Approximate Methods for Nuclei II

RPA, MEC, 2p2h... (**pionic modes of excitation in nuclei**)

Juan Nieves (IFIC, CSIC & UV)

Bibliography:

- Fetter A.L. and Walecka J.D.: *Quantum Theory on Many Particle Systems.* McGraw-Hill, New York, 1971
- *Mattuck R.D..: A guide to Feynman diagrams in the Many-Body Problem.* McGraw-Hill, New York, 1974
- Ericson T. and Weise W.: *Pions in Nuclei.* Oxford University Press, 1988.
- Oset E., Toki H. and Weise W.: Phys. Rep. 83 (1982) 281
- Oset E., Fernández de Córdoba, Salcedo, L.L. and Brockmann R.: Phys. Rep. 188 (1990) 79
- Nieves J., *Quantum Field Theoretical Methods in Many Body Systems, Czech. Jour. Phys. 46 (1996) 673.*

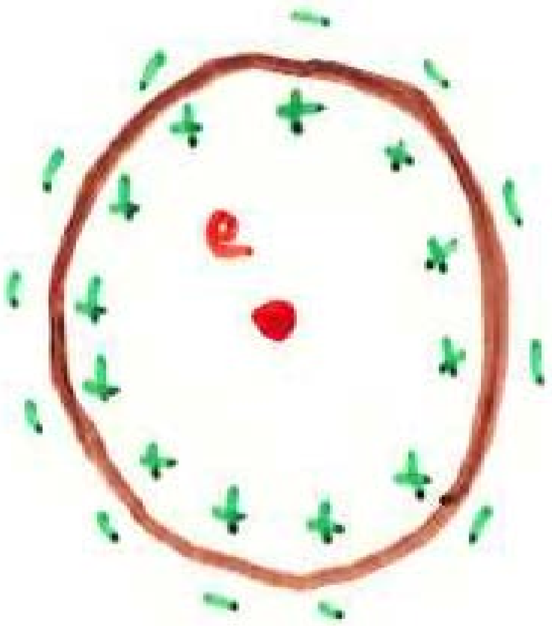
Outline:

- Introduction: Properties of the Coulomb interaction in an electron gas: screening of the interaction
- Particle and particle-hole propagators in a Fermi sea: occupation number and the Lindhard function
- Nuclear matter. Pion propagation in a nuclear medium: πN interaction at intermediate energies
- Induced spin-isospin NN interaction in a nuclear medium
- Examples:
  - inclusive muon capture in nuclei
  - pion-nucleus interactions: pionic atoms, pion nucleus scattering, …
  - inclusive electron-nucleus scattering

One electron inside of an electron gas (metal) **polarizes the medium** in such a way that, in a region around it, the negative charges are slightly displaced away from the electron, leaving behind the background charge of positive ions

SCREENING of the Coulomb Interaction

$$\frac{1}{4\pi}\frac{1}{r} \longrightarrow \frac{1}{4\pi}\frac{e^{-\mu(\rho)r}}{r}$$



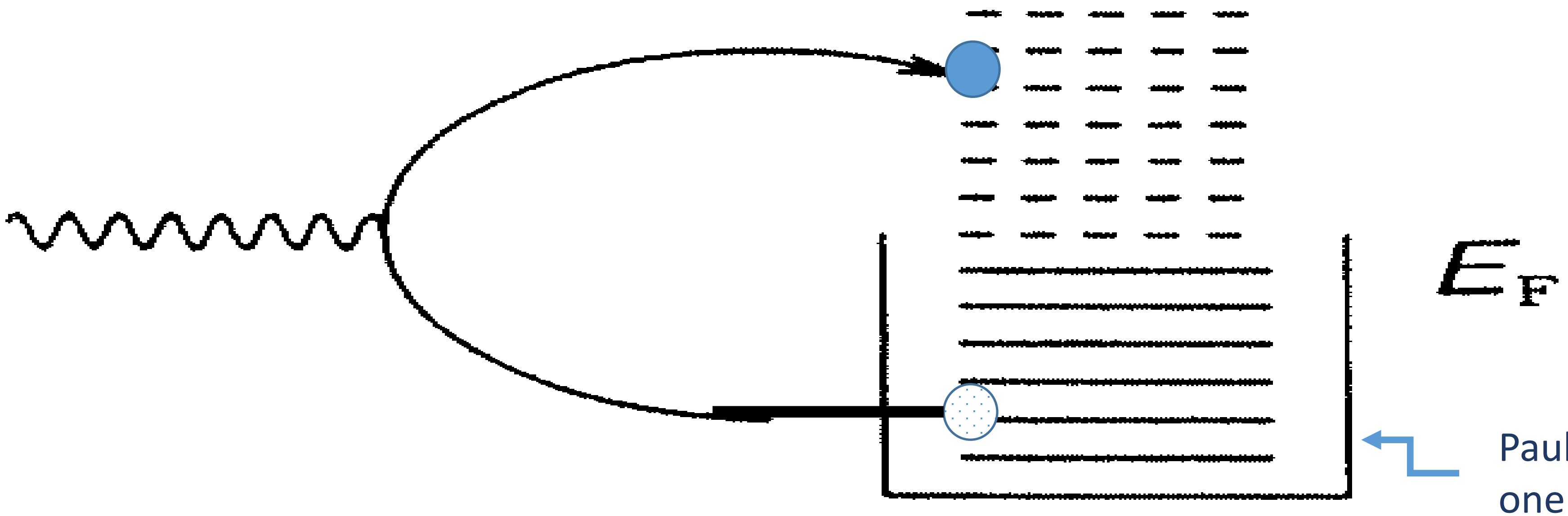
ρ is the electron density of the gas and μ a certain function of it. The effect of the polarizations has been to **convert the infinite range interaction into one of finite range.** The positive charge around one electron, coming from the polarization of the medium, cancels the electron negative charge, and a at large distances we see an effective charge zero.

In momentum space (static photon propagator, $q^0=0$)

$$\frac{1}{\vec{q}^{\,2}} \rightarrow \frac{1}{\vec{q}^{\,2}+\mu^2(\rho)}$$

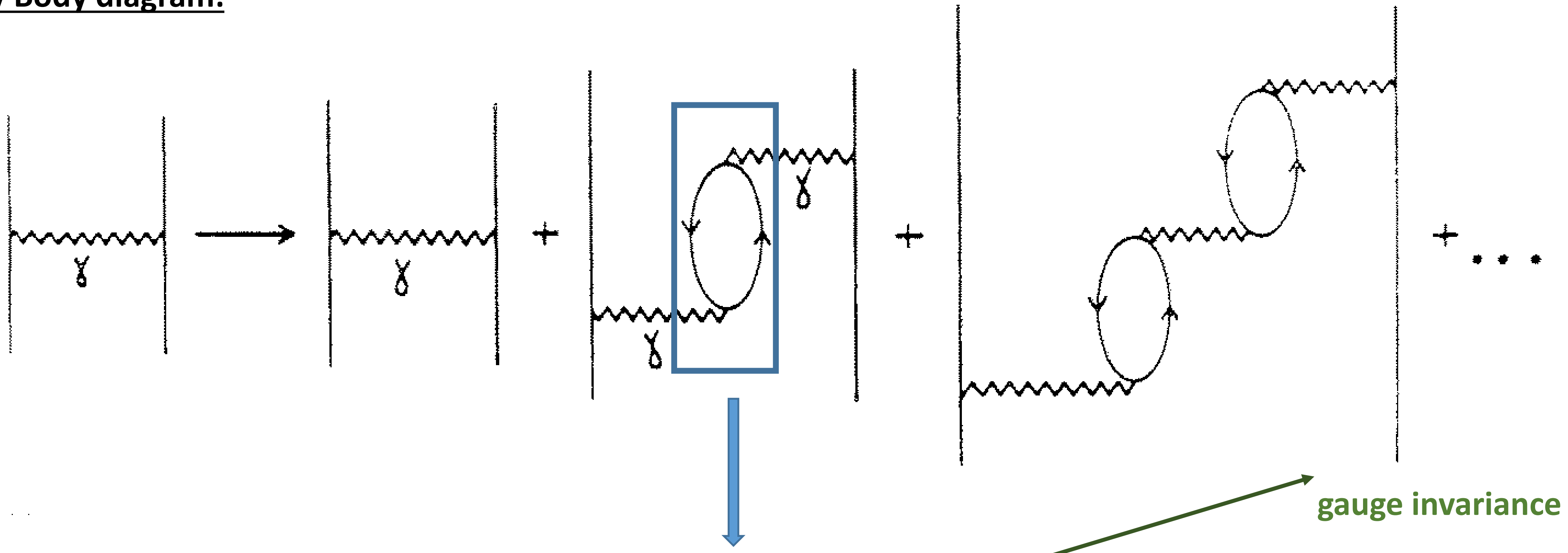
Photon acquires an effective mass: interaction becomes of <u>shorter range</u> !

The physical mechanism for the polarization consists in a transfer of some electrons from occupied states of a Fermi sea to some unoccupied states: **<u>producing particle-hole excitations</u>**:



Pauli-Fermi statistics: only one fermion per state

Many Body diagram:



$$\Pi^{\mu\nu}(q) = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\Pi(q)$$

photon selfenergy corresponding to a single ph excitation

Solving the Dyson equation in the Landau gauge,

$$\Pi^{\mu\nu}(q) = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\Pi(q)$$

$$iD_0^{\mu\nu}(q^2) = -i\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\frac{1}{q^2+i\epsilon}$$

$$iD^{\mu\nu}(q) = iD_0^{\mu\nu}(q) + iD_0^{\mu\rho}(q)\, i\Pi^{\rho\sigma}(q)\, iD_0^{\sigma\nu}(q) + \cdots$$

$$iD^{\mu\nu}(q) = -i\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\frac{1}{q^2+\Pi(q)}$$

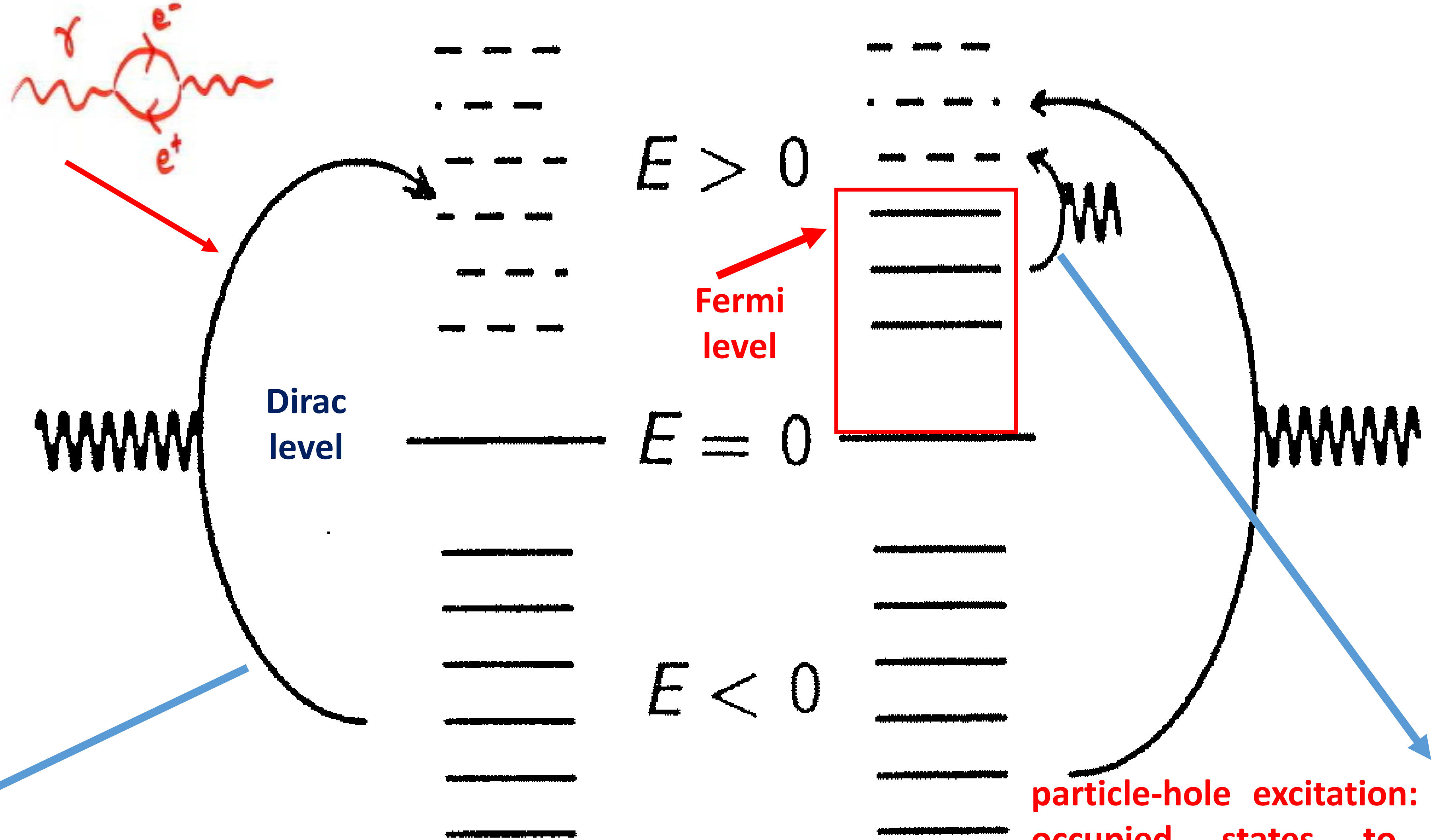
$$\frac{1}{4\pi}\frac{1}{r} \longrightarrow \frac{1}{4\pi}\frac{e^{-\mu(\rho)r}}{r}$$

Coordinate space (Coulomb static, $q^0=0$)

Modifies the propagation of the photon inside of the Fermi sea

Determined by $\Pi(q)$!

Dirac sea:



all the states of negative energy are filled by electrons, a hole in the Dirac sea behaves like a positive charge particle e^+

A particle-antiparticle excitation is represented by a transition of one electron from an occupied state of negative energy to an occupied state of positive energy

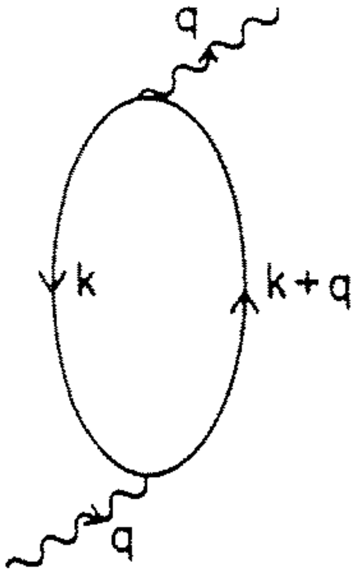
e^+e^- excitation: free photon selfenergy renormalize electron mass and charge and the γe^+e^- coupling

particle-hole excitation: excite occupied states to other unoccupied states of the Fermi sea. *These transitions are additional to those from the negative energy states to the positive energy ones.*

Fermi gas particle-hole propagator: Lindhard function

For the electromagnetic interaction
In the static limit…

$$H_{\rm em}(x) = -e\,\overline{\Psi}(x)\gamma^\mu\,\Psi(x)A_\mu(x)$$



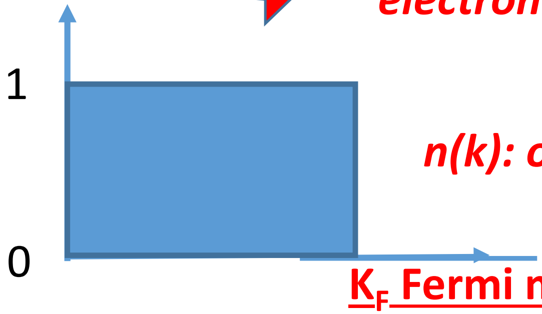
vertex
non-relat.

spin

$$-\mathrm{i}\Pi(q^0,\vec{q}) = (-\mathrm{i}e)(-\mathrm{i}e)(-2)\int\frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{i}G_0(k)\,\mathrm{i}G_0(k+q)$$

$$G_0(k) = \frac{1-n(\vec{k})}{k^0-\epsilon(\vec{k})+\mathrm{i}\eta} + \frac{n(\vec{k})}{k^0-\epsilon(\vec{k})-\mathrm{i}\eta}$$

electron propagator in a Fermi sea (non relativistic)



n(k): occupation number for a free Fermi gas.

electron kinetic energy

FERMION PROPAGATOR

$px = p^\mu x_\mu = p^0 x_0 - \vec{p}\,\vec{x}$

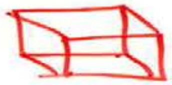
A) GROUND STATE = VACUUM $\equiv |0\rangle$ (CONTAINS NO PARTICLES)

$$\psi(x) = \sum_{\vec{p},r} \frac{1}{\sqrt{2E(p)V}} \left(c_r(\vec{p})\, u_r(\vec{p})\, e^{-ip\cdot x} + d_r^+(\vec{p})\, v_r(\vec{p})\, e^{ipx} \right)$$

$$\bar{\psi}(x) = \sum_{\vec{p},r} \frac{1}{\sqrt{2E(p)V}} \left(d_r(\vec{p})\, \bar{v}_r(\vec{p})\, e^{-ipx} + c_r^+(\vec{p})\, \bar{u}_r(\vec{p})\, e^{ipx} \right)$$

$(p^0 = E(\vec{p}) = \sqrt{\vec{p}^{\,2} + m^2}\,)$

FINITE VOLUME V



$V \qquad \frac{1}{V}\sum_{\vec{p}} \longrightarrow \int \frac{d^3p}{(2\pi)^3}$, $\quad \vec{p}$: momentum.
r: helicity.

$c_r^+(\vec{p})$ CREATES A FERMION WITH MOMENTUM $\vec{p}$ AND HELICITY r.

$$c_r^+(\vec{p})|0\rangle = |\text{fermion}, \vec{p}, r\rangle$$

$d_r^+(\vec{p})$ CREATES A ANTI-FERMION WITH MOMENTUM $\vec{p}$ AND HELICITY r

$$d_r^+(\vec{p})|0\rangle = |\overline{\text{fermion}}, \vec{p}, r\rangle$$

$c_r(\vec{p}), d_r(\vec{p})$ ASSOCIATED ANNHILATION OPERATORS

$$c_r(\vec{p})|0\rangle = d_r(\vec{p})|0\rangle = 0.$$

$$i\, G^{\alpha\beta}(x', x) = i \int \frac{d^4p}{(2\pi)^4}\, e^{-ip(x'-x)}\, G^{\alpha\beta}(p) \equiv \langle 0| T(\psi_\alpha(x')\, \bar{\psi}_\beta(x))|0\rangle =$$

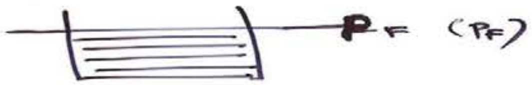
$$= \Theta(x'^0 - x^0)\, \langle 0|\psi_\alpha(x')\, \bar{\psi}_\beta(x)|0\rangle - \Theta(x^0 - x'^0)\, \langle 0|\bar{\psi}_\beta(x)\, \psi_\alpha(x')|0\rangle$$

$$\Theta(x'^0 - x^0)\left\{ \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E(p)} e^{iE(p)(x^0 - x'^0)} e^{-i\vec{p}(\vec{x} - \vec{x}')} (\not{p} + m) \right\}$$

$$+\, \Theta(x^0 - x'^0)\left\{ \int \frac{d^3p}{(2\pi)^3} \frac{1}{2E(p)} e^{iE(p)(x'^0 - x^0)} e^{i\vec{p}(\vec{x}' - \vec{x})} (-E\gamma^0 - \vec{p}\vec{\gamma} + m) \right\}$$

$$= i \int \frac{d^4p}{(2\pi)^4} e^{-ip(x'-x)}\, G(p)$$

$$\Rightarrow G(p) = \frac{\not{p} + m}{p^2 - m^2 + i\epsilon} .$$

B) GROUND STATE ≡ FERMI SEA ≡ 

$|0\rangle \to |\psi_0\rangle$

$d_r(p)\,|\psi_0\rangle = 0$

$c^\dagger_r(p)\,|\psi_0\rangle = 0 \quad (p < k_F)$ (CAN NOT CREATE A PARTICLE BELOW THE FERMI SURFACE)

$c_r(p)\,|\psi_0\rangle \neq 0 \quad (p < k_F)$ (CAN ANNIHILATE A PARTICLE BELOW THE FERMI SURFACE) → CREATES A HOLE.

$c_r(p)$ 

$$c_r(p) = \begin{cases} b_r(p) \; ; & p > k_F \quad \text{annihilates particles} \\ b^\dagger_r(p) \; ; & p < k_F \quad \text{creates holes.} \end{cases}$$

WE CAN REWRITE THE FIELD $\psi(x)$ IN TERMS OF OPERATORS WHICH DESTROY THE VACCUUM.

⇓

$$c_r(\vec{p}) = \begin{cases} b_r(\vec{p}), & p > p_F \\ b_r^\dagger(\vec{p}), & p \le p_F \end{cases} \qquad c_r^\dagger(\vec{p}) = \begin{cases} b_r^\dagger(\vec{p}), & p > p_F \\ b_r(\vec{p}), & p \le p_F \end{cases}$$

$$\begin{aligned}
\{c_r(\vec{p}), c_s(\vec{p}\,')\} &= \{d_r(\vec{p}), d_s(\vec{p}\,')\} = 0 \\
\{c_r(\vec{p}), d_s(\vec{p}\,')\} &= \{c_r^\dagger(\vec{p}), d_s^\dagger(\vec{p}\,')\} = 0 \\
\{c_r^\dagger(\vec{p}), c_s^\dagger(\vec{p}\,')\} &= \{d_r^\dagger(\vec{p}), d_s^\dagger(\vec{p}\,')\} = 0 \\
\{c_r(\vec{p}), d_s^\dagger(\vec{p}\,')\} &= \{c_r^\dagger(\vec{p}), d_s(\vec{p}\,')\} = 0 \\
\{c_r(\vec{p}), c_s^\dagger(\vec{p}\,')\} &= \{d_r(\vec{p}), d_s^\dagger(\vec{p}\,')\} = \delta_{rs}\delta_{\vec{p}\vec{p}\,'}
\end{aligned}$$

$$\begin{aligned}
\{b_r(\vec{p}), b_s(\vec{p}\,')\} &= \{b_r^\dagger(\vec{p}), b_s^\dagger(\vec{p}\,')\} = 0 \\
\{b_r(\vec{p}), b_s^\dagger(\vec{p}\,')\} &= \delta_{rs}\delta_{\vec{p}\vec{p}\,'}
\end{aligned}$$

$$\psi(x) = \sum_{\vec{p},r} \frac{1}{\sqrt{2E(p)V}} d_r^\dagger(p)\, v_r(p)\, e^{ipx} + \sum_{\vec{p},r} \frac{1}{\sqrt{2E(p)V}} e^{-ipx} \left(\Theta(|\vec{p}|-p_F)\, b_r(\vec{p})\, u_r(p) + \Theta(p_F-|\vec{p}|)\, b_r^\dagger(p)\, u_r(p) \right)$$

$$\bar{\psi}(x) = \sum_{\vec{p},r} \frac{1}{\sqrt{2E(p)V}} \left(d_r(p)\, \bar{v}_r(p)\, e^{-ipx} + e^{ipx} \cdot \left[\Theta(|\vec{p}|-p_F)\, b_r^\dagger(p)\, \bar{u}_r(p) + \Theta(p_F-|\vec{p}|)\, b_r(p)\, \bar{u}_r(p) \right) \right.$$

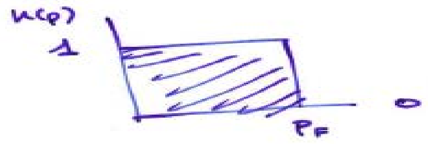
PROPAGATOR:

$$\langle \psi_0 | T(\psi(x')\bar{\psi}(x)) | \psi_0 \rangle = \cdots\cdots \langle 0 | T(\psi(x')\bar{\psi}(x)) | 0 \rangle + \left[-\int \frac{d^3p}{(2\pi)^3} \frac{1}{2E(p)} \Theta(p_F - p)\, (\not{p}+m)\, e^{-iE(p)(x'^0-x^0)} e^{i\vec{p}(\vec{x}\,'-\vec{x})} \right]$$

$$\Rightarrow G^0(p) = \frac{\not{p}+m}{p^2-m^2+i\epsilon} + 2\pi i\, n(p) \frac{\not{p}+m}{2E(p)} \delta(p^0 - E(p))$$

$$E(\vec{p}) = \sqrt{\vec{p}^{\,2}+m^2}$$

$$n(p) = \Theta(p_F - p)$$



Blocking only affects to particles on-shell.

NON-RELATIVISTIC REDUCTION

$$\frac{1}{p^0-\epsilon(\vec{p})\pm i\epsilon} = \mathcal{P}\left(\frac{1}{p^0-\epsilon(\vec{p})}\right) \mp i\pi\delta(p^0-\epsilon(\vec{p}))$$

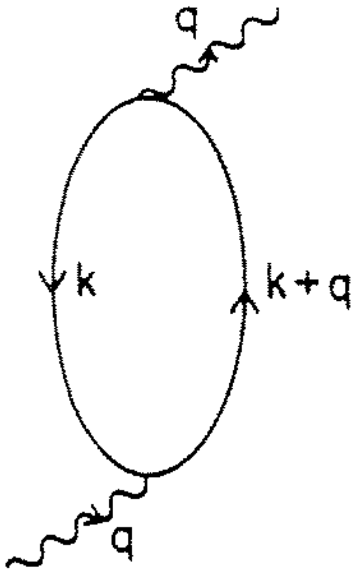
$$G^0(p) = \frac{1}{p^0-\epsilon(p)+i\epsilon} + 2\pi i\, n(p)\, \delta(p^0-\epsilon(p)) = \frac{1-n(p)}{p^0-\epsilon(p)+i\epsilon} + \frac{n(p)}{p^0-\epsilon(p)-i\epsilon},$$

$$\epsilon(p) = m + \vec{p}^{\,2}/2m$$

particle-hole propagator: Lindhard function

For the electromagnetic interaction
In the static limit…

$$H_{\rm em}(x) = -e\,\overline{\Psi}(x)\gamma^\mu\,\Psi(x)A_\mu(x)$$



vertex
non-relat.

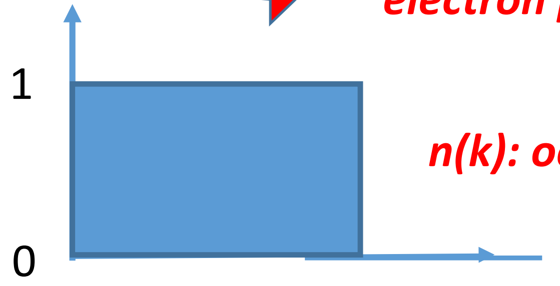
spin

$$-\mathrm{i}\Pi(q^0,\vec{q}) = (-\mathrm{i}e)(-\mathrm{i}e)(-2)\int\frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{i}G_0(k)\,\mathrm{i}G_0(k+q)$$

$$G_0(k) = \frac{1-n(\vec{k})}{k^0-\epsilon(\vec{k})+\mathrm{i}\eta}+\frac{n(\vec{k})}{k^0-\epsilon(\vec{k})-\mathrm{i}\eta}$$

electron propagator in a Fermi sea (non-relativistic)

electron kinetic energy



n(k): occupation number for a free Fermi gas.

K_F Fermi momentum

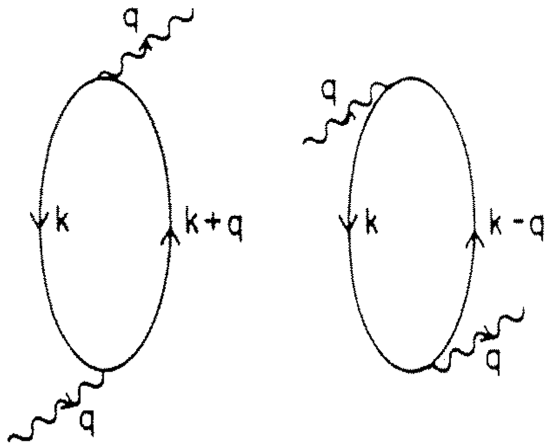
Juan Nieves, IFIC (CSIC & UV)

The k^0 integration can be done in the complex plane…

$$\Pi(q^0,\vec{q}) = e^2 U_e(q^0,\vec{q}),$$

where $U_e(q^0,\vec{q})$, called the Lindhard function is given by

$$U_e(q^0,\vec{q}) = 2\int \frac{d^3k}{(2\pi)^3}\left[\frac{n(\vec{k})(1-n(\vec{k}+\vec{q}))}{q^0-\epsilon(\vec{k}+\vec{q})+\epsilon(\vec{k})+i\eta} + \frac{n(\vec{k}+\vec{q})(1-n(\vec{k}))}{-q^0+\epsilon(\vec{k}+\vec{q})-\epsilon(\vec{k})+i\eta}\right].$$



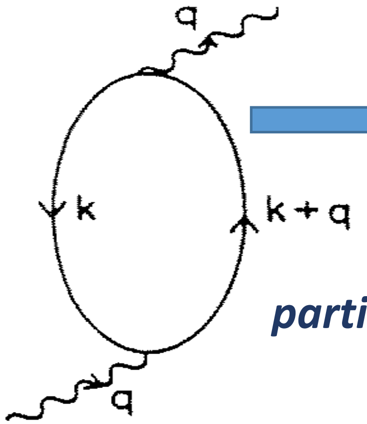
Direct and cross terms of the ph excitation

$U_e(q)$ can have both real and imaginary parts. The imaginary part comes from situations in the intermediate states integration, where the are placed on shell [*Cutkowsky's rule* (Itzykson & Zuber, Quantum Field Theory, McGraw-Hill, New York, 194)].

The imaginary part can be obtained using

$$\frac{1}{p^0 - \epsilon(\vec{p}) \pm i\epsilon} = \mathcal{P}\left(\frac{1}{p^0 - \epsilon(\vec{p})}\right) \mp i\pi\delta(p^0 - \epsilon(\vec{p}))$$

For **$q^0 > 0$**, only the **direct term** gives rise to an imaginary part

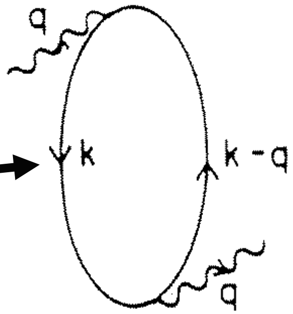


hole

particle

$$\mathrm{Im}\, U_e(q) = -2\int \frac{d^3k}{(2\pi)^3}\pi\delta\left[q^0 - \epsilon(\vec{k}+\vec{q}) + \epsilon(\vec{k})\right]\; n(\vec{k})\left[1 - n(\vec{k}+\vec{q})\right]$$

For $q^0 < 0$, the crossed term gives rise to an imaginary part



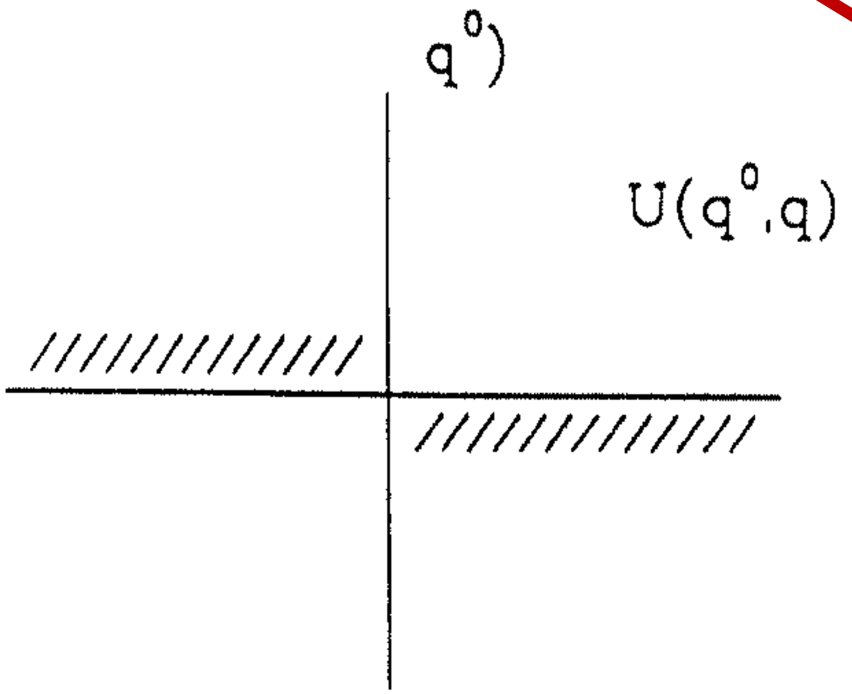
$$U_e(q^0,\vec{q}) = 2\int \frac{d^3k}{(2\pi)^3}\left[\frac{n(\vec{k})(1-n(\vec{k}+\vec{q}))}{q^0-\epsilon(\vec{k}+\vec{q})+\epsilon(\vec{k})+i\eta} + \frac{n(\vec{k}+\vec{q})(1-n(\vec{k}))}{-q^0+\epsilon(\vec{k}+\vec{q})-\epsilon(\vec{k})+i\eta}\right].$$

Analytical structure: In the complex q^0 plane, it has a continuous set of poles ***(cut) in the four quadrant***

$$q^0 = \epsilon_{par} - \epsilon_{hole} - i\eta$$

and in the second quadrant

$$q^0 = \epsilon_{hole} - \epsilon_{par} + i\eta$$

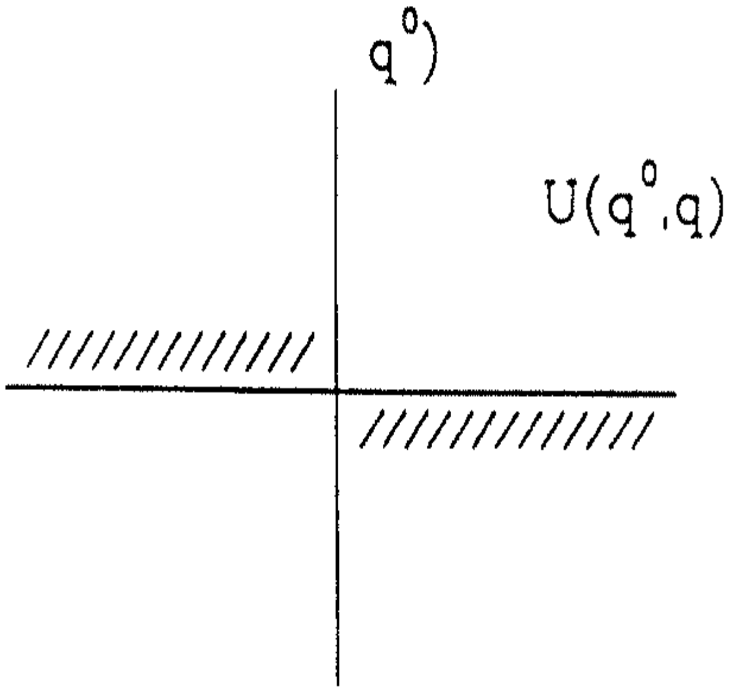


Analytical cuts in the second and fourth quadrants, and it has an imaginary part for real values of q^0 situated in the analytical cuts.

For complex values of q^0

$$2U_e(\nu,\hat q) = \frac{mk_F}{\pi^2}\left\{-1+\frac{1}{2\hat q}\left[1-\left(\frac{\nu}{\hat q}-\frac{\hat q}{2}\right)^2\right]\ln\frac{\nu/\hat q-\hat q/2+1}{\nu/\hat q-\hat q/2-1} - \frac{1}{2\hat q}\left[1-\left(\frac{\nu}{\hat q}+\frac{\hat q}{2}\right)^2\right]\ln\frac{\nu/\hat q+\hat q/2+1}{\nu/\hat q+\hat q/2-1}\right\}$$

$$\nu = \frac{q^0 m}{k_F^2}, \qquad \hat q = \frac{q}{k_F}$$



Analytical structure: In the complex q^0 plane, it has a continuous set of poles ***(cut) in the four quadrant and in the second quadrant***

$$2ImU_e(\nu,\hat q) = \frac{-2mk_F}{4\pi\hat q}\left[1-\left(\frac{|\nu|}{\hat q}-\frac{\hat q}{2}\right)^2\right]$$

for $\hat q > 2$ and $\frac12\hat q^2+\hat q \ge |\nu| \ge \frac12\hat q^2-\hat q$ or $\hat q < 2$ and $\frac12\hat q^2+\hat q \ge |\nu| \ge \hat q - \frac12\hat q^2$,

$$2ImU_e(\nu,\hat q) = -\frac{mk_F}{\pi\hat q}|\nu|$$

for $\hat q < 2$ and $0 \le |\nu| \le \hat q - \frac12\hat q^2$, and $\mathrm{Im}\,U_N(\nu,\hat q) = 0$ otherwise.

Analytical cuts in the second and fourth quadrants, and it has an imaginary part for real values of q^0 situated in the analytical cuts.

Fully relativistic expressions:

$$S(p) = (p^\mu\gamma_\mu + m)\left(\frac{1}{p^2 - m^2 + i\,\varepsilon} + i\,\frac{\pi}{E(\vec{p}\,)}\,\delta\left(p^0 - E(\vec{p}\,)\right) n(\vec{p}\,)\right)$$

$$E(\vec{p}) = \sqrt{\vec{p}^2 + m^2}$$

$$G(p) = \left(\frac{1}{p^2 - m^2 + i\,\varepsilon} + i\,\frac{\pi}{E(\vec{p}\,)}\,\delta\left(p^0 - E(\vec{p}\,)\right) n(\vec{p}\,)\right)$$

$$U_e(q) = -2i\int\frac{d^4p}{(2\pi)^4}\,G(p)G(p+q) = 2\int\frac{d^3p}{(2\pi)^3}\frac{m}{E(\vec{p}\,)}\frac{m}{E(\vec{p}+\vec{q})}\,\frac{n(\vec{p}\,)\left(1-n(\vec{p}+\vec{q})\right)}{q^0 + E(\vec{p}) - E(\vec{p}+\vec{q}) + i\varepsilon} + (q \leftrightarrow -q)$$

low density limit

$$Im U_e(q) = -m^2\,\frac{\Theta(q^0)\Theta(-q^2)}{2\pi|\vec{q}|}\,\Theta(E_F - \varepsilon_R)\,(E_F - \epsilon_R) \approx -\pi\rho\,\frac{m}{E(\vec{q}\,)}\,\delta\left(q^0 + M - E(\vec{q})\right)$$

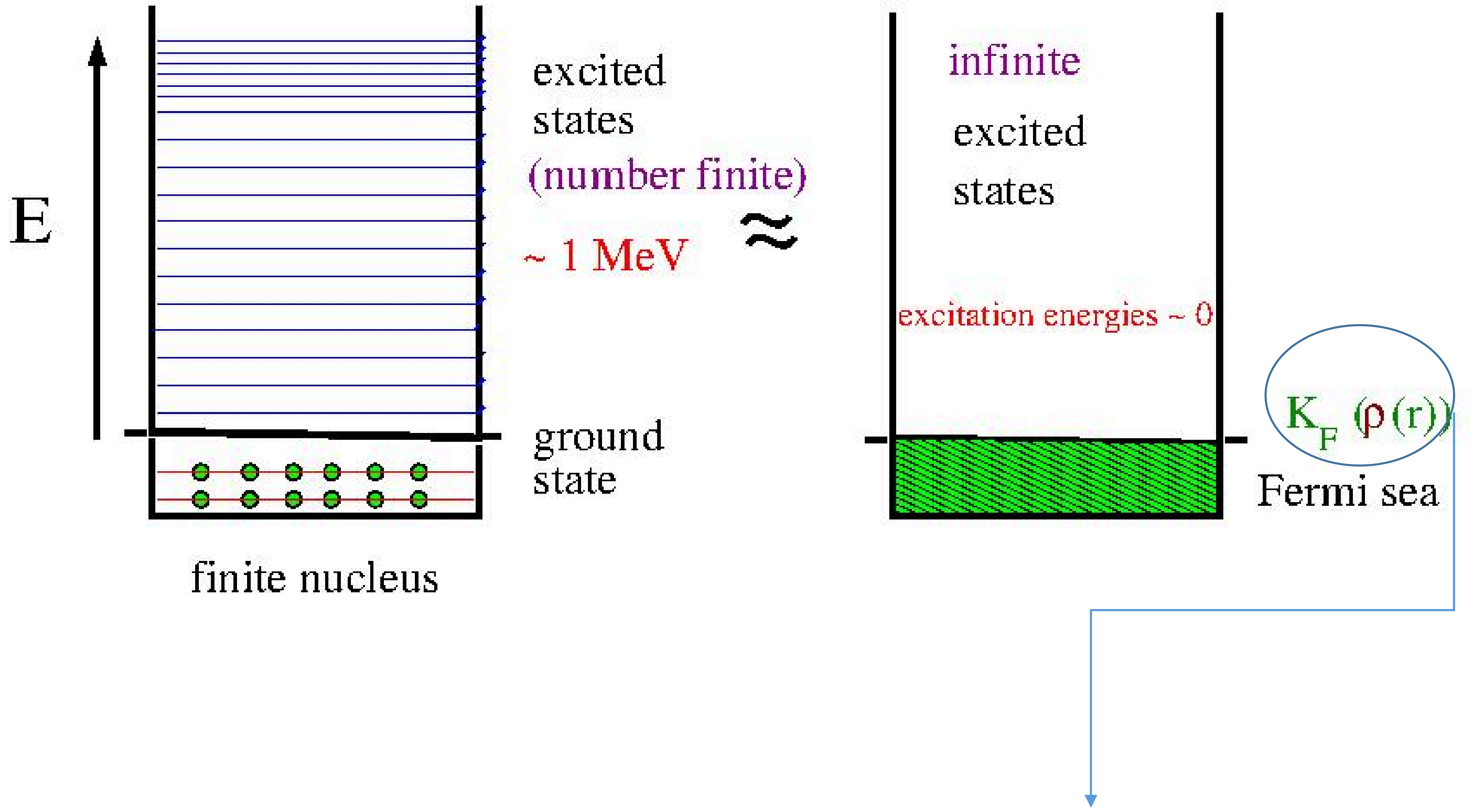
$$E_F(\vec{p}) = \sqrt{k_F^2 + m^2}, \qquad \varepsilon_R = Max\left\{m, E_F - q^0, \frac{-q^0 + |\vec{q}|\sqrt{1-4m^2/q^2}}{2}\right\}$$

Dictionary:

- Propagation of photons through an electron gas (metal)
- Electric charge of the electron and screening
- Photon carrier of the electromagnetic interaction

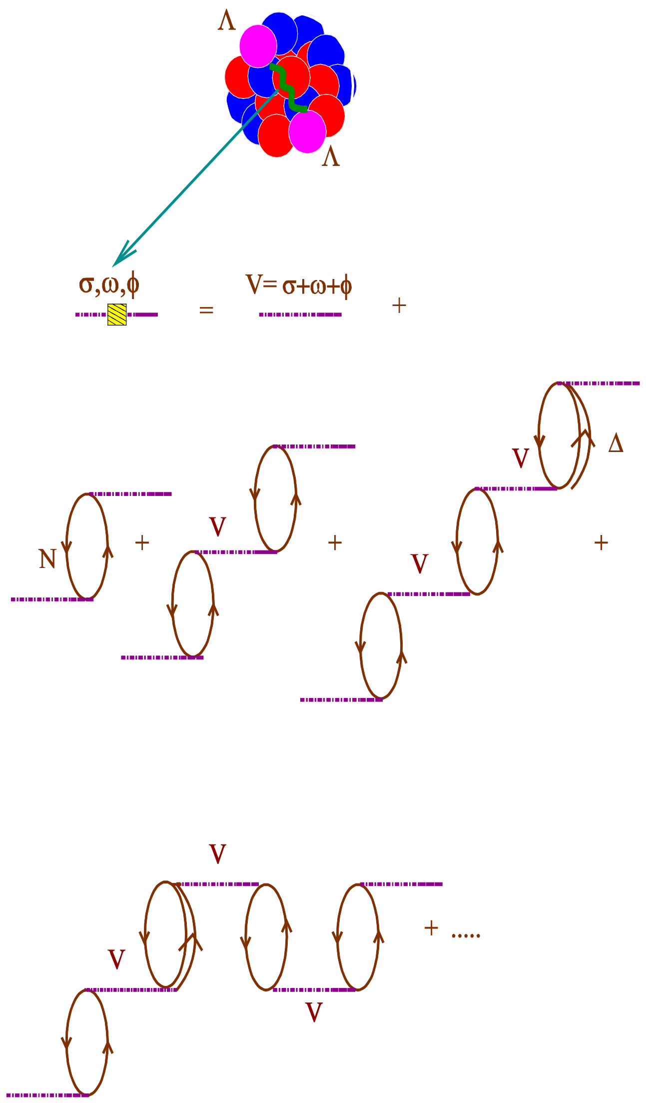
→

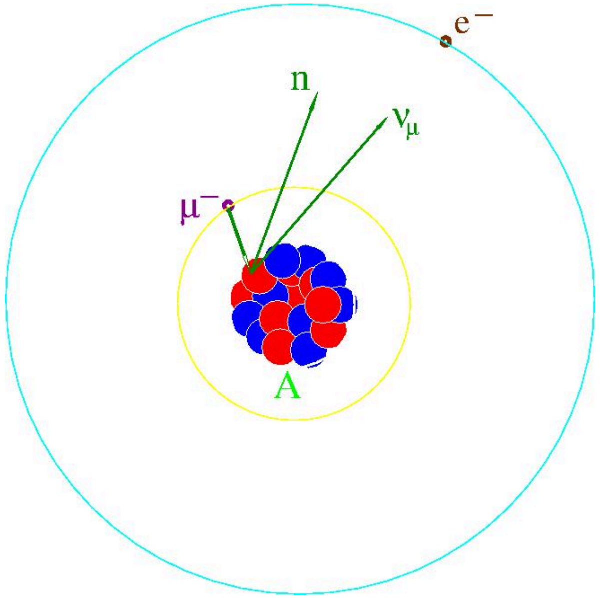
- Propagation of γ, W, Z^0 or pions (mesons in general) through a nuclear medium
- Axial charge of the nucleon and axial polarization
- π,ρ,… carriers of the NN interaction

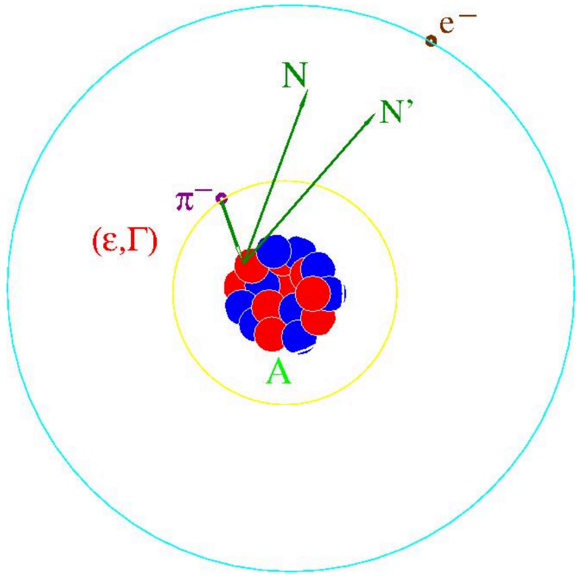


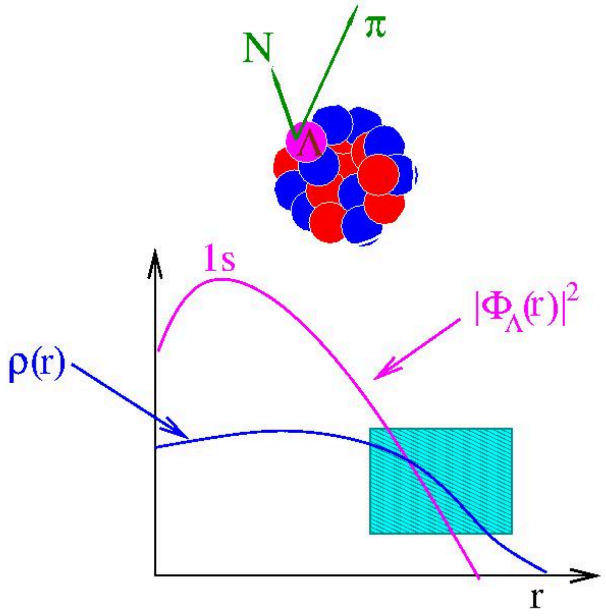
For inclusive processes , with large excitation energies (> 50 MeV) and spherical nuclei…

Local density approximation (LDA)

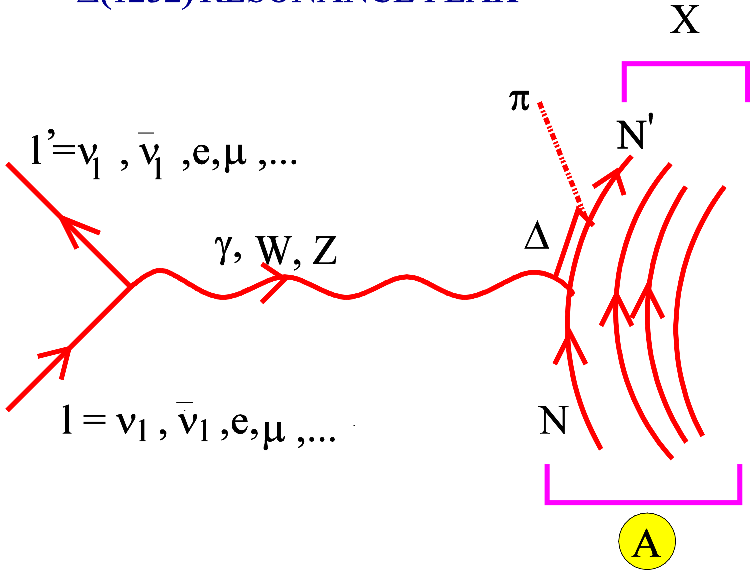








Δ(1232) RESONANCE PEAK



EXCITATION OF Δ (1232) DEGREES OF FREEDOM

Juan Nieves, IFIC (CSIC & UV)

Nuclear medium: Nuclear matter (approximation)

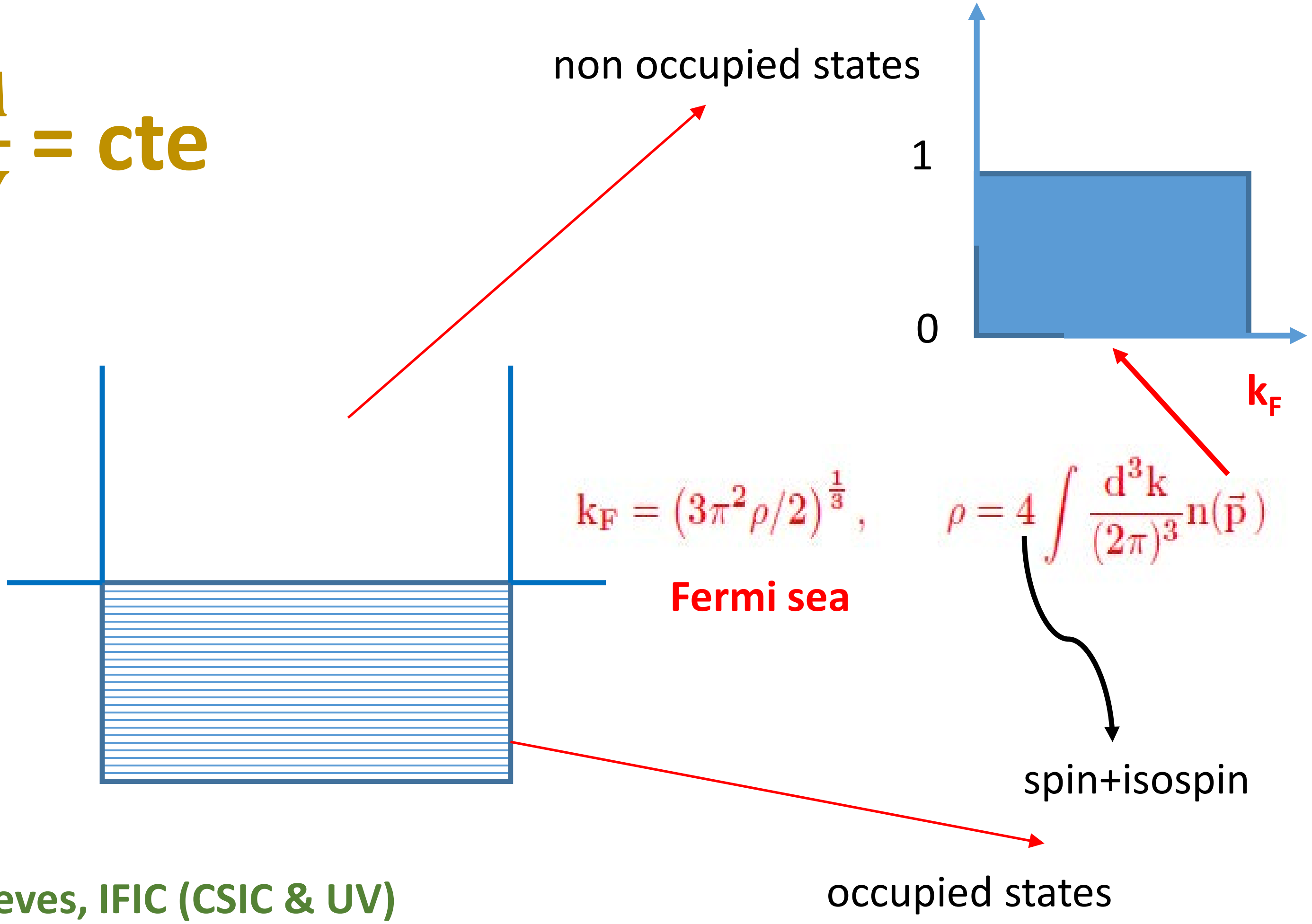
- Infinite system homogeneous and isotropic
- Infinite volume with an infinite number of nucleons, but with a constant density

$$\rho = \frac{A}{V} = \text{cte}$$

- ***Momentum and energy are conserved***
- Symmetric Z=N=A/2 (no necessary)
- ***Pauli's exclusion principle***
- ***Finite nuclei**: LDA (local density appx)*

ρ_{NM} (cte) $\rightarrow \rho(\vec{r})$ (s-wave)

$\vec{q}^2 \rho_{NM} \rightarrow \vec{\nabla}\rho(\vec{r})\,\vec{\nabla}$ (p-wave)

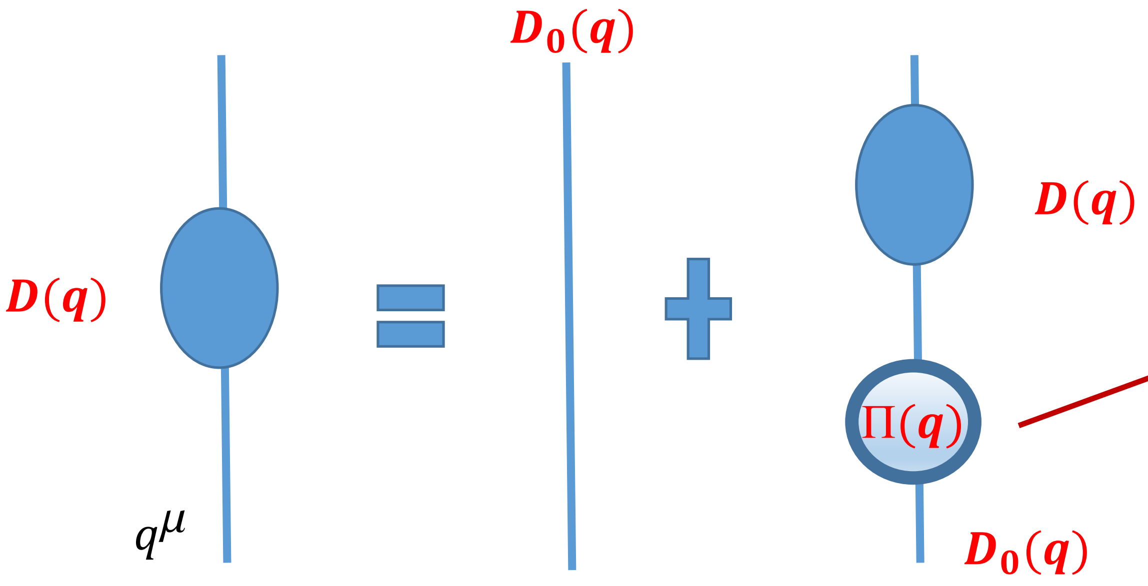


$$k_F = (3\pi^2\rho/2)^{\frac{1}{3}}, \qquad \rho = 4\int \frac{d^3k}{(2\pi)^3} n(\vec{p})$$

propagation of photons in the electron gas → propagation of pions in the nuclear medium *(carriers of the interaction)*

Dyson equation

$$iD = iD_0 + iD_0\, i\Pi\, iD \Longrightarrow D(q) = \frac{D_0}{1 - D_0\Pi} = \frac{1}{q^2 - m_\pi^2 - \Pi(q)}$$



Full pion propagator

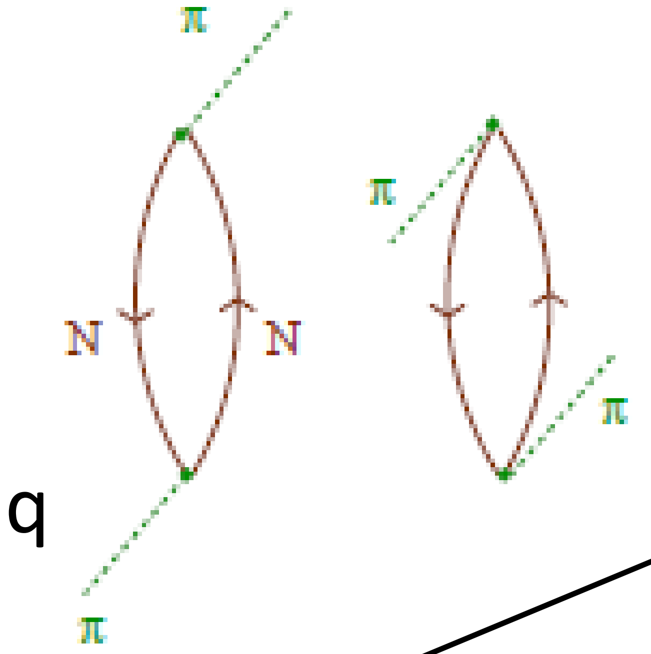
$D(q)$

Free pion propagator

$D_0(q)$

pion selfenergy: contains only irreducible diagrams

Pion selfenergy: first approximation πNN vertex



$$f = \frac{m_\pi}{2 f_\pi} g_A$$

$$\Pi(q) = \frac{f^2}{m_\pi^2} \vec{q}^{\,2}\, U_N(q)$$

$$U_N(q) = 2U_e(q), \quad (m_e \to m_N)$$

Chiral symmetry

$$\mathcal{L}^{\sigma}_{\rm int} = \frac{g_A}{f_\pi}\bar{\Psi}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}(\partial_\mu\vec{\phi})\Psi - \frac{1}{4f_\pi^2}\bar{\Psi}\gamma_\mu\vec{\tau}(\vec{\phi}\times\partial^\mu\vec{\phi})\Psi$$

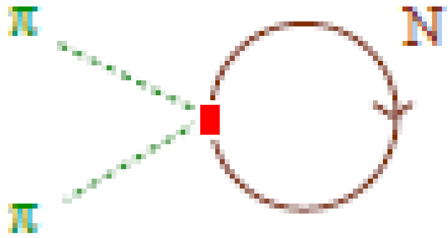
P-wave — *S-wave (Weinberg-Tomozawa)*

$\psi(x)$: SU(2) ⇒ bi-SPINOR : $\begin{bmatrix}\psi_p(x)\\ \psi_n(x)\end{bmatrix}$

$\phi_+(x) = \phi_-^\dagger(x) = \frac{1}{\sqrt{2}}(\phi_1(x) - i\,\phi_2(x))$; $\phi_0(x) = \phi_3(x)$

$\phi_-^\dagger(x)$ CREATES A π^- AND ANNIHILATES A π^+ MESON.

