ν Deeply Inelastic Scattering

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Lecture I - Introduction to Deeply Inelastic Scattering

- Kinematics
- Cross sections and structure functions
- Lowest order results - parton model
- Parton distributions functions (PDFs)
- Examples, interpretations, and applications

Lecture II - Beyond the Parton Model

- Higher order corrections
- Factorization schemes
- PDF scale dependence and DGLAP evolution equations
- QCD-improved parton model
- Global Fits for PDFs

Lecture III - Life in the Real World: low Q^2, nuclear effects and more

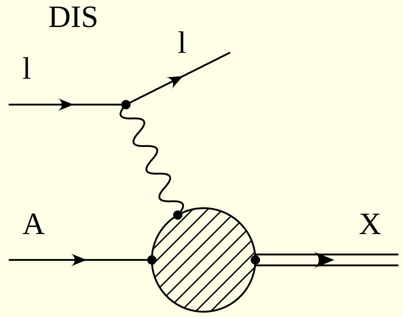
- Leading twist versus power suppressed corrections
- Higher Twist contributions
- Target Mass Corrections
- Heavy Quarks
- Nuclear Effects

References

While the present lectures are designed to be self contained, the following may provide useful additional information and details.

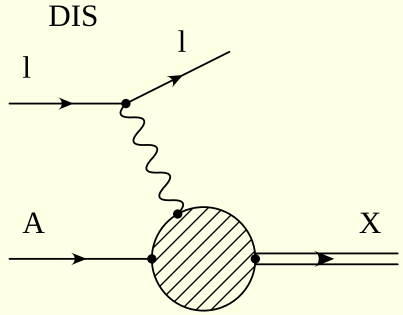
1. CTEQ Summer School Lectures at www.cteq.org
   - My lectures from 2013 may be useful for additional details on the material to be covered here
   - Lectures are available on a wide variety of topics
2. Handbook of Perturbative QCD - also at the CTEQ website www.cteq.org
3. Deep Inelastic Scattering - R. Devenish and A. Cooper-Sarkar
4. QCD and Collider Physics - R.K. Ellis, W.J. Stirling, and B. Webber
5. Nuclear PDFs: arXiv:hep-ph/0404093, arXiv:0710.4897[hep-ph], arXiv:0907.2357[hep-ph]

Brief Overview of DIS

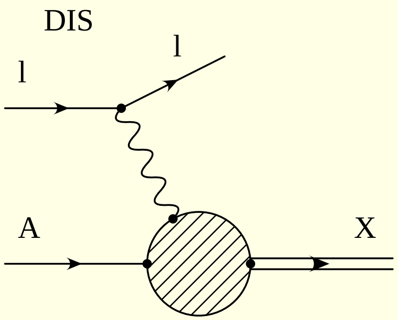


Charged lepton, neutral current DIS

- The basic idea is to use the known interaction of a photon to probe the structure of the target particle
- Elastic lepton scattering from a point-like target particle can be calculated using QED
- If the target is an extended object the cross section is modified by one or more form factors, e.g., one for a spinless target, two for a proton.
- These form factors depend on the squared four-momentum transfer Q^2
- The Fourier transform of the electric form factor gives the spatial dependence of the charge density

