Outline

Introduction

Nuclei made up by protons and neutrons (nucleons). These interact via the strong interactions (QCD), and also via the electroweak (EW) interactions, which are much weaker.

Nucleons are not the fundamental degrees of freedom (d.o.f.) of QCD.

These lectures: nuclei as bound states of nucleons interacting amongst themselves via two- and three-body forces and with external EW fields via one-, two-, and three-body currents; this is the "basic model". Other nuclear models, for example the shell model, are approximations of the basic model valid in certain mass number ranges and/or energy regimes.

Basic model:

dominant pair potential (theory + data)

$$H = \sum_{i=1}^{A} \Big(\underbrace{m_i}_{\text{rest mass}} + \underbrace{\frac{\vec{p}_i^{\,2}}{2m_i}}_{\text{NR kinetic energy}} \Big) + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

weaker three-body potential (theory + data)

Protons and neutrons are spin $s = \frac{1}{2}$ particles. If the mass difference between neutron and proton ($m_n - m_p \simeq 1.3$ MeV) is ignored, then the proton and neutron can be considered as two possible states of a particle we call the nucleon (N), of rest mass

$$m = (m_p + m_n)/2 ,$$

having "isospin" $t = 1/2$ and "isospin projections" $t_z = 1/2$ for $|p\rangle$ and $t_z = -1/2$ for $|n\rangle$. The operator $\vec{t} = \frac{1}{2}\vec{\tau}$ is mathematically identical to $\vec{s} = \frac{1}{2}\vec{\sigma}$,

$$[t_i, t_j] = i\,\varepsilon_{ijk}\, t_k ,$$

but acts in "isospin space" rather than in ordinary space like $\vec{s}$.

Strong interactions have approximate isospin invariance:

$$\underbrace{H}_{} = \underbrace{H_0}_{\text{scalar in isospin space}} + H_{IB} \leftarrow \text{isospin symmetry breaking terms (small)}$$

State of N: $\vec{r}, \sigma_z \tau_z \equiv \vec{x}$; v_{ij} depends on $\vec{x}_i, \vec{x}_j$ and V_{ijk} depends on $\vec{x}_i, \vec{x}_j, \vec{x}_k$

The basic model assumes that the series of potentials converge rapidly; this is expected from theory (χEFT) and borne out by calculations.

Nucleons are baryons; baryons and meson, i.e. the hadrons, are bound states of quarks (q) and gluons (g), the fundamental d.o.f. of QCD; q and g are confined within the hadrons (confinement)

q come in 6 flavors $\underbrace{\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}}_{\text{3 families}}$ $\begin{matrix} \text{charge} = 2/3\, e \\ = -1/3\, e \end{matrix}$

and each q carries a unit of color charge, and is in one of 3 possible color states, conventionally R, B, G.

Hadrons consist of color singlet states of q and g:

baryons: $|qqq\rangle + |qqqq'\bar{q}'\rangle + \dots$

mesons: $\underbrace{|q\bar{q}\rangle}_{\text{valence structure}} + |q\bar{q}\,\underbrace{q'\bar{q}'}_{\text{additional } q'\bar{q}' \text{ pairs}}\rangle + \dots$

For example,

$$|p\rangle = |uud\rangle + \underbrace{|uuddd\rangle}_{\simeq |n\rangle|\pi^+\rangle} + \overbrace{|uuds\bar{s}\rangle}^{\simeq |\Lambda\rangle|K^+\rangle} + \dots$$

and therefore nucleons have "meson clouds".

Lowest energy baryons:

E(MeV)		J^π	t
1440	N(1440)	$\frac{1}{2}^+$	$\frac{1}{2}$
1232	Δ	$\frac{3}{2}^+$	$\frac{3}{2}$
939	N	$\frac{1}{2}^+$	$\frac{1}{2}$

300 MeV

The basic model assumes that the q and $\bar{q}$ in nuclei are confined in color singlet states close to those of free nucleons. They need not be exactly in free N states in nuclei. The v_{ij} is constrained by experimental data and hence contains all the effects of the quarks in the interacting clusters being in excited states of the nucleon.

Energy scales in nuclei: (binding energy)/A $\lesssim$ 9 MeV

$$\frac{1}{A}\left(\sum_{i<j} \langle v_{ij} \rangle + \sum_{i<j<k} \langle V_{ijk} \rangle\right) \sim -(10-50)\text{ MeV}$$

largely cancelled by (kinetic energy)/A $\simeq$ 300 MeV

|(potential energy)/A|, (kinetic energy)/A $\ll m_\Delta - m$, and this presumably helps the series of potentials converge rapidly.

The Yukawa potential

Dominant terms in v_{ij} due to exchange of pions; they have a complex dependence on spins and isospins of interacting NN pair because pions have $J^\pi = 0^-$ and $t=1$ (see later)

We want to investigate the connection between meson exchange interactions and their representation in terms of an instantaneous potential. We simplify the problem (for the time being) by ignoring spins and isospins and by considering scalar meson exchange.

The Yukawa potential in classical mechanics

Consider a scalar field $\phi(\vec{r},t)$ interacting with static particles at positions $\vec{r}_1, \ldots, \vec{r}_A$

$$\mathcal{L} = \underbrace{\frac{1}{2}\left[\dot{\phi}^2 - (\vec{\nabla}\phi)\cdot(\vec{\nabla}\phi) - \mu^2\phi^2\right]}_{\mathcal{L}_0} \underbrace{- g\phi\sum_{i=1}^{A}\delta(\vec{r}-\vec{r}_i)}_{\mathcal{L}_{int}}$$

