



# Electroweak interactions on the nucleon

Luis Alvarez Ruso



# Outline

- General Introduction
- Electroweak interactions in the Standard Model
- Strong interactions in the Standard Model
  - Example: charged pion decay
- Inclusive neutrino-nucleon(nucleus) cross section
  - Example: EM scattering on a pointlike particle
- Derivation of the nucleon EW current. Flavor structure. Form factors.
- Analysis of the CCQE and NCE cross sections

# Introduction

- Neutrino interactions with matter are at the heart many interesting and relevant physical processes
  - Astrophysics
    - Dynamics of the core-collapse in supernovae
    - r-process nucleosynthesis
  - Hadronic physics
    - Nucleon and Nucleon-Resonance (N- $\Delta$ , N-N\*) axial form factors
    - Strangeness content of the nucleon spin
  - Nuclear physics
    - Information about: nuclear correlations, MEC, spectral functions
    - Complement electron scattering studies
  - Beyond Standard Model
    - Non-standard  $\nu$  interactions

# Introduction

- Neutrino interactions with matter are at the heart of all experiments seeking to unravel its nature.
- Oscillation experiments (with accelerator  $\nu$  in the few-GeV region)
  - Good understanding of neutrino interactions are important for:
    - $\nu$  detection,  $E_\nu$  reconstruction,  $\nu$  flux calibration
    - determination of (irreducible) backgrounds
    - reduction of systematic errors
    - needed in the quest for CP violation and  $\nu$  mass hierarchy
  - Near detectors help to reduce systematic errors but:
  - ND vs FD:
    - exposed to different fluxes with different flavor composition
    - different targets
  - All modern experiments are performed with nuclear targets
    - nuclear effects: essential for the interpretation of the data

# Relevance for oscillation experiments

- (Kinematic)  $E_\nu$  reconstruction:

$$E_\nu = \frac{2m_n E_\mu - m_\mu^2 - m_n^2 + m_p^2}{2(m_n - E_\mu + p_\mu \cos \theta_\mu)}$$

- **Important** for **oscillations**:  $P(\nu_\mu \rightarrow \nu_\tau) = \sin^2 2\theta_{23} \sin^2 \frac{\Delta m_{23}^2 L}{4E_\nu}$

# Relevance for oscillation experiments

- (Kinematic)  $E_\nu$  reconstruction:

$$E_\nu = \frac{2m_n E_\mu - m_\mu^2 - m_n^2 + m_p^2}{2(m_n - E_\mu + p_\mu \cos \theta_\mu)}$$

- Not exact on nuclear targets

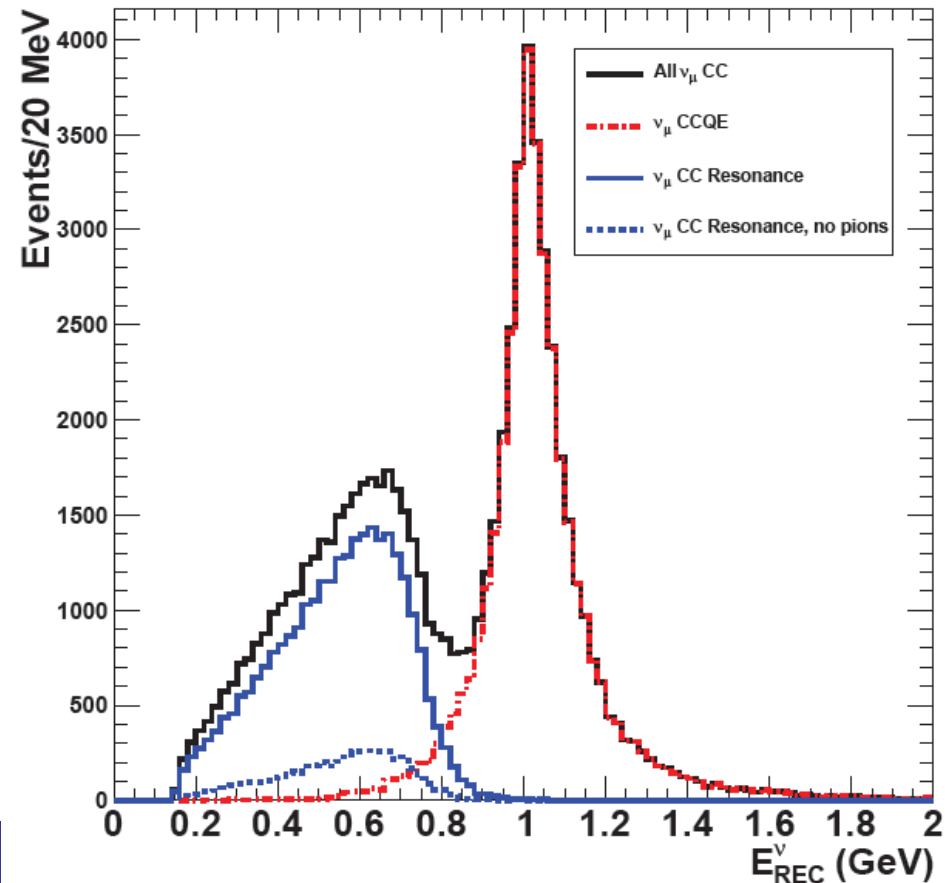
- CCQE-like events from

- absorbed pions

- 2p2h

- ...

GENIE  $E_\nu = 1$  GeV



# Relevance for oscillation experiments

- (Calorimetric)  $E_\nu$  reconstruction (e.g. MINOS)

- $E_\nu = E_{\text{lep}} + E_{\text{had}}$

but

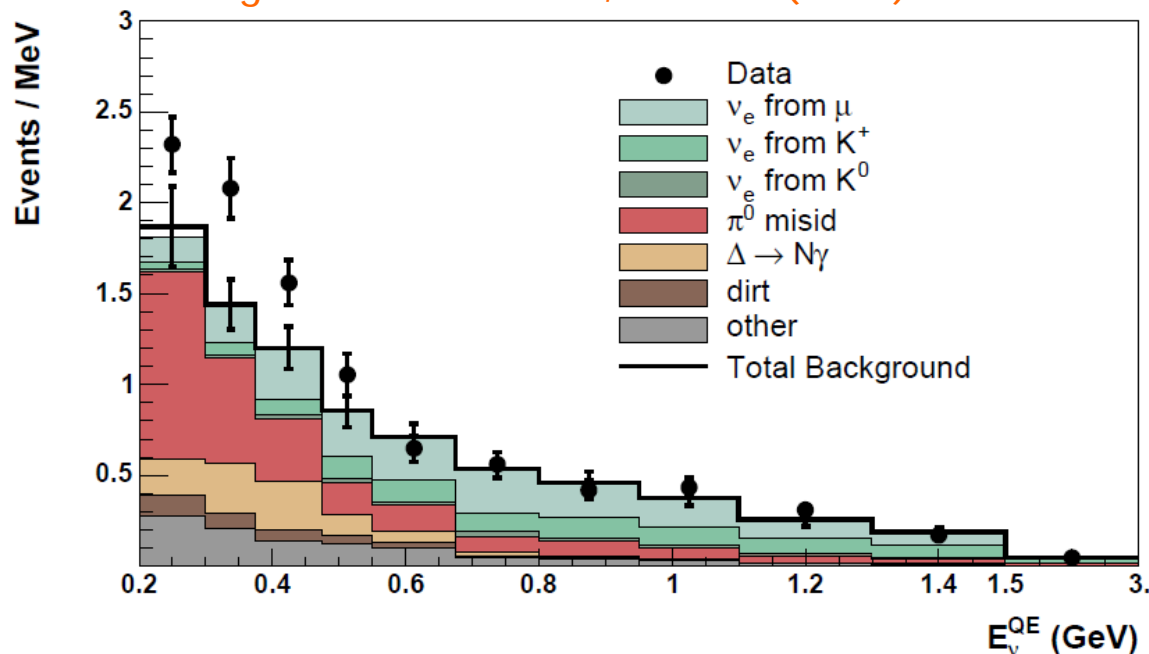
- There are invisible heavy fragments, neutrons or other undetected particles:  $E_{\text{vis}} < E_{\text{had}}$
  - $E_{\text{vis}} \rightarrow E_{\text{had}}$  relies on the simulation

# Relevance for oscillation experiments

## ■ Backgrounds

- E.g. in the MiniBooNE  $\nu_\mu \rightarrow \nu_e$  search

Aguilar-Arevalo et al., PRL102 (2009) 101802



- NC backgrounds:  $\nu_l N \rightarrow \nu_l \pi^0 N'$   
 $\nu_l N \rightarrow \nu_l \gamma N'$

- Also important for  $\nu_\mu \rightarrow \nu_e$  measurements at T2K



# Electroweak interactions in the SM

- Spontaneously broken SU(2) x U(1) gauge symmetry

$$\mathcal{L}_{EW} = -e J_{em}^\mu A_\mu - \frac{g}{2 \cos \theta_W} J_{nc}^\mu Z_\mu - \frac{g}{2\sqrt{2}} J_{cc}^\mu W_\mu^\dagger + h.c.$$

$$\sin \theta_W = \frac{e}{g} \quad \cos \theta_W = \frac{M_W}{M_Z} \quad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

in the leptonic sector:

$$J_{em}^\mu = \bar{l}_i \gamma^\mu l_i \quad i = e, \mu, \tau$$

$$J_{cc}^\mu = \bar{\nu}_i \gamma^\mu (1 - \gamma_5) l_i$$

$$J_{nc}^\mu = \frac{1}{2} \bar{l}_i \gamma^\mu (g_V - g_A \gamma_5) l_i + \frac{1}{2} \bar{\nu}_i \gamma^\mu (1 - \gamma_5) \nu_i$$

$$g_V = -1 + 4 \sin^2 \theta_W, \quad g_A = -1$$

$$|g_V| \approx 0.04 \ll |g_A|$$

# Electroweak interactions in the SM

- Spontaneously broken SU(2) x U(1) gauge symmetry

$$\mathcal{L}_{EW} = -e J_{em}^\mu A_\mu - \frac{g}{2 \cos \theta_W} J_{nc}^\mu Z_\mu - \frac{g}{2\sqrt{2}} J_{cc}^\mu W_\mu^\dagger + h.c.$$

$$\sin \theta_W = \frac{e}{g} \quad \cos \theta_W = \frac{M_W}{M_Z} \quad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

in the **quark** sector:

$$J_{em}^\mu = Q_i \bar{q}_i \gamma^\mu q_i = \frac{2}{3} \bar{q}_u \gamma^\mu q_u - \frac{1}{3} (\bar{q}_d \gamma^\mu q_d + \bar{q}_s \gamma^\mu q_s) + \dots$$

$$J_{nc}^\mu = \bar{q}_u \gamma^\mu \left[ \frac{1}{2} - \left( \frac{2}{3} \right) 2 \sin^2 \theta_W - \frac{1}{2} \gamma_5 \right] q_u + (u \rightarrow c) + (u \rightarrow t) \\ + \bar{q}_d \gamma^\mu \left[ -\frac{1}{2} - \left( -\frac{1}{3} \right) 2 \sin^2 \theta_W + \frac{1}{2} \gamma_5 \right] q_d + (d \rightarrow s) + (d \rightarrow b)$$

# Electroweak interactions in the SM

- Spontaneously broken SU(2) x U(1) gauge symmetry

$$\mathcal{L}_{EW} = -e J_{em}^\mu A_\mu - \frac{g}{2 \cos \theta_W} J_{nc}^\mu Z_\mu - \frac{g}{2\sqrt{2}} J_{cc}^\mu W_\mu^\dagger + h.c.$$

$$\sin \theta_W = \frac{e}{g} \quad \cos \theta_W = \frac{M_W}{M_Z} \quad \frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}$$

in the **quark** sector:

$$J_{cc}^\mu = (\bar{q}_u \bar{q}_c \bar{q}_t) \gamma^\mu (1 - \gamma_5) U \begin{pmatrix} q_d \\ q_s \\ q_b \end{pmatrix} \quad U \leftarrow \text{CKM matrix}$$

$$U \approx \begin{pmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{pmatrix} \quad \theta_C \approx 13 \text{ deg} \leftarrow \text{Cabibbo angle}$$

$$W^- p(ud) \rightarrow n(ud) \quad \sim \cos^2 \theta_C$$

$$\left. \begin{array}{l} W^- p(ud) \rightarrow \Lambda(ud) \\ W^- p(ud) \rightarrow p(ud) K^-(\bar{u}s) \end{array} \right\} \sim \sin^2 \theta_C$$

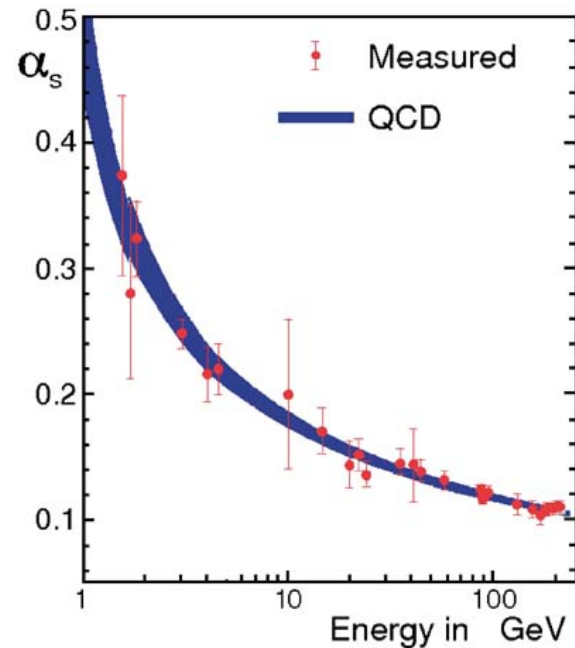
# Strong interactions in the SM

- SU(3) (color) gauge symmetry: QCD

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}_q (i\gamma^\mu D_\mu - m_q) \psi_q - \frac{1}{4} G_a^{\mu\nu} G_{a\mu\nu} \quad q = u, d, s, \dots \quad a = 1 - 8$$

$$D_\mu \psi = \left( \partial_\mu - ig \frac{\lambda_a}{2} A_\mu^a \right) \psi \quad G_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + gf_{abc} A_b^\mu A_c^\nu$$

- Asymptotically free  $\Rightarrow$  perturbative at high energies
- Nonperturbative at low energies
- Confining



$$\alpha_s = \frac{g^2}{4\pi}$$

# Strong interactions in the SM

- Approximate symmetries of  $N_f = 3$  QCD

- $m_u = m_d = m_s \Leftrightarrow$  Global  $SU(3)_{\text{flavor}}$  symmetry

$$V_a^\mu = \bar{q} \gamma^\mu \frac{\lambda_a}{2} q \Leftrightarrow \partial_\mu V_a^\mu = 0 \quad a = 1 - 8 \quad \bar{q} = (\bar{q}_u \bar{q}_d \bar{q}_s) \quad q = \begin{pmatrix} q_u \\ q_d \\ q_s \end{pmatrix}$$

$$m_u (1 \text{ GeV}) = 4 \pm 2 \text{ MeV}$$

$$m_d (1 \text{ GeV}) = 8 \pm 4 \text{ MeV}$$

$$m_s (1 \text{ GeV}) = 164 \pm 33 \text{ MeV}$$

$$\partial_\mu V_a^\mu = \bar{q} \left[ m, \frac{\lambda_a}{2} \right] q \quad m = \text{diag}(m_u, m_d, m_s)$$

