Pion-exchange interactions

Pions are the lightest of all mesons: $m_{\pi^+} = m_{\pi^-} \simeq 139.6$ MeV and $m_{\pi^0} \simeq 135.0$ MeV. They are pseudoscalar $J^{\pi} = 0^-$

The one pion exchange is responsible for the long range component of the nuclear force (strong evidence from NN scattering data).

Neglect mass difference between $m_{\pi^\pm}$ and m_{π^0}, and consider $|\pi^{\pm,0}\rangle$ as $t=1$ states with

$$t_z = \begin{cases} +1 & |\pi^+\rangle \\ 0 & |\pi^0\rangle \\ -1 & |\pi^-\rangle \end{cases}$$

The pion field can be thought of as a vector in isospin space:

$$\vec{\phi}(\vec{r}) = \boxed{\phi_x(\vec{r}), \phi_y(\vec{r}), \phi_z(\vec{r})}$$

$$\phi_a(\vec{r}) = \sum_{\vec{k}} \frac{1}{\sqrt{2\omega_k}} \left(a_{\vec{k}a}\, e^{i\vec{k}\cdot\vec{r}} + a^\dagger_{\vec{k}a}\, e^{-i\vec{k}\cdot\vec{r}} \right) \quad (\text{set } V=1)$$

$$\left[a_{\vec{k}a}, a^\dagger_{\vec{k}'a'} \right] = \delta_{\vec{k},\vec{k}'}\, \delta_{a,a'}$$

Construct the fields corresponding to charged pions. In ordinary space

$$L_z Y_{1m} = m Y_{1m}, \quad Y_{1\pm1} \propto \frac{x \pm iy}{\sqrt{2}}, \quad Y_{10} \propto z$$

Thus a $\pi^\pm$ with momentum $\vec{k}$ is given by

$$|\pi^\pm, \vec{k}\rangle = \frac{1}{\sqrt{2}}\left(a^\dagger_{\vec{k}x} \pm i\, a^\dagger_{\vec{k}y}\right)|0\rangle \equiv a^\dagger_{\vec{k}\pm}|0\rangle$$

while a π^0 with momentum $\vec{k}$ is given by

$$|\pi^0, \vec{k}\rangle = a^\dagger_{\vec{k}z}|0\rangle$$

In terms of the ϕ_a fields, it follows

$$\phi_+(\vec{r}) = \frac{1}{\sqrt{2}}\left[\phi_x(\vec{r}) + i\,\phi_y(\vec{r})\right]$$

$$= \sum_{\vec{k}} \frac{1}{\sqrt{2\omega_k}}\left[\underbrace{\frac{a_{\vec{k}x} + i\,a_{\vec{k}y}}{\sqrt{2}}}_{a_{\vec{k}-}} e^{i\vec{k}\cdot\vec{r}} + \underbrace{\frac{a^\dagger_{\vec{k}x} + i\,a^\dagger_{\vec{k}y}}{\sqrt{2}}}_{a^\dagger_{\vec{k}+}} e^{-i\vec{k}\cdot\vec{r}}\right]$$

destroys π^- — creates π^+

$$\phi_-(\vec{r}) = \left[\phi_+(\vec{r})\right]^\dagger = \sum_{\vec{k}} \frac{1}{\sqrt{2\omega_k}}\left[a_{\vec{k}+}\, e^{i\vec{k}\cdot\vec{r}} + a^\dagger_{\vec{k}-}\, e^{-i\vec{k}\cdot\vec{r}}\right]$$

destroys π^+ — creates π^-

Strong interactions conserve (approximately) isospin, i.e. they are invariant under rotations in isospin space

Hence the isospin structure of the πN interaction must be an isoscalar

$$\underset{\text{pion field}}{\vec{\tau}\cdot\vec{\phi}} = \underbrace{\frac{\tau_x - i\tau_y}{\sqrt{2}}}_{\tau_-}\phi_+ + \underbrace{\frac{\tau_x + i\tau_y}{\sqrt{2}}}_{\tau_+}\phi_- + \tau_z\phi_z$$

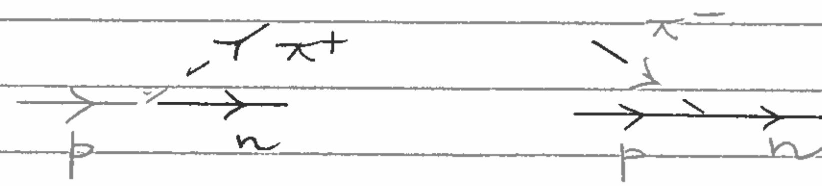
$$= \tau_-\phi_+ + \tau_+\phi_- + \tau_0\phi_0$$

and $\tau_{+/-}$ are $\propto$ to the isospin raising/lowering operators

$$\tau_+|p\rangle = 0 \qquad \tau_+|n\rangle = \sqrt{2}\,|p\rangle$$

$$\tau_-|p\rangle = \sqrt{2}\,|n\rangle \qquad \tau_-|n\rangle = 0$$

$\vec{\tau}\cdot\vec{\phi}$ conserves charge: for example, $\tau_-\phi_+$ changes a proton into a neutron and either creates a π^+ or destroys a π^-



Strong interactions conserve angular momentum and parity, hence

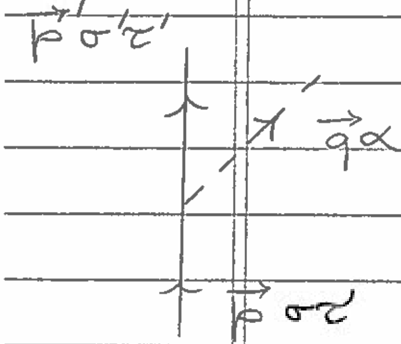
$$H_{\pi NN} = -\frac{f_{\pi NN}}{m_\pi}\,\vec{\sigma}\cdot\vec{\nabla}\,\boxed{\vec{\phi}(\vec{r})\cdot\vec{\tau}}$$

pseudovector (since ϕ is a pseudoscalar)

$\frac{1}{2}\vec{\sigma} = \vec{s}$ is a pseudovector

$H_{\pi NN}$ gives the coupling of a nucleon at $\vec{r}$ with the pion field. It conserves linear and angular momentum, parity, and isospin.