Euler-Lagrange equation gives

- $\frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial\dot{\phi}} + \vec{\nabla}\cdot\frac{\partial \mathcal{L}}{\partial\vec{\nabla}\phi} - \frac{\partial \mathcal{L}}{\partial\phi} = 0$
- $\mathcal{H} = \dot{\phi}\pi - \mathcal{L}$
- $\pi = \frac{\partial \mathcal{L}}{\partial\dot{\phi}}$
- $\mathcal{H} = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}[\vec{\nabla}\phi\cdot\vec{\nabla}\phi^2 + \mu^2\phi^2] + \ldots$

$$\ddot{\phi} - \vec{\nabla}^2\phi + \mu^2\phi = -g\sum_i \delta(\vec{r}-\vec{r}_i)$$

and the lowest-energy configuration of the field occurs in the static limit

$$\phi(\vec{r},t) \longrightarrow \bar{\phi}(\vec{r})$$

with

$$\vec{\nabla}^2\bar{\phi} - \mu^2\bar{\phi} = g\sum_i \delta(\vec{r}-\vec{r}_i),$$

which, apart from the mass term, looks like the Poisson equation for the electrostatic potential of a system of point-like charges at rest. To solve this equation, assume there is one particle at $\vec{r}_j$ and obtain $\bar{\phi}_j$. Then use superposition

$$\bar{\phi} = \sum_{j=1}^{A}\bar{\phi}_j$$

Translational invariance requires $\bar{\phi}_j \equiv \bar{\phi}_j(\vec{r}-\vec{r}_j)$, and introducing the Fourier transform

$$\bar{\phi}_j(\vec{r}-\vec{r}_j) = \int_{\vec{k}} e^{i\vec{k}\cdot(\vec{r}-\vec{r}_j)} \bar{\phi}_j(\vec{k}) \quad , \quad \int_{\vec{k}} \equiv \int \frac{d\vec{k}}{(2\pi)^3}$$

one finds

$$(k^2+\mu^2)\,\phi_j(\vec{k}) = -g$$

and

$$\bar{\phi}_j(\vec{r}-\vec{r}_j) = -\frac{g}{4\pi}\,\frac{e^{-\mu|\vec{r}-\vec{r}_j|}}{|\vec{r}-\vec{r}_j|}$$

The energy of the field is given by

$$E_\phi = \int d\vec{r}\, \mathcal{H} \leftarrow \text{Hamiltonian density}$$

which in the static limit reduces to

$$E_{\bar{\phi}} = \int d\vec{r}\left\{\frac{1}{2}\Big[\underbrace{(\vec{\nabla}\bar{\phi})\cdot(\vec{\nabla}\bar{\phi})}_{\vec{\nabla}\cdot(\bar{\phi}\,\vec{\nabla}\bar{\phi})-\bar{\phi}\,\vec{\nabla}^2\bar{\phi}} + \mu^2\bar{\phi}^2\Big] + g\bar{\phi}\sum_i \delta(\vec{r}-\vec{r}_i)\right\}$$

$$= \int d\vec{r}\,\frac{1}{2}g\bar{\phi}\sum_i \delta(\vec{r}-\vec{r}_i)$$

$$= \frac{1}{2}g\sum_i \bar{\phi}(\vec{r}_i) = -\frac{g^2}{8\pi}\sum_{i,j}\frac{e^{-\mu r_{ij}}}{r_{ij}}$$

or

$$E_{\bar{\phi}} = \sum_{i<j} v_Y(r_{ij}) + \text{self energies}$$

$$v_Y(r) = -\frac{g^2}{4\pi}\frac{e^{-\mu r}}{r}$$

The Yukawa potential in quantum mechanics

We now treat the problem in quantum mechanics, and show that in the limit of static particles the ground-state energy of the field (i.e., the exact ground state in the interacting theory) is the same as in the classical treatment

$$E_0 = -\sum_{i<j} \frac{g^2}{4\pi} \frac{e^{-\mu r_{ij}}}{r_{ij}}$$

This result is important because it implies that the concept of potential is also valid in quantum mechanics, at least for static or slowly moving particles.

The Hamiltonian reads

$$H = H_0 + H_{int}$$

$$H_0 = \sum_{\vec{k}} \omega_k a^\dagger_{\vec{k}} a_{\vec{k}} \quad , \quad \omega_k = (k^2 + \mu^2)^{1/2}$$

$$\phi(\vec{r}) = \sum_{k} \frac{1}{\sqrt{2\omega_k V}} \left(a_k e^{i\vec{k}\cdot\vec{r}} + a_k^\dagger e^{-i\vec{k}\cdot\vec{r}} \right)$$

$$H_{int} = \int d\vec{r}\, g\phi(\vec{r}) \sum_i \delta(\vec{r}-\vec{r}_i)$$

$$= g \sum_i \sum_{\vec{k}} \frac{1}{\sqrt{2\omega_k V}} \left(a_{\vec{k}} e^{i\vec{k}\cdot\vec{r}_i} + a^\dagger_{\vec{k}} e^{-i\vec{k}\cdot\vec{r}_i} \right)$$

One could use standard perturbation theory to calculate E_0 (all orders but for the 2^{nd} vanish). Instead, define

$$\underset{\text{c-number}}{\alpha_{\vec{k}}} \equiv \frac{g}{\sqrt{2\omega_k V}} \frac{1}{\omega_k} \sum_i e^{-i\vec{k}\cdot\vec{r}_i}$$

and express the full H as

$$H = \sum_{\vec{k}} \omega_k \left[\underbrace{(a^\dagger_{\vec{k}} + \alpha^*_{\vec{k}})}_{A^\dagger_{\vec{k}}} \underbrace{(a_{\vec{k}} + \alpha_{\vec{k}})}_{A_{\vec{k}}} - |\alpha_{\vec{k}}|^2 \right]$$

$$= \underbrace{\sum_{\vec{k}} \omega_k A^\dagger_{\vec{k}} A_{\vec{k}}}_{\text{same spectrum of } H_0} \underbrace{- \sum_{\vec{k}} \omega_k |\alpha_{\vec{k}}|^2}_{\text{overall energy shift}}$$