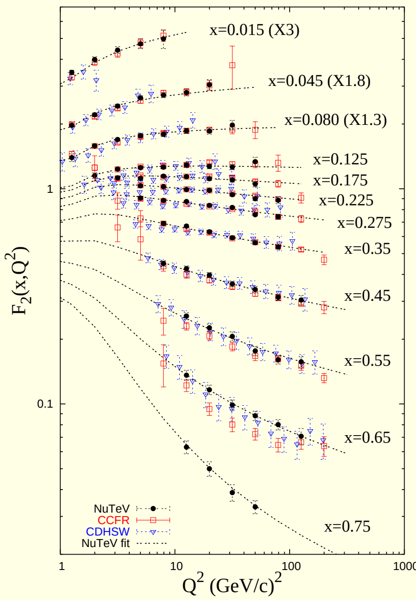


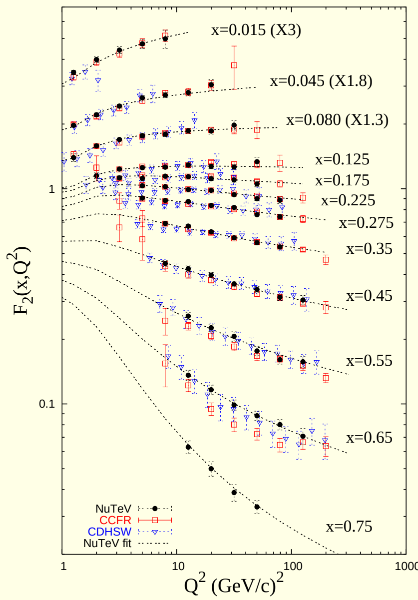
- The generalization to inelastic scattering from a proton introduces two "structure functions" (three if one is considering neutrino scattering)
- These structure functions depend on two kinematic variables - Q^2 and the energy transfer ν, for example.
- Early measurements at SLAC (1968) showed that for fixed values of Q^2/ν the structure functions showed no Q^2 dependence - that is, they only depended on one variable. This was called "scaling."
- Feynman's parton model (today's lowest order QCD) provided an intuitive description of scaling
- Higher order QCD corrections provide an excellent description of the observed deviations from exact scaling



Charged current DIS

- Flavor changing weak interaction allows separation of different flavor PDFs
- Parity violation introduces a third structure function
- Allows separation of q and $\bar{q}$ PDFs
- Complementary probe of hadron structure

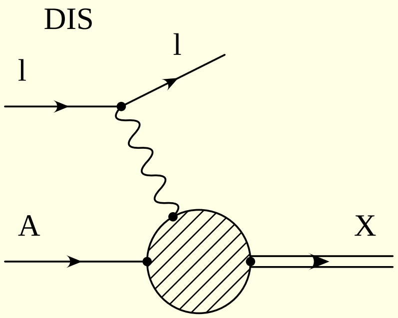




- NuTeV structure functions versus Q^2 at fixed values of $x = Q^2/2M\nu$
- Notice the near constant values for $x \approx 0.1 - 0.3$
- Structure functions increase with Q^2 at low values of x and decrease at high values of x
- Goal is to understand what these are, how they are extracted, and how theory describes their behavior

- Could simply calculate the cross section in terms of the interaction of the virtual boson with the partons in the target
- Historical approach has been based on structure functions
- The basic idea is to remove as much of the known physics of the lepton vertex as possible, constrain the remaining hadronic piece using gauge invariance, current conservation, parity invariance (for the electromagnetic interaction) and time reversal invariance and then express what is left in terms of the hadronic structure functions F_1 and F_2 (plus F_3 for weak interactions)
- For some purposes it is often preferable to work directly with the cross sections since that avoids any model-dependent assumptions associated the extraction of the structure functions
- On the other hand, the structure functions are easy to interpret in terms of the parton structure of the target
- I will summarize here the structure function approach

Start with a few definitions for the process $l(k) + A(P) \to l'(k') + X$ in the target rest frame where M denotes the target mass



$$
\begin{aligned}
q^2 &= -Q^2 = (k-k')^2 \qquad x = Q^2/2P\cdot q = Q^2/2M\nu \\
E &= k\cdot P/M \qquad E' = k'\cdot P/M \\
\nu &= P\cdot q/M = E - E' \qquad W^2 = (P+q)^2 = M^2 + Q^2\left(\frac{1}{x} - 1\right) \\
y &= \nu/E = 1 - E'/E \quad \text{(evaluated in the lab frame where } P^\mu = (M,0,0,0)) \\
&= P\cdot q/P\cdot k \quad \text{(in an arbitrary frame)} \\
s &= (k+P)^2 = M^2 + 2ME \quad \text{Can also write } Q^2 = 2MExy
\end{aligned}
$$

The cross section can be written as

$$\sigma = \frac{1}{4ME} \int \frac{d^3k'}{(2\pi)^3 2E'} \overline{\sum}_{spins} \sum_X \prod_{n=1}^{N_X} \int \frac{d^3p_n}{(2\pi)^3 2E_n} |T_{fi}|^2 (2\pi)^4 \delta(k+P-k'-p_X)$$

- The leptonic and hadronic phase space variables have been written separately
- Can simplify this by being differential in the scattered lepton energy and scattering solid angle.
- Can also express T_{fi} as

$$T_{fi} = \left(\frac{g_W}{2\sqrt{2}}\right)^2 \overline{u}(k')\gamma_\mu(1-\gamma_5)u(k)\frac{1}{q^2 - M_W^2} J^\mu$$

- Here J^μ is the matrix element of the weak charged current operator between the initial and final hadronic states with a factor of $\frac{g_W}{2\sqrt{2}}$ removed.