Matrix elements of $H_{\pi NN}$



$$\langle \vec{p}\,'\sigma'\tau';\vec{q}\alpha|H_{\pi NN}|\vec{p}\sigma\tau;0\rangle = \langle\ldots| -\frac{f_{\pi NN}}{m_\pi}\vec{\sigma}\cdot\vec{\nabla}\phi_\alpha\,\tau_{-\alpha}|\ldots\rangle$$

$$=\sum_{\vec{k}}\langle\ldots| -i\frac{f_{\pi NN}}{m_\pi}\frac{1}{\sqrt{2\omega_k}}\,\vec{\sigma}\cdot\vec{k}\,\tau_{-\alpha}\left(e^{i\vec{k}\cdot\vec{r}}a_{\vec{k}-\alpha} - e^{-i\vec{k}\cdot\vec{r}}a^\dagger_{\vec{k}\alpha}\right)|\ldots\rangle$$

$$=\langle\vec{p}\,'\sigma'\tau'|\,i\frac{f_{\pi NN}}{m_\pi}\frac{1}{\sqrt{2\omega_q}}e^{-i\vec{q}\cdot\vec{r}}\,\vec{\sigma}\cdot\vec{q}\,\tau_{-\alpha}|\vec{p}\sigma\tau\rangle$$

$$=\int d\vec{r}\;e^{i\vec{p}\,'\cdot\vec{r}}\chi^\dagger_{\sigma'}\eta^\dagger_{\tau'}\,e^{-i\vec{q}\cdot\vec{r}}\,i\frac{f_{\pi NN}}{m_\pi}\frac{1}{\sqrt{2\omega_q}}\vec{\sigma}\cdot\vec{q}\,\tau_{-\alpha}\,e^{i\vec{p}\cdot\vec{r}}\chi_\sigma\eta_\tau$$

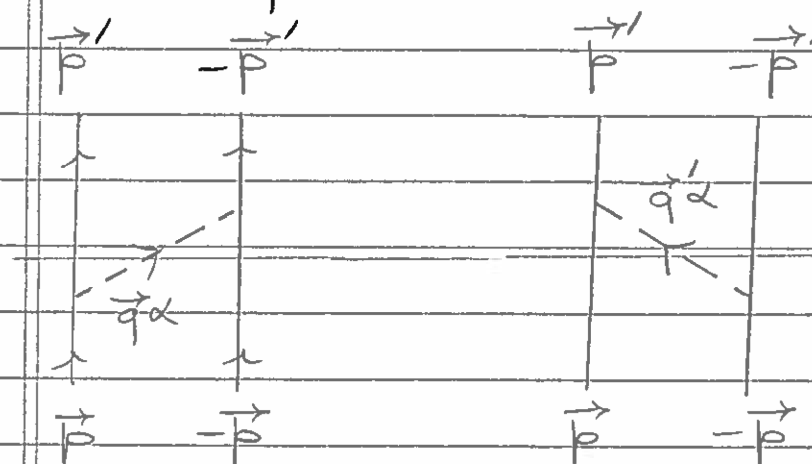
$$=\underbrace{\chi^\dagger_{\sigma'}\eta^\dagger_{\tau'}}_{\text{dropped}}\boxed{i\frac{f_{\pi NN}}{m_\pi}\frac{1}{\sqrt{2\omega_q}}\vec{\sigma}\cdot\vec{q}\,\tau_{-\alpha}}\underbrace{\chi_\sigma\eta_\tau}_{\text{dropped}}\;\underbrace{\delta_{\vec{p}\,',\vec{p}+\vec{q}}}_{\text{understood}}$$

$$i\frac{f_{\pi NN}}{m_\pi}\frac{1}{\sqrt{2\omega_q}}\,\vec{\sigma}\cdot\vec{q}\,\tau_{-\alpha}$$

$$-i\frac{f_{\pi NN}}{m_\pi}\frac{1}{\sqrt{2\omega_q}}\,\vec{\sigma}\cdot\vec{q}\,\tau_{\alpha}$$

OPEP

Consider the amplitudes described by the diagrams



(a) (b)

$$(a) = \left(i\frac{f_{\pi NN}}{m_\pi}\frac{1}{\sqrt{2\omega_q}}\,\vec{\sigma}_1\cdot\vec{q}\,\tau_{-\alpha,1}\right)\frac{1}{E_i - E_I}\left(-i\frac{f_{\pi NN}}{m_\pi}\frac{1}{\sqrt{2\omega_q}}\,\vec{\sigma}_2\cdot\vec{q}\,\tau_{\alpha,2}\right)$$

Energy is conserved $E_i = \frac{p^2}{m} = \frac{p'^2}{m} = E_f$. The energy of the intermediate state is

$$E_I = \frac{p'^2}{2m} + \omega_q + \frac{p^2}{2m}$$

and

$$E_i - E_I = -\omega_q .$$

Diagram (a) is then given by $(\vec{q} = \vec{p} - \vec{p}\,')$

$$(a) = \frac{f_{\pi NN}^2}{m_\pi^2} \frac{1}{2\omega_q} \cdot \frac{-1}{\omega_q} \cdot \vec{\sigma}_1 \cdot \vec{q}\; \vec{\sigma}_2 \cdot \vec{q}\; \tau_{-\alpha,1}\; \tau_{\alpha,2}$$

Diagram (b) leads to a similar expression involving $\tau_{\alpha,1}\,\tau_{-\alpha,2}$ $(\vec{q}\,' = -\vec{q}\,)$