The ground-state energy is then given by the energy shift

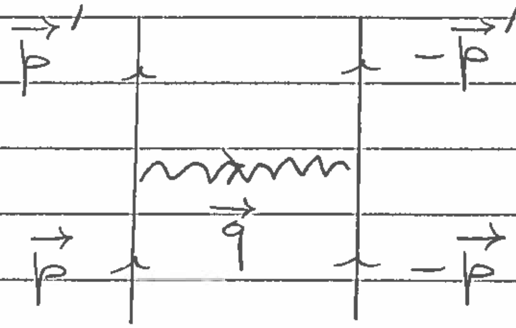
$$E_0 = -\sum_{\vec{k}} \omega_k |\alpha_{\vec{k}}|^2 = -\frac{g^2}{2} \sum_{ij} \int_{\vec{k}} \frac{e^{i\vec{k}\cdot(\vec{r}_i - \vec{r}_j)}}{\omega_k^2}$$

which is exactly the same as the classical result. Therefore in both classical and quantum theories the scalar field energy in the presence of static particles can be replaced by a sum of Yukawa potentials between the particles.

Scattering in Born approximation

Consider scattering between two slowly moving particles interacting via the coupling to the scalar field

$$H = -\frac{1}{2m}\left(\vec{\nabla}_1^2 + \vec{\nabla}_2^2\right) + v_Y(r)$$ ← assumes that ϕ remains in ground state (slow particles)



$$E_i = \frac{p^2}{m}, \quad E_f = \frac{p'^2}{m}$$

$$T^B_{fi} = \int d\vec{r}\, e^{-i\vec{p}\,'\cdot\vec{r}}\, v_Y(r)\, e^{i\vec{p}\cdot\vec{r}}$$

$$= -g^2/(q^2+\mu^2) = \tilde{v}_Y(q), \qquad \vec{q} = \vec{p} - \vec{p}\,'$$

One-meson exchange scattering

Calculate the scattering amplitude to order g^2 from meson exchange processes, i.e. in field theory (without using v_Y). We have

$$H_0 = \frac{1}{2m}(\vec{\nabla}_1^2 + \vec{\nabla}_2^2) + \sum_{\vec{k}} \omega_k a^\dagger_{\vec{k}} a_{\vec{k}}$$

$$H_{int} = g \sum_{i=1,2} \sum_{\vec{k}} \frac{1}{\sqrt{2\omega_k}} \left(a_{\vec{k}} e^{i\vec{k}\cdot\vec{r}_i} + h.c. \right)$$

and volumes have been set to 1, since they drop out.
Scattering theory gives

$$T_{fi} \underset{\eta \to 0^+}{=} \langle f | H_{int} + H_{int} \frac{1}{E_i - H_0 + i\eta} H_{int} + \ldots | i \rangle$$

with $E_i = E_f$, and no meson in initial or final state

$|i\rangle = |\vec{p}, -\vec{p}; 0\rangle$ and $|f\rangle = |\vec{p}\,', -\vec{p}\,'; 0\rangle$.

The first order vanishes, and to order g^2

$$T_{fi} \underset{\eta \to 0^+}{=} \sum_I \frac{\langle \vec{p}\,', -\vec{p}\,'; 0 | H_{int} | I \rangle \langle I | H_{int} | \vec{p}, -\vec{p}; 0 \rangle}{E_i - E_I + i\eta}$$

By considering $H_{int} |\vec{p}, -\vec{p}; 0\rangle$, it is clear that the only intermediate states that can contribute are those with one meson. Since three-momentum is conserved by H_{int}, they are of the type

$|I_1\rangle = |\vec{p} - \vec{q}, -\vec{p}; \vec{q}\rangle$ or $|I_2\rangle = |\vec{p}\,', -\vec{p} - \vec{q}\,'; \vec{q}\,'\rangle$

Now $\langle \vec{p}\,', -\vec{p}\,'; 0 | H_{int} | I_1 \rangle \neq 0$ only if the meson in $|I_1\rangle$ is absorbed by particle 2 and $\vec{q} = \vec{p} - \vec{p}\,'$.
Similarly, $\langle \vec{p}\,', -\vec{p}\,'; 0 | H_{int} | I_2 \rangle \neq 0$ only if the meson in $|I_2\rangle$ is absorbed by particle 1 and $\vec{q}\,' = \vec{p}\,' - \vec{p} = -\vec{q}$

$$T_{fi} = \lim_{\eta\to 0^+} \underbrace{\frac{\langle \vec{p}\,', -\vec{p}\,'; 0 | H_{int} | \vec{p}\,', -\vec{p}; \vec{q}\rangle \langle \vec{p}\,', -\vec{p}; \vec{q} | H_{int} | \vec{p}, -\vec{p}; 0\rangle}{E_i - E_{I_1} + i\eta}}_{(a)} \qquad \vec{q} = \vec{p} - \vec{p}\,'$$

$$+ \underbrace{\frac{\langle \vec{p}\,', -\vec{p}\,'; 0 | H_{int} | \vec{p}, -\vec{p}\,'; \vec{q}\,'\rangle \langle \vec{p}, -\vec{p}\,'; \vec{q}\,' | H_{int} | \vec{p}, -\vec{p}; 0\rangle}{E_i - E_{I_2} + i\eta}}_{(b)}$$

$$\vec{q} = \vec{p}\,' - \vec{p} = -\vec{q}$$

$$E_{I_1} = \omega_q + \frac{p'^2}{2m} + \frac{p^2}{2m} \qquad E_{I_2} = \omega_q + \frac{p^2}{2m} + \frac{p'^2}{2m}$$

$$= \omega_q + E_i \qquad\qquad = \omega_q + E_i$$

since $p = p'$ because of overall energy conservation.

