Neutrino induced pion production reaction 2

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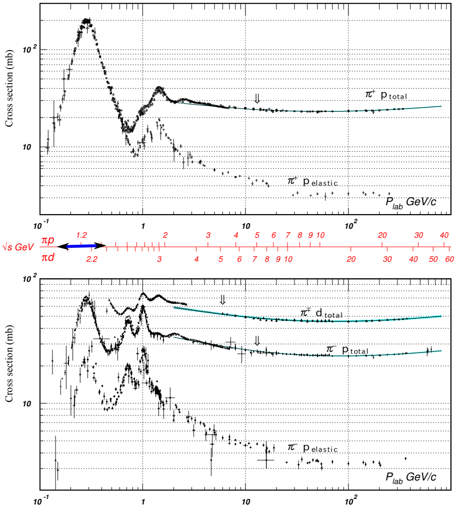
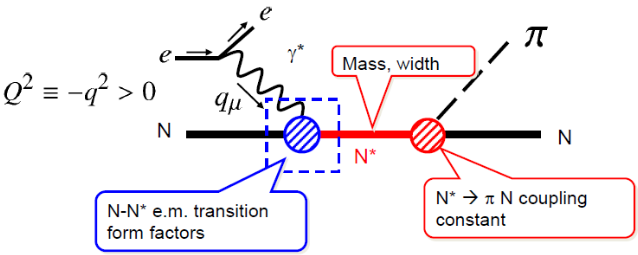
Oct. 27 2014

π-nucleon scattering and excited nucleons



PDG

$$\sqrt{s} = W = \sqrt{(p_N + k_\pi)^2}$$



πN scattering

↓

$N(\gamma, \pi)N\ (Q^2 = 0)$

↓

$N(e, e'\pi)$ Transtion form factor

Contents

- $N\Delta$ electromagnetic transition form factor

$\Delta(1232)$

Total cross seciton of $\pi^- p, \pi^+ p$

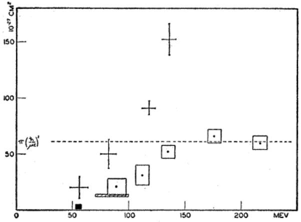
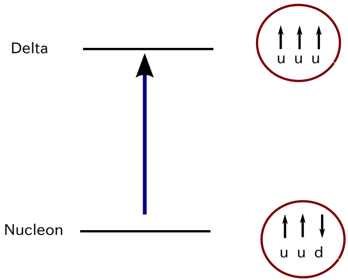


FIG. 1. Total cross sections of negative pions in hydrogen (sides of the rectangle represent the error) and positive pions in hydrogen (arms of the cross represent the error). The cross-hatched rectangle is the Columbia result. The black square is the Brookhaven result and does not include the charge exchange contribution.

H. L. Anderson et al. Phys. Rev. 85 934,936 (1952)

- First resonance to be discovered.
- $J^P = 3/2^+$,$I = 3/2$, p-wave πN resonance
- $\delta_{P33} = \pi/2$ at $W = 1.232$GeV (Pole at $1.210 - 0.05i$GeV)

Naive picture of Delta excitation in simple constituent quark model



- Magnetic dipole(Vector) $G_M i\vec{S} \times \vec{q}\, T^i \leftrightarrow \mu_N^V i\vec{\sigma} \times \vec{q}\tau^i$
  'Gamow-Teller'(Axial vector) $G_A \vec{S}\, T^i \leftrightarrow g_A \vec{\sigma}\tau^i$

(transition spin, isospin)

$$< \Delta(3/2, m_\Delta)|S_m|N(1/2, m_N) >= (1/2, m_N, 1, m|3/2 m_\Delta)$$

$$< \Delta(3/2, t_\Delta)|T_t|N(1/2, t_N) >= (1/2, t_N, 1, t|3/2 t_\Delta)$$

$N\Delta$ electromagnetic transition form factor

Parametrization of electromagnetic (iso-vector vector current) $N\Delta$ transition form factors

$$< \Delta(p_\Delta)|\vec{V}^\mu(q)|N(p) >= \bar{u}^\nu(p_\Delta)\Gamma_{\mu\nu}\vec{T}u(p)$$