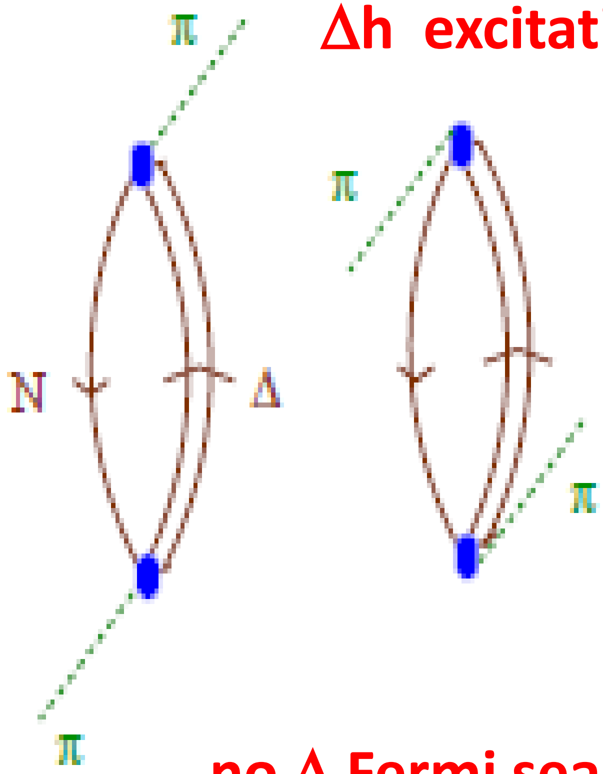
[J.D. Bjorken and S.D. Drell, Relativistic Quantum Fields, McGraw-Hill, New York, 1965]



S-wave selfenergy: small for isoscalar nuclei !

The pion cannot only excite nucleons above the Fermi sea, but it can also excite the internal degrees of freedom of the nucleon since it is a composite particle made out of quarks. Hence a nucleon can be converted into a $\Delta, N^*, \Delta^*, etc.$.

Δh excitation



Δ(1232) [spin 3/2 and isospin 3/2] plays an important role at intermediate energies because of its lower mass and strong coupling → contribution to

$$\mathcal{L}_{\pi N\Delta} = \frac{f^*}{m_\pi}\bar{\Psi}_\mu \vec{T}^\dagger(\partial^\mu \vec{\phi})\Psi + \text{h.c.}$$

where Ψ_μ is a Rarita-Schwinger $J^\pi = 3/2^+$ field, $\vec{T}^\dagger$ is the isospin transition operator from isospin 1/2 to 3/2, and $f^* = 2.13 \times f = 2.14.$

no Δ Fermi sea !

Juan Nieves, IFIC (CSIC & UV)

Some technical aspects about the Δh excitation

- $\left\langle \frac{3}{2}\; M_T \left| T_\nu^\dagger \right| \frac{1}{2}\; m_T \right\rangle = \left(\frac{1}{2}, 1, \frac{3}{2} \middle| m_T, \nu, M_T \right) \left\langle \frac{3}{2} \middle\| T_\nu^\dagger \middle\| \frac{1}{2} \right\rangle$ (Wigner-Eckart)

  (the reduced matrix element is crossed out with an arrow pointing to: 1)

- Δ propagator (unstable particle)

$P^{\mu\nu}$: spin 3/2 projector

$$G^{\mu\nu}(p_\Delta) = \frac{P^{\mu\nu}(p_\Delta)}{p_\Delta^2 - M_\Delta^2 + iM_\Delta \Gamma_\Delta},$$

$$P^{\mu\nu}(p_\Delta) = -(\not p_\Delta + M_\Delta)\left[g^{\mu\nu} - \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{2}{3}\frac{p_\Delta^\mu p_\Delta^\nu}{M_\Delta^2} + \frac{1}{3}\frac{p_\Delta^\mu \gamma^\nu - p_\Delta^\nu \gamma^\mu}{M_\Delta} \right],$$

$$\Gamma_\Delta(s) = \frac{1}{6\pi}\left(\frac{f^*}{m_\pi}\right)^2 \frac{M}{\sqrt{s}} \left[\frac{\lambda^{1/2}(s, m_\pi^2, M^2)}{2\sqrt{s}} \right]^3 \times \Theta(\sqrt{s} - M - m_\pi), \qquad s = p_\Delta^2,$$

$(\pi N\ CM\ momentum)^3$

$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$

Some technical aspects about the Δh excitation: non-relativistic expressions

- πNΔ transition: $$H_{\pi N\Delta} = \frac{f^*}{\mu} \Psi_N^\dagger(x)\, S_i\, \partial_i \phi^\lambda(x)\, T^\lambda\, \Psi_\Delta(x) + \text{h.c.}\,,$$

spin transition operator.

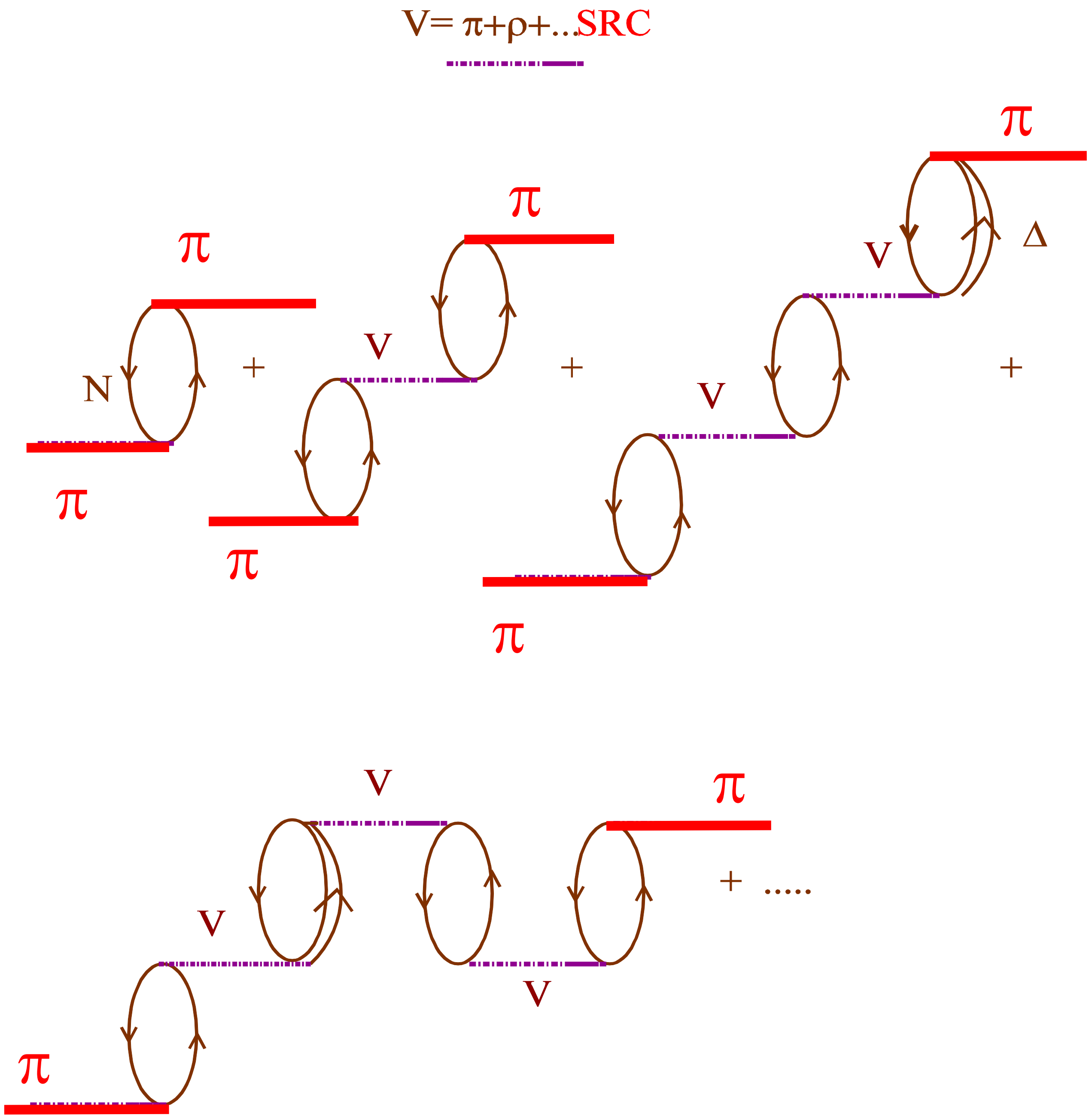
- Δ propagator

$$G_\Delta(k) = \frac{1}{k^0 - w_R - T_\Delta + \frac{1}{2}\mathrm{i}\Gamma_\Delta}$$

with $w_R = M_\Delta - M_N$, T_Δ the Δ kinetic energy a

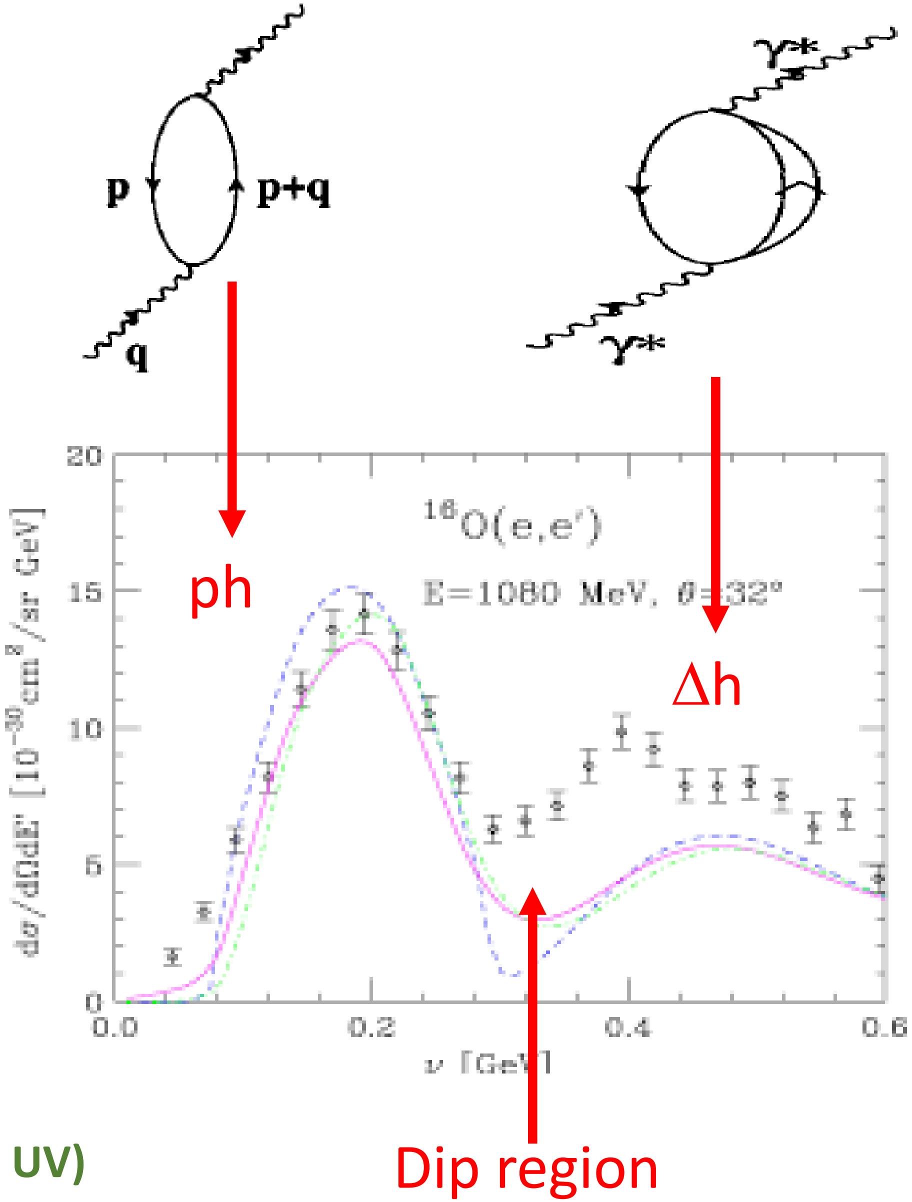
The pion selfenergy reads: $\boldsymbol{\Pi(q) = \frac{f^2}{m_\pi^2}\,\vec{q}^{\,2}\, U(q)}$ ***with*** $\boldsymbol{U(q) = U_N(q) + U_\Delta(q)}$

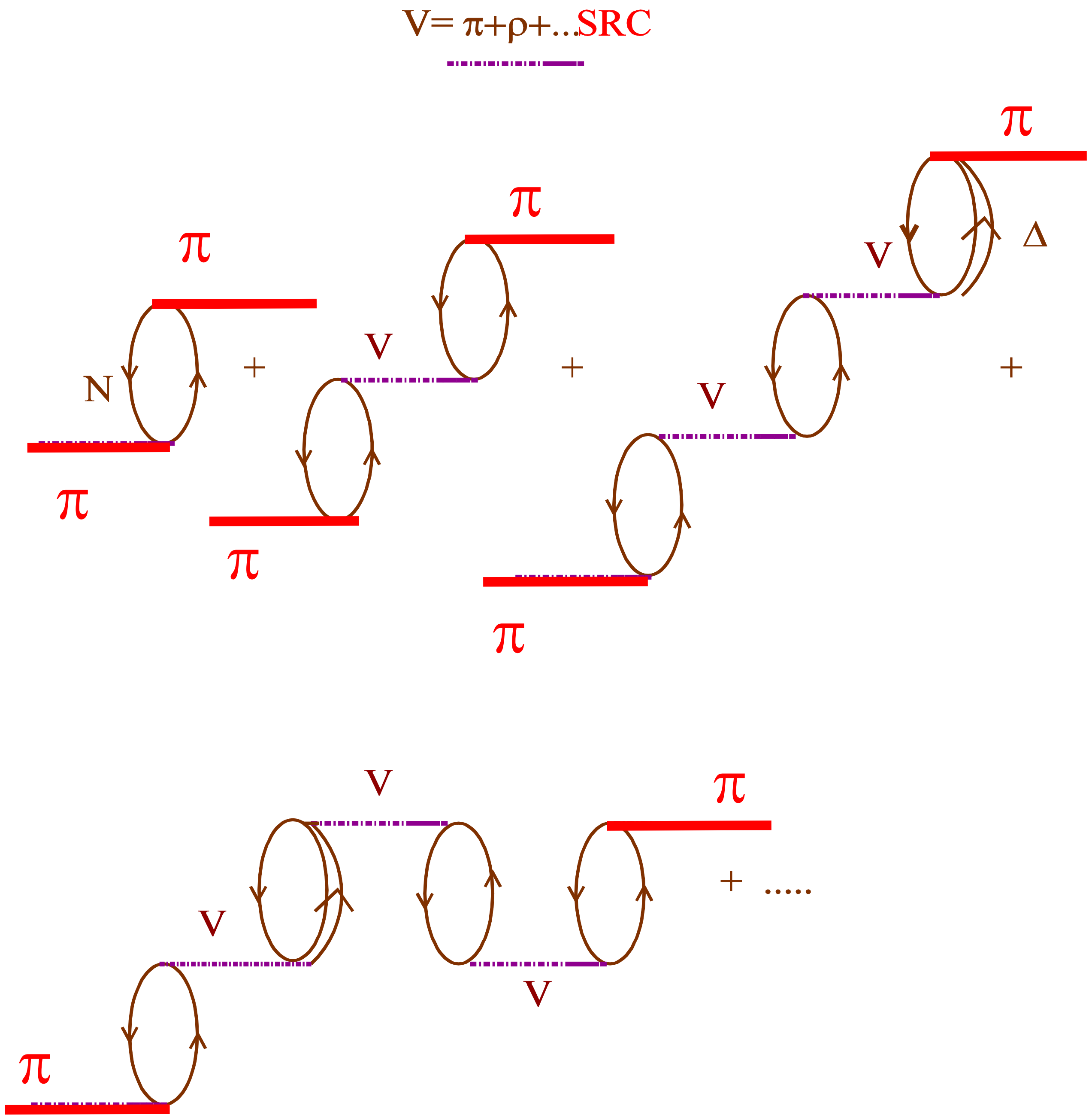
$$U_\Delta(q) = -\mathrm{i}\left(\frac{4}{3}\right)^2 \left(\frac{f^*}{f}\right)^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \left[G^0(k) G_\Delta(k+q) + G^0(k) G_\Delta(k-q)\right]$$



Remarks:

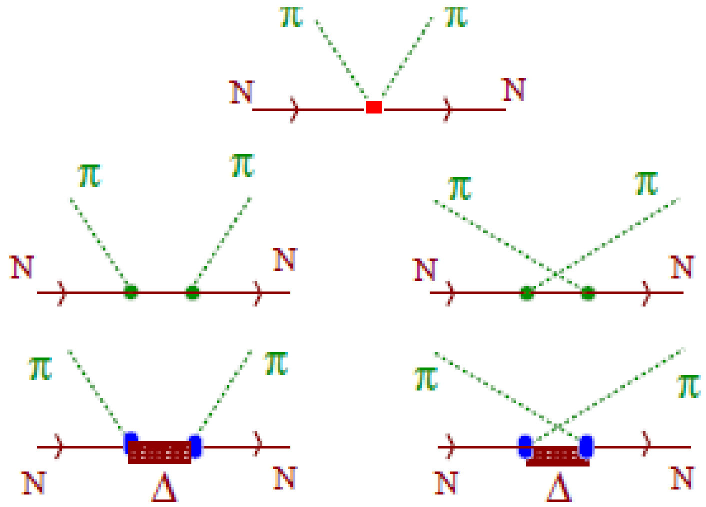
- External γ can also excite Δh components





Remarks:

- *V is an effective ph-ph, ph-Δh and Δh-Δh interaction in the nuclear medium*
- *V=π + ρ+ other mesons (Short Range Correlations). Starting point πN → πN in free space (πNN and πNΔ couplings)* [Ericson+Weise, *Pions in nuclei*] *and NN → NN Bonn potential*



NN INTERACTION: SIMPLE MODEL: (Oset + Weise, 1979)

$\pi + \rho$ exchanges + nuclear correlations

- $H_{\rho NN}(q) = i \dfrac{f_\rho}{m_\rho} (\vec{\sigma} \times \vec{q}) \vec{\epsilon}\, \tau^{\alpha}$

($\mathcal{L}(x) = -g_{\rho NN} \bar{\Psi} (\gamma^\mu - \frac{k}{2M} \sigma^{\mu\nu} \partial_\nu) \vec{\tau} \vec{\rho}_\mu \Psi$)

- $H_{\rho N\Delta}(q) = \dfrac{i f^*_\rho}{m_\rho} (\vec{S}^{+} \times \vec{q}) \vec{\epsilon}\, T^{\lambda}$

with $C_\rho \equiv \dfrac{f_\rho^2/m_\rho^2}{f^2/m_\pi^2} \simeq 2$, $f^*_\rho / f_\rho = f^*/f$ ← QUARK MODEL.

Now: $\pi + \rho$ EXCHANGES

- $\hat{V}_{s-i} = \hat{V}_\pi(q) + \hat{V}_\rho(q)$

$\hat{V}_\pi(q) = \sigma_1^i \sigma_2^j \vec{\tau}_1 \vec{\tau}_2 \left(\frac{f}{m_\pi}\right)^2 V^\pi_{ij}(q)$; $V^\pi_{ij} = F_\pi^2 \vec{q}^{\,2} D_\pi(q) \hat{q}_i \hat{q}_j$ (Longitudinal !!)

$\hat{V}_\rho(q) = \sigma_1^i \sigma_2^j \vec{\tau}_1 \vec{\tau}_2 \left(\frac{f}{m_\pi}\right)^2 V^\rho_{ij}(q)$; $V^\rho_{ij} = F_\rho^2 \vec{q}^{\,2} D_\rho(q) C_\rho \times (\delta_{ij} - \hat{q}_i \hat{q}_j)$

!! (Transversal ⟹

where the form factors (off-shell extrapolations)

$$F_i(q^2) = \frac{\Lambda_i^2 - \mu_i^2}{\Lambda_i^2 - q^2} \qquad (q^2 = q^{0\,2} - \vec{q}^{\,2})$$

$\Lambda_\pi = 1250$ MeV $= \Lambda^*_\pi$

$\Lambda_\rho = 2500$ MeV $= \Lambda^*_\rho$

- BECAUSE $C_\rho = C^*_\rho$, $\Lambda^*_\pi = \Lambda_\pi$ AND $\Lambda^*_\rho = \Lambda_\rho$ THE FORMER POTENTIALS DESCRIBE $\Delta N \to NN$, $NN \to \Delta N$, $\Delta N \to \Delta N$ AND $\Delta\Delta \to NN$ INTERACTION WITH THE FOLLOWING REPLACEMENTS

$$\frac{f}{m_\pi}\sigma\tau \to \frac{f^*}{m_\pi} S T \quad \text{or} \quad \frac{f^*}{m_\pi} S^+ T^+$$

- V^π_{ij} AND V^ρ_{ij} ARE ORTHOGONALS

$\Rightarrow$ SHORT RANGE CORRELATIONS

- ATTRIBUTED TO THE EXCHANGE OF THE ω-MESON.
- CORRELATED POTENTIAL IN COORDINATE SPACE

$$\tilde{V}(r) = V(r)\, g(r) \; ; \; g(r) = 1 - j_0(q_c r) \; ; \; q_c \simeq m_\omega = 783$$

$\Rightarrow \frac{1}{m_\omega}$ defines the range of the correlations

- MOMENTUM SPACE

$$\tilde{V}(q) = \int \frac{d^3k}{(2\pi)^3}\, g(\vec{k}-\vec{q})\, V(\vec{k})$$

$$g(\vec{k}-\vec{q}) = (2\pi)^3 \delta^3(\vec{k}-\vec{q}) - 2\pi^2 \delta(|\vec{q}-\vec{k}| - q_c)/q_c^2$$

NN potential $V(q) = c_0\{f_0(\rho) + f_0'(\rho)\vec{\tau}_1\vec{\tau}_2 + g_0(\rho)\vec{\sigma}_1\vec{\sigma}_2\} + \vec{\tau}_1\vec{\tau}_2 \sum_{i,j=1}^{3} \sigma_1^i \sigma_2^j V_{ij}^{\sigma\tau}$

zero range Landau force
J. Speth et al., Phys. Rep. 33 (1977) 127

$$V_{ij}^{\sigma\tau} = (\hat{q}_i\hat{q}_j V_l(q) + (\delta_{ij} - \hat{q}_i\hat{q}_j)V_t(q))$$

with $\hat{q}_i = q_i/|\vec{q}|$

V_π

$$V_l(q^0,\vec{q}) = \frac{f^2}{m_\pi^2}\left\{\left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - q^2}\right)^2 \frac{\vec{q}^2}{q^2 - m_\pi^2} + g_l'(q)\right\},$$

V_ρ

$$\frac{f^2}{4\pi} = 0.08,\ \Lambda_\pi = 1200\ \text{MeV},$$

$$V_t(q^0,\vec{q}) = \frac{f^2}{m_\pi^2}\left\{C_\rho\left(\frac{\Lambda_\rho^2 - m_\rho^2}{\Lambda_\rho^2 - q^2}\right)^2 \frac{\vec{q}^2}{q^2 - m_\rho^2} + g_t'(q)\right\},$$

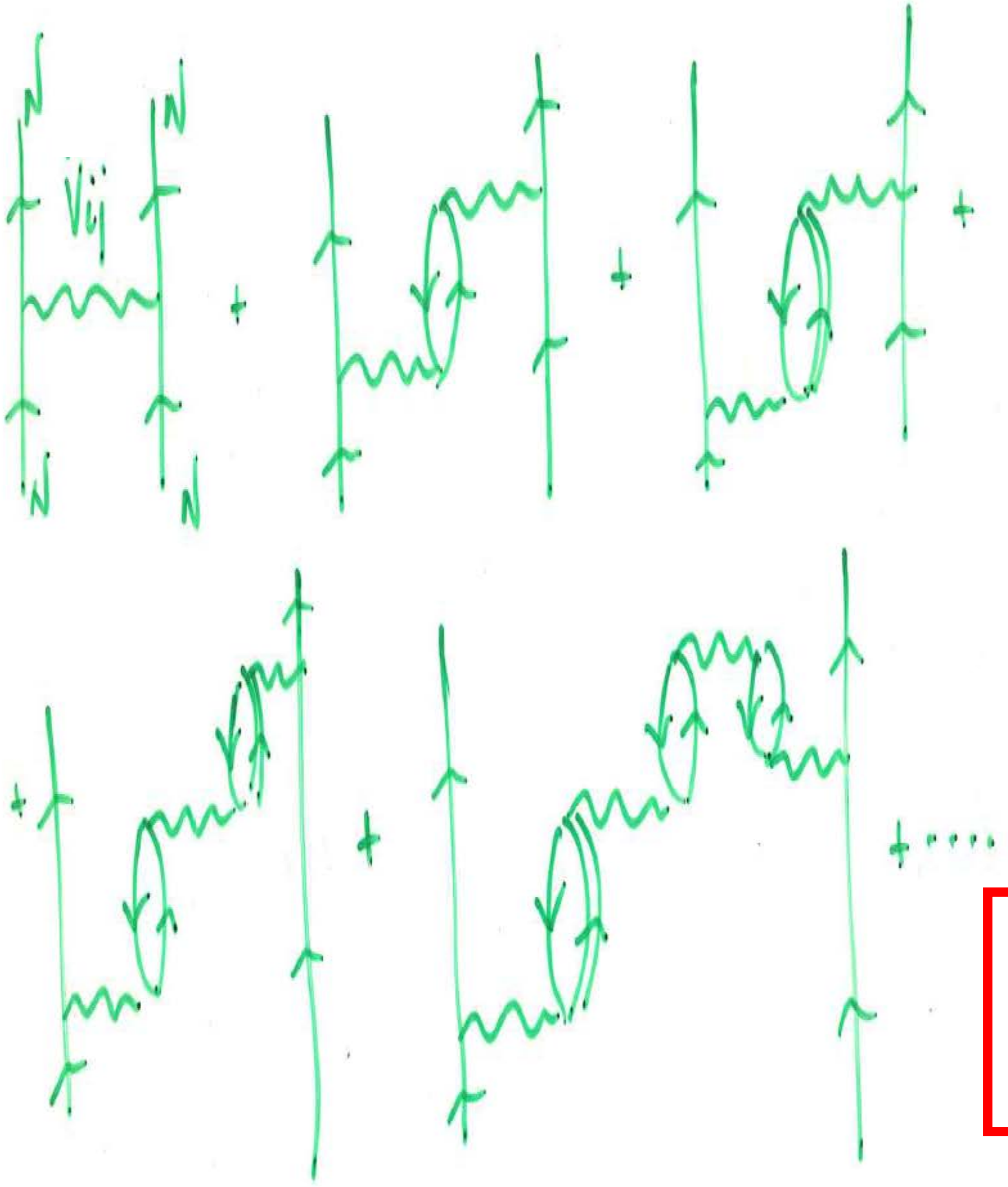
$$C_\rho = 2,\ \Lambda_\rho = 2500\ \text{MeV},\ m_\rho = 770\ \text{MeV}.$$

SRC

The $N\Delta$ and the $\Delta\Delta$ potentials are obtained from V_l and V_t by replacing

$$\vec{\sigma} \to \vec{S},\ \vec{\tau} \to \vec{T}.$$

$$f \to f^*$$



The spin-isospin part of the interaction, taking into account the propagation of the mesons through the medium

$$W_{\sigma\tau}(q) = \sigma_1^i \sigma_2^j \, \vec{\tau}_1 \vec{\tau}_2 V_{ij}^{\sigma\tau}(q) +$$
$$\sigma_1^i \sigma_2^j \, \vec{\tau}_1 \vec{\tau}_2 \{ V_{ik}^{\sigma\tau}(q) \, U(q) V_{kj}^{\sigma\tau}(q) \} +$$
$$\sigma_1^i \sigma_2^j \, \vec{\tau}_1 \vec{\tau}_2 \{ V_{ik}^{\sigma\tau}(q) \, U(q) V_{km}^{\sigma\tau}(q) U(q) V_{mj}^{\sigma\tau}(q) \} +$$
$$\ldots \;=\; \sigma_1^i \sigma_2^j \, \vec{\tau}_1 \vec{\tau}_2 W_{ij}^{\sigma\tau}(q)$$

$$U(q) = U_N(q) + U_\Delta(q)$$

(direct + crossed terms)

$$W_{ij}^{\sigma\tau}(q) = \frac{V_l(q)}{1 - U(q) V_l(q)} \, \hat{q}_i \hat{q}_j \; + \frac{V_t(q)}{1 - U(q) V_t(q)} \left(\delta_{ij} - \hat{q}_i \hat{q}_j \right)$$

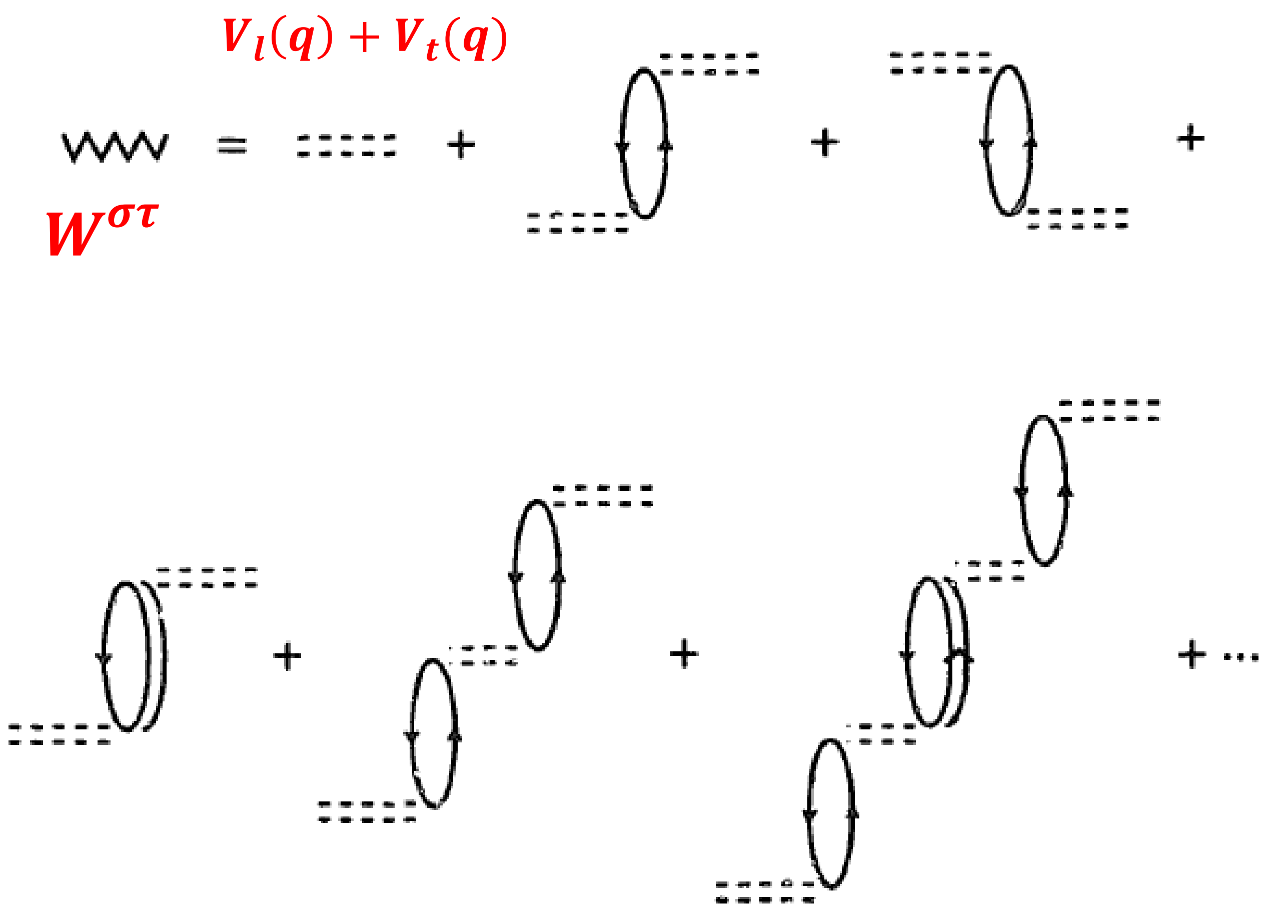
$$\hat{q}_i \hat{q}_j \perp \left(\delta_{ij} - \hat{q}_i \hat{q}_j \right)$$

Induced spin-isospin NN interaction in a nuclear medium

Diagrammatically,

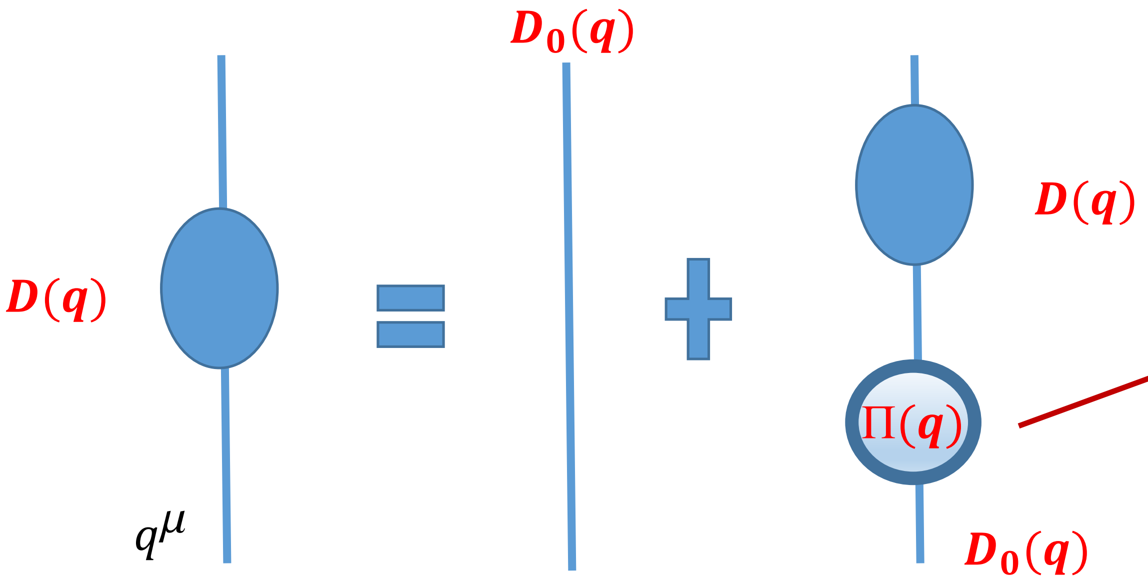
$$W_{ij}^{\sigma\tau}(q) = \frac{V_l(q)}{1-U(q)V_l(q)}\ \hat{q}_i\hat{q}_j\ + \frac{V_t(q)}{1-U(q)V_t(q)}\left(\delta_{ij}\ -\hat{q}_i\hat{q}_j\ \right)$$

From the spin-isospin interaction, we construct the induced interaction by exciting ph and Δh components in a RPA sense



Dyson equation

$$iD = iD_0 + iD_0\, i\Pi\, iD \Longrightarrow \boldsymbol{D(q)} = \frac{\boldsymbol{D_0}}{\boldsymbol{1 - D_0\Pi}} = \frac{\boldsymbol{1}}{\boldsymbol{q^2 - m_\pi^2 - \Pi(q)}}$$



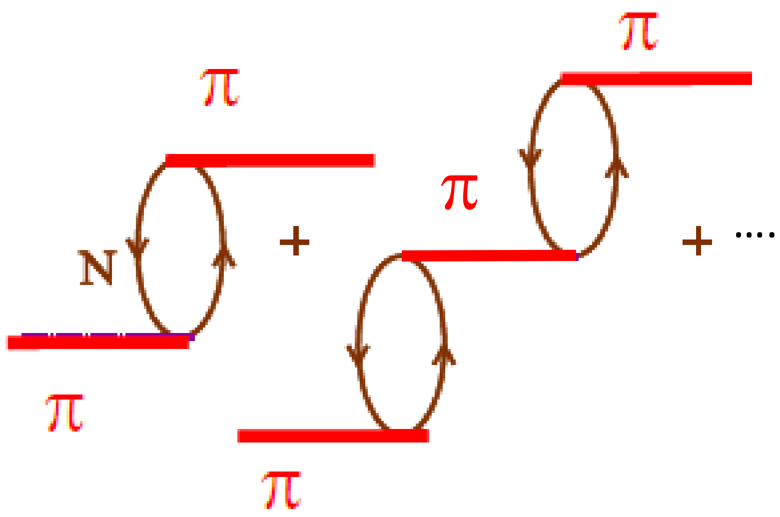
pion selfenergy: contains only
irreducible diagrams

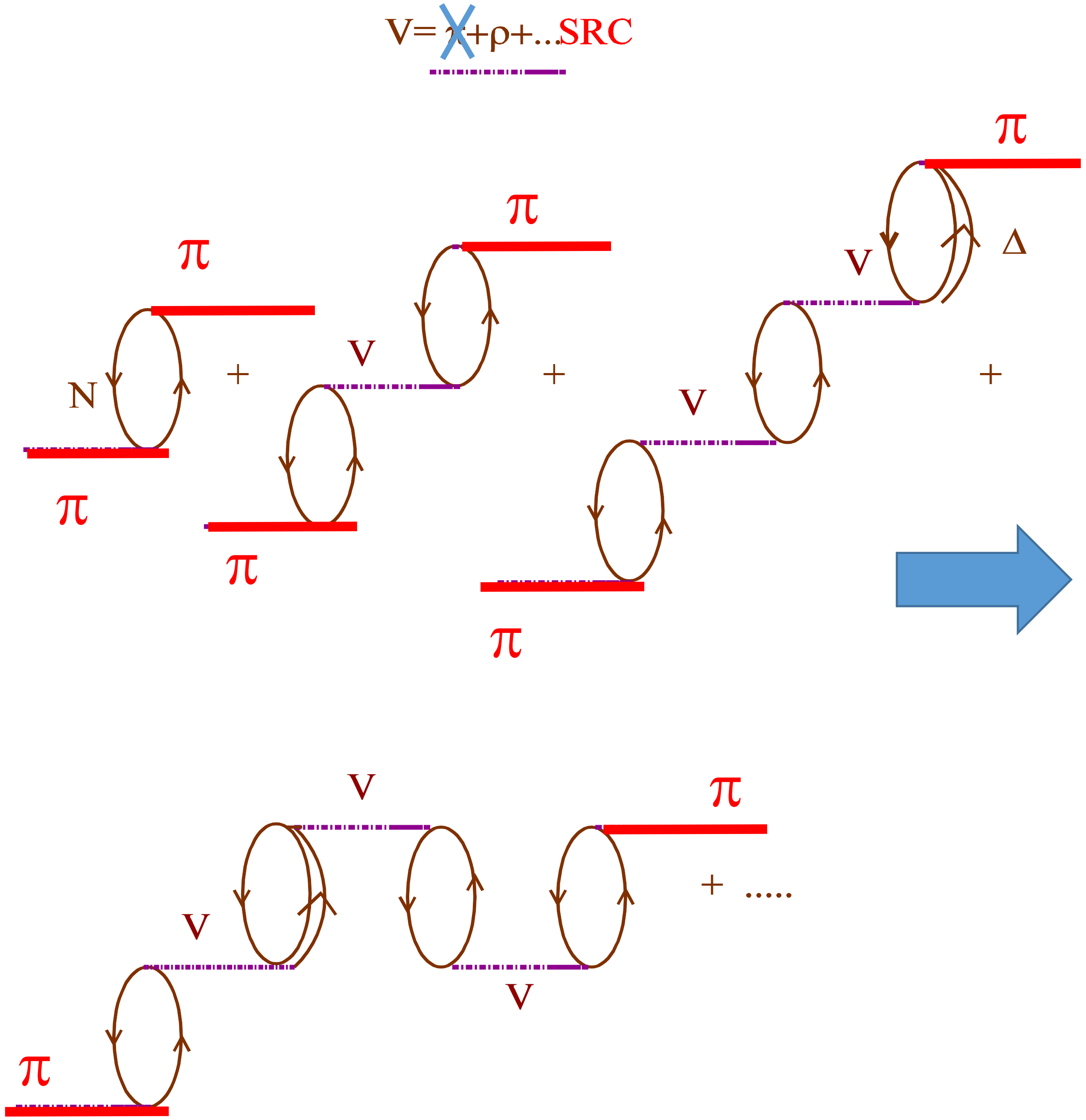
Full pion propagator

$D(q)$

Free pion propagator

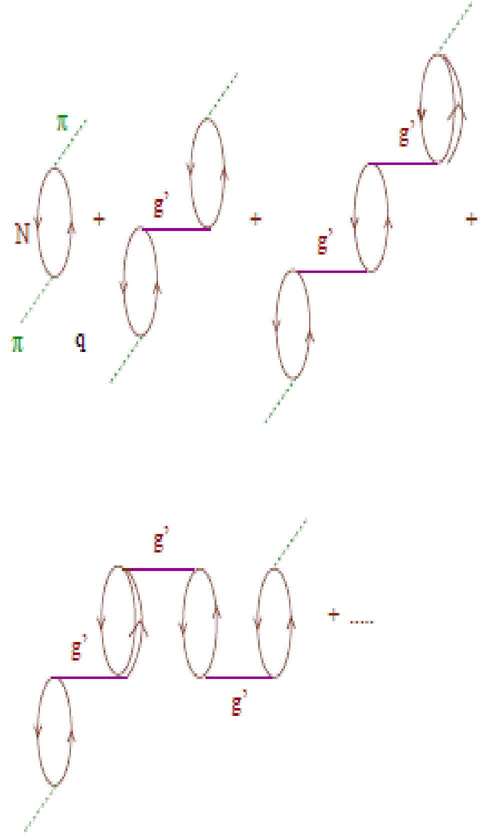
$D_0(q)$

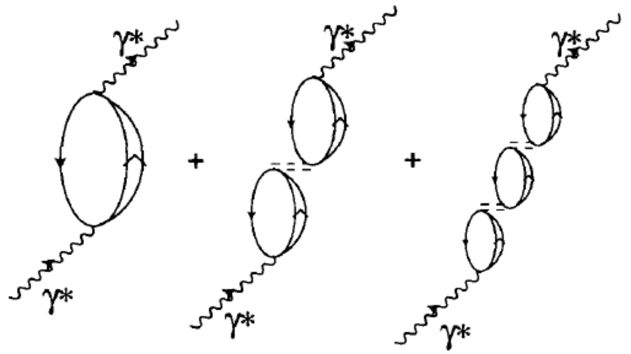




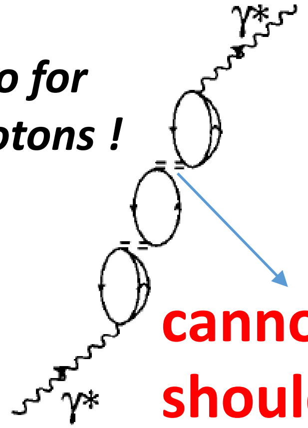
Remarks:

- To compute the pion selfenergy, only irreducible diagrams should be considered: V → ρ + SRC (g’) [the initial pion selects the longitudinal channel]

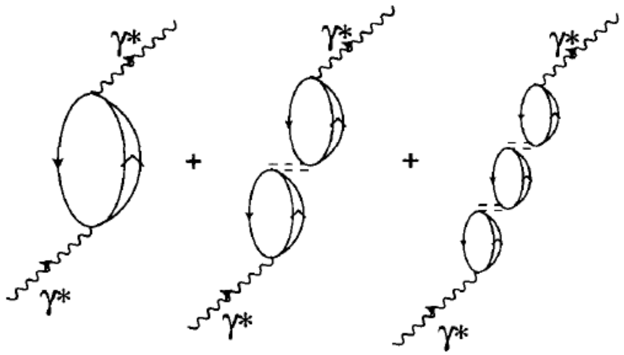




Also for photons !



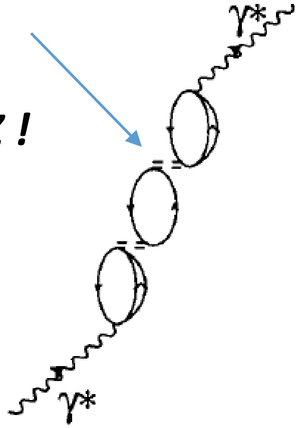
cannot be γ, it should be V: ph-ph, ph-Δh interaction



Remarks:

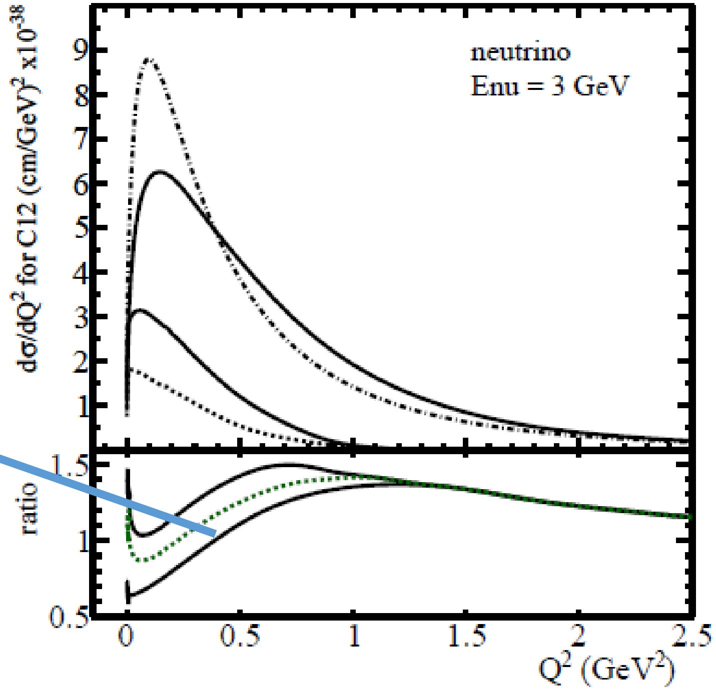
RPA corrections should disappear (ratio goes to 1.0) **at very large Q^2 values, because this is a collective effect which strength decreases when sizes larger than one nucleon are no longer being probed**. Hence in any realistic model, one should expect a qualitative Q^2 behavior similar to that exhibited by the QE_{RPA}/QE_{noRPA} ratio line depicted in the figure.: **low Q^2 suppression, followed by an enhancement that could even give rise to a net increase of the cross section, and finally all RPA effects should disappear for sufficiently high Q^2 values**

Also for photons, W, Z !



