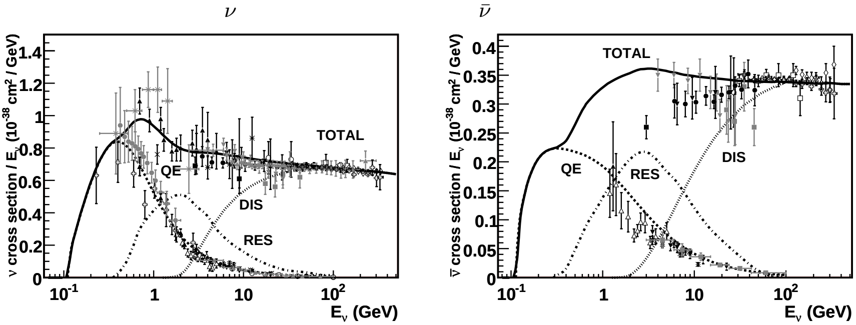
# Neutrino induced pion production reaction 1

Toru Sato

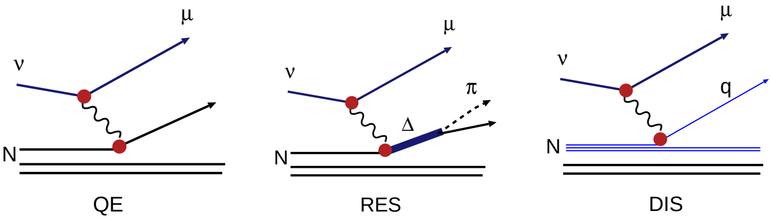
Osaka University

Oct. 26 2014 NuSTEC14

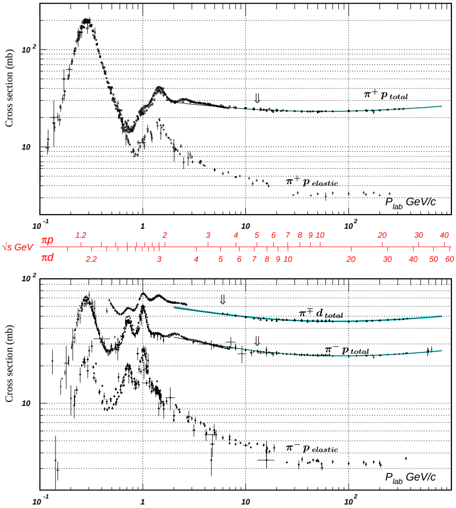
# Introduction




J. A. Formagglo and G. P. Zeller Rev. Mod. Phys. 84 (2012) 1307.



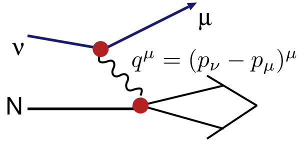
## $\pi$-nucleon scattering and excited nucleons



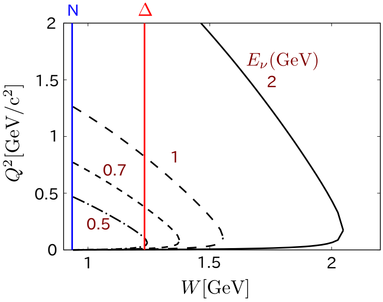

PDG

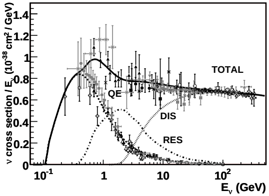
$$\sqrt{s} = W = \sqrt{(p_N + k_\pi)^2}$$

# Kinematical region covered by neutrino reaction




$$W = \sqrt{(q+p_N)^2}, Q^2 = -q^2$$




- Hadron dynamics characterised by $W, Q^2$
- Resonance region : $W < 2GeV$
- $\sigma_\nu(tot)$ : sum of strength in the $W$ region covered by $E_\nu$

