Semi-leptonic Electroweak Interactions with Nuclei

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Outline:

Lecture 1

- Basics (this may overlap with other speakers' talks)
- The Relativistic Fermi Gas (RFG)
- Superscaling

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- More sophisticated model of inclusive scattering
- 2p-2h Meson –Exchange Currents (MEC)
- Comparisons with data
  - Inclusive electron scattering
  - Inclusive charge-changing neutrino reactions

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Lecture 3

- Semi-inclusive semi-leptonic electroweak processes
- Reduction to t- and u-channel inclusive processes

DEFINITIONS (Taken from the December, 2013 INT Workshop on neutrino reactions)

Quasielastic

From very early work in electron scattering from nuclei the inclusive quasielastic contribution is usually taken to be the peak seen at roughly $Q^2/2m$. The reason for the terminology comes from a simple (and somewhat naïve) model: were the process to be simply electron scattering from a non-interacting nucleon at rest in a nucleus, a delta-function peak at the above energy loss would be the answer. The nucleons in the nucleus are in fact interacting and moving, and thus the delta function is smeared out (Fermi smearing). Actually things are more complicated than this and both initial- and final-state interactions are important; furthermore, the nucleons are not on-shell and thus their energies and momenta are not trivially related as some models suggest.

quasielastic, continued…

For many theorists the quasielastic contributions are distinguished by their being produced by one-body operators, in contrast to effects arising from two-body operators, such as meson exchange current contributions (see below). Note that this does not equate to one-, two-, three-, etc. nucleon knockout, however, as one type of current operator can give rise to different numbers of nucleons in the final state, dependent on what channels are open.

In neutrino studies, in contrast to electron scattering, different signatures occur for contributions where a pion is detected versus where it is not; the latter is called quasielastic, but is in that usage really the net effect obtained using the full electroweak current with one- and two-body contributions. In fact, even then there is an issue that pions can be produced, but be absorbed and so not detected and accordingly these contributions are counted as "quasielastic". Naturally some model is typically invoked to account for these effects that corrupt the strict meaning of quasielastic, although this means some model dependence has been introduced.

Inelastic contributions

Above pion production threshold one has inelastic contributions coming from various sources: non-resonant pion production, production in the region where the Delta dominates, or where other baryon resonances are expected to play significant roles, multi-pion production, kaon production, etc., eventually to deep inelastic scattering. In the inclusive cross section these are not really distinguishable, but all pieces of the total cross section. For instance, duality studies indicate that on the average effects from specific hadrons in the final state give rise to the same overall result as does DIS. Thus modeling in this region must be done with care to avoid double counting.

Meson exchange current contributions

In models such as the relativisitic Fermi gas (RFG) one can catalogue the various contributions from one-body current operators, two-body contributions counted as MEC versus correlation effects, with both single-nucleon and two-nucleon ejection (the former interfere with the one-body single-nucleon amplitudes), and so on. This can be done because the many-body wave functions are especially simple, namely, on-shell (non-interacting) plane-wave states in Slater determinants. For models with interactions present one has a problem separating the MEC and correlation effects computed as matrix elements of two-body operators from effects already present in the wave functions themselves. In fact, the very concepts are not observables but are representation-dependent. A sophisticated interacting many-body description may already have some (but likely not all) correlation and MEC effects incorporated, in contrast to a simple model where they may not already be present.
[In discussion, examples can be given.]

Another comment on MEC effects: These are not optional, but are required for any interacting system by gauge invariance. Any model with interactions must confront the requirement of having the corresponding two-body MEC contributions… and many models (almost all) cannot do this consistently.

Correlations, both long- and short-range

In naïve models such as the RFG one can include long-range p-h correlation effects within the context of perturbation theory, and can make things gauge invariant to a given order. These arise typically from the longest-range part of the NN interaction, namely that arising from pion exchange. Short-range effects are sometimes also included, although there may be issues with their validity at high energies.

Once one goes to more sophisticated models the meaning of short- and long-range correlations change… these are not observables and are representation dependent. For instance, in one approach a strong repulsive core might be included to allow saturation of nuclear matter and might thereby influence the electroweak cross sections, especially in promoting strength to high missing energy (this needs defining as well; it can be covered in discussions). Alternatively, in relativistic mean field approaches (a la Walecka) many of the correlation effects are already present via strong scalar and vector meson exchanges and therefore one should not be adding them willy nilly or one will run the risk of double counting. Note that this approach also saturates nuclear matter.

Inclusive versus semi-inclusive and more exclusive reactions

Inclusive electroweak cross sections are total hadronic cross sections: only the final-state lepton is presumed to be detected, but nothing from the nuclear side of the scattering diagram. As such, even very naïve models such as the RFG can give reasonable answers. The models tend to satisfy basic symmetries such as unitarity, Lorentz covariance (often, but not always which is serious), maybe gauge invariance, etc. This being so, one tends to get roughly the correct answers since sum rules are being enforced. The main issues with such simple modeling for inclusive reactions is that the strength is often not quite correctly distributed in energy-momentum.

In contrast, when (say) a nucleon in the final state is detected in coincidence with the final-state lepton one has a very different problem. The details of how that nucleon interacts with the rest of the nucleons in the final state is a much more complicated problem. Typical modeling that may be adequate for inclusive cross sections can be very bad for semi-inclusive studies.

See lecture #3

Typically we assume that if one had a viable model for semi-inclusive scattering, for example, for both (e,e'p) and (e,e'n) reactions, then one could integrate each over the outgoing-nucleon, add the results, correct for double counting (which rarely done, in fact) and should then recover the inclusive cross section.

A few basics:

The cross section takes on its characteristic form involving the contraction of two second-rank Lorentz tensors, $d\sigma \sim \eta_{\mu\nu} W^{\mu\nu}$, corresponding to the leptonic and the hadronic contributions which are thus factorized and dealt with independently. The leptonic tensor is defined as

$$\eta_{\mu\nu} \equiv 2mm' \overline{\sum_{if}} j^*_\mu j_\nu, .$$

Its hadronic counterpart is

$$W^{\mu\nu} \equiv \overline{\sum_{if}} J^{\mu *}_{fi}(\mathbf{q}) J^{\nu}_{fi}(\mathbf{q}),$$

where the operations $\overline{\sum}_{if}$ in the two cases correspond to sums and averages over the appropriate sets of leptonic quantum numbers (the helicities, in fact) or hadron quantum numbers (helicities or spins, *etc.*) and integration over all unobserved particles in the final state of the $A-1$ system for hadrons.

It proves useful to decompose both leptonic and hadronic tensors into pieces which are symmetric (s) or antisymmetric (a) under index interchange $\mu \leftrightarrow \nu$, since in contracting them no symmetric-antisymmetric cross-terms are allowed. Both tensors can thus be decomposed as $\eta_{\mu\nu} = \eta^s_{\mu\nu} + \eta^a_{\mu\nu}$ and $W^{\mu\nu} = W^{\mu\nu}_s + W^{\mu\nu}_a$, where the terms are defined as

$$\begin{aligned} \eta^s_{\mu\nu} &= \tfrac{1}{2}(\eta_{\mu\nu} + \eta_{\nu\mu}) \qquad & \eta^a_{\mu\nu} &= \tfrac{1}{2}(\eta_{\mu\nu} - \eta_{\nu\mu}) \\ W^{\mu\nu}_s &= \tfrac{1}{2}(W^{\mu\nu} + W^{\nu\mu}) \qquad & W^{\mu\nu}_a &= \tfrac{1}{2}(W^{\mu\nu} - W^{\nu\mu}). \end{aligned}$$

Clearly one has that $\eta^s_{\mu\mu} = \eta_{\mu\mu}$ and $W^{\mu\mu}_s = W^{\mu\mu}$, whereas $\eta^a_{\mu\mu} = W^{\mu\mu}_a = 0$ (no summation over μ implied in these expressions). In addition, since each tensor is proportional to the bilinear combinations of the electroweak currents in the forms $\eta_{\mu\nu} \sim j^*_\mu j_\nu$ and $W^{\mu\nu} \sim J^{\mu *} J^\nu$, one has that $\eta^*_{\mu\nu} = \eta_{\nu\mu}$ and $W^{\mu\nu *} = W^{\nu\mu}$, and thus that

$$\begin{aligned} \eta^s_{\mu\nu} &= \mathrm{Re}\eta_{\mu\nu} \qquad & \eta^a_{\mu\nu} &= i\mathrm{Im}\eta_{\mu\nu} \\ W^{\mu\nu}_s &= \mathrm{Re}W^{\mu\nu} \qquad & W^{\mu\nu}_a &= i\mathrm{Im}W^{\mu\nu}. \end{aligned}$$

Inclusive Scattering of Leptons from Hadronic Systems: General Hadronic Tensor

Let us consider the inclusive scattering of leptons from hadrons (nucleons or nuclei, to be specific) in the one-boson-exchange approximation (exchange of a $\gamma, W^{\pm}$ or $Z^{\mathbf{0}}$) where the boson brings in 4-momentum $Q^\mu = (\omega, \mathbf{q})$, the initial hadronic system has 4-momentum P_i^μ, and by 4-momentum conservation, the unobserved final-state system has 4-momentum $P_f^\mu = Q^\mu + P_i^\mu$. The allowed form of the hadronic tensor can be deduced from the general developments of the hadronic tensor as it is constructed from the available four-momenta which for inclusive scattering where no final-state aspect of the hadronic system is measured are only Q^μ and P_i^μ (see T. W. Donnelly, *Prog. in Nuclear and Particle Physics* **13** (1985) 183 for extended arguments on hadronic tensors). Three invariants can be constructed, $I_1 \equiv Q^2$, $I_2 \equiv Q \cdot P_i$ and $P_i^2 = M_i^2$, the first two being dynamical variables, while the latter is simply a constant. In the rest frame of the target one has $P_i^\mu = (M_i, 0, 0, 0)$ and so $I_2 = M_i \omega$, and accordingly one can write all response functions as functions of (I_1, I_2), (q, ω) or $(Q^2, x \equiv |Q^2|/2m\nu)$, where ν (particle physics) is the same as ω (nuclear physics).

Next one can write symmetric and antisymmetric hadronic tensors as functions of the two independent four-momenta Q^μ and P_i^μ. In fact, it proves to be more convenient to introduce a projected 4-momentum to replace the last one, namely,

$$U^\mu \equiv \frac{1}{M_i}\left[P_i^\mu - \left(\frac{Q\cdot P_i}{Q^2}\right)Q^\mu\right],$$

where then $Q \cdot U = 0$. Also, to keep the dimensions consistent in the developments below let us introduce a dimensionless four-momentum transfer

$$\widetilde{Q}^\mu \equiv \frac{Q^\mu}{\sqrt{|Q^2|}}.$$

The symmetric hadronic tensor may then be written

$$W_s^{\mu\nu} = X_1 g^{\mu\nu} + X_2\widetilde{Q}^\mu\widetilde{Q}^\nu + X_3 U^\mu U^\nu + X_4(\widetilde{Q}^\mu U^\nu + U^\mu\widetilde{Q}^\nu),$$

where X_i, $i = 1\ldots4$ are invariant functions of the invariants discussed above.

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where X_i, $i = 1 \ldots 4$ are invariant functions of the invariants discussed above. Likewise the antisymmetric tensor can be constructed from the basic 4-momenta

$$W_a^{\mu\nu} = i \left\{ Y_1 (\widetilde{Q}^\mu U^\nu - U^\mu \widetilde{Q}^\nu) + Z_1 \varepsilon^{\mu\nu\alpha\beta} \widetilde{Q}_\alpha U_\beta \right\},$$

where again Y_1 and Z_1 are invariant functions of the invariants above. The terms having no $\varepsilon^{\mu\nu\alpha\beta}$, namely the Y_1 term as well as the X_i terms, arise from VV and AA contributions, whereas those with $\varepsilon^{\mu\nu\alpha\beta}$, namely the Z_1 term, come from VA interferences.

For a conserved vector current (CVC) situation such as here for the VV terms or for purely polar-vector electron scattering the continuity equation in momentum space requires that

$$Q_\mu \left(W_s^{\mu\nu}\right)_{VV} = Q_\mu \left(W_a^{\mu\nu}\right)_{VV} = 0.$$

This contraction removes the terms with $X_3$and Z_1, leaving the conditions

$$\begin{aligned} &\left(-X_1^{VV} + X_2^{VV}\right)\tilde{Q}^\nu + X_4^{VV} U^\nu = 0 \\ &Y_1^{VV} U^\nu = 0. \end{aligned}$$

Since the basic four-momenta are linearly independent of each other the coefficients above must all be independently zero, namely $X_1^{VV} - X_2^{VV} = X_4^{VV} = Y_1^{VV} = 0$. Accordingly, one has

$$\begin{aligned} \left(W_s^{\mu\nu}\right)_{VV} &= X_1^{VV}\left[g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right] + X_3^{VV} U^\mu U^\nu \\ \left(W_a^{\mu\nu}\right)_{VV} &= 0. \end{aligned}$$

Conventionally X_1^{VV} is usually called $-W_1$ and X_3^{VV} is called W_2. Or, alternatively, linear conbinations are called R_L and R_T (L for longitudinal and T for transverse).

A side note: the fact that for a purely CVC situation one has no antisymmetric hadronic tensor with which to contract the antisymmetric leptonic tensor, the latter having an antisymmetric term when (say) considering inclusive polarized electron scattering from unpolarized nuclei, means that any leptonic helicity asymmetry must arise from the VA Z_1 term above, and provides the basic reason why parity-violating electron scattering probes the weak neutral current in an especially clean way.

The Relativistic Fermi Gas (RFG)

Inclusive Electron Scattering

Following W. M. Alberico, A. Molinari,T. W. Donnelly, E. L. Kronenberg and J. W. Van Orden, *Phys. Rev.* **C34** (1988) 1801, one can write the inclusive electron scattering cross section for scattering of an electron with initial 4-momentum $K^\mu = (\epsilon, \mathbf{k})$ to a final-state scattered electron with 4-momentum $K'^\mu = (\epsilon', \mathbf{k}')$ through scattering angle θ in the form

$$\frac{d^2\sigma}{d\Omega d\epsilon'} = \sigma_M \left[v_L R_L + v_T R_T \right],$$

where

$$\sigma_M = \left[\frac{\alpha \cos\theta/2}{2\epsilon \sin^2\theta/2} \right]^2$$

is the Mott cross section and the lepton Rosenbluth kinematic factors in the extreme relativistic limit are given by

$$\begin{aligned} v_L &= \left| \frac{Q^2}{q^2} \right|^2 \\ v_T &= \frac{1}{2} \left| \frac{Q^2}{q^2} \right| + \tan^2\theta/2, \end{aligned}$$

where the 4-momentum transfer is given by

$$Q^\mu = K^\mu - K'^\mu = (\omega, \mathbf{q})$$

with

$$Q^2 = \omega^2 - q^2 < 0 \text{ (spacelike)}.$$

The factors R_L and R_T are the longitudinal and transverse inclusive response functions, respectively. They are obtained from the hadronic (nuclear) tensor $W^{\mu\nu}$ through the relationships

$$\begin{aligned} R_L &= W^{00} \\ R_T &= W^{11} + W^{22}. \end{aligned}$$

As discussed above, the single-nucleon (eN) elastic scattering hadronic tensor for scattering from a nucleon moving with 4-momentum P^μ is given by

$$W^{\mu\nu}_{eN}(P' = P + Q, P) = -W_1(\tau)\left[g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right] + W_2(\tau)U^\mu U^\nu$$

with

$$U^\mu = \frac{1}{m_N}\left[P^\mu - \frac{P\cdot Q}{Q^2}\right]$$

and with invariant structure functions

$$
\begin{aligned}
W_1(\tau) &= \tau G_M^2(\tau) \\
W_2(\tau) &= \frac{1}{1+\tau}\left(G_E^2(\tau) + \tau G_M^2(\tau)\right)
\end{aligned}
$$

namely, functions of the dimensionless 4-momentum transfer $\tau = |Q^2|/4m_N^2$. Upon conserving 4-momentum, promoting a particle from a filled Fermi sea to above it (making a 1p-1h state) and integrating over the Fermi sea one obtains an expression for the RFG hadronic tensor:

$$
\begin{aligned}
W^{\mu\nu}_{RFG} &= \frac{3\kappa}{4\pi\xi_F} R_0 \int \frac{d^3p}{E(\mathbf{p})E(\mathbf{p}+\mathbf{q})} W^{\mu\nu}_{eN}(P' = P + Q, P) \\
&\times \theta(k_F - |\mathbf{p}|)\theta(|\mathbf{p}+\mathbf{q}| - k_F)\delta\left[\omega - (E(\mathbf{p}+\mathbf{q}) - E(\mathbf{p}))\right],
\end{aligned}
$$

where the overall factor R_0 is given by

$$
R_0 = \frac{\mathcal{N}\xi_F}{m_N \kappa \eta_F^3}
$$

with $\mathcal{N} = Z, N$ the number of nucleons in the nucleus taking part in the interaction (one must sum over protons with a factor Z and add this to the sum over neutron with factor N).

Here and below we use the dimensionless variables

$$
\begin{aligned}
\lambda &\equiv \omega/2m_N \\
\kappa &\equiv q/2m_N \\
\eta_F &\equiv k_F/m_N \\
\epsilon_F &\equiv \sqrt{1+\eta_F^2} \\
\xi_F &\equiv \sqrt{1+\eta_F^2}-1
\end{aligned}
$$

with $\tau = \kappa^2 - \lambda^2 > 0$. After a bit of work one can perform the integrals above to obtain complete analytic expressions for the RFG response functions:

$$R_{L,T} = R_0 \widetilde{f}_{RFG} U_{L,T}$$

The function $\widetilde{f}_{RFG}$ is given by

$$\widetilde{f}_{RFG} = \frac{3}{4\xi_F}\left(\epsilon_F - \Gamma\right)\theta\left(\epsilon_F - \Gamma\right),$$

where

$$\Gamma = \begin{cases} \epsilon_F - 2\lambda & q \le 2k_F \qquad \text{Pauli}-\text{blocked} \\ \gamma_- = \kappa\sqrt{1+1/\tau} - \lambda = 1 + \xi_F\psi^2 & q \ge 2k_F \quad \text{non}-\text{Pauli}-\text{blocked} \end{cases}$$

In the latter case one obtains the scaling function f_{RFG} :

$$\widetilde{f}_{RFG} \underset{npb}{\rightarrow} f_{RFG} = \frac{3}{4}(1-\psi^2)\theta(1-\psi^2),$$

where the scaling variable is given by

$$\psi = \frac{1}{\sqrt{\xi_f}} \frac{\lambda - \tau}{\sqrt{\tau(1+\lambda) + \kappa\sqrt{\tau(1+\tau)}}}$$

The factors U are

$$\begin{aligned} U_L &= \frac{\kappa^2}{\tau}\left[G_E^2 + W_2\Delta\right] \\ U_T &= 2\tau G_M^2 + W_2\Delta \end{aligned}$$

where the following definition (the result of performing the integrations over the Fermi sea above) has been used:

$$\Delta = \frac{\tau}{\kappa^2}\left[\frac{1}{3}\left(\epsilon_F^2 + \epsilon_F\Gamma + \Gamma^2\right) + \lambda(\epsilon_F + \Gamma) + \lambda^2\right] - (1+\tau)\,.$$

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Note: The RFG has also been derived starting from a spectral function.
See R. Cenni, T. W. Donnelly and A. Molinari, *Phys. Rev.* **C56** (1997) 276.