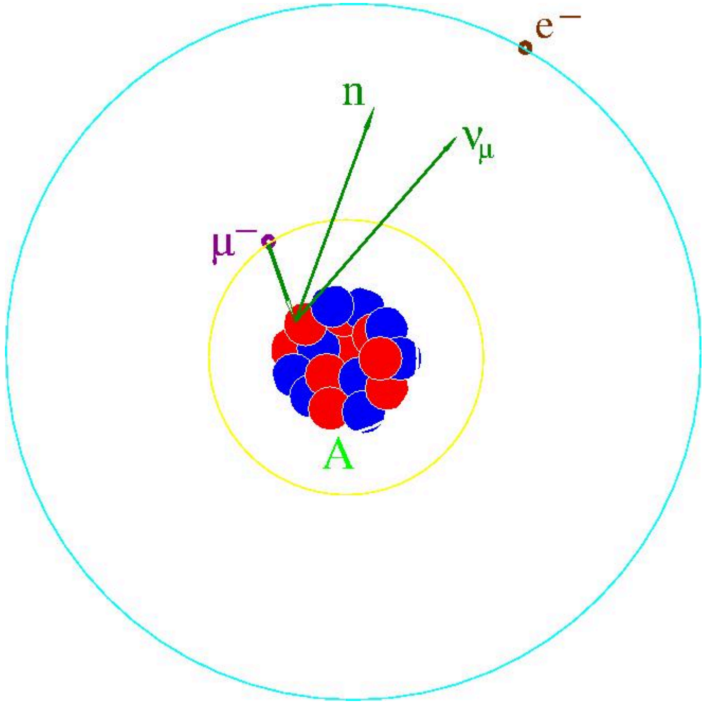
cannot be γ, *W, Z,* it should be V: ph-ph, ph-Δh interaction

QE_{RPA}/QE_{noRPA}



Inclusive muon capture in nuclei

[E. Oset et al., NPA 510 (1990) 591; J. Nieves et al., PRC 70 (2004) 055503; J.E. Amaro et al., EJPA 24 (2005) 343]



- Hydrogen-like atom, but $R_{\mu^-} \ll R_{e^-}$, since $m_\mu \gg m_e$
- There are screening and relativistic effects (solve Dirac equation)
- μ^- can be absorbed by the nucleus

$$(\mu^- A_Z)_{\text{bound}} \rightarrow \nu_\mu X \qquad [\mu^- p \rightarrow n \, \nu_\mu]$$

$$\Gamma = -2 \, Im \, \Pi_{\mu^-}$$

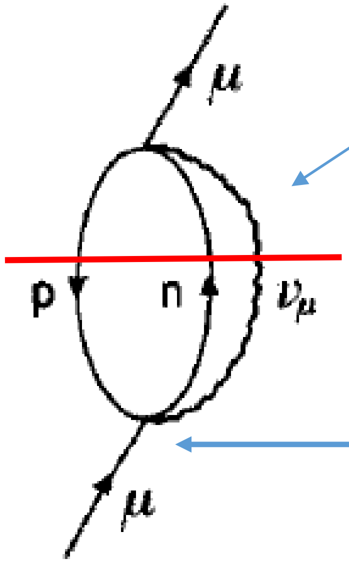
$$L(x) = \sqrt{\tfrac{1}{2}} G \, J^{\mu}(x) L^{+}_{\mu}(x)$$

Leptonic current

$$\tilde{L}^{\mu^+} \rightarrow \bar{u}_\nu(p_\nu)\gamma^\mu(1-\gamma_5)u_\mu(p_\mu) \, ,$$

$$\tilde{J}^\mu \rightarrow \bar{u}_n(p_n)[g_V\gamma^\mu + i\frac{g_M}{2m_p}\sigma^{\mu\nu}q_\nu + g_A\gamma^\mu\gamma_5 + \frac{g_P}{m_\mu}q^\mu\gamma_5]u_p(p_p)$$

Hadron current



Lowest order contribution: *imaginary part of the Lindhard function*

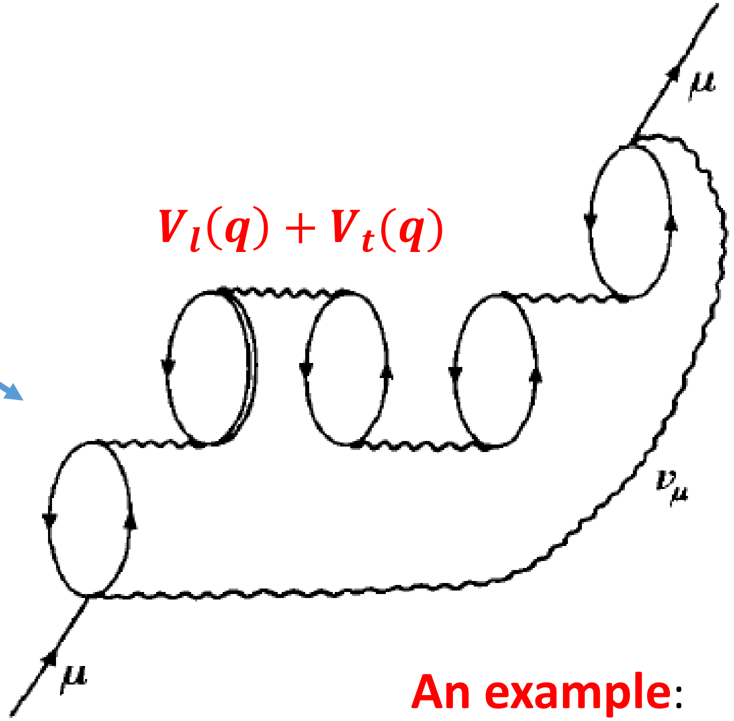
$$\Gamma = -2\int \frac{d^3p_\nu}{(2\pi)^3}\frac{2m_\nu}{2E_\nu}\frac{2m_\mu}{2E_\mu}\frac{2m_p}{2E_p}\frac{2m_n}{2E_n}\overline{\sum}\sum |T|^2\,\mathrm{Im}\,\bar{U}(p_\nu - p_\mu)$$

- *Finite nuclei: LDA (local density appx)*

$$\Gamma = \int d^3r\,|\Phi_{1s}(r)|^2\,\tilde{\Gamma}(\rho_p(r),\rho_n(r))$$

Strong renormalization effects:

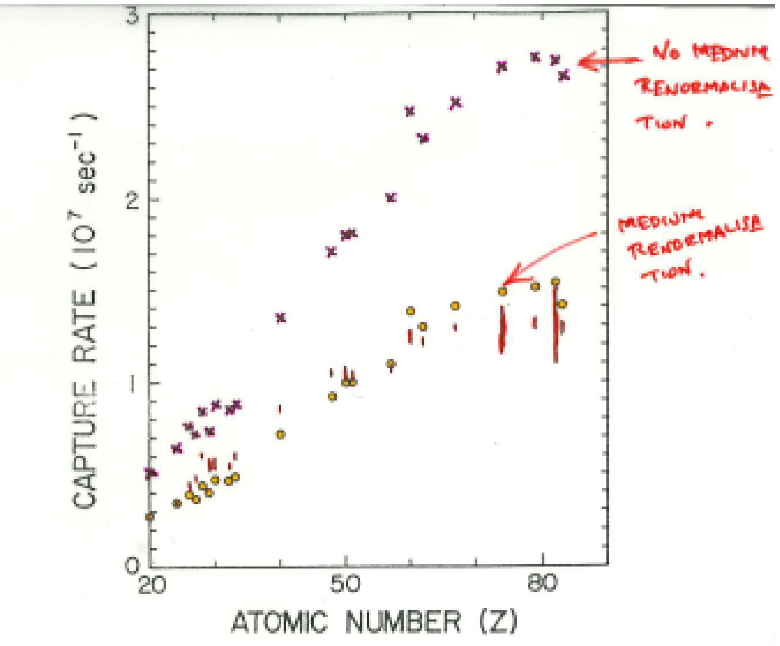
$V_l(q) + V_t(q)$

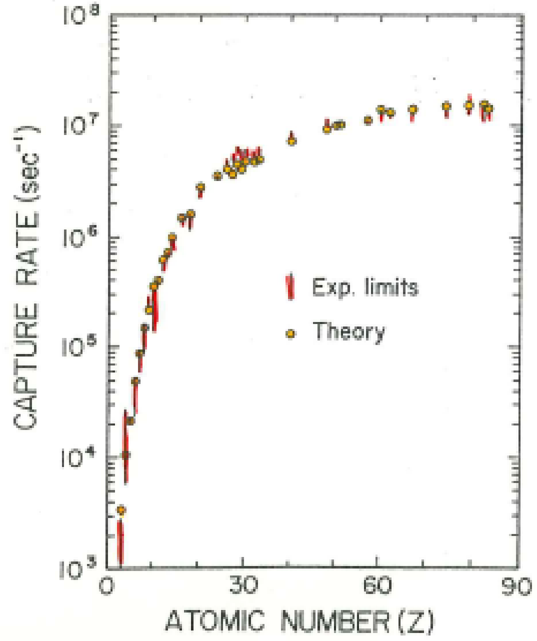


	Pauli $[10^4\ s^{-1}]$	RPA $[10^4\ s^{-1}]$	Exp $[10^4\ s^{-1}]$	$\left(\Gamma^{Exp} - \Gamma^{Th}\right)/\Gamma^{Exp}$
^{12}C	5.42	3.21	3.78 ± 0.03	0.15
^{16}O	17.56	10.41	10.24 ± 0.06	−0.02
^{18}O	11.94	7.77	8.80 ± 0.15	0.12
^{23}Na	58.38	35.03	37.73 ± 0.14	0.07
^{40}Ca	465.5	257.9	252.5 ± 0.6	−0.02
^{44}Ca	318	189	179 ± 4	−0.06
^{75}As	1148	679	609±4	−0.11
^{112}Cd	1825	1078	1061±9	−0.02
^{208}Pb	1939	1310	1311±8	0.00

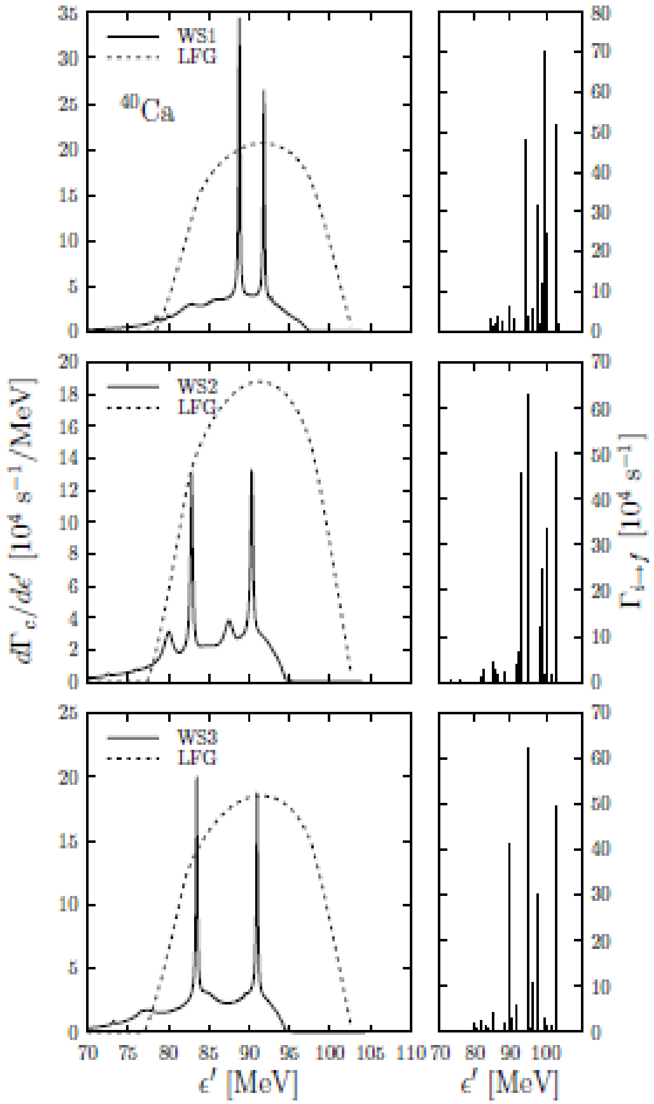
An example:

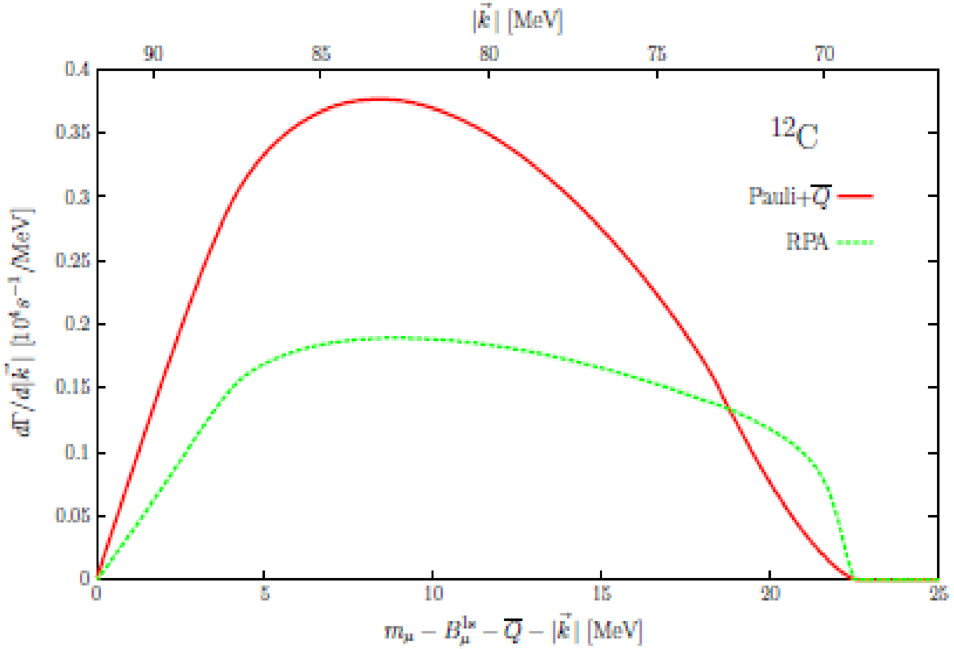
$$g_A^2\,\mathrm{Im}\,2\bar{U} \rightarrow g_A^2\left\{\frac{1}{3}\frac{\mathrm{Im}\,2\bar{U}}{|1-UV_\ell|^2} + \frac{2}{3}\frac{\mathrm{Im}\,2\bar{U}}{|1-UV_t|^2}\right\}$$





Shell model





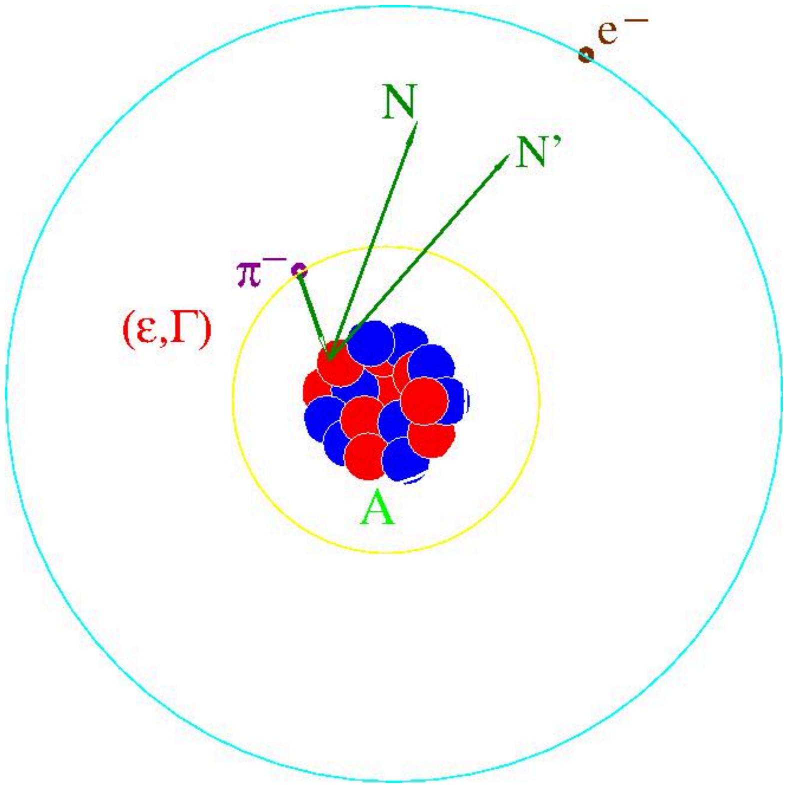
	discrete	total	LFG	%
WS1	29.10	37.12	36.73	−1.1
WS2	27.79	33.79	34.90	3.3
WS3	26.28	32.73	35.03	7.0

Nuclear Effects? SM vs FG

(Amaro, Nieves, Maieron and Valverde EPJA 24, 343)

Pionic Atoms

[J. Nieves et al., NPA 554 (1993) 509]



- Precise experimental measurements (spectroscopy techniques): **shifts $\varepsilon = B_{exp} - B_{em}$ and widths Γ** → information on the pion-nucleus interaction

- $\Pi(q^0, \vec{q}, \rho(r)) = 2q^0\, V_{opt}(r)$, we solve the Klein-Gordon equation
- Impulse approximation

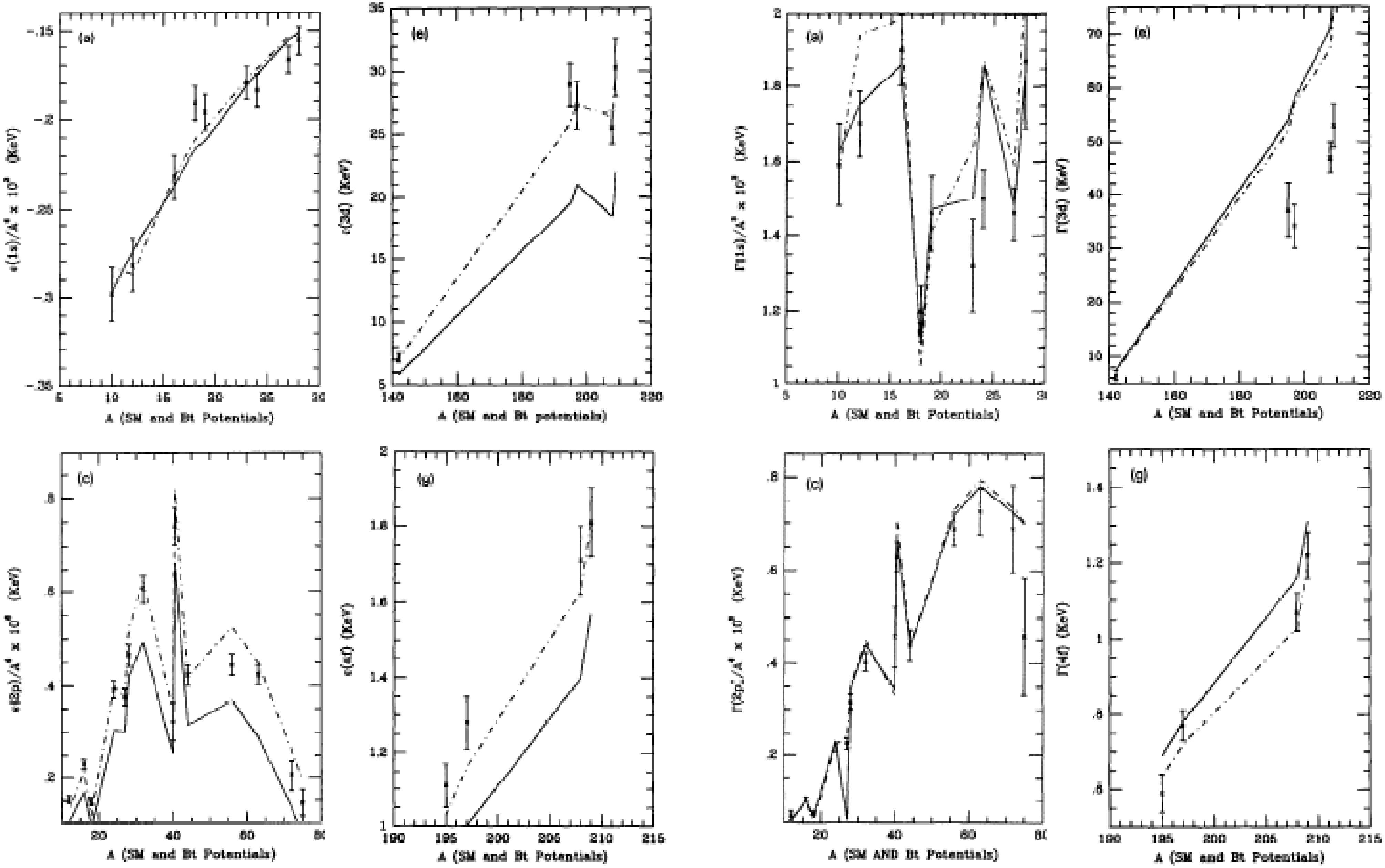
$$\Pi(q^0, \vec{q}) \approx T(q^0, \vec{q})\rho \Rightarrow \Gamma = 0$$

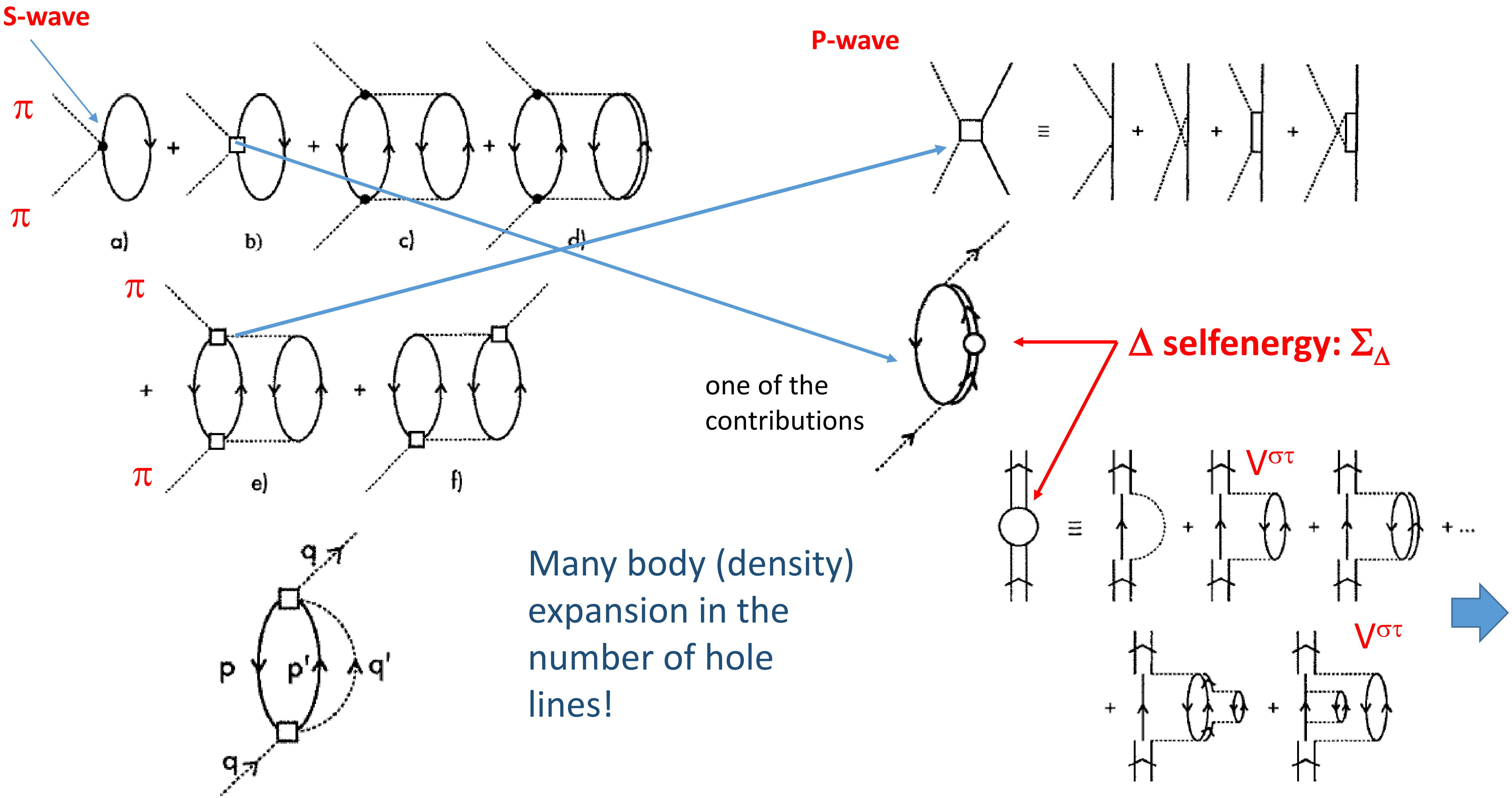
- Widths come from absorption by two nucleons: $\pi^- NN \to NN$ that give rise to complex terms, proportional to ρ^2 in $\Pi(q^0, \vec{q}, \rho(r))$.

- There exists experimental information on 1S and 2P levels in light nuclei and 3D and 4F in heavy nuclei. Phenomenological potentials fail to describe these data: **problem of the anomalies in pionic atoms.**

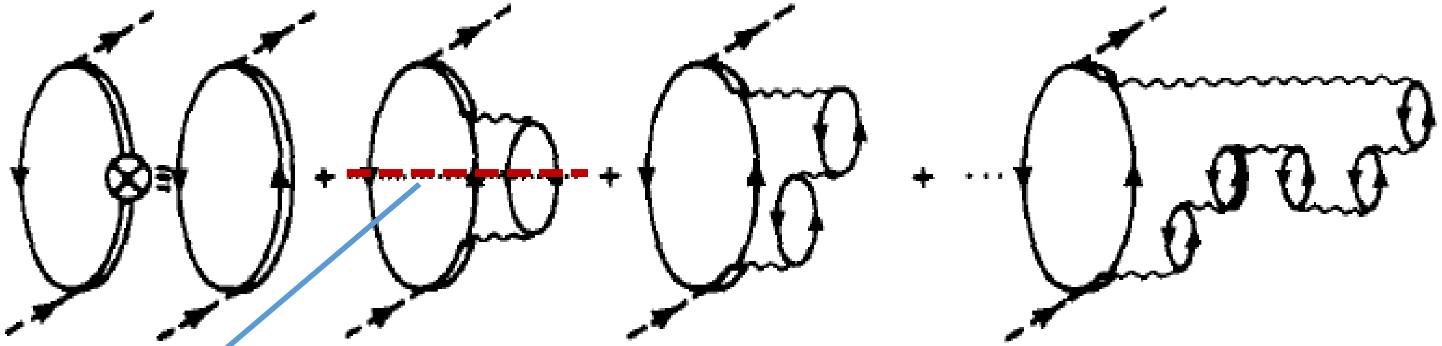
Precisions of the order of 5% through the whole periodic table.

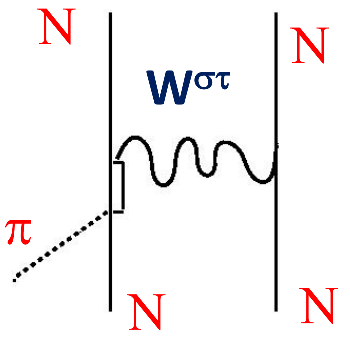
Range: eV to 20-30 KeV





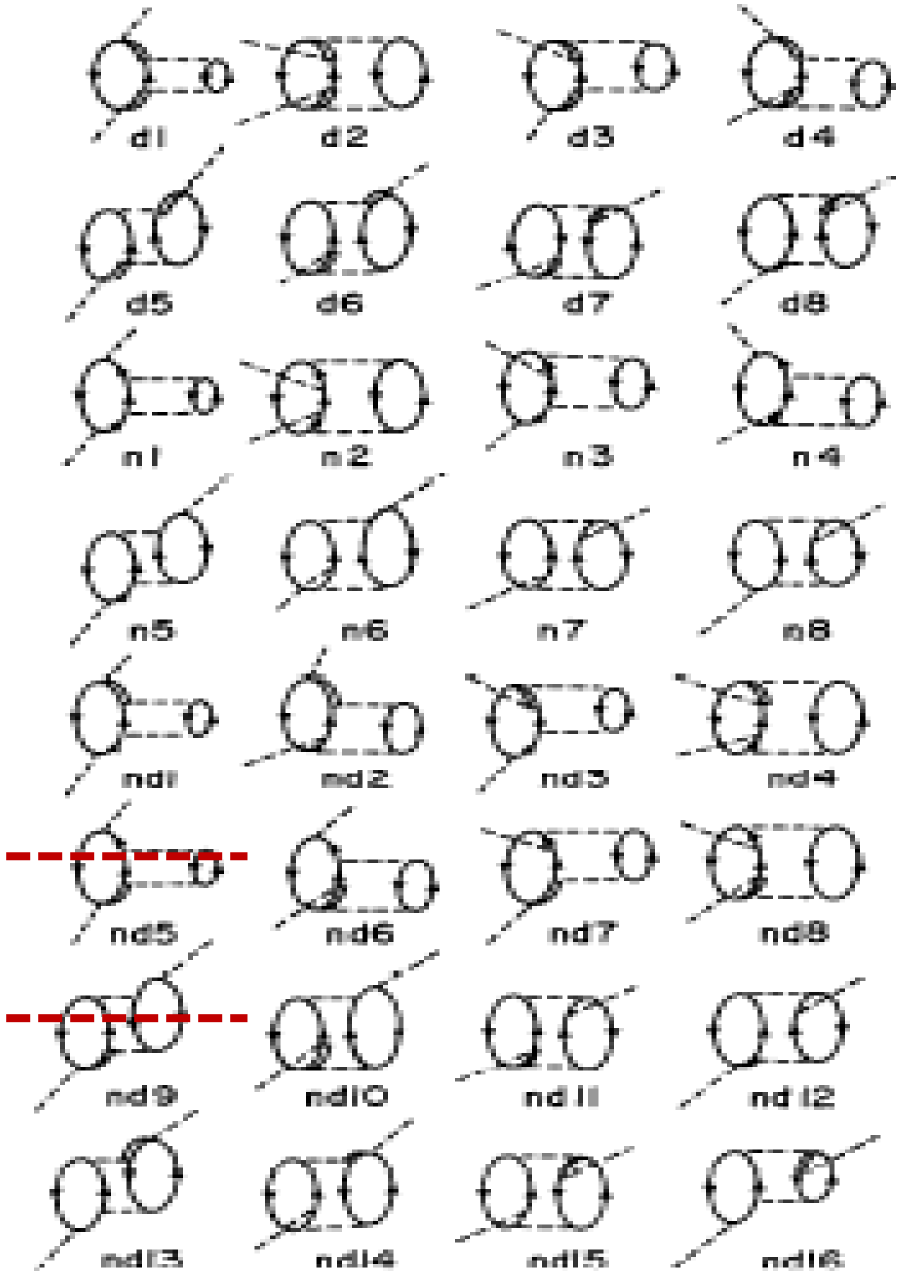
Many body (density) expansion in the number of hole lines!





$\pi NN \rightarrow NN$

2p2h



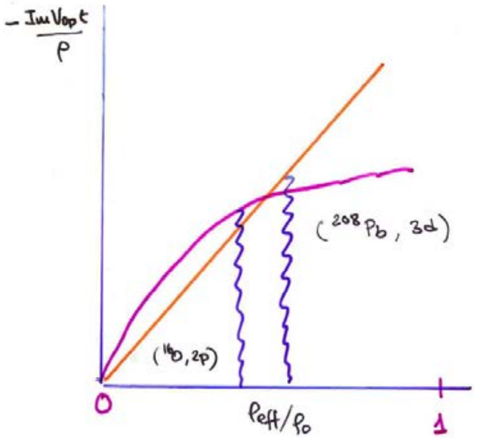
$$2\omega V_1^{(s)}(\boldsymbol{r}) = -4\pi[(1+\varepsilon)(b_0+\Delta b_0(\boldsymbol{r}))f(T)\rho + (1+\varepsilon)b_1(\rho_n-\rho_p)$$
$$+\, i(\mathrm{Im}\, B_0\,(1+\tfrac{1}{2}\varepsilon)2(\rho_p^2+\rho_p\rho_n) + \mathrm{Im}\, B_0^Q(T)(1+\tfrac{1}{2}\varepsilon)\rho^2)]$$

$$2\omega V_{\mathrm{opt}}^{(p)}(\boldsymbol{r}) = 4\pi \frac{M_N}{s}\left[\nabla \frac{P(\boldsymbol{r})}{1+4\pi g' P(\boldsymbol{r})}\nabla - \tfrac{1}{2}\varepsilon\Delta\left(\frac{P(\boldsymbol{r})}{1+4\pi g' P(\boldsymbol{r})}\right)\right]$$

Theoretical potential

$\mathrm{Im}\, V_{opt}^{STANDARD} \propto \rho^2$

$\mathrm{Im}\, V_{opt}^{NEW} \propto \frac{\rho_0}{2.72}\, \rho\, \mathrm{arctg}\, \frac{2.72\,\rho}{\rho_0}$ + L.L. Effect.



ANOMALIES: SIMULTANEOUS DESCRIPTION OF BOTH LIGHT AND HEAVY ATOMS

⇓

$\Gamma_{2p} \uparrow$ and $\Gamma_{3d} \downarrow$

$$\left.\frac{\Gamma_{3d}^{HEAVY}}{\Gamma_{2p}^{LIGHT}}\right|_{NEW} < \left.\frac{\Gamma_{3d}^{Heavy}}{\Gamma_{2p}^{light}}\right|_{Standard}$$

↑ QUENCHING PROPERTIES

low densities

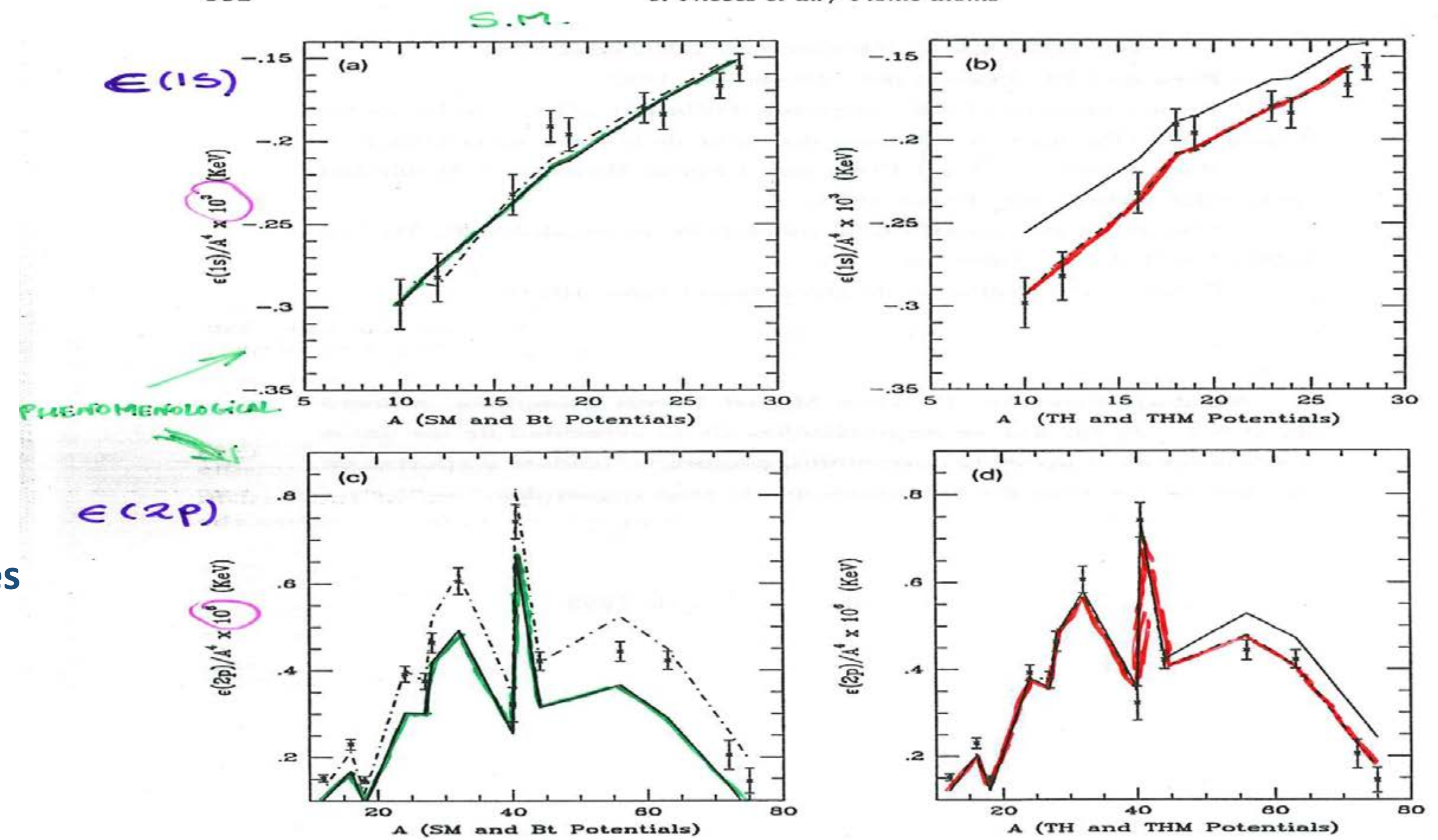
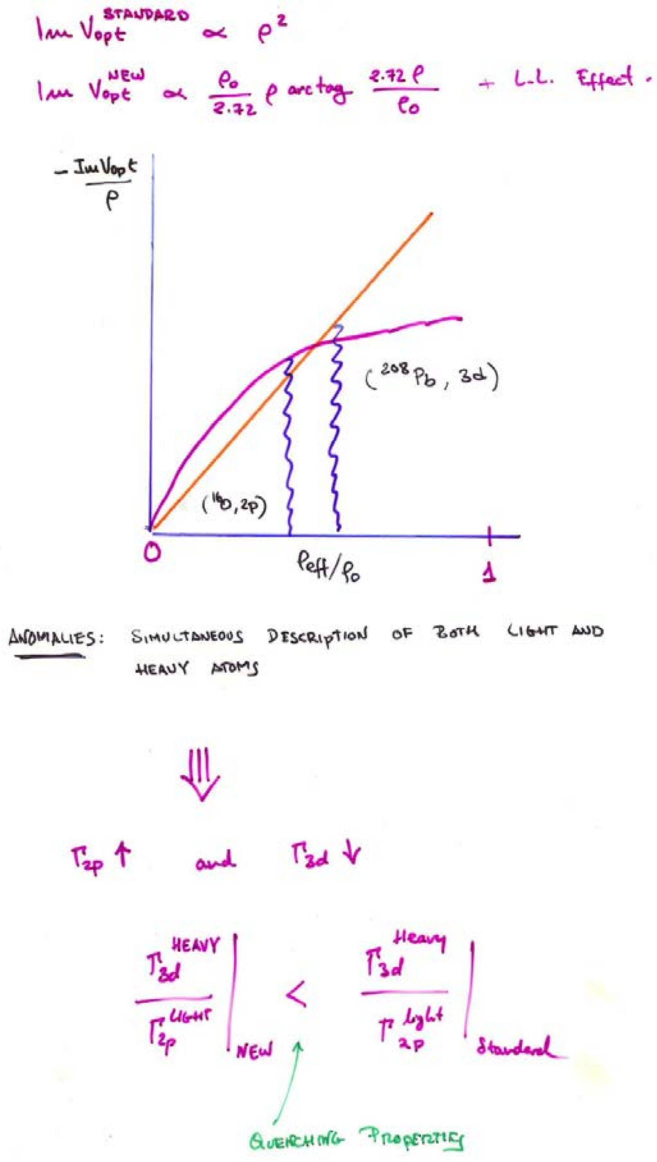


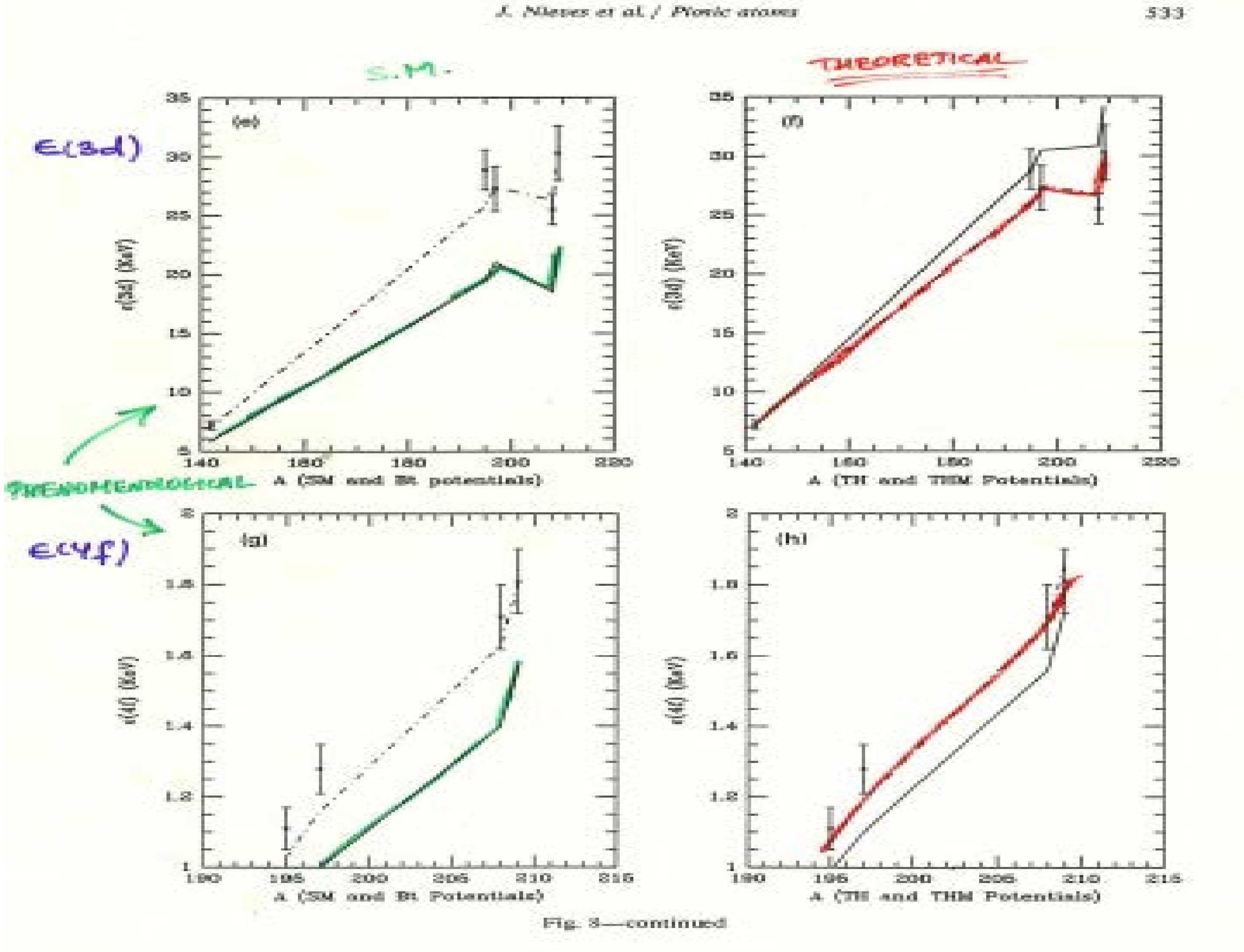
Fig. 8. Results for the shifts of different pionic states with different potentials. (a), (c), (e), (g): SM potential (solid line) of ref. [42]) given by eqs. (57) and (58), and Bt potential (dashed line) of ref. [1]) given by eqs. (59) and (60); (b), (d), (f), (h): Our potentials, theoretical potential (TH, solid line) and the semiphenomenological potential (THM, dashed line). The experimental values and the errors are statistical averages [see ref. [67]) for detailed values of the experimental results]. The lines between data points are just to guide the eye.

⇑ THEORETICAL.

Theoretical potential



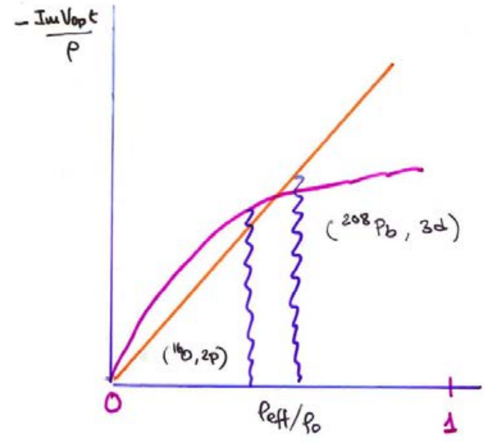
medium densities



Juan Nieves, IFIC (CSIC & UV)

$\mathrm{Im}\, V_{opt}^{STANDARD} \propto \rho^2$

$\mathrm{Im}\, V_{opt}^{NEW} \propto \frac{\rho_0}{2.72}\, \rho \,\mathrm{arctg}\, \frac{2.72\,\rho}{\rho_0}$ + L.L. Effect.



ANOMALIES: SIMULTANEOUS DESCRIPTION OF BOTH LIGHT AND HEAVY ATOMS

⇓

$\Gamma_{2p} \uparrow$ and $\Gamma_{3d} \downarrow$

$$\left.\frac{\Gamma_{3d}^{HEAVY}}{\Gamma_{2p}^{LIGHT}}\right|_{NEW} < \left.\frac{\Gamma_{3d}^{Heavy}}{\Gamma_{2p}^{light}}\right|_{Standard}$$

QUENCHING PROPERTIES

medium densities

Theoretical solution to the problem of the anomalies in pionic atoms !

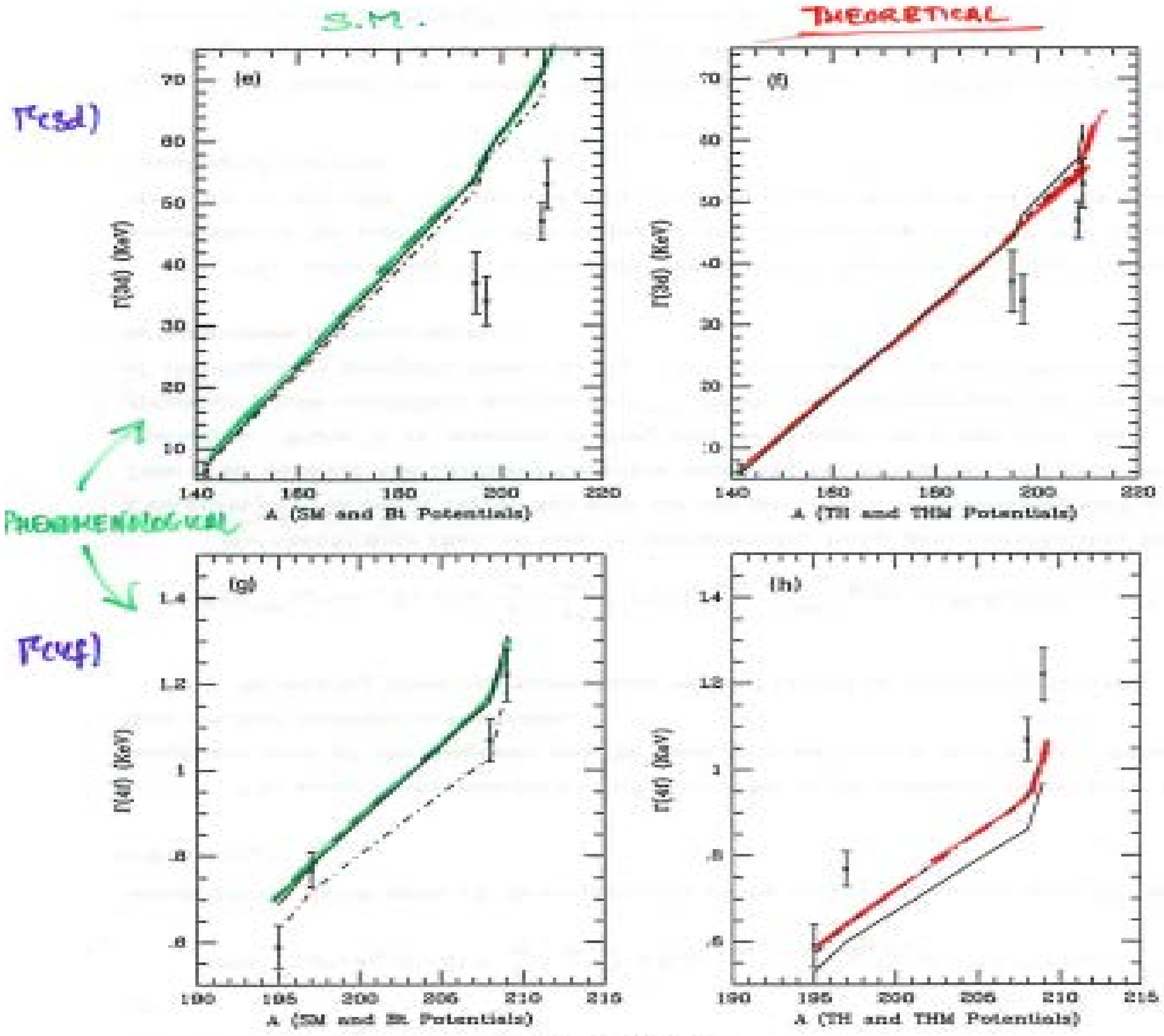


Fig. 9—continued

Theoretical potential

Juan Nieves, IFIC (CSIC & UV)

$\pi^{\pm}$ – nucleus reactions

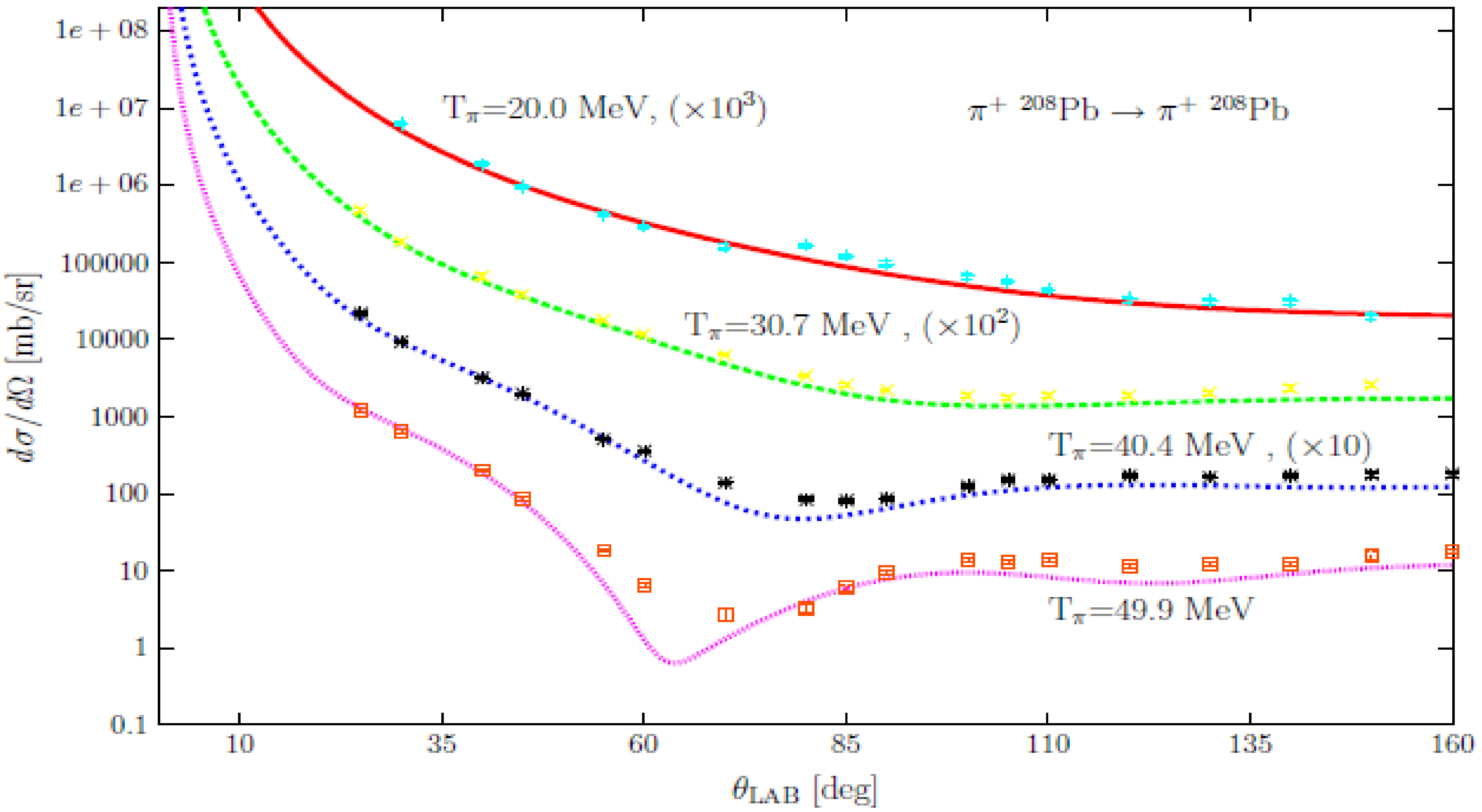
- $\pi^{\pm}$ – nucleus reactions [J. Nieves et al., NPA 554 (1993) 554]
  - ✓ $\pi^{\pm} A_Z \to \pi^{\pm} A_Z$ [elastic]
  - ✓ $\pi^{\pm} A_Z \to \pi' X$ [quasielastic]
  - ✓ $\pi^{\pm} A_Z \to X$ (no pions) [absorption]

- Determination of neutron distributions from pionic atom data [C. García-Recio et al., NPA 547 (1992) 473]
-
- Radiative pion capture [H.C. Chiang et al., NPA 510 (1990) 573]

$$(\pi^- A_Z)_{\text{bound}} \to \gamma X$$

- Chiral symmetry restoration [C. García-Recio et al., PLB 541 (2002) 64]

$$f_\pi(\rho)/f_\pi \to 0, \rho >> 0$$

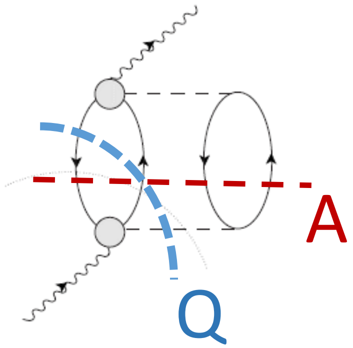


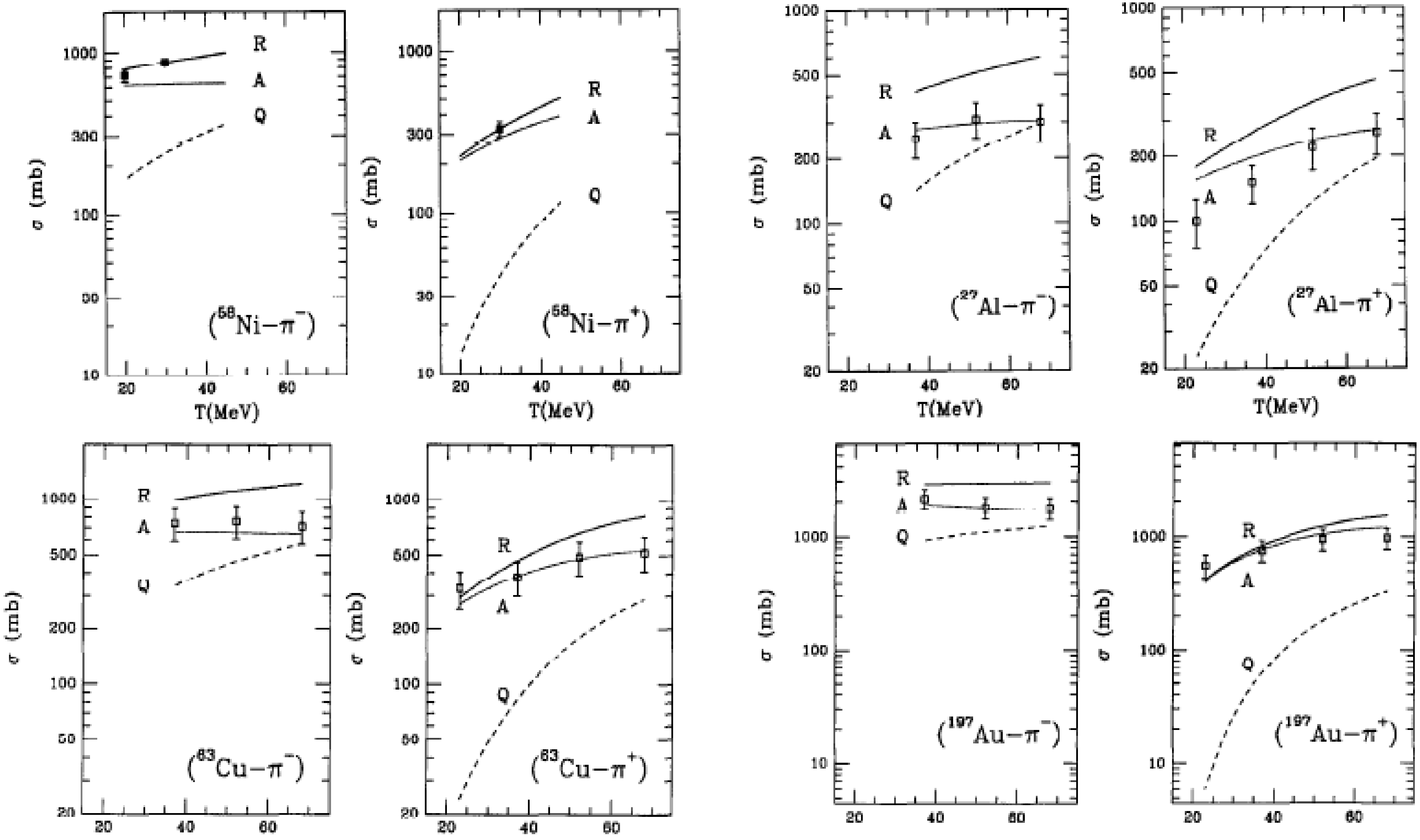
Juan Nieves, IFIC (CSIC & UV)

Q+R = A

Absorption + Quasielastic= Reaction

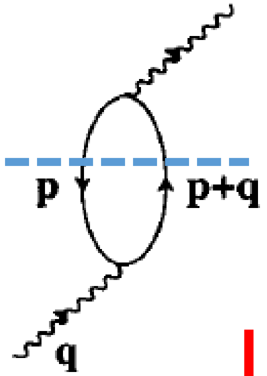
Q= pions which have changed either charge, energy or momentum

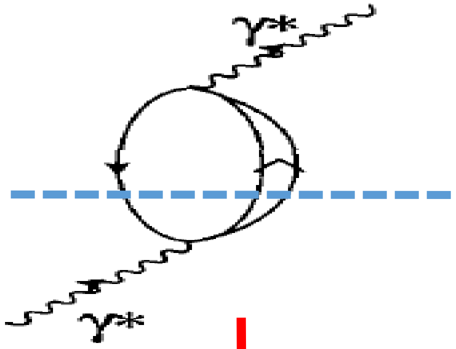


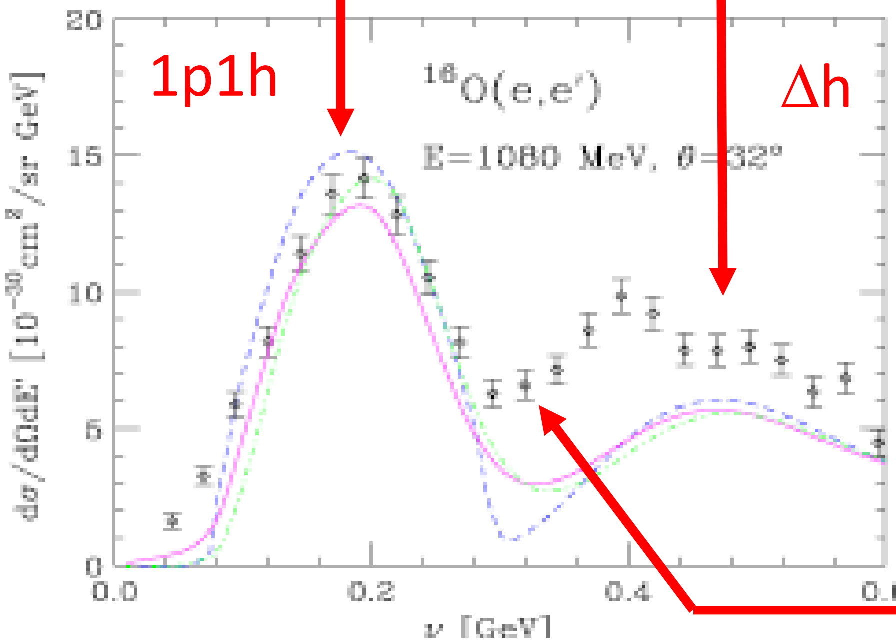


Inclusive electron-nucleus scattering

[A. Gil et al.,NPA 627 (1997) 543; NPA 627 (1997) 599]







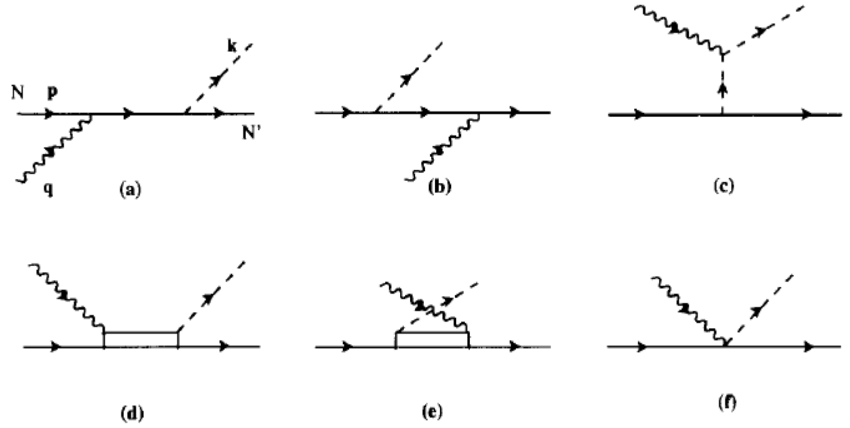
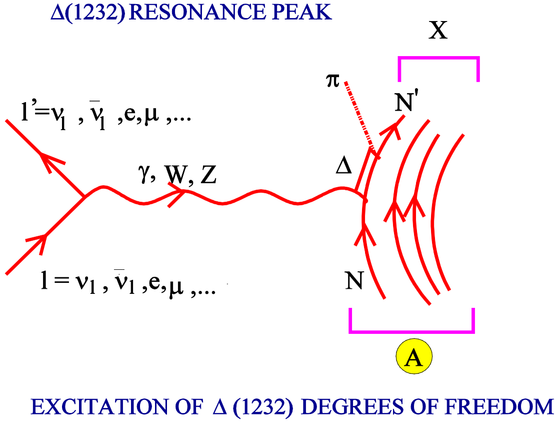
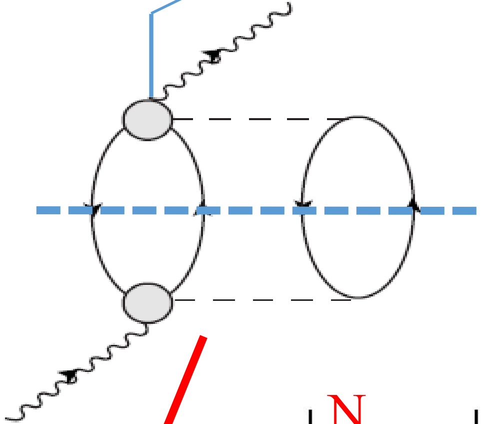
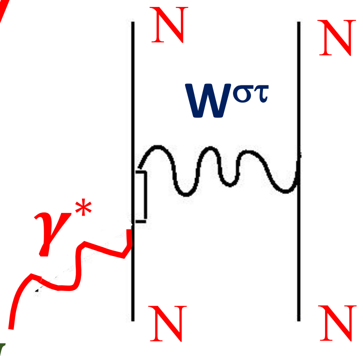


Fig. 2. Feynman diagrams considered for the $\gamma^* N \to \pi N$ reaction.