## Weak and electromagnetic current

$$< \pi N | j^\mu_\alpha | N >$$

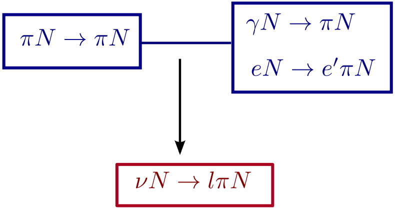
$$\begin{aligned} j^\mu_{cc} &= (V^\mu_1 + iV^\mu_2) - (A^\mu_1 + iA^\mu_2)\,, \\ j^\mu_{nc} &= (1 - 2\sin^2\theta_W) j^\mu_{em} - V^\mu_{IS} - A^\mu_3 \\ j^\mu_{em} &= V^\mu_3 + V^\mu_{IS} \end{aligned}$$

Iso spin

$$\begin{aligned} j^0_{em} &= p^\dagger p = N^\dagger [\frac{1}{2} + \frac{\tau_3}{2}] N \\ V^0_{cc} &= p^\dagger n = N^\dagger \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} N = N^\dagger [\frac{\tau_1}{2} + i\frac{\tau_2}{2}] N \end{aligned}$$

- vector current of em and cc, nc are related by rotation in isospin space

How pion is created from neutrino, electron, photon induced reactions and interacts with nucleon?




1. Near threshold pion production: Low energy theorem, chiral symmetry, unitarity
2. Delta resonance region: resonance vs non-resonance, $N\Delta$ transition form factor
3. Pion interaction in nuclei and coherent pion production
4. Electroweak meson production in resonance region

# Near threshold pion production

- Pion and chiral symmetry
- Effective chiral Lagrangian
- S-wave $\pi N$ scattering
- Pion photo and electroproduction
- Pion production by axial vector current
- Unitarity and Fermi-Watson theorem

# pion and chiral symmetry

Pion $(\pi^+, \pi^0, \pi^-)$

- mass : $m_\pi \sim 139 MeV \sim \frac{1}{7} m_N$
- Spin-Parity :$J^P = 0^-$
- Isospin : $I = 1$

$$
\begin{aligned}
|\pi^\pm > &= \mp \frac{1}{\sqrt{2}} [|\pi^1 > \pm i |\pi^2 >] \\
|\pi^0 > &= |\pi^3 >
\end{aligned}
$$

## Chiral symmetry

Symmetry of $L_{QCD}$ (massless quark) under transformation ($SU(2)_L \otimes SU(2)_R$)

$$\psi_{L/R} \to e^{i\vec{\theta}_{L/R}\cdot\vec{\tau}}\psi_{L/R}$$

where $\psi_{L/R} = \left( \begin{array}{c} \frac{1\mp\gamma_5}{2}u \\ \frac{1\mp\gamma_5}{2}d \end{array} \right)$

The symmetry realized in nature is $SU(2)_V$. $SU(2)_A$ is sponatneously broken.

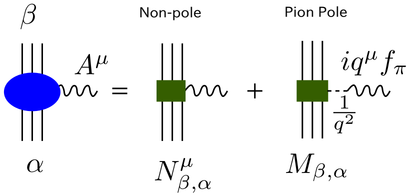
- pion as massless Nambu-Goldstone boson
- axial vector current annihilates pion

$$< 0|A^i_\mu|\pi^i(q) >= if_\pi q_\mu, \quad f_\pi \sim 93MeV$$

## Emission of soft pion ($m_\pi = 0$, $q \to 0$)

Simple example on how pion production matrix element is constrained from chiral symmetry:
Matrix element of axial vector current $<\beta|A^i_\mu|\alpha>$

$$<\beta|A^\mu_i|\alpha> = N^\mu_{\beta,\alpha} + \frac{i f_\pi q^\mu}{q^2} M_{\beta,\alpha}$$




Requirement of current conservation $q \cdot A = 0$ gives

$$M_{\beta,\alpha} = \frac{i q_\mu}{f_\pi} N^\mu_{\beta,\alpha}$$

Relation between matrix element of axial vector current and pion production amplitude