Charge-changing Quasielastic (CCQE) Reactions

The developments are completely analogous to the inclusive electron scattering case discussed above (see J. E. Amaro, M. B. Barbaro, J. A. Caballero, T. W. Donnelly, A. Molinari and I. Sick, *Phys. Rev.* **C71** (2005) 015501). The CCQE response functions are simply the extensions of $R_{L,T}$ derived above:

$$R_X^Y = R_0 \widetilde{f}_{RFG} U_X^Y$$

where the superscripts Y represent the vector and axial-vector combinations of the responses (VV, AA, VA) and the subscripts X represent their spacetime projections $(CC, CL, \ldots)$. The overall factor R_0 is as above but now with only $\mathcal{N} = N(Z)$ for neutrinos (anti-neutrinos). The purely isovector factors U are

$$\begin{aligned}
U_L^{VV} &= \frac{\kappa^2}{\tau}\left((G_E^{(1)})^2 + W_2\Delta\right)\\
U_T^{VV} &= 2\tau(G_M^{(1)})^2 + W_2\Delta\\
U_{CC}^{AA} &= \frac{\kappa^2}{\tau}\left(\frac{\lambda^2}{\kappa^2}(G_A'^{(1)})^2 + (G_A^{(1)})^2\Delta\right)\\
U_{LL}^{AA} &= \frac{\kappa^2}{\tau}\left((G_A'^{(1)})^2 + \frac{\lambda^2}{\kappa^2}(G_A^{(1)})^2\Delta\right)\\
U_{CL}^{AA} &= -\frac{\kappa\lambda}{\tau}\left((G_A'^{(1)})^2 + (G_A^{(1)})^2\Delta\right)\\
U_{TT}^{AA} &= (G_A^{(1)})^2\left[2(1+\tau) + \Delta\right]\\
U_{T'}^{VA} &= 2\sqrt{\tau(1+\tau)}H_{VA}^{(1)}\left[1 + \Delta'\right]
\end{aligned}$$

where, in addition to Δ defined above, the following definition has been used:

$$\Delta' = \frac{1}{\kappa}\sqrt{\frac{\tau}{1+\tau}}\left(\lambda + \frac{1}{2}(\epsilon_F + \Gamma)\right) - 1.$$

The isovector nucleon form factors $G_E^{(1)} = G_{E_p} - G_{E_n}$, $G_M^{(1)} = G_{M_p} - G_{M_n}$, $G_A^{(1)}$ and $G_P^{(1)}$ enter above through the combinations

$$\begin{aligned}
W_2 &= \frac{1}{1+\tau}\left((G_E^{(1)})^2 + \tau(G_M^{(1)})^2\right) \\
G_A'^{(1)} &= G_A^{(1)} - \tau G_P^{(1)} \\
H_{VA}^{(1)} &= G_M^{(1)} G_A^{(1)}
\end{aligned}$$

By defining the following components of the matrix element squared

$$\begin{aligned}
X_L &= V_L R_L^{VV} \\
X_{CL} &= V_{CC} R_{CC}^{AA} + 2V_{CL} R_{CL}^{AA} + V_{LL} R_{LL}^{AA} \\
X_T &= V_T (R_T^{VV} + R_T^{AA}) \\
X_{T'} &= 2V_{T'} R_{T'}^{VA}
\end{aligned}$$

which contain leptonic tensor components V given by

$$
\begin{aligned}
V_{CC} &= 1 - \tan^2\tilde{\theta}/2 \cdot \delta^2 \\
V_{CL} &= \nu + \tan^2\tilde{\theta}/2 \cdot \frac{\delta^2}{\rho'} \\
V_{LL} &= \nu^2 + \tan^2\tilde{\theta}/2 \cdot \left(1 + \frac{2\nu}{\rho'} + \rho\delta^2\right) \cdot \delta^2 \\
V_L &= V_{CC} - 2\nu V_{CL} + \nu^2 V_{LL} \\
V_T &= \frac{1}{2}\rho + \tan^2\tilde{\theta}/2 - \frac{1}{\rho'}\tan^2\tilde{\theta}/2 \cdot \left(\nu + \frac{1}{2}\rho\rho'\delta^2\right) \cdot \delta^2 \\
V_{T'} &= \frac{1}{\rho'}\tan^2\tilde{\theta}/2 \cdot (1 - \nu\rho'\delta^2)
\end{aligned}
$$

and involving the following additional dimensionless variables:

$$
\begin{aligned}
\nu &\equiv \omega/q \\
\rho &\equiv 1 - \nu^2 \\
\rho' &\equiv q/(\epsilon + \epsilon') \\
\delta &= m'/\sqrt{|Q^2|} \\
v_0 &\equiv (\epsilon + \epsilon')^2 - q^2 \\
\tan^2\tilde{\theta}/2 &= |Q^2|/v_0
\end{aligned}
$$

Note that now one in general should not invoke the extreme relativistic limit for an outgoing muon, but should retain its mass, leading to the somewhat more complicated lepton kinematic factors above. The ERL is recoveded simply by setting δ to zero and $\tilde{\theta}$ to θ.

The CCQE cross section in the RFG model is then

$$\frac{d\sigma}{d\Omega dk} = \frac{G_F^2 \cos^2\theta_C}{2\pi^2}\frac{k'^2 v_0}{4\epsilon\epsilon'}\left[X_L + X_{CL} + X_T + \chi X_{T'}\right],$$

where θ_C is the Cabbibo angle and where $\chi = 1$ for neutrino scattering and $\chi = -1$ for antineutrino scattering.

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In this talk I will freely switch between EM responses and CC/NC weak interaction responses.

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3. Dynamical effects in the wave functions themselves

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1. Kinematic effects:

At high energies the final-state ejected nucleon should obey relativistic kinematics, $E = (p^2 + m^2)^{1/2}$ when on-shell. Of course, when interacting the initial- and final-state nucleons in the nucleus are off-shell. A non-relativistic model can be roughly relativized for such effects by replacing the energy transfer ω by $\omega\ (1 + \omega/2m)$, which places the QE peak at essentially the correct position, namely, $|Q^2|/2m$ rather than $q^2/2m$.

Relativistic effects arise from three sources (which are not distinct):

1. Kinematic effects
2. **Boost effects on the single-particle current matrix elements**
3. Dynamical effects in the wave functions themselves

2. Boost effects on the single-particle current matrix elements:

When making a non-relativistic approximation to the (on-shell) single-particle matrix elements of the vector and axial-vector currents there are boost factors that should be included. To leading order these are multiplicative factors typically γ or $1/\gamma$, where $\gamma = |q^2/Q^2|$.

So, for instance the charge response is enhanced by the factor γ (note that this becomes very large as one approaches the lightcone where $\omega = q$ and so Q^2 goes to zero); this is a Lorentz contraction effect on the charge density. The transverse response goes the other way, namely, is decreased by the factor $1/\gamma$.

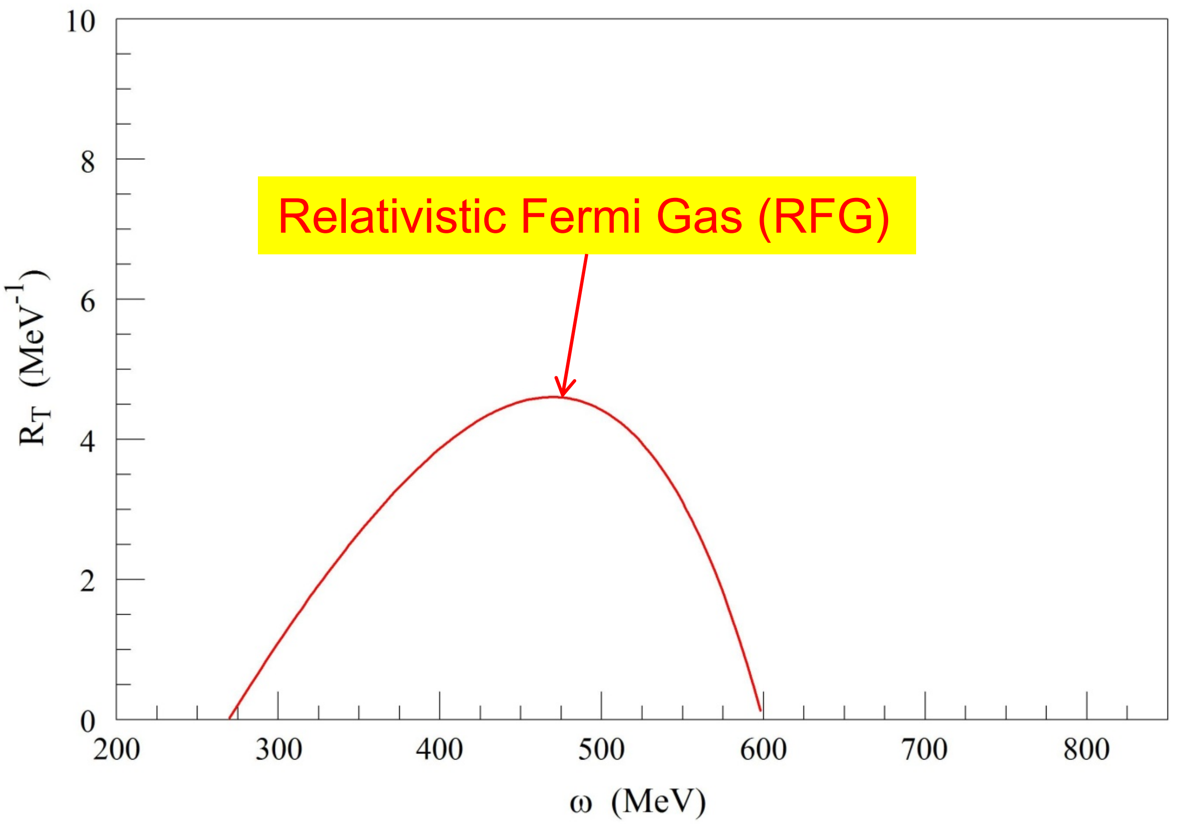
Relativistic effects arise from three sources (which are not distinct):

1. Kinematic effects
2. Boost effects on the single-particle current matrix elements
3. **Dynamical effects in the wave functions themselves**

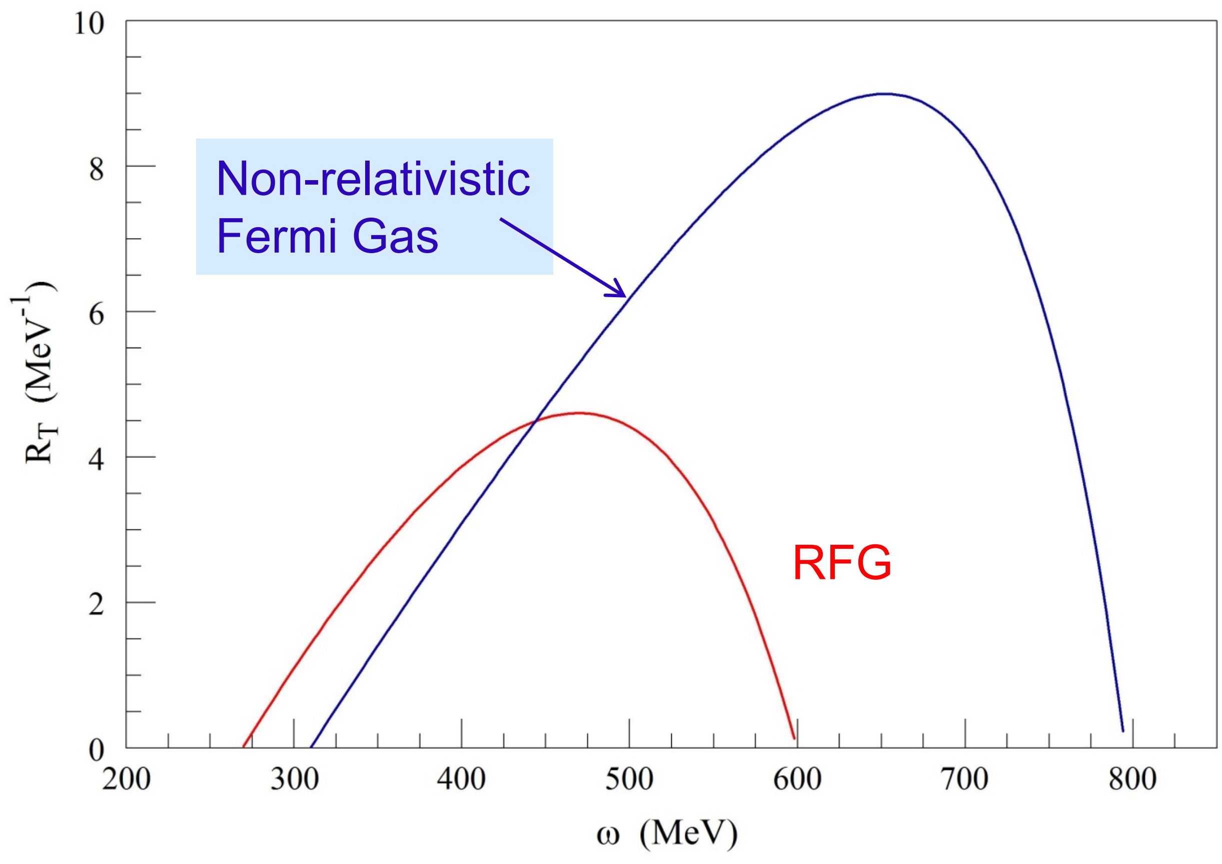
3. Dynamical effects in the wave functions themselves:

The initial-and final-state nucleons in the nucleus are interacting and are therefore off-shell. When relativistic bound and scattering wave functions are employed (for instance in a Dirac Hartree approach) the lower components of the 4-spinors are not related to the upper components by the free-particle relationship and this is manifested in the electroweak responses; typically these amount to 15-20% differences between the various types of response, namely, violations of the so-called scaling of the **zeroth kind** where all of the various responses (longitudinal, vector transverse, axial transverse, VA interference, etc.) scale to a universal function.

Transverse vector response at q = 1 GeV/c



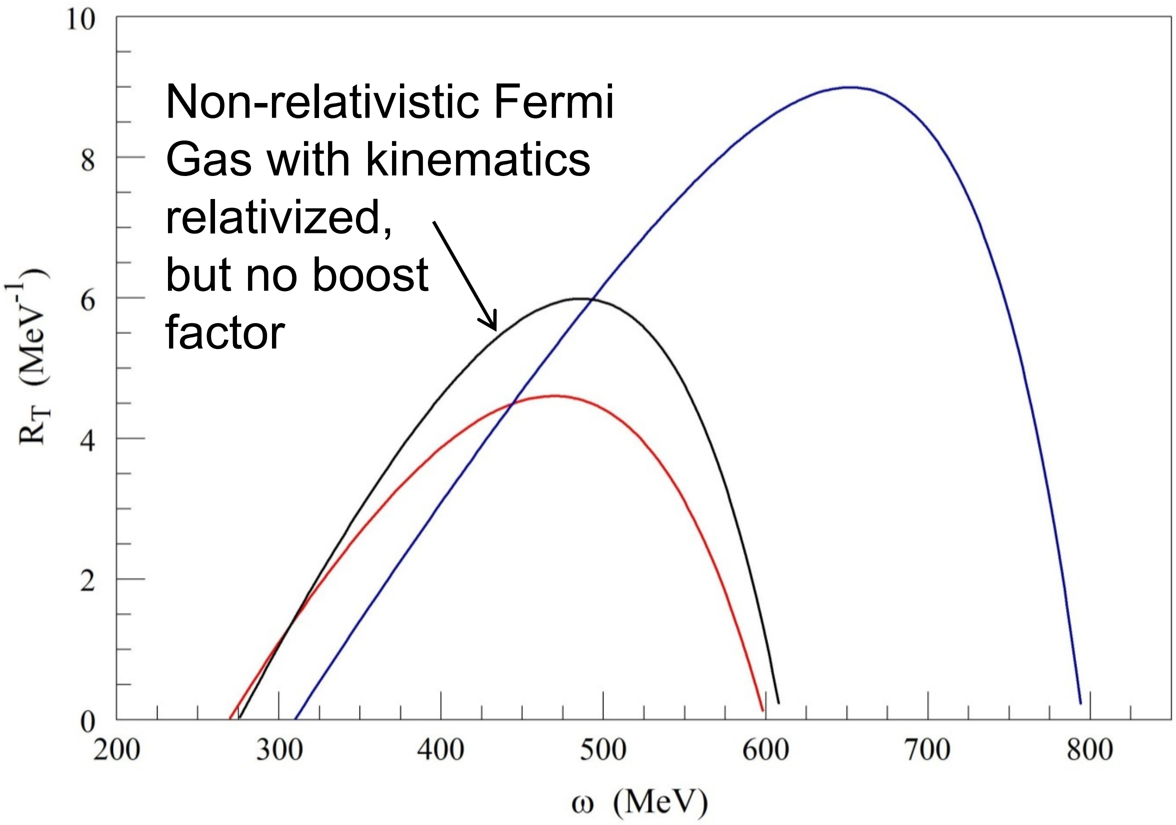
Transverse vector response at q = 1 GeV/c