Dip region 2p2h



$\gamma^* NN \to NN$

Juan Nieves, IFIC (CSIC & UV)

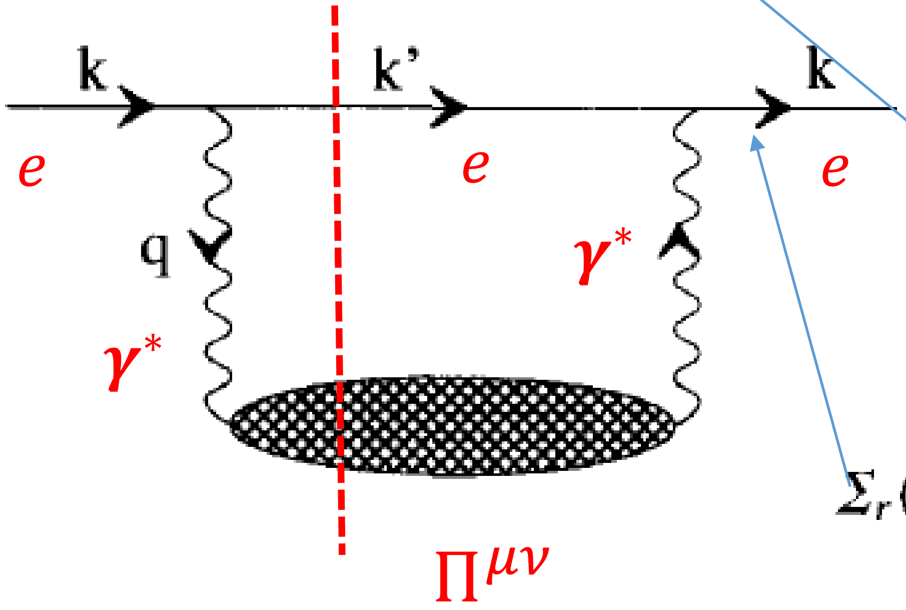
$$e + A_Z \to e'X$$

$$\frac{d^2\sigma}{d\Omega'_e dE'_e} = \frac{\alpha^2}{q^4}\frac{|\boldsymbol{k}'|}{|\boldsymbol{k}|} L^{\mu\nu} W_{\mu\nu}$$

$$L_{\mu\nu}(e,e') = 2\left(k'_{e\mu}k_{e\nu} + k'_{e\nu}k_{e\mu} + \frac{q^2}{2}g_{\mu\nu}\right)$$

$$W^{\mu\nu} = -\frac{1}{\pi e^2}\int d^3r \frac{1}{2}\left(\mathrm{Im}\,\Pi^{\mu\nu} + \mathrm{Im}\,\Pi^{\nu\mu}\right)$$

virtual photon selfenergy in the nuclear medium



$$\Gamma(k) = -2\frac{m_e}{E_e}\mathrm{Im}\,\Sigma$$

$$\Sigma_r(k) = ie^2\int \frac{d^4q}{(2\pi)^4}\bar{u}_r(k)\gamma_\mu \frac{(\not{k}' + m_e)}{k'^2 - m_e^2 + i\epsilon}\gamma_\nu u_r(k)\frac{\Pi_\gamma^{\mu\nu}(q)}{(q^2+i\epsilon)^2}$$

$$d\sigma = \Gamma(k)\,dt\,dS = -\frac{2m}{E_e}\mathrm{Im}\,\Sigma\,dl\,dS = -\frac{2m}{|\boldsymbol{k}|}\mathrm{Im}\,\Sigma\,d^3r$$

$$\sigma = -\int d^3r \frac{2m}{|\boldsymbol{k}|}\mathrm{Im}\,\Sigma(k,\rho(\boldsymbol{r}))$$

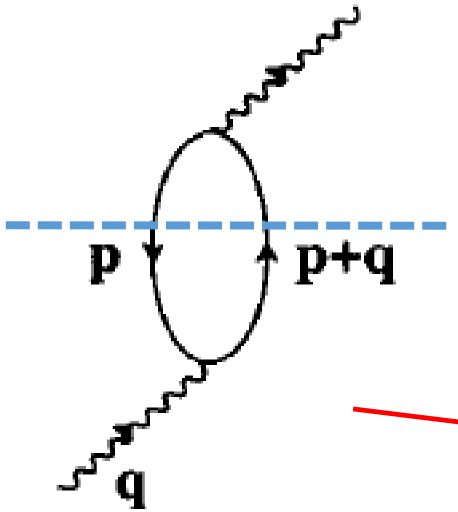
$$\Sigma(k) \to 2i\mathrm{Im}\,\Sigma(k)\Theta(k^0),$$
$$\Xi(k') \to 2i\mathrm{Im}\,\Xi(k')\Theta(k'^0),$$
$$\Pi^{\mu\nu}(q) \to 2i\mathrm{Im}\,\Pi^{\mu\nu}(q)\Theta(q^0),$$

where

$$\Xi(k') = \frac{1}{k'^2 - m_e^2 + i\epsilon}$$

Technically Cutkowsky's rules are used to obtain the imaginary part of the many body Feynman diagrams

Quasielastic peak



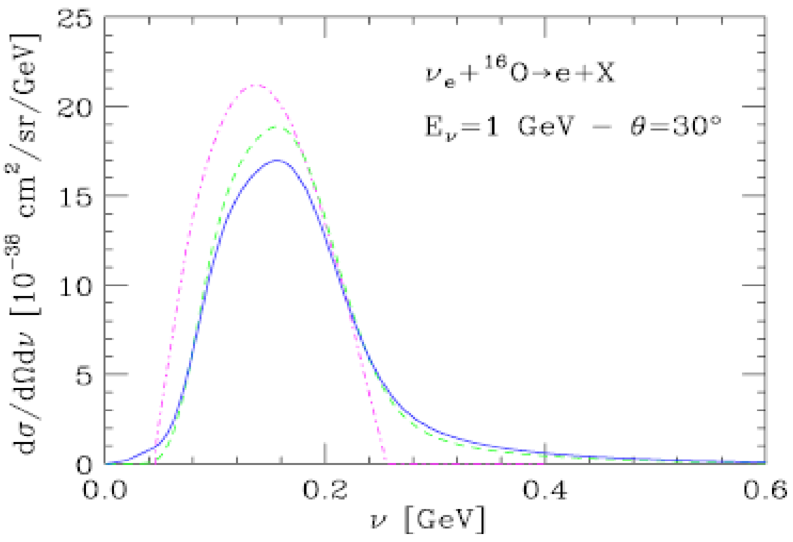
(lowest oder: **imaginary part of the Lindhard function**)

$$\mathrm{Im}\,\Pi^{00} = \tfrac{1}{2}\mathrm{Im}\,\bar{U}(q,\rho)\,\langle \mathrm{Tr}(V^0 V^{\dagger 0})\rangle,$$
$$\mathrm{Im}\,\Pi^{xx} = \tfrac{1}{2}\mathrm{Im}\,\bar{U}(q,\rho)\,\langle \mathrm{Tr}(V^x V^{\dagger x})\rangle.$$

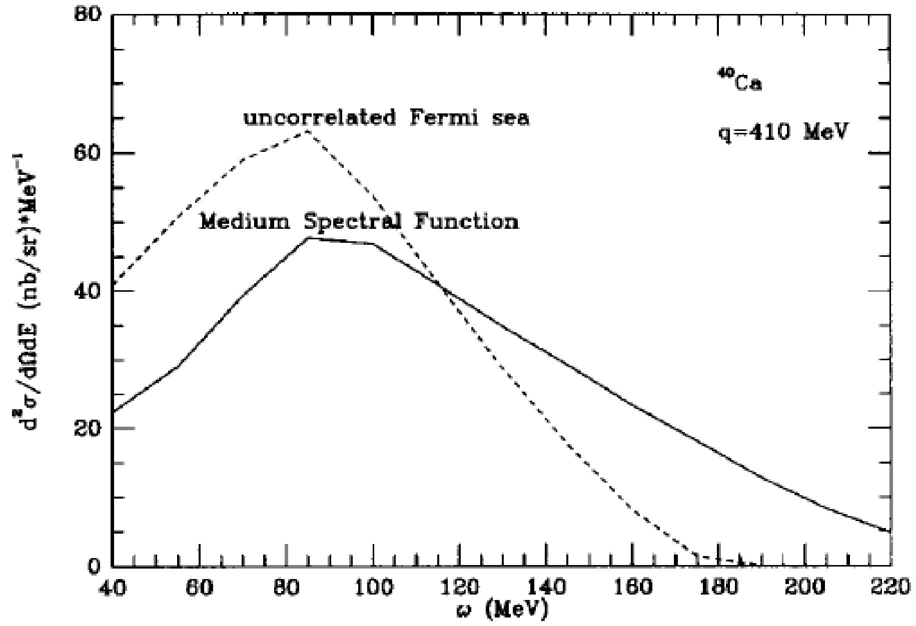
neglecting Fermi motion

$$p^2 = (p+q)^2 = M^2 \rightarrow q^2 = -2pq \rightarrow q^0 = -\,{q^2}/_{2M}$$

Cutkowsky's rules

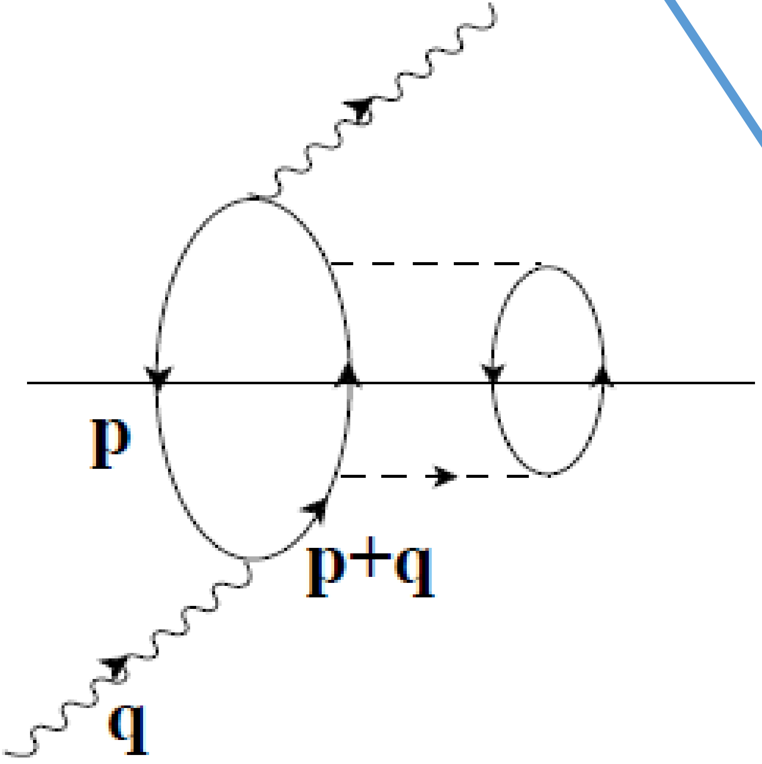


spectral function ?



- **Spectral Function (SF) + Final State Interaction (FSI):** dressing up the nucleon propagator of the hole (SF) and particle (FSI) states in the *ph* excitation

$$G(p) = \frac{n(\vec{p})}{p^0-\varepsilon(\vec{p})-i\epsilon} + \frac{1-n(\vec{p})}{p^0-\varepsilon(\vec{p})+i\epsilon}$$



- Change of nucleon dispersion relation:
  - ∗ hole ⇒ Interacting Fermi sea (SF)
  - ∗ particle ⇒ Interaction of the ejected nucleon with the final nuclear state (FSI)

$$G(p) \to \int_{-\infty}^{\mu} d\omega \frac{S_h(\omega,\vec{p})}{p^0-\omega-i\epsilon} + \int_{\mu}^{+\infty} d\omega \frac{S_p(\omega,\vec{p})}{p^0-\omega+i\epsilon}$$

The hole and particle spectral functions are related to nucleon self-energy Σ in the medium,

$$S_{p,h}(\omega,\vec{p}) = \mp\frac{1}{\pi}\frac{\mathrm{Im}\Sigma(\omega,\vec{p}\,)}{[\omega^2-\vec{p}^2-M^2-\mathrm{Re}\Sigma(\omega,\vec{p})]^2+[\mathrm{Im}\Sigma(\omega,\vec{p})]^2}$$

with $\omega \geq \mu$ or $\omega \leq \mu$ for S_p and S_h, respectively (μ is the chemical potential).

To take into account SF+FSI → replace $\mathrm{Im}\bar{U}_R^N(q)$ by the response function:

$$-\frac{1}{2\pi}\int_0^{+\infty} dp p^2 \int_{-1}^{+1} dx \int_{\mu-q^0}^{\mu} d\omega \mathbf{S_h}(\omega,\vec{\mathbf{p}})\mathbf{S_p}(\mathbf{q^0}+\omega,\mathbf{t})$$

with $t^2 = \vec{p}^{\,2} + \vec{q}^{\,2} + 2|\vec{p}||\vec{q}|x$.

This nuclear effect is additional to those due to RPA (long range) correlations !!

$$S_{p,h}(\omega,\vec{p}) \;=\; \mp\frac{1}{\pi}\frac{\mathrm{Im}\Sigma(\omega,\vec{p}\,)}{[\omega^2-\vec{p}^{\,2}-M^2-\mathrm{Re}\Sigma(\omega,\vec{p})]^2+[\mathrm{Im}\Sigma(\omega,\vec{p})]^2}$$

with $\omega \geq \mu$ or $\omega \leq \mu$ for S_p and S_h, respectively (μ is the chemical potential).

For non interacting fermions $\boxed{\Sigma = 0}$,

$$S_p(\omega,\vec{p}\,) \;=\; \frac{\theta(|\vec{p}\,|-k_F)}{2E(\vec{p}\,)}\delta(\omega-E(\vec{p}\,))$$

$$S_h(\omega,\vec{p}\,) \;=\; \frac{\theta(k_F-|\vec{p}\,|)}{2E(\vec{p}\,)}\delta(\omega-E(\vec{p}\,))$$

and only Pauli blocking is incorporated!!

The simplest description ⇒ relativistic Fermi Gas with non interacting fermions $\boxed{\Sigma = 0}$,

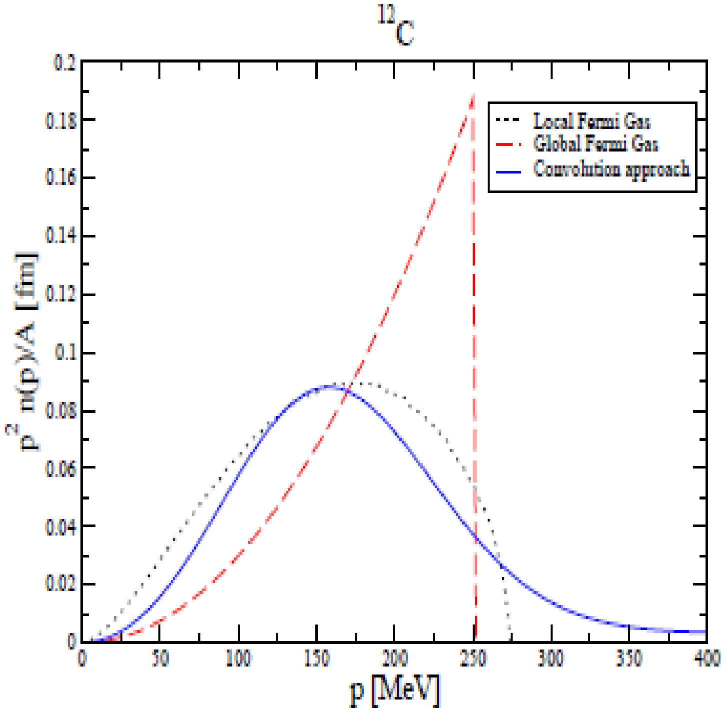
$$S_p(\omega,\vec{p}) = \frac{\theta(|\vec{p}\,| - k_F)}{2E(\vec{p})}\delta(\omega - E(\vec{p}))$$

$$S_h(\omega,\vec{p}) = \frac{\theta(k_F - |\vec{p}\,|)}{2E(\vec{p})}\delta(\omega - E(\vec{p}))$$

and only Pauli blocking is incorporated!!

Local vs Global Fermi Gas ?

$$k_F^{p,n}(r) = [3\pi^2\rho^{p,n}(r)]^{1/3} \text{ vs } k_F^{p,n} = \text{cte ?}$$



Local vs Global Fermi Gas ?

$$k_F(r) = \left[3\pi^2\rho(r)/2\right]^{1/3} \text{ vs } k_F = \text{cte ?}$$

$$S_h(\omega,\vec{p}) = \delta(\omega - E(\vec{p}))\theta(k_F - |\vec{p}|)/2\omega$$

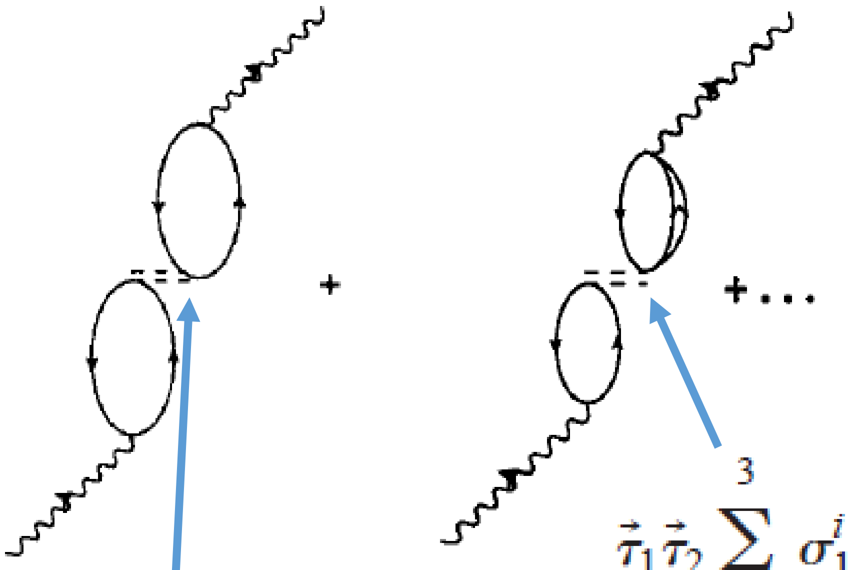
$$n^{\rm RgFG}(|\vec{p}|) = \frac{4V}{(2\pi)^3}\int d\omega 2\omega S_h(\omega,\vec{p}) = \frac{3A}{4\pi k_F^3}\theta(k_F - |\vec{p}|)$$

$$n^{\rm LDA}(|\vec{p}|) = 4\int\frac{d^3r}{(2\pi)^3}\int d\omega 2\omega S_h(\omega,\vec{p}) = 4\int\frac{d^3r}{(2\pi)^3}\theta(\mathbf{k_F(r)} - |\vec{p}|)$$

$$\left(\int d^3p\, n(|\vec{p}|) = A\right)$$

Convolution approach: C. Ciofi degli Atti, S. Liuti, and S. Simula, PRC 53, 1689 (1996), provide realistic distribution due to short-range correlations !

Polarization effects (RPA) at QE peak



$$\vec{\tau}_1 \vec{\tau}_2 \sum_{i,j=1}^{3} \sigma_1^i \sigma_2^j V_{ij}^{\sigma\tau}$$

$$c_0\{f_0(\rho) + f_0'(\rho)\vec{\tau}_1\vec{\tau}_2 + g_0(\rho)\vec{\sigma}_1\vec{\sigma}_2$$

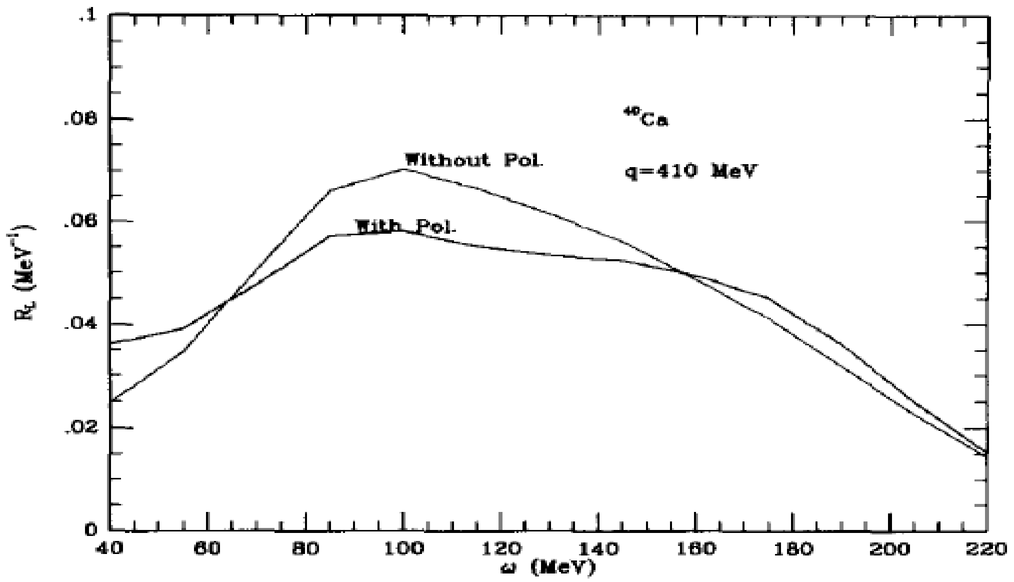
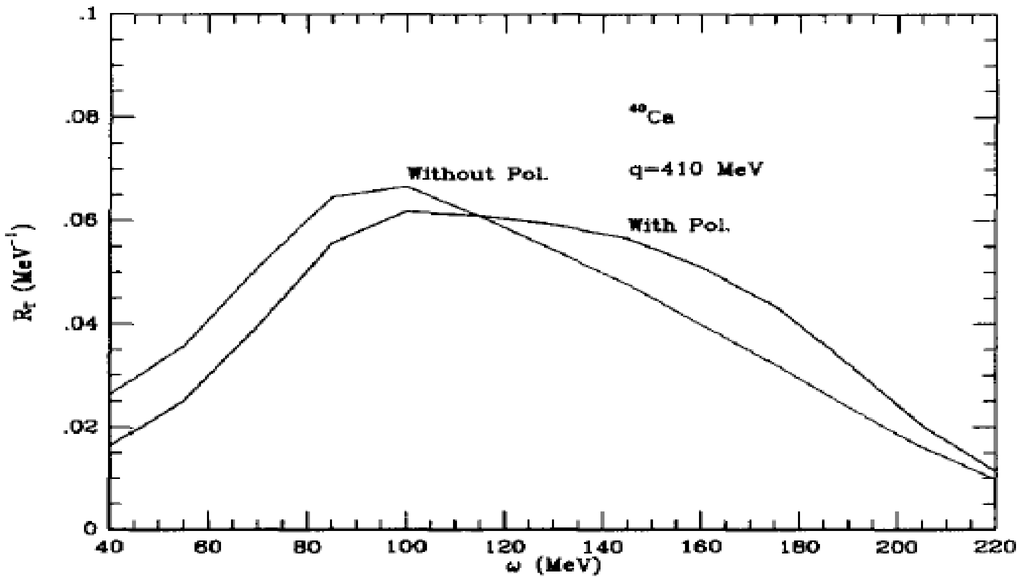
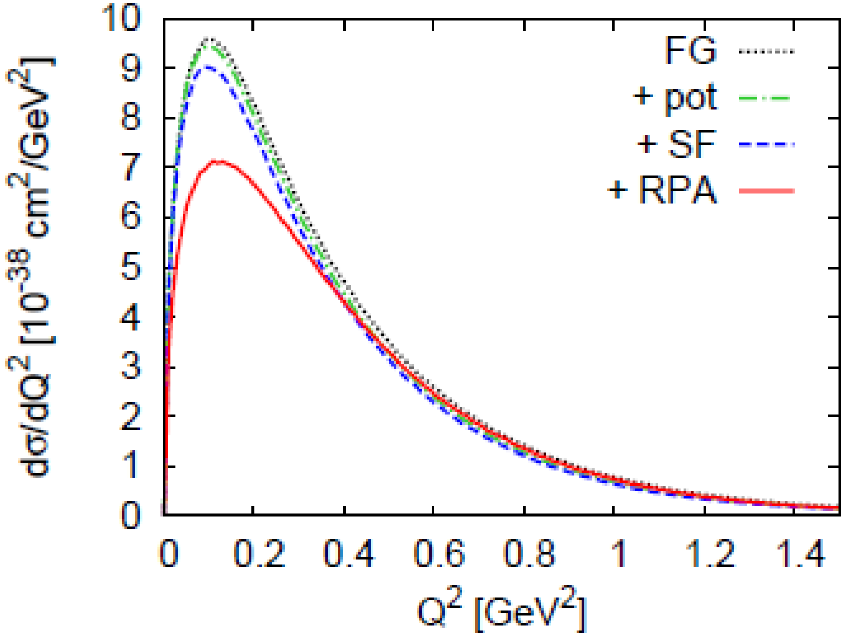


Fig. 37. Polarization (RPA) effect in the evaluation of R_L.

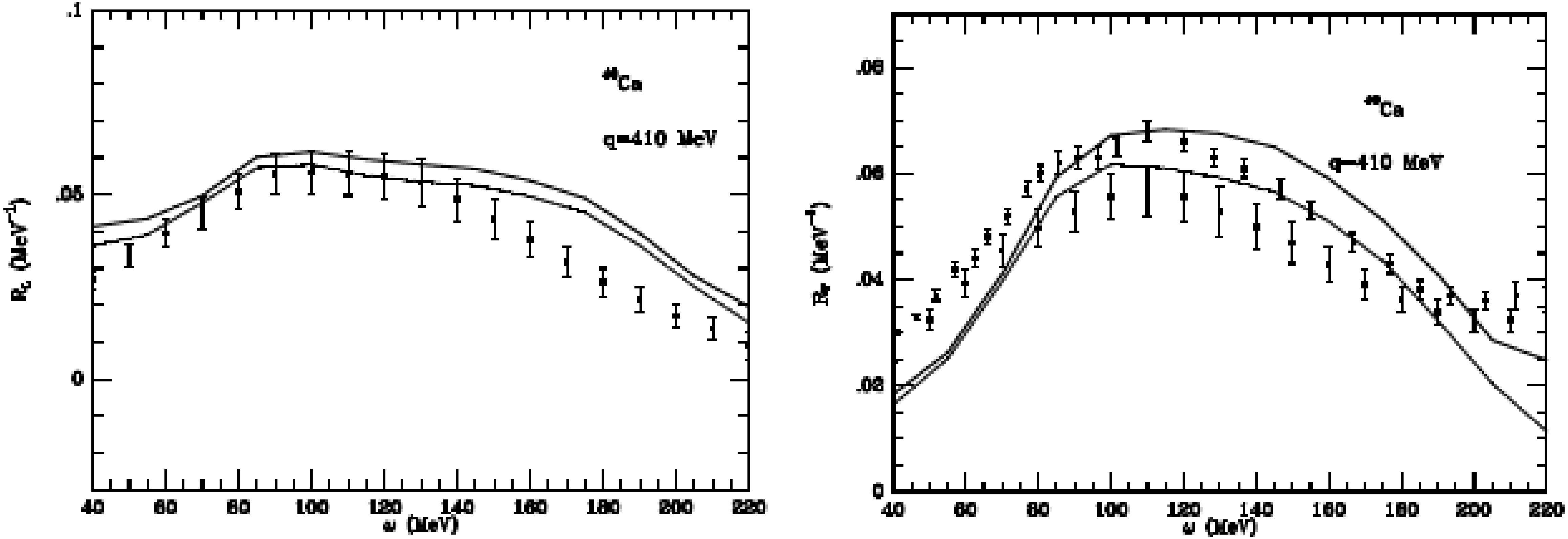




RPA vs SF effects: Differential cross sections for the CCQE reaction on ^{12}C averaged over the MiniBooNE flux

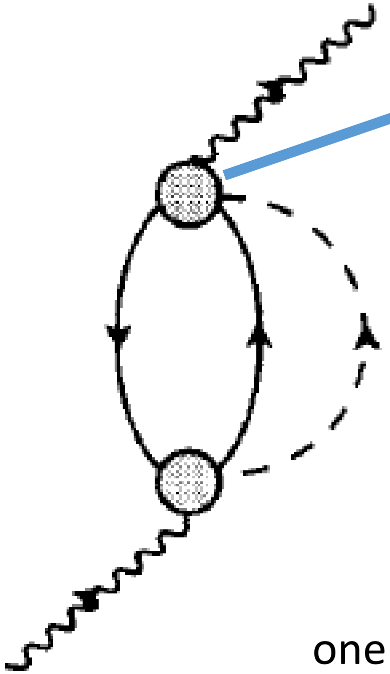
(Alvarez-Ruso L et al., 2009 AIP Conf. Proc. 1189 151)

$RPA \gg SF$ ← **It depends on the specific kinematics and observable !**



R_L and R_T QE response functions for $e + {}^{40}\mathrm{Ca} \to e' + X$

Pion production



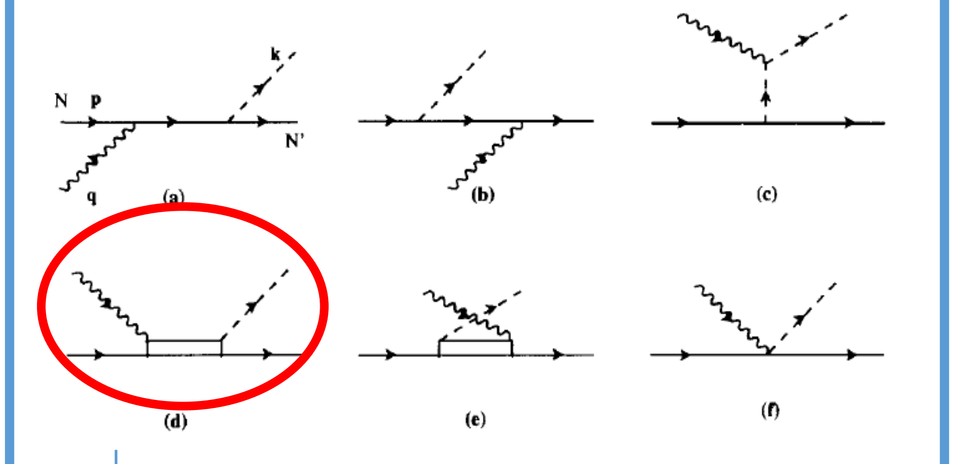
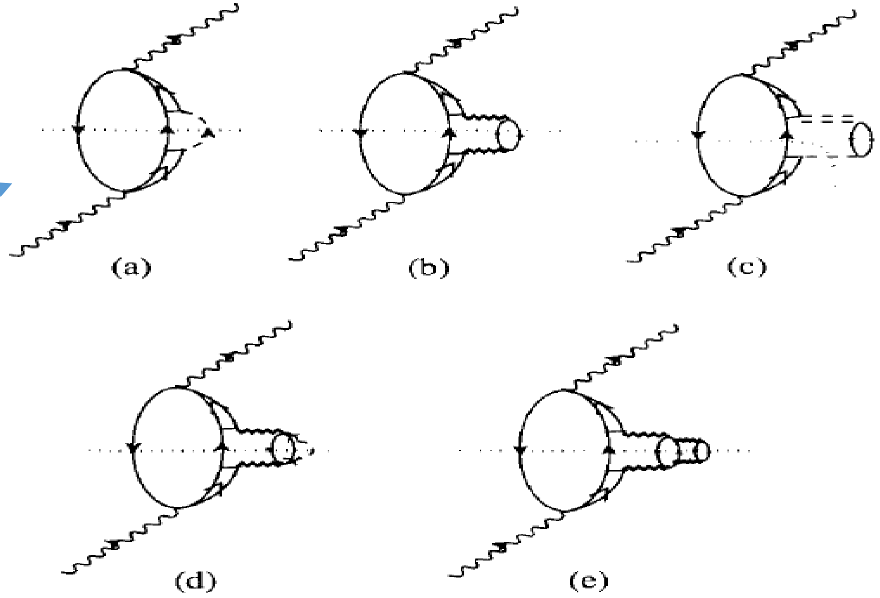
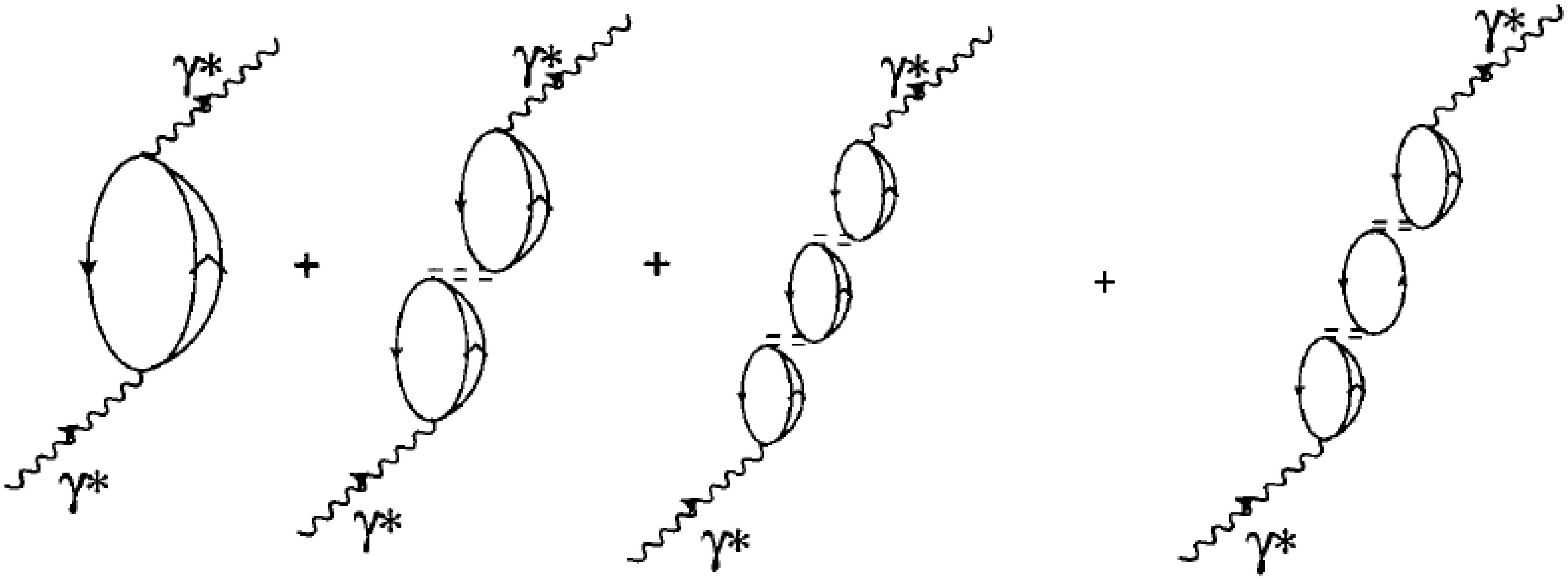


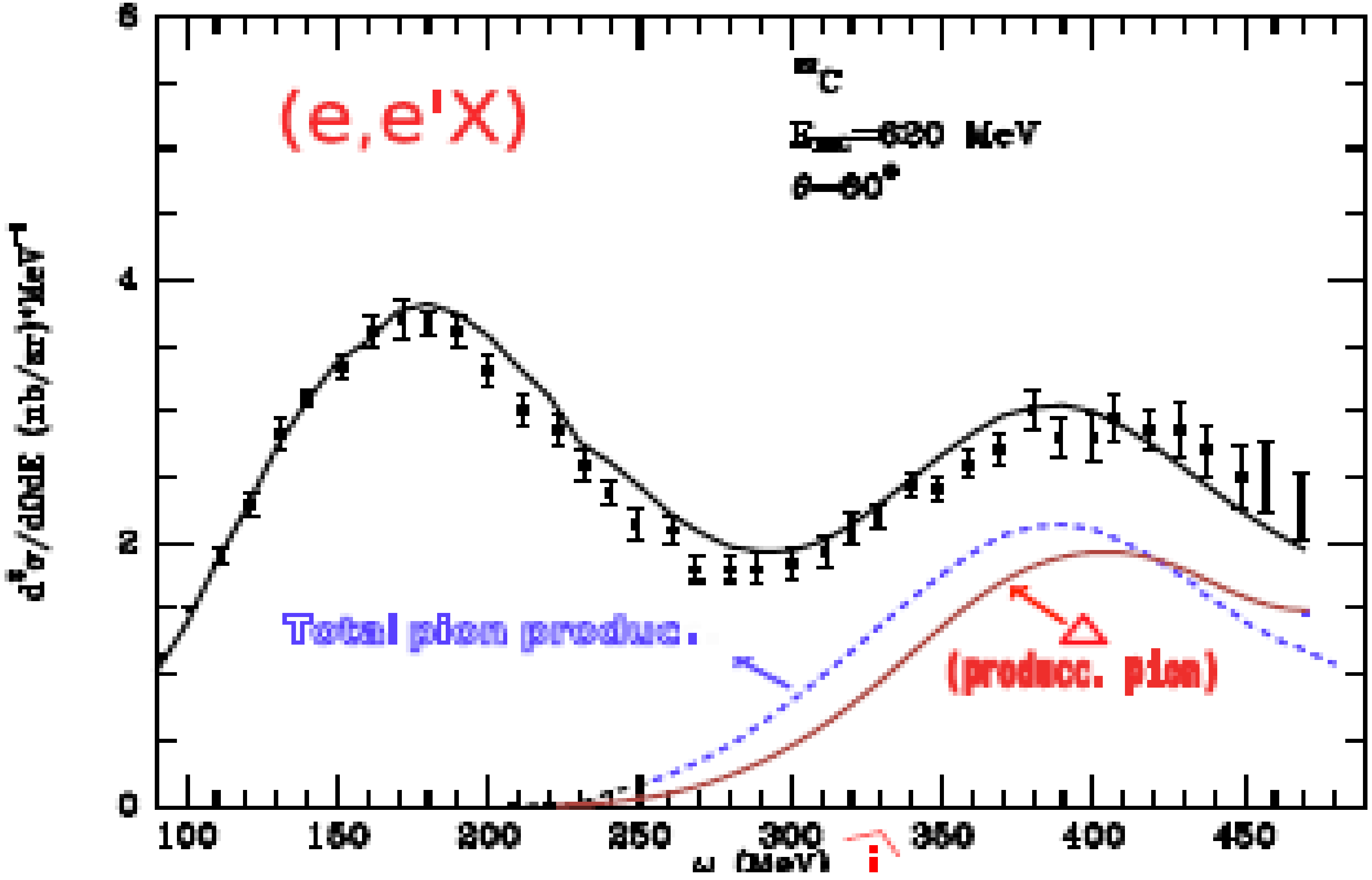
Fig. 2. Feynman diagrams considered for the $\gamma^* N \to \pi N$ reaction.

one of the terms generates the
Δ contribution



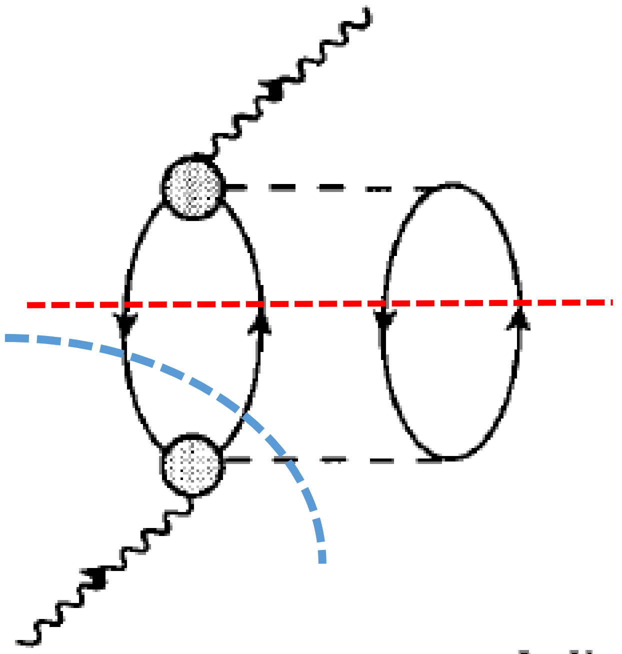


RPA corrections to the dominant Δh term

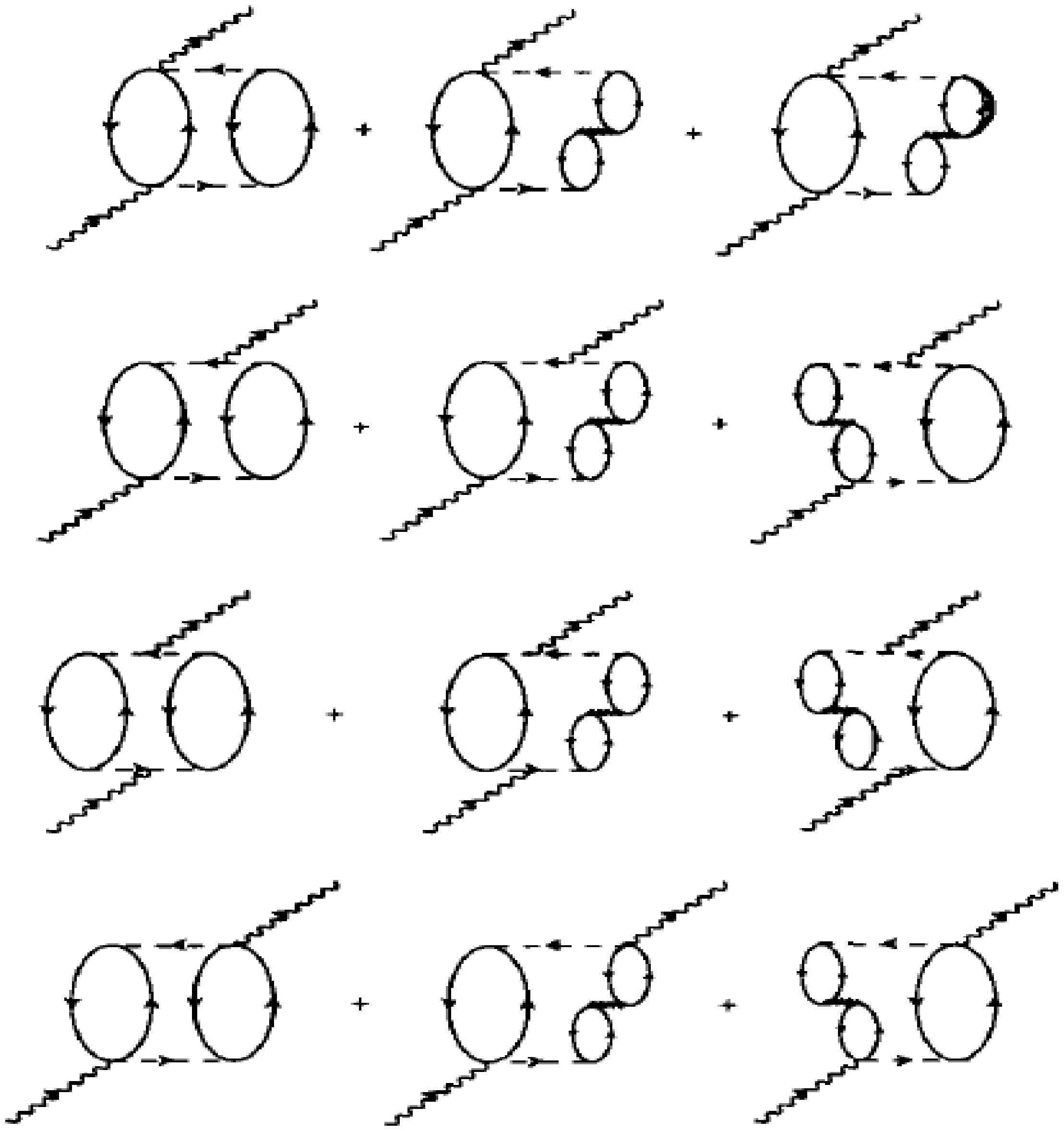


- Δ dominant component of the pion production contribution
- Missing strength both at the dip region and the Δ peak

2p2h (two body absorption) contributions



RPA corrections to 2p2h contributions



$$\mathrm{Im}\,U_N \to a\frac{\mathrm{Im}\,U_N}{|1-U_\lambda(q)V_l|^2} + b\frac{\mathrm{Im}\,U_N}{|1-U_\lambda V_t|^2}$$

+.....

Two cuts: $\gamma^* NN \to NN$

$\gamma^* N \to N\pi\ (dressed)$

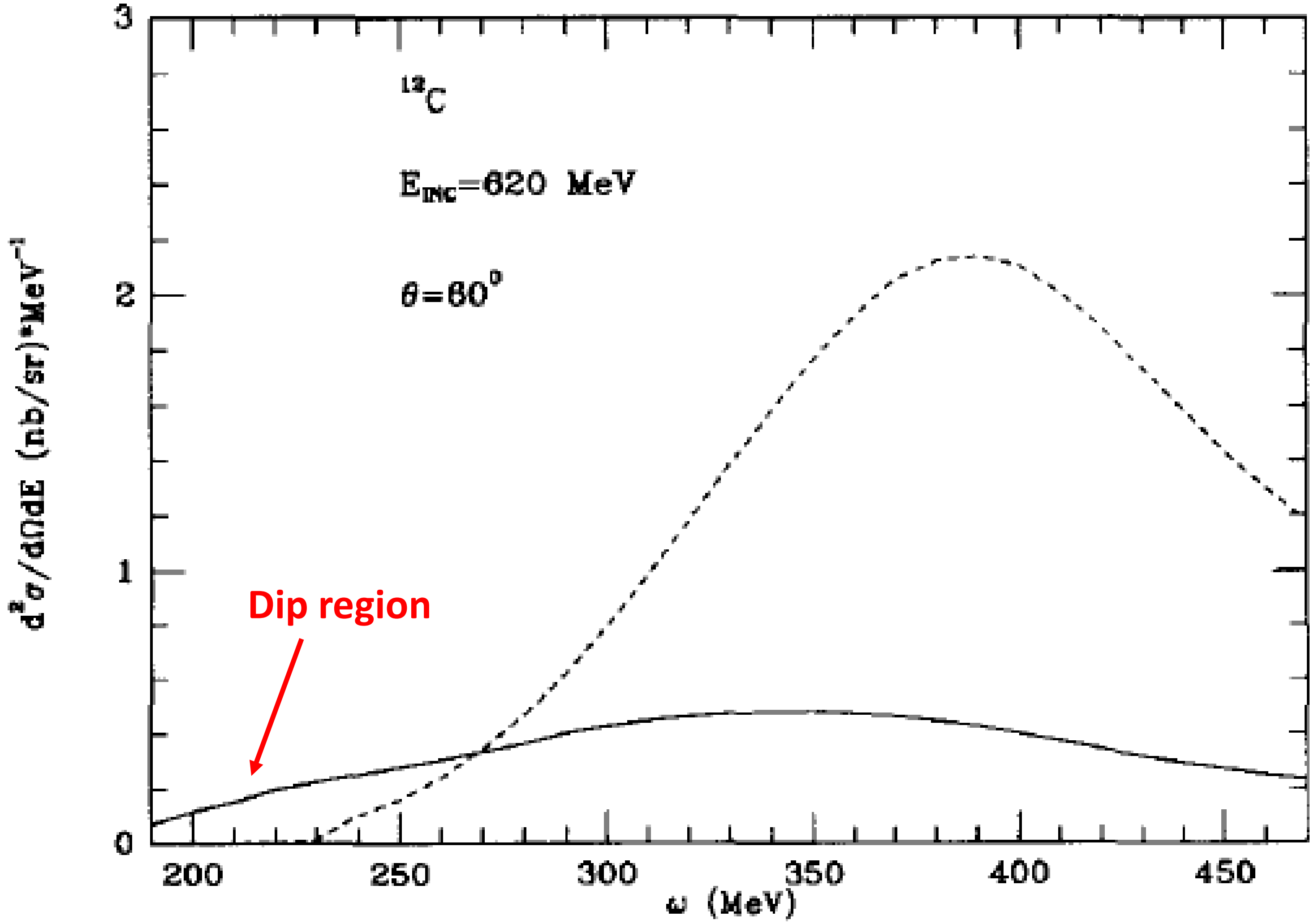


Fig. 44. Two-body photon absorption (solid line) versus pion production (dotted line) for ^{12}C.