The OPEP is defined as

$$\tilde{v}^\pi(\vec{q}) = \sum_{\alpha=\pm1,0} \left[(a) + (b) \right] \qquad \omega_q = \sqrt{q^2 + m_\pi^2}$$

$$= -\frac{f_{\pi NN}^2}{m_\pi^2} \frac{\vec{\sigma}_1 \cdot \vec{q}\; \vec{\sigma}_2 \cdot \vec{q}}{q^2 + m_\pi^2}\; \vec{\tau}_1 \cdot \vec{\tau}_2$$

Since $\tilde{v}^\pi$ depends on $\vec{q} = \vec{p} - \vec{p}\,'$, its r-space representation is local

$$v^\pi(\vec{r}) = \int_{\vec{q}} e^{-i\vec{q}\cdot\vec{r}}\, \tilde{v}^\pi(\vec{q}) \qquad \frac{e^{-m_\pi r}}{4\pi r}$$

$$= \frac{f_{\pi NN}}{m^2} \vec{\tau}_1 \cdot \vec{\tau}_2\; \vec{\sigma}_1 \cdot \vec{\nabla}\; \vec{\sigma}_2 \cdot \vec{\nabla} \underbrace{\int_{\vec{q}} e^{-i\vec{q}\cdot\vec{r}} \frac{1}{q^2 + m_\pi^2}}$$

Care must be taken in evaluating the Fourier transform, there is a δ-function singularity

$$v^{\pi}_{(\vec{r})} = \frac{f^2_{\pi NN}}{4\pi}\frac{m_\pi}{3}\vec{\tau}_1\cdot\vec{\tau}_2\left\{T_\pi(r)\,S_{12} + \left[Y_\pi(r) - \frac{4\pi\,\delta(\vec{r})}{m_\pi^3}\right]\vec{\sigma}_1\cdot\vec{\sigma}_2\right\}$$

$$Y_\pi(r) = \frac{e^{-m_\pi r}}{m_\pi r}, \quad T_\pi(r) = Y_\pi(r)\left(1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2}\right).$$

Range of $v^\pi \sim 1/m_\pi \simeq 1.4$ fm and within this range

$$T_\pi(r) >> Y_\pi(r)$$

i.e. the tensor component dominates,

$$S_{12} = 3\,\sigma_1\cdot\hat{r}\;\sigma_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2 .$$

Strong coupling between spin-space and isospin variables. For comparison, in atomic and molecular systems the dominant term of the interaction is the Coulomb potential

$$v^c_{(r)} = -e^2/r$$

which depends on the relative distance. OPEP is not spherically symmetric. This fact has profound consequences on the structure of nuclei; for example, the deuteron, the simplest nucleus consisting of a pn pair bound at 2.25 MeV, is not spherically symmetric: it has a quadrupole moment and a tensor polarization, measured in elastic electron scattering experiments. See class notes pp. 24-25

Δ-resonance in πN scattering

E (GeV)		
1.44	N(1440)	$J^\pi, I = \frac{1}{2}^+, \frac{1}{2}$
1.23	Δ	$= \frac{3}{2}^+, \frac{3}{2}$
0.94	N	$= \frac{1}{2}^+, \frac{1}{2}$

The lowest energy baryons are listed above. One way to study these excited baryon states is via πN scattering (similarly one can study the excited states of the hydrogen atom by scattering photons off it) In particular, the $\pi^\pm$ are relatively long-lived mesons ($\tau \simeq 2.6 \cdot 10^{-8}$ s), and their scattering by protons can be studied in the lab.

$\pi^+ p$ and $\pi^- p$ cross sections are quite different:

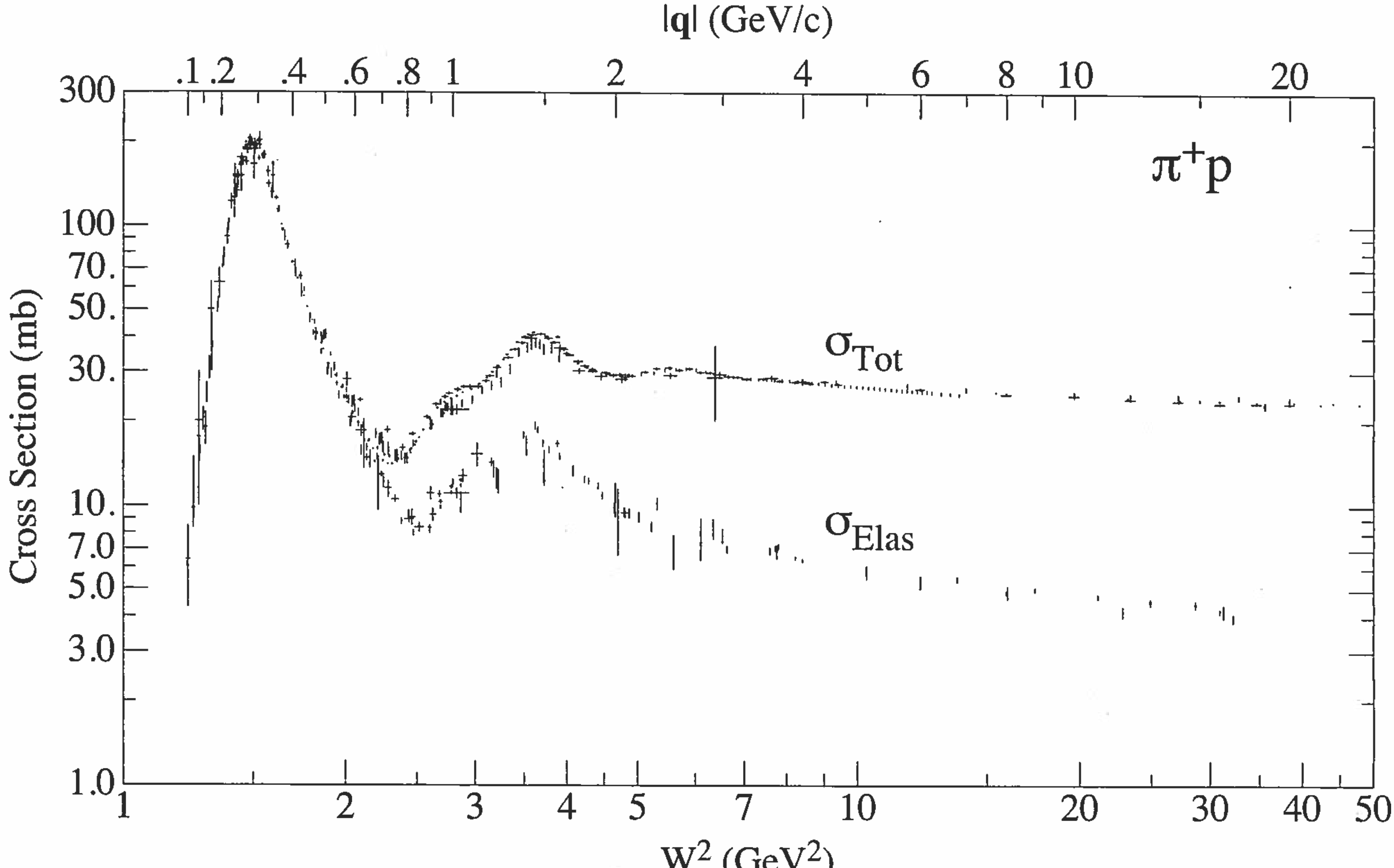
1. σ_{TOT} for $\pi^+ p$ scattering is mostly elastic for $W^2 \lesssim 2\ \text{GeV}^2$;
2. For $W^2 \lesssim 2\ \text{GeV}^2$, σ_{TOT} for $\pi^- p$ scattering has a dominant contribution from inelastic processes.