As an approximation, one can consider “semi-relativistic” modeling where, starting with a non-relativistic model, two steps are made:

1. The kinematic shift introduced above is implemented,
   placing the QE peak in roughly the correct position

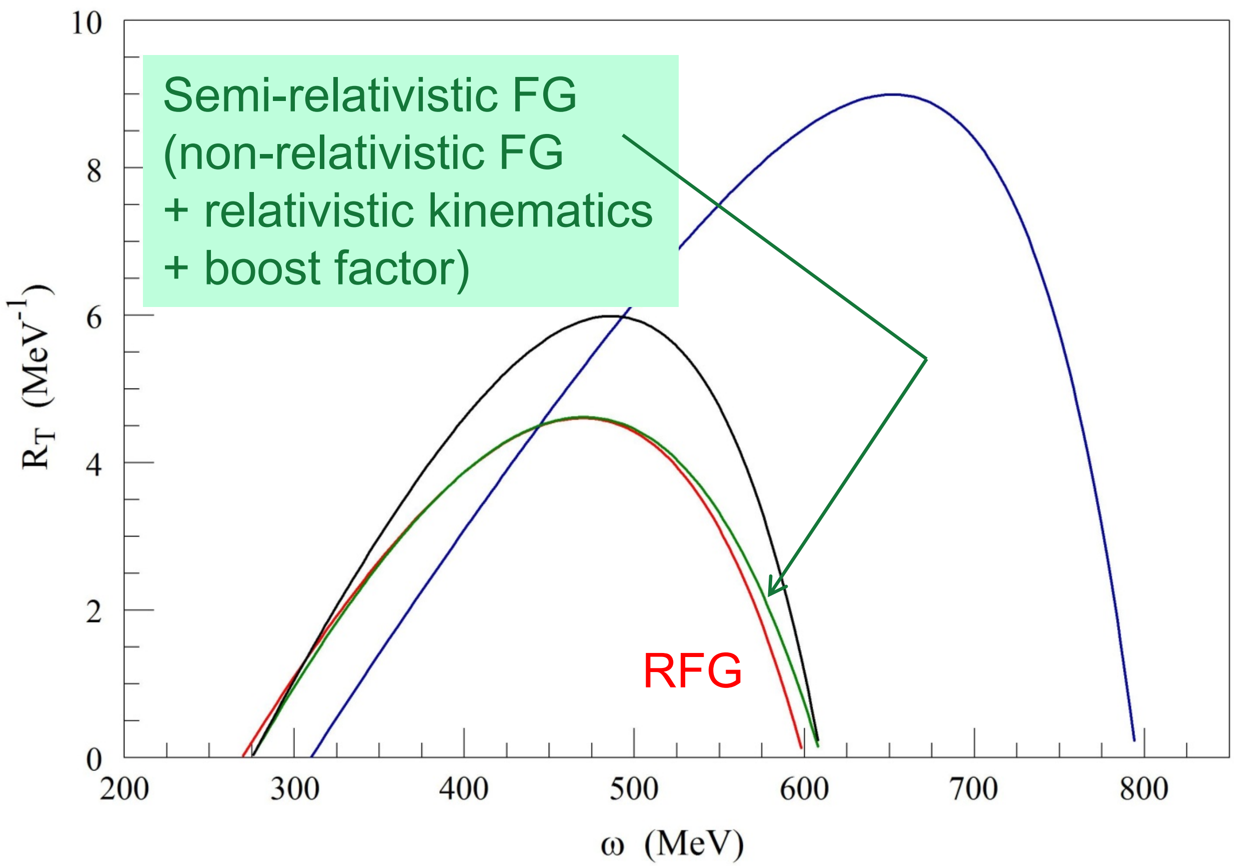
Transverse vector response at q = 1 GeV/c



As an approximation, one can consider "semi-relativistic" modeling where, starting with a non-relativistic model, two steps are made:

1. The kinematic shift introduced above is implemented, placing the QE peak in roughly the correct position
2. The boost factors are included in leading order

Transverse vector response at q = 1 GeV/c



Scaling phenomena are seen in many quantum many-body systems:

- Condensed matter physics (electron scattering, neutron scattering)
- Nuclear physics (lepton scattering, hadron scattering from nucleons)
- Particle physics (lepton-parton scattering)

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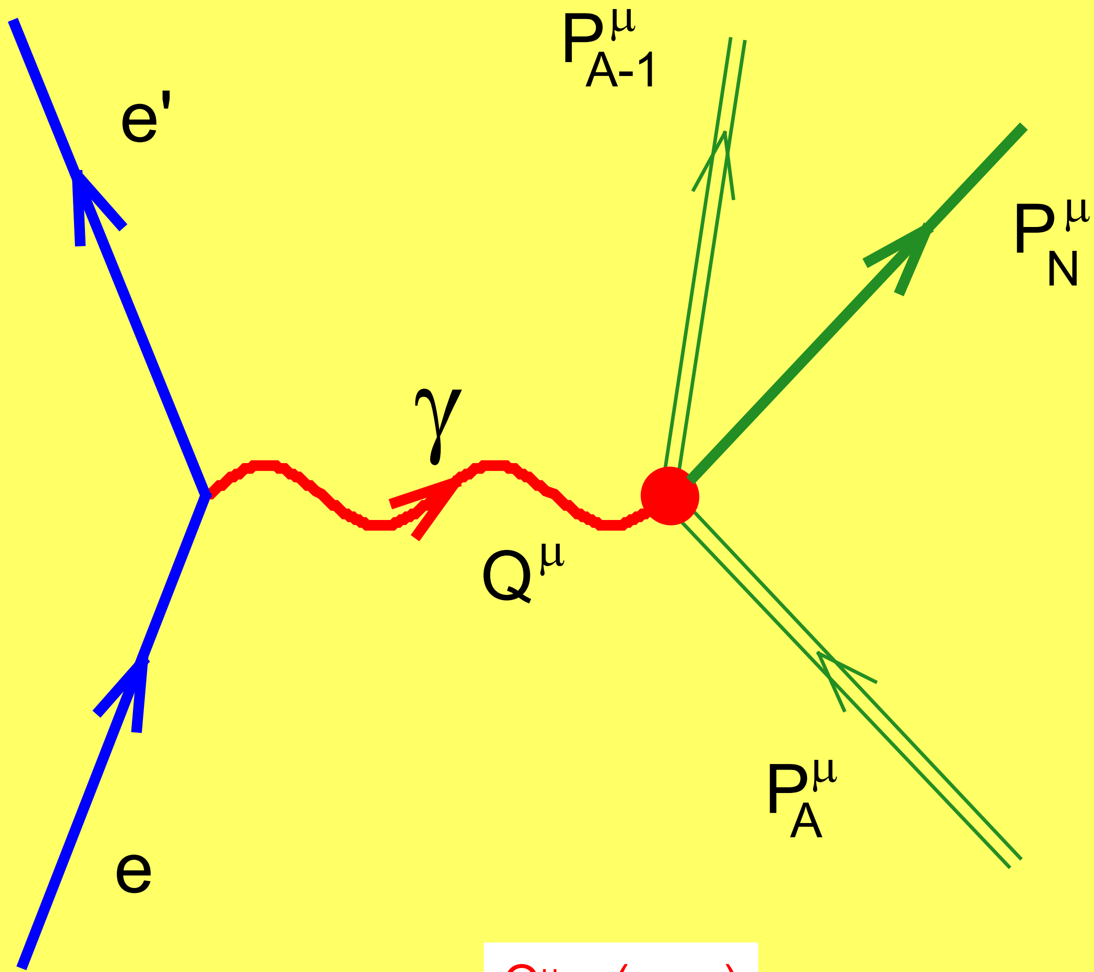
… in this talk I will focus on lepton scattering from nuclei

Begin by assuming that QE scattering is dominated by (e,e´N):



$Q^{\mu} = (\omega, \mathbf{q})$

The daughter nucleus has 4-momentum

$$P_{A-1}^{\mu} = (E_{A-1}, \mathbf{p}_{A-1}) = Q^{\mu} + P_A^{\mu} - P_N^{\mu}$$

In the lab. system we define the **missing momentum**

$$p = |\mathbf{p}| \equiv |\mathbf{p}_N - \mathbf{q}| = |\mathbf{p}_{A-1}|$$

and an “excitation energy” (essentially **missing energy** – separation energy)

$$\mathcal{E}(p) \equiv \sqrt{(M_{A-1})^2 + p^2} - \sqrt{(M_{A-1}^0)^2 + p^2}$$

where

$$M_{A-1}^0 = M_A^0 - m_N + E_s$$

with E_s the separation energy and M^0_{A-1} the daughter rest mass

Energy conservation gives

$$
\begin{aligned}
M_A^0 + \omega &= E_N + E_{A-1} \\
&= \sqrt{m_N{}^2 + p_N{}^2} + E_{A-1}^0 + \mathcal{E} \\
&= \sqrt{m_N{}^2 + (\mathbf{q} + \mathbf{p})^2} + \sqrt{(M_{A-1}^0)^2 + p^2} + \mathcal{E}
\end{aligned}
$$

which can be turned around to yield an expression for the excitation energy:

$$
\mathcal{E} = M_A^0 + \omega - \sqrt{(M_{A-1}^0)^2 + p^2} - \sqrt{m_N{}^2 + q^2 + p^2 + 2pq\cos\theta}
$$

One can let the angle between *p* and *q* vary over all values and impose the constraints

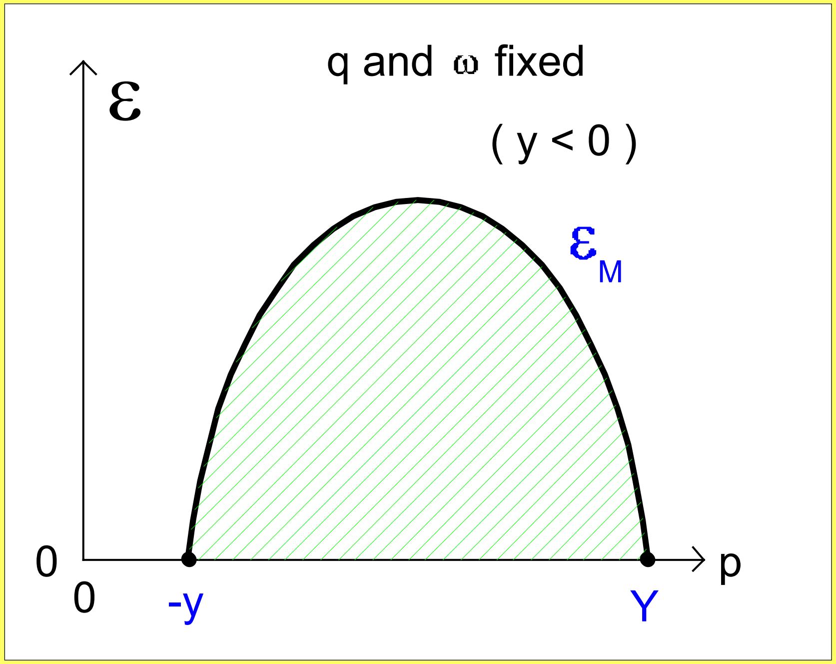
$$p \geq 0$$

$$\varepsilon \geq 0$$

to find the allowed region in the missing-energy, missing-momentum plane. When

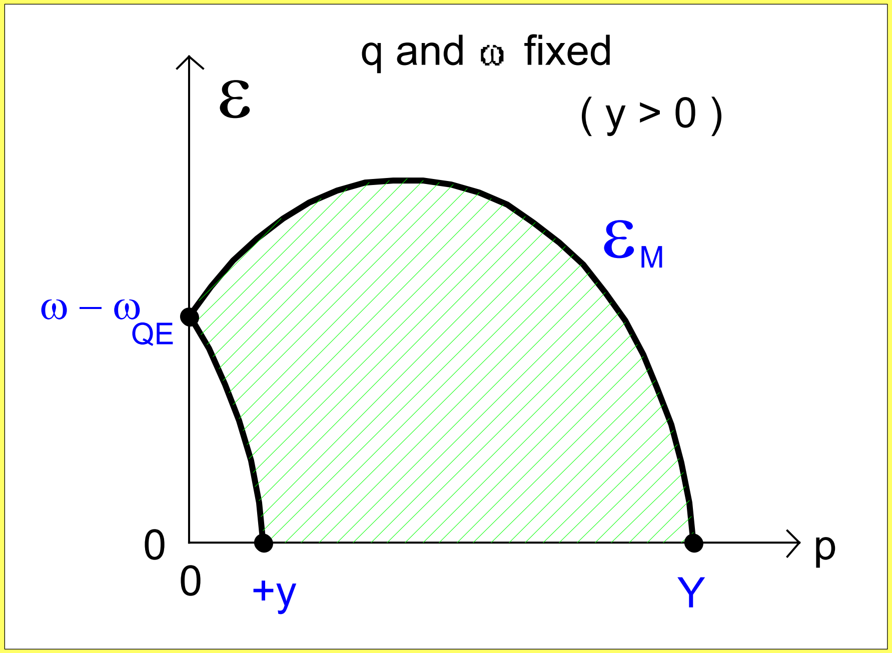
$$\omega < \omega_{QE} = \left|Q^2\right| / 2m_N$$

one finds



… and when

$$\omega > \omega_{QE} = |Q^2| / 2m_N$$ one has



where one has the smallest and largest values of the missing momentum at zero excitation energy occurring at

$$y = \frac{1}{2W^2}\left[\alpha - \beta\right]$$

$$Y = \frac{1}{2W^2}\left[\alpha + \beta\right]$$

with

$$W = \sqrt{(M_A^0 + \omega)^2 - q^2} \geq W_T = M_{A-1}^0 + m_N$$

$$\alpha = (M_A^0 + \omega)\sqrt{W^2 - W_T^{\,2}}\sqrt{W^2 - (W_T - 2m_N)^2}$$

$$\beta = q\left[W^2 + W_T\left(W_T - 2m_N\right)\right]$$

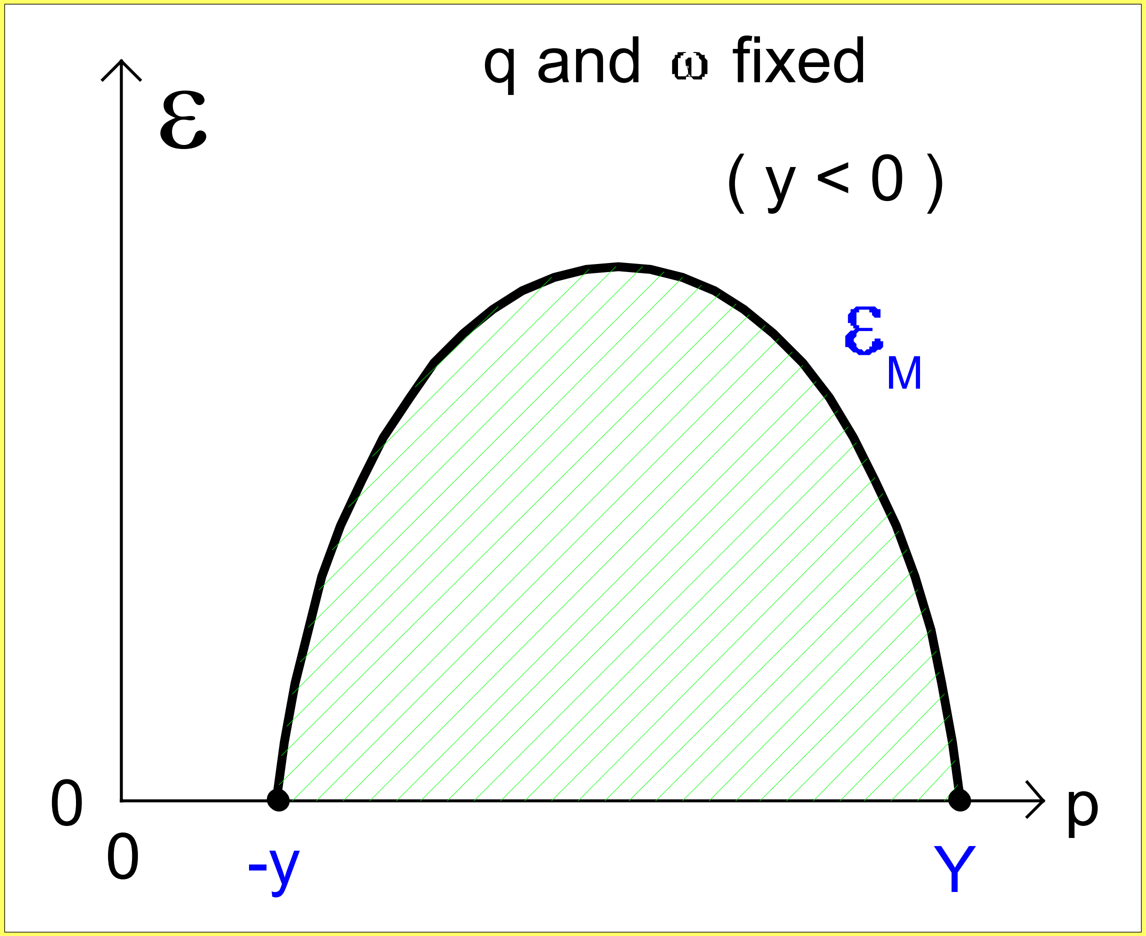
The so-called **y-scaling variable** is approximately given by