- $m_u = m_d \Leftrightarrow$  Global  $SU(2)_{\text{flavor}}$  isospin symmetry

$$V_a^\mu = \bar{q} \gamma^\mu \frac{\lambda_a}{2} q = \bar{q}' \gamma^\mu \frac{\tau_a}{2} q' \Leftrightarrow \partial_\mu V_a^\mu = 0 \quad a = 1 - 3 \quad q' = \begin{pmatrix} q_u \\ q_d \end{pmatrix}$$

# Strong interactions in the SM

## Gell-Mann matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ +i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ +i & 0 & 0 \end{pmatrix},$$
$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & +i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

# Strong interactions in the SM

- Approximate symmetries of  $N_f = 3$  QCD

- $m_u = m_d = m_s \Leftrightarrow$  Global  $SU(3)_{\text{flavor}}$  symmetry

$$V_a^\mu = \bar{q} \gamma^\mu \frac{\lambda_a}{2} q \Leftrightarrow \partial_\mu V_a^\mu = 0 \quad a = 1 - 8 \quad \bar{q} = (\bar{q}_u \bar{q}_d \bar{q}_s) \quad q = \begin{pmatrix} q_u \\ q_d \\ q_s \end{pmatrix}$$

$$m_u (1 \text{ GeV}) = 4 \pm 2 \text{ MeV}$$

$$m_d (1 \text{ GeV}) = 8 \pm 4 \text{ MeV}$$

$$m_s (1 \text{ GeV}) = 164 \pm 33 \text{ MeV}$$

$$\partial_\mu V_a^\mu = \bar{q} \left[ m, \frac{\lambda_a}{2} \right] q \quad m = \text{diag}(m_u, m_d, m_s)$$

- $m_u = m_d \Leftrightarrow$  Global  $SU(2)_{\text{flavor}}$  isospin symmetry

$$V_a^\mu = \bar{q} \gamma^\mu \frac{\lambda_a}{2} q = \bar{q}' \gamma^\mu \frac{\tau_a}{2} q' \Leftrightarrow \partial_\mu V_a^\mu = 0 \quad a = 1 - 3 \quad q' = \begin{pmatrix} q_u \\ q_d \end{pmatrix}$$

# Strong interactions in the SM

- **Flavor** structure of the EW quark currents:

$$\mathcal{L}_{EW} = -eJ_{em}^\mu A_\mu - \frac{g}{2\cos\theta_W} J_{nc}^\mu Z_\mu - \frac{g}{2\sqrt{2}} J_{cc}^\mu W_\mu^\dagger + h.c.$$

$$\begin{aligned} J_{em}^\mu &= \frac{2}{3}\bar{q}_u\gamma^\mu q_u - \frac{1}{3}(\bar{q}_d\gamma^\mu q_d + \bar{q}_s\gamma^\mu q_s) = \bar{q}Q\gamma^\mu q \quad Q = \text{diag}\left(\frac{2}{3} \quad -\frac{1}{3} \quad -\frac{1}{3}\right) \\ &= \frac{1}{2}\bar{q}\frac{\lambda_8}{\sqrt{3}}\gamma^\mu q + \bar{q}\frac{\lambda_3}{2}\gamma^\mu q = \frac{1}{2}V_Y^\mu + V_3^\mu \end{aligned}$$

$$J_{cc}^\mu = \bar{q}_u\gamma^\mu(1 - \gamma_5)(q_d \cos\theta_C + q_s \sin\theta_C)$$

$$V_+^\mu = \bar{q}_u\gamma^\mu q_d = \bar{q}\gamma^\mu \frac{\lambda_1 + i\lambda_2}{2} q = V_1^\mu + iV_2^\mu$$

$V_{1,2,3}$ : components of the same conserved flavor vector current

$$J_{nc}^\mu = \bar{q}_u\gamma^\mu \left[ \frac{1}{2} - \left(\frac{2}{3}\right) 2\sin^2\theta_W - \frac{1}{2}\gamma_5 \right] q_u + \bar{q}_d\gamma^\mu \left[ -\frac{1}{2} - \left(-\frac{1}{3}\right) 2\sin^2\theta_W + \frac{1}{2}\gamma_5 \right] q_d + (d \rightarrow s)$$

$$V_{nc}^\mu = (1 - 2\sin^2\theta_W)V_3^\mu - 2\sin^2\theta_W \frac{1}{2}V_Y^\mu - \frac{1}{2}\bar{q}_s\gamma^\mu q_s$$



# Strong interactions in the SM

- Approximate symmetries of  $N_f = 3$  QCD

- $m_u = m_d = m_s = 0 \Leftrightarrow$  Chiral  $SU(3)_L \times SU(3)_R$  symmetry

$$m_u (1 \text{ GeV}) = 4 \pm 2 \text{ MeV}$$

$$m_d (1 \text{ GeV}) = 8 \pm 4 \text{ MeV}$$

$$m_s (1 \text{ GeV}) = 164 \pm 33 \text{ MeV}$$

$\ll$  rho meson, nucleon mass

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}_{qL} i \gamma^\mu D_\mu \psi_{qL} + \bar{\psi}_{qR} i \gamma^\mu D_\mu \psi_{qR} - m_q (\bar{\psi}_{qL} \psi_{qR} + \bar{\psi}_{qR} \psi_{qL}) + \dots$$

- Conserved currents:

$$R_a^\mu = \bar{q}_R \gamma^\mu \frac{\lambda_a}{2} q_R$$

$$V_a^\mu = R_a^\mu + L_a^\mu = \bar{q} \gamma^\mu \frac{\lambda_a}{2} q$$

$$q = \begin{pmatrix} q_u \\ q_d \\ q_s \end{pmatrix}$$

$$L_a^\mu = \bar{q}_L \gamma^\mu \frac{\lambda_a}{2} q_L$$

$$A_a^\mu = R_a^\mu - L_a^\mu = \bar{q} \gamma^\mu \gamma_5 \frac{\lambda_a}{2} q$$

$$\bar{q} = (\bar{q}_u \bar{q}_d \bar{q}_s)$$

- Explicit chiral symmetry breaking:

$$\partial_\mu V_a^\mu = \bar{q} \left[ m, \frac{\lambda_a}{2} \right] q$$

$$\partial_\mu A_a^\mu = i \bar{q} \left\{ m, \frac{\lambda_a}{2} \right\} \gamma_5 q \leftarrow \text{PCAC}$$

# Electroweak nucleon current

- **Flavor** structure of the quark currents:

$$\mathcal{L}_{\text{EW}} = -e J_{em}^\mu A_\mu - \frac{g}{2 \cos \theta_W} J_{nc}^\mu Z_\mu - \frac{g}{2\sqrt{2}} J_{cc}^\mu W_\mu^\dagger + h.c.$$

$$J_{cc}^\mu = \bar{q}_u \gamma^\mu (1 - \gamma_5) (q_d \cos \theta_C + q_s \sin \theta_C)$$

$$A_+^\mu = \bar{q}_u \gamma^\mu \gamma_5 q_d = \bar{q}_u \gamma^\mu \gamma_5 \frac{\lambda_1 + i\lambda_2}{2} q_d = A_1^\mu + iA_2^\mu$$

$$J_{nc}^\mu = \bar{q}_u \gamma^\mu \left[ \frac{1}{2} - \left( \frac{2}{3} \right) 2 \sin^2 \theta_W - \frac{1}{2} \gamma_5 \right] q_u + \bar{q}_d \gamma^\mu \left[ -\frac{1}{2} - \left( -\frac{1}{3} \right) 2 \sin^2 \theta_W + \frac{1}{2} \gamma_5 \right] q_d + (d \rightarrow s)$$

$$A_{nc}^\mu = A_3^\mu + \frac{1}{2} \bar{q}_s \gamma^\mu \gamma_5 q_s$$

$A_{1,2,3}$  : components of the same **partially** conserved flavor axial current

# Strong interactions in the SM

- Approximate symmetries of  $N_f = 3$  QCD

- $m_u = m_d = m_s = 0 \Leftrightarrow$  Chiral  $SU(3)_L \times SU(3)_R$  symmetry

- **Explicit** chiral symmetry **breaking**:

$$\partial_\mu V_a^\mu = \bar{q} \left[ m, \frac{\lambda_a}{2} \right] q \quad \partial_\mu A_a^\mu = i\bar{q} \left\{ m, \frac{\lambda_a}{2} \right\} \gamma_5 q \quad \leftarrow \text{PCAC}$$

- **Spontaneous** chiral symmetry **breaking**:

- the ground state does **not** have the chiral symmetry of the Lagrangian

- $m_\rho = 770 \text{ MeV } (1^-) \neq m_{a_1} = 1230 \text{ MeV } (1^+)$

- $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$

- Goldstone bosons:  $\pi, K, \eta$

$$\mathcal{L}_{\text{QCD}} \rightarrow \mathcal{L}_{\text{effective}}(\pi, K, \eta, \dots)$$

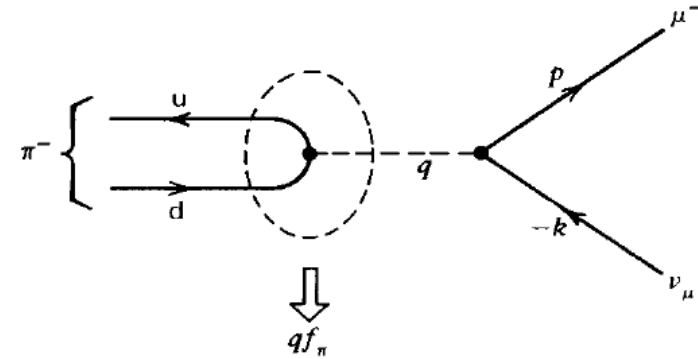
- In terms of hadronic degrees of freedom:  $A_a^\mu = -f_\pi \partial^\mu \pi_a + \dots$

$$\langle 0 | A_+^\mu | \pi^- \rangle = \sqrt{2} f_\pi i q^\mu$$

# Electroweak interactions in the SM

- Example:  $\pi^- (q) \rightarrow \mu^- (p) + \bar{\nu}_\mu (k)$

$$\mathcal{L}_{\text{EW}} = -\frac{g}{2\sqrt{2}} J_{cc}^\mu W_\mu^+ + h.c.$$



$$(-i)\mathcal{M} = (-i) \langle \mu^- \bar{\nu}_\mu | \left( -\frac{g}{2\sqrt{2}} \right) J_{cc}^\mu | 0 \rangle (-i) D_{\mu\nu}(q) (-i) \langle 0 | \left( -\frac{g}{2\sqrt{2}} \right) J_{cc}^\nu | \pi^- \rangle$$

$$D_{\mu\nu} = \frac{1}{q^2 - M_W^2} \left( g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2} \right) \approx -\frac{g_{\mu\nu}}{M_W^2} \quad q^2 = m_\pi^2 \ll M_W^2$$

$$\langle \mu^- \bar{\nu}_\mu | J_{cc}^\mu | 0 \rangle = \bar{u}(p) \gamma_\mu (1 - \gamma_5) v(k)$$

$$\langle 0 | J_{cc}^\nu | \pi^- \rangle = \bar{v}_u \gamma^\nu (1 - \gamma_5) u_d = \sqrt{2} f_\pi i q^\nu$$

$$\left( \frac{g}{2\sqrt{2}} \right)^2 \frac{1}{M_W^2} = \frac{G_F}{\sqrt{2}}$$

# Electroweak interactions in the SM

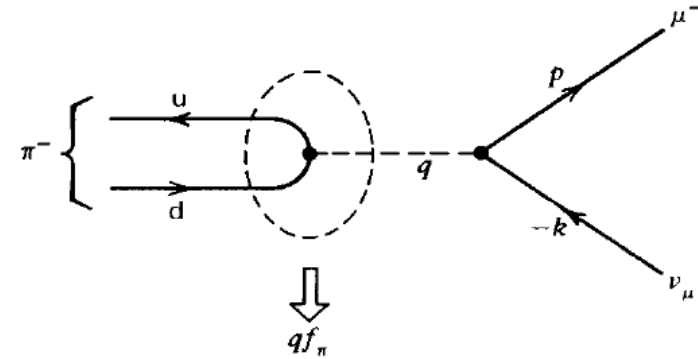
- Example:  $\pi^- (q) \rightarrow \mu^- (p) + \bar{\nu}_\mu (k)$

$$\overline{|\mathcal{M}|^2} = \overline{\sum_{\text{polar.}} |\mathcal{M}|^2} = 4G_F^2 L_{\mu\nu} H^{\mu\nu}$$

$$\text{Tr} [(\not{p} + m_\mu) \gamma_\mu (1 - \gamma_5) \not{k} \gamma_\nu (1 - \gamma_5)] = 8L_{\mu\nu}$$

$$L_{\mu\nu} = p_\mu k_\nu + p_\nu k_\mu - g_{\mu\nu} k \cdot p + i\epsilon_{\mu\nu\alpha\beta} p^\alpha k^\beta$$

$$H^{\mu\nu} = 2f_\pi^2 q^\mu q^\nu$$

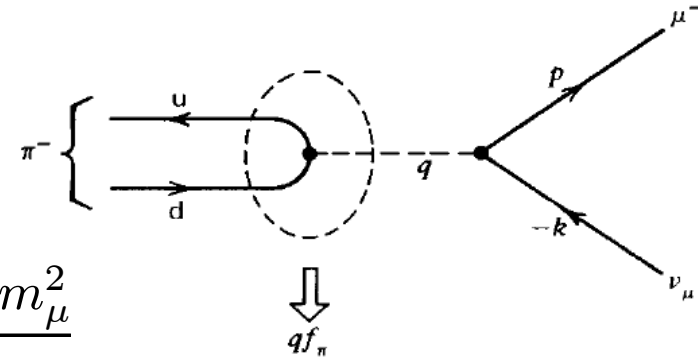


# Electroweak interactions in the SM

- Example:  $\pi^- (q) \rightarrow \mu^- (p) + \bar{\nu}_\mu (k)$

$$|\overline{\mathcal{M}}|^2 = 8G_F^2 f_\pi^2 m_\mu^2 (p \cdot k)$$

$$q^2 = (p + k)^2 = m_\mu^2 + 2(p \cdot k) \Rightarrow (p \cdot k) = \frac{m_\pi^2 - m_\mu^2}{2}$$



Decay width, in the  $\pi$  rest frame:

$$\Gamma = \frac{1}{2m_\pi} \int \frac{d^3 p}{2p^0 (2\pi)^3} \frac{d^3 k}{2k^0 (2\pi)^3} (2\pi)^4 \delta^4(k + p - q) |\overline{\mathcal{M}}|^2$$

$$\Gamma = \frac{G_F^2}{4\pi} f_\pi^2 m_\pi m_\mu^2 \left( 1 - \frac{m_\mu^2}{m_\pi^2} \right)^2$$

$$\tau = \frac{1}{\Gamma} = 2.6 \cdot 10^{-8} \text{ s} \Rightarrow f_\pi = 92.4 \text{ MeV}$$