Elastic — Inelastic

a. $p\pi^+ \to p\pi^+$ — d. $p\pi^+ \to p\pi^+\pi^0, \ldots$

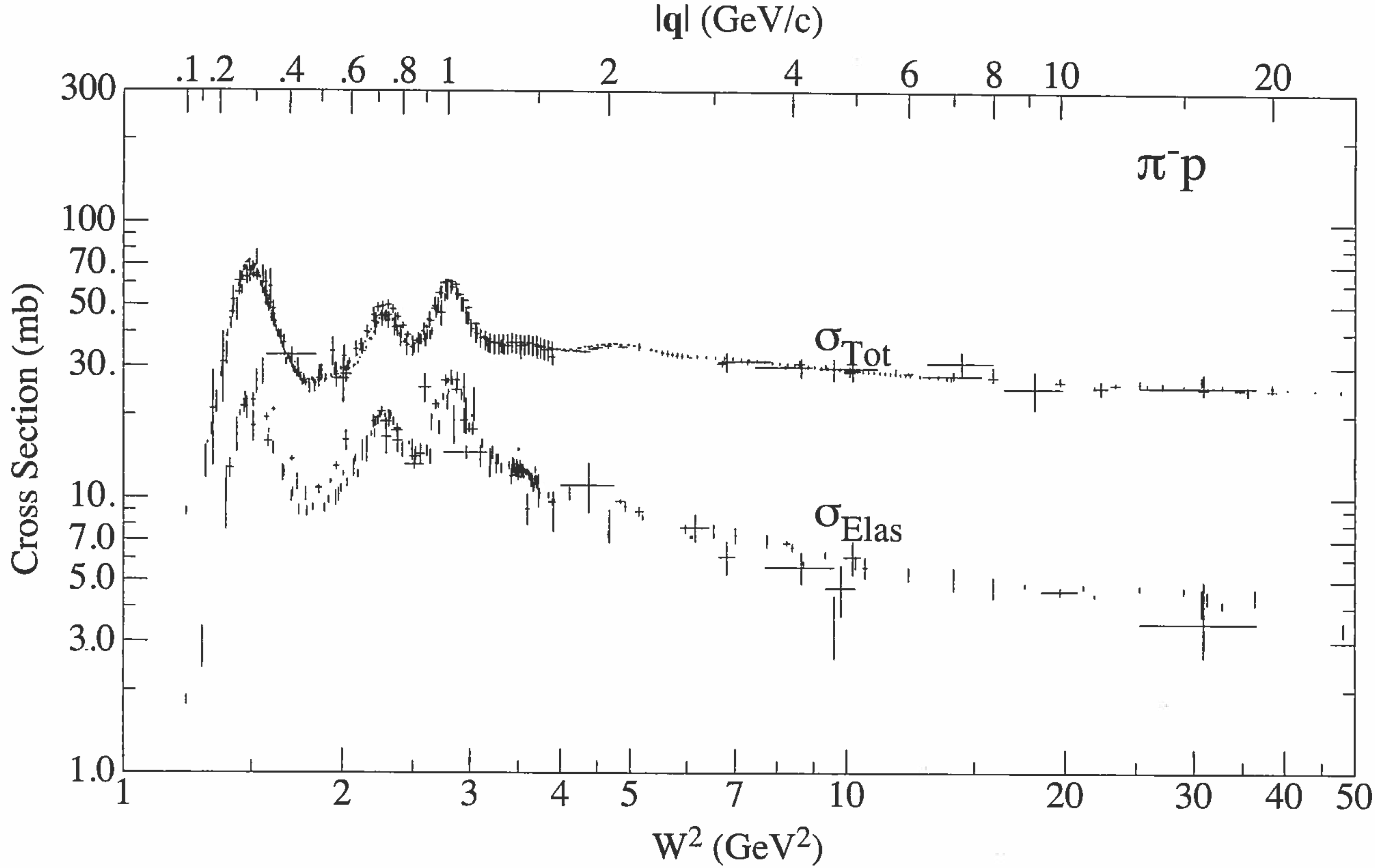
b. $p\pi^- \to p\pi^-$ — c. $p\pi^- \to n\pi^0$, $p\pi^- \to n\pi^0\pi^0, \ldots$

$W > m_p + m_\pi$ for a., b., and c. ; $W > m_\pi + 2m_\pi$ for d., ...



$$W^2 = p^\mu p_\mu = (m_p + \omega_q)^2 - q^2$$

W is the invariant mass


|q| (GeV/c)
.1 .2 .4 .6 .8 1 2 4 6 8 10 20
300
100
70.
50.
30.
10.
7.0
5.0
3.0
1.0
Cross Section (mb)
π^-p
σ_{Tot}
σ_{Elas}
1 2 3 4 5 7 10 20 30 40 50
W^2 (GeV^2)

where π^+ and π^0 mass differences are ignored. Inelastic scattering for $\pi^- p$ occurs at all values of W, while for $\pi^+ p$ it will occur only if W is large enough to have π production.