The matrix elements of Hint are readily evaluated:

$$\langle \vec{p}\,', -\vec{p}; \vec{q} | H_{int} | \vec{p}, -\vec{p}; 0\rangle = g \sum_{\vec{k}} \frac{1}{\sqrt{2\omega_k}} \langle \dots; \vec{q} | e^{-i\vec{k}\cdot\vec{r}_1} a^\dagger_{\vec{k}} | \dots; 0\rangle$$

$$= \frac{g}{\sqrt{2\omega_q}} \langle \vec{p}\,', -\vec{p} | e^{-i\vec{q}\cdot\vec{r}_1} | \vec{p}, -\vec{p}\rangle$$

$$= \frac{g}{\sqrt{2\omega}}$$

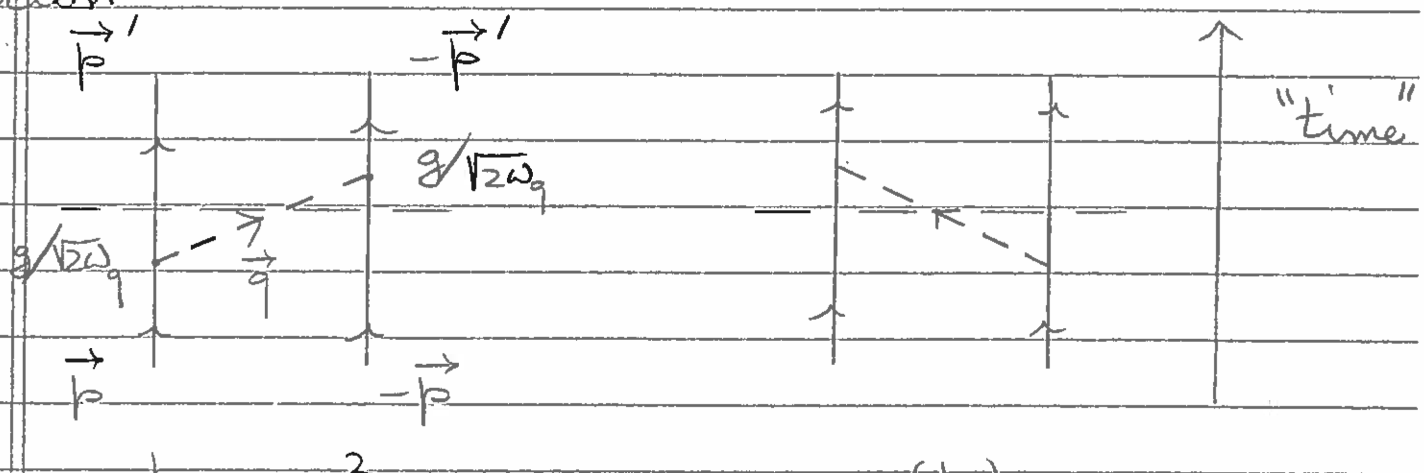
and similarly for the remaining matrix elements. Therefore we find

$$T_{fi} = \underbrace{g/\sqrt{2\omega_q}\,\frac{1}{-\omega_q}\,g/\sqrt{2\omega_q}}_{(a)} + \underbrace{g/\sqrt{2\omega_q}\,\frac{1}{-\omega_q}\,g/\sqrt{2\omega_q}}_{(b)}$$

and the $\eta \to 0^+$ limit can be taken safely here. The two terms (a) and (b) combine to

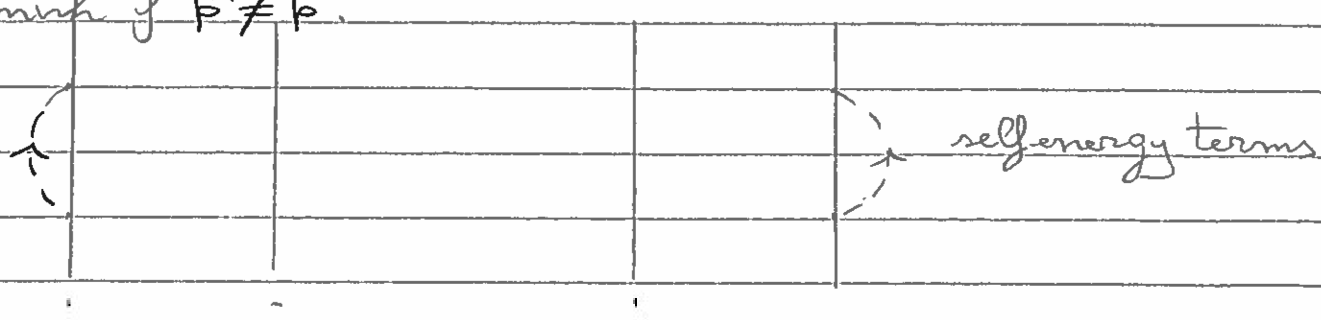
$$T_{fi} = -g^2/\omega_q^2 \equiv \tilde{v}_Y(q)$$

The terms (a) and (b) can be given a diagrammatic representation



(a)   (b)

Note that in T_{fi} are also present the two contributions, which vanish if $\vec{p}\,' \neq \vec{p}$.