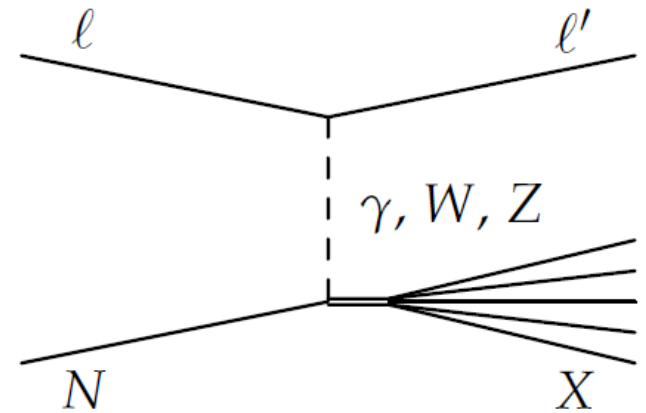
# Inclusive cross section

$$l(k) + N(p) \rightarrow l'(k') + X(p')$$

$$k = (k_0, \vec{k}) \quad p = (E, \vec{p})$$

$$k' = (k'_0, \vec{k}') \quad p' = (E', \vec{p}')$$

$$q = k - k' = p' - p = (\omega, \vec{q}) \quad q^2 = -Q^2 < 0$$



In Lab:  $p = (M, \vec{0})$

For CC: 
$$\frac{d\sigma}{dk'_0 d\Omega(\vec{k}')} = \frac{G_F^2}{(2\pi)^2} \frac{|\vec{k}'|}{k_0} L_{\mu\nu} W^{\mu\nu}$$

$$L_{\mu\nu} = k'_\mu k'_\nu + k'_\nu k'_\mu - g_{\mu\nu} k \cdot k' + i\epsilon_{\mu\nu\alpha\beta} k'^\alpha k^\beta$$

$$W^{\mu\nu} = \frac{1}{2M} \sum_{\text{polar.}} \sum_i \left( \int \frac{d^3 p_i}{2E'_i (2\pi)^3} \right) (2\pi)^3 \delta^4(k' + p' - k - p) \langle X | J^\mu | N \rangle \langle X | J^\nu | N \rangle^*$$

For EM:  $L_{\mu\nu} \rightarrow L_{\mu\nu}^{(\text{sym})} \quad \frac{G_F^2}{(2\pi)^2} \rightarrow \frac{\alpha^2}{q^4}$

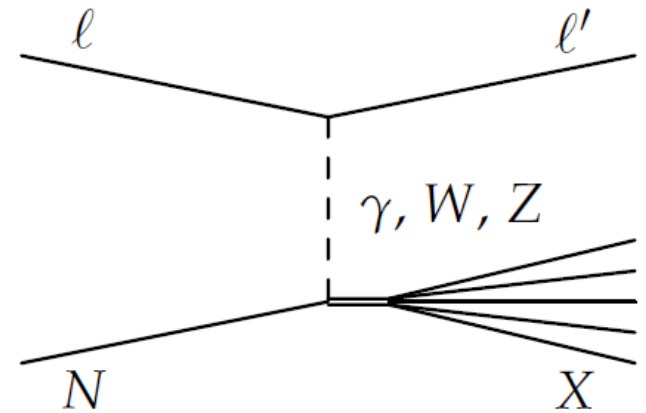
# Inclusive cross section

$$l(k) + N(p) \rightarrow l'(k') + X(p')$$

$$k = (k_0, \vec{k}) \quad p = (E, \vec{p})$$

$$k' = (k'_0, \vec{k}') \quad p' = (E', \vec{p}')$$

$$q = k - k' = p' - p = (\omega, \vec{q}) \quad q^2 = -Q^2 < 0$$



General structure of the **hadronic** tensor:

Ingredients:  $g^{\mu\nu}$ ,  $q^\mu$ ,  $p^\mu$ ,  $\epsilon^{\alpha\beta\mu\nu}$   $p' = p + q$  ← not independent

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + W_2 \frac{p^\mu p^\nu}{M^2} + W_4 \frac{q^\mu q^\nu}{M^2} + W_5 \frac{p^\mu q^\nu + q^\mu p^\nu}{M^2} \\ + W_3 i \epsilon^{\mu\nu\alpha\beta} \frac{p_\alpha q_\beta}{2M^2} + W_6 \frac{p^\mu q^\nu - q^\mu p^\nu}{M^2}$$

Structure functions:  $W_i = W_i(p^2 = M^2, q \cdot p = \omega M, q^2) = W_i(\omega, q^2)$



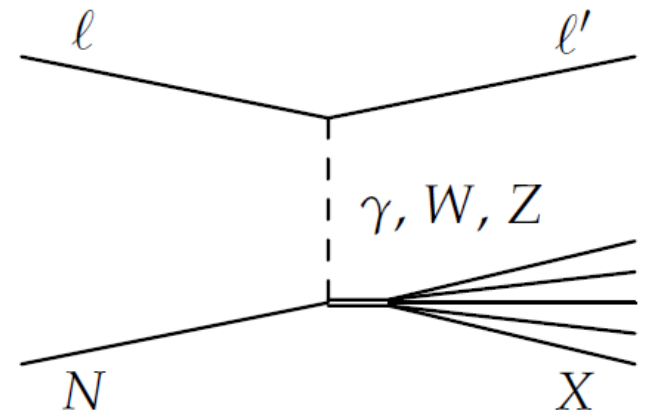
# Inclusive cross section

$$l(k) + N(p) \rightarrow l'(k') + X(p')$$

$$k = (k_0, \vec{k}) \quad p = (E, \vec{p})$$

$$k' = (k'_0, \vec{k}') \quad p' = (E', \vec{p}')$$

$$q = k - k' = p' - p = (\omega, \vec{q}) \quad q^2 = -Q^2 < 0$$



General structure of the **hadronic** tensor:

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + W_2 \frac{p^\mu p^\nu}{M^2} + W_4 \frac{q^\mu q^\nu}{M^2} + W_5 \frac{p^\mu q^\nu + q^\mu p^\nu}{M^2} \\ + W_3 i\epsilon^{\mu\nu\alpha\beta} \frac{p_\alpha q_\beta}{2M^2} + W_6 \frac{p^\mu q^\nu - q^\mu p^\nu}{M^2}$$

Structure functions:  $W_i = W_i(\omega, q^2)$

For EM interactions:  $q_\mu J^\mu = 0 \Rightarrow q_\mu W_{em}^{\mu\nu} = W_{em}^{\mu\nu} q_\nu = 0$

$$W_{em}^{\mu\nu} = W_1 \left( \frac{q^\mu q^\nu}{q^2} - g^{\mu\nu} \right) + \frac{W_2}{M^2} \left( p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left( p^\nu - \frac{p \cdot q}{q^2} q^\nu \right)$$

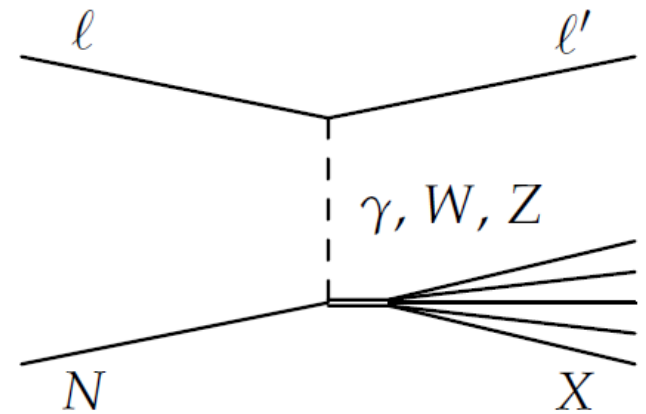
# Inclusive cross section

$$l(k) + N(p) \rightarrow l'(k') + X(p')$$

$$k = (k_0, \vec{k}) \quad p = (E, \vec{p})$$

$$k' = (k'_0, \vec{k}') \quad p' = (E', \vec{p}')$$

$$q = k - k' = p' - p = (\omega, \vec{q}) \quad q^2 = -Q^2 < 0$$



In Lab:  $p = (M, \vec{0})$

$$\frac{d\sigma}{dk'_0 d\Omega(\vec{k}')} = \frac{G_F^2}{(2\pi)^2} \frac{|\vec{k}'|}{k_0} \left\{ W_1 2k \cdot k' + W_2 (2k'_0 k_0 - k \cdot k') \right. \\ \left. + 2 \frac{m_l^2}{M^2} [W_4 k \cdot k' - W_5 M k_0] + \frac{W_3}{M} [(k_0 + k'_0) k \cdot k' - k_0 m_l^2] \right\}$$

$$m_l \rightarrow 0$$

$$\frac{d\sigma}{dk'_0 d\Omega(\vec{k}')} = \frac{G_F^2}{2\pi^2} (k'_0)^2 \left[ W_1 2 \sin^2 \frac{\theta'}{2} + W_2 \cos^2 \frac{\theta'}{2} \pm W_3 \frac{(k_0 + k'_0)}{M} \sin^2 \frac{\theta'}{2} \right]$$

# Inclusive cross section

- Example: EM scattering on a point-like spin 1/2 particle

$$\langle X | J^\mu | N \rangle \rightarrow \bar{u}(p') \gamma^\mu u(p)$$

$$W^{\mu\nu} = \frac{1}{2M} \int \frac{d^3 p'}{2E'} \delta^4(k' + p' - k - p) 4H^{\mu\nu}$$

$$H^{\mu\nu} = p'^\mu p^\nu + p'^\nu p^\mu - g^{\mu\nu} (p \cdot p' - M^2)$$

Using that:

$$\delta(E' + k'_0 - M - k_0) = \frac{E'}{M} \delta\left(k'_0 - k_0 - \frac{q^2}{2M}\right)$$

$$q^2 = (p' - p)^2 = 2M^2 - 2p \cdot p' \Rightarrow p \cdot p' - M^2 = -\frac{q^2}{2}$$

$$p'^2 = M^2 = (q + p)^2 = q^2 + 2(p \cdot q) + M^2 \Rightarrow \frac{p \cdot q}{q^2} = \frac{M\omega}{q^2} = -\frac{1}{2}$$

# Inclusive cross section

- Example: EM scattering on a point-like spin 1/2 particle

$$\langle X | J^\mu | N \rangle \rightarrow \bar{u}(p') \gamma^\mu u(p)$$

$$W^{\mu\nu} = \frac{1}{2M} \int \frac{d^3 p'}{2E'} \delta^4(k' + p' - k - p) 4H^{\mu\nu}$$

$$H^{\mu\nu} = p'^\mu p^\nu + p'^\nu p^\mu - g^{\mu\nu} (p \cdot p' - M^2)$$

one finds:

$$\begin{aligned} W_1 &= -\frac{q^2}{4M^2\omega} \delta\left(1 + \frac{q^2}{2M\omega}\right) \\ W_2 &= \frac{1}{\omega} \delta\left(1 + \frac{q^2}{2M\omega}\right) \end{aligned} \quad \Leftrightarrow \quad \begin{aligned} 2MW_1 &\equiv F_1 = x\delta(1-x) = F_1(x) \\ \omega W_2 &\equiv \delta(1-x) = F_2(x) \\ x &= -\frac{q^2}{2M\omega} \end{aligned}$$

# Inclusive cross section

- Example: EM scattering on a point-like spin 1/2 particle

$$\langle X | J^\mu | N \rangle \rightarrow \bar{u}(p') \gamma^\mu u(p)$$

$$W^{\mu\nu} = \frac{1}{2M} \int \frac{d^3 p'}{2E'} \delta^4(k' + p' - k - p) 4H^{\mu\nu}$$

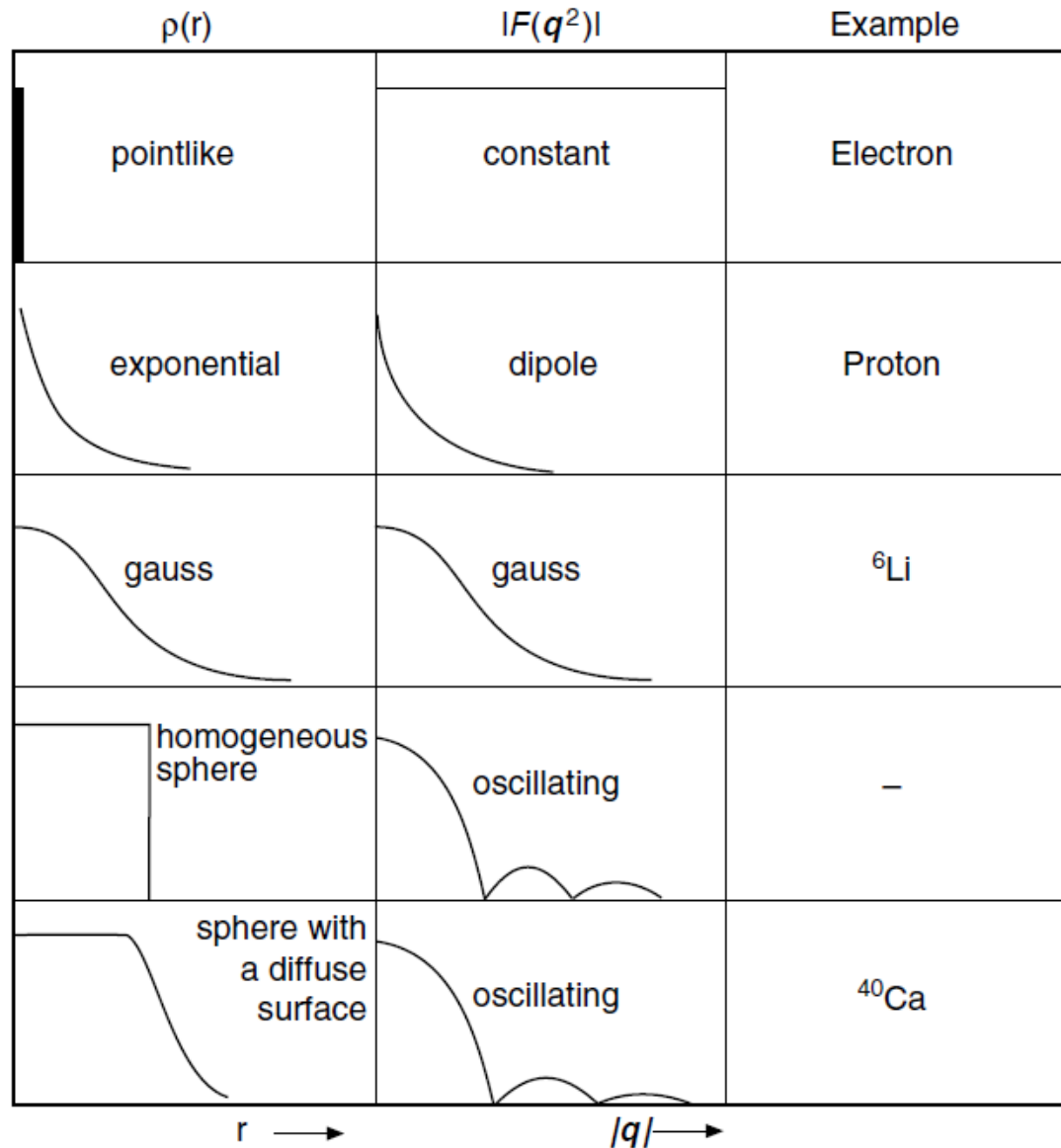
$$H^{\mu\nu} = p'^\mu p^\nu + p'^\nu p^\mu - g^{\mu\nu} (p \cdot p' - M^2)$$

for real nucleons, at low  $\vec{q}^2$

$$W_1 = -\frac{q^2}{4M^2\omega} \delta \left( 1 + \frac{q^2}{2M\omega} \right) \rightarrow -\frac{q^2}{4M^2\omega} G^2(q^2) \delta \left( 1 + \frac{q^2}{2M\omega} \right)$$

$$W_2 = \frac{1}{\omega} \delta \left( 1 + \frac{q^2}{2M\omega} \right) \rightarrow \frac{1}{\omega} G^2(q^2) \delta \left( 1 + \frac{q^2}{2M\omega} \right)$$