At $W^2 \lesssim 2\ \mathrm{GeV}^2$, πN scattering is dominated by resonances, which occur when W equals the mass of an excited state of the nucleon. The widths are proportional to their decay rates.

At low energy, πN scattering is dominated by the Δ resonance

$$\Delta : J^{\pi} = 3/2^{+},\ I = 3/2$$

with four charge states

$$\begin{aligned} &\Delta^{++} \quad I_z = +3/2 \\ &\Delta^{+} \qquad = +1/2 \\ &\Delta^{0} \qquad = -1/2 \\ &\Delta^{-} \qquad = -3/2 \end{aligned}$$

Strong interactions conserve isospin (approximately), and

$$|\pi^+ p\rangle = |T=3/2, T_z=+3/2\rangle$$

$$|\pi^- p\rangle = \frac{1}{\sqrt{3}}|T=3/2, T_z=-1/2\rangle - \sqrt{\frac{2}{3}}|T=1/2, T_z=-\frac{1}{2}\rangle$$

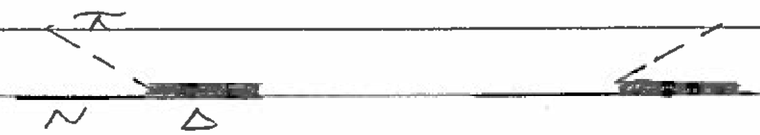
and only the $T=3/2$ components can form the Δ resonance. In the Δ region, one has

$$\langle \pi^- p|T|\pi^- p\rangle \simeq \frac{1}{3}\langle T=3/2, T_z=-1/2|T|T=3/2, T_z=-1/2\rangle$$
$$\simeq \frac{1}{3}\langle \pi^+ p|T|\pi^+ b\rangle$$

and $\sigma_{EL}(\pi^- p) \simeq \sigma_{EL}(\pi^+ p)/9$ under the Δ peak.

<u>The $\pi N \Delta$ interaction</u>

$$H_{\pi N \Delta} = -\frac{f_{\pi N \Delta}}{m_\pi} \Big\{ \underbrace{\vec{S} \cdot \vec{\nabla} \left[\phi(\vec{r}) \cdot \vec{T} \right]}_{①} + \underbrace{\vec{S}^{\dagger} \cdot \vec{\nabla} \left[\phi(\vec{r}) \cdot \vec{T}^{\dagger} \right]}_{②} \Big\}$$

① describes 

② describes 

The operator $\vec{S}$ converts a spin ½ particle to a spin 3/2 particle. The spin 3/2 has four possible spin projections $m' = 3/2, 1/2, -1/2, -3/2$, while a spin ½ particle has two, $m = 1/2, -1/2$.

Each component S_a is represented by a 4×2 matrix

$$(S_a)_{m'm} = \langle 3/2\, m' | S_a | 1/2\, m \rangle$$

$$S_a \longrightarrow \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \\ a_4 & b_4 \end{pmatrix} \quad \text{and} \quad S_a \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \end{pmatrix}$$

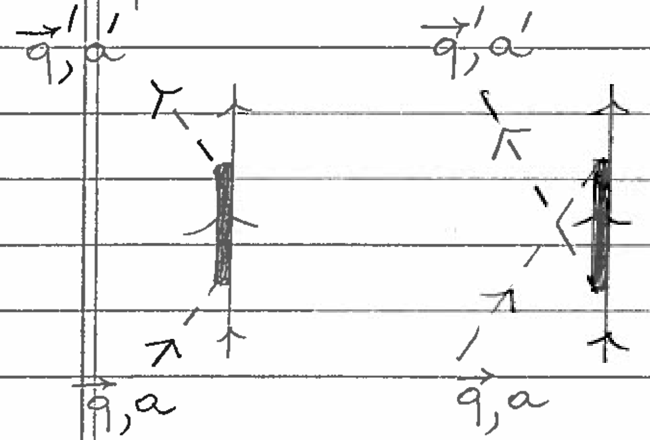
Explicit representation is constructed in class notes (p. 36). One rarely needs it. More useful are the identities

$$\vec{S}^\dagger \cdot \vec{S} = \frac{2}{3}\underbrace{\mathbb{1}}_{\substack{2\times 2 \text{ identity}\\ \text{matrix}}}\,, \qquad \vec{S}^\dagger \times \vec{S} = -\frac{2}{3}i\vec{\sigma}\,,$$

$$\vec{S}^\dagger\cdot\vec{A}\;\vec{S}\cdot\vec{B} = \frac{2}{3}\vec{A}\cdot\vec{B} - \frac{i}{3}\vec{\sigma}\cdot(\vec{A}\times\vec{B})$$

πN scattering in the Δ region.