$$\begin{aligned}
\Gamma^V_{\mu\nu} &= \frac{m_\Delta + m_N}{2m_N}\frac{1}{(m_\Delta + m_N)^2 - q^2} \\
&\times [(G_M - G_E)3\epsilon_{\mu\nu\alpha\beta}P^\alpha q^\beta + G_E i\gamma_5 \frac{12}{(m_\Delta - m_N)^2 - q^2}\epsilon_{\mu\lambda\alpha\beta}P^\alpha q^\beta \epsilon^\lambda{}_{\nu\alpha\delta}p^\gamma_\Delta q^\delta \\
&+ G_C i\gamma_5 \frac{6}{(m_\Delta - m_N)^2 - q^2} q_\mu (q^2 P_\nu - q\cdot P q_\nu)]
\end{aligned}$$

Momentum of nucleon(delta) p (p_Δ)

$$< \Delta^+(p_\Delta)|\vec{V}^\mu(q)|p(p) >= \bar{u}^\nu(p_\Delta)\Gamma_{\mu\nu}u(p)$$

$$\Gamma^V_{\mu\nu} = [\frac{C^V_3}{m_N}(g_{\mu\nu} \not{q} - q_\mu\gamma_\nu) + \frac{C^V_4}{m^2_N}(g_{\mu\nu}(q\cdot p_\Delta) - q_\mu p_{\Delta\nu}) + \frac{C^V_5}{m^2_N}(g_{\mu\nu}(q\cdot p) - q_\mu p_\nu)]$$

T. Sato, T. -S. H. Lee, PRC 63, 055201 (2001)
E. Hernandez, J.Nieves, M. Valverde, PRD76, 033005 (2007)

Parametrization of electromagnetic (iso-vector vector current) $N\Delta$ transition form factors
Simplified expression at Delta at rest, $W = m_\Delta$

$$< \Delta|J_{em}\cdot\epsilon|N >= F\frac{e}{2m_N}T_3[iG_M(q^2)\vec{S}\times\vec{q}\cdot\vec{\epsilon}+G_E(q^2)(\vec{S}\cdot\vec{\epsilon}\vec{\sigma}\cdot\vec{q}+\vec{S}\cdot\vec{q}\vec{\sigma}\cdot\vec{\epsilon})+\frac{G_C(q^2)}{m_\Delta}\vec{S}\cdot\vec{q}\vec{\sigma}\cdot\vec{q}\epsilon_0$$

At $W = 1.232$Gev ($e^{i\delta_{\pi N}} = e^{i\pi/2} = i$),

$$\begin{aligned} G_M &= NIm(M_{1+}^{3/2}) \quad Magnetic\ \ Dipole \\ G_E &= NIm(E_{1+}^{3/2}) \quad Electric\ \ Quadrupole \\ G_C &= NIm(S_{1+}^{3/2}) \quad Electric\ \ Quadrupole(timecomponent) \end{aligned}$$

- G_M, G_E, G_C are directly related to the pion photoproduction amplitude at resonance energy.
- G_M: main term
- G_E, G_C : related to deformation of $N\Delta$ transition density.

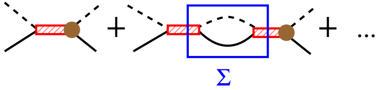
Breit-Wigner form

Breit-Wigner formula for resonance amplitude

$$\frac{\sqrt{\Gamma_\pi/2}\sqrt{\Gamma_\gamma/2}}{W-M+i\frac{\Gamma}{2}}$$

Δ is unstable particle ($\Delta \to \pi N$). Decay width(Γ) is by using Fermi's Golden rule,

$$\Gamma = 2\pi \sum_q \delta(W - E_N(q) - E_\pi(q)) | < \pi N|V|\Delta > |^2 = 2Im(\Sigma)$$



$$< \pi N|V|\Delta > [\frac{1}{W-M^0} + \frac{1}{W-M^0}\Sigma\frac{1}{W-M^0} + ...] < \Delta|V|\pi N >$$