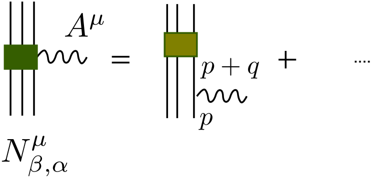
$$M_{\beta,\alpha} = \frac{iq_\mu}{f_\pi} N^\mu_{\beta,\alpha}$$

1) one-body: $\alpha = \beta = N$, $\pi NN$ interaction is given as

$$i\frac{g_A}{f_\pi}\bar{N}\gamma^\mu\gamma_5\frac{\tau^i}{2}Nq_\mu$$

2) two-body: most singular contribution of $N_{\beta,\alpha}$ in $q \to 0$ comes from axial vector current attached external line.

$\frac{1}{(p+q)^2-M^2} = \frac{1}{2p\cdot q+q^2} \sim O(\frac{1}{q})$




$$\lim_{q\to 0} M_{\beta,\alpha} = i\frac{q_\mu}{f_\pi}N^\mu_{\beta,\alpha}(ext)$$

Soft pion emission amplitude can be calculated from axial vector current attached at external line.

# Effective Chiral Lagrangian

1. Soft pion theorem for weak pion production
   Starting from:

$$T[\partial \cdot A_i J(0)] = \partial_\mu T[A_i^\mu(x) J(0)] - \delta(t)[A_i^0(x), J(0)]$$

   'Master formula' is derived by using PCAC + Current Algebra($[Q_i^5, V_j^\mu] = i\epsilon_{ijk} A_k^\mu$)
   S. L. Adler, Ann. Phys. 50 168 (1968).
   S. L. Adler, 'neutrino interaction phenomenology and neutral current', Proc. of the Sixth Hawaii topical conference in particle physics (1975).

2. Effective Chiral Lagrangian
   Effective field theory which describes interaction of Goldstone boson from the chiral symmetry. Nonlinear representation of pseudoscalar field

$$U = e^{i\vec{\pi}\cdot\vec{\tau}/f_\pi} = \xi^2 \rightarrow LUR^\dagger$$

   for $\psi_{L/R} \rightarrow L/R\psi_{L/R}$

"a theorem, which as far as I know has never been proven, but which I cannot imagine could be wrong" (S. Weinberg, Physicsa 96A 327 (1979))

## Effective Chiral Lagrangian

$$
\begin{aligned}
L &= \bar{N}[i\gamma^\mu D_\mu - m_N + i g_A \gamma^\mu \gamma_5 \Delta_\mu] N \\
&+ \frac{f_\pi^2}{4} Tr[\nabla_\mu U^\dagger \nabla^\mu U] + \frac{f_\pi^2 m_\pi^2}{4} Tr(U + U^\dagger)
\end{aligned}
$$

where

$$
\begin{aligned}
D_\mu &= \partial_\mu + \frac{1}{2}[\xi^\dagger, \partial_\mu \xi] - \frac{i}{2}\xi^\dagger (V_\mu + A_\mu)\xi - \frac{i}{2}\xi(V_\mu - A_\mu)\xi^\dagger \\
\Delta_\mu &= \frac{1}{2}\{\xi^\dagger, \partial_\mu \xi\} - \frac{i}{2}\xi^\dagger (V_\mu + A_\mu)\xi + \frac{i}{2}\xi(V_\mu - A_\mu)\xi^\dagger \\
\nabla_\mu U &= \partial_\mu U - i(V_\mu + A_\mu)U + iU(V_\mu - A_\mu)
\end{aligned}
$$

H. Georgi, 'weak interactions and modern particle theory' (Dover) chap 5,6
S. Weinberg, The quantum theory of field Vol II, chap. 19
J. Donoghue, E. Golowich, B. R. Holstein, 'Dynamics of the Standard Model' (Cambridge) Chap. IV, VI