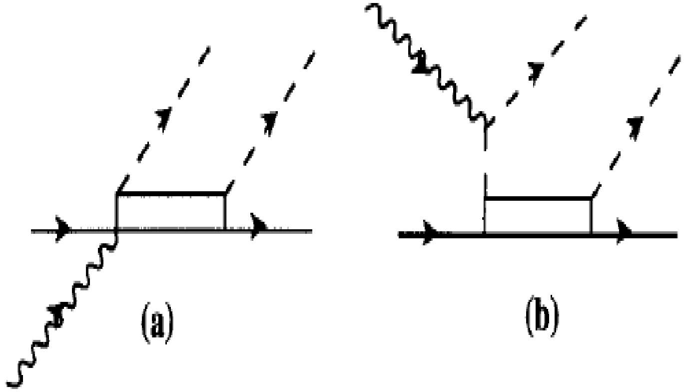
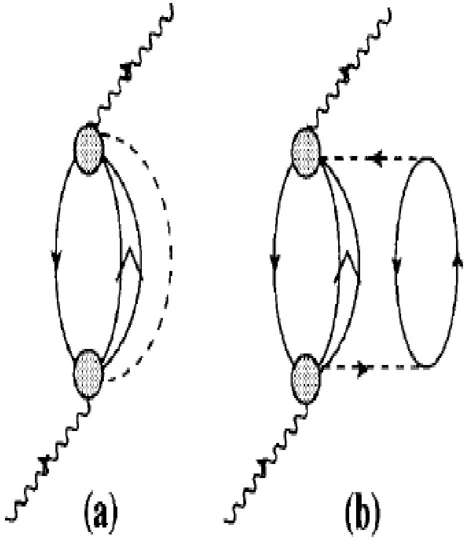
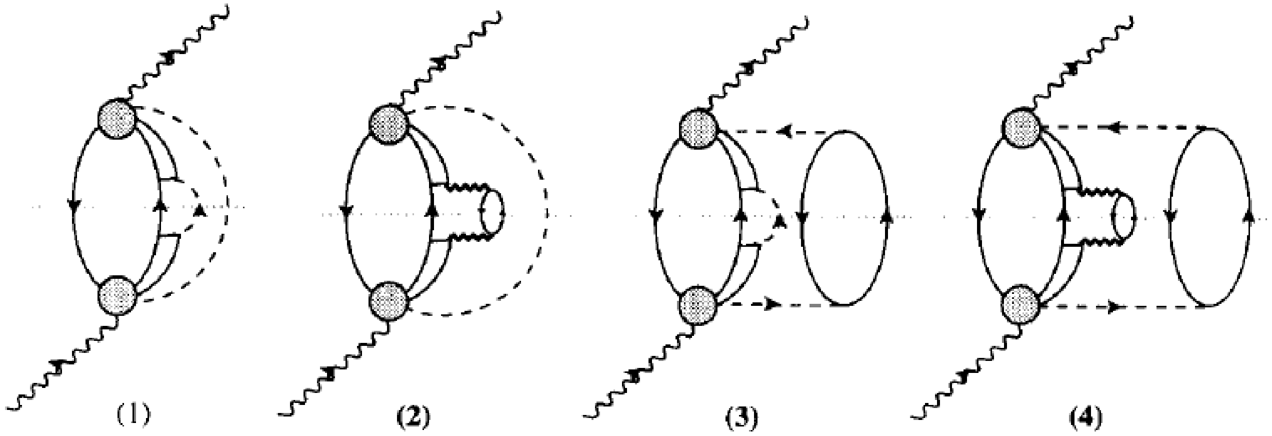


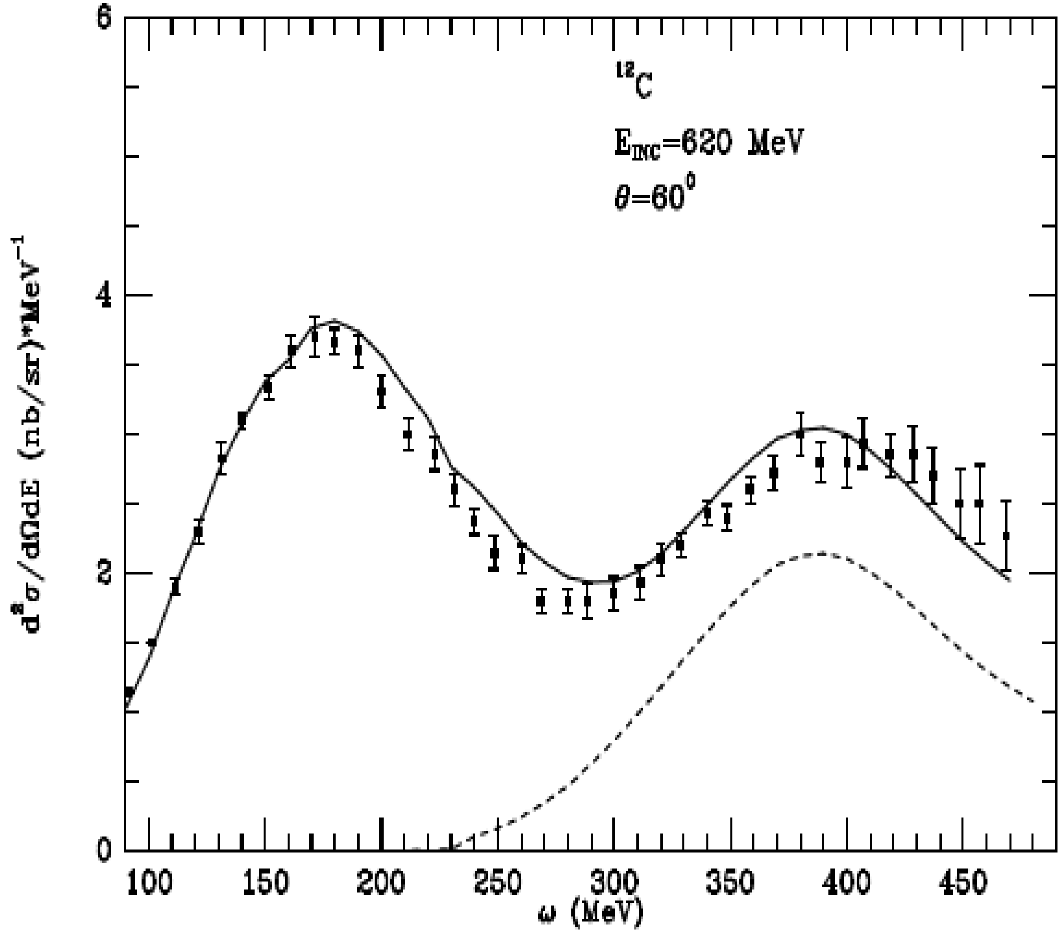
Fig. 23. Relevant Feynman diagrams that enter in the evaluation of the $\gamma^* N \to N2\pi$ cross section.

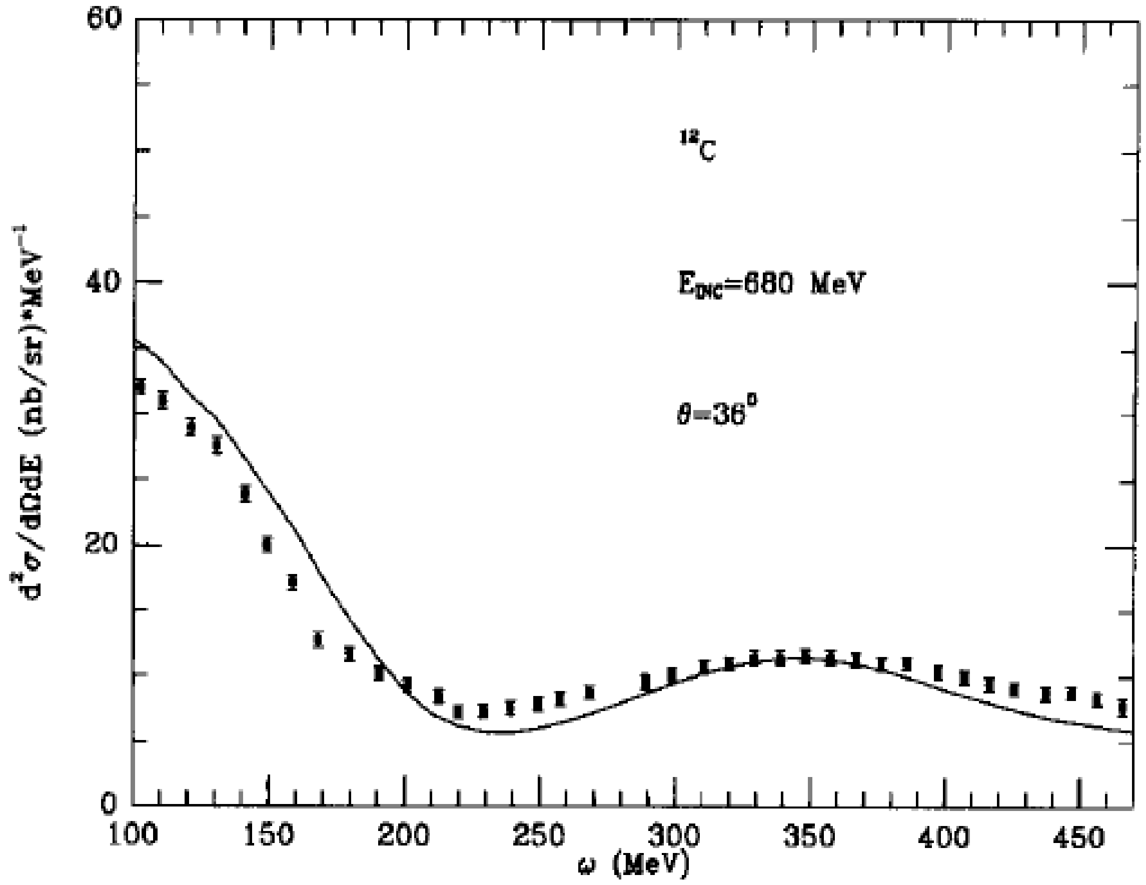
Additional contributions generated from the channel

$$\gamma^* N \to N\pi\pi$$



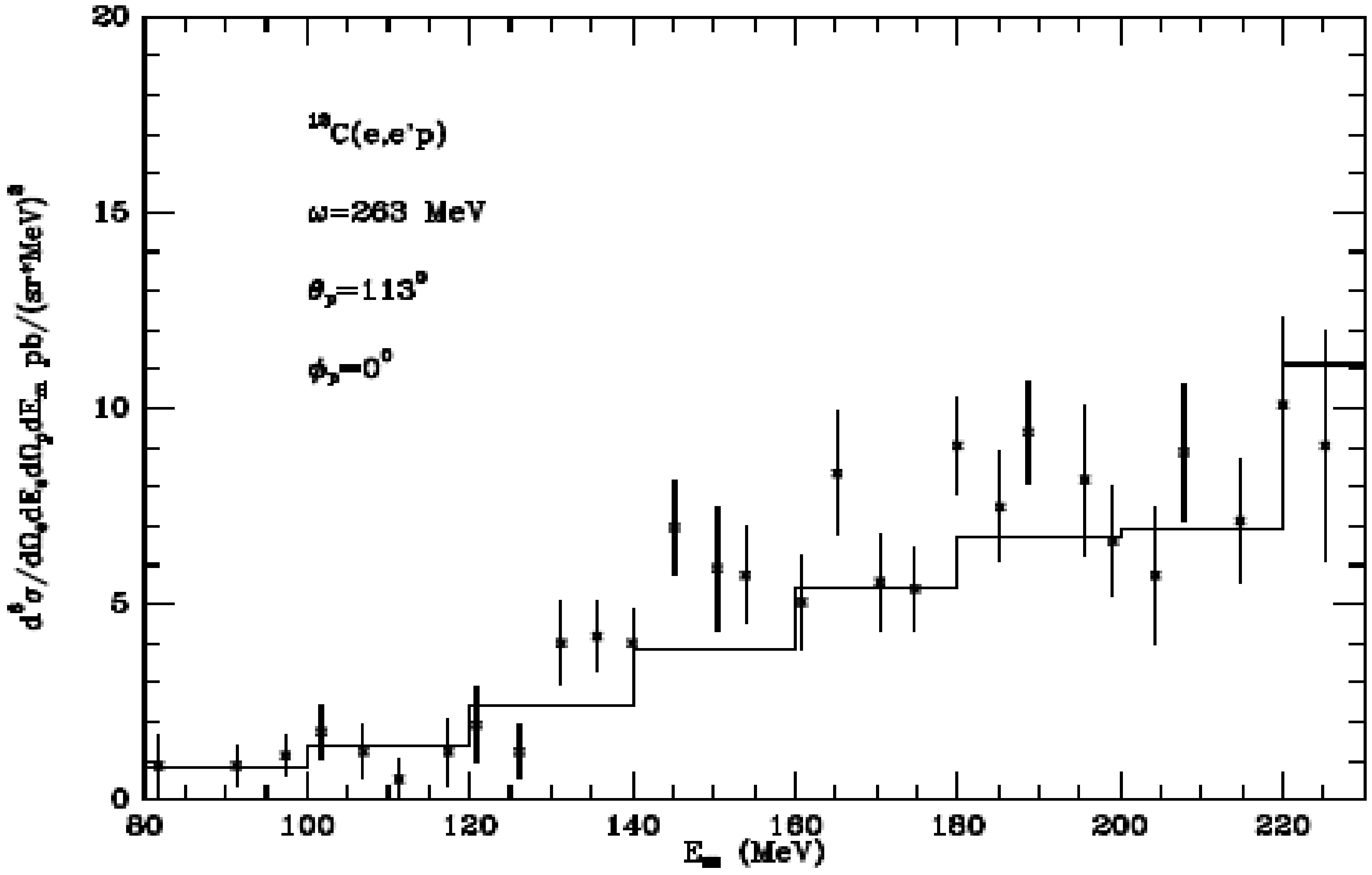




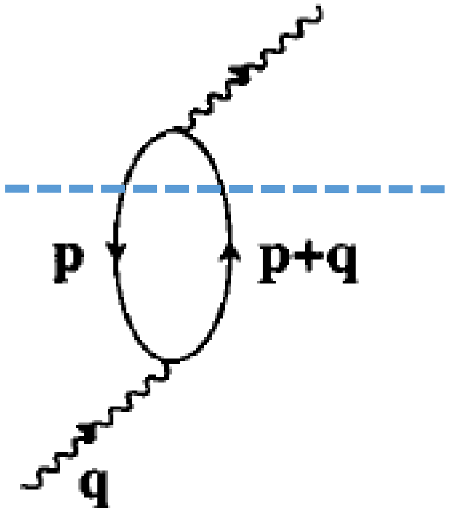


$$e + {}^{12}\mathrm{C} \rightarrow e' + X$$

and by means of a Monte Carlo simulation we obtain cross sections for the processes $(e, e'N)$, $(e, e'NN)$, $(e, e'\pi)$, ...



Important difference:



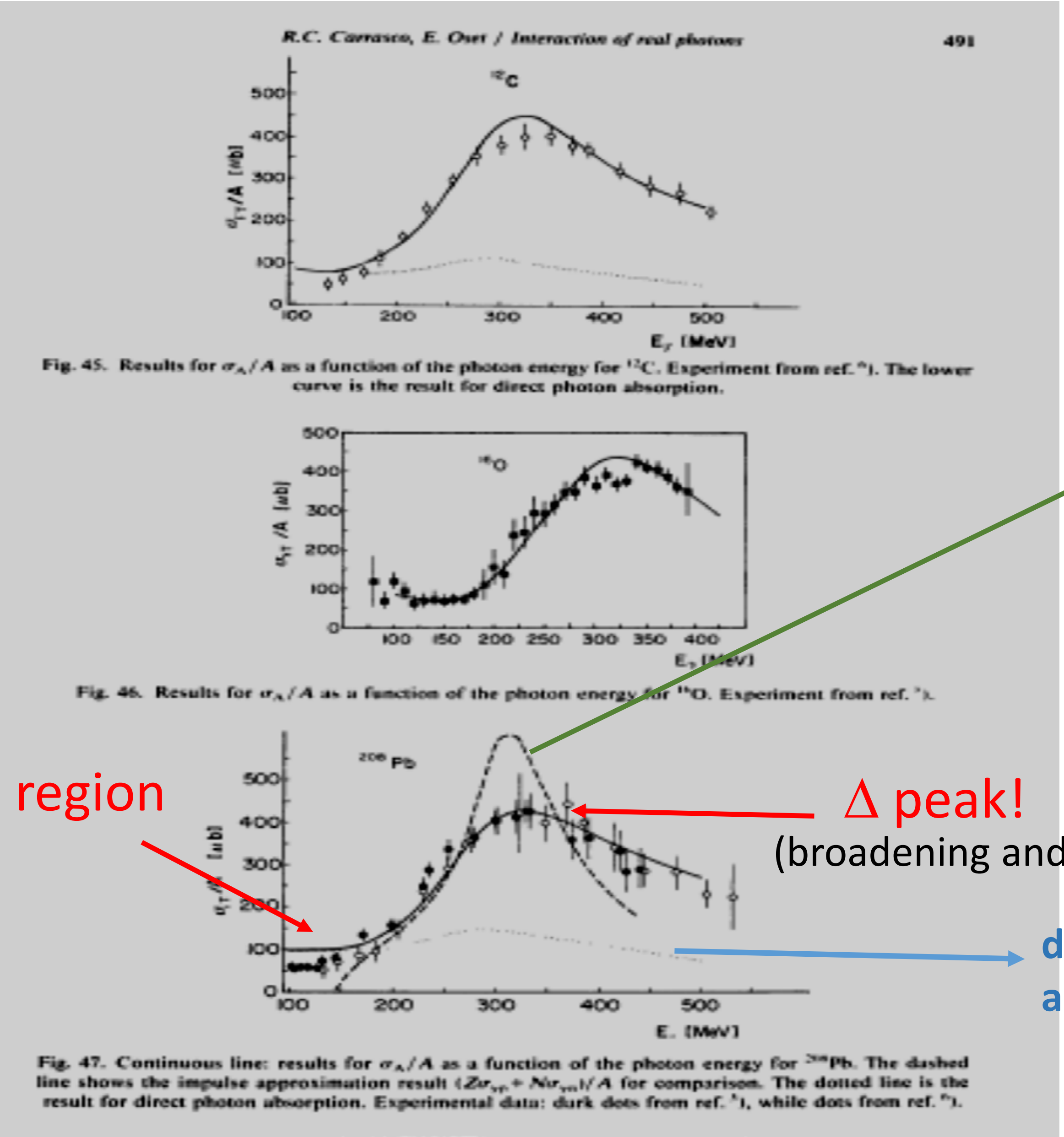
is forbidden !

$$p^2 = (p+q)^2$$

does not have solution when all particles are in the mass shell, i.e. $p^2 = (p+q)^2 = M^2$ **and** $q^2 = 0$.

Real Photon Results

Same formalism applied to the study of the interaction of Real Photons with Nuclei at Intermediate Energies: Total Photo-absorption cross section $\gamma A_Z \to X$ [Carrasco + Oset, NPA 536 (1992) 445] and Inclusive (γ,π), (γ,N), (γ,NN) and $(\gamma,N\pi)$ reactions [Carrasco + Oset + Salcedo NPA 541 (1992) 585 and Carrasco+Vicente-Vacas+ Oset NPA 570 (1994) 701]

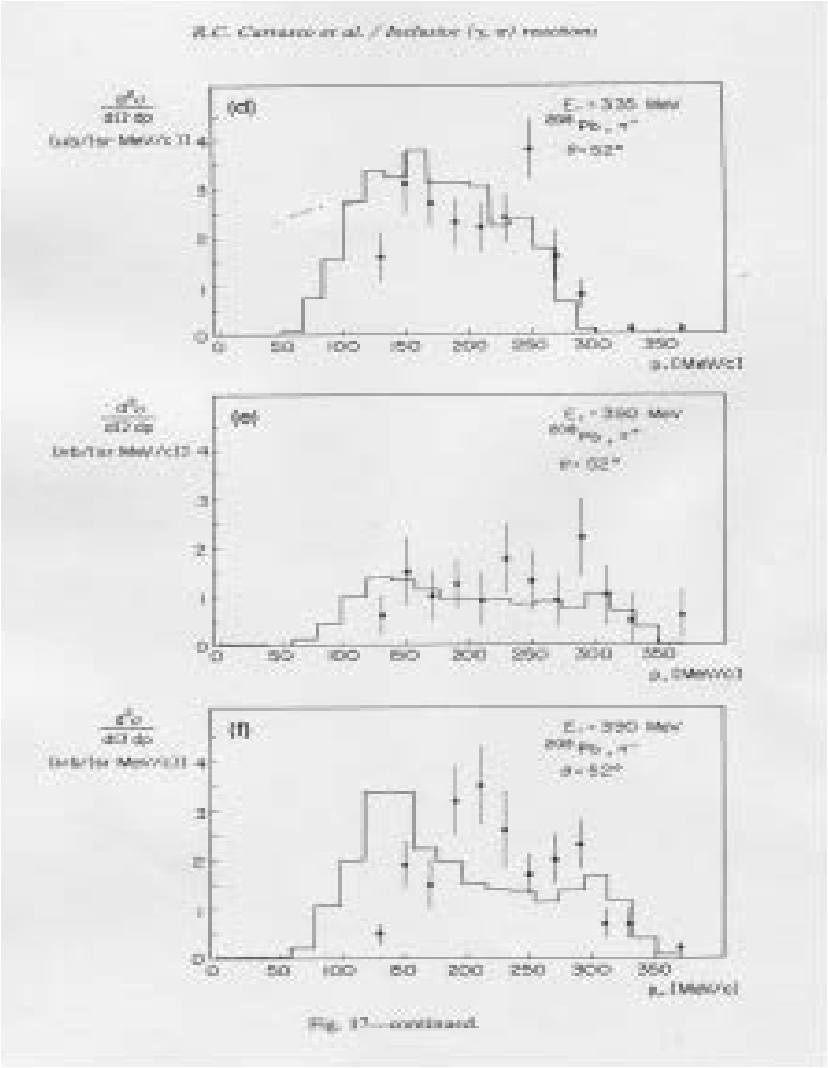


dip region

free Δ contribution

Δ peak!
(broadening and shift)

direct photon absorption



$$\gamma + {}^{208}\mathrm{Pb} \to X + \pi^{\pm}$$

$$\gamma + A \to X$$

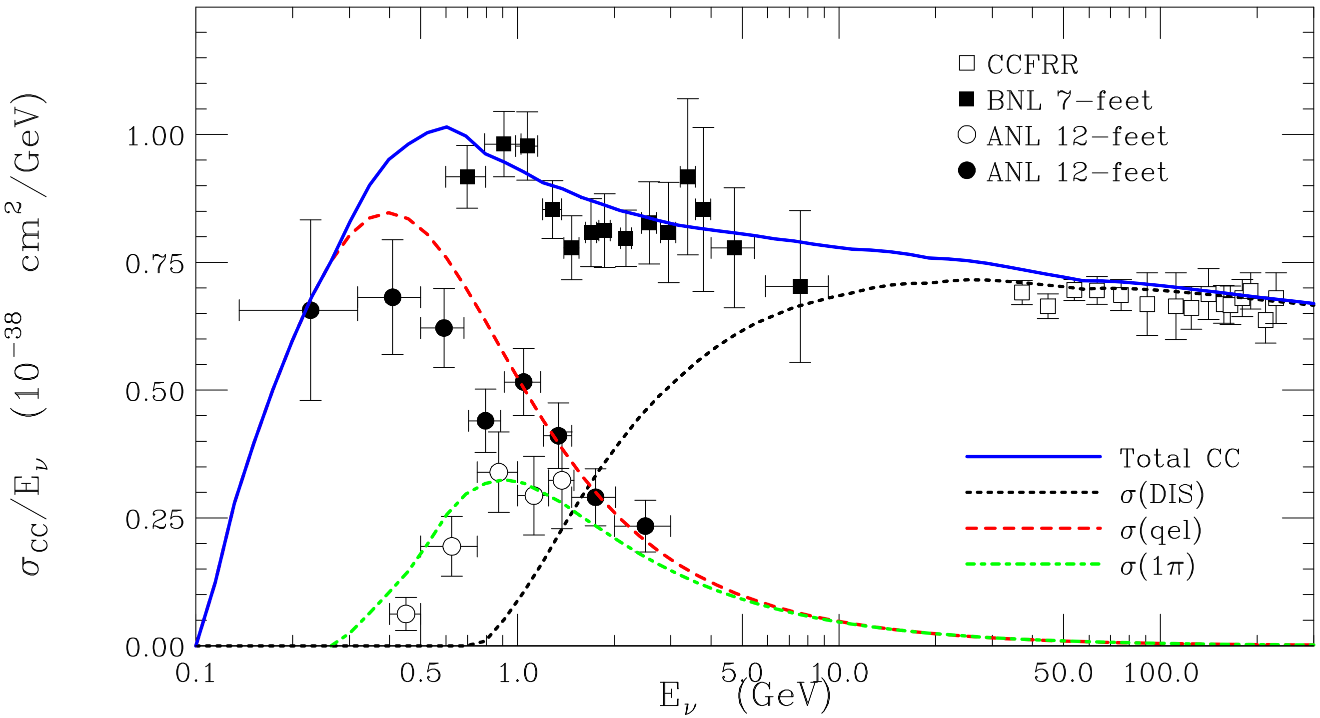
Juan Nieves, IFIC (CSIC & UV)

QE and QE-like scattering

- arXiv:1403.2673: (Progress and open questions in the physics of neutrino cross sections)
- arXiv:1307.8105: PRD 88 (2013) 113007 ($\nu, \bar{\nu}$ CCQE-like up to 10 GeV)
- arXiv:1302.0703: PLB 721 (2013) 90 ($\bar{\nu}$ CCQE-like)
- arXiv:1204.5404: PRD 85 (2012) 113008 (E_ν reconstruct.)
- arXiv:1106.5374: PLB 707 (2012) 72 (ν CCQE-like)
- arXiv:1102.2777: PRC 83 (2011) 045501 (CCQE, 2p2h, ...)

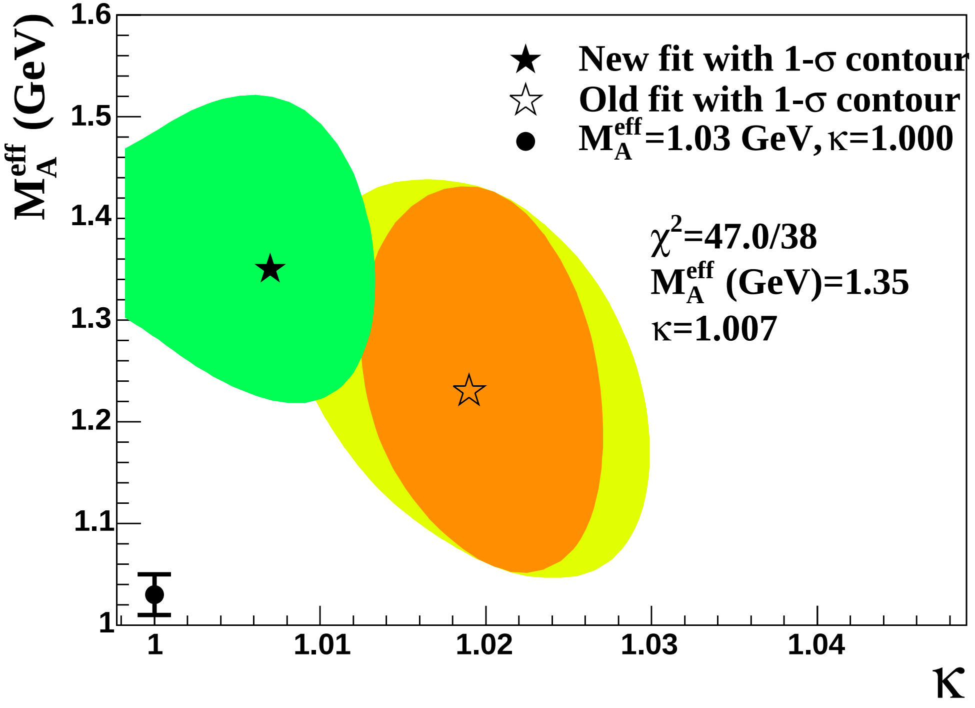
- nucl-th/0408005: PRC 70 (2004) 055503 (CCQE)
- hep-ph/0604042: PLB 638 (2006) 325 (Errors in CCQE)
- hep-ph/0511204 : PRC 73 (2006) 025504 (NCQE & MC)

Motivation: Details on the axial structure of hadrons in the free space and inside of nuclei, and



Theoretical knowledge of QE and 1π cross sections is important to carry out a precise neutrino oscillation data analysis...

Motivation: MiniBooNE CCQE (PRD 81, 092005)

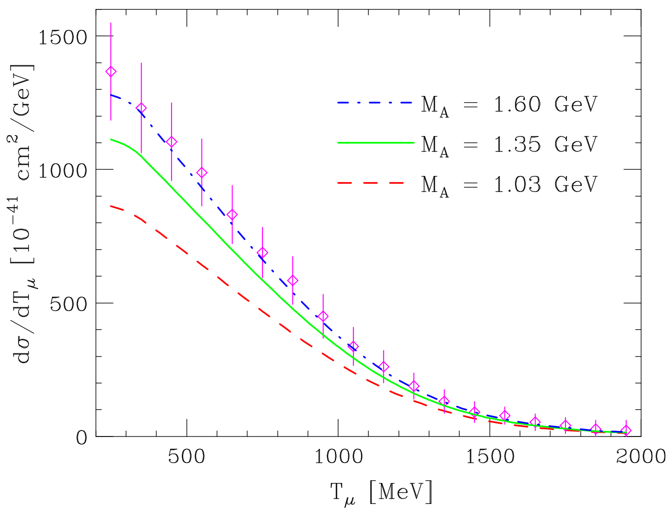


$$M_A^{\rm eff} = 1.35 \textbf{ GeV}$$

vs

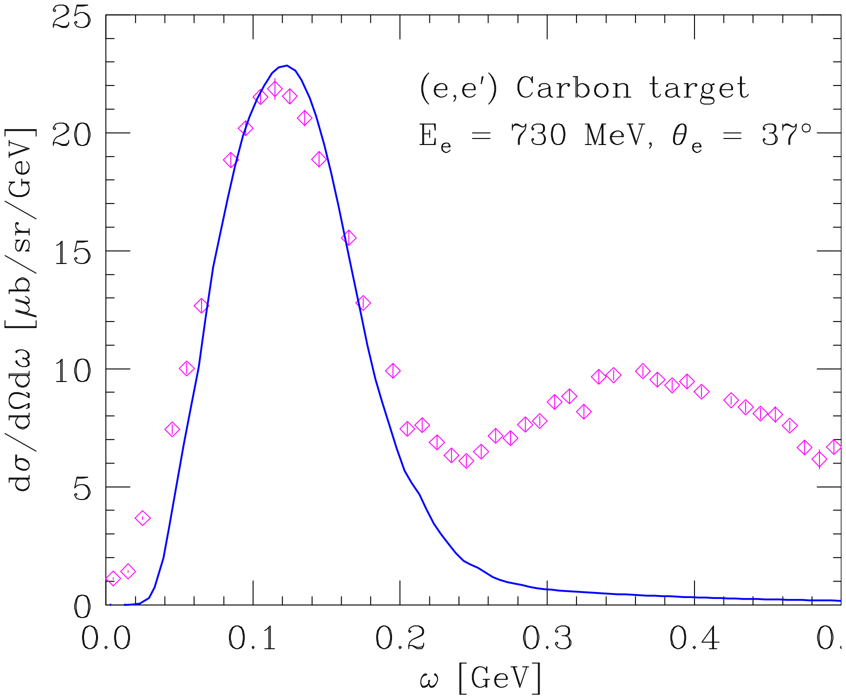
1.03 GeV (world avg)

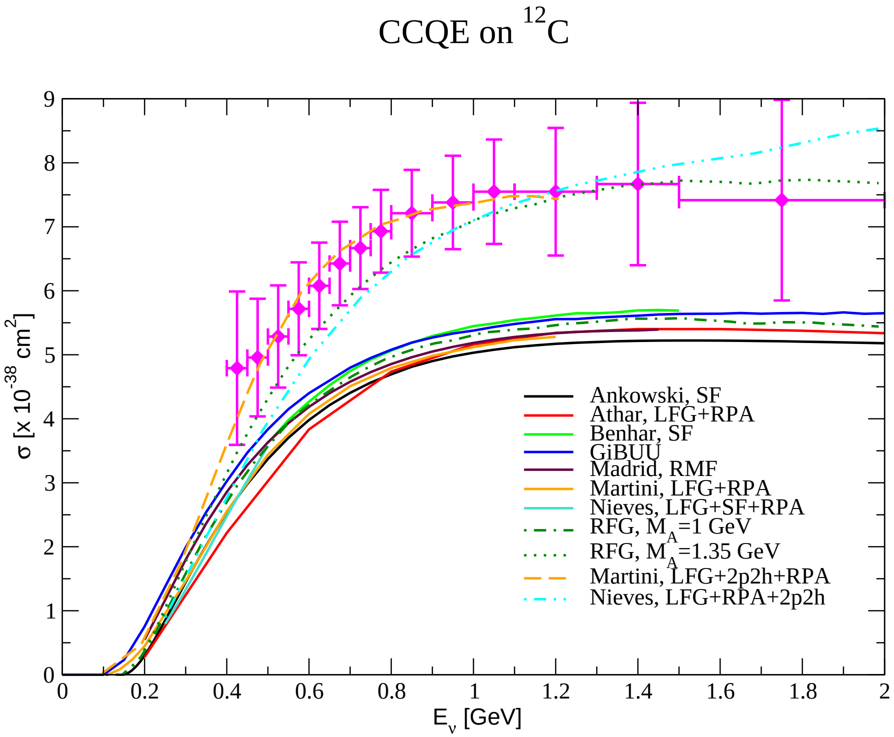
confirmed by many other groups, for instance by Benhar et al. (PRL 105, 132301)



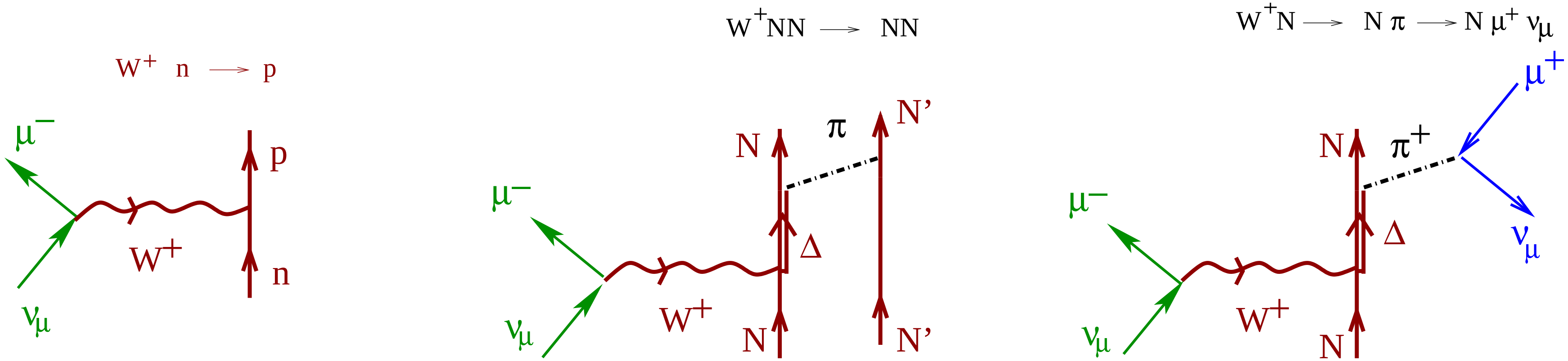
- ChPT $\mathcal{O}(p^3)$ + single pion electroproduction data: $M_A = 1.014 \pm 0.016$ GeV (V. Bernard, N. Kaiser, and U. G. Meissner, Phys.Rev.Lett. 69, 1877 (1992))
- CCQE measurements on deuterium and, to lesser extent, hydrogen targets is $M_A = 1.016 \pm 0.026$ GeV (A. Bodek, S. Avvakumov, R. Bradford, and H. S. Budd, Eur.Phys.J. C53, 349 (2008))

The problem turned out to **even more worrying** since the height, position, and width of the **QE peak in the case of electron scattering are well reproduced in most of used models,** for instance see results of Benhar et al. at similar energies and in carbon



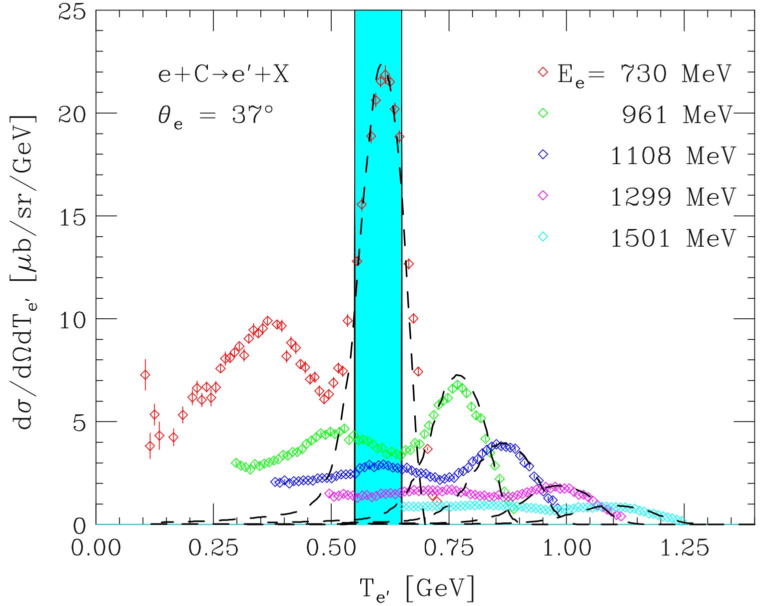


...but key observation (Martini et al., PRC 81, 045502): in most **theoretical** works QE is used for processes where the gauge boson **$W^{\pm}$ or Z^0 is absorbed by just one nucleon**, which together with a lepton is emitted, **however in the recent MiniBooNE measurements, QE is related to processes in which only a muon is detected** (ejected nucleons are not detected !) $\equiv$ **CCQE-like**



It **includes multinucleon processes and others like π production followed by absorption** (MBooNE analysis Monte Carlo corrects for those events). **It discards pions coming off the nucleus, since they will give rise to additional leptons after their decay.**

O. Benhar@NuFacT11: [arXiv : 1110.1835] measured electron-carbon scattering cross sections for a fixed outgoing electron angle $\theta = 37^o$ and different beam energies $\in [730, 1501]$ GeV, plotted as a function of E_e,

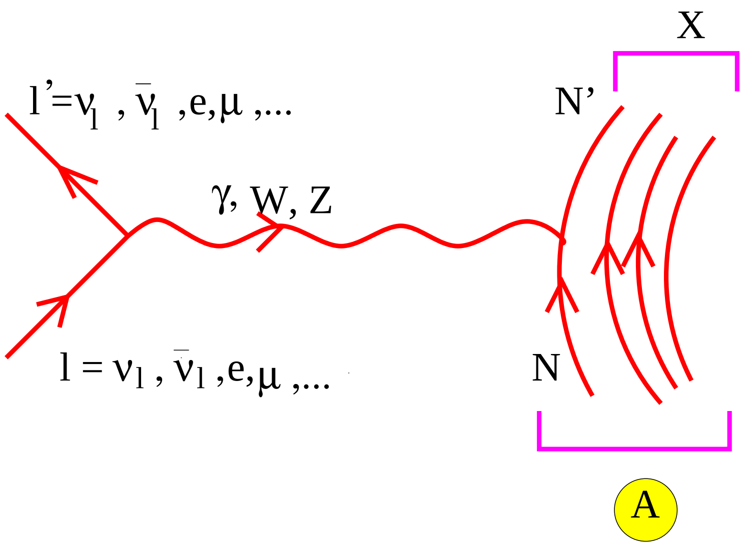


The energy bin corresponding to **the top of the QE peak at $E_e =$ 730 MeV receives significant contributions from** cross sections corresponding to different beam energies and **different mechanisms!**

- **MiniBooNE experimental results cannot be directly compared to most theoretical previous calculations!**
- **We present a microscopic calculation of the ν and $\bar{\nu}$ CCQE-like double differential cross sections $\frac{d^2\sigma}{dT_\mu d\cos\theta_\mu}$ measured by MiniBooNE and we will use the ν data to extract M_A**
- **Neutrino Energy Reconstruction and the Shape of the CCQE-like Total Cross Section**
- **Neutrino-nucleus quasi-elastic and 2p2h interactions up to 10 GeV**

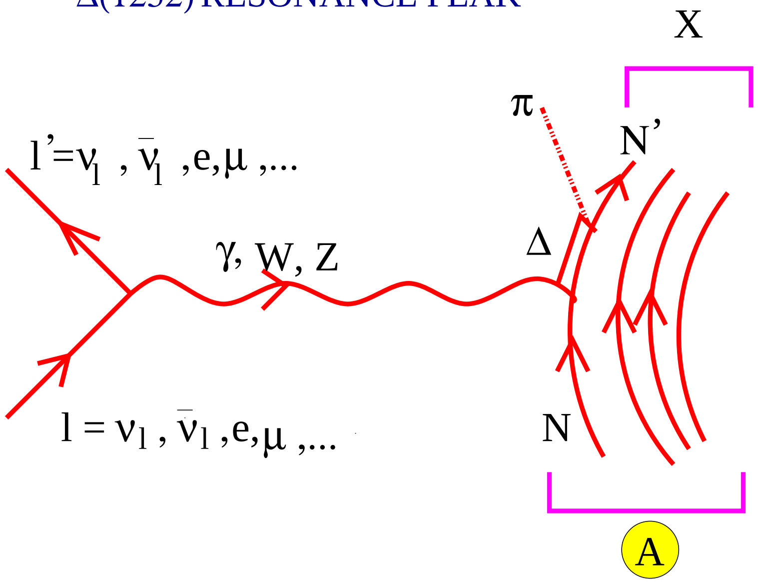
Nuclear renormalization effects on electroweak inclusive reactions in nuclei at intermediate energies

QUASIELASTIC PEAK



γ
VIRTUAL W BY ONE NUCLEON
Z

Δ(1232) RESONANCE PEAK



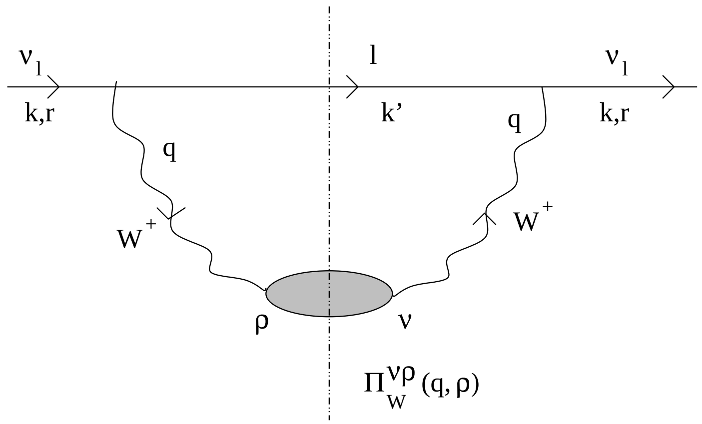
EXCITATION OF Δ (1232) DEGREES OF FREEDOM

To describe the propagation of particles inside of the nuclear medium $\Rightarrow$ microscopic framework:

- Pauli Blocking
- RPA and Short Range Correlations (SRC)
- $\Delta(1232)$–Degrees of Freedom
- Spectral Function (SF) + Final State Interaction (FSI)
- Meson Exchange Currents (MEC)

compute the imaginary part of the lepton-selfenergy inside of the **nucleus**:

For instance, let's look at $\nu_l + A_Z \longrightarrow l + X$



$$\frac{d^2\sigma}{d\Omega(\hat{k}')dE'} = \frac{|\vec{k}'|}{|\vec{k}\,|}\frac{G^2}{4\pi^2}L_{\mu\sigma}W^{\mu\sigma}$$

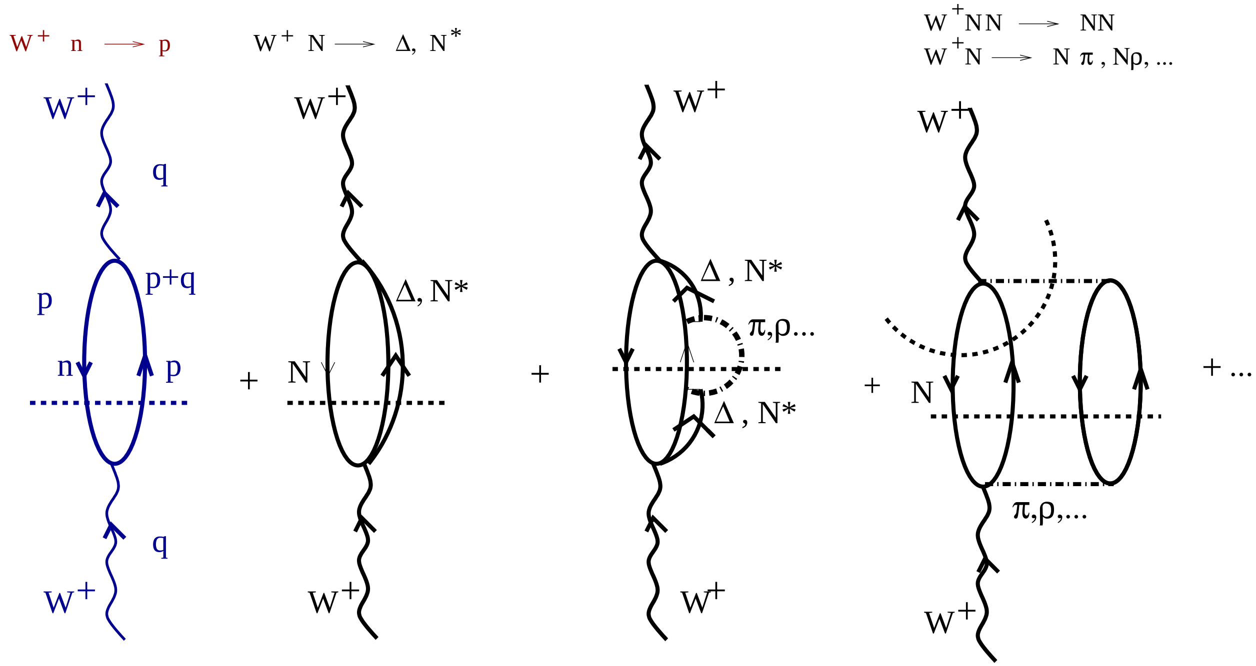
$$L_{\mu\sigma} = k'_\mu k_\sigma + k'_\sigma k_\mu - g_{\mu\sigma}k\cdot k' + \mathrm{i}\epsilon_{\mu\sigma\alpha\beta}k'^\alpha k^\beta$$

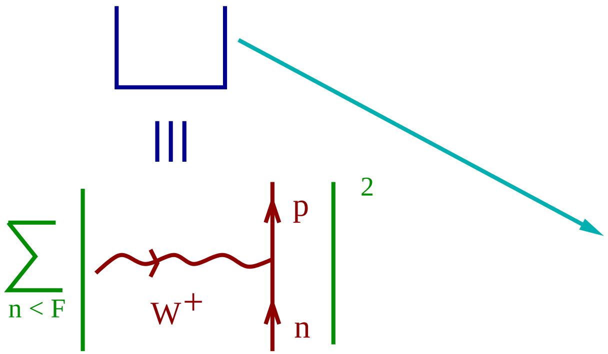
$$W^{\mu\sigma} = W^{\mu\sigma}_s + \mathrm{i}W^{\mu\sigma}_a$$

$$W^{\mu\sigma}_s \propto \int \frac{d^3r}{2\pi}\,\mathrm{Im}\,\left\{\Pi^{\mu\sigma}_W(q,\rho) + \Pi^{\sigma\mu}_W(q,\rho)\right\}\Theta(q^0)$$

$$W^{\mu\sigma}_a \propto \int \frac{d^3r}{2\pi}\,\mathrm{Re}\,\left\{\Pi^{\mu\sigma}_W(q,\rho) - \Pi^{\sigma\mu}_W(q,\rho)\right\}\Theta(q^0)$$

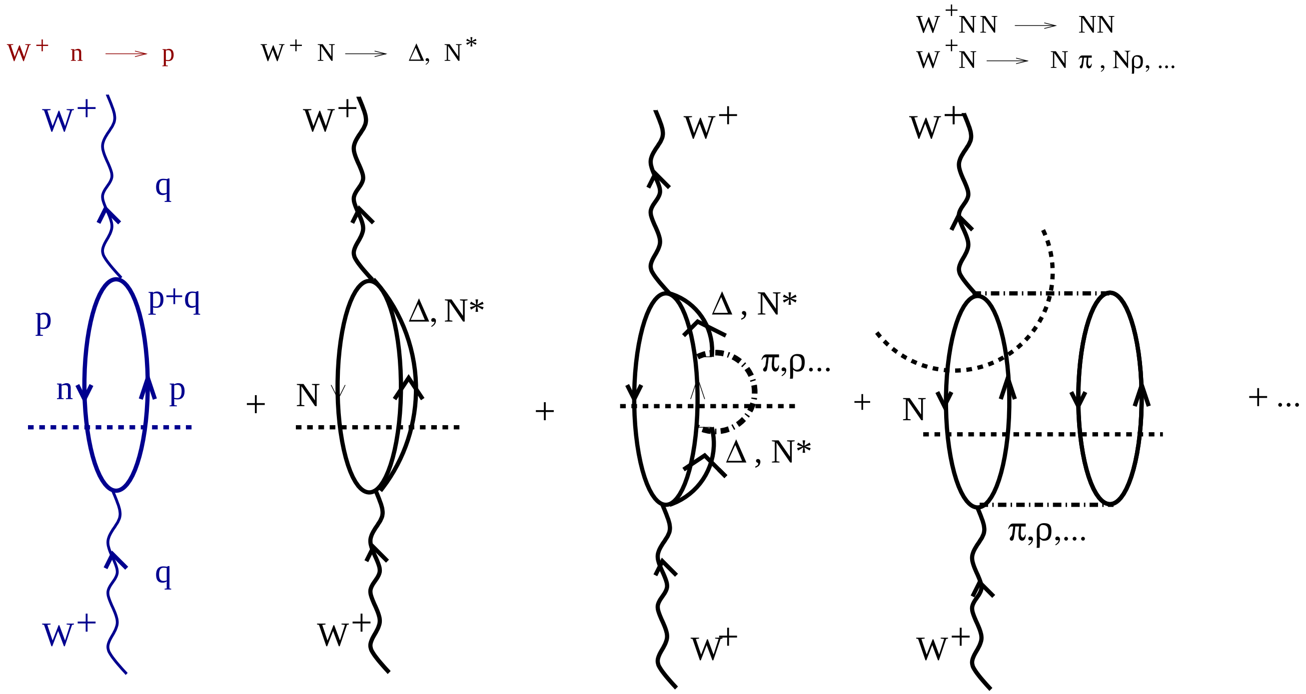
Basic object $\boxed{\Pi^{\nu\rho}_{W,Z^0,\gamma}(q,\rho)}$ $\equiv$ Selfenergy of the Gauge Boson $(W^\pm, Z^0, \gamma)$ inside of the nuclear medium. Perform a Many Body expansion, where the relevant gauge boson absorption modes should be systematically incorporated: absorption by one N, or NN or even 3N, real and virtual (MEC) meson $(\pi,\ \rho,\ \cdots)$ production, Δ excitation, etc...

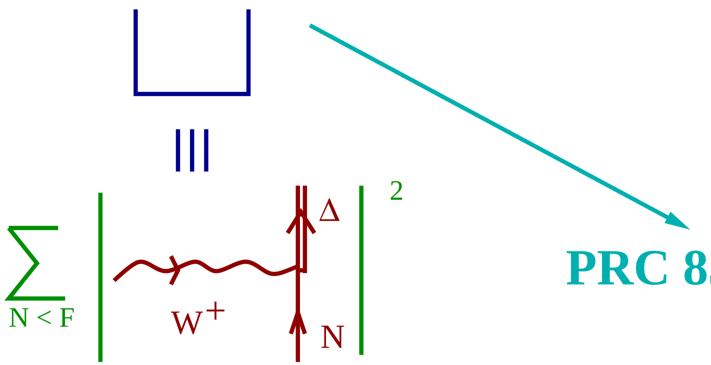




QE

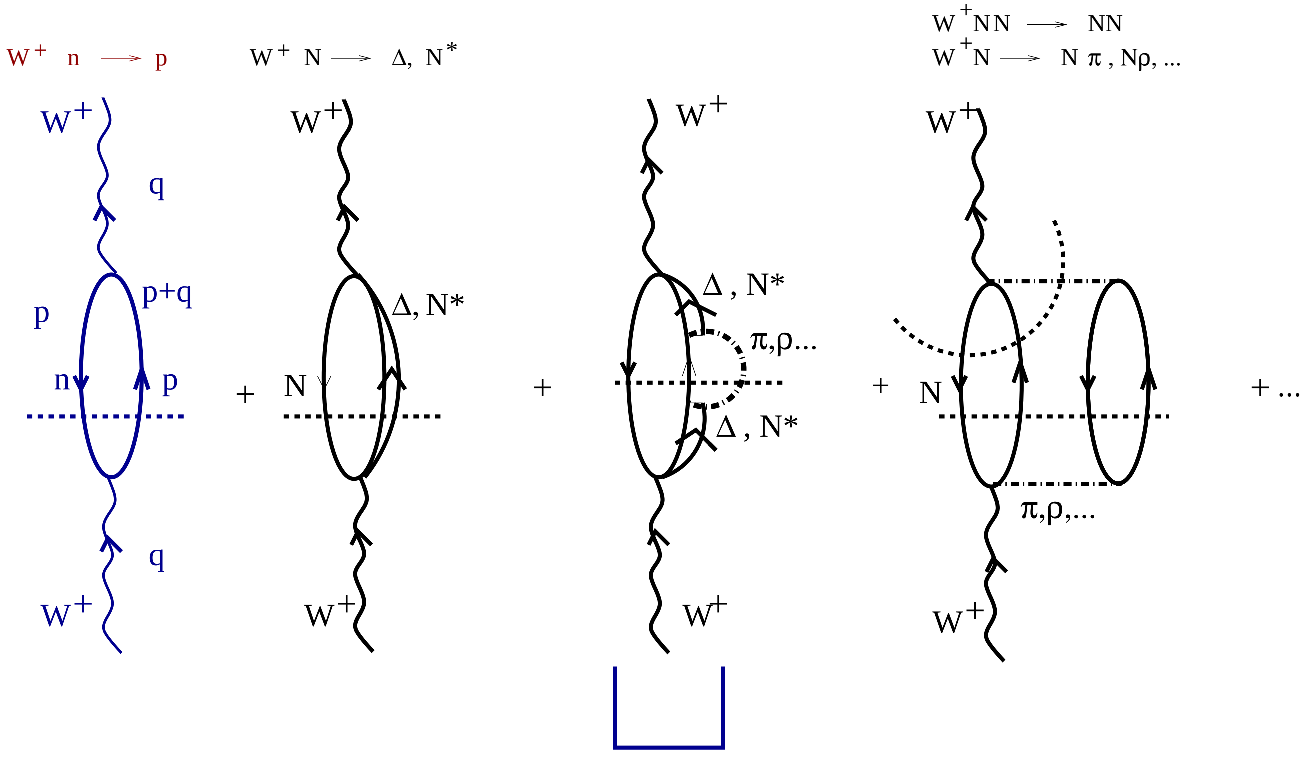
PRC 70 (2004) 055503 (CC)
PRC 73 (2006) 025504 (NC)





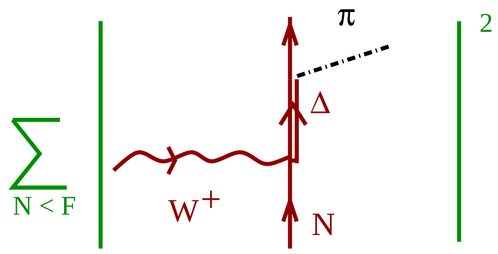
Δh prod.