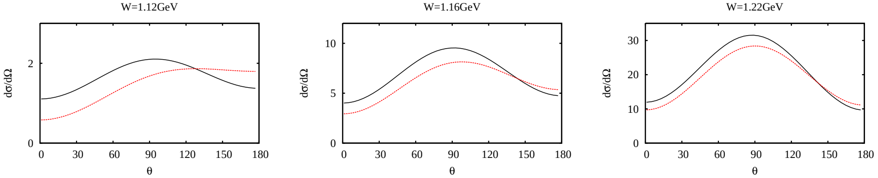
$$\to \frac{1}{W-M^0-\Sigma(W)} \quad \text{where } \Sigma = \sum_q \frac{< \Delta|V|\pi N >< \pi N|V|\Delta >}{W - E_N(q) + E_\pi(q) + i\epsilon}$$

The Breit-Wigner formula satisfies unitarity. The resonance amplitude of pion photoproduction near resonance energy is (M, Γ are constant.)

$$\frac{\gamma_\pi \gamma_\gamma}{W-M+i\Gamma/2}$$

Importance of phase when we extract multipole amplitudes from 'data'

Differential cross section of $\gamma + p \to \pi^0 + p$



Solid(Black): Full   Dash(red) : Born+Res

Unitaritization of pion photoproduction amplitude

Our pion photoproduction amplitude consists of non-resonant tree-diagram and Brei-Wigner type resonant term.

$$t_{tree} + \frac{\gamma_\pi \gamma_\gamma}{W - M + i\Gamma/2}$$

How to unitarize?

- non-resonant partial waves

$$t_{\pi,\gamma} = e^{i\delta_{\pi N}} t_{tree}$$

- resonant partial waves ($P33$)
  Introduce $\delta_{bg}, \delta_{res}$ and tune them so that the phase of whole amplitude is given by $\delta_{\pi N}(P33)$ (recipe of MAID)

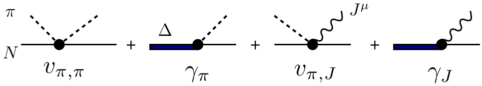
$$t_{tree} e^{i\delta_{bg}} + \frac{\gamma_\pi \gamma_\gamma}{W - M + i\Gamma/2} e^{i\delta_{res}} \rightarrow e^{i\delta_{\pi N}}$$

Another approach: dynamical model of πN and (γ, π) reactions.

Reaction model of pion production

Hamiltonian of π, N, Δ system and current $J^\mu = J^\mu_{em}, V^\mu, A^\mu$

$H = H_0 +$



Solve Lippman-Schwinger equation : Fock space $|\pi N>, |\gamma N>$, $|\Delta>$

$$T(W) = V + V\frac{1}{W - H_0 + i\epsilon}T$$

T. Sato, T.-S. H. Lee, J. Phys. G 36, 073001(2009)
T. Sato, T. -S. H. Lee, PRC 63, 055201 (2001)
T. Sato, T. -S. H. Lee, PRC 52, 2660 (1996)

scattering amplitude

Formal solution:

$$T_{\pi,\pi}(W) = t_{\pi,\pi}(W) + \frac{\tilde{\gamma}_\pi(W)\tilde{\gamma}_\pi(W)}{W - m^0_\Delta - \Sigma(W)}$$

Full amplitude can be written as sum of 'non-resonant' amplitude and 'resonant' amplitude.

$$\begin{aligned}
\text{non-resonant amplitude} \;\; t_{\pi,\pi}(W) &= v_{\pi,\pi} + v_{\pi,\pi}\frac{1}{W - H_0 + i\epsilon}t_{\pi,\pi}(W) \\
\text{dressed } \Delta N\pi \text{ vertex} \;\; \tilde{\gamma}_\pi &= (1 + t_{\pi,\pi}(W)\frac{1}{W - H_0 + i\epsilon})\gamma_\pi \\
\text{self-energy} \;\; \Sigma(W) &= \gamma_\pi \frac{1}{W - H_0 + i\epsilon}\tilde{\gamma}_\pi(W)
\end{aligned}$$