As the experimental data indicate, πN scattering at $W \sim 1.2$ GeV is dominated by the Δ excitation of the nucleon. The amplitude (2^{nd} order in $H_{\pi N\Delta}$) is described by



(a) (b)

The contribution (b) is suppressed by the energy denominator. The contribution (a) is

$$a) = \sum_I \frac{\langle f | H_{\pi N\Delta} | I\rangle \langle I | H_{\pi N\Delta} | i\rangle}{E_i - E_I}$$

The sum over intermediate states is over Δ states of momentum $\vec{p}_\Delta$ and spin-isospin state $\chi_{\sigma_\Delta}\,\eta_{\tau_\Delta}$. However,

$\vec{p}_\Delta = \vec{q}$ by momentum conservation, and hence

$$(a) = \sum_{\sigma_\Delta \tau_\Delta} \frac{\langle f | H_{\pi N \Delta} | \vec{q}\, \sigma_\Delta \tau_\Delta \rangle \langle \vec{q}\, \sigma_\Delta \tau_\Delta | H_{\pi N \Delta} | i \rangle}{m_N + \omega_q - m_\Delta}$$

$\nwarrow$ neglect kinetic energy of Δ

The Δ has a very short lifetime, and therefore a large uncertainty in its energy (a large width):

$$m_\Delta = \bar{m}_\Delta - i\Gamma_\Delta/2$$

and the wave function of a Δ at rest is

$$e^{-im_\Delta t} \chi_{\sigma_\Delta} \eta_{\tau_\Delta} = e^{-i\bar{m}_\Delta t}\, e^{-\Gamma_\Delta/2\, t} \chi_{\sigma_\Delta} \eta_{\tau_\Delta}$$

and the probability of finding a Δ vanishes as $e^{-\Gamma_\Delta t}$

Empirically

$$\bar{m}_\Delta = 1.232 \text{ GeV} \qquad \Gamma_\Delta \simeq 0.118 \text{ GeV}$$

and the amplitude in (a) reads

$$(a) = \frac{f_{\pi N \Delta}^2}{m_\pi^2} \frac{1}{\sqrt{2\omega_q}} \frac{1}{\sqrt{2\omega_{q'}}} \frac{M_\sigma M_\tau}{\omega_q - (\bar{m}_\Delta - m_N) + i\Gamma_\Delta/2}$$

and the cross section behaves as

$$\frac{d\sigma}{d\Omega_{q'}} \propto 1/\left\{ \left[\omega_q - (\bar{m} - m_N)\right]^2 + \Gamma_\Delta^2/4 \right\}$$

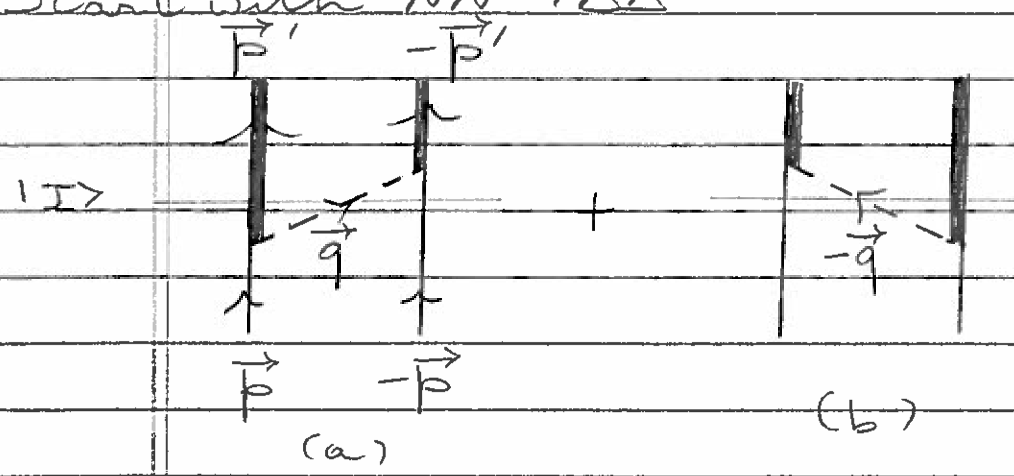
One can determine the $\pi N\Delta$ coupling constant from the decay rate by considering

One finds $f^2_{\pi N\Delta}/4\pi \simeq 0.36$ to be compared to $f^2_{\pi NN}/4\pi \simeq 0.075$ from NN elastic scattering.

One-pion-exchange transition potentials (OPETP)

Consider the processes $NN \to \Delta N$ and $NN \to \Delta\Delta$. These processes occur at CM energies ~ 300 MeV, at which energies pions are produced in NN collisions.

Start with $NN \to \Delta\Delta$



$$= \tilde{v}^{\pi}_{NN\to\Delta\Delta}(\vec{q})$$

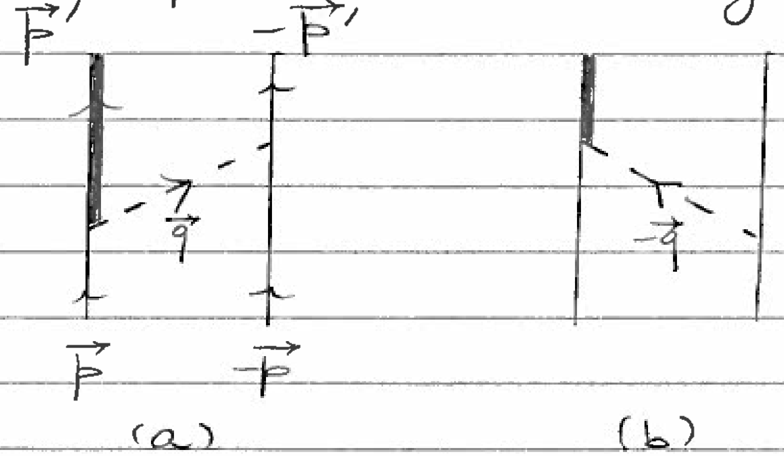
Identical calculation as for OPEP. Note

$$E_i = \frac{p^2}{m_N} + 2m_N = E_f = \frac{p'^2}{m_\Delta} + 2m_\Delta$$

$$E_I = \frac{p'^2}{2m_\Delta} + \omega_q + \frac{p^2}{2m_N} + m_\Delta + m_N = E_i + \omega_q$$

and
$$\tilde{v}^{\pi}_{NN\to\Delta\Delta}(\vec{q}) = -\frac{f^2_{\pi N\Delta}}{m^2_\pi}\,\vec{T}_1\cdot\vec{T}_2\,\frac{\vec{S}_1\cdot\vec{q}\;\vec{S}_2\cdot\vec{q}}{q^2+m^2_\pi}.$$