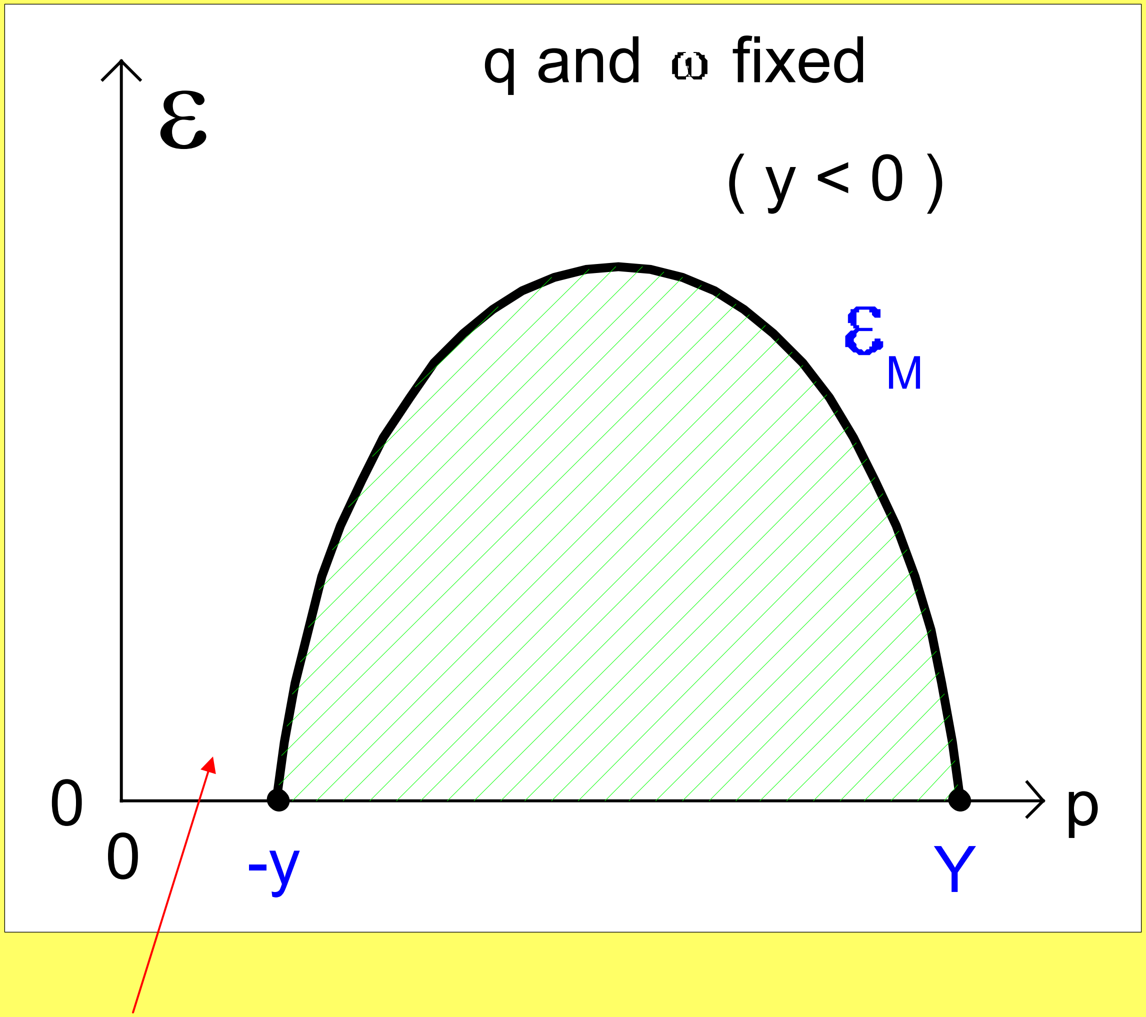
$$y \cong \sqrt{\nu(2m_N + \nu)} - q$$

$$\nu \equiv \omega - E_s$$

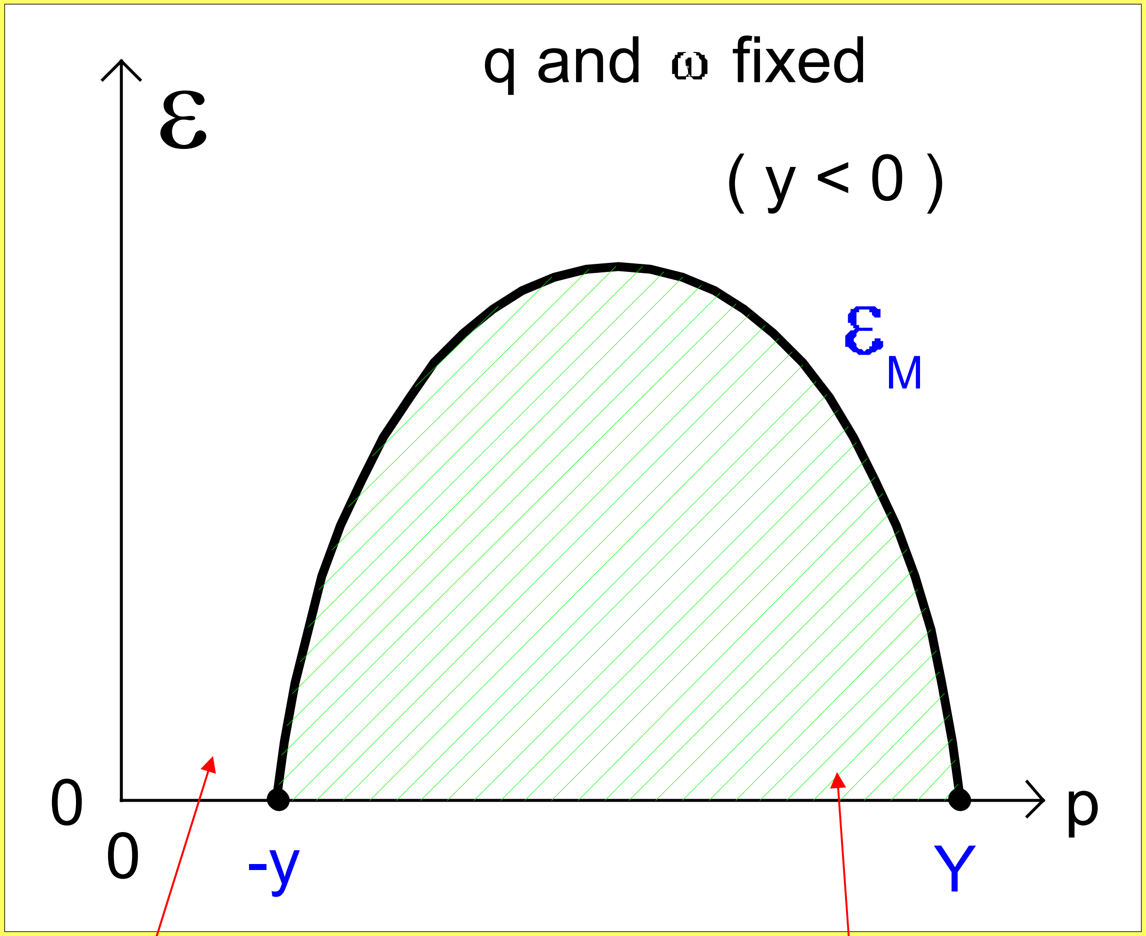
Scaling of the 1st Kind

- First, one uses ***(q,y)*** rather than ***(q,ω)*** for the functional dependence of the inclusive cross section. The inclusive cross section is assumed to be the sum of the integrals over the semi-inclusive (e,e'p) and (e,e'n) cross sections, *i.e.,* over the momentum of the ejected nucleon $\mathbf{p}_N$. These can be turned into integrals over p and ε covering the regions discussed above.


q and ω fixed
(y < 0)
ε
ε_M
0
0
-y
Y
p



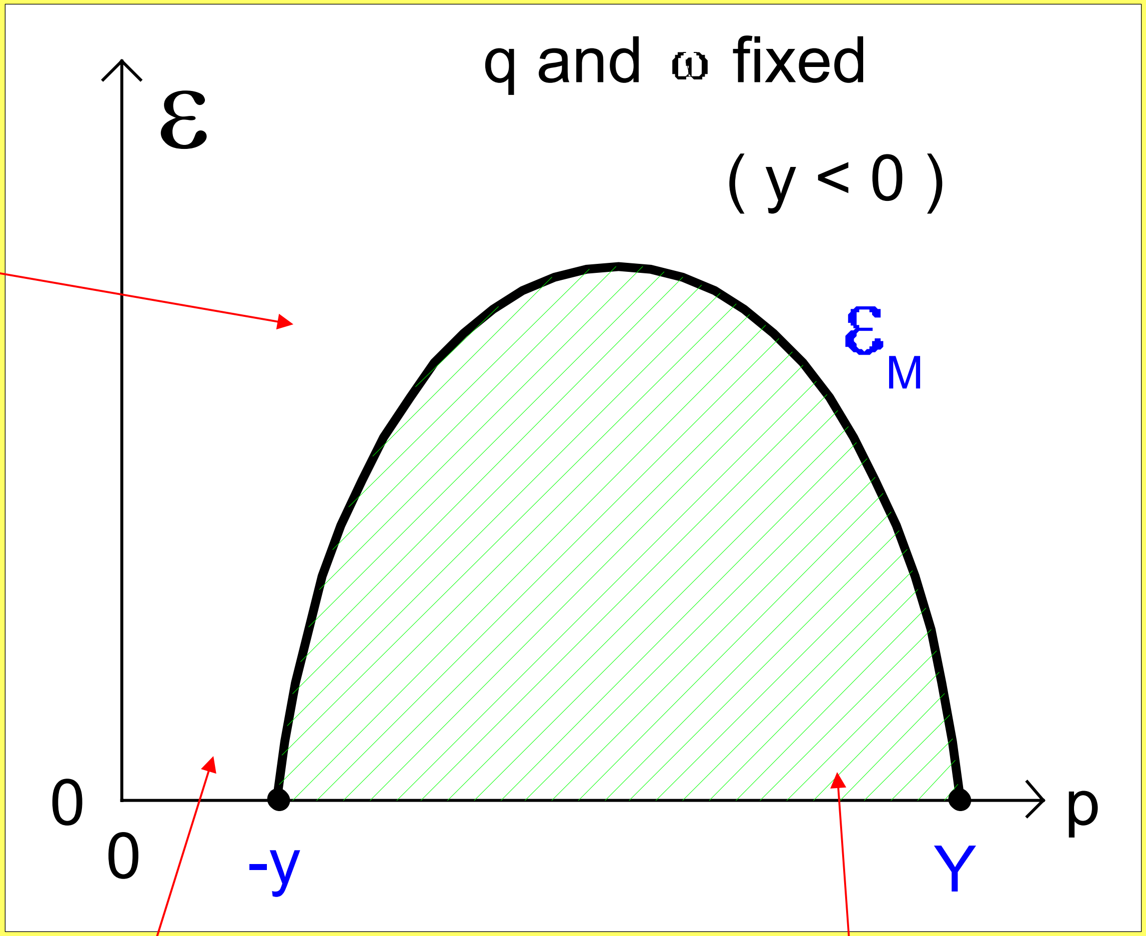
The semi-inclusive cross section is
typically largest at small **p** and ε



The semi-inclusive cross section is typically largest at small **p** and ε

… and is very small at large **p** and small ε

For given **y<0** the region at small **p**, but high $\mathcal{E}$ is inaccessible



The semi-inclusive cross section is typically largest at small **p** and $\mathcal{E}$

… and is very small at large **p** and small $\mathcal{E}$

- First, one uses ***(q,y)*** rather than ***(q,ω)***

- Second, one notes that the typical parametrizations for the off-shell single-nucleon cross sections (functions of q, ω, p, ε, and ϕ_N) vary rather slowly as functions of (p, ε) for fixed (q, ω, ϕ_N). This suggests integrating over ϕ_N (leaving only L and T responses) and then removing the result evaluated at an "optimal" choice of p and ε.

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What is optimal?

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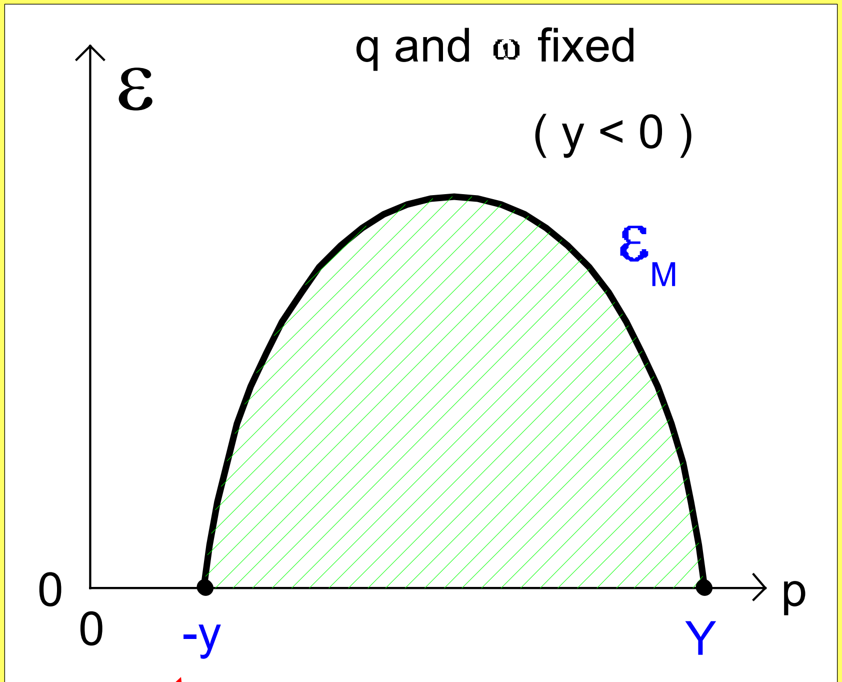
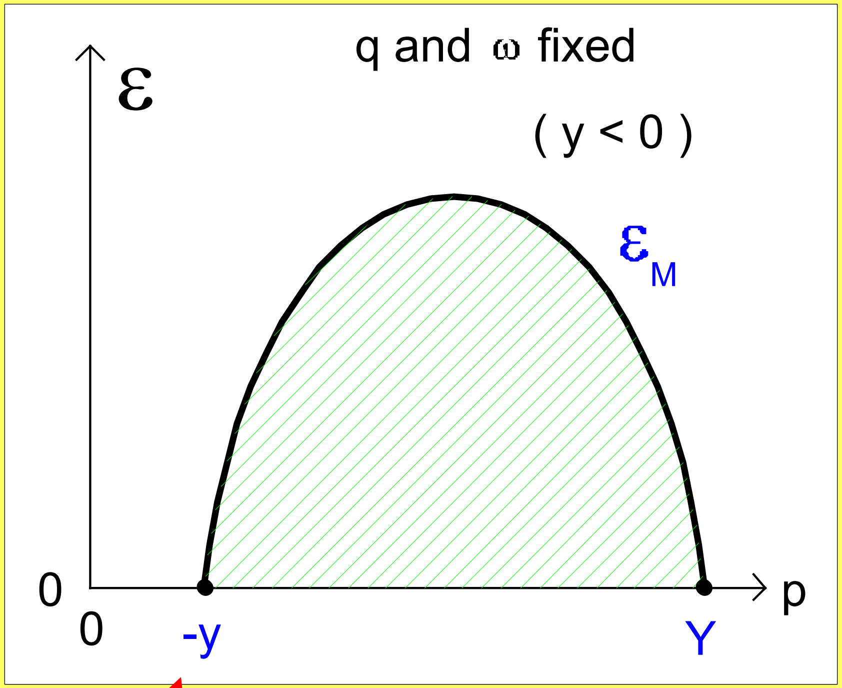
What is optimal?

From the discussions above one is led to a choice such as the one made in many analyses of scaling, namely, set **p to |y|** and ε **to 0**:

$$\Sigma_{eN}^{eff} = \frac{1}{A}\left[Z\overline{\sigma}_{ep}^{elastic} + N\overline{\sigma}_{en}^{elastic} \right]_{p=|y|,\,\varepsilon=0}$$



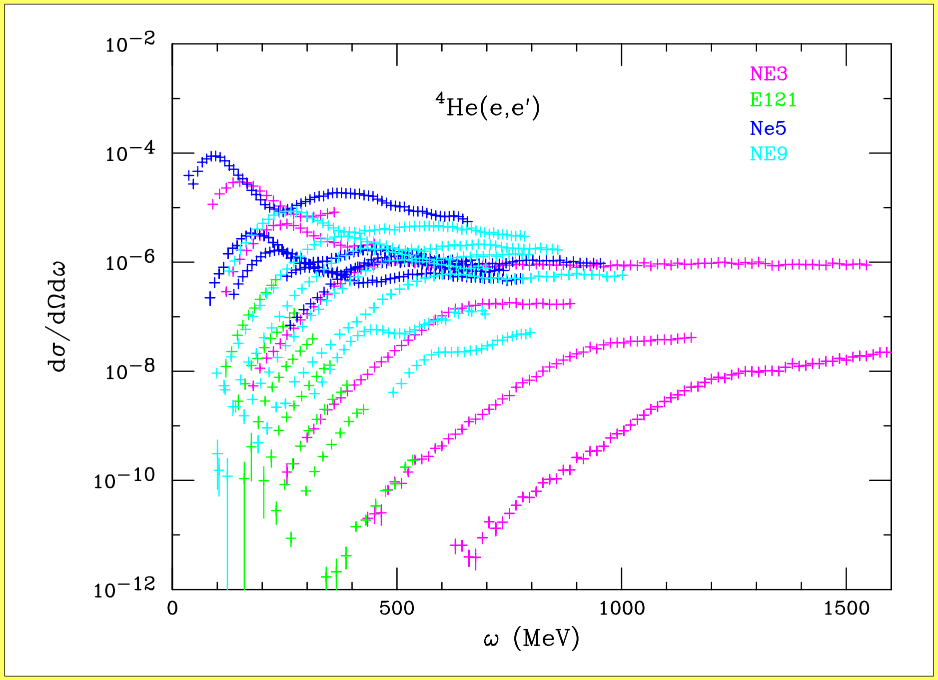
Evaluate the single-nucleon cross section at this point and remove from integral



Evaluate the single-nucleon cross section at this point and remove from integral

… then, dividing by the effective single-nucleon cross section leads to the definition of the **scaling function**:

$$F(q,y) \equiv \frac{d^2\sigma / d\Omega_e d\omega}{A\Sigma_{eN}^{eff}}$$



Example using ^{4}He data from SLAC:

when the inclusive cross section for various beam energies and electron scattering angles