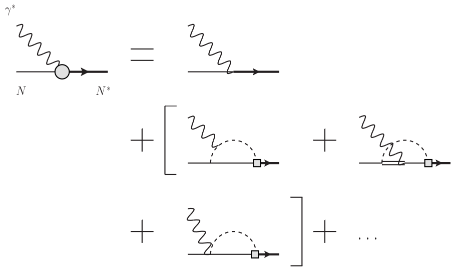
- Non-resonant channel: $t_{\pi,\pi}$ carries whole πN phase.
- Resonant channel: non-resonant phase generated from $t_{\pi,\pi}$, which enter in resonance amplitude so that the phase of whole amplitude is $\delta_{\pi N}$. (Note $W(\delta_{\pi N} = \pi/2) \neq Re(M(pole))$

Electroweak pion production amplitude (first order in em/weak interaction J^μ)

$$T_{\pi,J}(W) = t_{\pi,J}(W) + \frac{\tilde{\gamma}_\pi(W)\tilde{\gamma}_J(W)}{W - m_\Delta^0 - \Sigma(W)}$$

$$\text{non-resonant amplitude } t_{\pi,J}(W) = [1 + t_{\pi,\pi}(W)\frac{1}{W - H_0 + i\epsilon}]v_{\pi,J}$$

$$\text{dressed } \Delta N J \text{ vertex } \tilde{\gamma}_J = \gamma_J + t_{\pi,\pi}(W)\frac{1}{W - H_0 + i\epsilon}v_{\pi,J}$$



Numerical task

1. Partial wave expansion of non-resonant interactions.

2. Solve integral equation for each partial wave

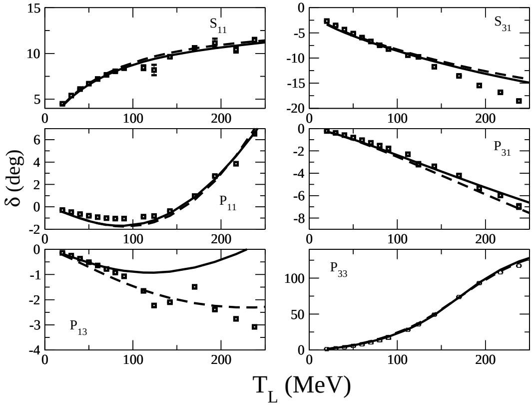
$$< k'|t^{\alpha}_{\pi,\pi}(W)|k > = < k'|v^{\alpha}_{\pi,\pi}|k > + \int_0^{\infty} dq q^2 \frac{< k'|v^{\alpha}_{\pi,\pi}|q >< q|t^{\alpha}_{\pi,\pi}(W)|k >}{W - E_N(q) - E_{\pi}(q) + i\epsilon}$$

Integral equation can be solved by matrix inversion method.

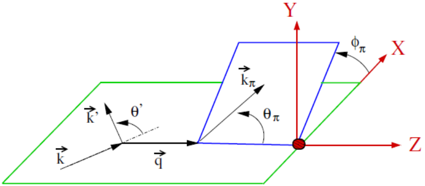
Note:Separate on-shell delta function part, and apply subtraction method to manage principal value integral

$$\frac{1}{W - E_N(q) - E_{\pi}(q) + i\epsilon} = \frac{P}{W - E_N(q) - E_{\pi}(q)} - i\pi\delta(W - E_N(q) - E_{\pi}(q)),$$

Some results of the model πN phase shift

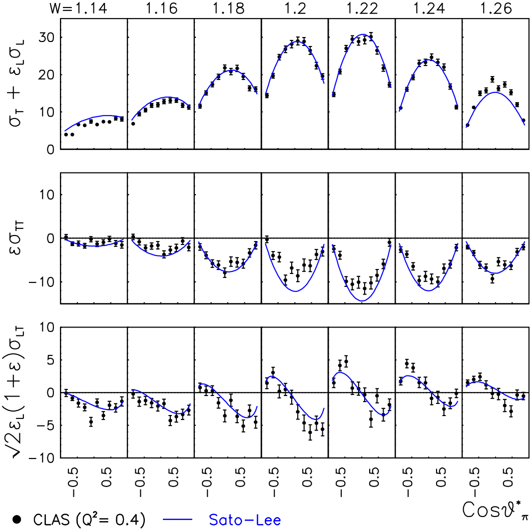


$$
\begin{aligned}
\frac{d\sigma}{dE_e d\Omega_e d\Omega_\pi} &= \Gamma \frac{d\sigma}{d\Omega_\pi} \\
\frac{d\sigma}{d\Omega_\pi} &= \frac{d\sigma_T}{d\Omega_\pi} + \epsilon \frac{d\sigma_L}{d\Omega_\pi} \\
&+ \cos\phi_\pi \sqrt{2\epsilon(1+\epsilon)} \frac{d\sigma_{LT}}{d\Omega_\pi} + \cos 2\phi_\pi \epsilon \frac{d\sigma_{TT}}{d\Omega_\pi}
\end{aligned}
$$