The Yukawa potential in Born approximation gives the same amplitude as the meson-exchange field theory in leading order. Indeed, one can define the Yukawa potential via

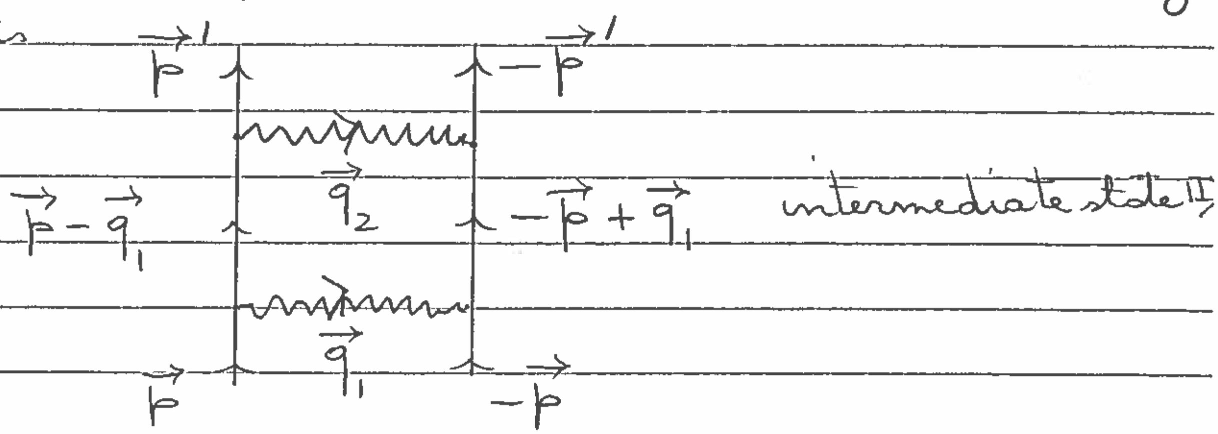
$$\langle f | v_Y | i \rangle \equiv \sum_I \frac{\langle f | H_{int} | I \rangle \langle I | H_{int} | i \rangle}{E_i - E_I}$$

The range of v_Y is $\sim 1/\mu$: $|I\rangle$ violates energy conservation by $\Delta E \sim \omega_q \sim \mu$, and can only exist for a time $\Delta t \sim 1/\mu$, during which the virtual meson propagates for a distance $\lesssim 1/\mu$.

Two-meson exchange amplitude

What happens beyond one-meson exchange? When the coupling constant g is "large", one needs to go beyond the leading order and consider two-meson exchange processes. To what extent is the iterated v_Y a good representation of these processes? We will try answering these questions here.

The 2^{nd} order Yukawa potential contribution to the scattering amplitude is



(Y2)

Note that $\vec{q}_1$ is free to change but, given $\vec{q}_1$, $\vec{q}_2$ is fixed by momentum conservation, since

$$\vec{p}-\vec{q}_1 = \vec{p}\,' + \vec{q}_2 \quad \text{or} \quad \vec{q}_2 = \underbrace{\vec{p}-\vec{p}\,'}_{\vec{q}} - \vec{q}_1$$

The contribution (Y2) reads

$$T_{fi}^{(Y2)} \underset{\eta\to 0^+}{=} \sum_I \frac{\langle f|v_Y|I\rangle\langle I|v_Y|i\rangle}{E_i - E_I + i\eta} = \sum_{\vec{q}_1} A_Y^{(2)}(\vec{q}_1,\vec{q}_2)$$

with

$$\langle \vec{p}-\vec{q}_1 | v_Y | \vec{p}\rangle = \int d\vec{r}\, e^{-i(\cancel{\vec{p}}-\vec{q}_1)\cdot\vec{r}}\, v_Y(r)\, e^{\cancel{i\vec{p}\cdot\vec{r}}} = -g^2/\omega_1^2$$

$$\langle \vec{p}\,' | v_Y | \vec{p}-\vec{q}_1\rangle = \int d\vec{r}\, e^{-i\vec{p}\,'\cdot\vec{r}}\, v_Y(r)\, e^{i(\vec{p}-\vec{q}_1)\cdot\vec{r}} = -g^2/\omega_2^2$$

$$E_i - E_I = \frac{p^2}{m} - \frac{(\vec{p}-\vec{q}_1)^2}{m} = (-q_1^2 + 2\vec{p}\cdot\vec{q}_1)/m$$

The amplitude $A_Y^{(2)}$ simply reads:

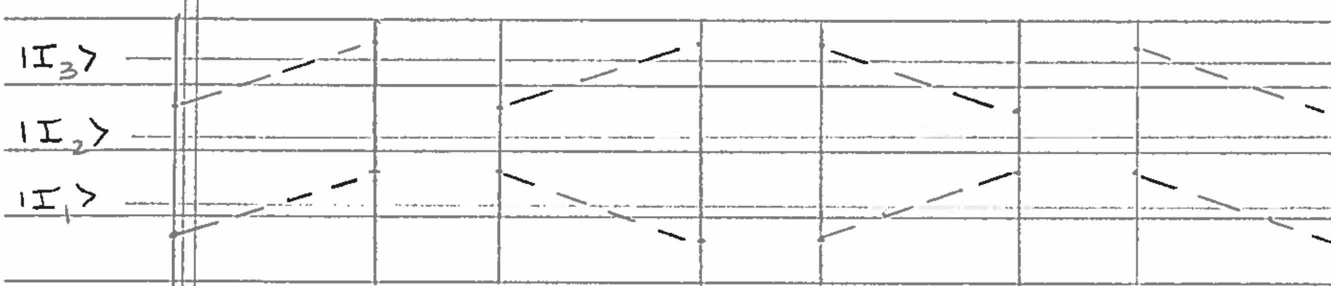
$$A_Y^{(2)} = \frac{g^4}{\omega_1^2\omega_2^2}\,\frac{1}{2\delta}, \quad 2\delta = (q_1^2 - 2\vec{p}\cdot\vec{q}_1)/m - i\eta$$

The exact two-meson exchange amplitude comes from

$$T^{(2)}_{fi} = \langle f | H_{int} \frac{1}{E_i - H_0 + i\eta} H_{int} \frac{1}{E_i - H_0 + i\eta} H_{int} \frac{1}{E_i - H_0 + i\eta} H_{int} | i \rangle$$