## Relevant Interaction and current

$\pi N$ interaction

$$-\frac{g_A}{2f_\pi}\bar{N}\gamma^\mu\gamma_5\vec{\tau}N\cdot\partial_\mu\vec{\pi}-\frac{1}{4f_\pi^2}\bar{N}\gamma^\mu\vec{\tau}N\cdot\vec{\pi}\times\partial_\mu\vec{\pi}$$

Pseudo vector $\pi NN$, s-wave $\pi N$(Weinberg-Tomozawa interaction)

Vector current

$$\vec{V}^\mu=\bar{N}\gamma^\mu\frac{\vec{\tau}}{2}N+\vec{\pi}\times\partial^\mu\vec{\pi}+\frac{g_A}{2f_\pi}\bar{N}\gamma^\mu\gamma_5\vec{\tau}N\times\vec{\pi}$$

Contact interaction can be generated from $\pi NN$ using Gauge invariance

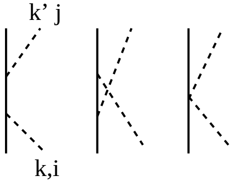
Axial vector current

$$\vec{A}^\mu=g_A\bar{N}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}N-f_\pi\partial^\mu\vec{\pi}+\frac{1}{2f_\pi}\bar{N}\gamma^\mu\vec{\tau}N\times\vec{\pi}$$

# S-wave pion-nucleon scattering

$\pi N$ T-matrix

$$-\frac{g_A}{2f_\pi}\bar{N}\gamma^\mu\gamma_5\vec{\tau}N\cdot\partial_\mu\vec{\pi} - \frac{1}{4f_\pi^2}\bar{N}\gamma^\mu\vec{\tau}N\cdot\vec{\pi}\times\partial_\mu\vec{\pi}$$



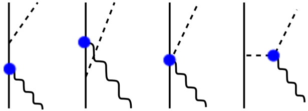

- isovector scattering length (proportional to $\epsilon_{ijk}$)

$$(1+\frac{m_\pi}{m_N})a^- = \frac{m_\pi}{8\pi f_\pi^2} \sim 0.13 fm$$

$$\begin{aligned} a^-_{exp} &= (0.128 \pm 0.002) fm \\ a^+_{exp} &= (-0.003 \pm 0.002) fm \end{aligned}$$

Vector Current $< \pi^i(k)N(p')|V^j_\mu(q)|N(p) >$

$$\vec{V}^\mu = \bar{N}\gamma^\mu\frac{\vec{\tau}}{2}N + \vec{\pi}\times\partial^\mu\vec{\pi} + \frac{g_A}{2f_\pi}\bar{N}\gamma^\mu\gamma_5\vec{\tau}N\times\vec{\pi}$$



$$i\frac{g_A}{2f_\pi}\bar{u}(p')[\not{k}\gamma_5\tau_i\frac{1}{\not{p}+\not{q}-m_N}\gamma_\mu\frac{\tau^j}{2} + \gamma_\mu\frac{\tau^j}{2}\frac{1}{\not{p}-\not{k}-m_N}\not{k}\gamma_5\tau_i$$
$$+\epsilon_{ijk}\tau^k\gamma_\mu\gamma_5 - \epsilon_{ijk}\tau^k(2k-q)^\mu\frac{1}{(k-q)^2-m_\pi^2}(\not{k}-\not{q})\gamma_5]u(p)$$

- Contact interaction becomes main term at low energy
- For electromagnetic current,
  - Nucleon current: $\tau^j/2 \to (1+\tau^3)/2$, Add anomalous magnetic moment term in practical calculation.
  - Contact and pion pole terms: $\epsilon_{ijk} \to \epsilon_{i3k}$ No contirbution for neutral pion production.($\epsilon_{33k} = 0$)

Examine low energy limit of the derived amplitudes and compare with 'data' from amplitude analysis

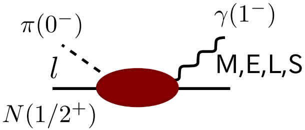
- partial wave expansion of the amplitude
  Use eigenvalue of constant of motion ($[H, O] = 0$, $O = J^2, I^2, P$).
  Example of scattering amplitude of non-relativistic quantum mechanics

$$[H, \vec{L}^2] = 0 \quad \rightarrow \quad F(E, \cos\theta) = \sum_{L=0}^{\infty} P_L(\cos\theta) f_L(E)$$

- From differential cross section and polarization data one extracts partial wave amplitudes.