The calculation of $\tilde{v}^{\pi}_{NN\to\Delta N}$ is similar to that above, but there is a subtle point. The scattering amplitude is given by



$$(a)+(b) = -\frac{f_{\pi N\Delta}f_{\pi NN}}{m^2_\pi}\,\vec{T}_1\cdot\vec{\tau}_2\,\vec{S}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}\,\frac{1}{2\omega_q}\left[\frac{1}{\omega_q+e}+\frac{1}{\omega_q-e}\right]$$

$$e = m_\Delta - m_N + \frac{p'^2}{2m_\Delta} - \frac{p^2}{2m_N}$$

and there is not only a dependence on $\vec{q}$ but also on e (or $\vec{p}$ or $\vec{p}\,'$). However, for threshold production of the Δ ($\vec{p}\,'\simeq 0$), one can show that $e^2 \ll \omega_q^2$, and hence one can define a

$$\tilde{v}^{\pi}_{NN\to\Delta N}(\vec{q}) \simeq -\frac{f_{\pi N\Delta}f_{\pi NN}}{m^2_\pi}\,\frac{\vec{T}_1\cdot\vec{\tau}_2\,\vec{S}_1\cdot\vec{q}\,\vec{\sigma}_2\cdot\vec{q}}{q^2+m^2_\pi}.$$

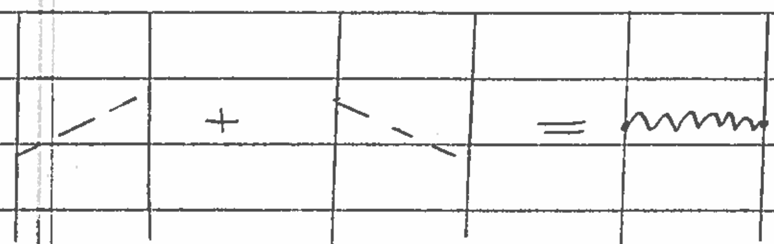
Of course, the processes $\Delta\Delta\to NN$ and $\Delta N\to NN$ due to one pion exchange are obtained by taking the adjoints of the

OPETP's above $\tilde{v}^{\pi}_{\Delta\Delta\to NN} = \left[\tilde{v}^{\pi}_{NN\to\Delta\Delta}\right]^{\dagger}$

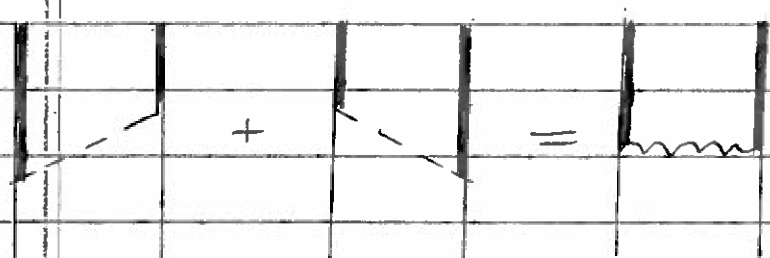
$$\tilde{v}^{\pi}_{\Delta N\to NN} = \left[\tilde{v}^{\pi}_{NN\to\Delta N}\right]^{\dagger}.$$

Two pion-exchange NN potential

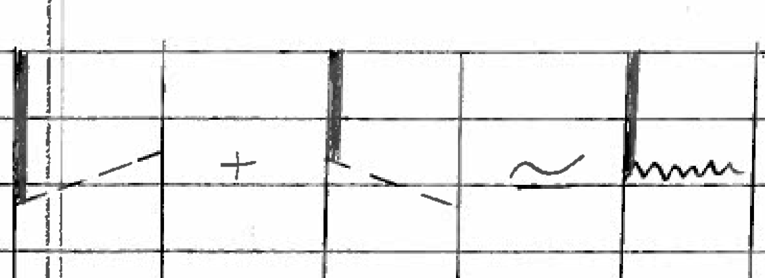
We want to study two-pion exchange contributions to NN scattering below pion production threshold (kinetic energy in CM < 140 MeV) At the one pion-exchange, we obtained



$\tilde{v}^{\pi}(\vec{q})$

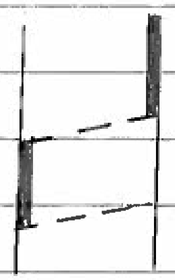


$\tilde{v}^{\pi}_{NN\to\Delta\Delta}(\vec{q})$

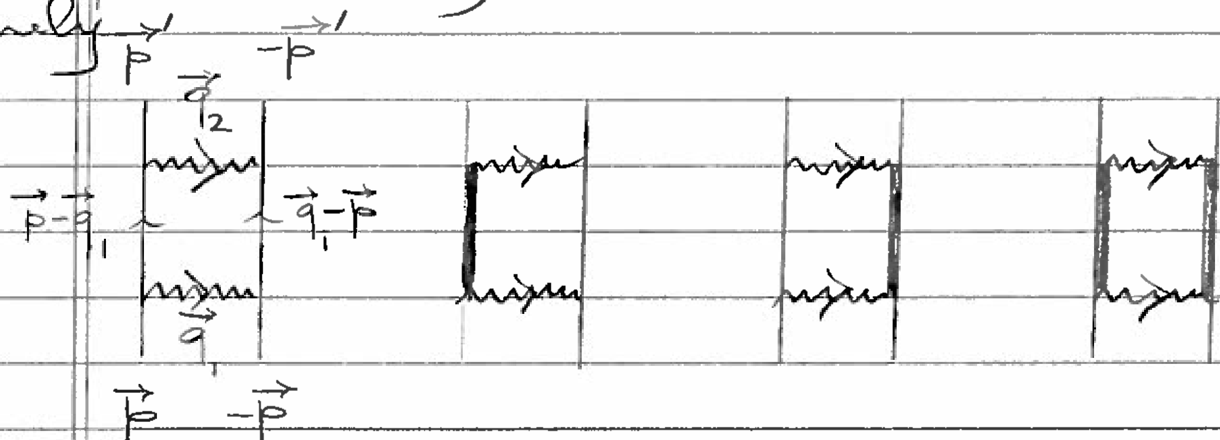


$\tilde{v}^{\pi}_{NN\to\Delta N}(\vec{q})$

At the two pion-exchange level, one needs to consider 48 time ordered diagrams (see p. 15). Since we are considering energies below 140 MeV, no pions can be produced. Therefore, no Δ's can be produced in the final state, for example