The leptonic phase space factor can be simplified using

$$\begin{aligned}\frac{d^3k'}{E'} &= \frac{k'^2 dk' d\Omega'}{E'} = k' dE' d\Omega' \\ &\approx E' dE' d\Omega'\end{aligned}$$

where the last line follows if the recoil lepton mass is neglected.
Convenient to use $\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2}$.
Exercise: show that

$$\frac{d\sigma}{dE' d\Omega'} = \frac{\pi}{4M} \frac{G_F^2}{2} R_W(Q^2) \left(\frac{E'}{E}\right) L_{\mu\nu} W^{\mu\nu}$$

where $L_{\mu\nu}$ is given by the trace of the leptonic current

$$L_{\mu\nu} = 8 \left(k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu} k \cdot k' \pm i\epsilon_{\mu\nu\alpha\beta} k^\alpha k'^\beta\right)$$

where the $+(-)$ corresponds to an incoming $\nu(\bar{\nu})$.

$W^{\mu\nu}$ is given by

$$W^{\mu\nu} = \frac{(2\pi)^3}{4} \sum_{spins} \sum_X \prod_{n=1}^{N_X} \frac{d^3 p_n}{(2\pi)^3 2E_n} J^{\mu\dagger} J^\nu \delta^4(q + P - p_X)$$

and the factor $R_W(Q^2)$ is given by $R_W(Q^2) = \left[\frac{M_W^2}{M_W^2+Q^2}\right]^2$.

The hadronic tensor, $W^{\mu\nu}$, contains all the information pertaining to the interaction of the exchanged vector boson with the target hadron.

Notice the part of $L_{\mu\nu}$ that contains $\epsilon_{\mu\nu\alpha\beta}$. This comes from a trace of four γ matrices with a γ_5. This represent an interference term between the vector and axial vector currents. Parity conservation in electromagnetic interactions would prevent such a term from being present, but it is present for the weak interactions.

The next step is to examine the tensor structure of $W^{\mu\nu}$.

- It can be constructed from $g^{\mu\nu}, P^{\mu}, q^{\mu},$ and $\epsilon^{\mu\nu\alpha\beta}$, where the last factor is allowed if parity is not conserved.

- Note that terms proportional to q^{μ} will give contributions proportional to lepton masses when contracted with $L_{\mu\nu}$. Exercise: Show this.

- There are then only three combinations that survive and the hadronic tensor can be written as follows

$$W^{\mu\nu} = -g^{\mu\nu}F_1 + \frac{F_2}{M\nu}P^{\mu}P^{\nu} \mp i\frac{F_3}{2M\nu}\epsilon^{\mu\nu\alpha\beta}P_{\alpha}q_{\beta}$$

- The structure functions $F_1, F_2,$ and F_3 contain information on the structure of the hadronic target
- They depend on the 4-vectors P and q through Lorentz scalars
- Since $P^2 = M^2$ and $q^2 = -Q^2$, they can depend on Q^2 and $x = \frac{Q^2}{2P\cdot q}$, for example.
- The signs and various factors of $M\nu$ have been chosen to make the final results simpler and to be in accordance with the usual (but by no means universal) conventions.

Interpretation of F_1 and F_2

- F_1 and F_2 exist for both electromagnetic and weak interactions
- F_3 results from axial vector-vector interference and is unique to the weak interactions

In what follows I use photons, but the same is true for W and Z bosons. For transverse photons

$$\epsilon^\mu(x) = (0,1,0,0) \quad \epsilon^\mu(y) = (0,0,1,0) \quad \text{but we need an expression for } \epsilon(0)$$

Consider a frame where the proton and virtual photon four-vectors are as follows:

$$P \longrightarrow \ \longleftarrow q$$

$$q^\mu = (0,0,0,-Q) \quad q^2 = -Q^2 \qquad P^\mu = (P_0, 0, 0, P_z)$$

Exercise: use $P \cdot q = M\nu = P_z Q$ and $P^2 = M^2$ to show that

$$P^\mu = M\left(\sqrt{\frac{\nu^2 + Q^2}{Q^2}}, 0, 0, \frac{\nu}{Q}\right)$$

We need $\epsilon^\mu(0)$ such that $\epsilon(0) \cdot q = 0,\ \ \epsilon(0) \cdot \epsilon(x) = 0,$ and $\epsilon(0) \cdot \epsilon(y) = 0$