# Form factors



# QE scattering on the nucleon

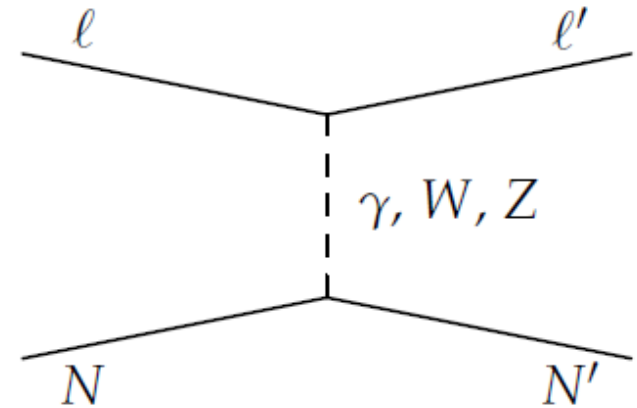
$$\text{EM} : l^\pm(k) + N(p) \rightarrow l^\pm(k') + N(p')$$

$$\text{CC} : \nu(k) + n(p) \rightarrow l^-(k') + p(p')$$

$$\bar{\nu}(k) + p(p) \rightarrow l^+(k') + n(p')$$

$$\text{NC} : \nu(k) + N(p) \rightarrow \nu(k') + N(p')$$

$$\bar{\nu}(k) + N(p) \rightarrow \bar{\nu}(k') + N(p')$$



## ■ QE kinematics:

$$(q + p)^2 = (p')^2$$

$$q^2 + 2M\omega + M^2 = M^2$$

$$\omega = -\frac{q^2}{2M}$$

$$x = 1$$

# QE scattering on the nucleon

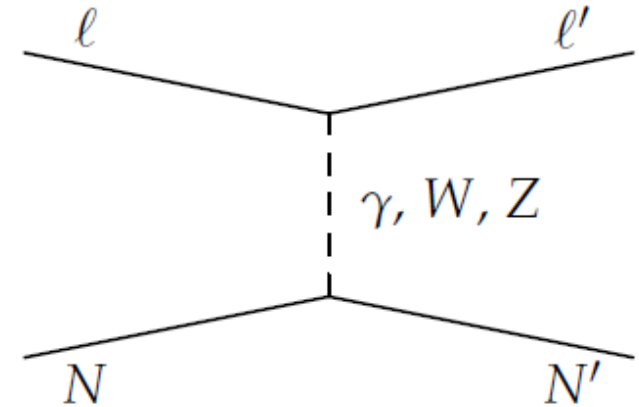
$$\text{EM} : l^\pm(k) + N(p) \rightarrow l^\pm(k') + N(p')$$

$$\text{CC} : \nu(k) + n(p) \rightarrow l^-(k') + p(p')$$

$$\bar{\nu}(k) + p(p) \rightarrow l^+(k') + n(p')$$

$$\text{NC} : \nu(k) + N(p) \rightarrow \nu(k') + N(p')$$

$$\bar{\nu}(k) + N(p) \rightarrow \bar{\nu}(k') + N(p')$$



## ■ Cross section:

$$\frac{d\sigma}{dk'_0 d\Omega(\vec{k}')} = \frac{G_F^2}{(2\pi)^2} \frac{|\vec{k}'|}{k_0} L_{\mu\nu} W^{\mu\nu} \quad \text{For EM: } L_{\mu\nu} \rightarrow L_{\mu\nu}^{(\text{sym})} \quad \frac{G_F^2}{(2\pi)^2} \rightarrow \frac{\alpha^2}{q^4}$$

$$W^{\mu\nu} = \frac{1}{2M} \int \frac{d^3 p'}{2E'} \delta^4(k' + p' - k - p) H^{\nu\mu}$$

$$H^{\alpha\beta} = \text{Tr} \left[ (\not{p} + M) \gamma^0 (\Gamma^\alpha)^\dagger \gamma^0 (\not{p}' + M) \Gamma^\beta \right]$$

$$\langle N' | J^\mu | N \rangle = \bar{u}(p') \Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$



# Electroweak nucleon current

$$\langle N' | J^\mu | N \rangle = \bar{u}(p') \Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

$$\mathcal{V}^\mu = \bar{u}(p') \left[ \gamma^\mu F_1 + \frac{i}{2M} \sigma^{\mu\nu} q_\nu F_2 \right] u(p)$$

$$\mathcal{A}^\mu = \bar{u}(p') \left[ \gamma^\mu \gamma_5 F_A + \frac{q^\mu}{M} \gamma_5 F_P \right] u(p)$$

- $F_1$  ← Dirac form factor
- $F_2$  ← Pauli form factor
- $F_A$  ← axial form factor
- $F_P$  ← pseudoscalar form factor

# Electroweak nucleon current

$$\langle N' | J^\mu | N \rangle = \bar{u}(p') \Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

■  $\Gamma^\mu \leftarrow$  4-vector constructed from:

(1)  $p^\mu, p'^\mu$

(2)  $\epsilon_{\alpha\beta\mu\nu}, g^{\mu\nu}$

(3)  $\left\{ \gamma_\mu, \gamma_5, \gamma_\mu \gamma_5, \sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu] \right\}$

# Electroweak nucleon current

$$\langle N' | J^\mu | N \rangle = \bar{u}(p') \Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

■  $\Gamma^\mu \leftarrow$  4-vector constructed from:

(1)  $p^\mu, p'^\mu$

(2)  $\epsilon_{\alpha\beta\mu\nu}, g^{\mu\nu}$

(3)  $\left\{ \gamma_\mu, \gamma_5, \gamma_\mu \gamma_5, \sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu] \right\}$

Any other combination of  $\gamma$  matrices can be reduced to (3).

For example:

$$\gamma_\mu \gamma_\nu = g_{\mu\nu} - i \sigma_{\mu\nu}$$

$$\gamma^\alpha \sigma^{\mu\nu} = i (g^{\alpha\mu} g^{\beta\nu} - g^{\alpha\nu} g^{\beta\mu}) \gamma_\beta + \epsilon^{\alpha\mu\nu\beta} \gamma_5 \gamma_\beta$$

# Electroweak nucleon current

$$\langle N' | J^\mu | N \rangle = \bar{u}(p') \Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

■  $\Gamma^\mu \leftarrow$  4-vector constructed from:

(1)  $p^\mu, p'^\mu$

(2)  $\epsilon_{\alpha\beta\mu\nu}, g^{\mu\nu}$

(3)  $\left\{ \gamma_\mu, \gamma_5, \gamma_\mu \gamma_5, \sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu] \right\}$

Using:

■ Dirac algebra

■ Dirac equation:  $(\not{p} - M)u(p) = \bar{u}(p')(\not{p}' - M) = 0$

one finds:

$$\Gamma^\mu = \gamma^\mu F_1 + \frac{i}{2M} \sigma^{\mu\nu} q_\nu F_2 + \frac{q^\mu}{M} F_S - \gamma^\mu \gamma_5 F_A - \frac{i}{2M} \sigma^{\mu\nu} q_\nu \gamma_5 F_T - \frac{q^\mu}{M} \gamma_5 F_P$$

# Electroweak nucleon current

$$\langle N' | J^\mu | N \rangle = \bar{u}(p') \Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

$$\Gamma^\mu = \gamma^\mu F_1 + \frac{i}{2M} \sigma^{\mu\nu} q_\nu F_2 + \frac{q^\mu}{M} F_S - \gamma^\mu \gamma_5 F_A - \frac{i}{2M} \sigma^{\mu\nu} q_\nu \gamma_5 F_T - \frac{q^\mu}{M} \gamma_5 F_P$$

■  $F_i = F_i(q^2) \Leftrightarrow 2 p \cdot q + q^2 = 0$

# Electroweak nucleon current

$$\langle N' | J^\mu | N \rangle = \bar{u}(p') \Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

$$\Gamma^\mu = \gamma^\mu F_1 + \frac{i}{2M} \sigma^{\mu\nu} q_\nu F_2 + \frac{q^\mu}{M} F_S - \gamma^\mu \gamma_5 F_A - \frac{i}{2M} \sigma^{\mu\nu} q_\nu \gamma_5 F_T - \frac{q^\mu}{M} \gamma_5 F_P$$

■ Time reversal (**T**) transformation:

$$T (\bar{u} \Gamma^\mu u) T^\dagger = (-1)^\mu \sum_i F_i^* \bar{u} \mathcal{O}_i^\mu u \quad T l_\mu T^\dagger = (-1)^\mu l_\mu$$

the interaction is proportional to:

$$l_\mu \bar{u} \Gamma^\mu u = \sum_i F_i l_\mu \bar{u} \mathcal{O}_i^\mu u$$

$$T (l_\mu \bar{u} \Gamma^\mu u) T^\dagger = (-1)^{2\mu} \sum_i F_i^* l_\mu \bar{u} \mathcal{O}_i^\mu u$$

■ T-inv  $\Rightarrow F_i = F_i^*$

# Electroweak nucleon current

$$\langle N' | J^\mu | N \rangle = \bar{u}(p') \Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

$$\Gamma^\mu = \gamma^\mu F_1 + \frac{i}{2M} \sigma^{\mu\nu} q_\nu F_2 + \frac{q^\mu}{M} F_S - \gamma^\mu \gamma_5 F_A - \frac{i}{2M} \sigma^{\mu\nu} q_\nu \gamma_5 F_T - \frac{q^\mu}{M} \gamma_5 F_P$$

■ Parity (P) transformation:

$$P \bar{u}(p'_0, \vec{p}') \Gamma^\mu (q_0, \vec{q}) u(p_0, \vec{p}) P^\dagger = \bar{u}(p'_0, -\vec{p}') \gamma_0 \Gamma^\mu (q_0, -\vec{q}) \gamma_0 u(p_0, -\vec{p})$$

$$\gamma^\mu, \sigma^{\mu\nu} q_\nu, q^\mu \quad \leftarrow \text{transform as vectors}$$

$$\gamma^\mu \gamma_5, \sigma^{\mu\nu} \gamma_5 q_\nu, q^\mu \gamma_5 \quad \leftarrow \text{transform as axial-vectors}$$

# Electroweak nucleon current

$$\langle N' | J^\mu | N \rangle = \bar{u}(p') \Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

$$\mathcal{V}^\mu = \bar{u}(p') \left[ \gamma^\mu F_1 + \frac{i}{2M} \sigma^{\mu\nu} q_\nu F_2 + \frac{q^\mu}{M} F_S \right] u(p)$$

$$\mathcal{A}^\mu = \bar{u}(p') \left[ \gamma^\mu \gamma_5 F_A + \frac{i}{2M} \sigma^{\mu\nu} q_\nu \gamma_5 F_T + \frac{q^\mu}{M} \gamma_5 F_P \right] u(p)$$

■ Current conservation + Dirac eq. :

$$q_\mu \mathcal{V}^\mu = \bar{u}(p') \left[ (\not{p}' - \not{p}) F_1 + \frac{q^2}{M} F_S \right] u(p) = 0 \Rightarrow F_S = 0$$



# Electroweak nucleon current

$$\langle N' | J^\mu | N \rangle = \bar{u}(p') \Gamma^\mu u(p) = \mathcal{V}^\mu - \mathcal{A}^\mu$$

$$\mathcal{V}^\mu = \bar{u}(p') \left[ \gamma^\mu F_1 + \frac{i}{2M} \sigma^{\mu\nu} q_\nu F_2 + \frac{q^\mu}{M} F_S \right] u(p)$$

$$\mathcal{A}^\mu = \bar{u}(p') \left[ \gamma^\mu \gamma_5 F_A + \frac{i}{2M} \sigma^{\mu\nu} q_\nu \gamma_5 F_T + \frac{q^\mu}{M} \gamma_5 F_P \right] u(p)$$

■ G-parity (**G**) transformation:  $G = C e^{i\pi \frac{\tau_2}{2}}$

■ Isospin rotation:  $e^{i\pi \frac{\tau_2}{2}} \begin{pmatrix} q_u \\ q_d \end{pmatrix} = i\tau_2 \begin{pmatrix} q_u \\ q_d \end{pmatrix} = \begin{pmatrix} q_d \\ -q_u \end{pmatrix}$

$$G \mathcal{V}^\mu G^\dagger = \mathcal{V}^\mu \quad \leftarrow \text{except for the } F_S \text{ term}$$

$$G \mathcal{A}^\mu G^\dagger = -\mathcal{A}^\mu \quad \leftarrow \text{except for the } F_T \text{ term}$$

■  $\Rightarrow F_S = F_T = 0 \Leftrightarrow$  absence of 2<sup>nd</sup> class currents

# Electroweak nucleon current

$$\mathcal{V}^\mu = \bar{u}(p') \left[ \gamma^\mu F_1 + \frac{i}{2M} \sigma^{\mu\nu} q_\nu F_2 \right] u(p)$$

$$\mathcal{A}^\mu = \bar{u}(p') \left[ \gamma^\mu \gamma_5 F_A + \frac{q^\mu}{M} \gamma_5 F_P \right] u(p)$$

■ Sachs form factors:  $G_E = F_1 + \frac{q^2}{2m_N} F_2$

$$G_M = F_1 + F_2$$

■ In the Breit frame:  $p = (E, -\vec{q}/2)$ ,  $p' = (E, \vec{q}/2)$ ,  $q = p' - p = (0, \vec{q})$ ,  $q^2 = -\vec{q}^2$

$$\langle N'_{s'} | \mathcal{V}^0 | N_s \rangle = G_E(\vec{q}^2) \delta_{ss'}$$

$$\langle N'_{s'} | \vec{\mathcal{V}} | N_s \rangle = G_M(\vec{q}^2) \frac{i}{2M} \chi_{s'} (\vec{\sigma} \times \vec{q}) \chi_s$$

$$\langle N'_{s'} | \mathcal{A}^0 | N_s \rangle = 0$$

$$\langle N'_{s'} | \vec{\mathcal{A}} | N_s \rangle = \chi_{s'} \left[ \frac{E}{M} F_A(\vec{q}^2) \vec{\sigma}_T + \left[ F_A(\vec{q}^2) - \frac{\vec{q}^2}{2M} F_P(\vec{q}^2) \right] \vec{\sigma}_L \right] \chi_s$$

$$\vec{\sigma}_T = \vec{\sigma} - \hat{q}(\vec{\sigma} \cdot \hat{q}), \quad \vec{\sigma}_L = \hat{q}(\vec{\sigma} \cdot \hat{q})$$