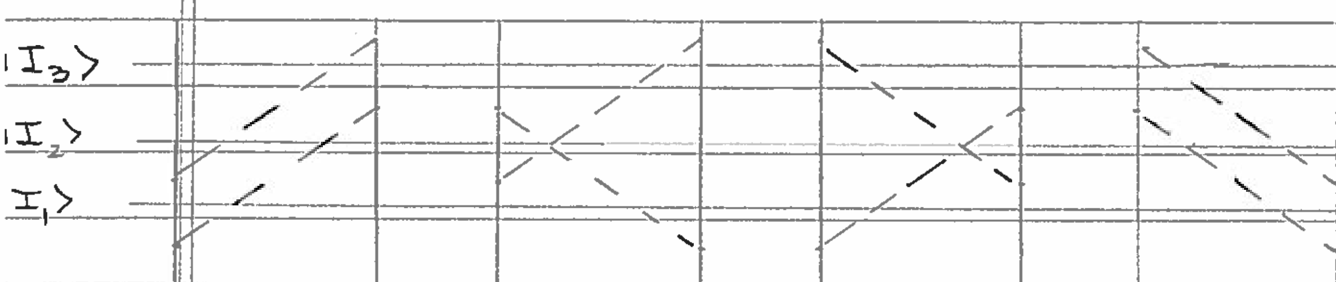
$$= \sum_{I_1 I_2 I_3} \underbrace{\frac{\langle f | H_{int} | I_3 \rangle \langle I_3 | H_{int} | I_2 \rangle \langle I_2 | H_{int} | I_1 \rangle \langle I_1 | H_{int} | i \rangle}{(E_i - E_{I_3} + i\eta)(E_i - E_{I_2} + i\eta)(E_i - E_{I_1} + i\eta)}}_{A^{(2)}}$$

It is easier to analyze the contribution $A^{(2)}$ diagrammatically. There are 12 diagrams that contribute

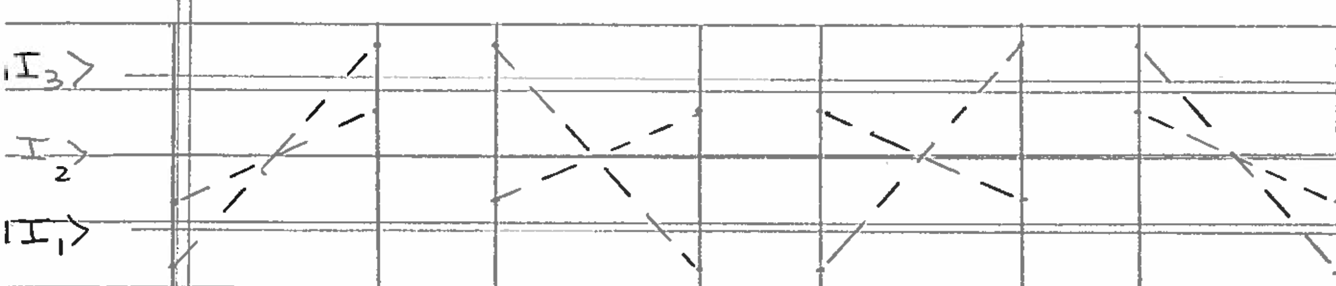


(a1) (a2) (a3) (a4)

reducible

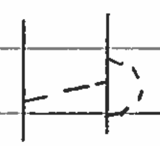


(b1) (b2) (b3) (b4)

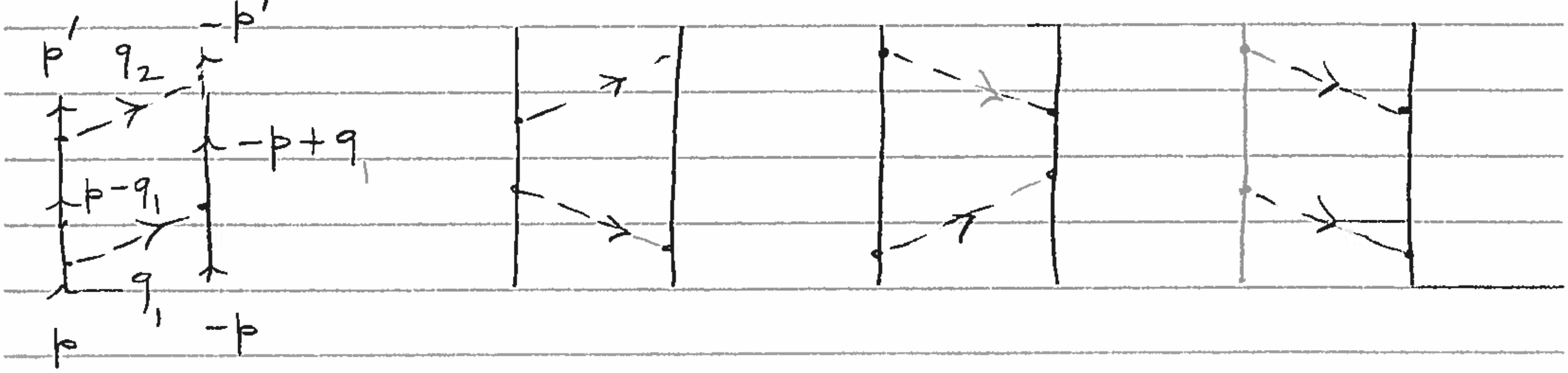


(c1) (c2) (c3) (c4)

irreducible

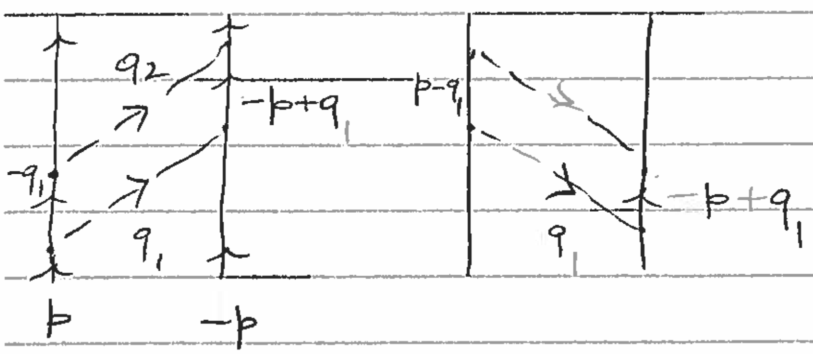
Note that $T^{(2)}_{fi}$ contains other processes as well, like 