Can choose $\epsilon^\mu(0) = (1, 0, 0, 0)$

For the electromagnetic interactions

$$W_{\mu\nu} = F_1(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}) + \frac{F_2}{P \cdot q}(P_\mu + \frac{q_\mu P.q}{q^2})(P_\nu + \frac{q_\nu P \cdot q}{q^2})$$

Transverse cross section: $\sigma_T \propto F_1$

Longitudinal cross section: $\sigma_L \propto -F_1 + F_2 M^2 \frac{\nu^2 + Q^2}{P \cdot q Q^2}$

Exercise: Derive these and rewrite the last result as

$$-F_1 + \frac{F_2}{2x}\left(1 + \frac{4M^2 x^2}{Q^2}\right)$$

Sometimes see the ratio

$$R = \frac{\sigma_L}{\sigma_T} = \frac{F_2\left(1 + \frac{4M^2x^2}{Q^2}\right) - 2xF_1}{2xF_1}$$

Interpretation:

- F_1 measures the interaction of transverse photons
- Up to corrections of $\mathcal{O}(1/Q^2)$, $F_2 - 2xF_1$ measures the interaction of longitudinal photons

Exercise: Show that $\frac{d\sigma}{dx\,dy} = \frac{d\sigma}{dE'd\Omega'}\frac{M(E-E')}{E'}$

Exercise: Work out the contraction of $L_{\mu\nu}$ with the hadronic tensor $W^{\mu\nu}$ thereby showing that the cross section can be written as

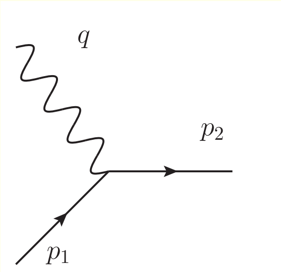
$$\frac{d\sigma}{dxdy} = \frac{G_F^2}{2\pi} R_W(Q^2) s \left[xy^2 F_1 + \left(1 - y - \frac{M^2x^2y^2}{Q^2}\right) F_2 \pm y(1-\frac{y}{2})xF_3 \right]$$

where the $+(-)$ sign refers to $\nu(\bar{\nu})$ scattering. Alternatively, using $F_L = F_2 - 2xF_1$ one can write

$$\frac{d\sigma}{dxdy} = \frac{G_F^2}{2\pi} R_W(Q^2) \frac{s}{2} \left[\left(1 + (1-y)^2\right) F_2 - y^2 F_L \pm y(2-y)xF_3 \right]$$

- To separate F_2 and F_L one needs to have data at fixed values of x and Q^2, but different values of y.
- Since $Q^2 = 2MExy$ this requires data from different beam energies

- With these definitions, we can now examine the form of the structure functions in the parton model
- Start with the basic definition of $W_{\mu\nu}$ using a parton target



$$W^{\mu\nu} = \frac{(2\pi)^3}{2}\frac{1}{2}\sum_{spins} N \int \frac{d^3p'}{(2\pi)^3 2E'}\delta^4(p'-q-p)\left(\overline{u}(p')\gamma^\mu(1-\gamma_5)u(p)\right)^\dagger \left(\overline{u}(p')\gamma^\nu(1-\gamma_5)u(p)\right)$$

- N is a normalization factor to be defined below

- Use $\frac{d^3p'}{2E'} = d^4p'\delta(p'^2)$ to get

$$W^{\mu\nu} = 2N\delta(p'^2)\left(p^\mu p'^\nu + p^\nu p'^\mu - 2p\cdot p' g^{\mu\nu} - i\epsilon^{\mu\nu\alpha\beta}p_\alpha p'_\beta\right)$$

- Next, assume that the parton carries a fraction η of the target's 4-momentum and neglect target mass effects. Thus, $p = \eta P$
- With this definition,

$$\begin{aligned}\delta(p'^2) &= \delta[(p+q)^2] = \delta(q^2 + 2p\cdot q) \\ &= \frac{1}{2M\nu}\delta(\eta - \frac{Q^2}{2M\nu}) = \frac{1}{2M\nu}\delta(\eta - x)\end{aligned}$$