# Electroweak nucleon current

- **Vector** and **EM** form factors:

$$V_a^\alpha = \mathcal{V}^\alpha \frac{\tau_a}{2} \leftarrow \text{isovector current} \quad V_Y^\alpha = \mathcal{V}_Y^\alpha I \leftarrow \text{hypercharge (isoscalar) current}$$

$$\langle p | V_{\text{EM}}^\alpha | p \rangle = \langle p | V_3^\alpha + \frac{1}{2} V_Y^\alpha | p \rangle = \frac{\mathcal{V}^\alpha + \mathcal{V}_Y^\alpha}{2} \equiv \mathcal{V}_p^\alpha$$

$$\langle n | V_{\text{EM}}^\alpha | n \rangle = \langle n | V_3^\alpha + \frac{1}{2} V_Y^\alpha | n \rangle = \frac{-\mathcal{V}^\alpha + \mathcal{V}_Y^\alpha}{2} \equiv \mathcal{V}_n^\alpha$$

Then:  $\langle p | V_{\text{CC}}^\alpha | n \rangle = \langle p | V_1^\alpha + i V_2^\alpha | n \rangle = \mathcal{V}^\alpha = \mathcal{V}_p^\alpha - \mathcal{V}_n^\alpha$

$$\begin{aligned} \langle p | V_{\text{NC}}^\alpha | p \rangle &= \langle p | (1 - 2 \sin^2 \theta_W) V_3^\alpha - \sin^2 \theta_W V_Y^\alpha | p \rangle \\ &= \left( \frac{1}{2} - \sin^2 \theta_W \right) \mathcal{V}^\alpha + \sin^2 \theta_W \mathcal{V}_Y^\alpha \\ &= \left( \frac{1}{2} - 2 \sin^2 \theta_W \right) \mathcal{V}_p^\alpha - \mathcal{V}_n^\alpha \end{aligned}$$

- **Vector CC** and **NC** form factors can be expressed in terms of **EM** ones

# Electroweak nucleon current

- Consequences of **PCAC** and **pion pole dominance**

$$\langle n | (-i) \mathcal{V}_\pi^\mu | p \rangle = \langle n | (-i) \mathcal{L}_{N\pi\pi} | p \pi^- \rangle (-i) D_\pi(q) (-i) \langle \pi^- | \mathcal{A}_-^\mu | 0 \rangle$$

$$\mathcal{L}_{NN\pi} = -\frac{g_{NN\pi}}{f_\pi} \bar{N} \gamma_\mu \gamma_5 (\partial^\mu \vec{\pi}) \vec{\tau} N \quad D_\pi = \frac{1}{q^2 - m_\pi^2} \quad \langle \pi^- | \mathcal{A}_-^\mu | 0 \rangle = -\sqrt{2} f_\pi i q^\mu$$

$$\begin{aligned} \langle n | \mathcal{V}_\pi^\mu | p \rangle &= -2g_{N\pi\pi} F_{N\pi\pi}(q^2) \frac{1}{q^2 - m_\pi^2} \bar{u} \not{q} \gamma_5 q^\mu u \quad F_{N\pi\pi}(0) = 1 \\ &= \underbrace{-2g_{N\pi\pi} F_{N\pi\pi}(q^2) \frac{2M}{q^2 - m_\pi^2} \bar{u} \gamma_5 q^\mu u}_{= \frac{F_P}{M}} \end{aligned}$$

# Electroweak nucleon current

- Consequences of **PCAC** and **pion pole** dominance

$$\begin{aligned}\langle n | \mathcal{V}_\pi^\mu | p \rangle &= -2g_{N\pi\pi} F_{N\pi\pi}(q^2) \frac{1}{q^2 - m_\pi^2} \bar{u} \not{q} \gamma_5 q^\mu u \\ &= -2g_{N\pi\pi} F_{N\pi\pi}(q^2) \frac{2M}{q^2 - m_\pi^2} \bar{u} \gamma_5 q^\mu u \\ &\quad \underbrace{\hspace{10em}} = \frac{F_P}{M}\end{aligned}$$

$$\text{PCAC: } \langle n | q_\mu \mathcal{A}^\mu | p \rangle = 0 \quad m_\pi \rightarrow 0$$

$$\bar{u} \left[ \not{q} \gamma_5 F_A - 2g_{N\pi\pi} F_{N\pi\pi}(q^2) \frac{2M}{q^2 - m_\pi^2} q^2 \gamma_5 \right] u = 0 \quad m_\pi \rightarrow 0$$

$$F_A(q^2) = 2g_{N\pi\pi} F_{N\pi\pi}(q^2)$$

$$F_A(0) \equiv g_A = 2g_{NN\pi} \leftarrow \text{Goldberger-Treiman relation}$$

# Electroweak nucleon current

- Consequences of **PCAC** and **pion pole** dominance

$$\mathcal{A}^\mu = \bar{u}(p') \left[ \gamma^\mu \gamma_5 F_A(q^2) + \frac{q^\mu}{M} \gamma_5 F_A(q^2) \frac{2M}{Q^2 + m_\pi^2} \right] u(p)$$

- $F_p$  has a small impact on **CCQE** cross sections (except for  $\nu_\tau$  !)
  - appear in terms proportional to  $(m_l/M)^4$
- $F_p$  does not contribute to **NCE** cross sections
- $F_p$  is studied in **muon capture**  $\mu^- + p \rightarrow \nu_\mu + n$ 
  - $F_p(0)/g_A$  consistent with the **PCAC**+**pion-pole** model
- $F_p$  is also studied in **radiative muon capture**  $\mu^- + p \rightarrow \nu_\mu + n + \gamma$ 
  - more **difficult** to measure and to **model**

# Electron scattering on the nucleon

$$\text{EM} : l^\pm(k) + N(p) \rightarrow l^\pm(k') + N(p')$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{\alpha^2}{4k_0^2} \frac{k'_0}{\sin^4 \frac{\theta}{2}} \cos^2 \frac{\theta}{2} \quad \leftarrow \text{Scattering on a point-like spinless target in Lab}$$

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[1 - \frac{q^2}{2M^2} \tan^2 \frac{\theta}{2}\right] \quad \leftarrow \text{Scattering on a point-like spin 1/2 target in Lab}$$

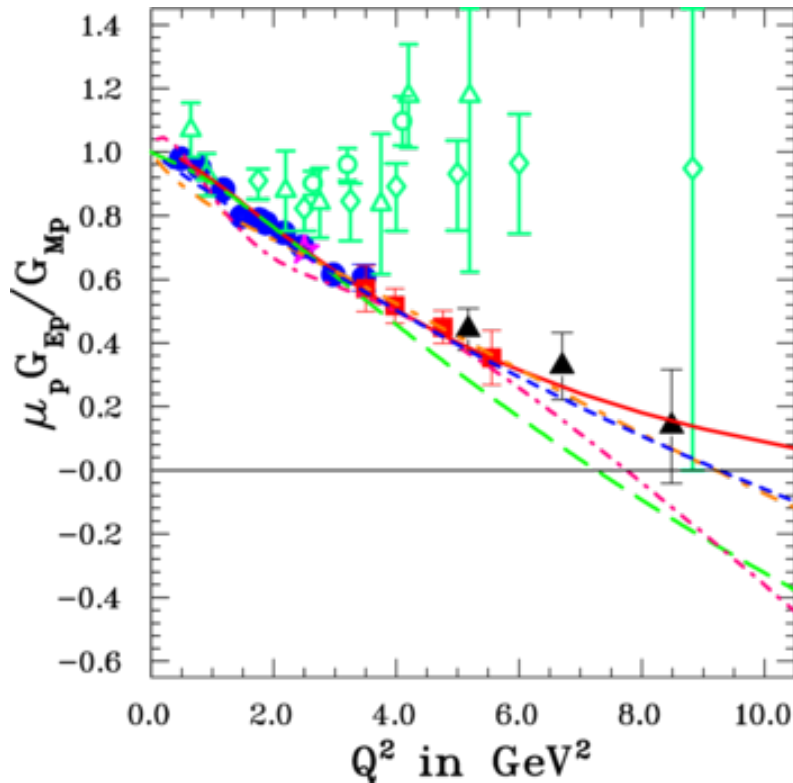
$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \left[ \frac{G_E^2 - \frac{q^2}{4M^2} G_M^2}{1 - \frac{q^2}{4M^2}} - \frac{q^2}{2M^2} G_M^2 \tan^2 \frac{\theta}{2} \right]$$

$$q^2 = -4k_0 k'_0 \sin^2 \frac{\theta}{2}$$

■ Rosenbluth separation  $\Rightarrow G_E, G_M$

# Electron scattering on the nucleon

- More precision, particularly at high  $Q^2$ , with polarization techniques (Jlab)
  - Polarization transfer:  $\vec{e} + p \rightarrow e + \vec{p}$
  - Double polarization:  $\vec{e} + \vec{p} \rightarrow e + p$



Perdrisat et al., Prog.Part.Nucl.Phys. 59 (2007) 694-764, Scholarpedia

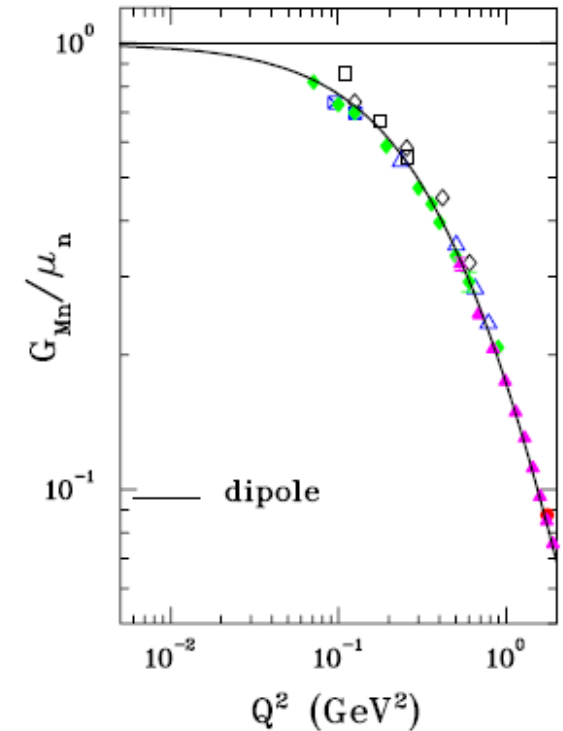
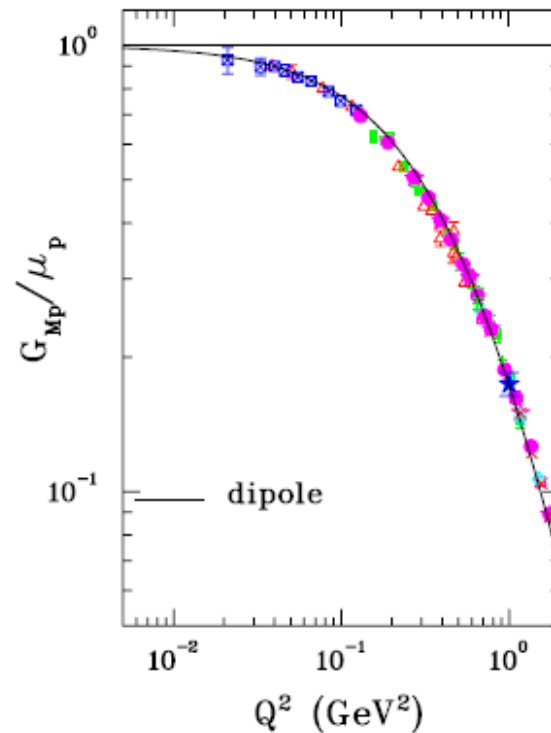
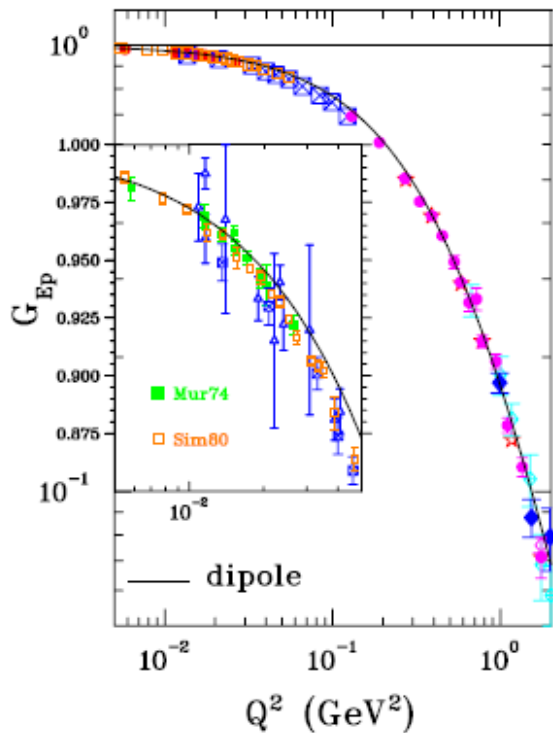