# Multipole expansion

amplitude for given $\vec{J}^2, P, \vec{I}^2$




$$j = l \pm 1/2, \quad P = (-1)^{l+1}$$

Important s-wave and p-wave amplitudes

| $lJ^P$ | Vector | Axial Vector |
|---|---|---|
| $s1/2^-$ | $E_{0+}, S_{0+}$ | $\mathcal{M}_{0+}, \mathcal{S}_{0+}, \mathcal{L}_{0+}$ |
| $p3/2^+$ | $M_{1+}, E_{1+}, S_{1+}$ | $\mathcal{E}_{1+}, \mathcal{M}_{1+}, \mathcal{S}_{1+}, \mathcal{L}_{1+}$ |

- Threshold pion photo production: $E_{0+}$
- Dominant contribution of $\Delta$ resonance, $M_{1+}, \mathcal{E}_{1+}$


F. A. Berends, A. Donnachie, D. L. Weaer, Nucl. Phys. B4 1(1967)
L. S. Adler ann. phys. 50 168 (1968)

# E0+ charged pion photoproduction

Main contribution for the threshold s-wave pion production is $E_{0+}$.
$E_{0+}$ for charge(not neutral) pion production is given by contact term (Kroll-Ruderman term)

($E_{0+}$ for $\gamma p \to \pi^0 p$ is factor $m_\pi / m_N$ smaller than charged pion production.)

$$\frac{g_A}{2f_\pi}(\bar{N}\gamma^\mu\gamma_5\vec{\tau}N \times \vec{\pi})^3$$

For $\gamma + p \to \pi^+ n$

$$E_{0+} = \frac{e g_A}{4\sqrt{2} f_\pi}(1 - \frac{3}{2}\frac{m_\pi}{m_N}) \sim 26.3 \times 10^{-3}/m_\pi$$

$$E_{0+}^{exp} = (27.9 \pm 0.5) \times 10^{-3}/m_\pi$$

V. Bernard, N. Kaiser, U. Maissner, arXiv 9501385[hep-ph]
B. R. Holstein, 'Chiral perturbation theory: a Primer

## $Q^2$ dependence of $E_{0+}$ and pion electroproduction

(Heavy baryon chiral perturbation theory)

$$E_{0+}^{-}(m_\pi = 0, q^2) = \frac{eg_A}{8\pi f_\pi}[1 + \frac{q^2}{6} < r^2 >_A + \frac{q^2}{4m_N^2}(\kappa_V + \frac{1}{2}) + \frac{k^2}{128 f_\pi^2}(1 - \frac{12}{\pi^2}]$$

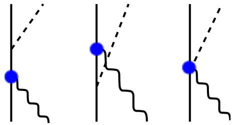
- Pion electroproduction can gives information of axial vector mass.
- $M_A = (1.026 \pm 0.021) GeV$ from neutirno scattering
  $M_A = (1.069 \pm 0.016) GeV$ from electron scattering
  higher order contibution from chiral perturbation theory gives $\Delta M_A = 0.055 GeV$ brings agreement between $M_A$ extracted from neutrino and electron scattering data.