- So, to this order, x is a measure of the momentum fraction carried by the struck parton
- The normalization factor N corrects for the flux factor being that of the parton, not the target hadron: $N = 1/\eta$

Exercise: Use $p' = p + q$ and $p = \eta P$ to get

$$W^{\mu\nu} = \frac{\eta}{M\nu}\delta(\eta - x)\left[2P^\mu P^\nu + \frac{P^\mu q^\nu + P^\nu q^\mu}{\eta} + \frac{q^2}{2\eta^2}g^{\mu\nu} - i\epsilon^{\mu\nu\alpha\beta}\frac{P_\alpha q_\beta}{\eta}\right]$$

- From this expression one can read off the results

$$\hat{F}_1 = \delta(\eta - x) \quad \hat{F}_2 = 2\eta\delta(\eta - x) \quad \hat{F}_3 = 2\delta(\eta - x)$$

- Note that $\hat{F}_L = \hat{F}_2 - 2x\hat{F}_1 = 0$ in lowest order
- I have used the ˆ symbol to denote the contributions to the structure functions at the parton level.
- The last relation above is called the Callan-Gross relation
- To calculate the *hadronic* structure function introduce a parton distribution function (PDF) defined such that $G_{a/A}(x)dx$ gives the probability of finding a parton a in a hadron A with a momentum fraction between x and $x + dx$

Simple interpretation of $\sigma^{\nu,\bar{\nu}}$

Simplifying assumptions

- Use $F_2 = 2xF_1$ (Callan-Gross relation for spin-1/2 particles)
- $xF_3 = F_2$ (valid at large x or if $\bar{q} = 0$)

$$\frac{d\sigma^{\nu,\bar{\nu}}}{dx\,dy} = \frac{G_F^2}{2\pi} R_W(Q^2) s F_2 \left[\frac{y^2}{2} + (1-y) \pm y(1-\frac{y}{2})\right]$$

- The bracketed factor is 1 for νq or $\bar{\nu}\bar{q}$ scattering
- It is $(1-y)^2$ for $\nu\bar{q}$ or $\bar{\nu}q$ scattering
- Suppose one only has quarks - integrating on x gives the same overall factor for ν or $\bar{\nu}$
- Integrating on y then gives $\sigma^{\nu} = 3\,\sigma^{\bar{\nu}}$
- Any deviation from 3 is evidence for antiquarks in the target

Exercise: let $R = \frac{\sigma^{\bar{\nu}}}{\sigma^{\nu}}$. Then show that

$$\frac{\int x\bar{Q}(x)dx}{\int xQ(x)dx} = \frac{3R-1}{3-R}$$

Here $Q(\bar{Q})$ represents the sum of the active quark (antiquark) PDFs. Thus, neutrino and antineutrino cross sections give a measure of the contribution of the antiquark PDFs relative to those of the quarks.

Why $(1-y)^2$?

Recall that $y = \frac{E-E'}{E} = \frac{P\cdot q}{P\cdot k} = \frac{p\cdot q}{p\cdot k}$

In the lepton-parton system

$$k = \frac{\sqrt{\hat{s}}}{2}(1,0,0,1) \quad p = \frac{\sqrt{\hat{s}}}{2}(1,0,0,-1) \quad k' = \frac{\sqrt{\hat{s}}}{2}(1,\sin\theta,0,\cos\theta)$$

Exercise: Show that $1-y = \frac{1+\cos\theta}{2}$

$\nu q \to \mu^- q'$

$\nu \quad \overrightarrow{\Leftarrow} \quad \overleftarrow{\Rightarrow} \quad q$

$\theta = \pi$

$\mu^- \quad \overleftarrow{\Rightarrow} \quad \overrightarrow{\Leftarrow} \quad q'$

$\bar{\nu} q \to \mu^+ q'$

$\bar{\nu} \quad \overrightarrow{\Rightarrow} \quad \overleftarrow{\Rightarrow} \quad q$

$\mu^+ \quad \overleftarrow{\Leftarrow} \quad \overrightarrow{\Leftarrow} \quad q'$

The z component of angular momentum is not conserved for the second case, so it must vanish for $\theta = \pi$.

PDF Sum Rules

PDFs are inherently non-perturbative and so can not be calculated using perturbative QCD. But we do know some properties they must satisfy. Denote the different PDFs by a symbol corresponding to their flavor.