# Nucleon EM form factors

■  $Q^2 \lesssim 1 \text{ GeV}^2$

$$G_E^p(q^2) = G_D(q^2), \quad G_M^p = \mu_p G_D(q^2), \quad G_M^n = \mu_n G_D(q^2), \quad G_D = \left(1 - \frac{q^2}{M_V^2}\right)^{-2}$$

$$\mu_p = 2.793, \quad \mu_n = -1.913, \quad M_V^2 = 0.71 \text{ GeV}^2$$



Perdrisat et al., Prog.Part.Nucl.Phys. 59 (2007) 694-764

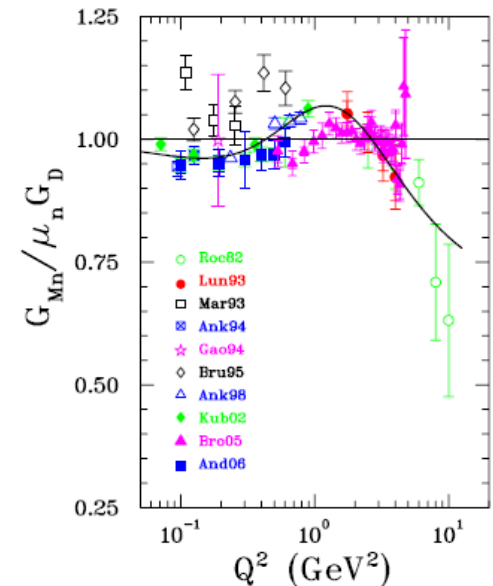
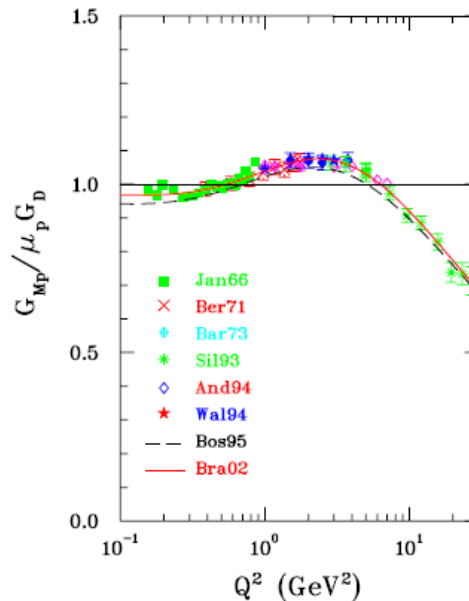
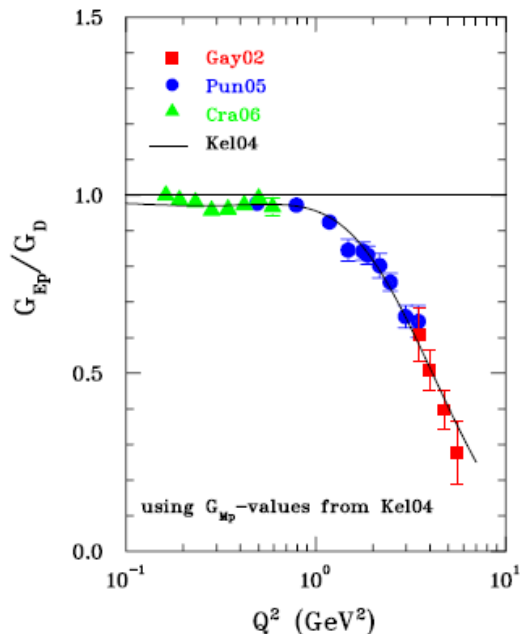
# Nucleon EM form factors

## ■ $Q^2 \lesssim 1 \text{ GeV}^2$

$$G_E^p(q^2) = G_D(q^2), \quad G_M^p = \mu_p G_D(q^2), \quad G_M^n = \mu_n G_D(q^2), \quad G_D = \left(1 - \frac{q^2}{M_V^2}\right)^{-2}$$

$$\mu_p = 2.793, \quad \mu_n = -1.913, \quad M_V^2 = 0.71 \text{ GeV}^2$$

## ■ $Q^2 \gtrsim \text{GeV}^2$

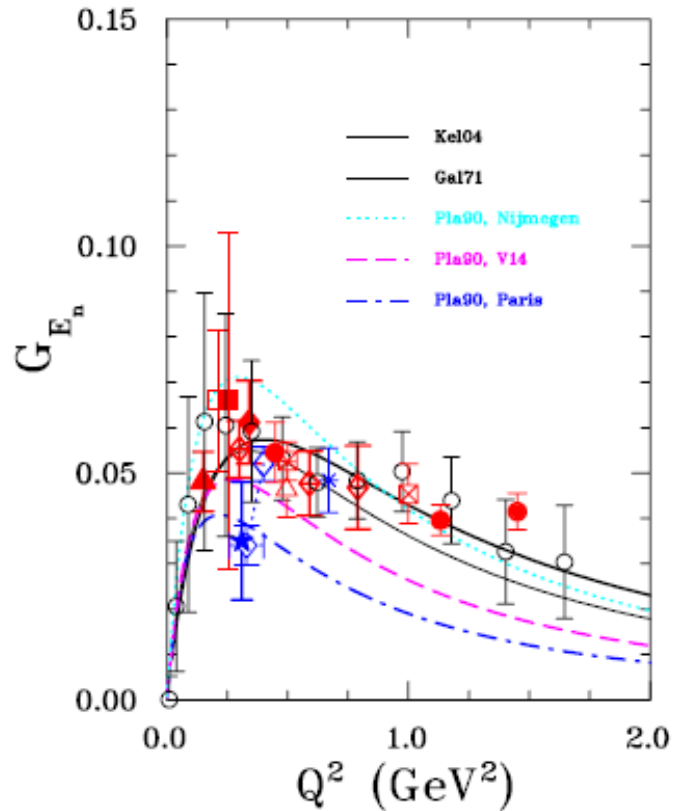


Perdrisat et al., Prog.Part.Nucl.Phys. 59 (2007) 694-764

# Nucleon EM form factors

## ■ Neutron electric form factor

$$G_E^n(0) = 0$$



Perdrisat et al., Prog.Part.Nucl.Phys. 59 (2007) 694-764, Scholarpedia

# $\nu$ QE scattering on the nucleon

- Cross section:

- As an expansion in small variables  $q^2, m_l^2 \ll M^2, E_\nu^2$

- Mean square radii

$$G_E^p(Q^2) = 1 - \frac{1}{6} \langle r_p^2 \rangle Q^2 + \dots$$

$$G_E^n(Q^2) = -\frac{1}{6} \langle r_n^2 \rangle Q^2 + \dots$$

$$F_A(Q^2) = g_A - \frac{1}{6} \langle r_A^2 \rangle Q^2 + \dots$$

# CCQE scattering on the nucleon

## ■ Cross section:

- As an expansion in small variables  $q^2, m_l^2 \ll M^2, E_\nu^2$

$$\frac{d\sigma}{dq^2} = \frac{1}{2\pi} G^2 c_{\text{EW}}^2 \left[ R - \frac{m_l^2}{4E_\nu^2} S + \frac{q^2}{4E_\nu^2} T \right] + \mathcal{O}(q^4, m_l^4, m_l^2 q^2)$$

- CC:  $c_{\text{CC}} = \cos \theta_C$

$$R_{\text{CC}} = 1 + g_A^2$$

$$S_{\text{CC}} = \frac{2E_\nu + M}{M} + g_A^2 \frac{2E_\nu - M}{M}$$

$$T_{\text{CC}} = 1 - g_A^2 + 2 \frac{E_\nu}{M} (1 \mp g_A)^2 \mp 4 \frac{E_\nu}{M} g_A \kappa^\nu - \left( \frac{E_\nu}{M} \kappa^\nu \right)^2 \\ + 4E_\nu^2 \left[ \frac{1}{3} (\langle r_p^2 \rangle - \langle r_n^2 \rangle + g_A^2 \langle r_A^2 \rangle) - \frac{1}{2M^2} \kappa^\nu \right]$$

$$\kappa^\nu = \mu_p - \mu_n - 1$$

# CCQE scattering on the nucleon

- Cross section:

- As an expansion in small variables  $q^2, m_l^2 \ll M^2, E_\nu^2$

$$\frac{d\sigma}{dq^2} = \frac{1}{2\pi} G^2 c_{\text{EW}}^2 \left[ R - \frac{m_l^2}{4E_\nu^2} S + \frac{q^2}{4E_\nu^2} T \right] + \mathcal{O}(q^4, m_l^4, m_l^2 q^2)$$

- CC:  $c_{\text{CC}} = \cos \theta_C$

- Large fraction of the **CCQE** cross section depends on a **small number** of **nucleon** properties:

- Charges, magnetic moments, charge mean squared radii
- axial coupling and axial radius

# CCQE scattering on the nucleon

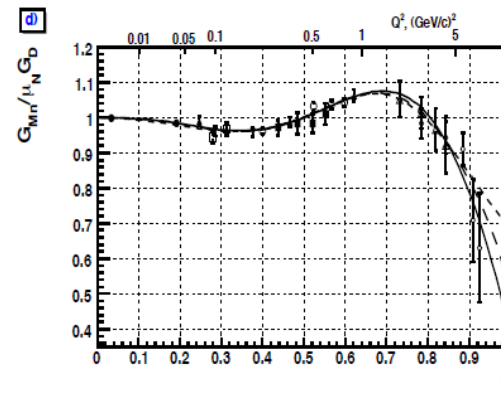
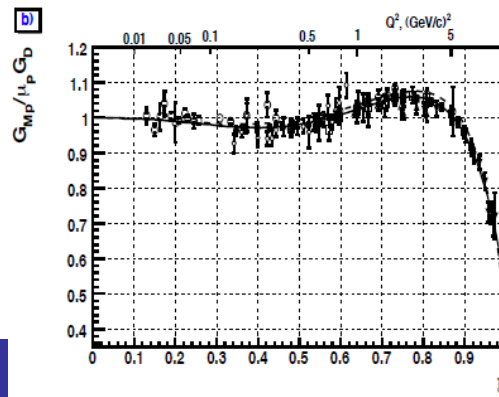
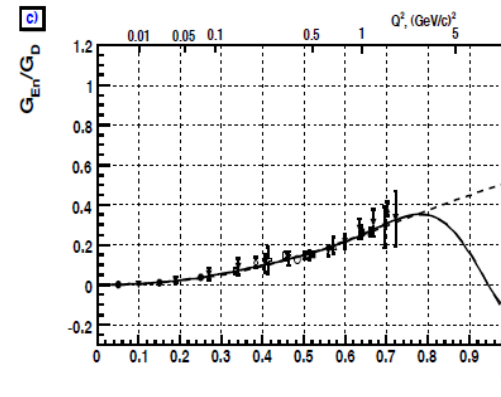
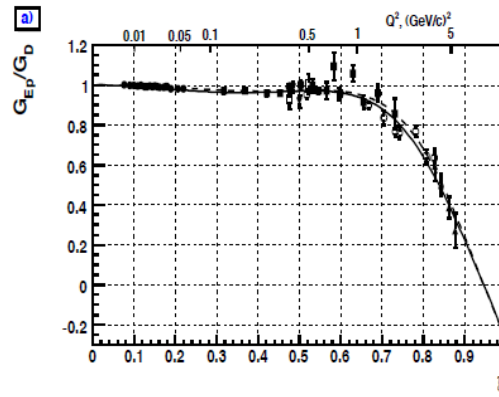
- Measurement of the axial radius:

- CCQE on H and D (BNL, ANL)

$$F_A(Q^2) = g_A \left( 1 + \frac{Q^2}{M_A^2} \right)^{-2} \quad \langle r_A^2 \rangle = \frac{12}{M_A^2}$$

- $M_A = 1.016 \pm 0.026$  GeV Bodek et al., EPJC 53 (2008)

- Using:



# CCQE scattering on the nucleon

- Measurement of the axial radius:

- CCQE on H and D (BNL, ANL)

$$F_A(Q^2) = g_A \left(1 + \frac{Q^2}{M_A^2}\right)^{-2} \quad \langle r_A^2 \rangle = \frac{12}{M_A^2}$$

- $M_A = 1.016 \pm 0.026$  GeV Bodek et al., EPJC 53 (2008)

- From  $\pi$  electroproduction on p:

$$6 \left. \frac{dE_{0+}^{(-)}}{dq^2} \right|_{q^2=0} = \langle r_A^2 \rangle + \frac{3}{M} \left( \kappa^V + \frac{1}{2} \right) + \frac{3}{64f_\pi^2} \left( 1 - \frac{12}{\pi^2} \right)$$

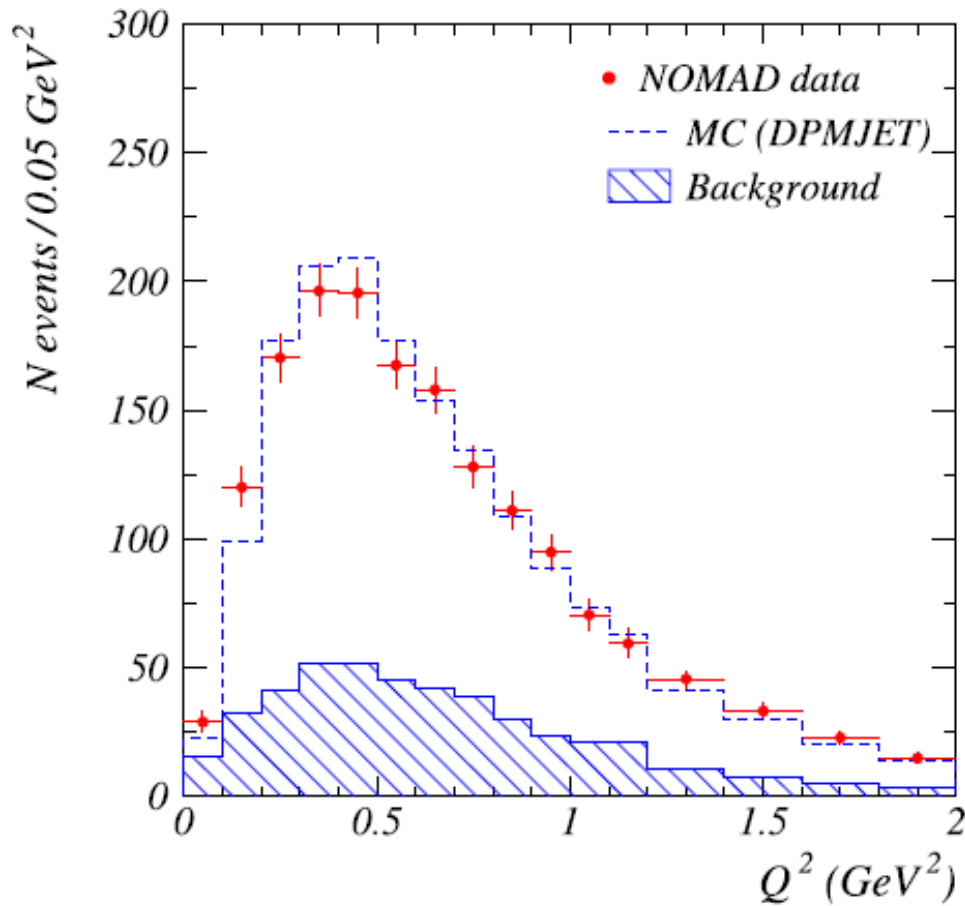
- $M_A = 1.014 \pm 0.016$  GeV Liesenfeld et al., PLB 468 (1999) 20

- No indication of deviations from the dipole dep. in BNL, ANL data



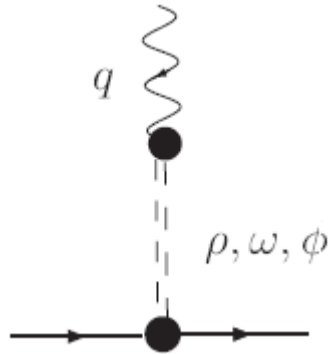
# CCQE scattering on the nucleon

- **No indication** of deviations from the dipole dep. in **NOMAD** data (on  $^{12}\text{C}$ )



# CCQE scattering on the nucleon

- Why dipoles?