Define $\delta_1 = (q_1^2 - 2p\cdot q_1)/2m - i\eta$, $\delta' = (q_2^2 - 2p\cdot q_2)/2m - i\eta$



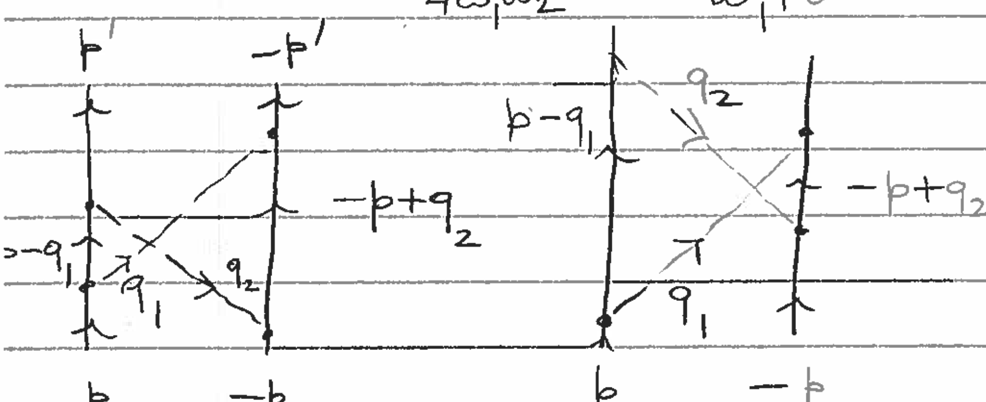
a1 a2 a3 a4

$$a1 + \cdots + a4 = -\frac{g^4}{4\omega_1\omega_2}\cdot 4 \cdot \frac{1}{\omega_1+\delta}\;\frac{1}{2\delta}\;\frac{1}{\omega_2+\delta}$$



b1 b4

$$b1 + b4 = -\frac{g^4}{4\omega_1\omega_2}\cdot 2 \cdot \frac{1}{\omega_1+\delta}\;\frac{1}{\omega_1+\omega_2}\;\frac{1}{\omega_2+\delta}$$



b2 b3

$$b2 = -\frac{g^4}{4\omega_1\omega_2}\frac{1}{\omega_2+\delta'}\frac{1}{\omega_1+\omega_2+\delta+\delta'}\frac{1}{\omega_1+\delta'}$$

$$b3 = -\frac{g^4}{4\omega_1\omega_2}\frac{1}{\omega_1+\delta}\frac{1}{\omega_1+\omega_2+\delta+\delta'}\frac{1}{\omega_2+\delta}$$

Note that each contribution associated with a given diagram is

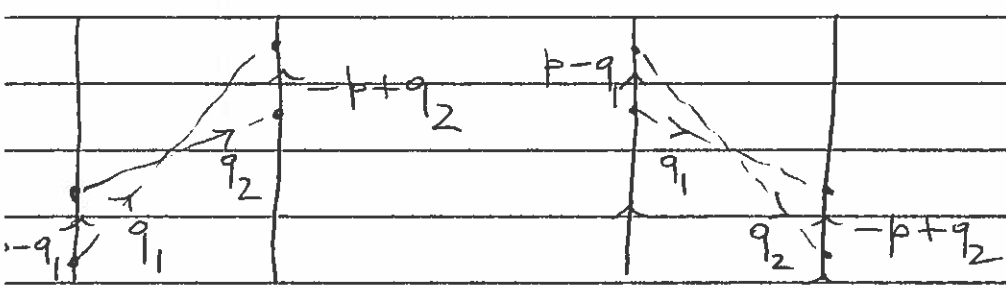
$$\int_{q_1}\int_{q_2} \bar{\delta}(\bar{q}_1+\bar{q}_2-\bar{p}+\bar{p}')\ \mathrm{contrib}(\bar{q}_1,\bar{q}_2)$$

so we can exchange $q_1 \rightleftarrows q_2$ (Jacobian is 1) in the integral, and hence ($\delta \rightleftarrows \delta'$)

$$b2 = b3$$

under the integral.

Finally

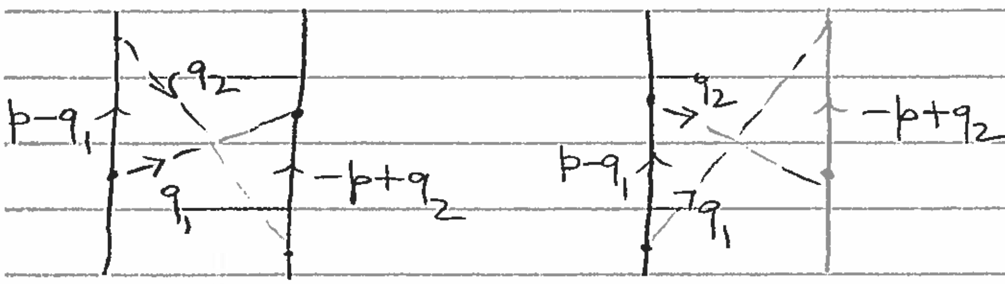


C1 C4

$$C1 = -\frac{g^4}{4\omega_1\omega_2}\frac{1}{\omega_1+\delta}\frac{1}{\omega_1+\omega_2}\frac{1}{\omega_1+\delta'}$$

$$C4 = -\frac{g^4}{4\omega_1\omega_2}\frac{1}{\omega_2+\delta'}\frac{1}{\omega_1+\omega_2}\frac{1}{\omega_2+\delta}$$

and $C1 = C4$ under the integral in $q_1 q_2$.