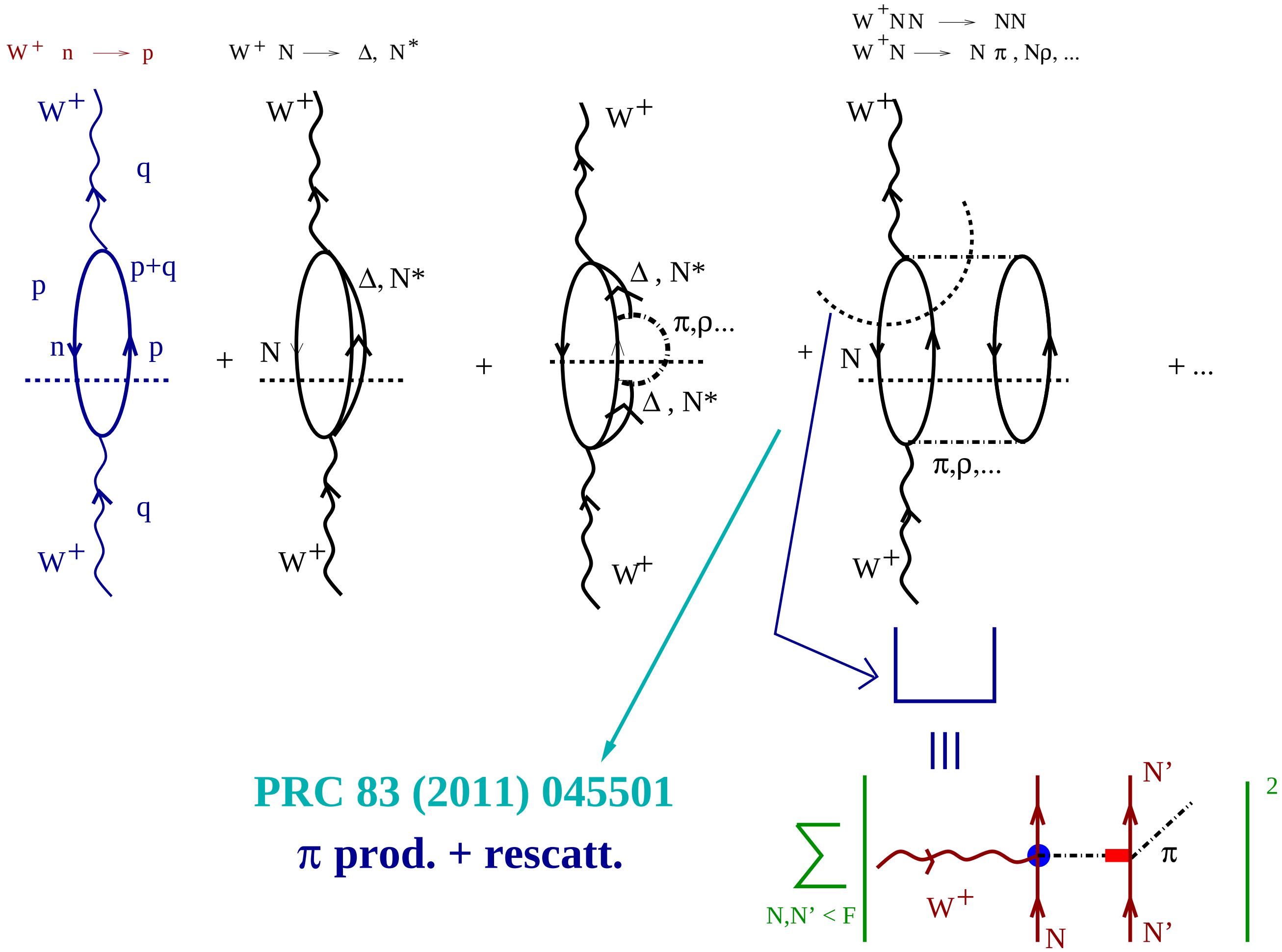
PRC 83 (2011) 045501



PRC 83 (2011) 045501

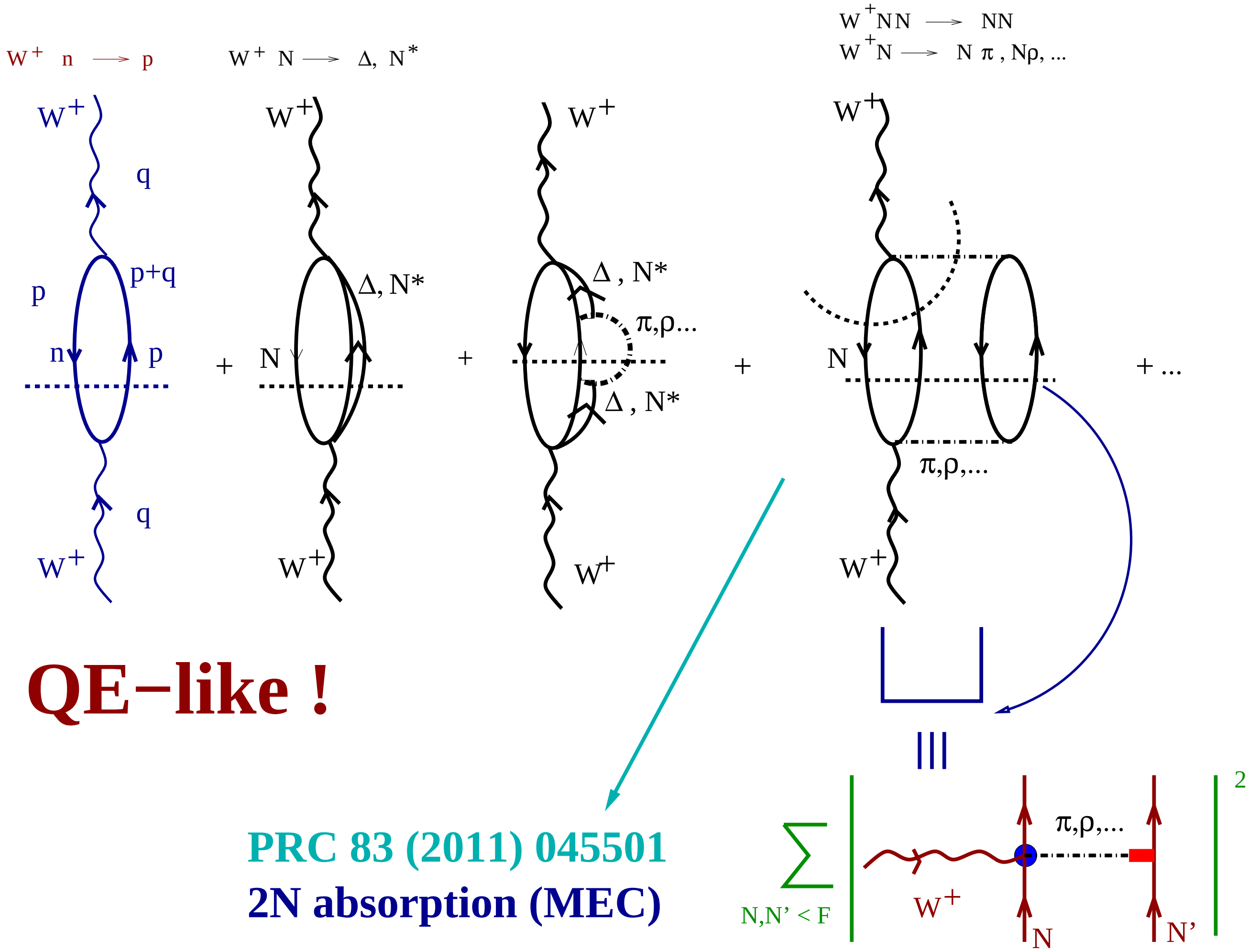
π prod.





PRC 83 (2011) 045501

π prod. + rescatt.



QE−like !

PRC 83 (2011) 045501

2N absorption (MEC)

Inclusive QE processes [f.i. (ν_l, l)]

($W^\pm$, Z^0 absorption by one nucleon)

First ingredient: M.E. of the CC/NC current between nucleons.

$$< p; \vec{p}\,' = \vec{p} + \vec{q}\, | j^\alpha_{\rm cc}(0) | n; \vec{p}\ > \quad = \quad \bar{u}(\vec{p}\,')\, [V^\alpha - A^\alpha]\, u(p)$$

$$V^\alpha \quad = \quad 2\cos\theta_c \times \left(F_1^V(q^2)\gamma^\alpha + {\rm i}\mu_V \frac{F_2^V(q^2)}{2M}\sigma^{\alpha\nu}q_\nu \right)$$

$$A^\alpha \quad = \quad \cos\theta_c G_A(q^2) \times \left(\gamma^\alpha\gamma_5 + \frac{2M}{m_\pi^2 - q^2} q^\alpha \gamma_5 \right) \qquad \textbf{(PCAC)}$$

with vector form factors related to the electromagnetic ones and

$$G_A(q^2) \quad = \quad \frac{g_A}{(1 - q^2/\boxed{M_A^2}\,)^2}, \quad g_A = 1.257$$

One finds (quasielastic peak)

$$\begin{aligned} W^{\mu\nu}_{s,a}(q) &= -\frac{1}{2M^2}\int_0^\infty \mathbf{dr r^2}\left\{2\int \frac{d^3p}{(2\pi)^3}\frac{M}{E(\vec p)}\frac{M}{E(\vec p+\vec q)}\Theta(q^0)\right. \\ &\times \boldsymbol{\Theta}(\mathbf{k_F^n}(\mathbf{r}) - |\vec{\mathbf{p}}\,|)\boldsymbol{\Theta}(|\vec{\mathbf{p}}+\vec{\mathbf{q}}\,| - \mathbf{k_F^p}(\mathbf{r})) \\ &\times \left.(-\pi)\delta(q^0+E(\vec p)-E(\vec p+\vec q\,))A^{\mu\nu}_{s,a}(p,q)\right\} \end{aligned}$$

Relativistic Local Fermi Gas that includes Pauli Blocking !

$$\begin{aligned} A^{\mu\nu}_s(p,q) &= 16(F_1^V)^2\left\{(p+q)^\mu p^\nu + (p+q)^\nu p^\mu + \frac{q^2}{2}g^{\mu\nu}\right\} \\ &+ 2q^2(\mu_V F_2^V)^2\left\{4g^{\mu\nu} - 4\frac{p^\mu p^\nu}{M^2} - 2\frac{p^\mu q^\nu + q^\mu p^\nu}{M^2}\right. \end{aligned}$$

$$
\begin{aligned}
&- \quad q^\mu q^\nu (\frac{4}{q^2} + \frac{1}{M^2})\Big\} - 16 F_1^V \mu_V F_2^V (q^\mu q^\nu - q^2 g^{\mu\nu}) \\
&+ \quad 4 G_A^2 \Big\{ 2p^\mu p^\nu + q^\mu p^\nu + p^\mu q^\nu + g^{\mu\nu} (\frac{q^2}{2} - 2M^2) \\
&- \quad \frac{2M^2(2m_\pi^2 - q^2)}{(m_\pi^2 - q^2)^2} q^\mu q^\nu \Big\} \\
A_a^{\mu\nu}(p,q) &= \quad 16 G_A \left(\mu_V F_2^V + F_1^V \right) \epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta
\end{aligned}
$$

- Nucleus dependence: $k_F^{p,n}(r) = [3\pi^2 \rho_{p,n}(r)]^{\frac{1}{3}}$, with $\rho_{p,n}(r)$ proton and neutron center densities.
- The leading contribution ($1ph$) in the density expansion, is fully relativistic.
- Analytical $\int d^3p$ integration in terms of the imaginary part of the relativistic ph Lindhard function:

$$\mathrm{Im}\bar{U}_R^N(q) = 2\int \frac{d^3p}{(2\pi)^3} \frac{M}{E(\vec{p})} \frac{M}{E(\vec{p}+\vec{q})} \Theta(k_F^n(r) - |\vec{p}\,|)$$
$$\Theta(q^0)\Theta(|\vec{p}+\vec{q}\,| - k_F^p(r))(-\pi)\delta(q^0 + E(\vec{p}) - E(\vec{p}+\vec{q}\,))$$

in addition we include some nuclear corrections...

- **Low Density Theorem.** For low densities

$$\mathrm{Im}\bar{U}_R^N(q) \approx -\pi\rho_n(r)\frac{M}{E(\vec{q})}\delta(q^0 + M - E(\vec{q})) + \cdots$$

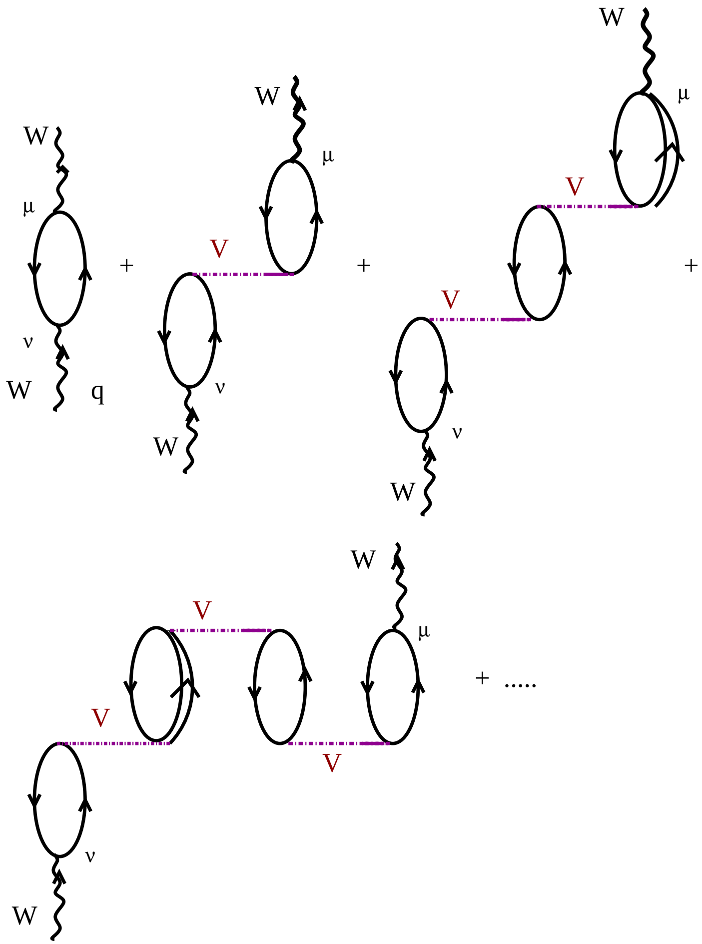
$\int d^3r \to$ N (number of neutrons) and $\sigma_{\nu_l A \to l X} = \mathrm{N}\sigma_{\nu_l n \to l^- p}$

- Low energies:

1. Correct Energy Balance, incorporating the experimental Q value, $\to \delta(q^0 - \boxed{\mathrm{Q}} + E(\vec{p}) - E(\vec{p} + \vec{q}\ ))$ with $Q = M(A_{Z+1}) - M(A_Z)$.
2. Coulomb distortion of outgoing lepton

$$(k'^2 - m_l^2 + \mathrm{i}\epsilon)^{-1} \to (k'^2 - m_l^2 - \boxed{\Sigma_{\mathrm{Coul}}} + \mathrm{i}\epsilon)^{-1}$$

- Polarization (RPA) effects. Substitute the *ph* excitation by an RPA response: series of *ph* and Δh excitations.



1. Effective Landau-Migdal interaction

$$V(\vec{r}_1,\vec{r}_2) = c_0\delta(\vec{r}_1-\vec{r}_2)\left\{\boxed{f_0(\rho)} + f_0'(\rho)\vec{\tau}_1\vec{\tau}_2 + \boxed{g_0(\rho)\vec{\sigma}_1\vec{\sigma}_2} + g_0'(\rho)\vec{\sigma}_1\vec{\sigma}_2\vec{\tau}_1\vec{\tau}_2\right\}$$

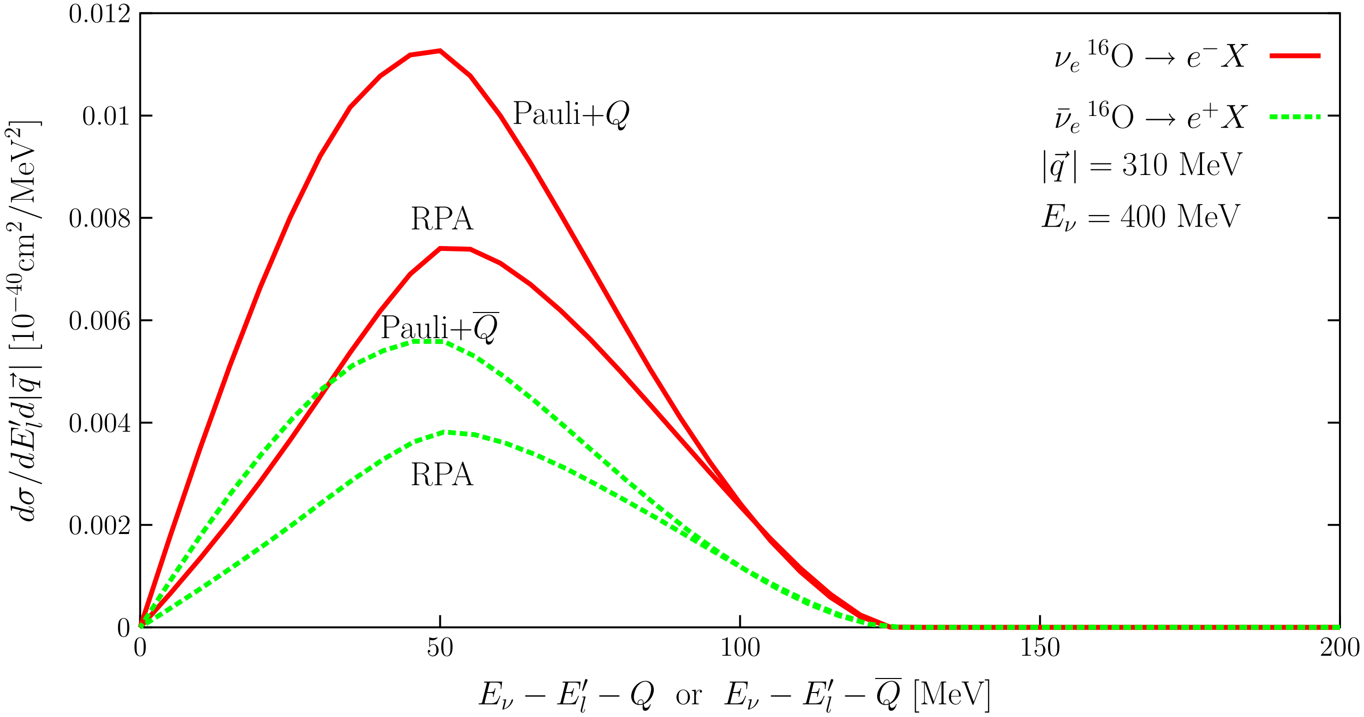
   Isoscalar terms $\boxed{}$ do not contribute to CC

2. $S=T=1$ channel of the *ph–ph* interaction $\to$ s longitudinal (π) and transverse (ρ) + **SRC**

$$g_0'\vec{\sigma}_1\vec{\sigma}_2\vec{\tau}_1\vec{\tau}_2 \to \left[V_l(q)\hat{q}_i\hat{q}_j + V_t(q)(\delta_{ij}-\hat{q}_i\hat{q}_j)\right]\sigma_1^i\sigma_2^j\vec{\tau}_1\vec{\tau}_2$$

$$V_{l,t}(q) = \frac{f_{\pi NN,\rho NN}}{m^2_{\pi,\rho}}\left(F_{\pi,\rho}(q^2)\frac{\vec{q}^{\,2}}{q^2-m^2_{\pi,\rho}} + g'_{l,t}(q)\right)$$

3. Contribution of Δh excitations important

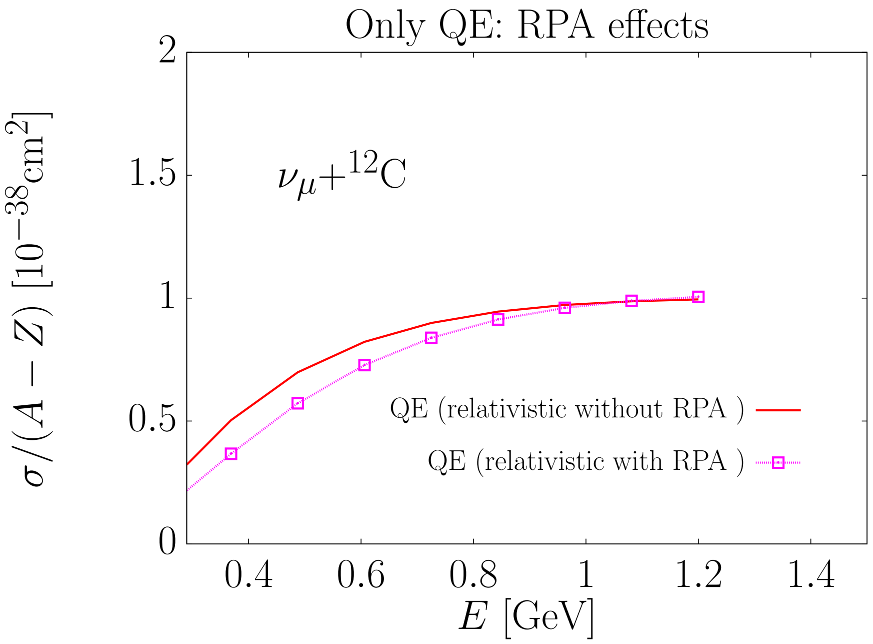


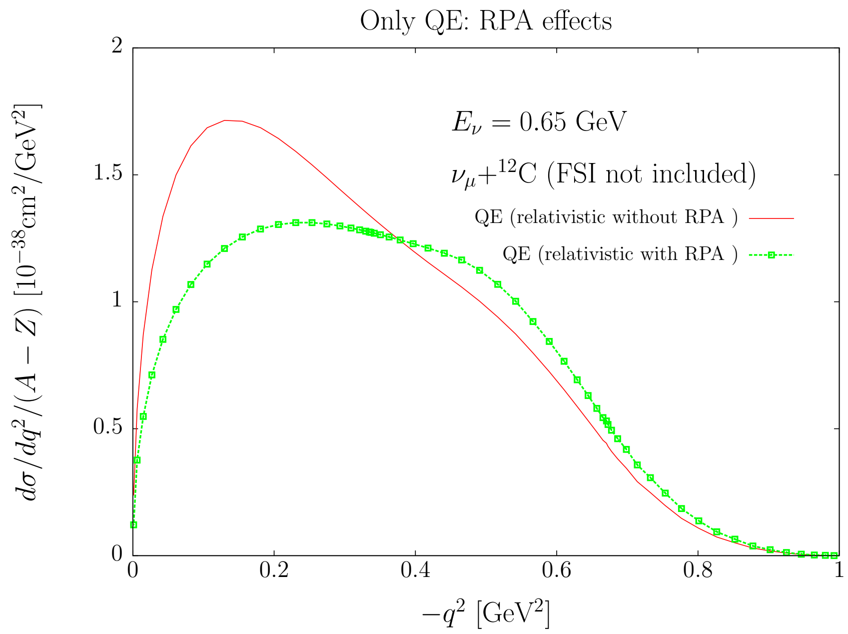
Examples of the RPA effect

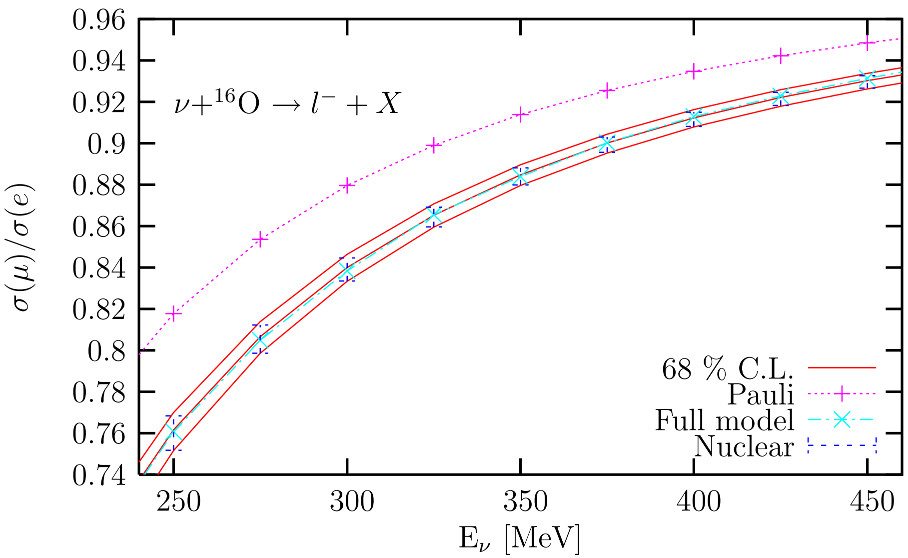
$$G_A^2 \delta^{ij} \quad \rightarrow \quad G_A^2 \left(\frac{\hat{q}^i \hat{q}^j}{|1 - U(q) V_l(q)|^2} + \frac{\delta^{ij} - \hat{q}^i \hat{q}^j}{|1 - U(q) V_t(q)|^2} \right)$$

$$(F_1^V)^2 \quad \rightarrow \quad \frac{(F_1^V)^2}{|1 - c_0 f_0'(\rho) U_N(q)|^2}, \quad \text{etc...}$$

The Lindhard function $U(q) = U_N + U_\Delta \quad [ph + \Delta h]$

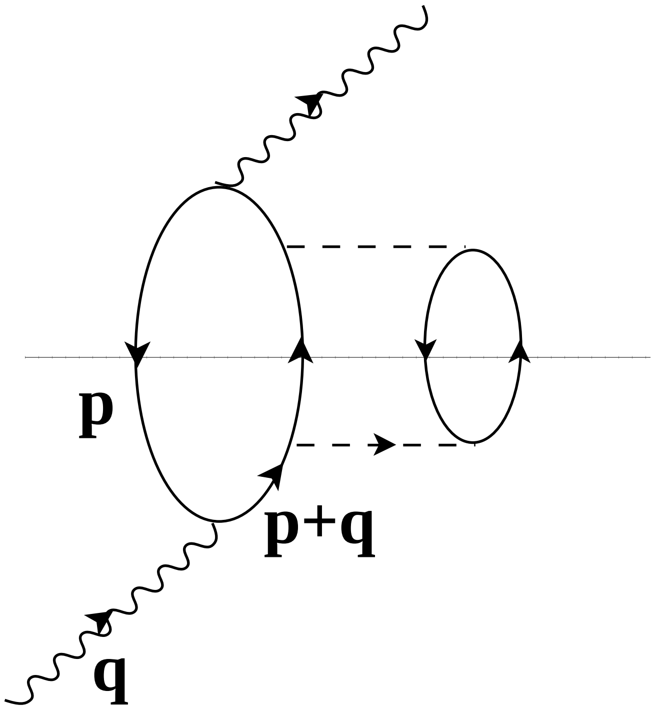






RPA corrections strongly decrease as the neutrino energy increases. However, their effects might account for a low Q^2 deficit of CCQE events and affect the σ_μ/σ_e ratio (~ 5 %)

- **Spectral Function (SF) + Final State Interaction (FSI):** dressing up the nucleon propagator of the hole (SF) and particle (FSI) states in the *ph* excitation



- **Change of nucleon dispersion relation:**
  - **hole ⇒ Interacting Fermi sea (SF)**
  - **particle ⇒ Interaction of the ejected nucleon with the final nuclear state (FSI)**

$$G(p) \to \int_{-\infty}^{\mu} d\omega \frac{S_h(\omega, \vec{p}\,)}{p^0 - \omega - i\epsilon} + \int_{\mu}^{+\infty} d\omega \frac{S_p(\omega, \vec{p}\,)}{p^0 - \omega + i\epsilon}$$

The hole and particle spectral functions are related to nucleon self-energy $\boxed{\Sigma}$ in the medium,

$$S_{p,h}(\omega,\vec{p}\,) \;=\; \mp\frac{1}{\pi}\frac{\mathrm{Im}\Sigma(\omega,\vec{p}\,)}{\left[\omega^2-\vec{p}^{\,2}-M^2-\mathrm{Re}\Sigma(\omega,\vec{p}\,)\right]^2+\left[\mathrm{Im}\Sigma(\omega,\vec{p}\,)\right]^2}$$

with $\omega \geq \mu$ or $\omega \leq \mu$ for S_p and S_h, respectively (μ is the chemical potential).

$$W^{\mu\nu}_{s,a}(q) \;\propto\; -\int_0^\infty \mathbf{d r r^2}\left\{\int \frac{d^4p}{(2\pi)^4} A^{\mu\nu}_{s,a}(p,q) S_p(p+q) S_h(p)\right\}$$

The simplest description $\Rightarrow$ relativistic Fermi Gas

$$\text{chemical potential}: \mu \sim \frac{k_F^2}{2M} + \frac{\mathrm{Re}\Sigma(\mu,k_F)}{2E(k_F)}$$

with $E(\vec{p}\,) = \sqrt{M^2+\vec{p}^{\,2}} - E_B, \cdots$ **(P. Fernández de Córdoba and E. Oset, PRC46 (1992) 1697))**

$$S_{p,h}(\omega,\vec{p}\,) = \mp\frac{1}{\pi}\frac{\mathrm{Im}\Sigma(\omega,\vec{p}\,)}{\left[\omega^2-\vec{p}^{\,2}-M^2-\mathrm{Re}\Sigma(\omega,\vec{p}\,)\right]^2+\left[\mathrm{Im}\Sigma(\omega,\vec{p}\,)\right]^2}$$

with $\omega \geq \mu$ or $\omega \leq \mu$ for S_p and S_h, respectively (μ is the chemical potential).

$$W^{\mu\nu}_{s,a}(q) \propto -\int_0^\infty dr r^2 \left\{ \int \frac{d^4p}{(2\pi)^4} A^{\mu\nu}_{s,a}(p,q) S_p(p+q) S_h(p) \right\}$$

For non interacting fermions $\boxed{\Sigma = 0}$,

$$S_p(\omega,\vec{p}\,) = \frac{\theta(|\vec{p}\,|-k_F)}{2E(\vec{p}\,)}\delta(\omega-E(\vec{p}\,))$$

$$S_h(\omega,\vec{p}\,) = \frac{\theta(k_F-|\vec{p}\,|)}{2E(\vec{p}\,)}\delta(\omega-E(\vec{p}\,))$$

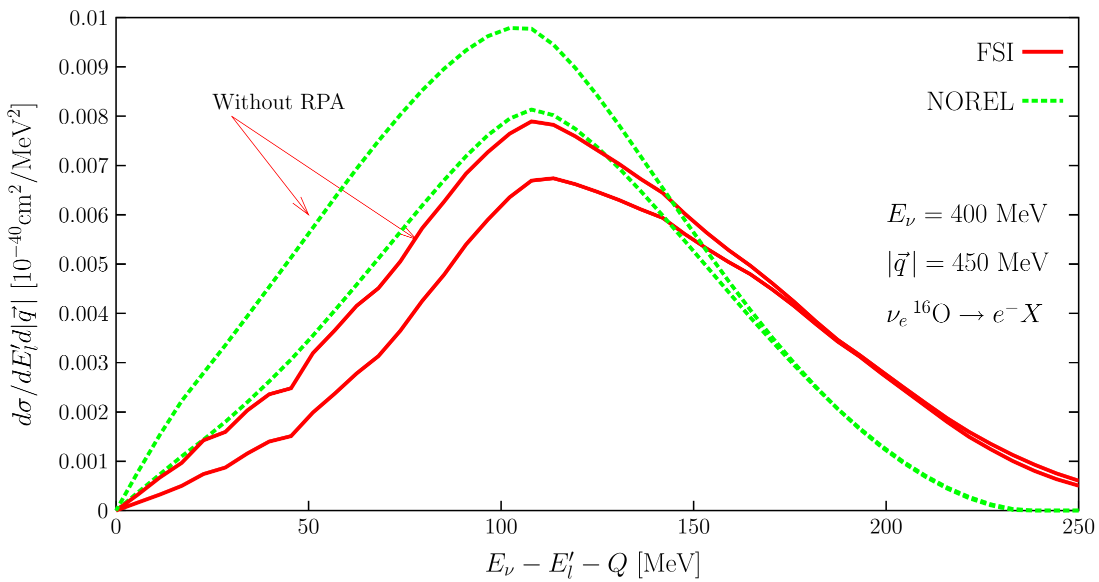
and only Pauli blocking is incorporated!!

To take into account SF+FSI $\rightarrow$ replace $\mathrm{Im}\bar{U}_R^N(q)$ **by the response function:**

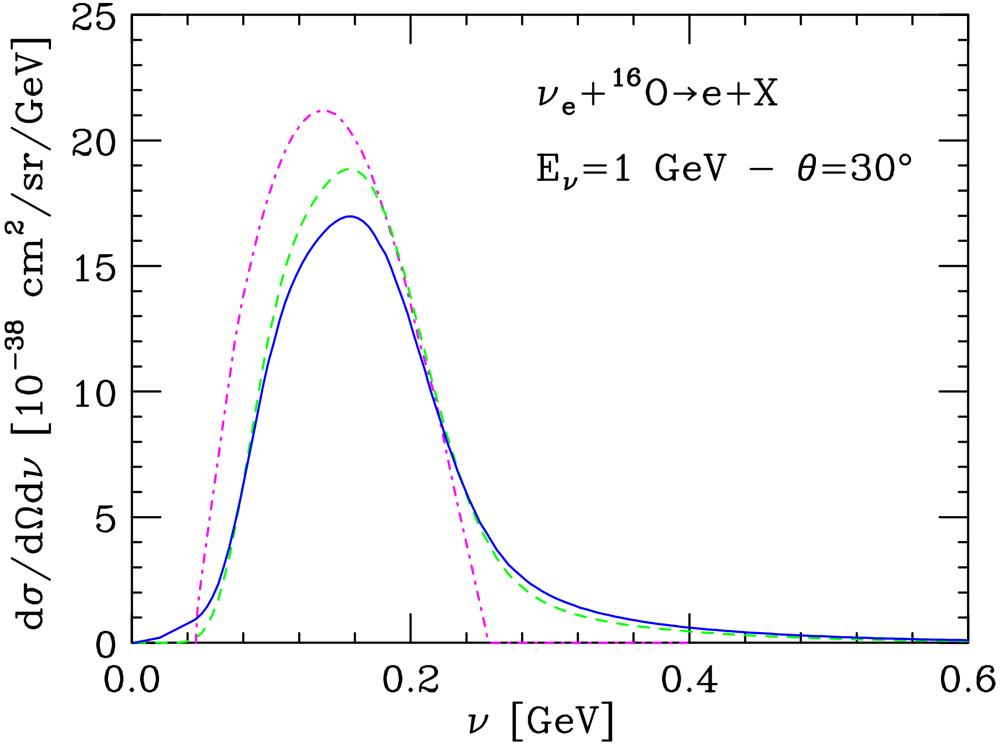
$$-\frac{1}{2\pi}\int_0^{+\infty} dp p^2 \int_{-1}^{+1} dx \int_{\mu-q^0}^{\mu} d\omega \mathbf{S_h}(\omega, \vec{\mathbf{p}}) \mathbf{S_p}(\mathbf{q^0}+\omega, \mathbf{t})$$

with $t^2 = \vec{p}^{\,2} + \vec{q}^{\,2} + 2|\vec{p}||\vec{q}|x$.

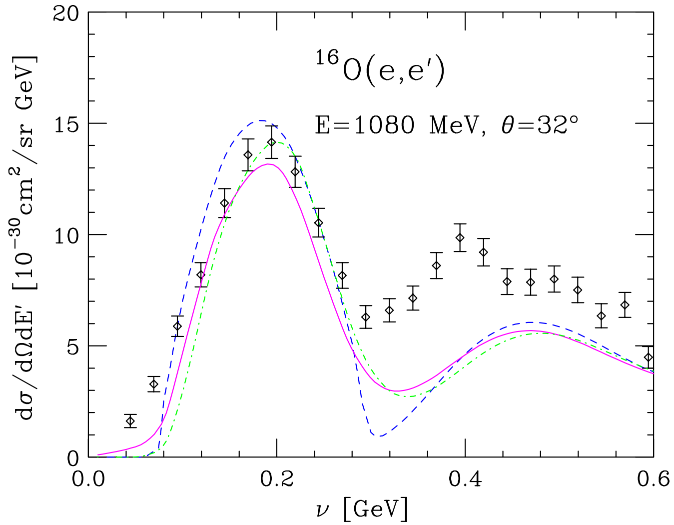
This nuclear effect is additional to those due to RPA (long range) correlations !!



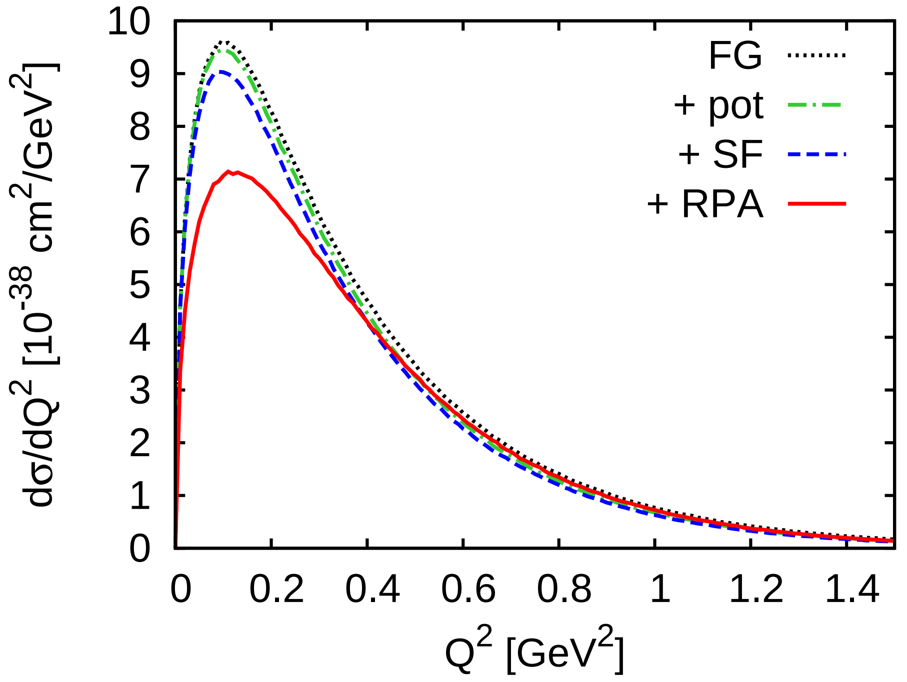
- **Sizeable reduction of the strength at the QE peak, which is slightly shifted. Neutrino energy re-construction uses $q^0 = -q^2/2M$, problems??**
- **Enhancement of the high energy transfer tail, which partially compensates the above reduction and thus the effect on the total (in-tegrated) cross section is smaller.**



Qualitatively agreement with Benhar, Farina, Nakamura, Sakuda and Seki [PRD 72 (2005) 053005]



- RPA corrections are not included, but probably small for $|\vec{q}| \geq 500$ MeV
- Pion production and 2N channels should be included in the “dip” and Δ regions.



RPA vs SF effects: Differential cross sections for the CCQE reaction on ^{12}C averaged over the MiniBooNE flux

(Alvarez-Ruso L et al., 2009 AIP Conf. Proc. 1189 151)

$$\text{RPA} \gg \text{SF}$$

The simplest description ⇒ relativistic Fermi Gas with **non interacting fermions** $\boxed{\Sigma = 0}$,

$$S_p(\omega, \vec{p}\,) = \frac{\theta(|\vec{p}\,| - k_F)}{2E(\vec{p}\,)}\delta(\omega - E(\vec{p}\,))$$
$$S_h(\omega, \vec{p}\,) = \frac{\theta(k_F - |\vec{p}\,|)}{2E(\vec{p}\,)}\delta(\omega - E(\vec{p}\,))$$

and only Pauli blocking is incorporated!!

Local vs Global Fermi Gas ?

$k_F^{p,n}(r) = \left[3\pi^2\rho^{p,n}(r)\right]^{1/3}$ **vs** $k_F^{p,n}$= **cte ?**

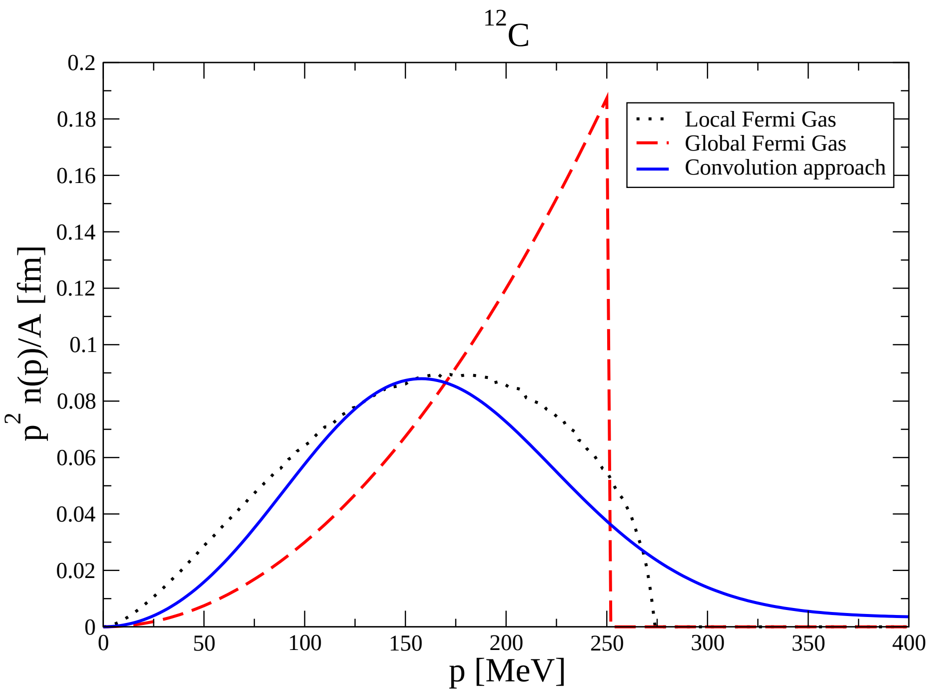
Local vs Global Fermi Gas ?

$$k_F(r) = \left[3\pi^2\rho(r)/2\right]^{1/3} \textbf{ vs } k_F\textbf{= cte ?}$$

$$S_h(\omega,\vec{p}) = \delta(\omega - E(\vec{p}))\theta(k_F - |\vec{p}\,|)/2\omega$$

$$n^{\rm RgFG}(|\vec{p}\,|) = \frac{4V}{(2\pi)^3}\int d\omega 2\omega S_h(\omega,\vec{p}) = \frac{3A}{4\pi k_F^3}\theta(k_F - |\vec{p}\,|)$$

$$n^{\rm LDA}(|\vec{p}\,|) = 4\int\frac{d^3r}{(2\pi)^3}\int d\omega 2\omega S_h(\omega,\vec{p}) = 4\int\frac{d^3r}{(2\pi)^3}\theta(\mathbf{k_F}(\mathbf{r}) - |\vec{p}\,|)$$

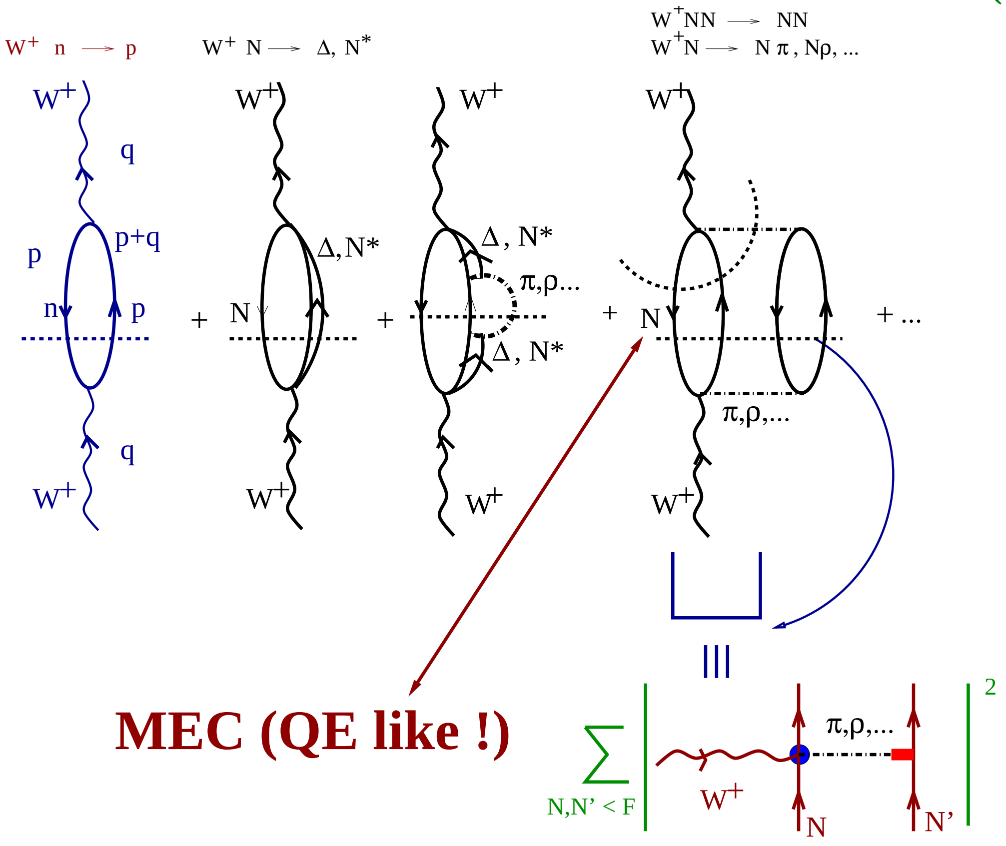


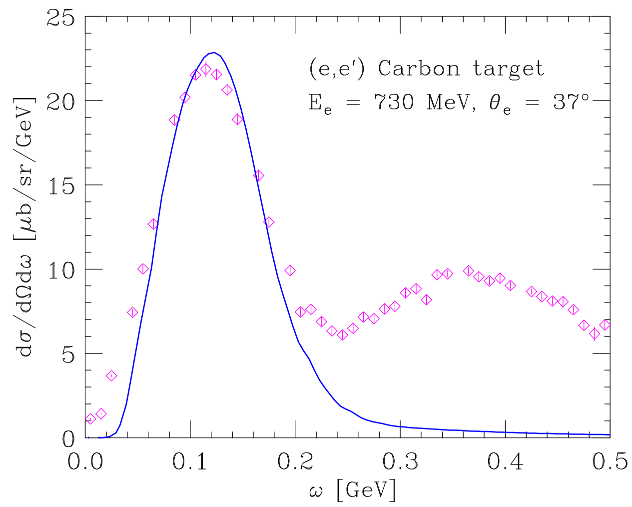
$$\left(\int d^3p\, n(|\vec{p}\,|) = A\right)$$

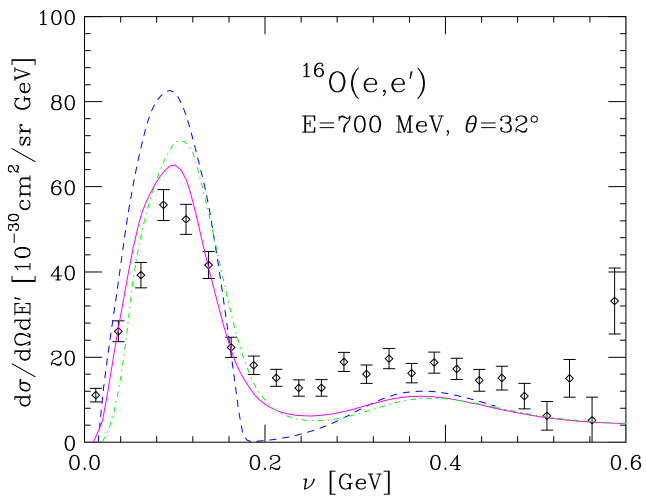
Convolution approach: C. Ciofi degli Atti, S. Liuti, and S. Simula, PRC 53, 1689 (1996), provide realistic distribution due to short-range correlations !

Above QE Region: π Production

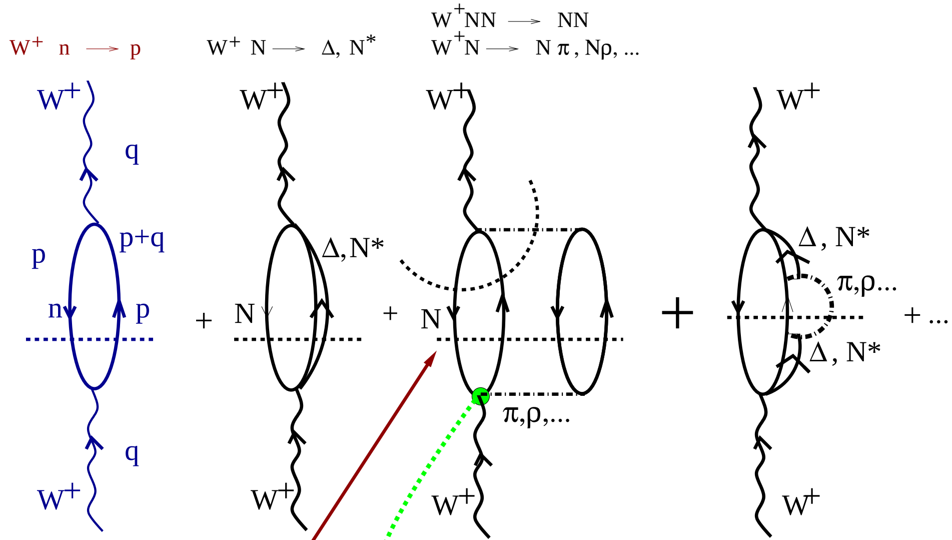
(e,e') PRL 105, 132301 & PRD 72 053005

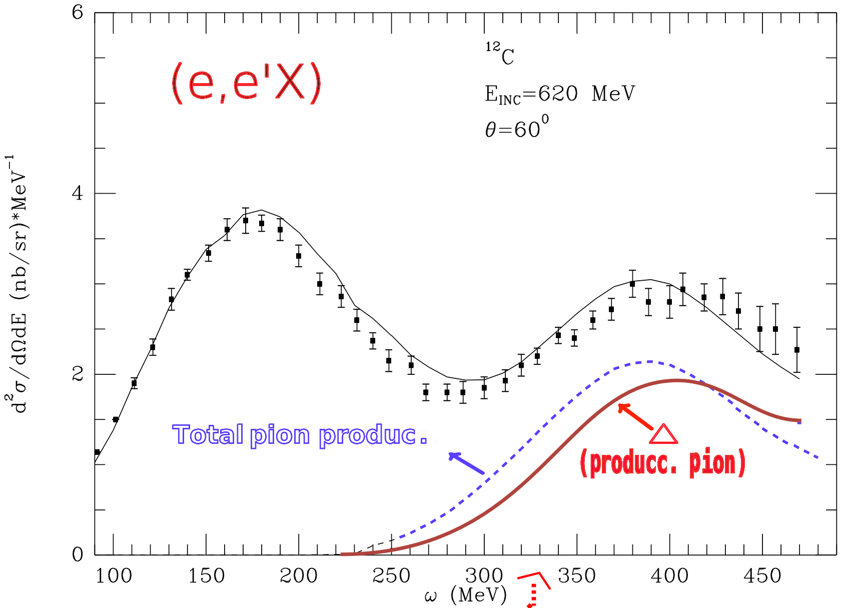






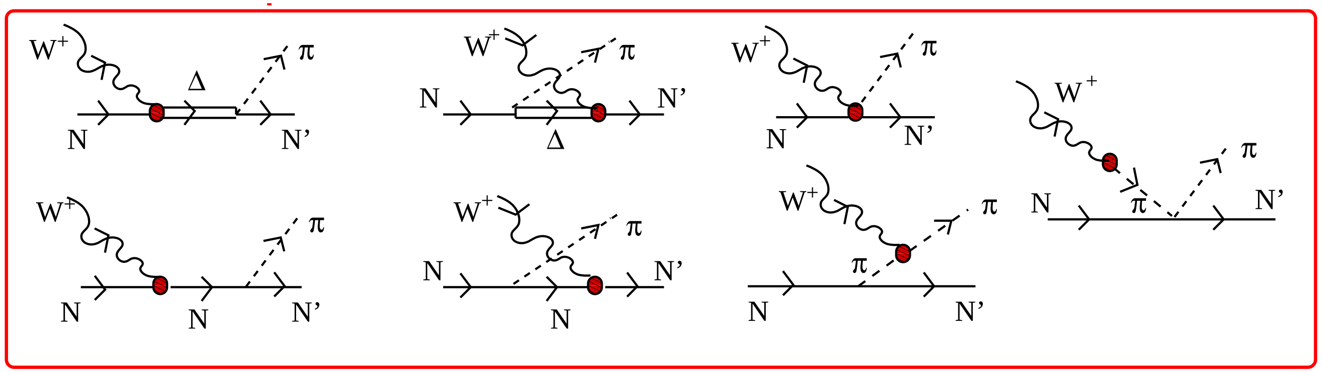
Above QE Region: π Production

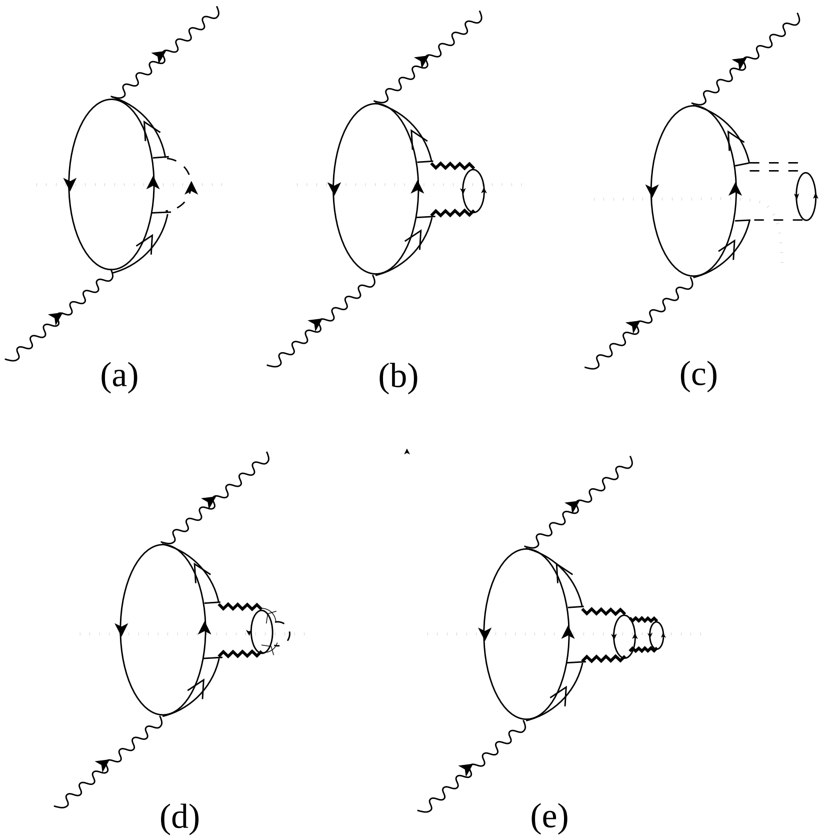


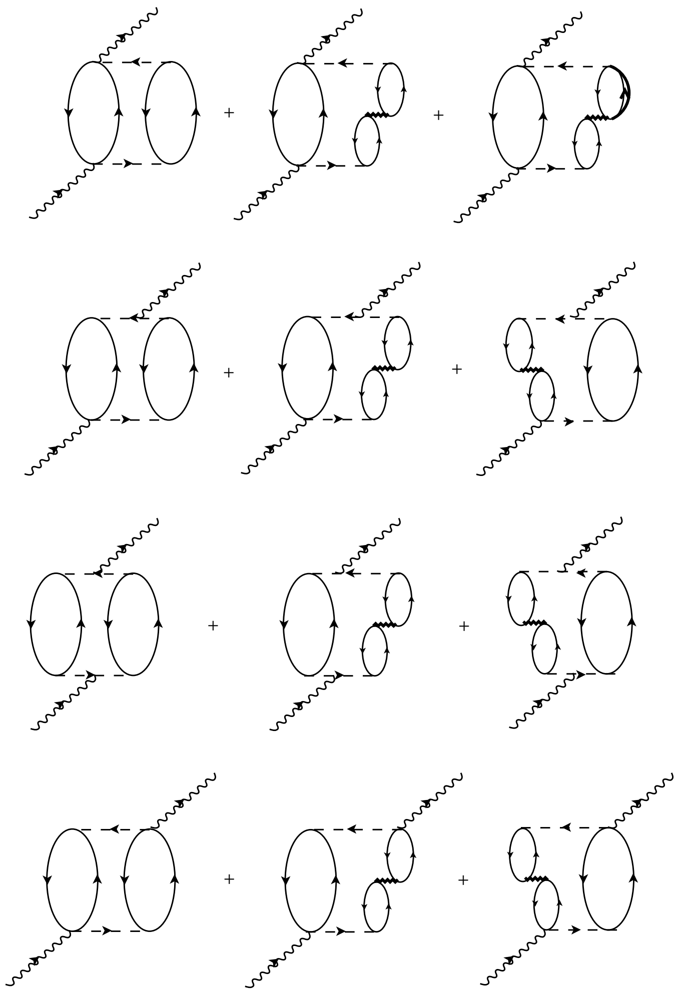


MEC ⟶ QE like !