- Dipole behavior might arise from the contribution of two mesons with similar masses and opposite couplings

$$F_{1,2}(q^2) \sim \frac{a}{q^2 - m_{V_1}^2} + \frac{(-a)}{q^2 - m_{V_2}^2} = \frac{a(m_{V_1}^2 - m_{V_2}^2)}{(q^2 - m_{V_1}^2)(q^2 - m_{V_2}^2)}$$

# QE scattering on the nucleon

## ■ Cross section:

- As an expansion in small variables  $q^2, m_l^2 \ll M^2, E_\nu^2$

$$\frac{d\sigma}{dq^2} = \frac{1}{2\pi} G^2 c_{\text{EW}}^2 \left[ R - \frac{m_l^2}{4E_\nu^2} S + \frac{q^2}{4E_\nu^2} T \right] + \mathcal{O}(q^4, m_l^4, m_l^2 q^2)$$

- NC:  $c_{\text{NC}} = 1/4$

$$R_{\text{NC}}^{(p)} = \alpha_V^2 + (g_A - \Delta s)^2$$

$$T_{\text{NC}}^{(p)} = \alpha_V^2 - (g_A - \Delta s)^2 + 2 \frac{E_\nu}{M} [\alpha_V \mp (g_A - \Delta s)]^2 \mp 4 \frac{E_\nu}{M} (g_A - \Delta s) \kappa_{\text{NC}}^{(p)} - \left( \frac{E_\nu}{M} \kappa_{\text{NC}}^{(p)} \right)^2$$

$$+ 4E_\nu^2 \left\{ \alpha_V \left[ \frac{1}{3} (\alpha_V \langle r_p^2 \rangle - \langle r_n^2 \rangle - \langle r_s^2 \rangle) - \frac{1}{2M^2} \kappa_{\text{NC}}^{(p)} \right] + \frac{1}{3} (g_A - \Delta s) (g_A \langle r_A^2 \rangle - \Delta s \langle r_{As}^2 \rangle) \right\}$$

$$R_{\text{NC}}^{(n)} = 1 + (g_A + \Delta s)^2$$

$$T_{\text{NC}}^{(n)} = 1 - (g_A + \Delta s)^2 + 2 \frac{E_\nu}{M} [1 \mp (g_A + \Delta s)]^2 \pm 4 \frac{E_\nu}{M} (g_A + \Delta s) \kappa_{\text{NC}}^{(n)} - \left( \frac{E_\nu}{M} \kappa_{\text{NC}}^{(n)} \right)^2$$

$$+ 4E_\nu^2 \left\{ -\frac{1}{3} (\alpha_V \langle r_n^2 \rangle - \langle r_p^2 \rangle - \langle r_s^2 \rangle) + \frac{1}{2M^2} \kappa_{\text{NC}}^{(n)} + \frac{1}{3} (g_A + \Delta s) (g_A \langle r_A^2 \rangle + \Delta s \langle r_{As}^2 \rangle) \right\}$$

$$\kappa_{\text{NC}}^{(p)} = \alpha_V (\mu_p - 1) - \mu_n - \mu_s \quad \kappa_{\text{NC}}^{(n)} = 1 - \mu_p + \alpha_V \mu_n - \mu_s \quad \alpha_V = 1 - 4 \sin^2 \theta_W$$

# QE scattering on the nucleon

- **Strangeness** content of the nucleon:

- $\langle r_s^2 \rangle, \mu_s, \langle r_{As}^2 \rangle \leftarrow$  insignificant

- $\Delta s$  strange axial coupling  $\Leftrightarrow$  strange quark contribution to the spin

$$\left. \frac{d\sigma_{\text{NC}}^{(p)} / dq^2}{d\sigma_{\text{NC}}^{(n)} / dq^2} \right|_{q^2=0} = \frac{\alpha_V^2 + (g_A - \Delta s)^2}{1 + (g_A + \Delta s)^2} \approx \frac{(g_A - \Delta s)^2}{1 + (g_A + \Delta s)^2} \approx \begin{cases} 0.62 & \text{if } \Delta s = 0 \\ 1.27 & \text{if } \Delta s = -0.3 \end{cases}$$

- A recent global fit: [Pate, Trujillo, arXiv:1308.5694](#)

# Weak Resonance excitation

- **Resonances** contribute to:

- the **inclusive**  $\nu_l N \rightarrow l X$  cross section

- several **exclusive** channels:  $\nu_l N \rightarrow l N' \pi$

$$\nu_l N \rightarrow l N' \gamma$$

$$\nu_l N \rightarrow l N' \eta$$

$$\nu_l N \rightarrow l \Lambda(\Sigma) \bar{K}$$

- At  $E_\nu \sim 1$  GeV (MiniBooNE, SciBooNE, T2K,...)  $\Delta(1232)$  is **dominant**

- At  $E_\nu > 1$  GeV (MINER $\nu$ A)  $N^*$  become also **important**

# Weak Resonance excitation

■ CC R excitation:  $\nu_l(k) N(p) \rightarrow l^-(k') R(p')$

$$\frac{d\sigma}{dk'_0 d\Omega'} = \frac{1}{32\pi^2} \frac{|\vec{k}'|}{k_0 M_N} \mathcal{A}(p') |\bar{\mathcal{M}}|^2 \quad \leftarrow \text{Inclusive cross section}$$

$$\mathcal{A}(p') = \frac{M^*}{\pi} \frac{\Gamma(p')}{(p'^2 - M^{*2})^2 + M^{*2} \Gamma^2(p')}$$

$\Gamma(p')$   $\leftarrow$  total momentum dependent **width**

$$\mathcal{M} = \frac{G_F \cos \theta_C}{\sqrt{2}} l^\alpha J_\alpha$$

$$l^\alpha = \bar{u}(k') \gamma^\alpha (1 - \gamma_5) u(k) \quad \leftarrow \text{leptonic current}$$

$$J_\alpha = V_\alpha - A_\alpha \quad \leftarrow \text{hadronic current}$$

can be parametrized in terms of  
N-R transition **form factors**

# Weak Resonance excitation

- $\Delta(1232) J^P=3/2^+$

$$J_\alpha = \bar{u}^\mu(p') \left[ \left( \frac{C_3^V}{M_N} (g_{\alpha\mu} \not{q} - q_\alpha \gamma_\mu) + \frac{C_4^V}{M_N^2} (g_{\alpha\mu} q \cdot p' - q_\alpha p'_\mu) + \frac{C_5^V}{M_N^2} (g_{\alpha\mu} q \cdot p - q_\alpha p_\mu) \right) \gamma_5 \right. \\ \left. + \frac{C_3^A}{M_N} (g_{\alpha\mu} \not{q} - q_\alpha \gamma_\mu) + \frac{C_4^A}{M_N^2} (g_{\alpha\mu} q \cdot p' - q_\beta p'_\mu) + C_5^A g_{\alpha\mu} + \frac{C_6^A}{M_N^2} q_\alpha q_\mu \right] u(p)$$

$C_{3-5}^V, C_{3-6}^A \leftarrow$  N- $\Delta$  transition form factors

- Rarita-Schwinger fields: spin 3/2

$$u_\mu(p, s_\Delta) = \sum_{\lambda, s} \left( 1\lambda \frac{1}{2}s \middle| \frac{3}{2}s_\Delta \right) \epsilon_\mu(p, \lambda) u(p, s)$$

- Eq. of motion:  $(\not{p} - M_\Delta) u_\mu = 0$
- with constrains:  $\gamma^\mu u_\mu = p^\mu u_\mu = 0$

# Weak Resonance excitation

- Second resonance peak:  $N^*(1440)$ ,  $N^*(1520)$ ,  $N^*(1535)$

- $N^*(1440)$   $J^P=1/2^+$

$$J_\alpha = \bar{u}(p') \left[ \frac{F_1^V}{(2M_N)^2} (\not{q}q_\alpha - q^2\gamma_\alpha) + i \frac{F_2^V}{2M_N} \sigma_{\alpha\beta} q^\beta - F_A \gamma_\alpha \gamma_5 - \frac{F_P}{M_N} \gamma_5 q_\alpha \right] u(p)$$

- $N^*(1535)$   $J^P=1/2^-$

$$J_\alpha = \bar{u}(p') \left[ \frac{F_1^V}{(2M_N)^2} (\not{q}q_\alpha - q^2\gamma_\alpha) \gamma_5 + i \frac{F_2^V}{2M_N} \sigma_{\alpha\beta} q^\beta \gamma_5 - F_A \gamma_\alpha - \frac{F_P}{M_N} q_\alpha \right] u(p)$$

- $N^*(1520)$   $J^P=3/2^-$

$$J_\alpha = \bar{u}^\mu(p') \left[ \frac{C_3^V}{M_N} (g_{\alpha\mu} \not{q} - q_\alpha \gamma_\mu) + \frac{C_4^V}{M_N^2} (g_{\alpha\mu} q \cdot p' - q_\alpha p'_\mu) + \frac{C_5^V}{M_N^2} (g_{\alpha\mu} q \cdot p - q_\alpha p_\mu) \right. \\ \left. + \left( \frac{C_3^A}{M_N} (g_{\alpha\mu} \not{q} - q_\alpha \gamma_\mu) + \frac{C_4^A}{M_N^2} (g_{\alpha\mu} q \cdot p' - q_\beta p'_\mu) + C_5^A g_{\alpha\mu} + \frac{C_6^A}{M_N^2} q_\alpha q_\mu \right) \gamma_5 \right] u(p)$$



# Weak Resonance excitation

- Vector CC and NC form factors can be expressed in terms of EM ones

- CC:  $F_{1,2}^V = F_{1,2}^p - F_{1,2}^n$

- NC:  $\tilde{F}_{1,2}^{p(n)} = \left( \frac{1}{2} - 2 \sin^2 \theta_W \right) F_{1,2}^{p(n)} - F_{1,2}^{n(p)}$

- The same applies for  $C_{1,2,3}^V$

- Helicity amplitudes from  $\pi$  photo- and electro-production data

$$A_{1/2} = \sqrt{\frac{2\pi\alpha}{k_R}} \langle R, J_z = 1/2 | \epsilon_\mu^+ J_{\text{EM}}^\mu | N, J_z = -1/2 \rangle \zeta$$

$$A_{3/2} = \sqrt{\frac{2\pi\alpha}{k_R}} \langle R, J_z = 3/2 | \epsilon_\mu^+ J_{\text{EM}}^\mu | N, J_z = 1/2 \rangle \zeta$$

$$S_{1/2} = -\sqrt{\frac{2\pi\alpha}{k_R}} \frac{|\mathbf{q}|}{\sqrt{Q^2}} \langle R, J_z = 1/2 | \epsilon_\mu^0 J_{\text{EM}}^\mu | N, J_z = 1/2 \rangle \zeta$$

- Helicity amplitudes  $\Rightarrow$  EM form factors

# Weak Resonance excitation

- Axial transition form factors

- Poorly known (if at all...)

- PCAC:  $q^\alpha A_\alpha \approx 0$

- $\pi$ -pole dominance of the pseudoscalar form factor:  $C_6^A$

- $\Delta(1232)$   $J^P=3/2^+$

$$\text{PCAC} \Rightarrow C_6^A = -\frac{M_N^2}{q^2} C_5^A$$

$$\text{Using } \mathcal{L}_{\Delta N \pi} = -\frac{g_{\Delta N \pi}}{f_\pi} \bar{\Delta}_\mu (\partial^\mu \vec{\pi}) \vec{T}^\dagger N \quad g_{\Delta N \pi} \Leftrightarrow \Gamma(N^* \rightarrow N \pi)$$

$$\pi\text{-pole dominance} \Rightarrow C_6^A = -\sqrt{\frac{2}{3}} g_{N^* N \pi} F(q^2) \frac{M_N^2}{q^2 - m_\pi^2} \quad F(0) = 1$$

$$\text{Therefore } C_5^A(0) = \sqrt{\frac{2}{3}} g_{\Delta N \pi} \leftarrow \text{Goldberger-Treiman relation}$$

$$C_4^A = -\frac{1}{4} C_5^A \quad C_3^A = 0 \leftarrow \text{Adler model}$$

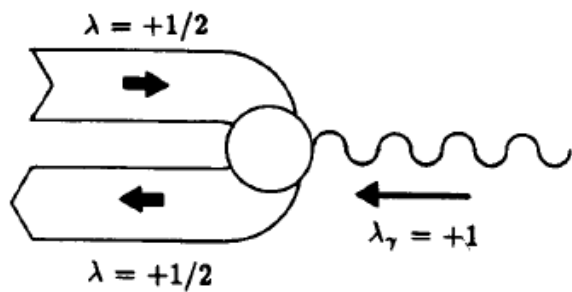
# MAID

## ■ Helicity amplitudes

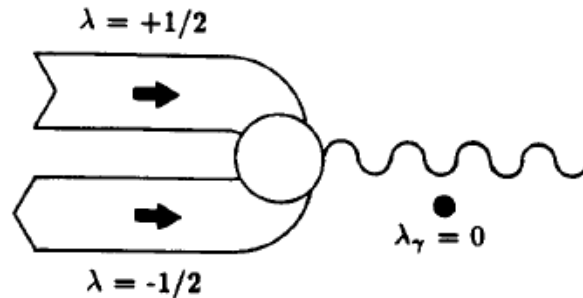
$$A_{1/2} = \sqrt{\frac{2\pi\alpha}{k_R}} \langle R, J_z = 1/2 | \epsilon_\mu^+ J_{EM}^\mu | N, J_z = -1/2 \rangle \zeta$$

$$A_{3/2} = \sqrt{\frac{2\pi\alpha}{k_R}} \langle R, J_z = 3/2 | \epsilon_\mu^+ J_{EM}^\mu | N, J_z = 1/2 \rangle \zeta$$

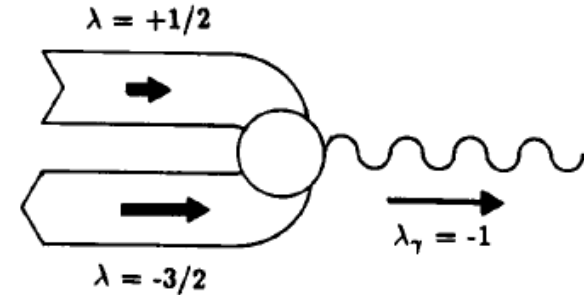
$$S_{1/2} = -\sqrt{\frac{2\pi\alpha}{k_R}} \frac{|\mathbf{q}|}{\sqrt{Q^2}} \langle R, J_z = 1/2 | \epsilon_\mu^0 J_{EM}^\mu | N, J_z = 1/2 \rangle \zeta$$