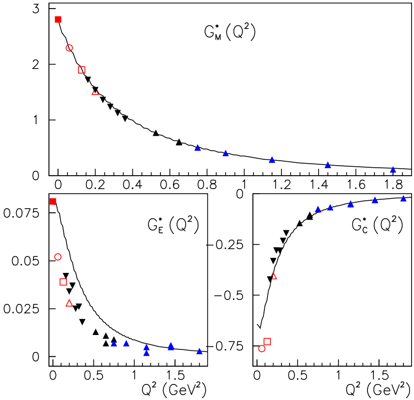


E. Hernandez, J.Nieves, M. Valverde, PRD76, 033005 (2007)

Pion electroproduction

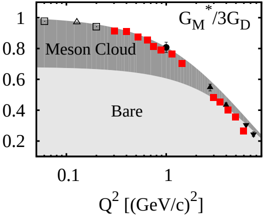


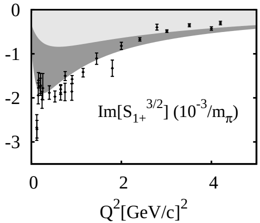
Extracted transition form factors(including vertex correction)

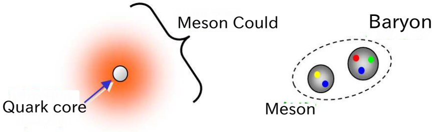


Sato,Lee PRC 63 (2001) 055201
B. Julia-diaz, T.-S. H. Lee, C. Smith, T. Sato PRC75(2007)015205
Extensive data from JLab, Mainz, Graal, MIT-Bates, LEGS of pion photo and electroproduction.

Interpretation of $N\Delta$ transition form factors







$|N^*\rangle = |qqq\rangle + |\text{m.c.}\rangle$ $\qquad$ $|N^*\rangle = |MB\rangle$

Neutrino reaction in the Delta resonance region

- Most important pion production mechanism below a few GeV E_ν region.
- Pion produciton of vector current is well studied around delta resonance region for $0 < Q^2 < 2GeV^2$.
- Study $N\Delta$ Axial vector current from neutrino reaction (Non-resonant + Delta resonant)

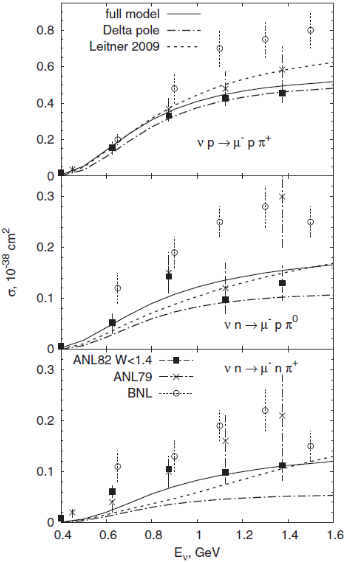
Theoretical analyses

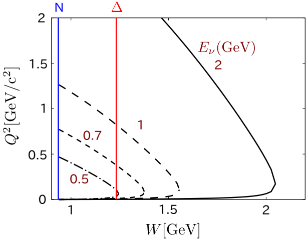
T. R. Hemmert, B. Holstein, N. C. Mukhopadhyay, PRD51,158 (1995)
O. Lalakulich, E. A. Paschos, PRD71, 074003 (2005)
M. O. Wascko (MinoBooNE Collaboration), Nucl. Phys. B, Proc. Suppl. **159**, 50 (2006).
E. Hernandez, J. Nieves, M. Valverde, PRD76, 033005 (2007)
T. Sato, D. Uno, T. -S. H. Lee, PRC67, 065201 (2003)
..