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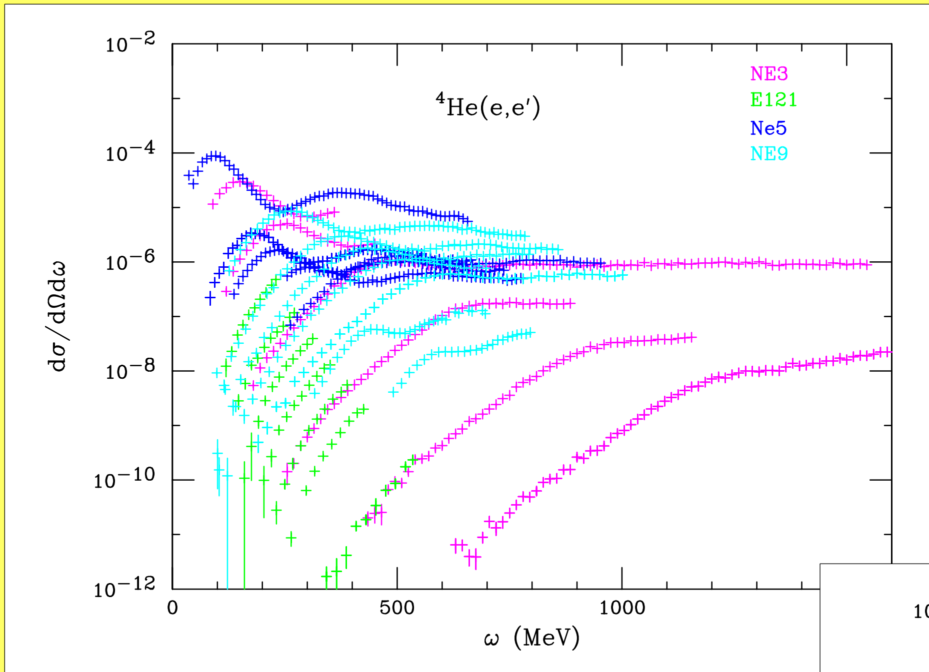
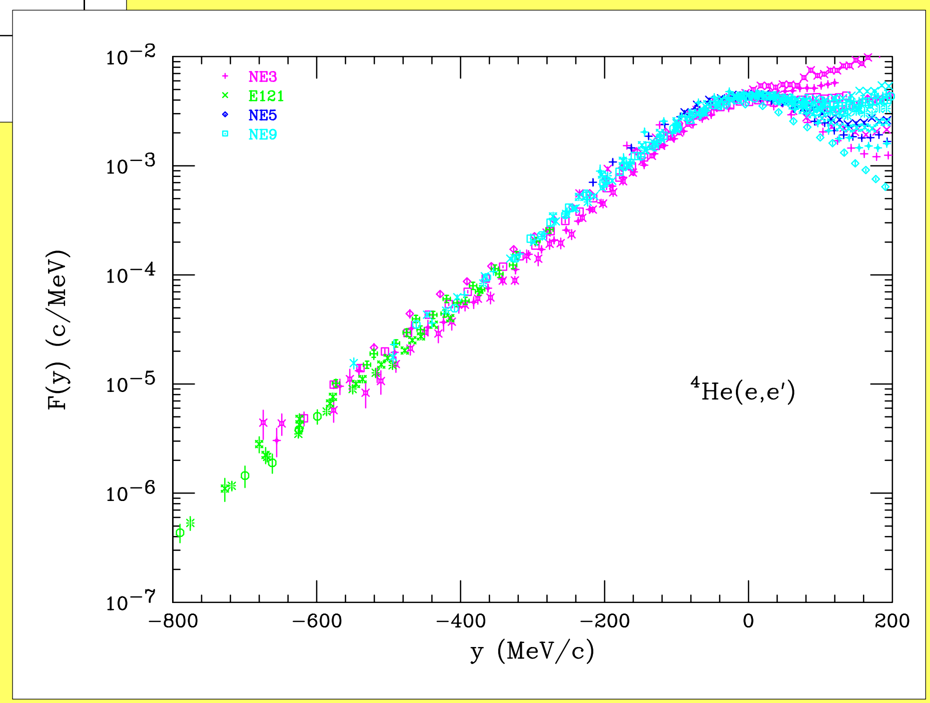


is used to obtain the function *F(q,y)*, and this is plotted as a function of *y* for various values of *q*, one finds



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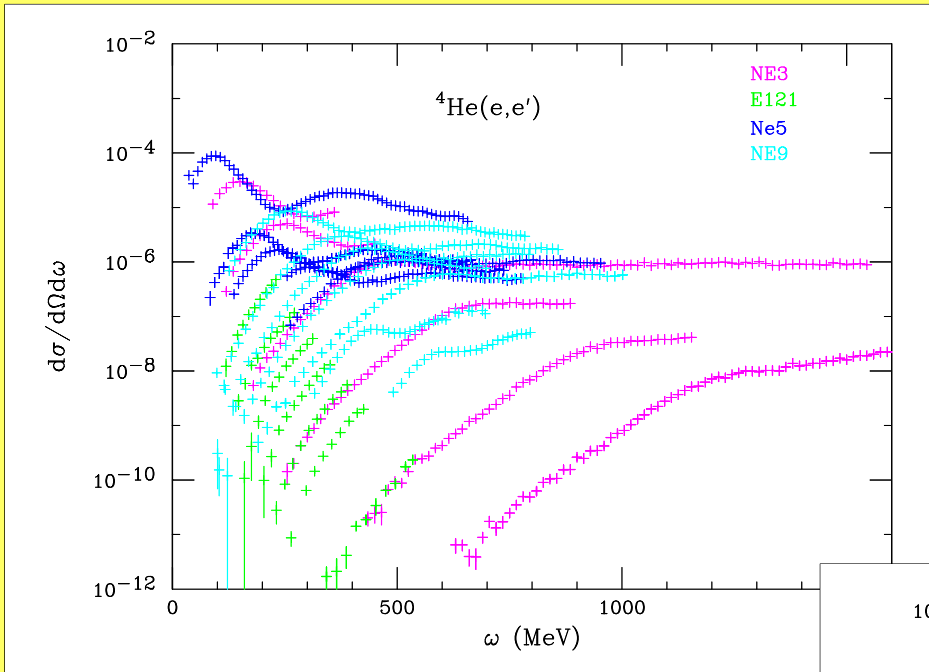
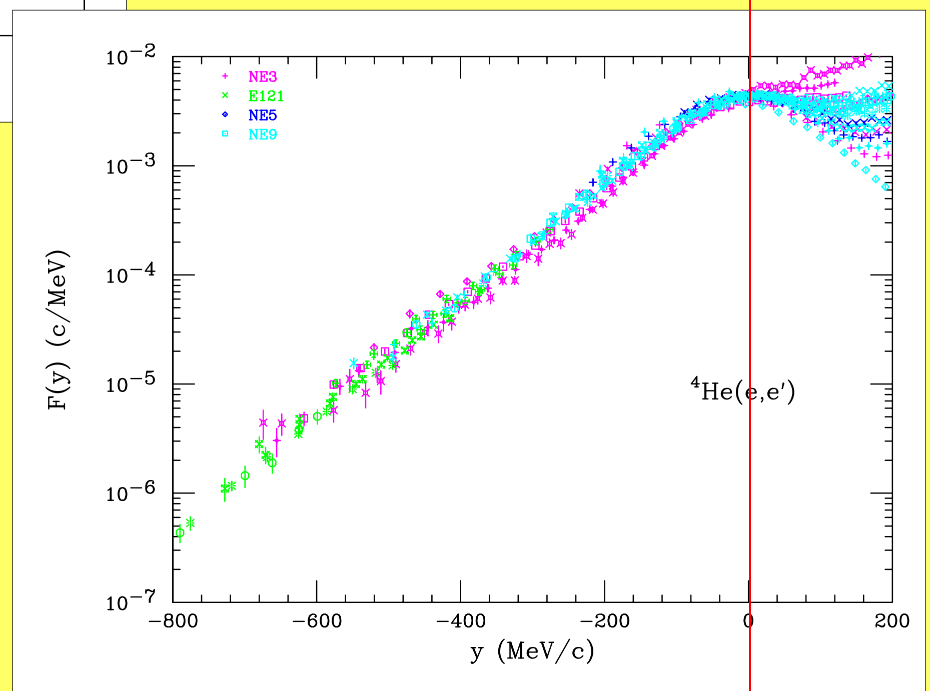


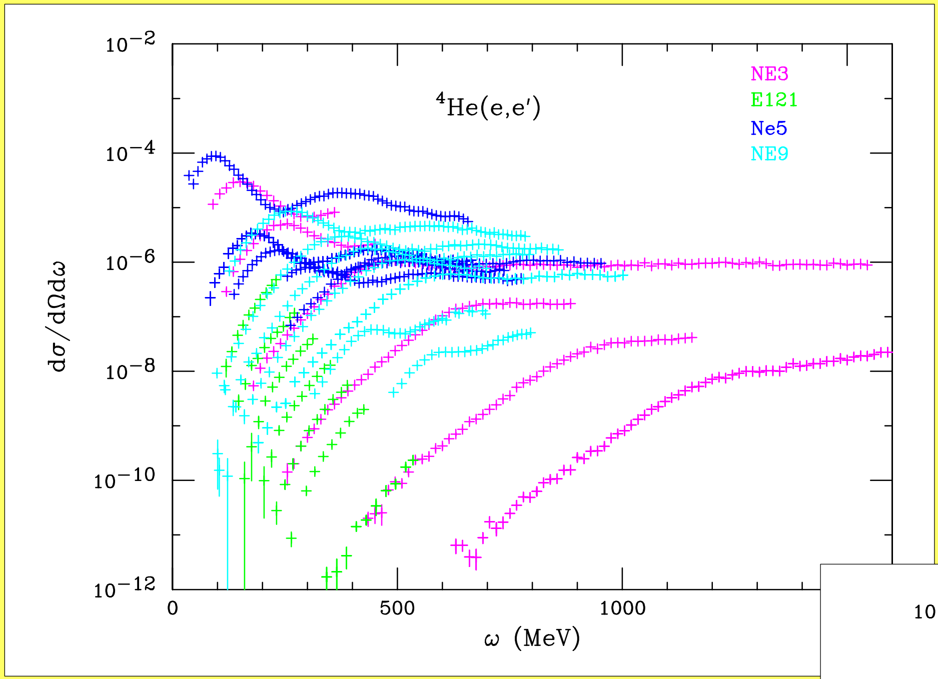
when the inclusive cross section for various beam energies and electron scattering angles

is used to obtain the function *F(q,y)*, and this is plotted as a function of *y* for various values of *q*, one finds

$\omega < \omega_{QE}$
(x > 1)

$\omega > \omega_{QE}$
(x < 1)





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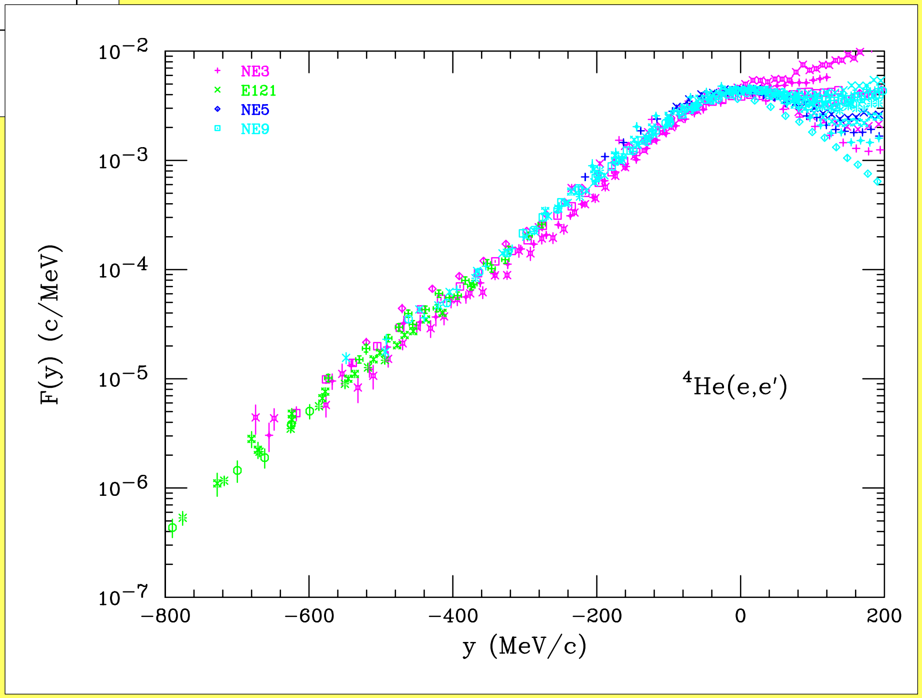


is used to obtain the function *F(q,y)*, and this is plotted as a function of *y* for various values of *q*, one finds

Independence of q

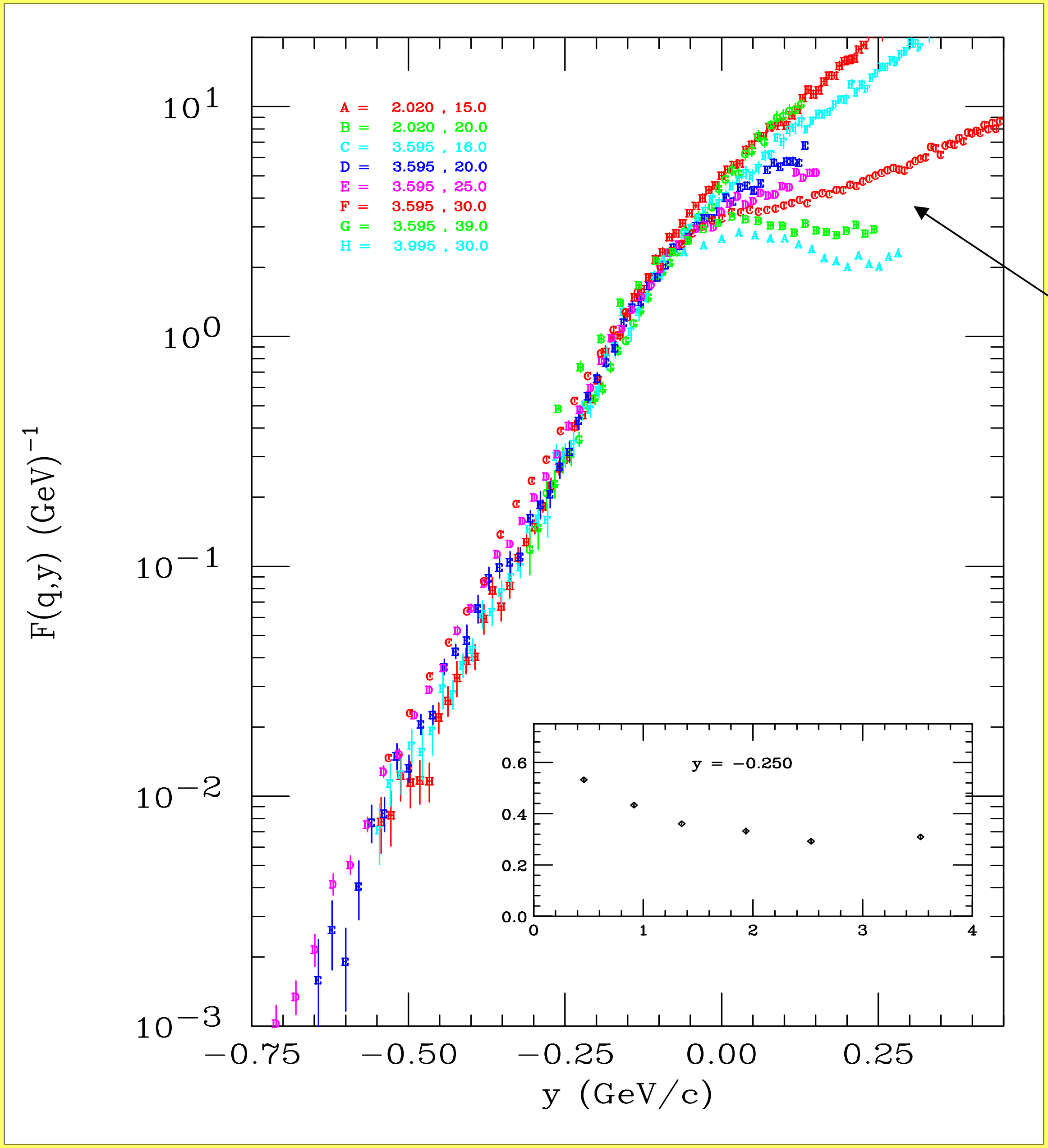
↕

SCALING OF THE 1st KIND

(y-scaling)

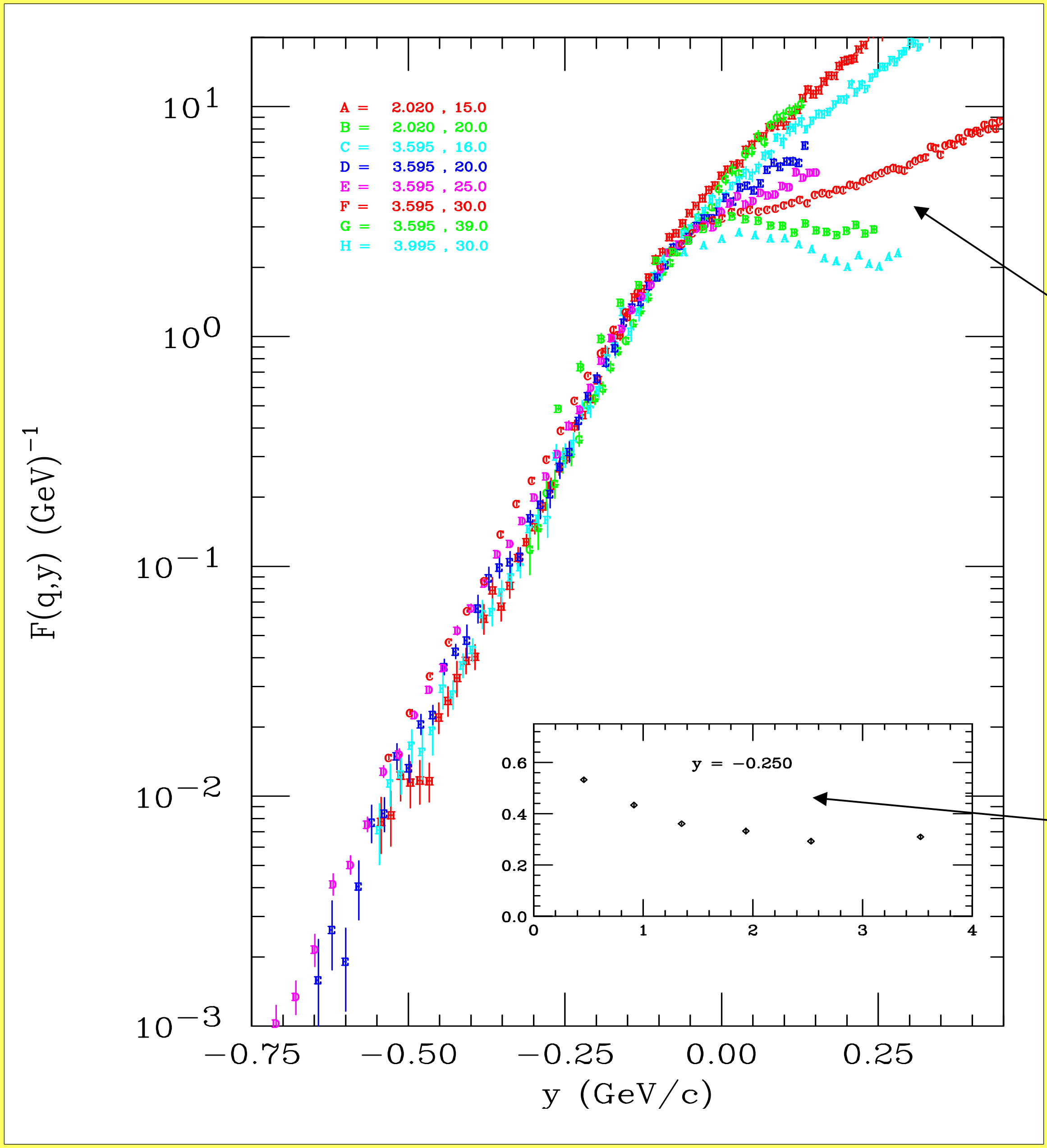
$$F(q,y) \xrightarrow[q\to\infty]{} F(y) \equiv F(\infty, y)$$

Example of ^{56}Fe



Note that at *y>0* the scaling is not good, due to the presence of resonances, meson production, *etc.* (see later, however)

Example of ^{56}Fe



Note that at *y>0* the scaling is not good, due to the presence of resonances, meson production, *etc.* (see later, however)

Scaling function at y = -250 MeV/c versus Q^2 in $(GeV/c)^2$

… approach to scaling

Next we introduce a characteristic momentum scale for a given nuclear species

$$k_A = \sqrt{\langle k^2 \rangle_A}$$

and use this to define a dimensionless function

$$f(q,y) \equiv k_A \bullet F(q,y)$$

Correspondingly, one wishes to introduce a dimensionless scaling variable ***ψ*** and then to plot ***f(q,ψ)*** versus *ψ* for various values of momentum transfer *q*

The **Relativistic Fermi Gas (RFG)** model is used to motivate the choice of scaling variable.