$A_{1/2}$



$S_{1/2}$

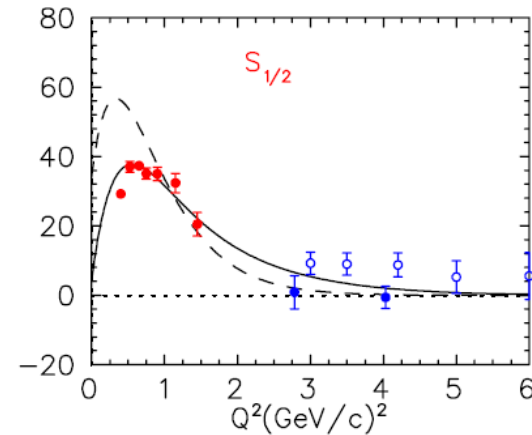
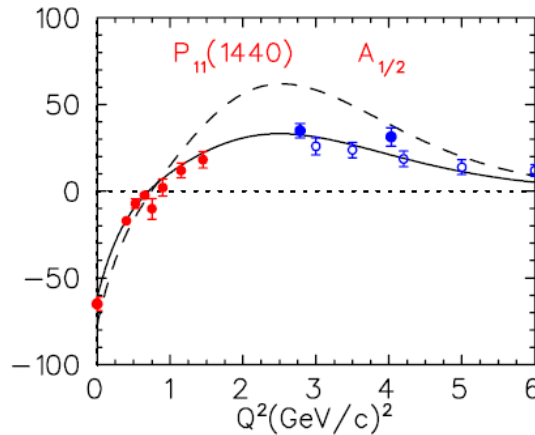
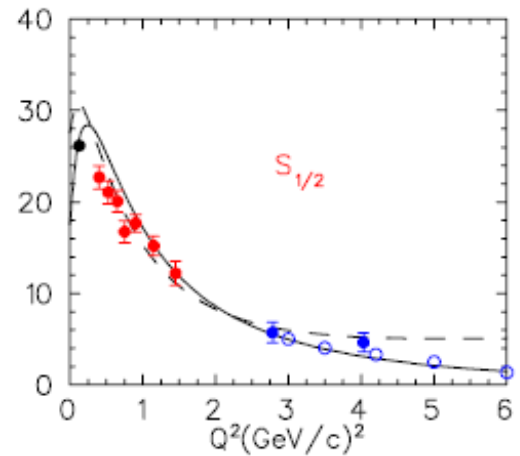
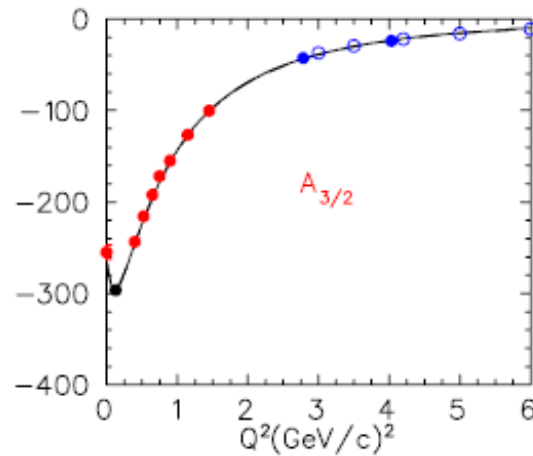
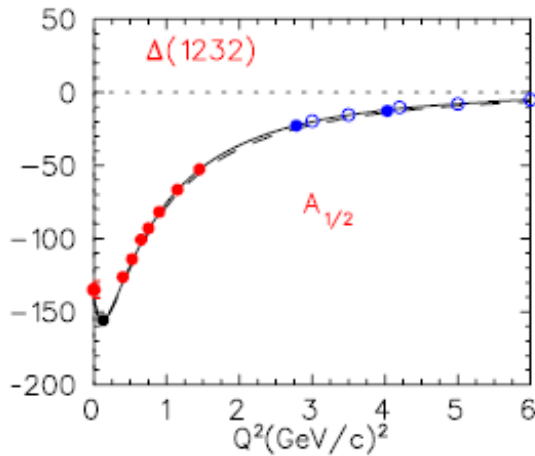


$A_{3/2}$

# MAID

- Transition N-R e.m. helicity amplitudes extracted for all 4-star resonances with  $W < 1.8$  GeV
- For example:

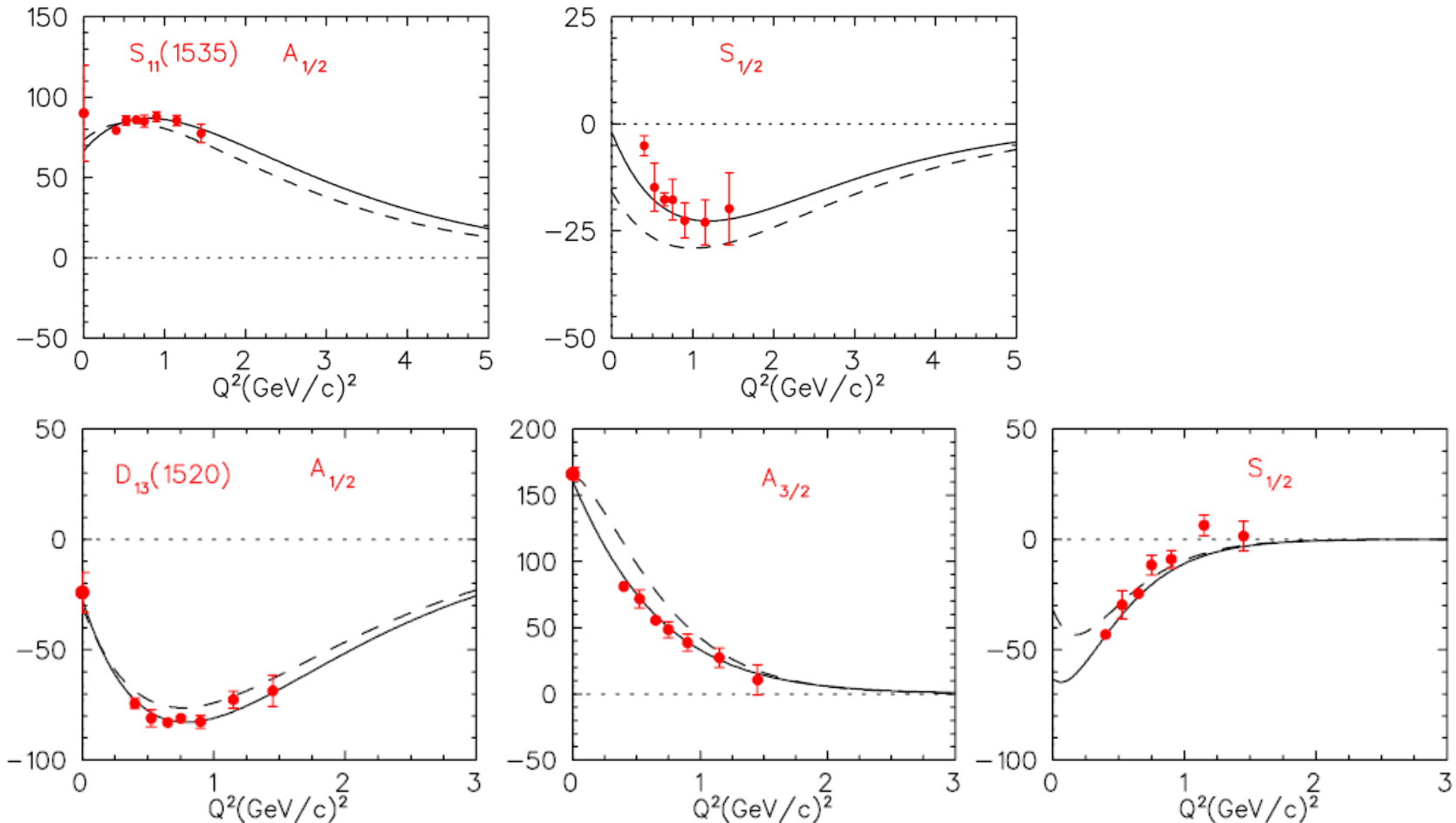
Tiator et al., EPJ Special Topics 198 (2011)



# MAID

- **Transition N-R e.m. helicity amplitudes extracted** for all 4-star resonances with  $W < 1.8$  GeV
- For example:

Tiator et al., EPJ Special Topics 198 (2011)



# Weak Resonance excitation

- Axial transition form factors

- Poorly known (if at all...)

- PCAC:  $q^\alpha A_\alpha \approx 0$

- $\pi$ -pole dominance of the pseudoscalar form factor:  $F_P$

- $N^*(1440)$   $J^P=1/2^+$

$$\text{PCAC} \Rightarrow F_P = -\frac{(M^* + M_N)M_N}{q^2} F_A$$

$$\text{Using } \mathcal{L}_{N^*N\pi} = -\frac{g_{N^*N\pi}}{f_\pi} \bar{N}^* \gamma_\mu \gamma_5 (\partial^\mu \vec{\pi}) \vec{\tau} N \quad g_{N^*N\pi} \Leftrightarrow \Gamma(N^* \rightarrow N\pi)$$

$$\pi\text{-pole dominance} \Rightarrow F_P = -2g_{N^*N\pi} F(q^2) \frac{(M^* + M_N)M_N}{q^2 - m_\pi^2} \quad F(0) = 1$$

Therefore  $F_A(0) = 2g_{N^*N\pi} \leftarrow$  Goldberger-Treiman relation

$$\text{Educated guess: } F_A(q^2) = F_A(0) \left(1 - \frac{q^2}{M_A^2}\right)^{-2} \quad M_A = 1 \text{ GeV}$$

# Weak Resonance excitation

- Axial transition form factors

- Poorly known (if at all...)

- PCAC:  $q^\alpha A_\alpha \approx 0$

- $\pi$ -pole dominance of the pseudoscalar form factor:  $F_P$

- $N^*(1535)$   $J^P=1/2^-$

$$\text{PCAC} \Rightarrow F_P = -\frac{(M^* - M_N)M_N}{q^2} F_A$$

$$\text{Using } \mathcal{L}_{N^*N\pi} = -\frac{g_{N^*N\pi}}{f_\pi} \bar{N}^* \gamma_\mu (\partial^\mu \vec{\pi}) \vec{\tau} N \quad g_{N^*N\pi} \Leftrightarrow \Gamma(N^* \rightarrow N\pi)$$

$$\pi\text{-pole dominance} \Rightarrow F_P = -2g_{N^*N\pi} F(q^2) \frac{(M^* - M_N)M_N}{q^2 - m_\pi^2} \quad F(0) = 1$$

Therefore  $F_A(0) = 2g_{N^*N\pi} \leftarrow$  Goldberger-Treiman relation

$$\text{Educated guess: } F_A(q^2) = F_A(0) \left(1 - \frac{q^2}{M_A^2}\right)^{-2} \quad M_A = 1 \text{ GeV}$$

# Weak Resonance excitation

- Axial transition form factors

- Poorly known (if at all...)

- PCAC:  $q^\alpha A_\alpha \approx 0$

- $\pi$ -pole dominance of the pseudoscalar form factor:  $C_6^A$

- $N^*(1520)$   $J^P=3/2^-$

$$\text{PCAC} \Rightarrow C_6^A = -\frac{M_N^2}{q^2 - m_\pi^2} C_5^A$$

$$\text{Using } \mathcal{L}_{N^*N\pi} = -\frac{g_{N^*N\pi}}{f_\pi} \bar{N}_\mu^* \gamma_5 (\partial^\mu \vec{\pi}) \vec{\tau} N \quad g_{N^*N\pi} \Leftrightarrow \Gamma(N^* \rightarrow N\pi)$$

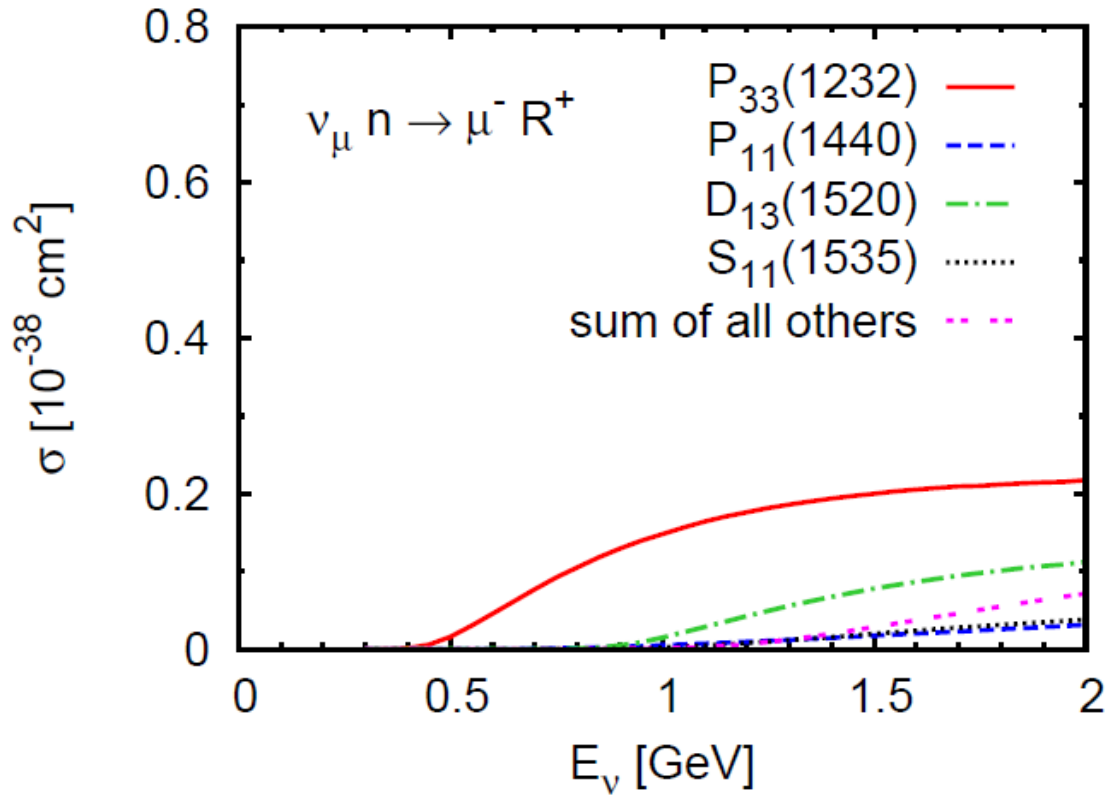
$$\pi\text{-pole dominance} \Rightarrow C_6^A = 2g_{N^*N\pi} F(q^2) \frac{M_N^2}{q^2 - m_\pi^2} \quad F(0) = 1$$

Therefore  $C_5^A(0) = -2g_{N^*N\pi}$  ← Goldberger-Treiman relation

$$\text{Educated guess: } C_5^A(q^2) = C_5^A(0) \left(1 - \frac{q^2}{M_A^2}\right)^{-2} \quad M_A = 1 \text{ GeV} \quad C_3^A = C_4^A = 0$$



# Inclusive resonance production



T. Leitner, O. Buss, LAR, U. Mosel, PRC 79 (2009)  
 T. Leitner, PhD Thesis, 2009

- At  $E_\nu = 2$  GeV,  $\text{CCN}^*(1520)/\text{CC}\Delta \sim 0.5$ ,  $\text{CCN}^*(1440,1535)/\text{CC}\Delta \sim 0.22$
- $\text{N}^*(1520)$  is important for  $\nu_l N \rightarrow l N' \pi$

# Bibliography

- A. W. Thomas, W. Weise, The structure of the nucleon
- F. Halzen, A. D. Martin, Quarks and leptons
- J. D. Walecka, Electron Scattering for Nuclear and Nucleon Structure
- C. Giunti, C. W. Kim, Fundamentals of Neutrino Physics and Astrophysics
- M. Nowakowski, E. A. Paschos, J. M. Rodriguez, All electromagnetic form factors, Eur.J.Phys. 26 (2005) 545-560.
- LAR, Y. Hayato, J. Nieves, Progress and open questions in the physics of neutrino cross sections, arXiv:1403.2673, New J. Phys.
- Perdrisat et al., Prog. Part. Nucl. Phys. 59 (2007) 694-764.