Data on neutrino reaction on proton oand deuteron

BNL T. Kitagaki et al., PRD 34, 2554 (1986).
ANL S. J. Barish et al. PRD19, 2521 (1979),G. M. Radecky et al, PRD 25, 1161 (1982)
BEBC P. Allen et al. NPB 176, 269 (1980), NPB 343, 285 (1990)

- BNL vs ANL
- rescattering and bound nucleon (deuteron reaction)





From O. Lalakulich, T. Leitner, O. Buss, U. Mosel, PRD 82, 093001 (2010) Fig. 5

Axial Vector current

Parametrization of Axial vector current (Charged current)

$$
\begin{aligned}
< \Delta^{+}(p_\Delta)|A_1^\mu + iA_2^\mu|n(p) > &= \bar{u}(p_\Delta)_\nu [\frac{C_3^A}{m_N}(g^{\mu\nu} \not{q} - q^\nu \gamma^\mu) \\
&+ \frac{C_4^A}{m_N^2}(g^{\mu\nu}(q\cdot p_\Delta) - q^\nu {p_\Delta}^\mu) + C_5^A g^{\nu\mu} + \frac{C_6^A}{m_N^2} q^\mu q^\nu] u(p)
\end{aligned}
$$

C. H. Lewellyn Smith, Phys. Rep. 3C, 261 (1972)

E. Hernandez, J. Nieves, M. Valverde, PRD76, 033005 (2007)

$$
\begin{aligned}
< \Delta(p_\Delta)|A_i^\mu|N(p) > &= \bar{u}(p_\Delta)_\nu [d_1 g^{\mu\nu} + \frac{d_2}{m_N^2}(p_\delta + p)_\alpha (q^\alpha g^{\mu\nu} - q^\nu g^{\alpha\mu}) \\
&- \frac{d_3}{m_N^2} p^\nu q^\mu + i\frac{d_4}{m_N^2}\epsilon^{\mu\nu\alpha\beta}(p_\Delta + p)_\alpha q_\beta \gamma_5] T_i u(p)
\end{aligned}
$$

T. R. Hemmert, B. Holstein, N. C. Mukhopadhyay, PRD51,158 (1995)

T. Sato, D. Uno, T. -S. H. Lee, PRC67, 065201 (2003)

- Leading term is C_5^A, d_1
- Pion pole term C_6^A, d_3
- $C_3^A = 0, C_4^A = -C_5^A/4$
- $d_4 = 0$, d_1 and d_2 relation
- relation between two parametrization is given in Appendix of Hemmert et al.

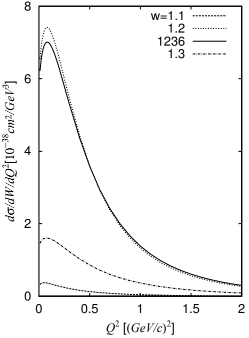
'Gamow-Teller' Operator of $N\Delta$ transition

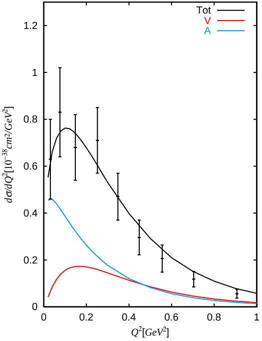
$$g_A \vec{\sigma} \tau^i \rightarrow G_A \vec{S} T^i$$

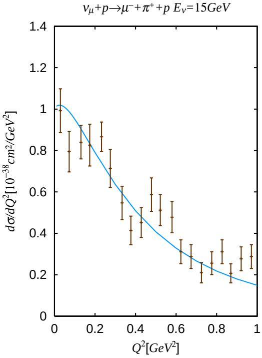
Rest frame of Delta $p_\Delta = (m_\Delta, \vec{0})$ at $Q^2 = 0$ ($|\vec{q}| = q_0 = \frac{m_\Delta^2 - m_N^2}{2m_\Delta}$)

$$\begin{aligned} G_A &= d_1(0) + \frac{m_\Delta^2 - m_N^2}{m_N^2} d_2(0) \\ &= \sqrt{\frac{3}{2}} (C_5^A(0) + \frac{m_\Delta^2 - m_N^2}{2m_N^2} C_4^A(0)) \end{aligned}$$