In the RFG one has

$$\left[k_A\right]^{RFG} = k_F$$

… and a dimensionless scaling variable ψ' which yields exact 1st-kind scaling for the RFG.

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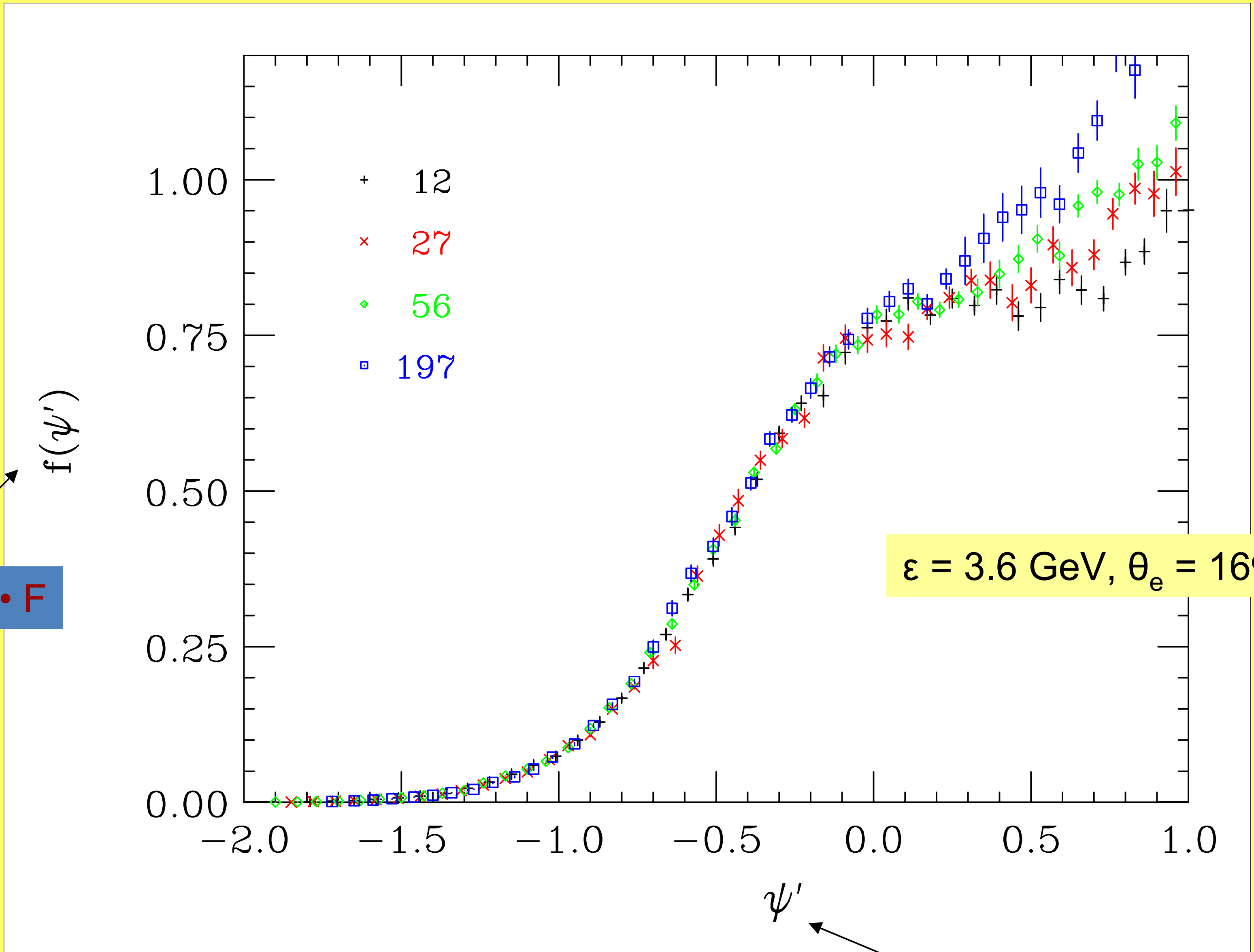
… and a dimensionless scaling variable ψ' which yields exact 1st-kind scaling for the RFG;

roughly $\psi' = y/k_A$

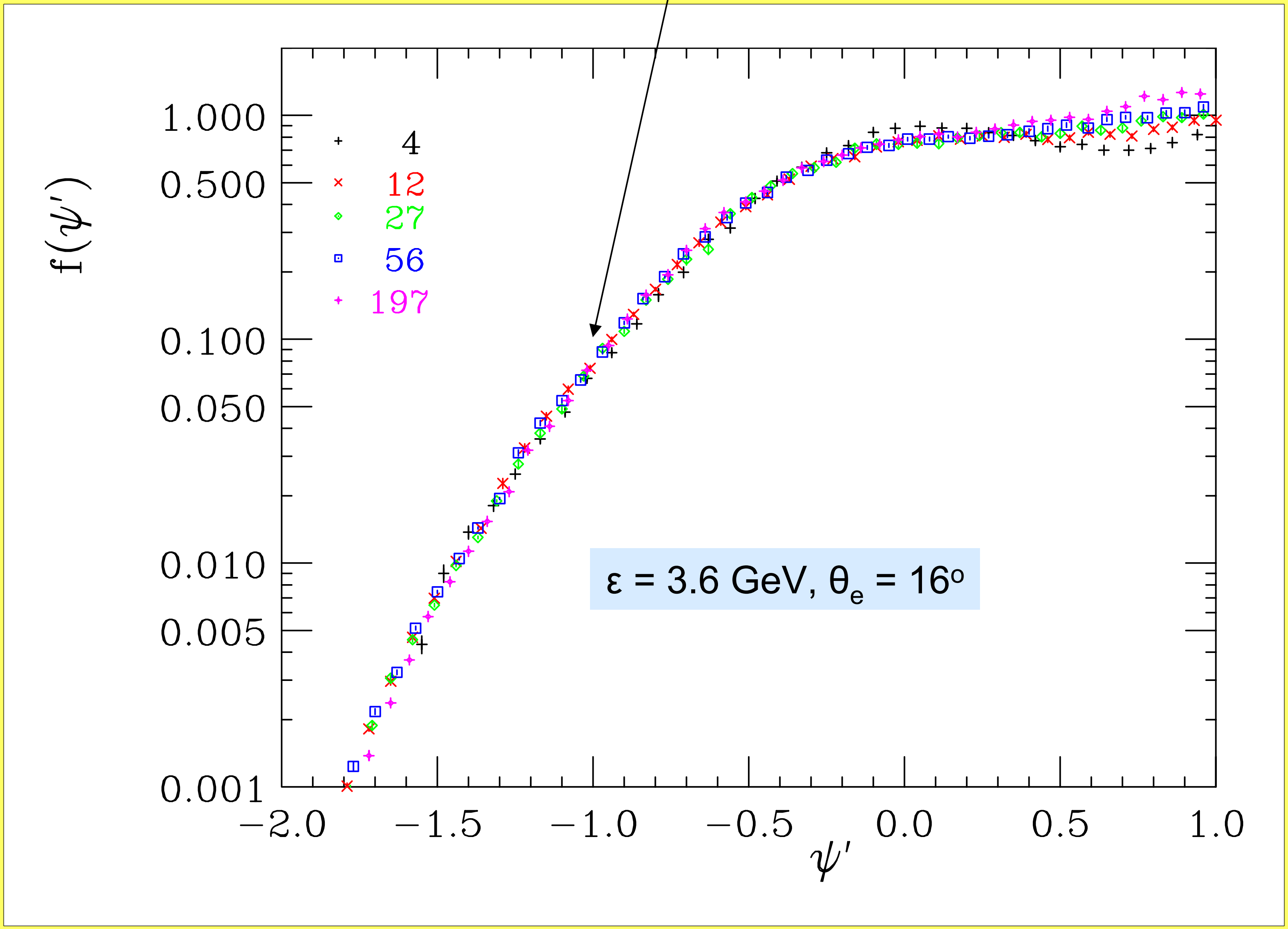


$k_A \bullet F$

$\sim y / k_A$

k_A = characteristic momentum scale for each nucleus

Scaling of the 2nd kind (A independence for ψ'<0)



In the scaling region ($\psi' < 0$) a universal behavior is seen, with

very little dependence on the nuclear species

↕

SCALING OF THE 2nd KIND

In the region above $\psi' = 0$ where resonances, meson production and the start of DIS enter the 2nd-kind scaling is not as good (see below)

Although the amount of data separated into longitudinal (L) and transverse (T) responses is small, one can attempt a scaling analysis with what does exist. The inclusive cross section may be written

$$\frac{d^2\sigma}{d\Omega_e d\omega} = \sigma_M \left[v_L R_L(q,\omega) + v_T R_T(q,\omega) \right]$$

$$v_L = \left| Q^2 / q^2 \right|^2$$

$$v_T = \tfrac{1}{2} \left| Q^2 / q^2 \right| + \tan^2 \theta_e / 2$$

From which L and T scaling functions can be defined as above

$$F_L(q,y) \equiv \frac{R_L(q,\omega)}{A\left[\Sigma_{eN}^{eff}\right]_L / \sigma_M v_L}$$

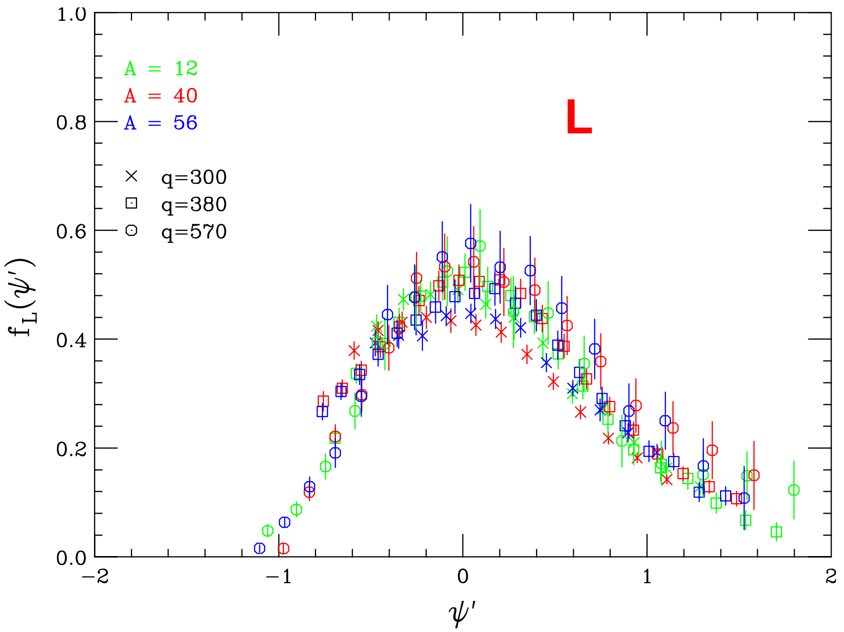
$$F_T(q,y) \equiv \frac{R_T(q,\omega)}{A\left[\Sigma_{eN}^{eff}\right]_T / \sigma_M v_T}$$

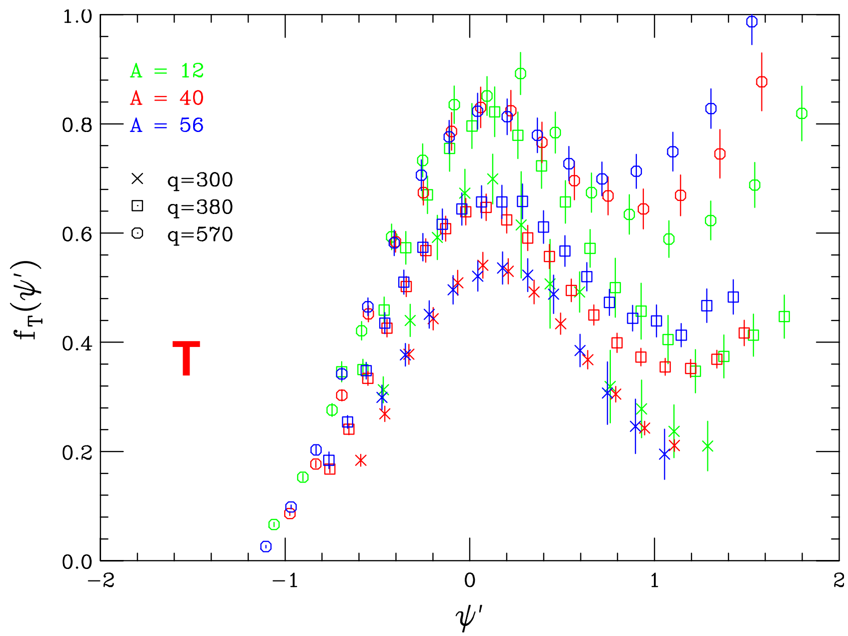
as can their dimensionless analogs

$$f_L(q,y) \equiv k_A \bullet F_L(q,y)$$

$$f_T(q,y) \equiv k_A \bullet F_T(q,y)$$

What results is the following:





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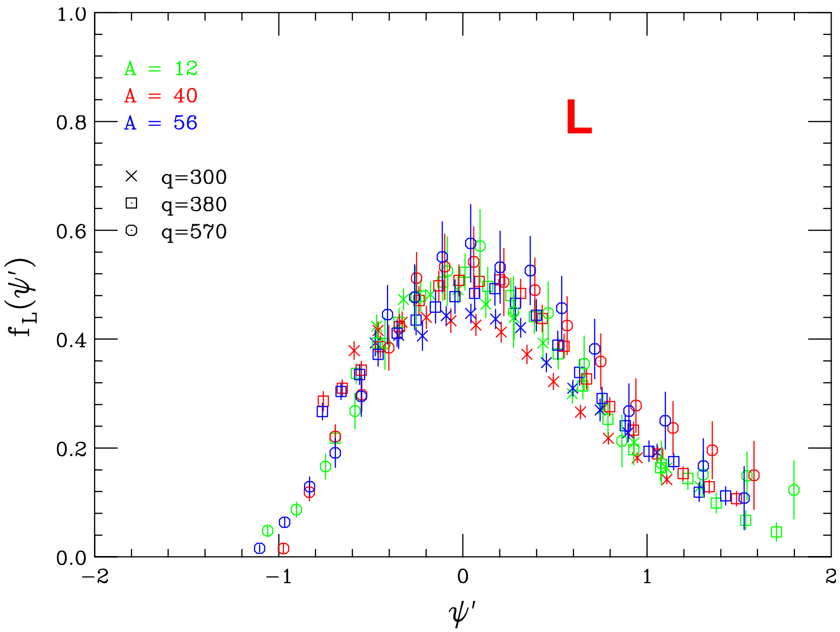
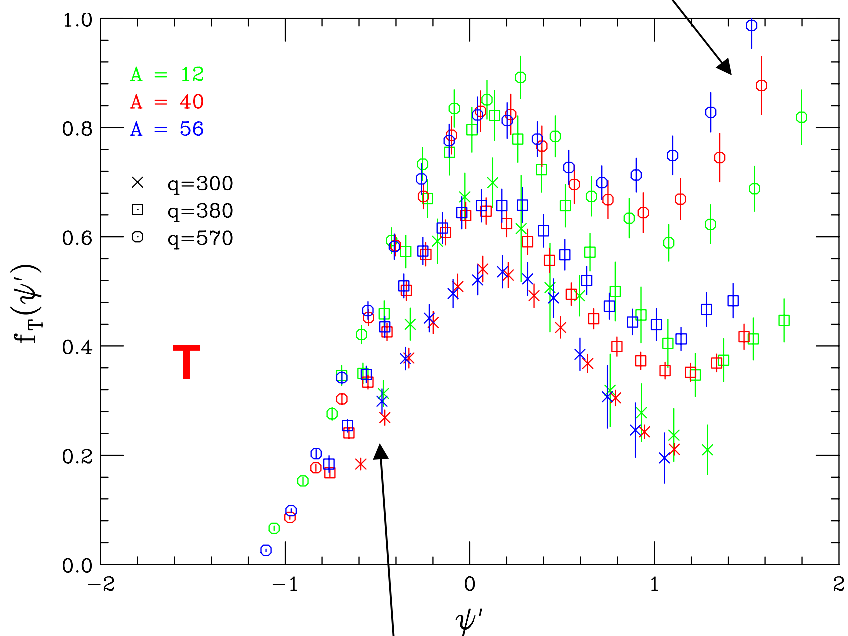
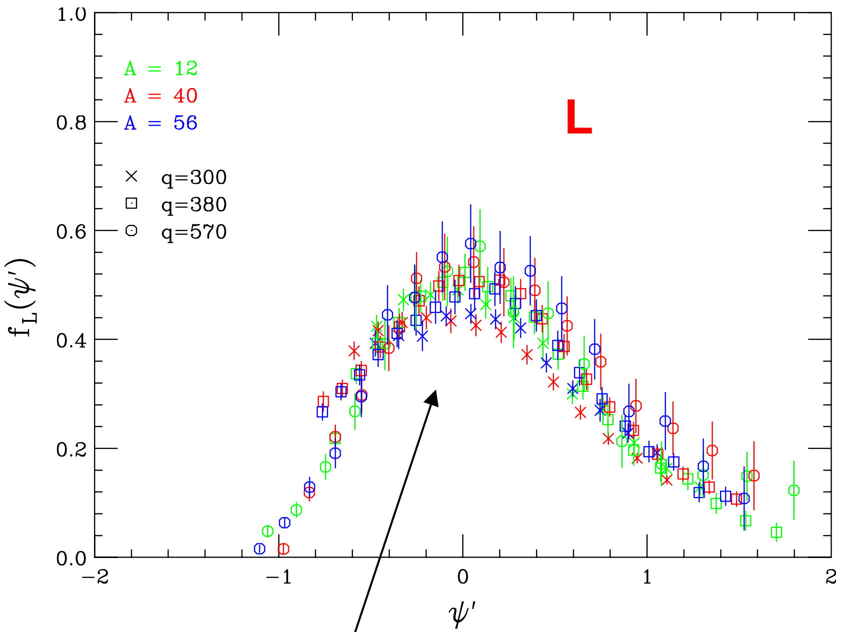