PRD D76 (2007) 033005
PRD D81 (2010) 085046

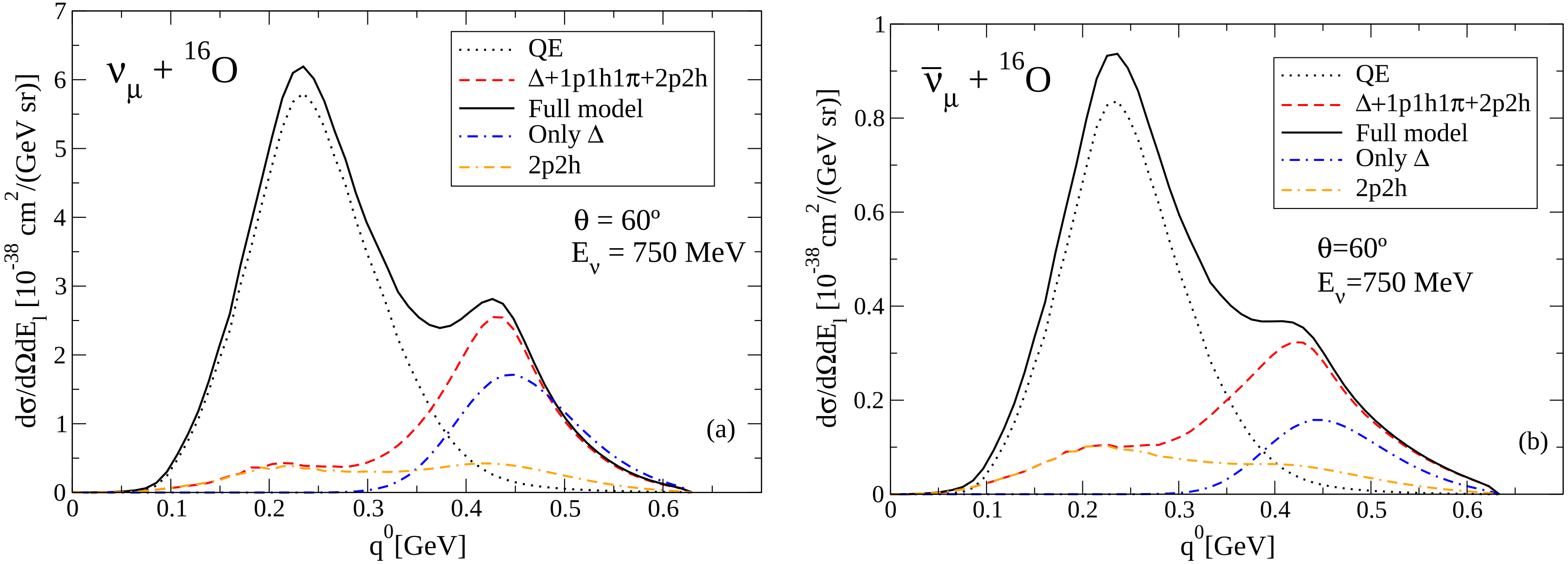


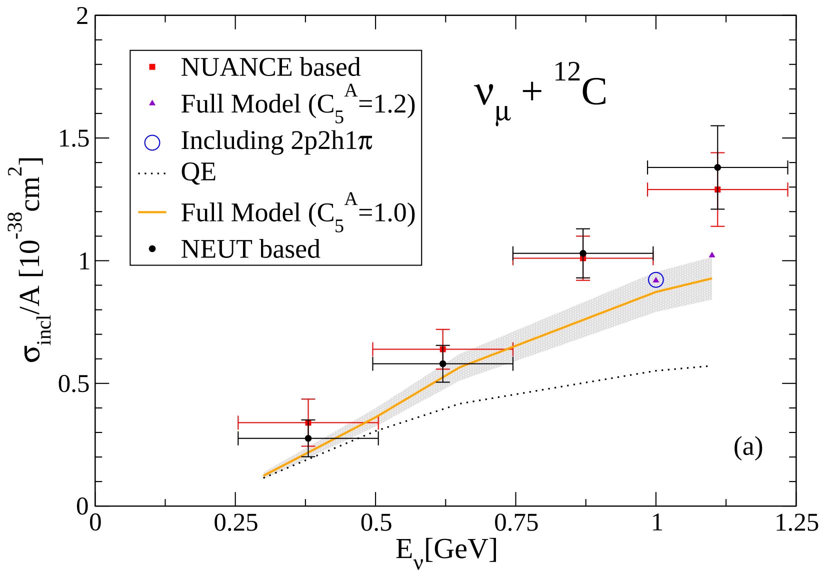

(a)
(b)
(c)
(d)
(e)



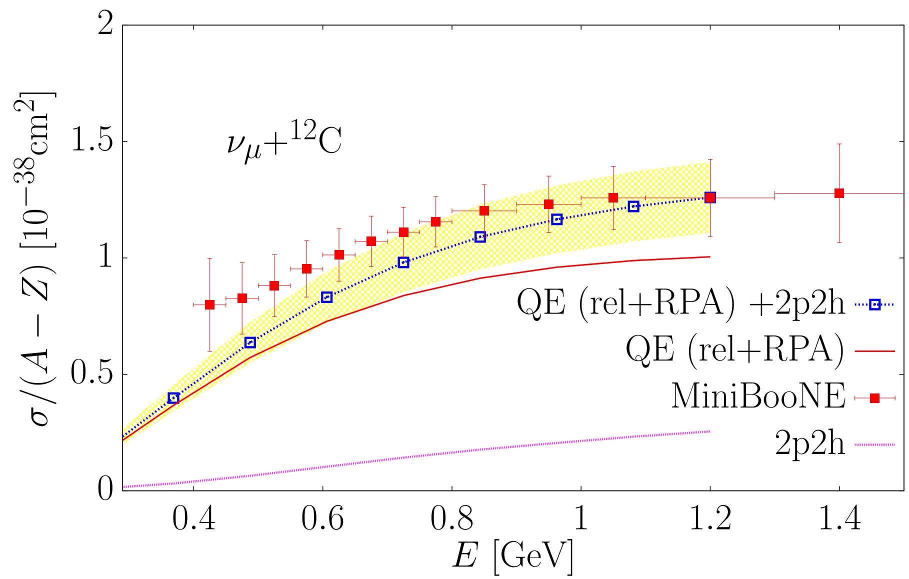
(ν_μ, μ^-) Results

PRC 83 (2011) 045501 [M_A= 1.049 GeV]





NUANCE based
Full Model (C_5^A=1.2)
Including 2p2h1π
QE
Full Model (C_5^A=1.0)
NEUT based
$\nu_\mu + {}^{12}C$
(a)
σ_{incl}/A [10^{-38} cm^2]
E_ν[GeV]



$\nu_\mu + {}^{12}C$
QE (rel+RPA) +2p2h
QE (rel+RPA)
MiniBooNE
2p2h
$\sigma/(A-Z)$ [10^{-38}cm^2]
E [GeV]

MiniBooNE CCQE-like double differential cross section $\frac{d^2\sigma}{dT_\mu \, d\cos\theta_\mu}$

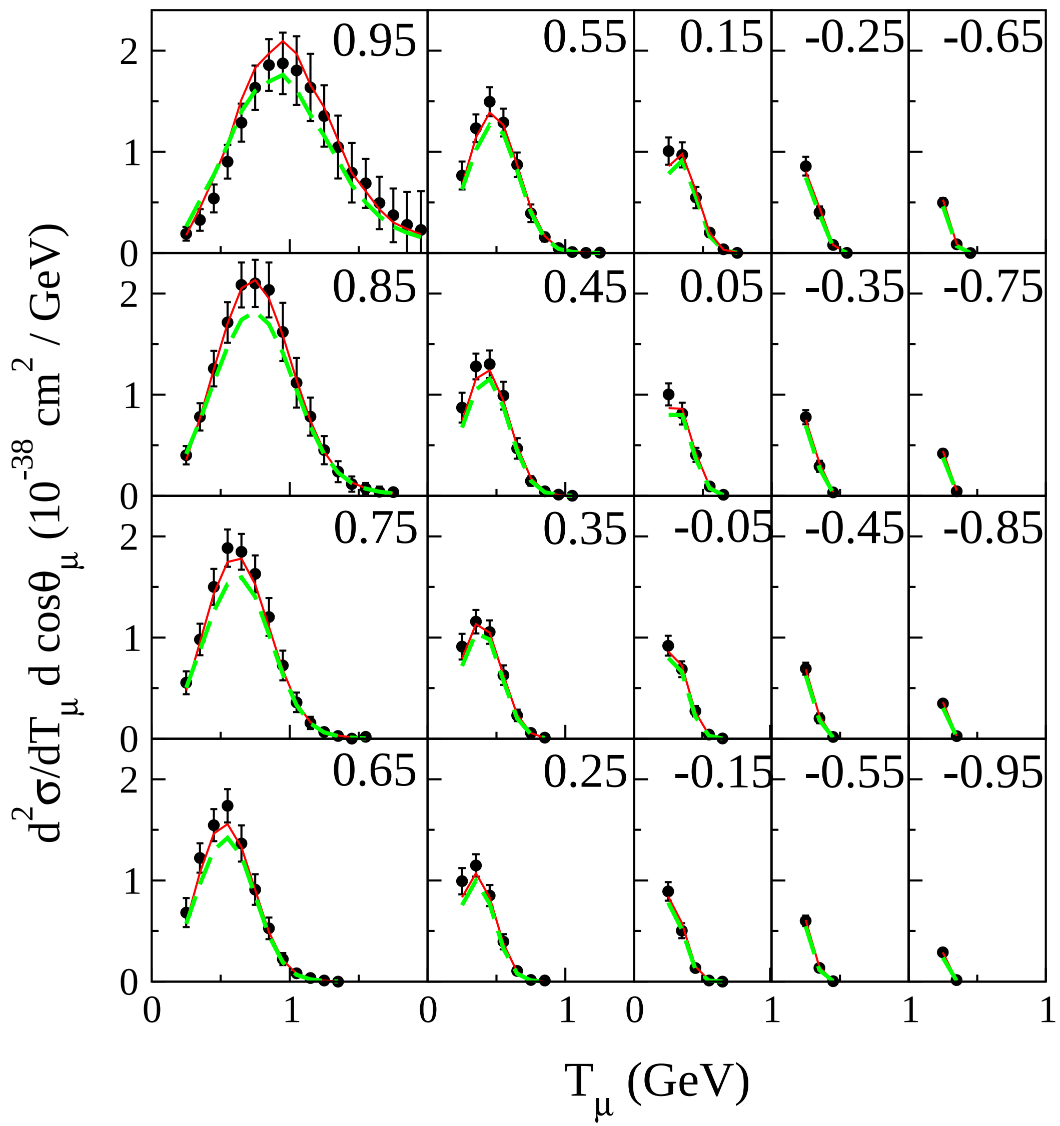
We define a **merit function** and consider our **QE+2p2h results**

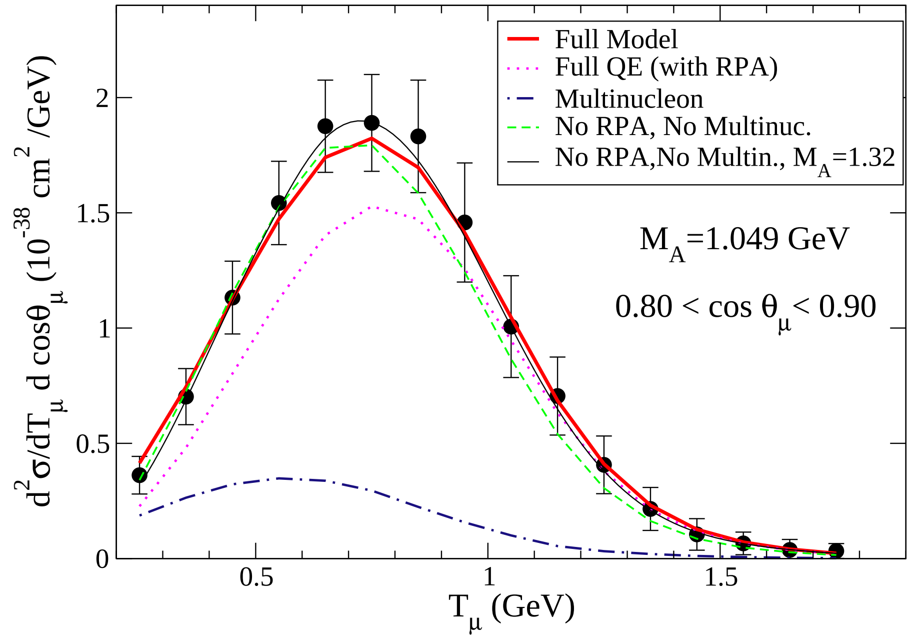
$$\chi^2 = \sum_{i=1}^{137} \left[\frac{\lambda \left(\frac{d^2\sigma^{exp}}{dT_\mu \, d\cos\theta} \right)_i - \left(\frac{d^2\sigma^{th}}{dT_\mu \, d\cos\theta} \right)_i}{\lambda \Delta \left(\frac{d^2\sigma}{dT_\mu \, d\cos\theta} \right)_i} \right]^2 + \left(\frac{\lambda - 1}{\Delta\lambda} \right)^2,$$

that takes into account the **global normalization uncertainty** ($\Delta\lambda = 0.107$) claimed by the MiniBooNE collaboration.

We fit λ to data with a fixed value of M_A (=1.049 GeV).
We obtain $\chi^2/\#$ bins =52/137 with $\lambda = 0.89 \pm 0.01$.

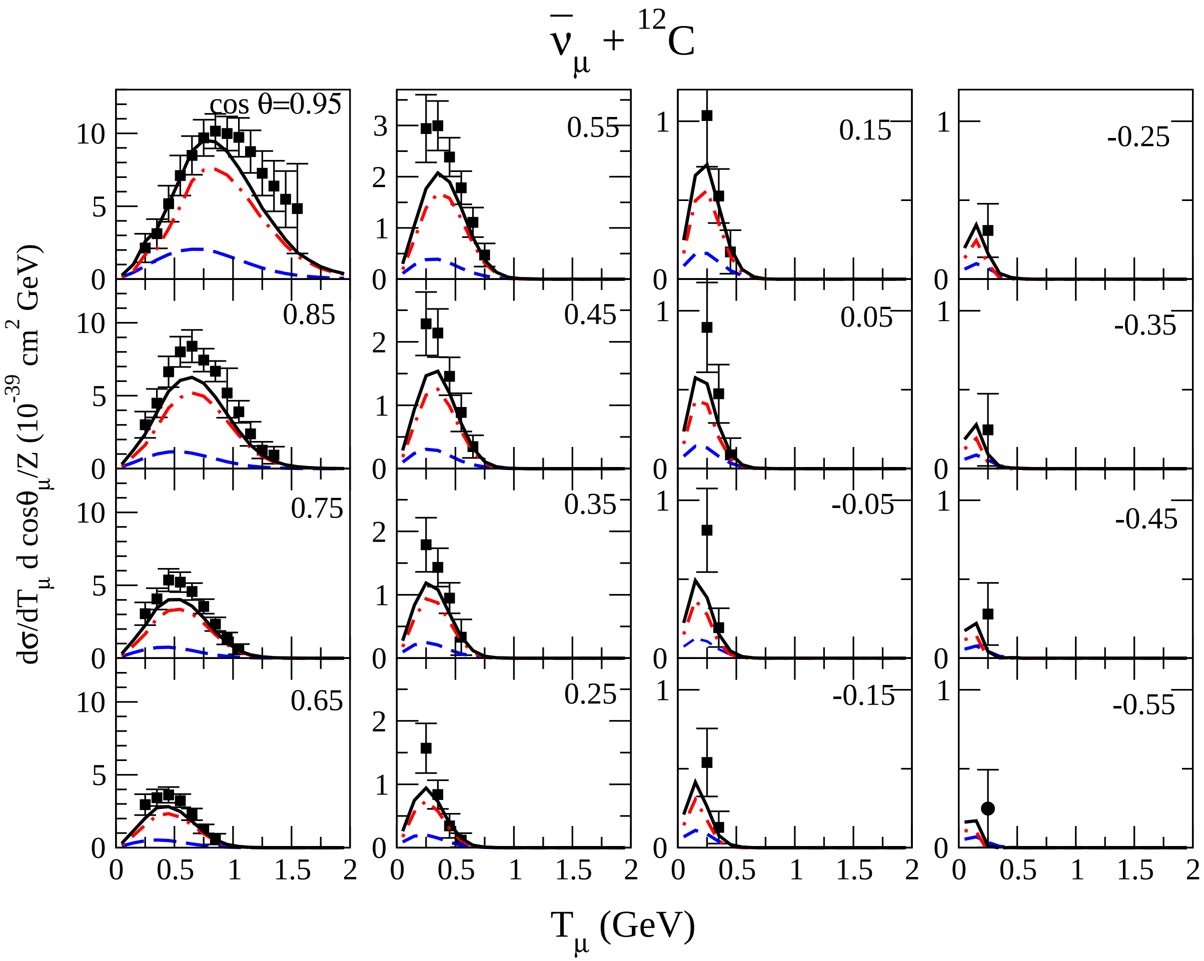
The microscopical model, with no free parameters, agrees remarkably well with data! The shape is very good and χ^2 strongly depends on λ, which is strongly correlated with M_A.

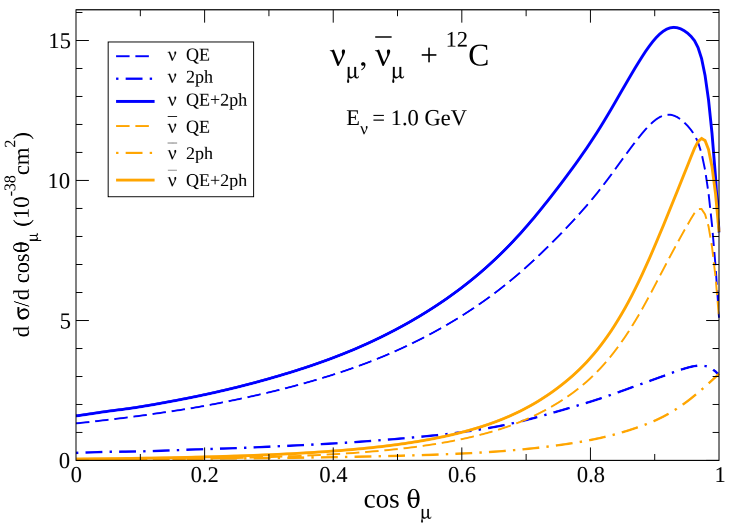


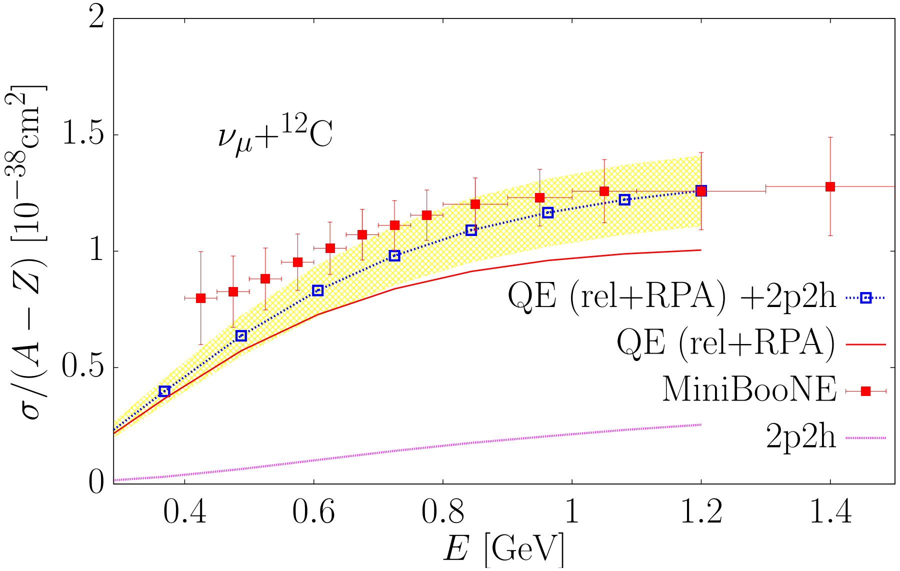


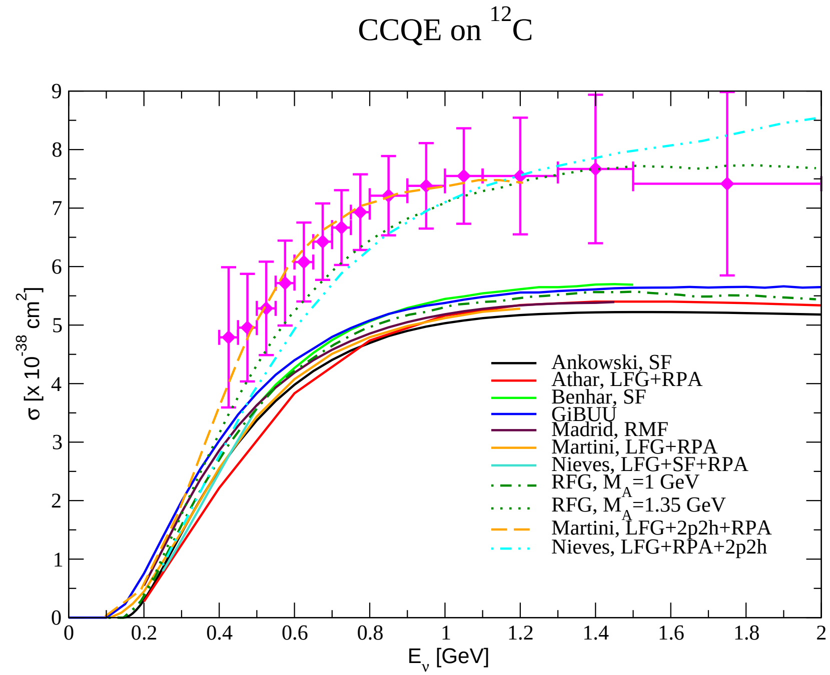
Model	Scale	M_A (GeV)	$\frac{\chi^2}{\#\text{bins}}$
LFG	0.96±0.03	1.32±0.03	35/137
Full	**0.92±0.03**	**1.08±0.03**	**50/137**
Full $\vert q\vert > 0.4^{\dagger}$ GeV	0.83±0.04	1.01±0.03	30/123

† : As suggested by Sobczyk et al. PRC 82, 045502


$\bar{\nu}_\mu + {}^{12}C$
cos θ=0.95
0.85
0.75
0.65
0.55
0.45
0.35
0.25
0.15
0.05
-0.05
-0.15
-0.25
-0.35
-0.45
-0.55
$d\sigma/dT_\mu\, d\cos\theta_\mu/Z\ (10^{-39}\ cm^2\ GeV)$
T_μ (GeV)


ν QE
ν 2ph
ν QE+2ph
$\bar{\nu}$ QE
$\bar{\nu}$ 2ph
$\bar{\nu}$ QE+2ph
$\nu_\mu, \bar{\nu}_\mu + {}^{12}C$
E_ν = 1.0 GeV
$d\sigma/d\cos\theta_\mu\ (10^{-38}\ cm^2)$
$\cos\theta_\mu$


$\nu_\mu + ^{12}C$
QE (rel+RPA) +2p2h
QE (rel+RPA)
MiniBooNE
2p2h
$\sigma/(A-Z)$ [10^{-38}cm^2]
E [GeV]

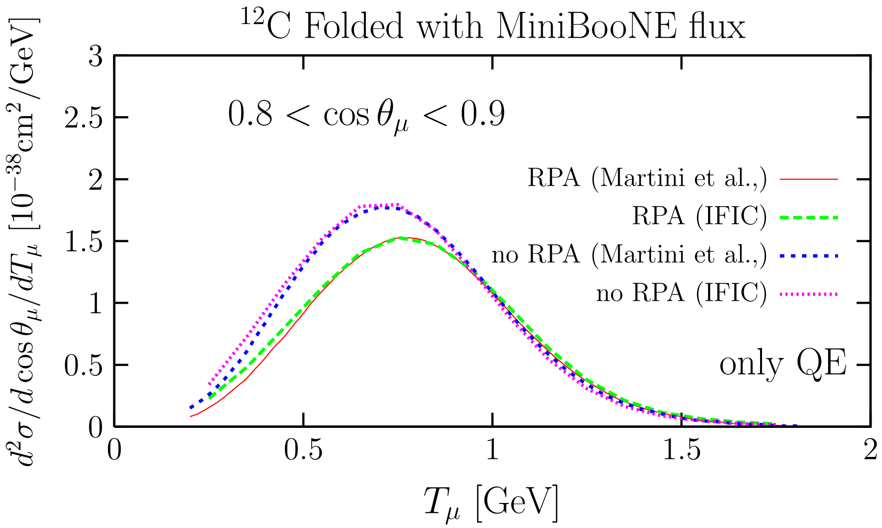

CCQE on ^{12}C
Ankowski, SF
Athar, LFG+RPA
Benhar, SF
GiBUU
Madrid, RMF
Martini, LFG+RPA
Nieves, LFG+SF+RPA
RFG, M_A=1 GeV
RFG, M_A=1.35 GeV
Martini, LFG+2p2h+RPA
Nieves, LFG+RPA+2p2h
σ [x 10^{-38} cm^2]
E_ν [GeV]

Differences with the work of Martini et al. (PRC80,065501)

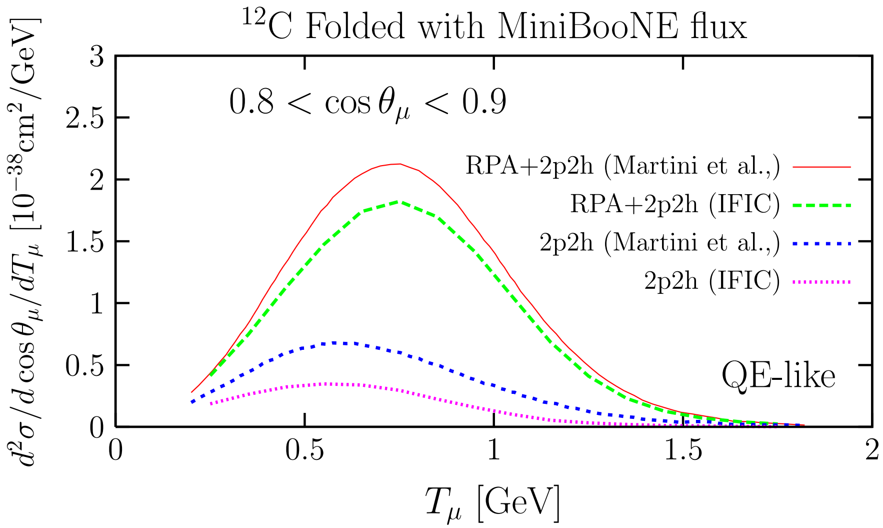
1. **Similar for the 2p2h contributions driven by Δh excitation** (both groups use the same model for the $\Delta-$selfenergy in the medium).

2. **Martini et al. do not consider 2p2h contributions driven by contact, pion pole and pion in flight terms.**

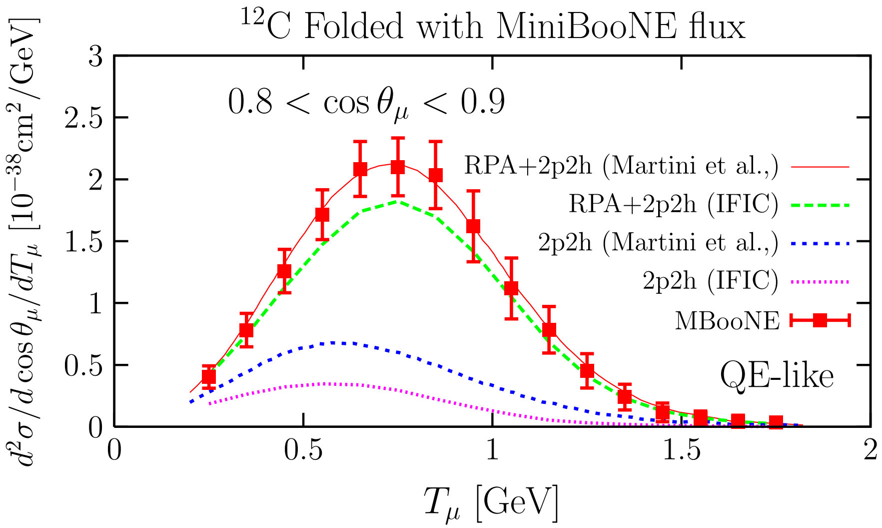
3. **Martini et al. give approximate estimates (no microscopical calculation) for the rest of 2p2h contributions** [relate them to the absorptive part of the $p-$wave pion-nucleus optical potential at threshold or to a microscopic calculation by Alberico et al. (Annals Phys. 154, 356) specifically aimed at the evaluation of the 2p-2h contribution to the isospin spin-transverse response, measured in inclusive (e, e') scattering].



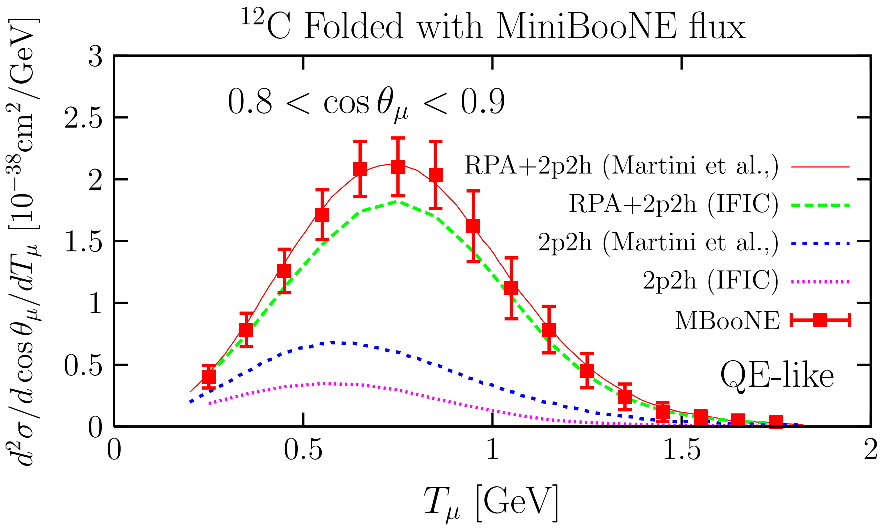
We compare rather well with Martini et al., PRC 84, 055502 for bare QE and QE+RPA



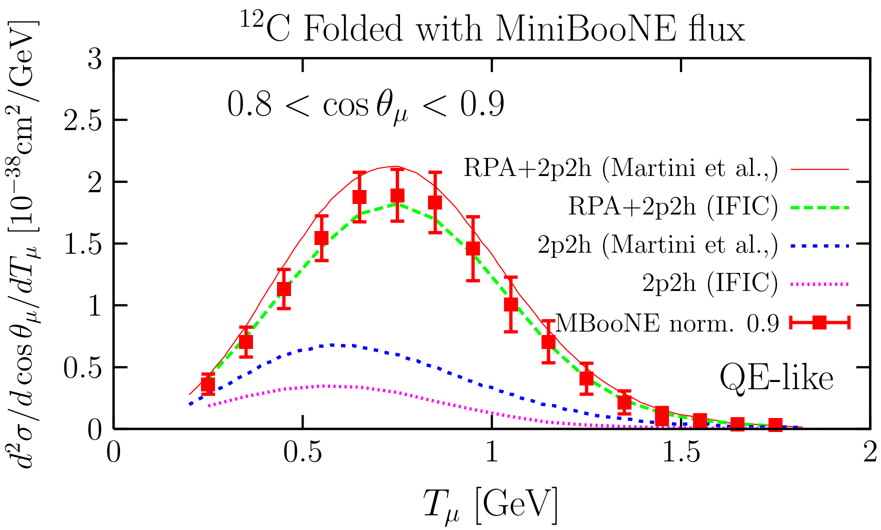
...however our 2p2h contribution is about a factor of 2 smaller!



Martini et al., predictions look consistent with MiniBooNE data ...



Martini et al., predictions look consistent with MiniBooNE data ..., **but their estimate rely on some computation of the 2p2h mechanisms for (e, e') (Alberico et al.,) $\Rightarrow$ no info on axial part of the interaction!**



...however our predictions for the 2p2h contribution would favor a global normalization scale of about 0.9. This would be consistent with the MiniBooNE estimate of a total normalization error of 10.7%.

Neutrino beams ARE NOT monochromatic. For QE-like events, only the charged lepton is observed and the only measurable quantities are then its direction (scattering angle θ_μ with respect to the neutrino beam direction) and its energy E_μ. **The energy of the neutrino that has originated the event is unknown.** Assuming QE dynamics is defined a **"reconstructed" energy**

$$E_{\rm rec} = \frac{ME_\mu - m_\mu^2/2}{M - E_\mu + |\vec{p}_\mu|\cos\theta_\mu}$$

(genuine quasielastic event on a nucleon at rest, ie. $E_{\rm rec}$ is determined by the QE-peak condition $q^0 = -q^2/2M$). Note that **each event contributing to the flux averaged double differential cross section** $d\sigma/dE_\mu d\cos\theta_\mu$ **defines unambiguously a value of** $E_{\rm rec}$**. The actual ("true") energy,** E**, of the neutrino that has produced the event will not be exactly** $E_{\rm rec}$**.**

Flux-folded $d\sigma/dT_\mu d\cos\theta_\mu \overbrace{\rightarrow}^{?}$ CCQE-like unfolded $\sigma(E)$

Unfolding procedure needs theoretical input!

$$P_{\rm true}(E) = \int dE_{\rm rec}\, \underbrace{P_{\rm rec}(E_{\rm rec})}_{\rm EXP}\, \underbrace{P(E|E_{\rm rec})}_{theory!}$$

$P_{\rm rec}(E_{\rm rec})$ is the *pd* of measuring an event with reconstructed energy $E_{\rm rec}$. $P(E|E_{\rm rec})$ is, given an event of reconstructed energy $E_{\rm rec}$, the conditional *pd* of being produced by a neutrino of energy E.

...using Bayes's theorem $P(E|E_{\rm rec})$ could be related to

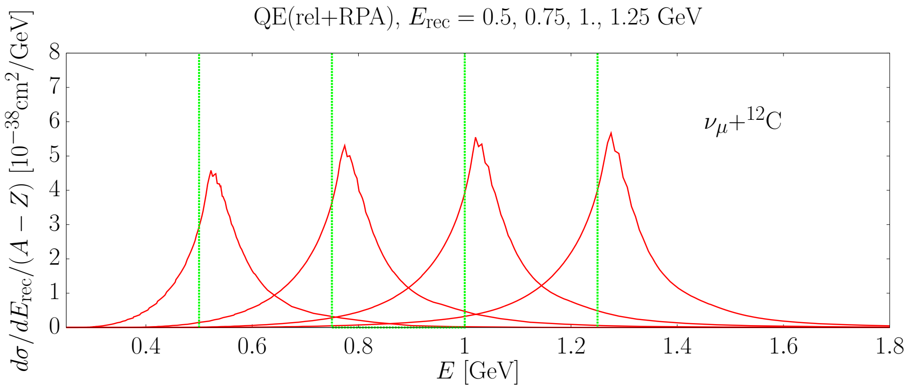
$$P(E_{\rm rec}|E) \quad \underline{\text{is determined by}} \quad \frac{d\sigma}{dE_{\rm rec}}(E;E_{\rm rec})$$

$$\mathbf{P(E|E_{rec})} = \frac{\mathbf{P(E_{rec}|E)} P_{\rm true}(E)}{P_{\rm rec}(E_{\rm rec})}$$

$P(E_{\rm rec}|E)$ is the conditional *pd* of measuring an event with reconstructed energy $E_{\rm rec}$ and induced by the interaction with the nuclear target of a neutrino of energy E.

$$P(E_{\rm rec}|E) = \frac{1}{\sigma(E)} \frac{d\sigma}{dE_{\rm rec}}(E; E_{\rm rec}), \qquad P_{\rm true}(E) \propto \Phi(E)\sigma(E)$$

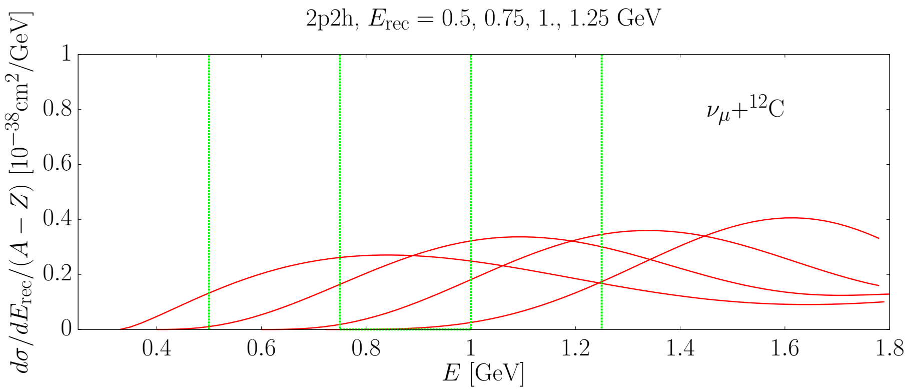
$$\frac{d\sigma}{dE_{\rm rec}}(E; E_{\rm rec}) = \int_{m_\mu}^{E} dE_\mu \left| \frac{\partial(\cos\theta_\mu)}{\partial E_{\rm rec}} \right| \underbrace{\frac{d^2\sigma}{d(\cos\theta_\mu) dE_\mu}(E; E_{\rm rec})}_{\text{theory!}}$$



Neutrino Energy Reconstruction and the Shape of the CCQE-like Total Cross Section

(qualitatively in agreement with Martini et al., PRD85 093012)

$$\frac{d\sigma}{dE_{\rm rec}}(E;E^0_{\rm rec}) \quad = \quad \int_{m_\mu}^{E} dE_\mu \, \frac{d^2\sigma}{dE_{\rm rec}dE_\mu}(E;E^0_{\rm rec}) = \int_{m_\mu}^{E} dE_\mu \left| \frac{\partial(\cos\theta_\mu)}{\partial E_{\rm rec}} \right| \frac{d^2\sigma}{d(\cos\theta_\mu)dE_\mu}(E;E^0_{\rm rec})$$



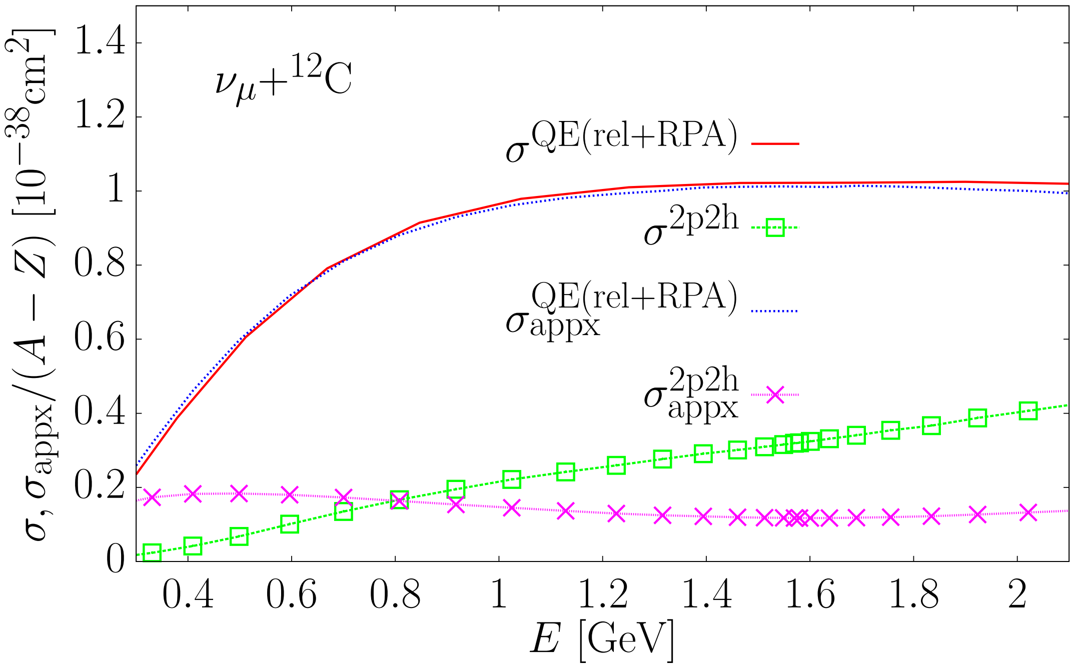
For each $E_{\rm rec}$, there exists a distribution of true neutrino energies that could give rise to events whose muon kinematics would lead to the given value of $E_{\rm rec}$.

$$\sigma(E) = \int dE_{\rm rec} \underbrace{\left[\langle\sigma\rangle P_{\rm rec}(E_{\rm rec})\right]}_{\textbf{EXP}} \times \overbrace{\underbrace{\left[\frac{d\sigma/dE_{\rm rec}(E;E_{\rm rec})}{\int dE''\Phi(E'')d\sigma/dE_{\rm rec}(E'';E_{\rm rec})}\right]}_{MODEL}}^{P(E|E_{\rm rec})}$$

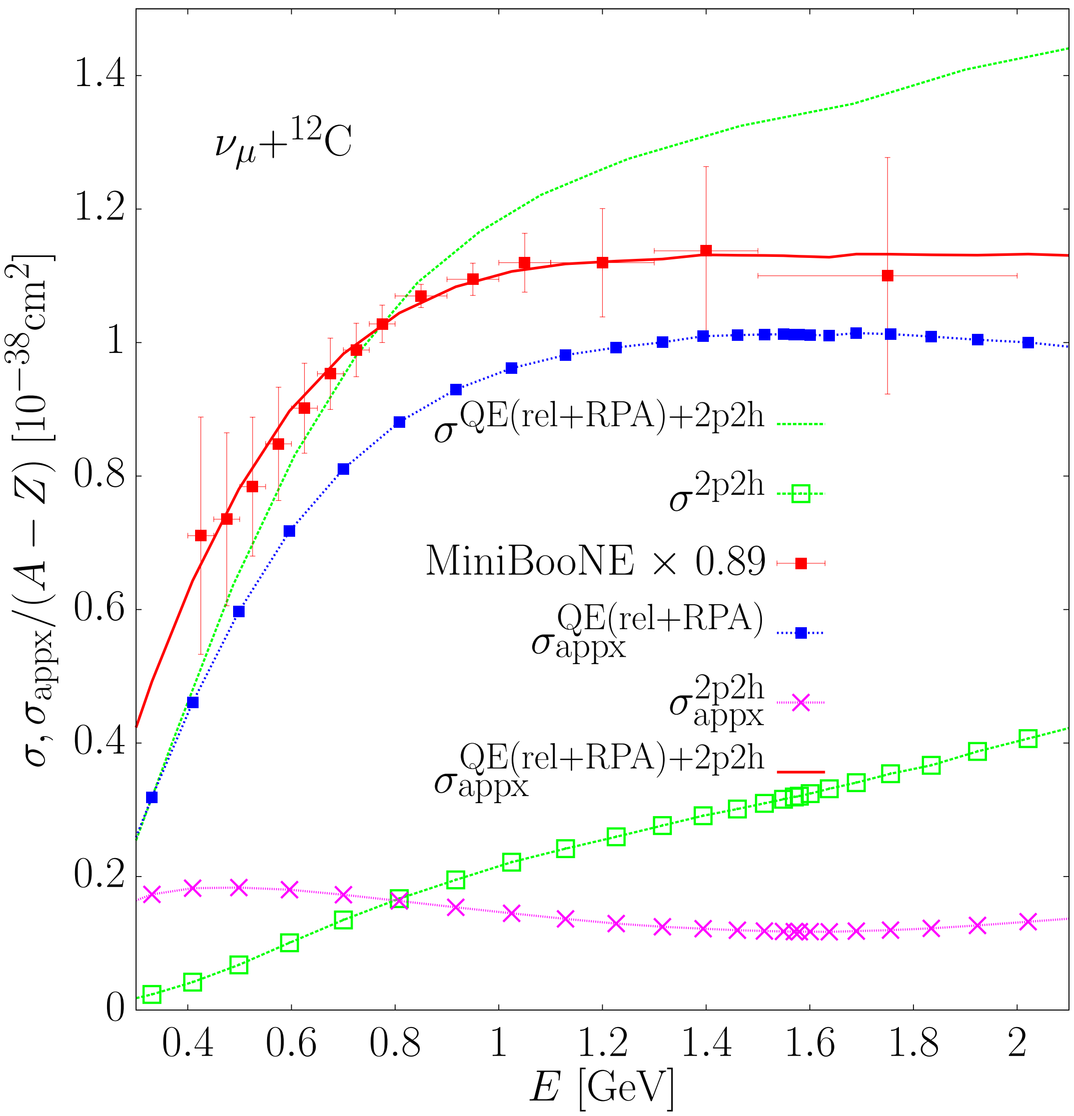
$$\sigma = \underbrace{\sigma^{\rm QE(\textbf{RPA})}}_{M_A=1.05\ \rm GeV} + \sigma^{\rm 2p2h}$$

$$\sigma(E) = \int dE_{\rm rec} \underbrace{\left[\langle\sigma\rangle P_{\rm rec}(E_{\rm rec})\right]}_{\textbf{EXP}} \times \underbrace{\left[\frac{d\sigma/dE_{\rm rec}(E;E_{\rm rec})}{\int dE''\Phi(E'')d\sigma/dE_{\rm rec}(E'';E_{\rm rec})}\right]}_{MODEL:\ \textbf{ONLY QE},\ \mathbf{M_A=1.32}\ \text{GeV}\ \textbf{and noRPA}}$$

$$\sigma = \underbrace{\sigma^{\rm QE(\mathbf{noRPA})}}_{M_A=1.32\ \text{GeV}} + \underbrace{\sigma^{\rm 2p2h}}_{\text{neglected!}}$$



$$\left[\langle\sigma\rangle P_{\rm rec}(E_{\rm rec})\right]_{\rm Exp} \sim \int \left(\frac{d\sigma}{dE_{\rm rec}}(E';E_{\rm rec})\Big|^{M_A=1.049\ \text{GeV}}_{\text{QE+RPA},} + \frac{d\sigma^{\rm 2p2h}}{dE_{\rm rec}}(E';E_{\rm rec})\right)\Phi(E')dE'$$



$$\left[\langle\sigma\rangle P_{\rm rec}(E_{\rm rec})\right]_{\rm Exp} \sim$$

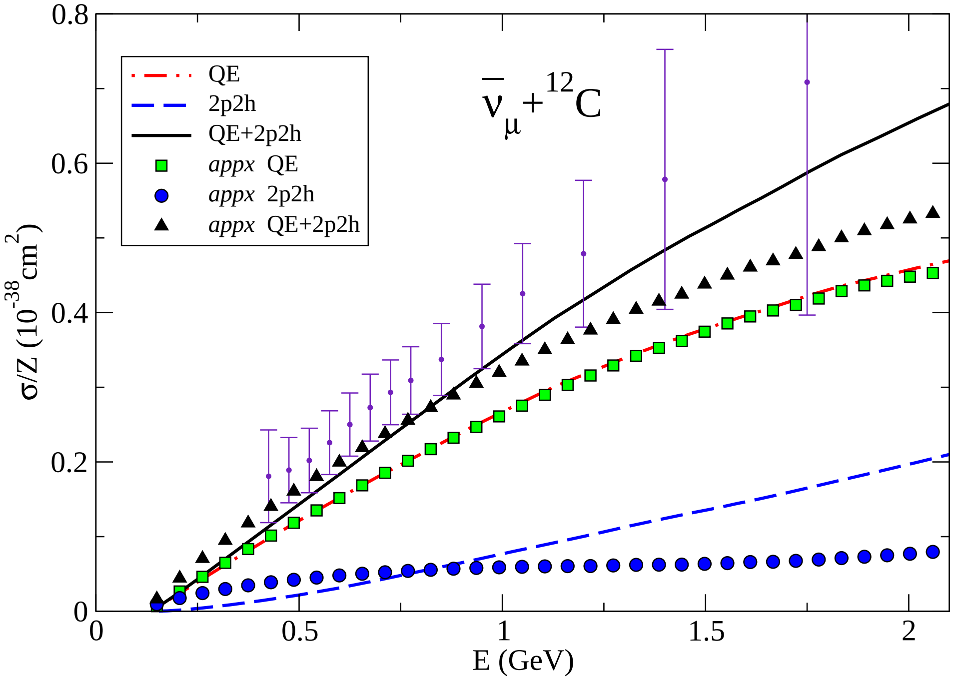
$$\int \left(\frac{d\sigma}{dE_{\rm rec}}(E';E_{\rm rec})\Big|^{M_A=1.049\ {\rm GeV}}_{\rm QE+RPA,} + \frac{d\sigma^{\rm 2p2h}}{dE_{\rm rec}}(E';E_{\rm rec})\right)\Phi(E')dE'$$

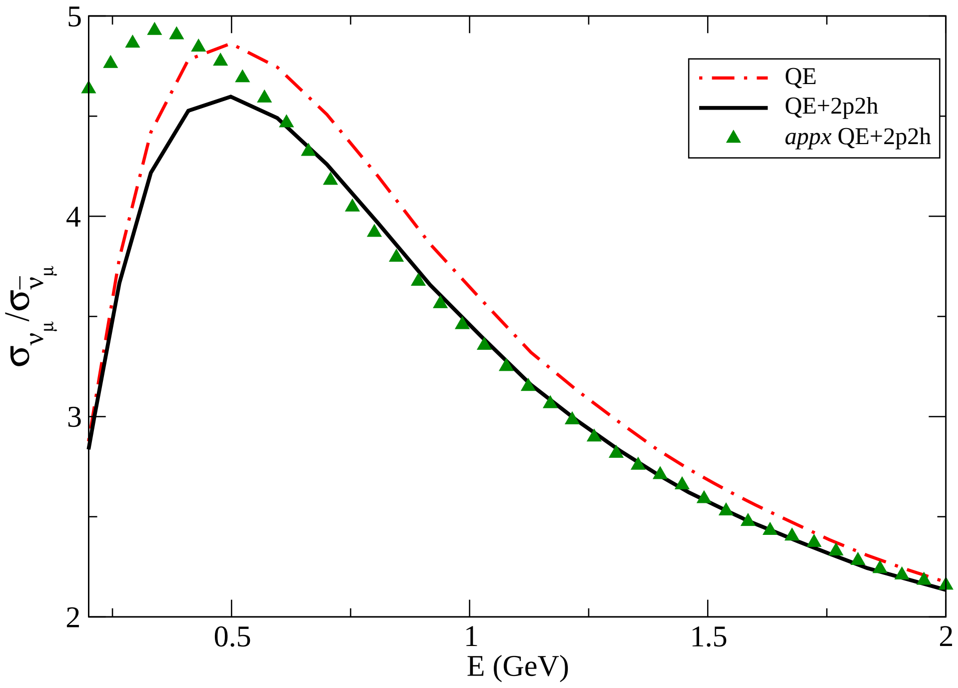
... and

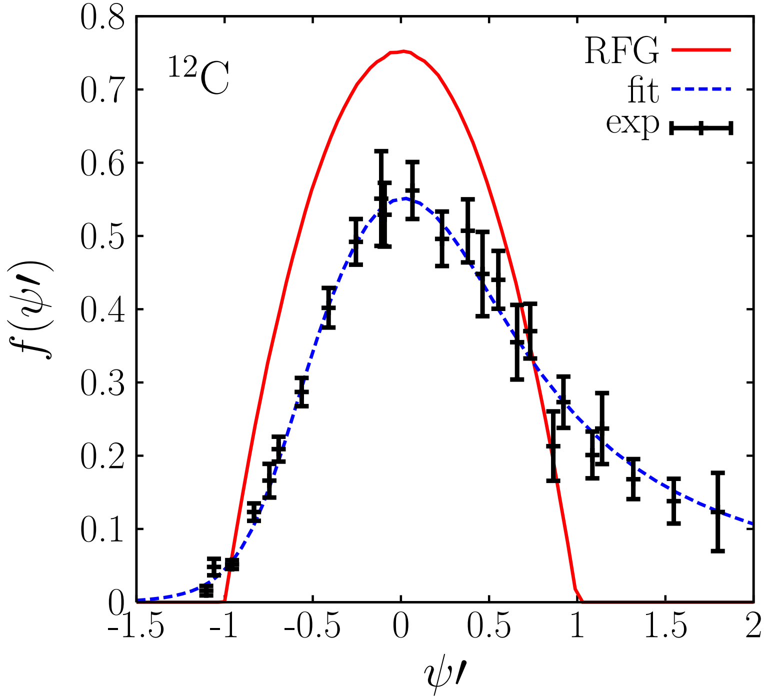
$$\underbrace{\left[\frac{d\sigma/dE_{\rm rec}(E;E_{\rm rec})}{\int dE''\Phi(E'')d\sigma/dE_{\rm rec}(E'';E_{\rm rec})}\right]}$$

ONLY QE ,M_A=1.32 GeV **and noRPA**

For $\bar{\nu}$


QE
2p2h
QE+2p2h
appx QE
appx 2p2h
appx QE+2p2h
$\bar{\nu}_\mu + {}^{12}C$
σ/Z (10^-38 cm^2)
E (GeV)


QE
QE+2p2h
appx QE+2p2h
$\sigma_{\nu_\mu}/\sigma_{\bar{\nu}_\mu}$
E (GeV)



Superscaling approach: Inclusive electron scattering data exhibit interesting systematics that can be used to predict (anti)neutrino-nucleus cross sections (T. Donnelly and I. Sick, PRL 82, 3212 (1999)),

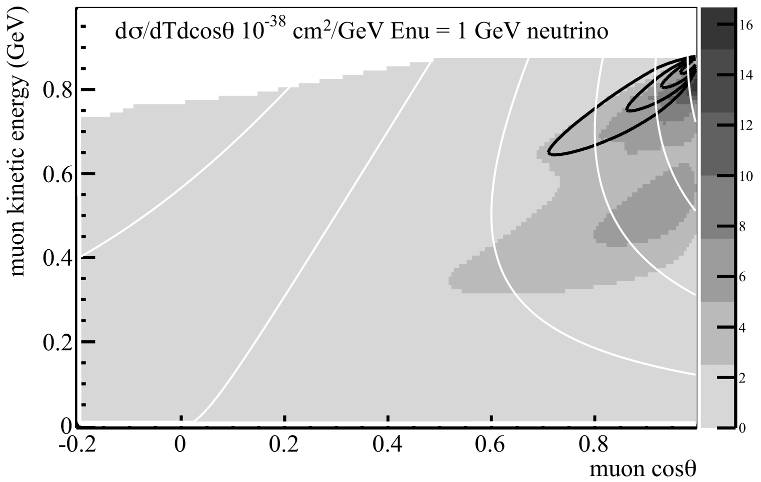
$$f = k_F \frac{\frac{d\sigma}{d\Omega' dE'}}{Z\sigma_{ep} + N\sigma_{en}}$$

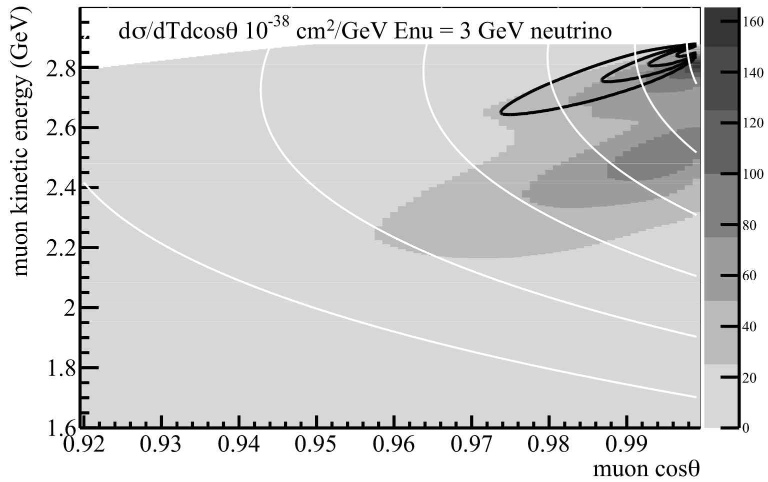
- $f = f(\psi')$, **with** $\psi' = \psi'(q^0, |\vec{q}\,|)$
- f **is largely independent of the specific nucleus**

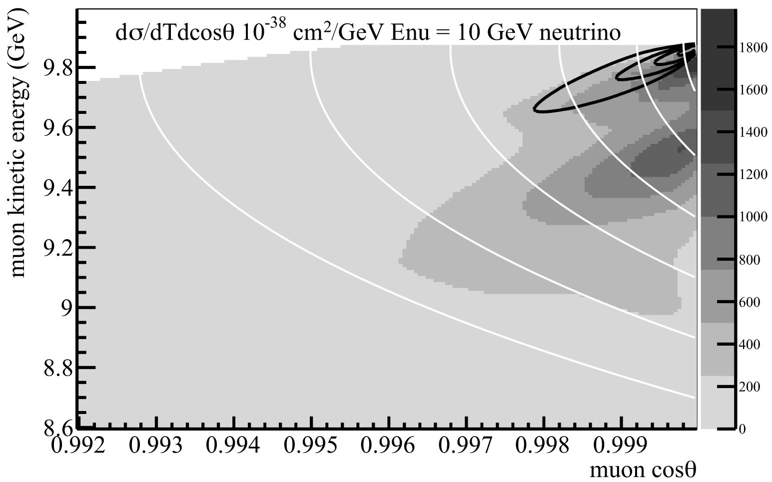
Scaling violations reside mainly in R_T: excitation of resonances, meson production, 2p2h mechanisms and even the tail of DIS. An experimental scaling function $f(\psi')$ could be reliably extracted by fitting the data for R_L.