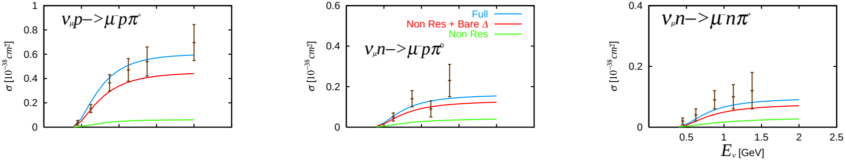
Q^2 of CC reaction $\nu_\mu + p \to \mu^- \pi^+ p$ (pure $I = 3/2$ reaction)







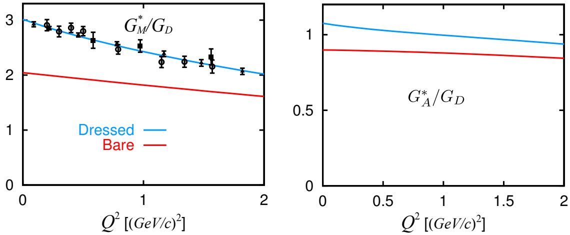
$E_\nu = 2GeV$ Flux average $0.5 < E_\nu < 6GeV$(ANL) $E_\nu = 15GeV$(CERN) total cross section of $\nu_\nu N \to \pi\mu^- N$ reactions



$$
\begin{aligned}
<\pi^+ p|J_{CC}|p> &= -\sqrt{2}J^{3/2} \\
<\pi^0 p|J_{CC}|n> &= -\frac{2}{3}(J^{3/2} - J^{1/2}) \\
<\pi^+ n|J_{CC}|n> &= -\frac{\sqrt{2}}{3}(J^{3/2} + 2J^{1/2})
\end{aligned}
$$

Assuming $J^{3/2}$ dominance, $\sigma(\pi^0 p) = \frac{2}{9}\sigma(\pi^+ p)$, $\sigma(\pi^+ n) = \frac{1}{9}\sigma(\pi^+ p)$

$N\Delta$ axial vector coupling



Parity violating asymmetry of electron scattering

Possibility to determine axial $N\Delta$ transition form factor

Parity violating asymmetry of electron scattering

$$A = \frac{\sigma(h_e = 1) - \sigma(h_e = -1)}{\sigma(h_e = 1) + \sigma(h_e = -1)}$$

Interference between electromagnetic and neutral current

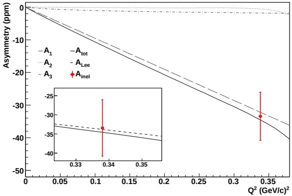
$$\begin{aligned} j^\mu_{em} &= j^\mu_e + J^\mu_{em} \\ j^\mu_{NC} &= \bar{e}((-1 + 4\sin^2\theta_W)\gamma^\mu + \gamma^\mu\gamma^5)e + (1 - 2\sin^2\theta_W)J^\mu_{em} - V^\mu_{IS} - A^\mu_3 \end{aligned}$$

$$A = -\frac{Q^2}{\sqrt{2}}\frac{G_F}{4\pi\alpha}(2 - 4\sin^2\theta_W + \Delta_V + \Delta_A)$$

$$\Delta_V = \frac{\cos^2\theta/2 W_2^{em-is} + 2\sin^2\theta/2 W_1^{em-is}}{D}$$

$$\Delta_A = \frac{\sin^2\theta/2(1-4\sin^2\theta_W)\frac{E+E'}{m_N}W_3^{em-nc}}{D}$$

$$D = \cos^2\theta/2 W_2^{em-em} + 2\sin^2\theta/2 W_1^{em-em}$$



K. Matsui, T. Sato, T.-S. H. Lee PRC72,025204 (2005)

D. Androic et al. arXiv:1212.1637 [nucl-ex] G^0

Summary of pion production at Delta resonance region

- $N\Delta$ transtion form factors are determined well for electromagnetic(Iso-vector Vector) current from pion photo and electroproduction. G_M is dominant term and G_E, G_C are small $< 10\%$.
- Axial $N\Delta$ form factor is not well determined as nucleon form factors from reaction model. (Lattice QCD and chiral perturbation theory)
- Model of electroweak pion production reaction is then used as a building block of the microscopic theoretical studies of neutrino-nucleus reaction.