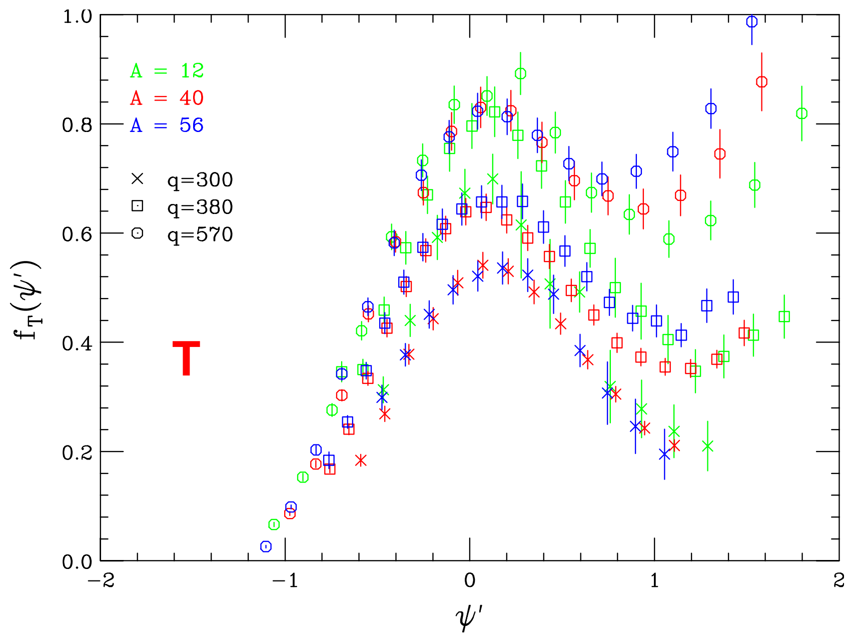
Inelastic contributions (mainly T)
+ MEC (dominantly T)



… however, still some residual below the QE peak

What results is the following:

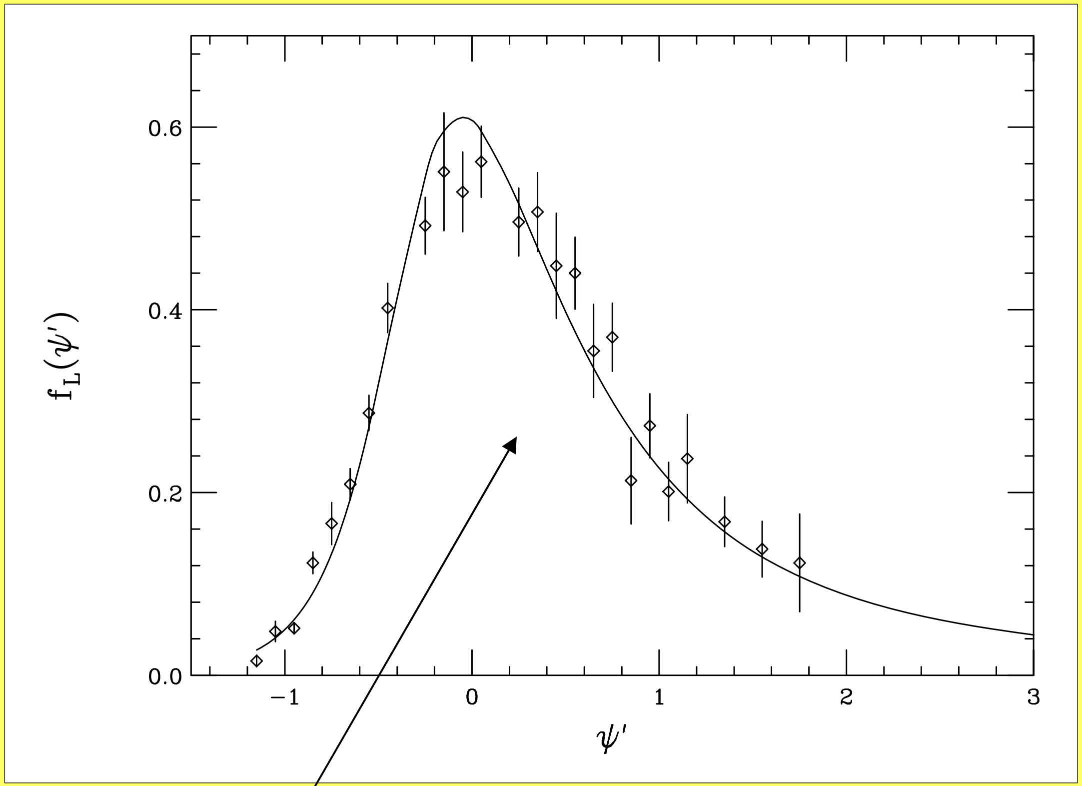




In contrast, the L results show a **universal behavior**

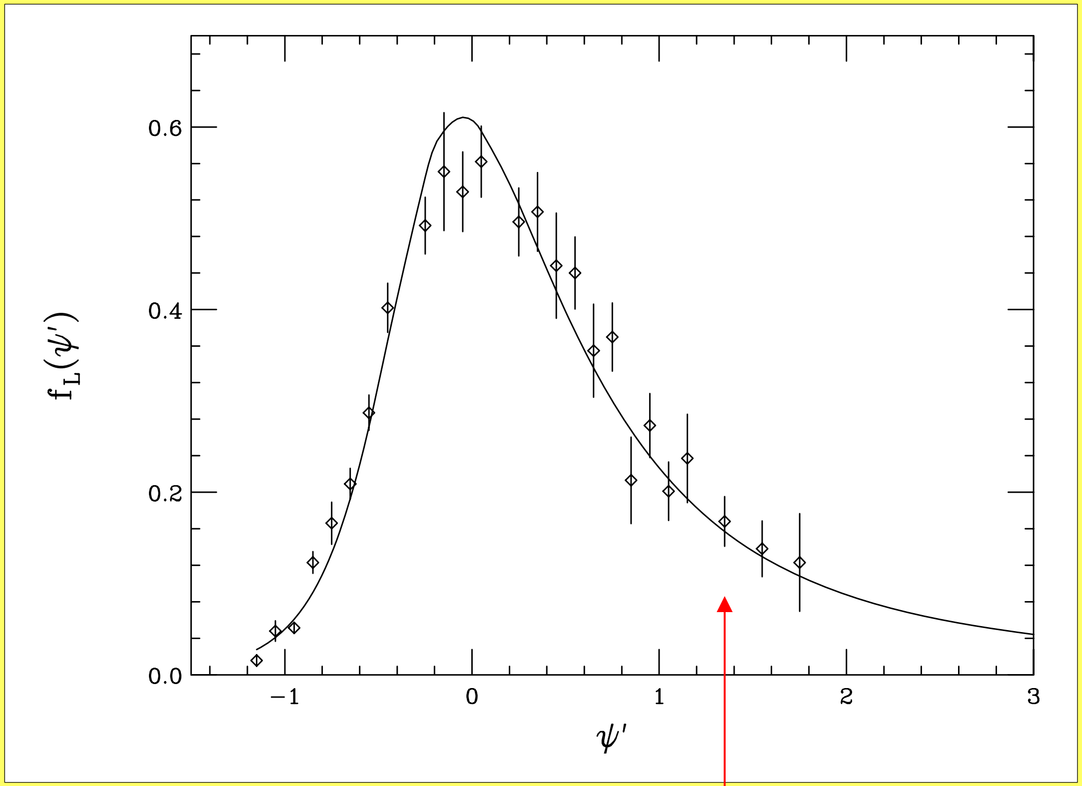
Longitudinal response only (little from MEC or pion production):

3 nuclei and
3 values of q



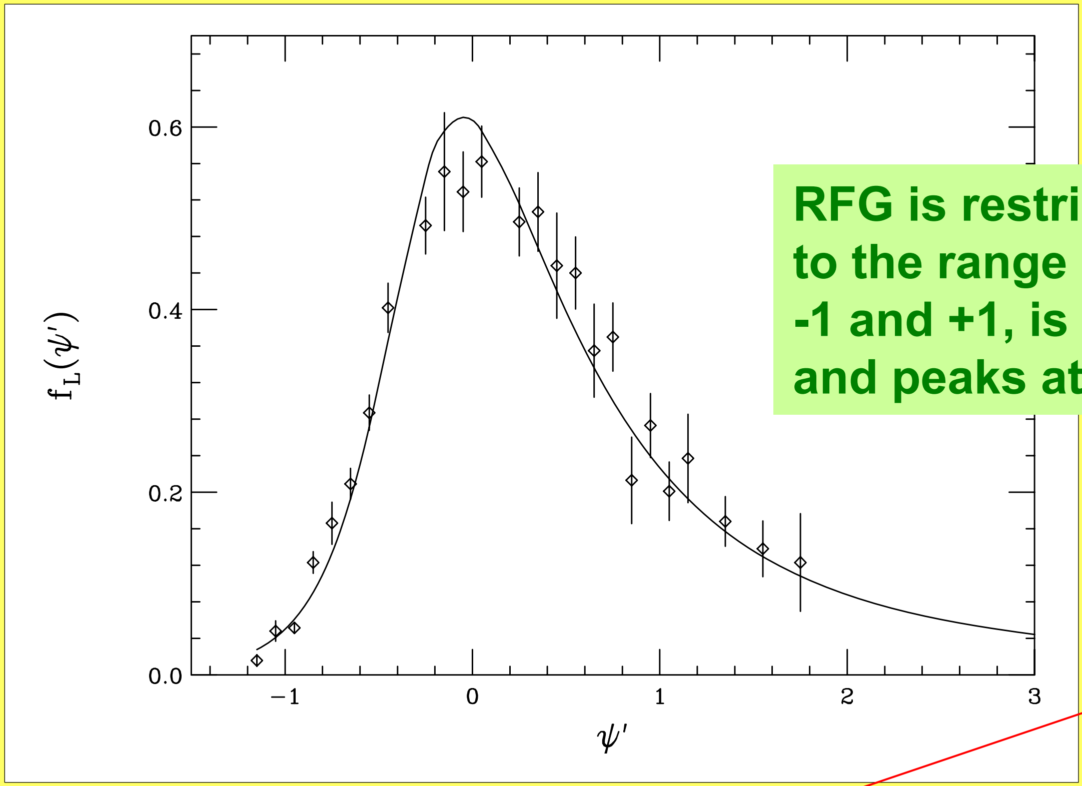
which is seen to be both independent of q (scaling of the 1st kind) and also independent of nuclear species (scaling of the 2nd kind)

$\leftrightarrow$ **SUPERSCALING**



Notes:

(1) Asymmetric shape; tail at high energy loss

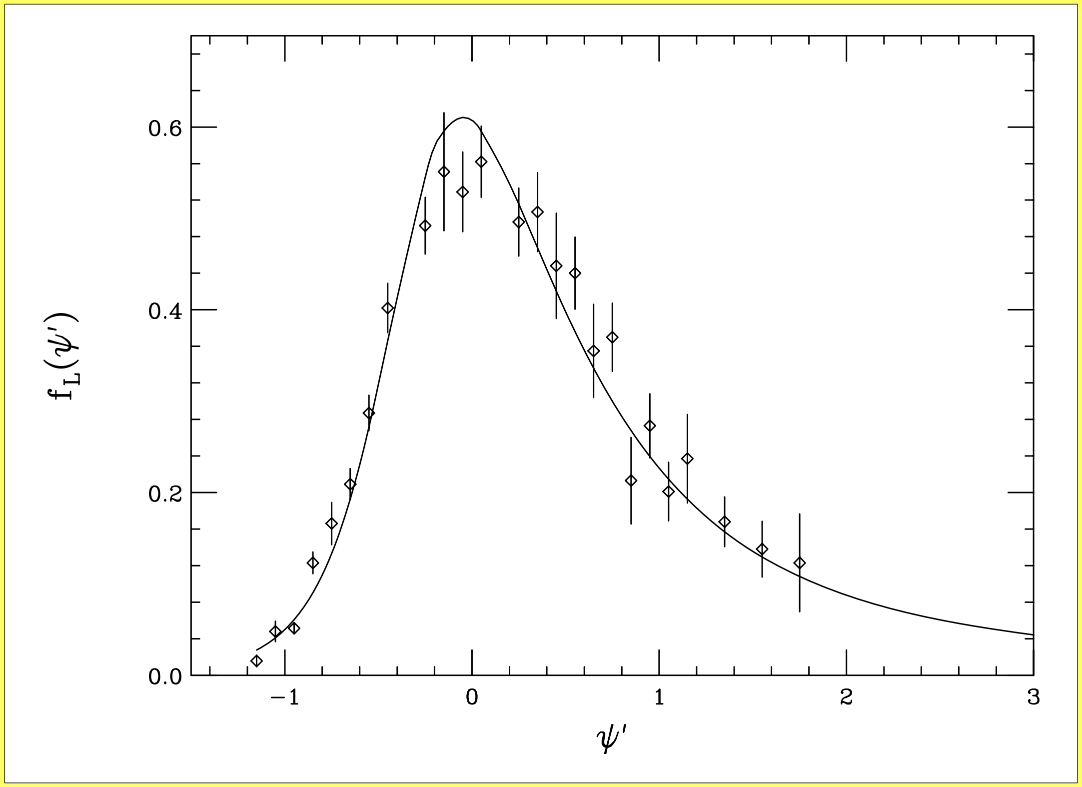


RFG is restricted to the range between -1 and +1, is symmetric and peaks at 0.75

Notes:

(1) Asymmetric shape; tail at high energy loss

(2) RFG very poor



Notes:

(1) Asymmetric shape; tail at high energy loss

(2) RFG very poor

(3) Best models yield this shape:

(a) **RMF approaches**

(b) Semi-rel approach

(c) BCS-inspired model

(d) Recent study with correlations

Note: in the RFG one has

$$\left[f_L \right]^{RFG} = \left[f_T \right]^{RFG} = \left[f \right]^{RFG}$$

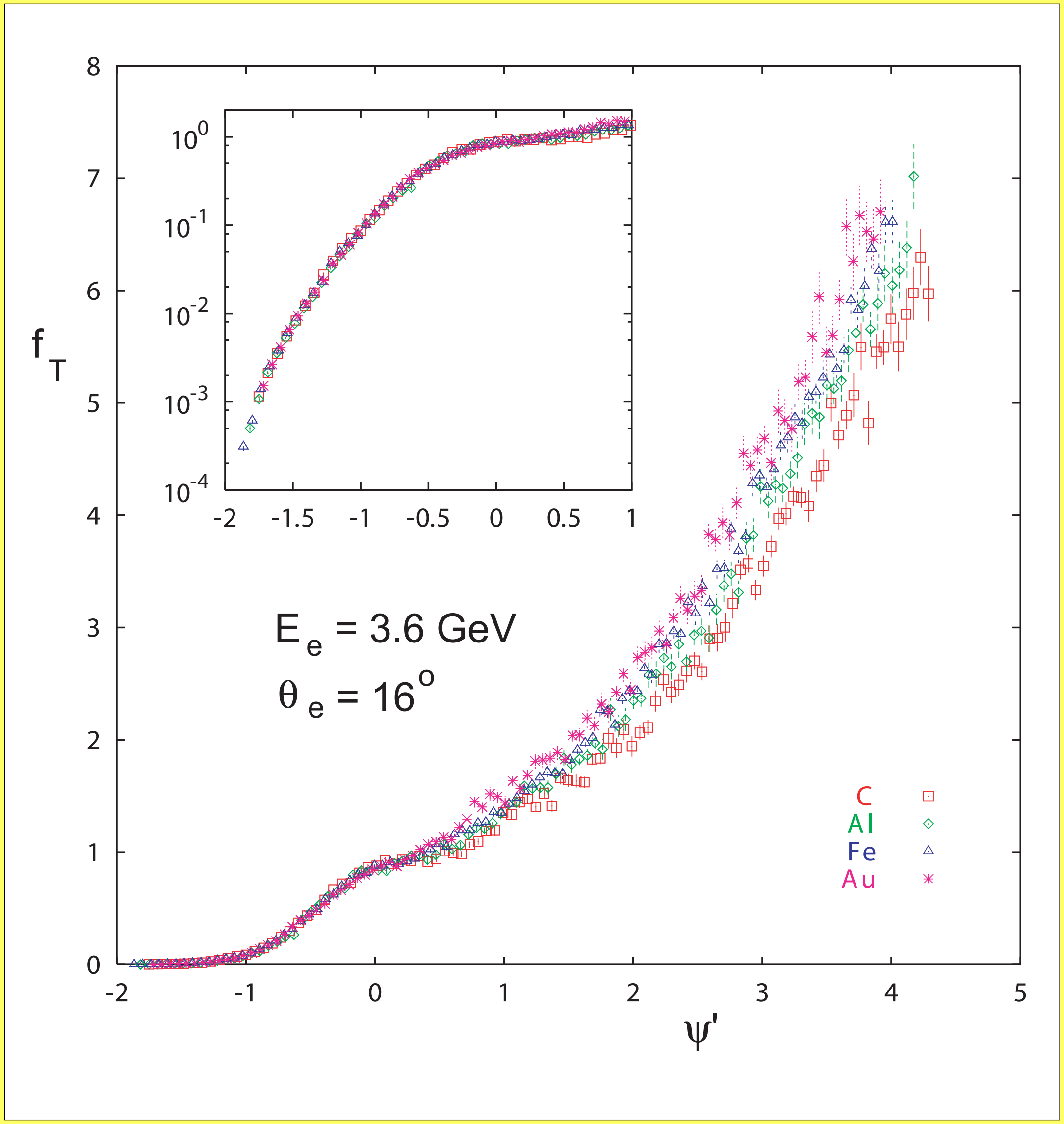
which has been called **SCALING OF THE 0th KIND**