$$C2 = -\frac{g^4}{4\omega_1\omega_2}\frac{1}{\omega_2+\delta'}\frac{1}{\omega_1+\omega_2+\delta+\delta'}\frac{1}{\omega_2+\delta}$$

$$C3 = -\frac{g^4}{4\omega_1\omega_2}\frac{1}{\omega_1+\delta}\frac{1}{\omega_1+\omega_2+\delta+\delta'}\frac{1}{\omega_1+\delta'}$$

and (with the understanding that the following amplitude is to be integrated over q_1 and q_2:

$$A^{(2)}_M(q_1,q_2) = \sum_{i=1}^{4}\left[(ai)+(bi)+(ci)\right]$$

$$= -\frac{g}{4\omega_1\omega_2}\,4\,\frac{1}{2\delta}\,\frac{1}{\omega_1\omega_2}\left[\frac{1}{1+\delta/\omega_1}\,\frac{1}{1+\delta/\omega_2}\right.$$

$$+\frac{\delta}{(1+\delta/\omega_1)\,(\omega_1+\omega_2)\,(1+\delta/\omega_2)}$$

$$+\frac{\delta}{(1+\delta/\omega_1)(\omega_1+\omega_2+\delta+\delta')(1+\delta/\omega_2)}$$

$$+\frac{\delta\,\omega_2/\omega_1}{(1+\delta/\omega_1)(\omega_1+\omega_2)(1+\delta'/\omega_1)}$$

$$\left.+\frac{\delta\,\omega_1/\omega_2}{(1+\delta'/\omega_2)(\omega_1+\omega_2+\delta+\delta')(1+\delta/\omega_2)}\right] *$$

To linear terms in δ, δ' ($\delta, \delta' \ll \omega_1, \omega_2$, slow particles)

$$A^{(2)}_M = A^{(2)}_Y\left[1-\frac{\delta}{\omega_1}-\frac{\delta}{\omega_2}+\frac{\delta}{\omega_1+\omega_2}+\frac{\delta}{\omega_1+\omega_2}\right.$$

$$\left.+\frac{\delta\,\omega_2}{\omega_1(\omega_1+\omega_2)}+\frac{\delta\,\omega_1}{\omega_2(\omega_1+\omega_2)}\right]$$

$$=A^{(2)}_Y\left[1+\delta\,\frac{-\omega_2(\omega_1+\omega_2)-\omega_1(\omega_1+\omega_2)+2\omega_1\omega_2+\omega_2^2+\omega_1^2}{\omega_1\omega_2(\omega_1+\omega_2)}\right.$$

$$\left.+\mathcal{O}(\delta^2)\right]$$

$$=A^{(2)}_Y+\mathcal{O}(\delta^2)$$

* correct (2.68) & (2.69)

Define
$$A_M^{(2)}(\vec{q}_1,\vec{q}_2) = \sum_{i=1}^{4}\left[(ai)+(bi)+(ci)\right]$$

and
$$T_{fi}^{(2)} = \sum_{\vec{q}_1} A_M^{(2)}(\vec{q}_1,\vec{q}_2)\ , \quad \vec{q}_2 = \vec{q}-\vec{q}_1$$

In the static limit (m "large"), one finds
$$A_M^{(2)} = A_Y^{(2)}\left[1+\mathcal{O}(\delta^2)\right]$$

Corrections to the Yukawa potential

From the scattering amplitudes $T_{fi}^{(Y2)}$ and $T_{fi}^{(2)}$ one obtains a correction term
$$\tilde{v}^{(2)}(\vec{p},\vec{q}) = \int_{\vec{q}_1}\left[A_M^{(2)}(\vec{q}_1,\vec{q}-\vec{q}_1) - A_Y^{(2)}(\vec{q}_1,\vec{q}-\vec{q}_1)\right]$$

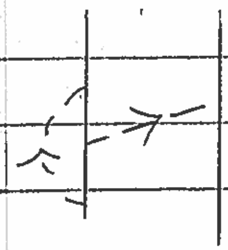
and can define a (non-local) potential
$$\tilde{v}(\vec{p},\vec{q}) = \tilde{v}_Y(q) + \tilde{v}^{(2)}(\vec{p},\vec{q})$$

such that it reproduces the exact scattering amplitude up to and including two-meson exchanges. One can obviously extend this procedure by considering the scattering amplitude originating from three meson exchanges, and so on. The series of potentials
$$\tilde{v} = \tilde{v}_Y + \sum_{\ell=2}^{n}\tilde{v}^{(\ell)}$$

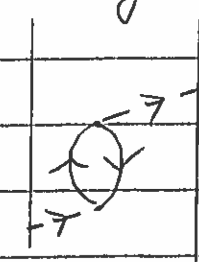
presumably converges if g is small and/or m is large. The range of $\tilde{v}^{(l)}$ is $1/(l\mu)$, and thus the $\tilde{v}^{(l)}$ form a series of potentials with decreasing ranges.

The above discussion is valid for point-like (truly elementary) particles (electrons and photons, for example). However, nucleons and mesons have internal structure and hence many additional processes contribute to $\tilde{v}^{(2)}$. Models of $\tilde{v}$ are obtained by reproducing NN scattering data, which naturally contain all possible many-meson exchange amplitudes. In contrast, the corrections to the Coulomb potential between electrons due to two-photon exchange processes can be calculated with the techniques presented above.

Addional corrections to $\tilde{v}_\gamma$ of order g^4 are obtained from vertex



and vacuum polarization



diagrams. These corrections change the shape of $v_\gamma(r)$ at small r and are important in both QED and QCD. Their role in shaping nuclear forces is not obvious. Nucleons and mesons have a finite size. The short-distance behavior of meson-exchange interactions is modified by their size and internal structure. It is likely that only the long-range part of nuclear forces can be conveniently described as due to exchange of mesons. Its short-range part is obtained from data.