- The number of quarks (or antiquarks) in a proton is indeterminate since quantum fluctuations can create $q\bar{q}$ pairs which subsequently annihilate
- But the net number of u quarks should be two:

$$\int_0^1 dx\,(u(x) - \bar{u}(x)) = 2$$

- The net number of d quarks should be one:

$$\int_0^1 dx\,\big(d(x) - \bar{d}(x)\big) = 1$$

- The net number of s quarks should be zero:

$$\int_0^1 dx\,(s(x) - \bar{s}(x)) = 0$$

  with similar relations for c and b quarks

- Note: This does not mean that $s(x) \equiv \bar{s}(x)$ – the s and $\bar{s}$ PDFs can have different x dependences

- Momentum must be conserved:

$$\int_0^1 dx x \left[\sum_q (q(x) + \bar{q}(x)) + g(x)\right] = 1$$

  where g(x) denotes the gluon PDF

- The hadronic structure functions are given by weighting the partonic structure function by the appropriate PDFs:

$$\begin{aligned} F_2(x,Q^2) &= 2xF_1(x,Q^2) \\ &= \sum_q \int d\eta\, 2\eta\, [q(\eta)+\bar{q}(\eta)]\, \delta(\eta - x) = \sum_q 2\, x\, [q(x)+\bar{q}(x)] \end{aligned}$$

and

$$\begin{aligned} xF_3(x,Q^2) &= \sum_q \int d\eta\, 2\, x\, [q(\eta)-\bar{q}(\eta)]\, \eta\delta(\eta - x) \\ &= \sum_q 2\, x\, [q(x)-\bar{q}(x)] \end{aligned}$$

- One can see that to this order the structure functions are independent of Q^2, which is the scaling result discussed earlier

Some Simple Examples

Remember the neutrino vertices: $\nu \to \mu^- W^+, \;\; \bar{\nu} \to \mu^+ W^-$

$$
\begin{aligned}
F_2^{\nu p} &= 2x\left[d + s + \bar{u} + \bar{c}\right] \\
F_2^{\nu n} &= 2x\left[u + s + \bar{d} + \bar{c}\right] \\
F_2^{\nu N} &= x\left[u + d + \bar{u} + \bar{d} + 2s + 2\bar{c}\right]
\end{aligned}
$$

where $N = \frac{p+n}{2}$ denotes an isoscalar target.

Exercise: Show that

$$
\begin{aligned}
F_2^{\bar{\nu} N} &= x\left[u + d + \bar{u} + \bar{d} + 2\bar{s} + 2c\right] \\
xF_3^{\nu N} &= x\left[u + d - \bar{u} - \bar{d} + 2s - 2\bar{c}\right] \\
xF_3^{\bar{\nu} N} &= x\left[u + d - \bar{u} - \bar{d} + 2c - 2\bar{s}\right]
\end{aligned}
$$

More Examples

- $F_2^{\nu N} = F_2^{\bar{\nu} N}$ if $s = \bar{s},\ c = \bar{c}$
- $xF_3^{\nu N} + xF_3^{\bar{\nu} N} = 2x\left[u - \bar{u} + d - \bar{d}\,\right]$
- $F_2^{\nu N} - xF_3^{\nu N} = 2x\left[\bar{u} + \bar{d} + 4\bar{c}\right]$
- $F_2^{\bar{\nu} N} - xF_3^{\bar{\nu} N} = 2x\left[\bar{u} + \bar{d} + 4\bar{s}\right]$

So, by taking various linear combinations of structure functions, one can separate the valence and antiquark distributions, at least in Leading Order.

Which is better - cross sections or structure functions?

- Structure functions have intuitive interpretations, as shown previously
  - Dependence on the vector boson polarization: F_L vs F_1
  - Can separate valence and sea terms (at least in Leading Order)
- Three variables in the cross section, but only two in the structure functions - easier to interpret

On the other hand...

- Extraction of structure functions is model dependent, e.g., need R to separate out F_2
- F_L is zero in lowest order - so a Leading Order calculation starts at $\mathcal{O}(\alpha_s)$ which is Next-to-Leading Order for F_2 and xF_3. This is because F_L is a difference of structure functions
- Cross sections are less model dependent - preferred for PDF global fitting
- No right answer for the question posed above