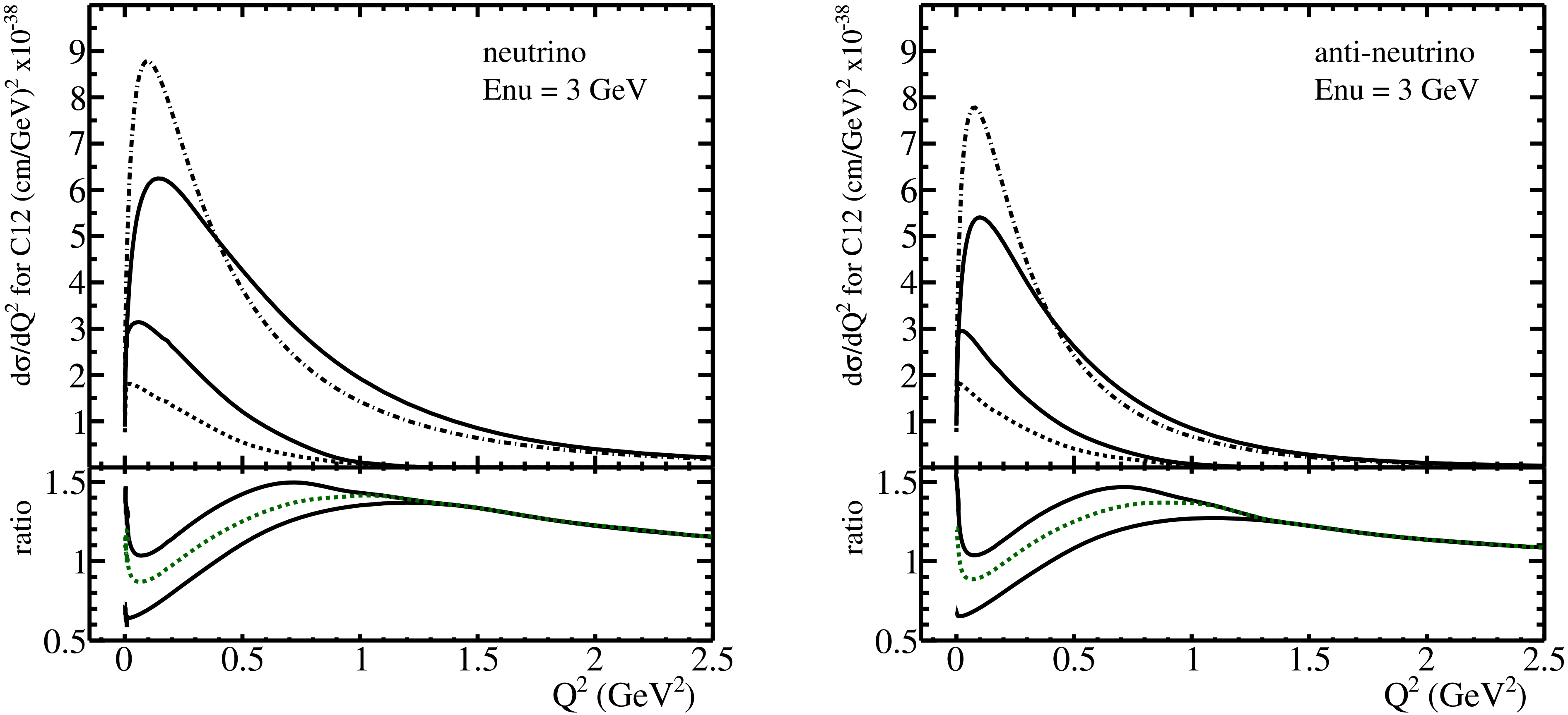
ν QE cross sections can be calculated with the simple RgFG model followed by the replacement $f_{RgFG} \to f_{\rm exp}$.

At higher ν energies

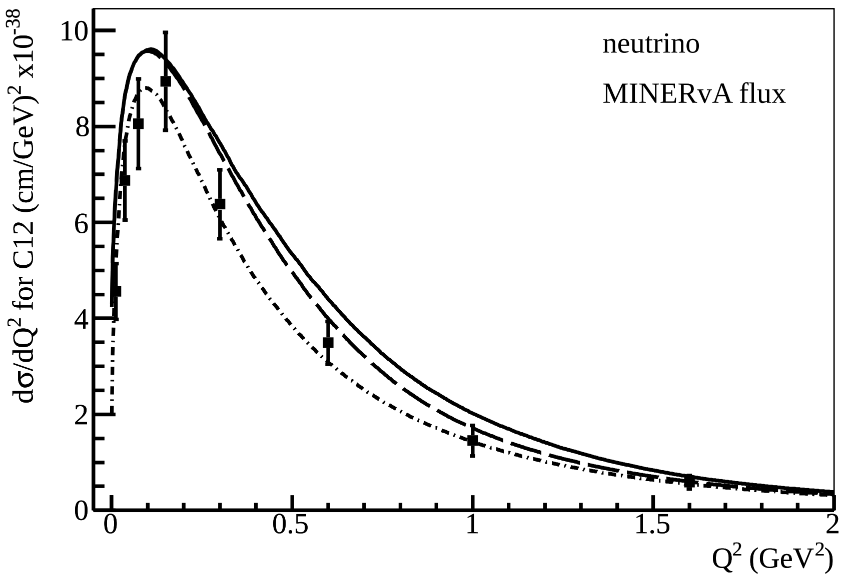


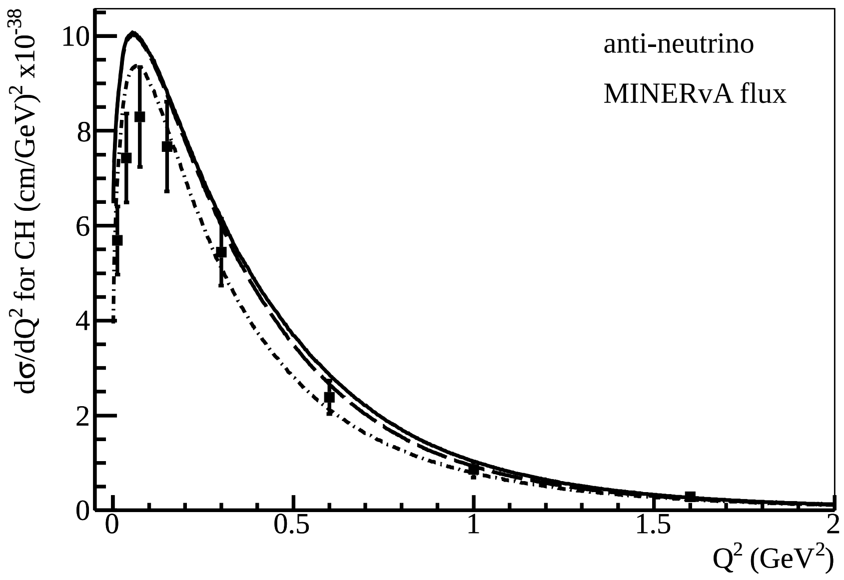


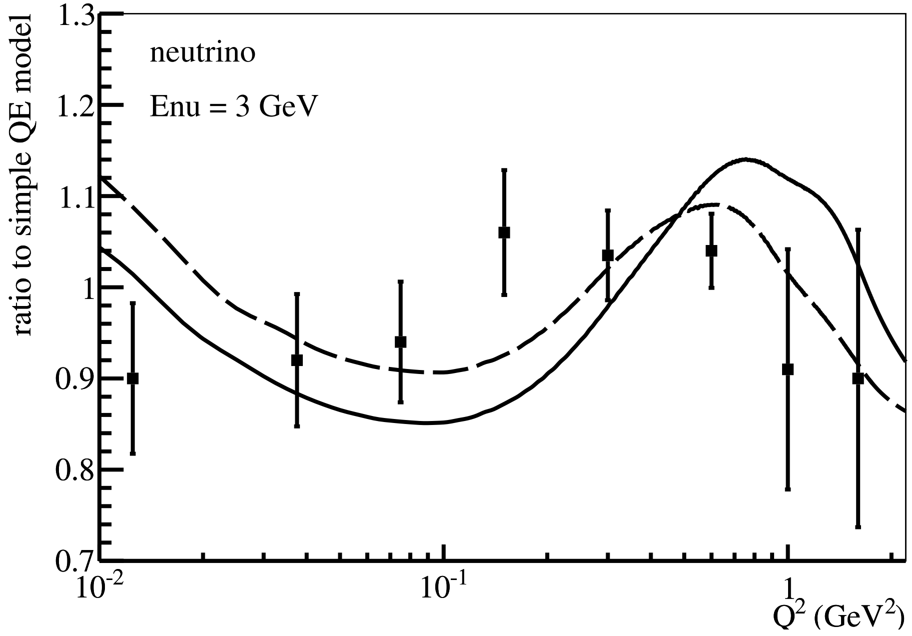


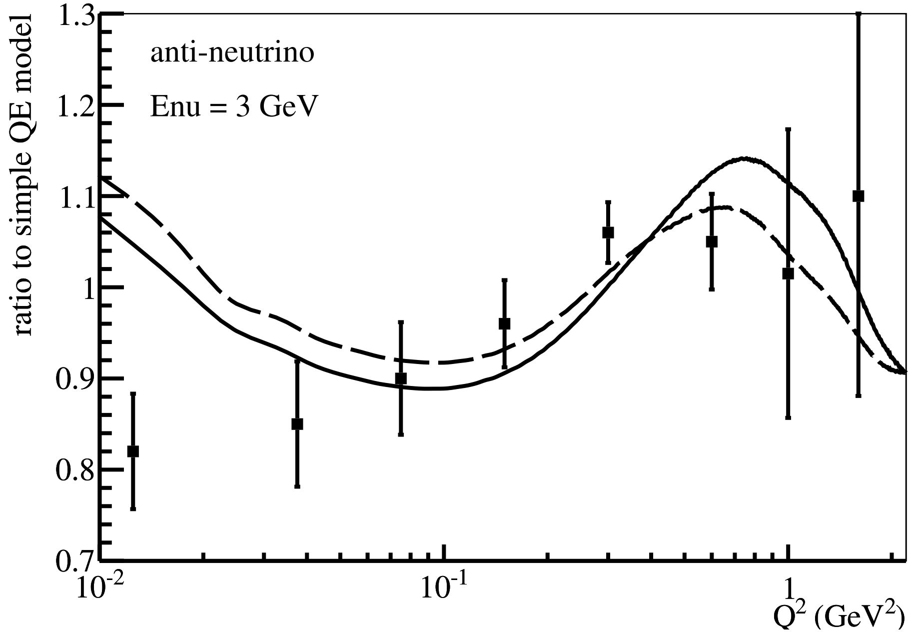

neutrino
Enu = 3 GeV
anti-neutrino
Enu = 3 GeV
dσ/dQ² for C12 (cm/GeV)² x10⁻³⁸
ratio
Q² (GeV²)
9
8
7
6
5
4
3
2
1
1.5
1
0.5
0
0.5
1
1.5
2
2.5

MINERνA


neutrino
MINERvA flux
dσ/dQ² for C12 (cm/GeV)² x10⁻³⁸
Q² (GeV²)
0
2
4
6
8
10
0.5
1
1.5


anti-neutrino
MINERvA flux
dσ/dQ² for CH (cm/GeV)² x10⁻³⁸
Q² (GeV²)
0
2
4
6
8
10
0.5
1
1.5


ratio to simple QE model
neutrino
Enu = 3 GeV
1.3
1.2
1.1
1
0.9
0.8
0.7
10^{-2}
10^{-1}
1
Q^2 (GeV^2)


ratio to simple QE model
anti-neutrino
Enu = 3 GeV
1.3
1.2
1.1
1
0.9
0.8
0.7
10^{-2}
10^{-1}
1
Q^2 (GeV^2)

Conclusions

- We have analyzed the MiniBooNE CCQE $\frac{d^2\sigma}{dT_\mu d\cos\theta_\mu}$ data using a theoretical model that has proved to be quite successful in the analysis of nuclear reactions with electron, photon and pion probes and contains no additional free parameters.
- RPA and multinucleon knockout have been found to be essential for the description of the data.
- MiniBooNE ν and $\bar{\nu}$ CCQE-like data are fully compatible with former determinations of M_A in contrast with several previous analyses. We find, $\boxed{M_A = 1.08 \pm 0.03}$.
- The ν_μ flux could have been underestimated ($\sim 10\%$)

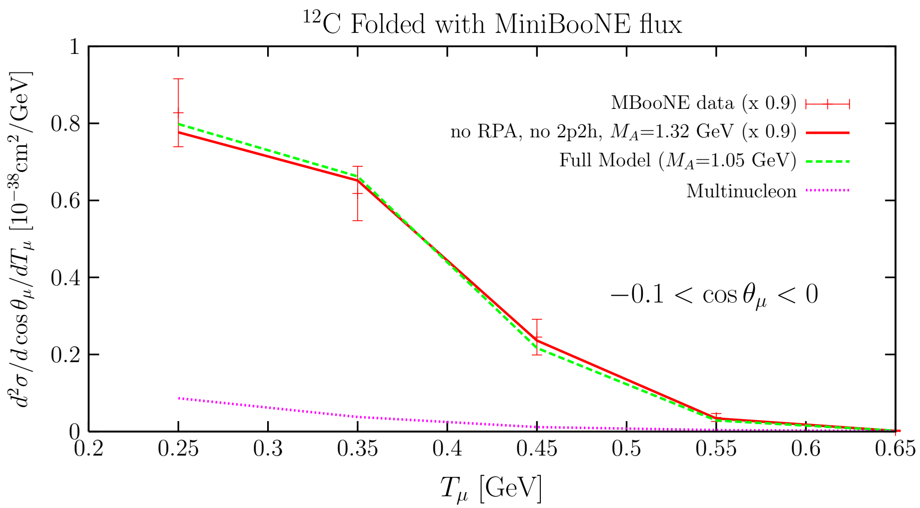
- **Because of the the multinucleon mechanism effects, the algorithm used to reconstruct the neutrino energy is not adequate when dealing with quasielastic-like events.**

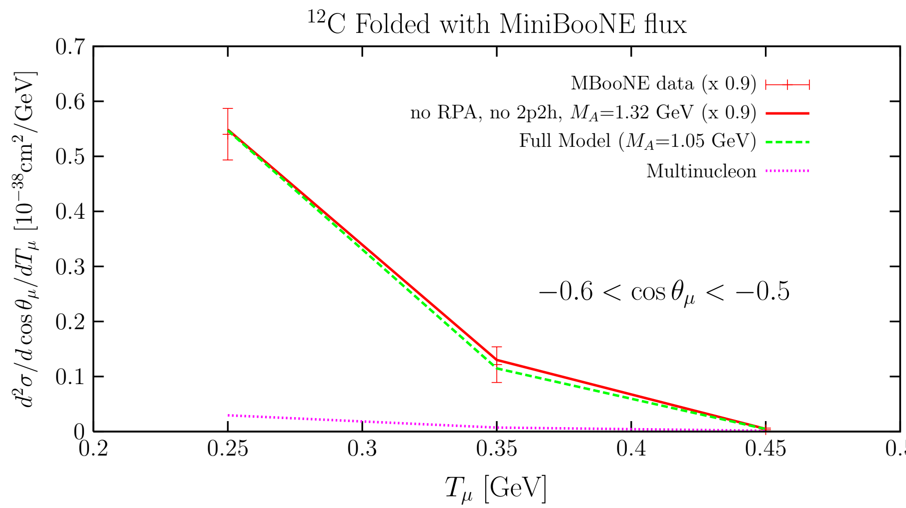
- **The inclusion of nucleon-nucleon correlation effects in the RPA series yields a much larger shape distortion toward relatively more high-q^2 interactions, with the 2p2h component filling in the suppression at very low q^2.**

- **When confronted with the MINERνA data and its small uncertainties, the model has the qualitative features and magnitude to give reasonable agreement.**

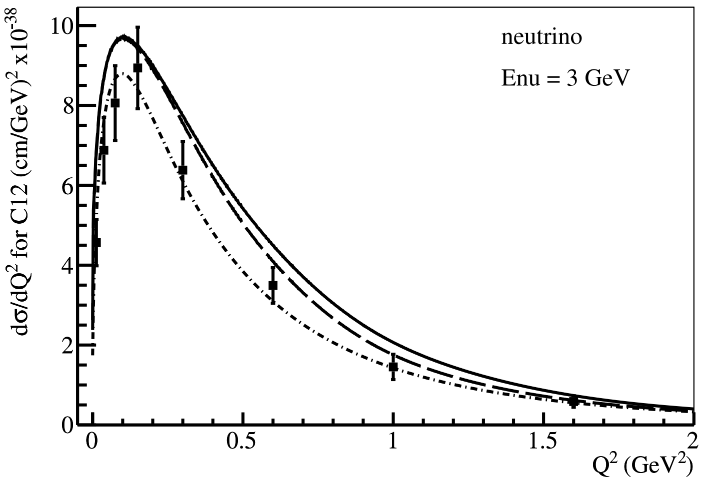
Back up material

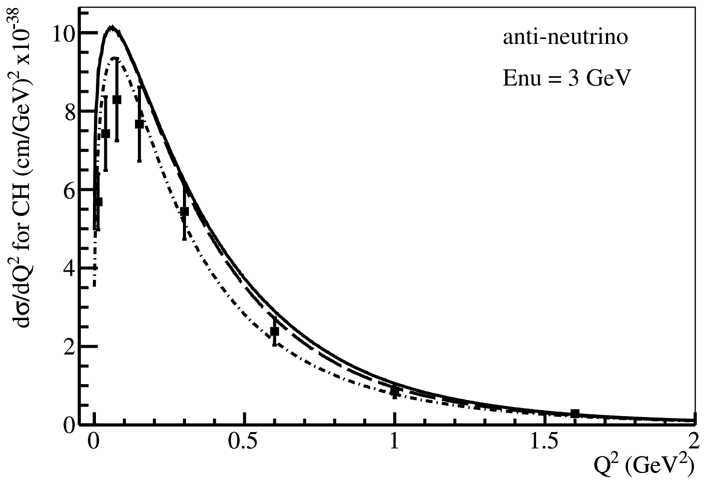
Dependence of the 2p2h contribution on $\cos\theta_\mu$

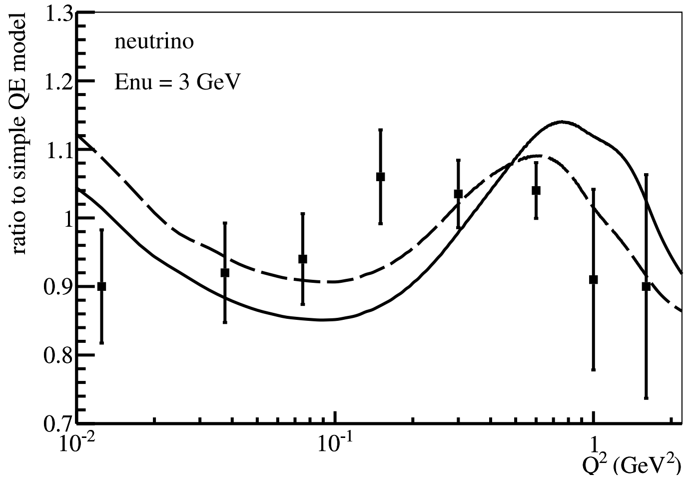


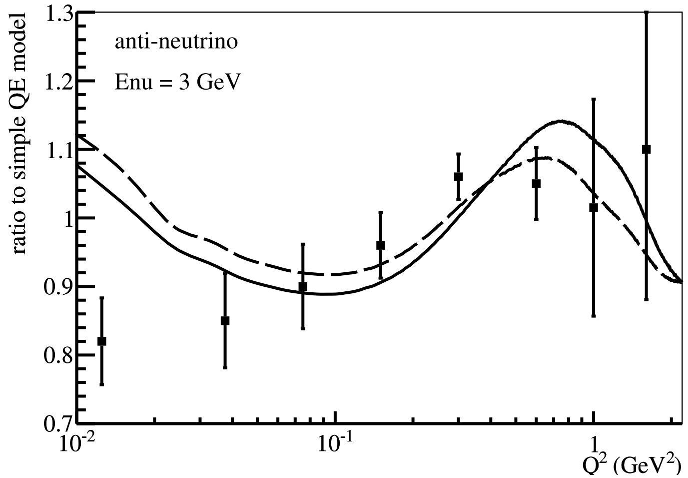


MINERνA


neutrino
Enu = 3 GeV
dσ/dQ² for C12 (cm/GeV)² x10⁻³⁸
Q² (GeV²)
0
0.5
1
1.5
2
10
8
6
4
2
0


anti-neutrino
Enu = 3 GeV
dσ/dQ² for CH (cm/GeV)² x10⁻³⁸
Q² (GeV²)
0
0.5
1
1.5
2
10
8
6
4
2
0


neutrino
Enu = 3 GeV
ratio to simple QE model
Q² (GeV²)
10⁻²
10⁻¹
1
1.3
1.2
1.1
1
0.9
0.8
0.7


anti-neutrino
Enu = 3 GeV
ratio to simple QE model
Q² (GeV²)
10⁻²
10⁻¹
1
1.3
1.2
1.1
1
0.9
0.8
0.7

CC ν Physics: PRC 70-055503, PLB 638,325

Low Energies

1. $\nu_\mu\, ^{12}\mathrm{C} \rightarrow \mu^- X \qquad (\overline{\sigma}[10^{-40}\mathrm{cm}^2])$

THEORY						EXP (LSND)		
LDT	Pauli	RPA	[A]	[B]	[C]	1995	1997	2002
66.1	20.7	11.9	13.2	15.2	19.2	8.3 ± 1.7	11.2 ± 1.8	10.6 ± 1.8

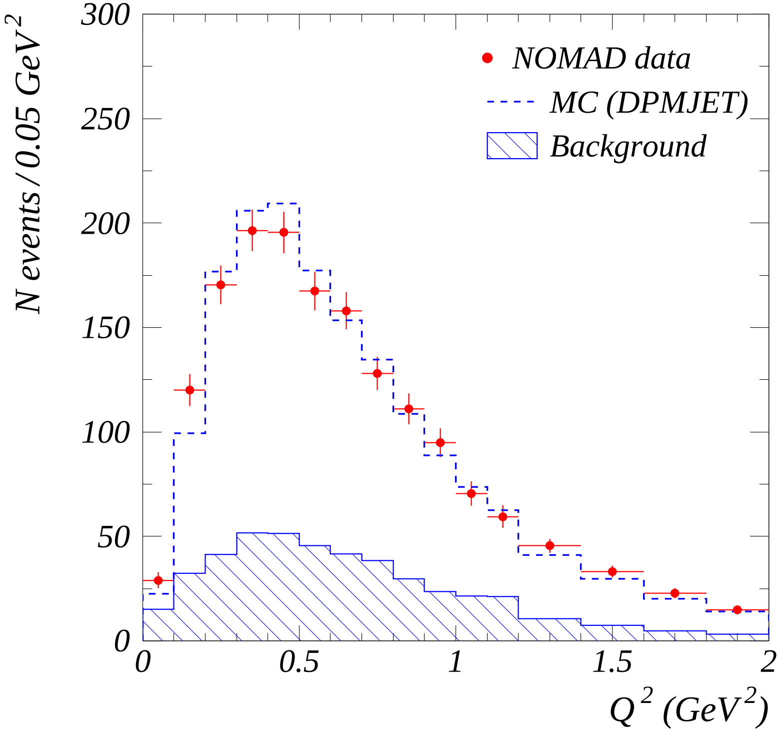
A Shell Model: A.C. Hayes and I.S. Towner, Phys. Rev. C61 (2000) 044603.

B Shell Model: C. Volpe, et al., Phys. Rev. C62 (2000) 015501.

C CRPA: E. Kolbe, et al., J. Phys. G29 (2003) 2569.

2. $\nu_e\, ^{12}\mathrm{C} \rightarrow e^- X \qquad (\overline{\sigma}[10^{-41}\mathrm{cm}^2])$

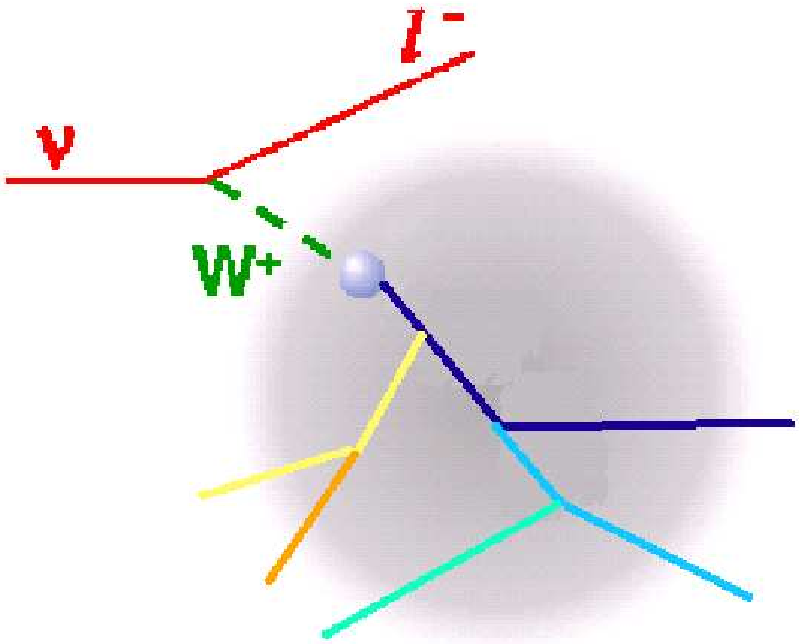
THEORY						EXP		
LDT	Pauli	RPA	[A]	[B]	[C]	KARMEN	LSND	LAMPF
59.7	1.9	1.4	1.2	1.6	1.5	1.50 ± 0.14	1.50 ± 0.14	1.41 ± 0.23



V. Lyubushkin et al. **(NOMAD Collaboration)**, Eur. Phys. J. C 63, 355 (2009). In the **two-track sample**, which is primarily Q^2 above 0.3 GeV^2, a **large fraction of the 2p2h component**, as well as QE and pion production where the hadrons rescattered as they exited the nucleus, are **rejected.**

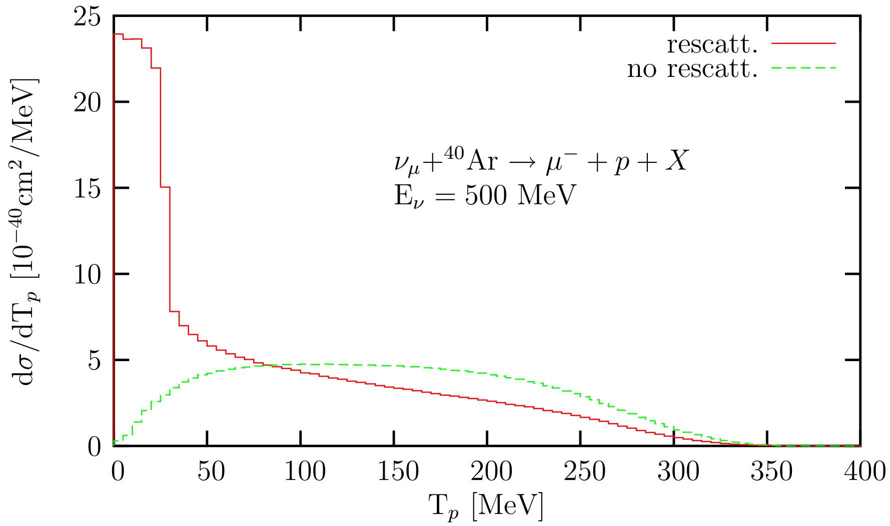
It is observed a relative **deficit at $Q^2 = 0.3$ and excess at 1.5 GeV^2 compared to QE without RPA**. If the first two or three points are eliminated, the distribution will be consistent with $M_A \sim 1.2$ GeV.

CC and NC Nucleon Emission: PRC 73-025504



- ⋆ Gauge boson ($W^{\pm}$ or Z^0), with four momentum q^μ, absorbed by one nucleon in a point of the nucleus $\vec{r} \to d^2\sigma/d\Omega' dE' d^3r$.
- ⋆ Kinematics of the outgoing nucleon: We generate a random $\vec{p}$ from the local Fermi sea and impose momentum conservation and take into account Pauli blocking.
- ⋆ We move the primary nucleon through the nucleus, considering NN collisions, according to the NN elastic cross section, incorporating some medium modifications (Fermi motion, Pauli blocking and polarization). We also move the **produced (secondary) nucleons** through the nucleus. **When one nucleon (primary or secondary) leaves the nucleus, it is counted as a contribution to σ**

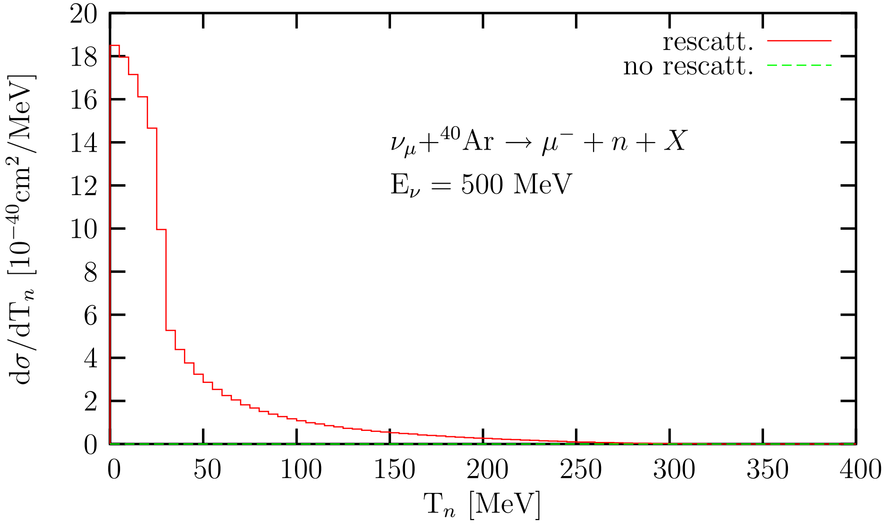
Why a MC Simulation?



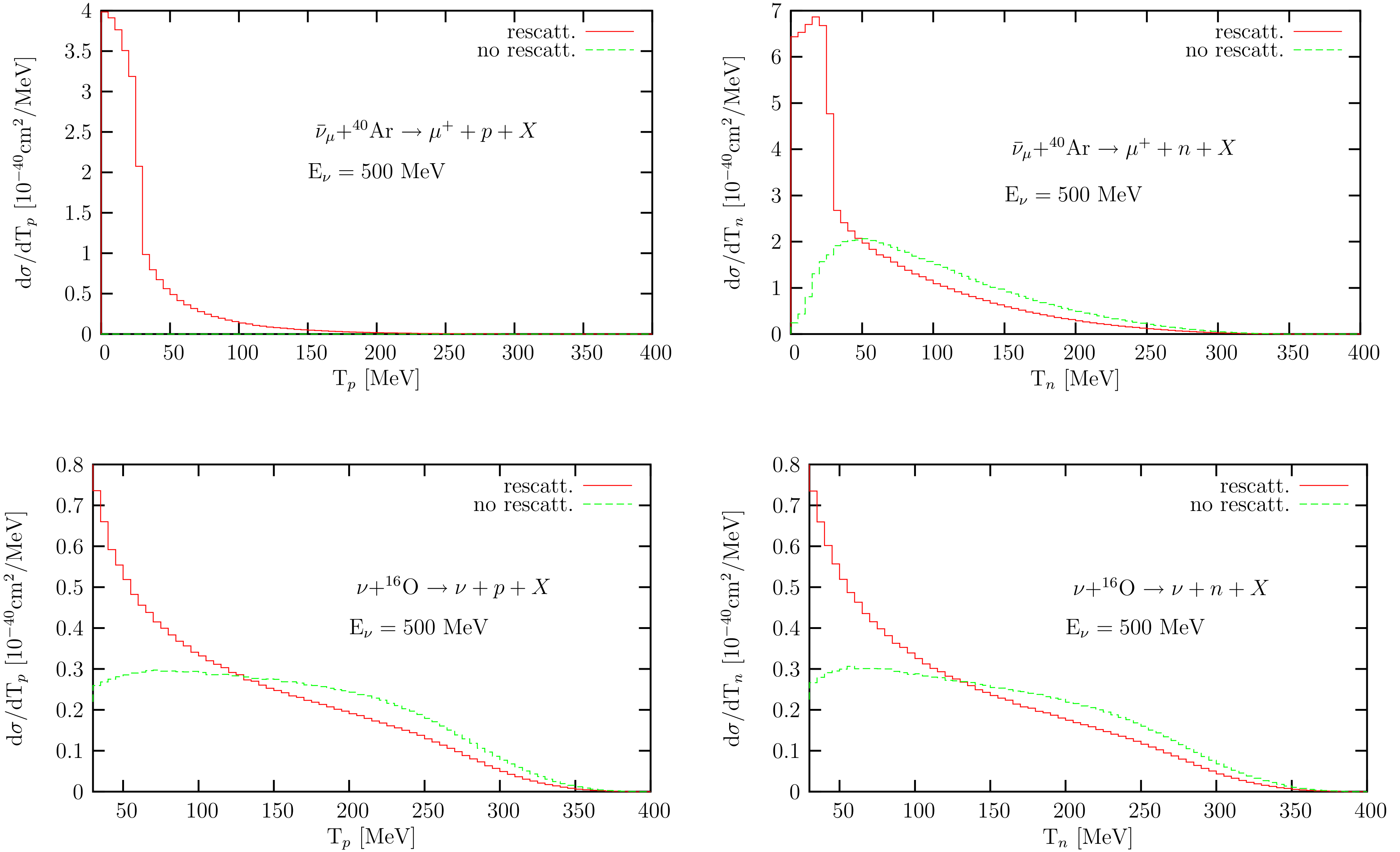
The **distortion** of the nucleon wave function by a **complex optical potential** **removes** all events where the nucleons collide with other nucleons:

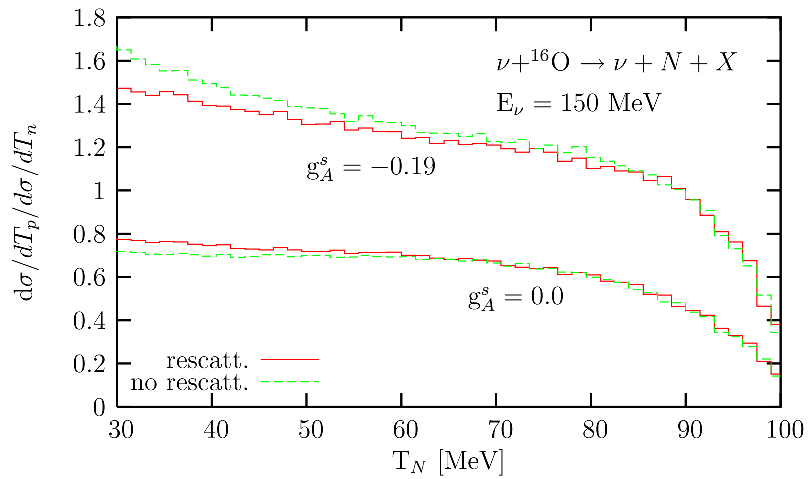
- This is correct when the final nucleus is left in the ground or in a particular excited state, but
- **not when the final nuclear state is unobserved**

DWIA $\rightarrow$ the nucleons that interact are **lost** when in the physical process **they simply come off the nucleus** with a different energy, angle, and may be charge, and they should definitely be **taken into account**.

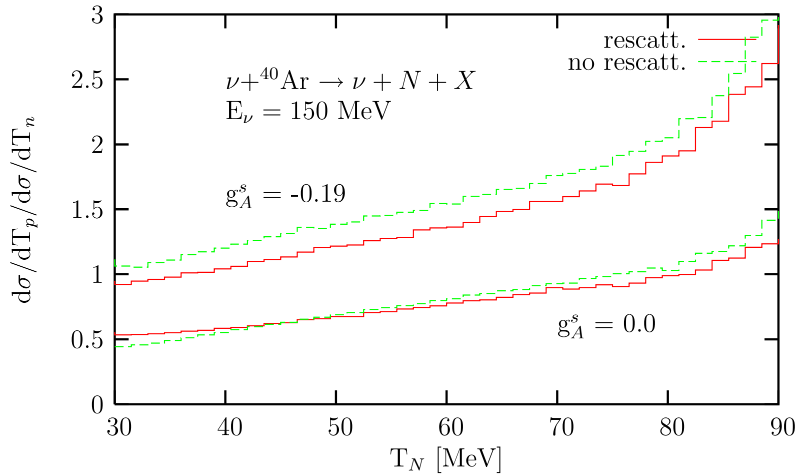


- Within the IA **neutrinos** only interact via CC with **neutrons** and would emit **protons** ($\nu_l n \to l^- p$), and therefore DWIA will predict **zero** cross sections for the neutron emission reaction: $(\nu_l, l^- n)$
- However, the primary **protons** interact strongly with the medium and collide with other nucleons which are also ejected. As a consequence there is a reduction of the flux of high energy **protons** but a large number of secondary nucleons, many of them **neutrons**, of lower energies appear.
- Similar for $(\bar{\nu}_l, l^+ p)$

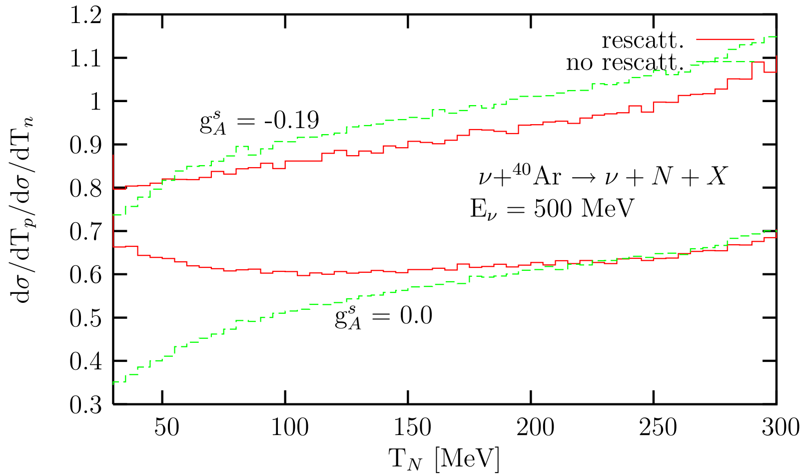

rescatt.
no rescatt.
$\bar{\nu}_\mu + {}^{40}\mathrm{Ar} \to \mu^+ + p + X$
$E_\nu = 500$ MeV
$d\sigma/dT_p$ [10^{-40}cm^2/MeV]
T_p [MeV]
rescatt.
no rescatt.
$\bar{\nu}_\mu + {}^{40}\mathrm{Ar} \to \mu^+ + n + X$
$E_\nu = 500$ MeV
$d\sigma/dT_n$ [10^{-40}cm^2/MeV]
T_n [MeV]
rescatt.
no rescatt.
$\nu + {}^{16}\mathrm{O} \to \nu + p + X$
$E_\nu = 500$ MeV
$d\sigma/dT_p$ [10^{-40}cm^2/MeV]
T_p [MeV]
rescatt.
no rescatt.
$\nu + {}^{16}\mathrm{O} \to \nu + n + X$
$E_\nu = 500$ MeV
$d\sigma/dT_n$ [10^{-40}cm^2/MeV]
T_n [MeV]



$\nu+^{16}O \rightarrow \nu + N + X$
$E_\nu = 150$ MeV
$g_A^s = -0.19$
$g_A^s = 0.0$
rescatt.
no rescatt.
$d\sigma/dT_p/d\sigma/dT_n$
T_N [MeV]



$\nu+^{40}Ar \rightarrow \nu + N + X$
$E_\nu = 150$ MeV
g_A^s = -0.19
$g_A^s = 0.0$
rescatt.
no rescatt.
$d\sigma/dT_p/d\sigma/dT_n$
T_N [MeV]



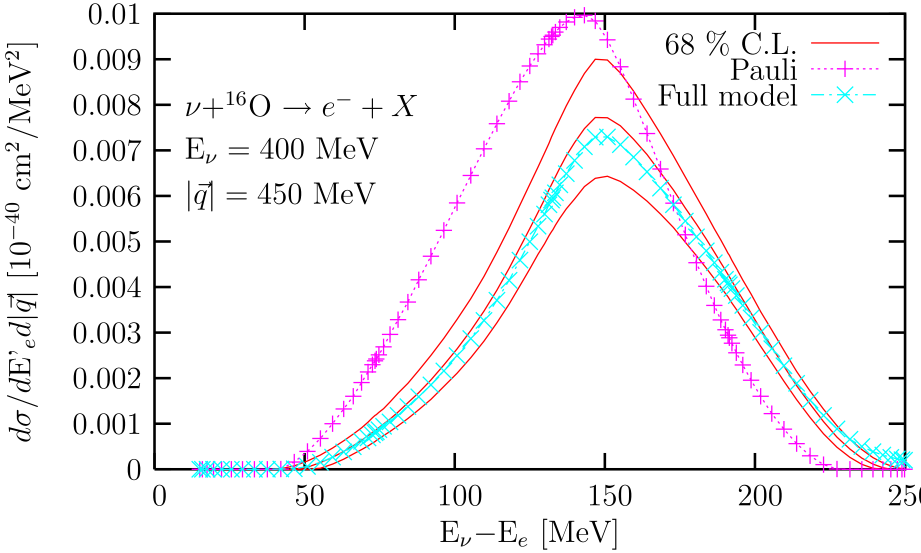
$\nu+^{40}Ar \rightarrow \nu + N + X$
$E_\nu = 500$ MeV
g_A^s = -0.19
$g_A^s = 0.0$
rescatt.
no rescatt.
$d\sigma/dT_p/d\sigma/dT_n$
T_N [MeV]

Theoretical Uncertainties: PLB 638,325

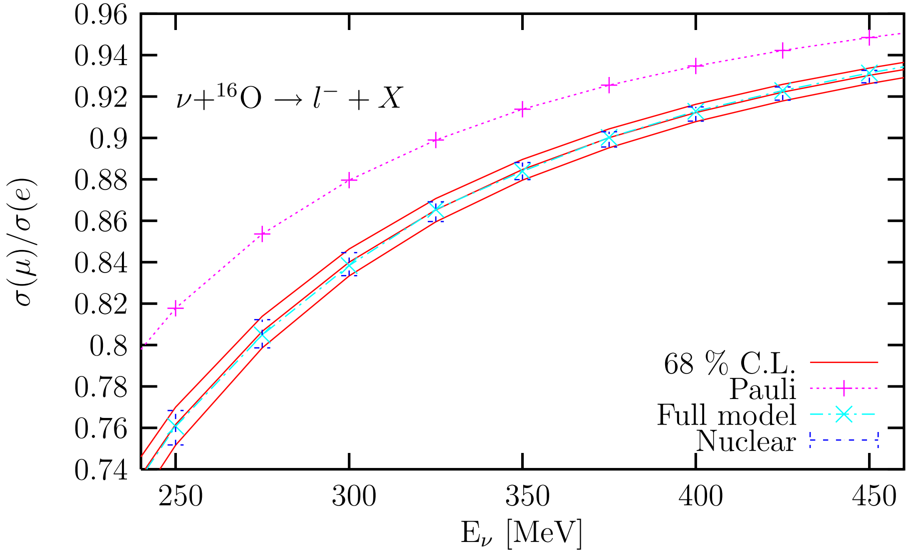
Predictions for CC and NC QE neutrino induced reactions in nuclei at intermediate energies of interest for future neutrino experiments. Uncertanties: $\sigma_{e,\mu} \sim 10 - 15\%$, $\sigma(\mu)/\sigma(e) \sim 5\%$

Form Factors					Nucleon Interaction				
M_D	$=$	0.843	$\pm$	0.042 GeV	$f_0^{\prime(in)}$	$=$	0.33	$\pm$	0.03
λ_n	$=$	5.6	$\pm$	0.6	$f_0^{(\prime ex)}$	$=$	0.45	$\pm$	0.05
M_A	$=$	1.05	$\pm$	0.14 GeV	f	$=$	1.00	$\pm$	0.10
g_A	$=$	1.26	$\pm$	0.01	f^*	$=$	2.13	$\pm$	0.21
					Λ_π	$=$	1200	$\pm$	120 MeV
					C_ρ	$=$	2.0	$\pm$	0.2
					Λ_ρ	$=$	2500	$\pm$	250 MeV
					g'	$=$	0.63	$\pm$	0.06

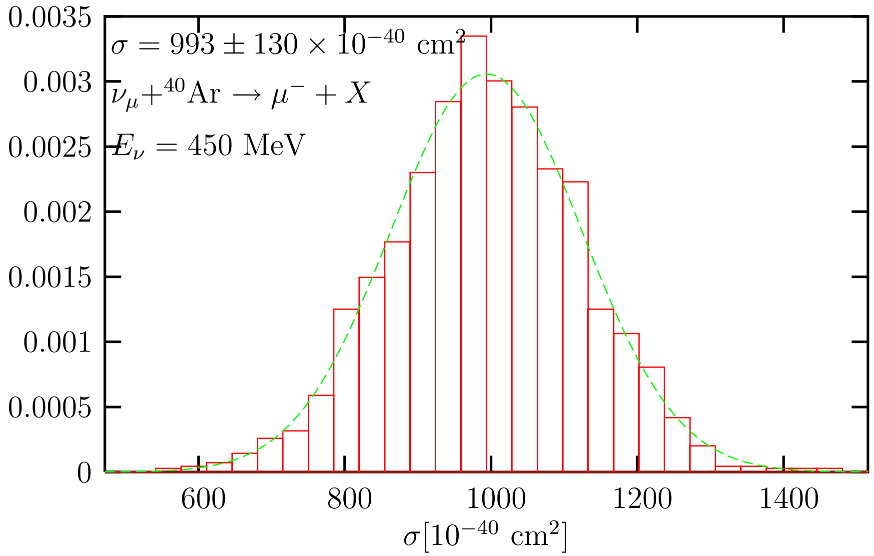
$+10\%$ in Σ (nucleon self-energy)



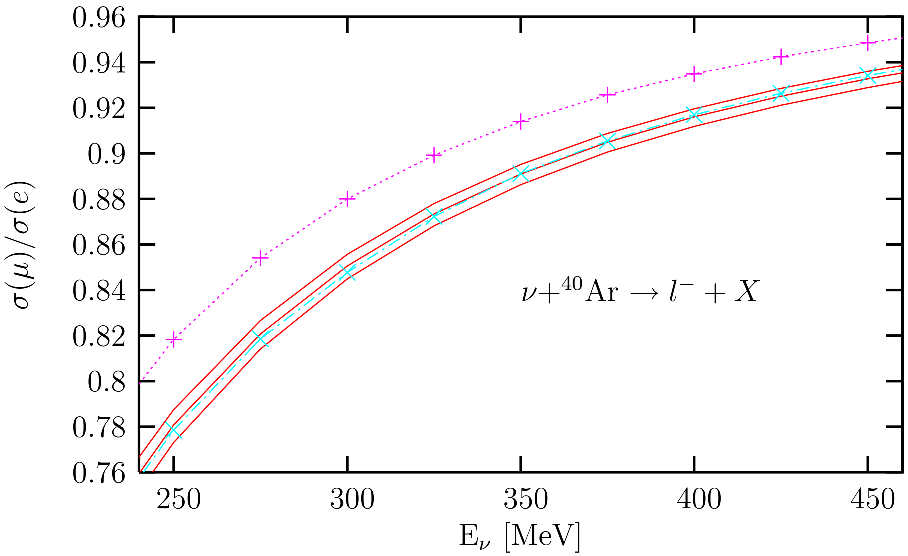
$\nu+^{16}\mathrm{O} \to e^- + X$
E_ν = 400 MeV
$|\vec{q}|$ = 450 MeV
68 % C.L.
Pauli
Full model
$d\sigma/d\mathrm{E}'_e d|\vec{q}|$ [10^{-40} cm^2/MeV^2]
$\mathrm{E}_\nu - \mathrm{E}_e$ [MeV]



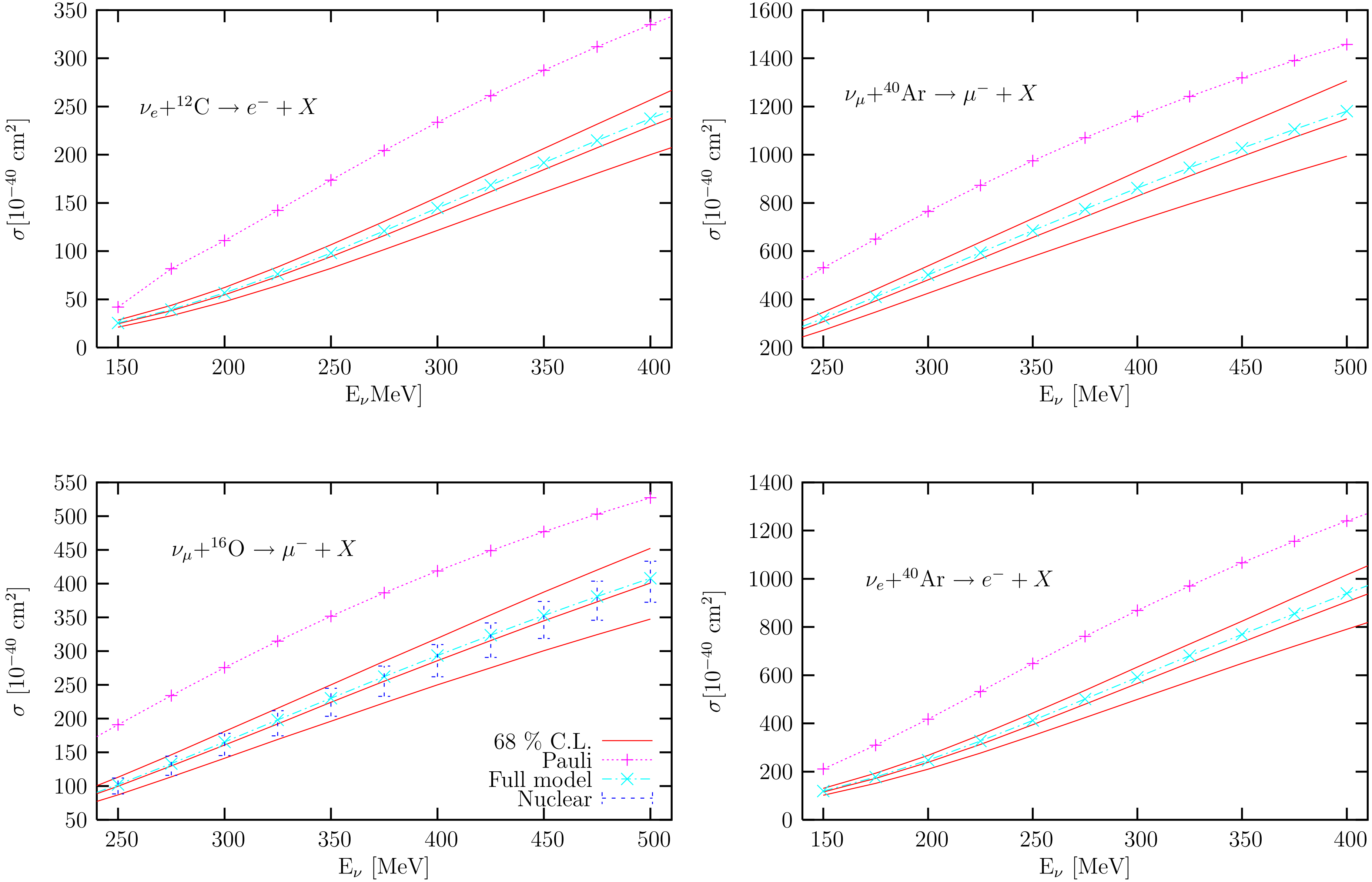
$\nu+^{16}\mathrm{O} \to l^- + X$
68 % C.L.
Pauli
Full model
Nuclear
$\sigma(\mu)/\sigma(e)$
E_ν [MeV]



$\sigma = 993 \pm 130 \times 10^{-40}$ cm^2
$\nu_\mu+^{40}\mathrm{Ar} \to \mu^- + X$
E_ν = 450 MeV
$\sigma[10^{-40}$ cm$^2]$



$\nu+^{40}\mathrm{Ar} \to l^- + X$
$\sigma(\mu)/\sigma(e)$
E_ν [MeV]


σ[10^−40 cm^2]
ν_e+^12C → e^− + X
E_νMeV]
σ[10^−40 cm^2]
ν_μ+^40Ar → μ^− + X
E_ν [MeV]
σ [10^−40 cm^2]
ν_μ+^16O → μ^− + X
68 % C.L.
Pauli
Full model
Nuclear
E_ν [MeV]
σ[10^−40 cm^2]
ν_e+^40Ar → e^− + X
E_ν [MeV]