit of the neutrino–nucleus cross section ...

Nowak et al (Wroclaw),
Acta Physica Polonica B 37



similar approach Naumov et al. (Dubna), hep-ph/0511308, Phys.Atom.Nucl.69:1857-1871,2006

"How to sum contributions into the total charged-current neutrino-nucleon cross section?"

The exclusive and inclusive (DIS) contributions are of the same order of magnitude within the few-GeV energy region. Thus, to avoid double counting, the phase space of the RES and DIS contributions have to be scratched by the conditions $W < W_{RES}$ cut and $W > W_{DIS}$ cut, the choice of these cutoffs is usually rather subjective.

... relies on duality

Results:

Dubna group:
$W_{RES} = W_{DIS} = 1.5\ \mathrm{GeV}$ means that only $P_{33}(1232)$ and $P_{11}(1440)$ resonances are considered exclusively (with parameters from Rein-Sehgal model), the contribution of higher resonances is simulated by DIS.
This is only possible if duality works, the success of the fit means that duality works !

Wroclaw group:
P_{33} is considered exclusively with phenomenological form factors, smooth transition to DIS single pion channel in the region of $1.3 < W < 1.6\ \mathrm{GeV}$. DIS contributions for $W > 1.6\ \mathrm{GeV}$
This is only possible if duality works, the success of the fit means that duality works !

Summary

- For proton and neutron targets separately duality holds in electron-nucleon scattering, but does not hold in neutrino–nucleon scattering
- Global and local duality hold for the average over proton and neutron targets. The accuracy is better for neutrino–nucleon reactions than for the electron–nucleon
- The degree to which the local duality is valid is high for F_2 structure functions. The results are similar within different models of resonance production and different parametrizations of DIS structure function.
- For $2xF_1$ global duality holds and local duality is fair for both electron- and neutrino- nucleon reactions. The quantitative agreement for local duality would require a more elaborate treatment of the target mass corrections for DIS structure functions
- For xF_3 the global duality holds, local duality is sensitive to the Q^2 behaviour of the resonance axial form factors. The accuracy of duality about 30% is consistent with the estimated uncertainly of the form factors.
- Adler Sum Rule holds with the 10% accuracy
- Duality is natural: quite often one relies on duality without making explicit mention of it