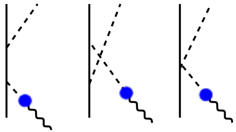
V. Bernard, L. Elouadrhiri, U. Meissner, J. Phys. G 28 R1 (2002)

# Axial vector current

$$< \pi^j(k)N|A^i_\mu(q)|N >$$

$$\vec{A}^\mu = g_A \bar{N}\gamma^\mu\gamma_5\frac{\vec{\tau}}{2}N - f_\pi\partial^\mu\vec{\pi} + \frac{1}{2f_\pi}\bar{N}\gamma^\mu\vec{\tau}N \times \vec{\pi}$$





- Pion pole and non-pole term.

$$A^\mu = A^\mu_{NP} - \frac{q^\mu q \cdot A_{NP}}{q^2 - m_\pi^2}$$

- Non-pole term and virtual( $q^2 \neq m_\pi^2$) pion-nucleon scattering amplitude

$$< \pi^j(k)N|T(W)|\pi^i(q)N >= \frac{iq^\mu}{f_\pi} < \pi^j(k)N|A^i_\mu(q)|N >$$

# Note on contact term

contribution of Contact term

$$\vec{V}^\mu \sim \bar{N}\gamma^\mu \frac{\vec{\tau}}{2} N + \frac{g_A}{2f_\pi}\bar{N}\gamma^\mu\gamma_5\vec{\tau}N \times \vec{\pi}$$

$$\vec{A}^\mu \sim g_A\bar{N}\gamma^\mu\gamma_5 \frac{\vec{\tau}}{2} N + \frac{1}{2f_\pi}\bar{N}\gamma^\mu\vec{\tau}N \times \vec{\pi}$$

| | | space-component | time component |
|---|---|---|---|
| $V^\mu$ | contact | $O(1)$ | $O(p/m_N)$ |
| | (Nucleon) | $O(p/m_N)$ | $O(1)$ |
| $A^\mu$ | contact | $O(p/m_N)$ | $O(1)$ |
| | (Nucleon) | $O(1)$ | $O(p/m_N)$ |

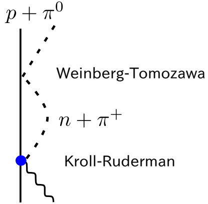
remarks: large contributions of pion exchange current are expected for space component of Vector current and time component of Axial vector current.
(K. Kubodera, J. Delorme, M. Rho, PRL 40, 755 (1978) )

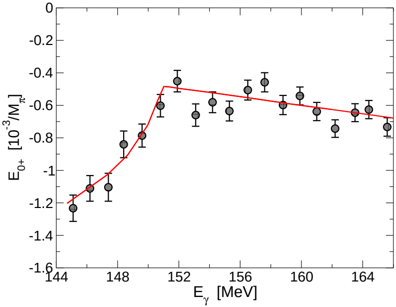
# E0+ neutral pion photproduction and rescattering

$E_{0+}$ for neutral pion photoproduction $\gamma + p \to \pi^0 + p$ is $m_\pi/m_N$ smaller than charged pion production.

$$E_{0+} = \frac{eg_A}{8\pi m_N}[\frac{m_\pi}{m_N} - (\frac{m_\pi}{m_N})^2(\frac{3+\kappa_p}{2} + \frac{m_N^2}{16 f_\pi^2})]$$

Last term is due to the one-loop correction.






V. Bernard, U-G Meissner, hep-ph 0611231

- Rescattering contirbution $\gamma + p \rightarrow \pi^+ + n \rightarrow \pi^0 + p$ (unitarity correction) produces cusp.

  $m_p + m_{\pi^0} = 1073.25$MeV, $m_n + m_{\pi^+} = 1079.14$MeV

  $E_{0+}^{\pi^+ n} >> E_{0+}^{\pi^0 p}$

# Unitarity and Fermi-Watson theorem

From unitarity, phase of pion photoproduction amplitude is given by the phase shift of pion-nucleon scattering

S-matrix of $\gamma N, \pi N, \pi\pi N, ...$ reactions. (for given channel $j^\pi i$)

$$S = \begin{pmatrix} S_{\gamma,\gamma} & S_{\gamma,\pi} & S_{\gamma,2\pi} & ... \\ S_{\pi,\gamma} & S_{\pi,\pi} & S_{\pi,2\pi} & ... \\ S_{2\pi,\gamma} & S_{2\pi,\pi} & S_{2\pi,2\pi} & ... \\ ... & ... & ... & ... \end{pmatrix}$$

Unitarity $S^\dagger S = 1$ gives relation among matrix elements.