Explicit calculation of the 48 diagrams shows they are well approximated by the iterated OPEP and OPETP, namely



(a) (b) (c) (d)

Contributions (b)–(d) are not included in (a), since they involve the excitation/de-excitation of Δ's. The two pion-exchange amplitude should include the contributions of higher mass resonances (not just the Δ, although the Δ is expected to play a dominant role). Thus the calculation of this amplitude is very complex. The analysis of diagrams (b)–(d) is only meant to provide the spin-isospin dependence and the radial shape of the two pion-exchange potential. Its strength is determined by fitting the observed NN scattering data.

We define

$$\langle f | v_{\Delta}^{2\pi} | i \rangle = \sum_{I_1} \frac{\langle f | v^{\pi}_{\Delta N \to NN} | I_1 \rangle \langle I_1 | v^{\pi}_{NN \to \Delta N} | i \rangle}{E_i - E_{I_1}} \qquad (b)$$

$$+ \left(\text{similar } \Delta N \to N\Delta \right) \qquad (c)$$

$$+ \sum_{I_2} \frac{\langle f | v^{\pi}_{\Delta\Delta \to NN} | I_2 \rangle \langle I_2 | v^{\pi}_{NN \to \Delta\Delta} | i \rangle}{E_i - E_{I_2}} \qquad (d)$$

In the static limit,

$$E_i - E_{I_1} \simeq -(m_\Delta - m_N), \quad E_i - E_{I_2} \simeq -2(m_\Delta - m_N)$$

and using closure over I_1 and I_2, one obtains

$$\langle f | v_{\Delta}^{2\pi} | i \rangle = \frac{2}{m_N - m_\Delta} \langle f | \left(v^{\pi}_{NN \to \Delta N} \right)^{\dagger} v^{\pi}_{NN \to \Delta N} | i \rangle$$

$$+ \frac{1}{2(m_N - m_\Delta)} \langle f | \left(v^{\pi}_{NN \to \Delta\Delta} \right)^{\dagger} v^{\pi}_{NN \to \Delta\Delta} | i \rangle$$

In configuration space, the $v_{\Delta}^{2\pi}$ above is particularly simple,

$$v_{\Delta}^{2\pi}(\vec{r}) = \frac{2}{m_N - m_\Delta} \left[v^{\pi}_{NN \to \Delta N}(\vec{r}) \right]^{\dagger} v^{\pi}_{NN \to \Delta N}(\vec{r})$$

$$+ \frac{1}{2(m_N - m_\Delta)} \left[v^{\pi}_{NN \to \Delta\Delta}(\vec{r}) \right]^{\dagger} v^{\pi}_{NN \to \Delta\Delta}(\vec{r})$$

Since for $r \lesssim 1/m_\pi$, the tensor radial function is largest, one can approximate ($T_\pi \gg Y_\pi$):

$$v_\Delta^{2\pi}(\vec{r}) \simeq \frac{2}{9} \frac{f_{\pi NN}^2}{4\pi} \frac{f_{\pi N\Delta}^2}{4\pi} \frac{m_\pi^2}{m_N - m_\Delta} T_\pi^2(r) \, \vec{T}_1^\dagger \cdot \vec{\tau}_2 \, \vec{T}_1 \cdot \vec{\tau}_2 \left(S_{12}^{II}\right)^\dagger S_{12}^{II}$$

$$+ \frac{1}{18} \frac{f_{\pi N\Delta}^4}{(4\pi)^2} \frac{m_\pi^2}{m_N - m_\Delta} T_\pi^2(r) \, T_1^\dagger \cdot T_2^\dagger \, T_1 \cdot T_2 \left(S_{12}^{III}\right)^\dagger S_{12}^{III}$$

The expression above can be reduced via the identities on p. 28 to a linear combination of the type

$$v_\Delta^{2\pi}(\vec{r}) = \sum_{p=1}^{6} I_\Delta^p \underbrace{T_\pi^2(r)}_{\text{range of } 1/(2m_\pi)} O_{12}^p$$

$$O_{12}^{1,\dots,6} = \mathbb{1},\ \vec{\tau}_1 \cdot \vec{\tau}_2,\ \vec{\sigma}_1 \cdot \vec{\sigma}_2,\ \vec{\sigma}_1 \cdot \vec{\sigma}_2 \, \vec{\tau}_1 \cdot \vec{\tau}_2,\ S_{12},\ S_{12} \, \vec{\tau}_1 \cdot \vec{\tau}_2$$

All NN "realistic" potentials have this operator structure and of course include $v^\pi(\vec{r})$). In these potentials the strength parameters I^p are determined by fitting NN elastic scattering data at $T_{CM} \lesssim 140$ MeV. The values for the I^p obtained in this way of course contain contributions of higher mass resonances as well as heavier meson exchange (like the η with $J^\pi, I = 0^-, 0$ and $m_\eta \simeq 548$ MeV or the f_0 with $J^\pi, I = 0^+, 0$ with mass 400–600 MeV) Nevertheless, the I_Δ^p account for most of the I^p which are obtained from data. This suggests that most of $v^{2\pi}(\vec{r})$ is from intermediate Δ's.