E. Fermi, Suppl. Nuovo Cimento 2 (1955), 17.
K. M. Watson, Phys. Rev. 95 (1954), 228.

Energy below $\pi\pi N$ threshold:

$$S = \begin{pmatrix} S_{\gamma,\gamma} & S_{\gamma,\pi} \\ S_{\pi,\gamma} & S_{\pi,\pi} \end{pmatrix}$$

The unitarity relation is given as

$$\begin{pmatrix} S^*_{\gamma,\gamma} & S^*_{\pi,\gamma} \\ S^*_{\gamma,\pi} & S^*_{\pi,\pi} \end{pmatrix} \times \begin{pmatrix} S_{\gamma,\gamma} & S_{\gamma,\pi} \\ S_{\pi,\gamma} & S_{\pi,\pi} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

we obtain

$$S^*_{\gamma,\gamma} S_{\gamma,\pi} + S^*_{\pi,\gamma} S_{\pi,\pi} = 0$$

- First order of $e$: $S_{\gamma,\gamma} \sim 1$
- T-invariance: $S_{\gamma,\pi} = S_{\pi,\gamma}$

$$S^*_{\gamma,\gamma} S_{\gamma,\pi} + S^*_{\pi,\gamma} S_{\pi,\pi} \sim S_{\pi,\gamma} + S^*_{\pi,\gamma} S_{\pi,\pi} = 0$$

- Below $\pi\pi N$ threshold: $S_{\pi,\pi} = e^{2i\delta_{\pi N}}$
- Pion photoproduction amplitude $t_{\pi,\gamma}$: $S_{\pi,\gamma} = 0 - it_{\pi,\gamma}$

$$\rightarrow \quad \frac{t_{\pi,\gamma}}{t^*_{\pi,\gamma}} = e^{2i\delta_{\pi N}}$$

Fermi-Watson theorem:Phase of the pion photoproduction amplitude is given by the phase shift of pion-nucleon elastic scattering.

$$t_{\pi,\gamma} = e^{i\delta_{\pi N}} |t_{\pi,\gamma}|$$

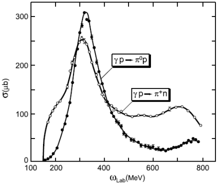
# Summary

# Summary

- Low energy pion-nucleon scattering and pion electroweak production amplitudes are constrained form chiral symmetry

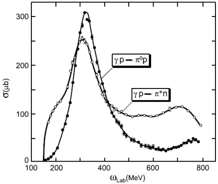
# Summary

- Low energy pion-nucleon scattering and pion electroweak production amplitudes are constrained form chiral symmetry
- Contact interaction explains larger charged pion photoproduction compared with neutral pion production on proton.



# Summary

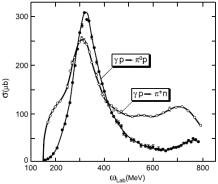
- Low energy pion-nucleon scattering and pion electroweak production amplitudes are constrained form chiral symmetry
- Contact interaction explains larger charged pion photoproduction compared with neutral pion production on proton.




- We adopt tree-diagram as the non-resonant mechanism of pion electroweak production in the following analysis.

# Summary

- Low energy pion-nucleon scattering and pion electroweak production amplitudes are constrained form chiral symmetry
- Contact interaction explains larger charged pion photoproduction compared with neutral pion production on proton.




- We adopt tree-diagram as the non-resonant mechanism of pion electroweak production in the following analysis.
- To include the phase of the amplitudes, rescattering or non-perturbative contirbution should be considered.