If it were not for

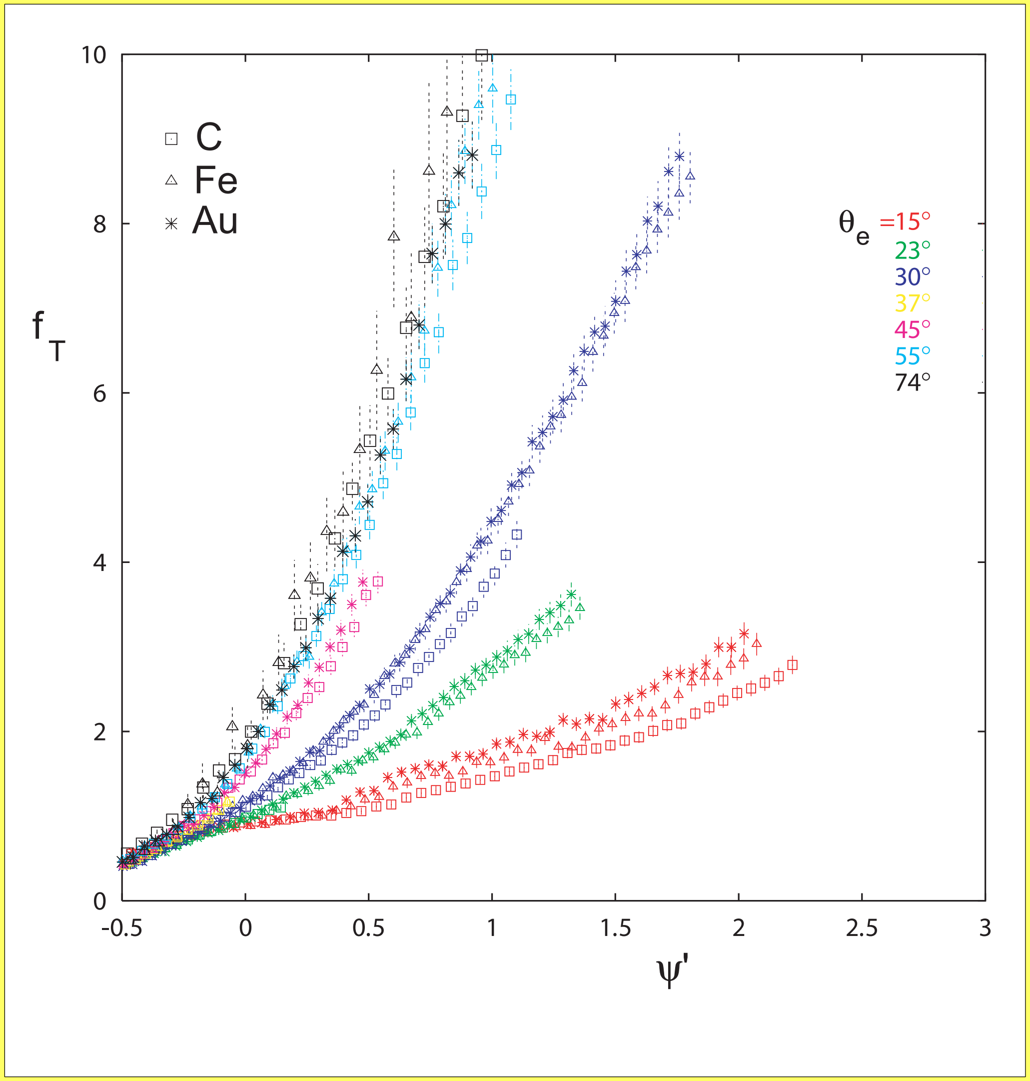
- contributions from resonances, meson production and DIS (which should not scale, since they involve different elementary cross sections, not elastic eN scattering, and since the scaling variables constructed above are appropriate only for QE scattering; see the discussions to follow), and for

- effects from meson-exchange currents (dominantly in T)

one might expect scaling of the 0th kind to be found.



Large ψ' region



Breaking of 1st and 2nd kind scaling at high ψ'

In the region above the QE peak one certainly expects **inelastic** contributions to be important in the T response, although not so in the L response

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One also expects to have **2p-2h MEC** contributions which add to the response discussed above; again, these are mainly T, not L. Typically they contribute 10-15% of the total and are one of the reasons for the scaling violations in the T response seen above.

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… see Lecture #2

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One also expects to have **2p-2h MEC** contributions which add to the response discussed above; again, these are mainly T, not L. Typically they contribute 10-15% of the total and are one of the reasons for the scaling violations in the T response seen above.

… the net result of adding together the **universal L scaling function**, the **inelastic contributions** obtained using this as well, and the **2p-2h MEC contributions** is in reasonable agreement with experiment (see below).

SuperScaling Approach (SuSA)

(1) Assume a **universal** scaling function, either phenomenological from the longitudinal results shown above, or from models
(2) Use this together with elastic eN as above or inelastic eN → e’X single-nucleon cross sections to obtain the QE and inel contributions
(3) Add 2-particle emission MEC contributions
(4) Use this universal approach to compare with inclusive ee’ data
(5) Replace the single-nucleon cross sections in (2) with CC or NC neutrino reaction cross sections to obtain the SuSA predictions for the neutrino-nucleus cross sections

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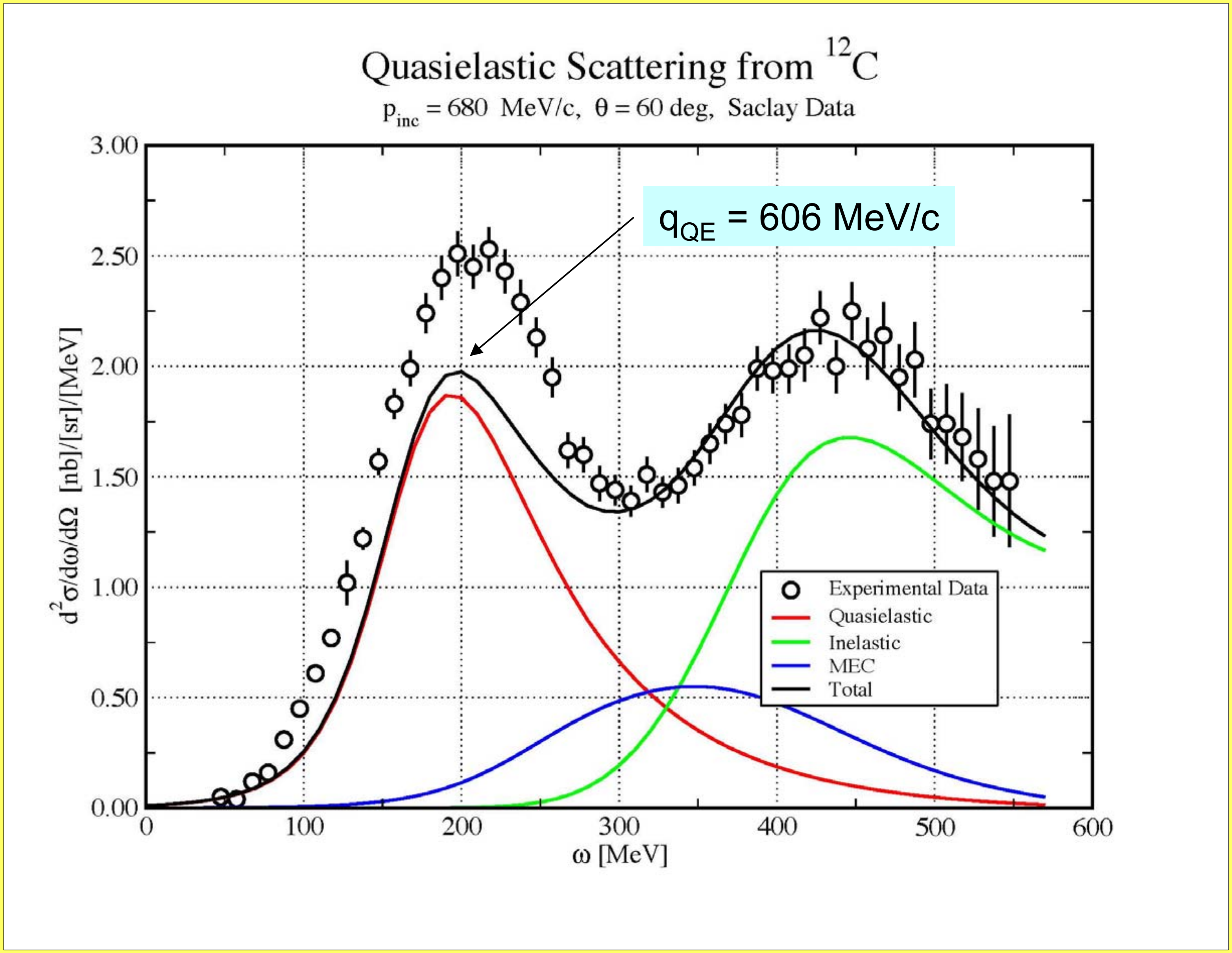
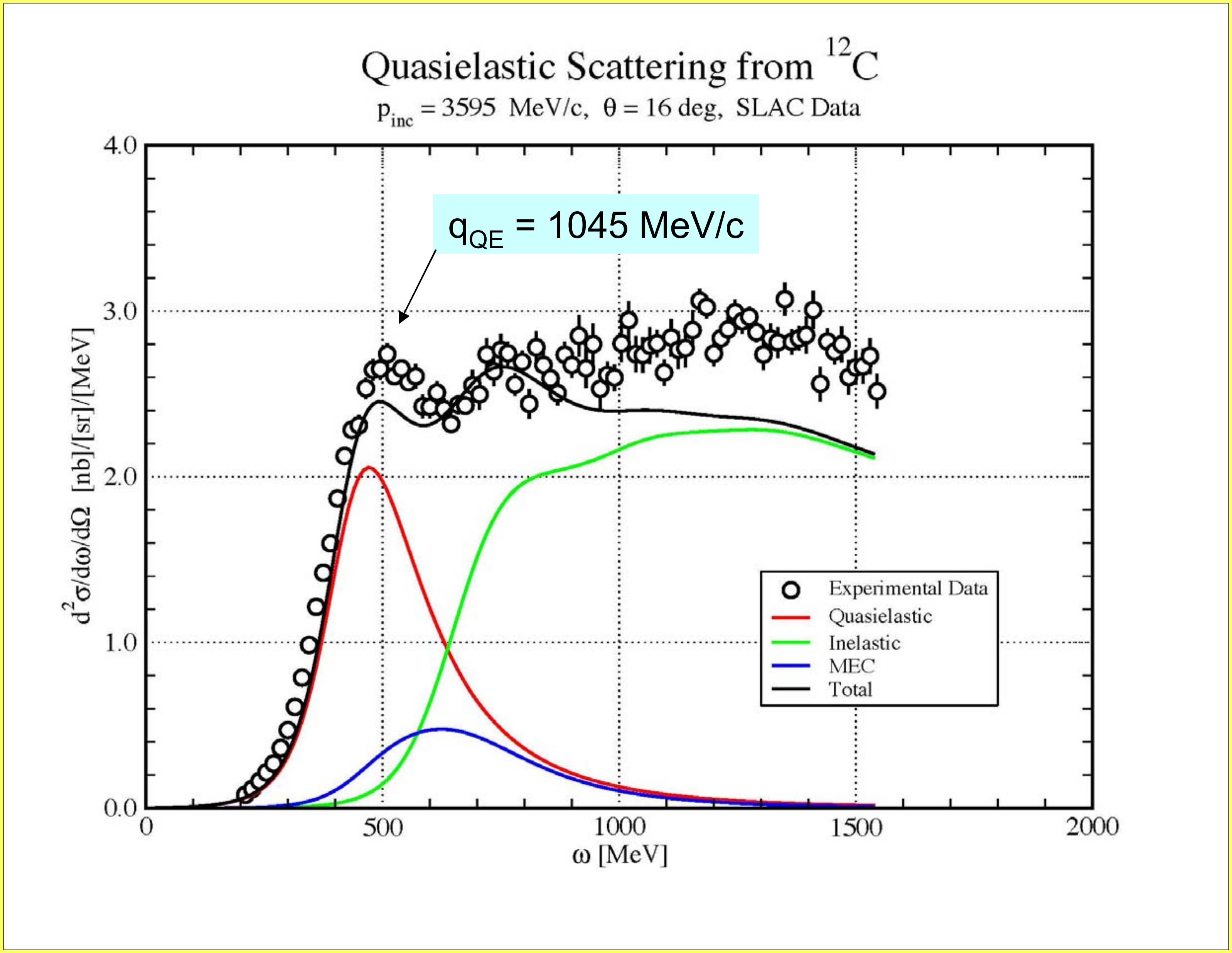
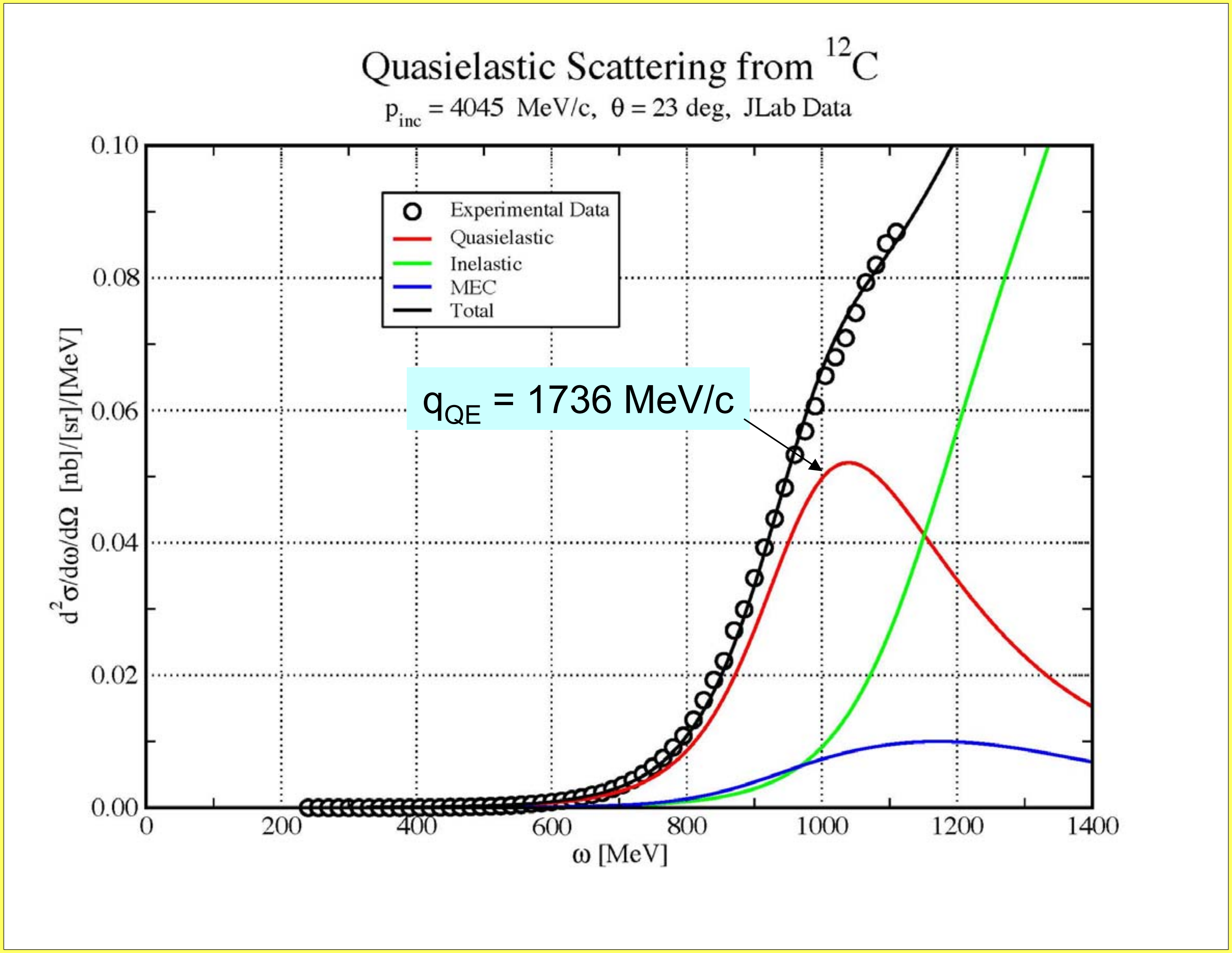
(5) Replace the single-nucleon cross sections in (2) with CC or NC neutrino reaction cross sections to obtain the SuSA predictions for the neutrino-nucleus cross sections

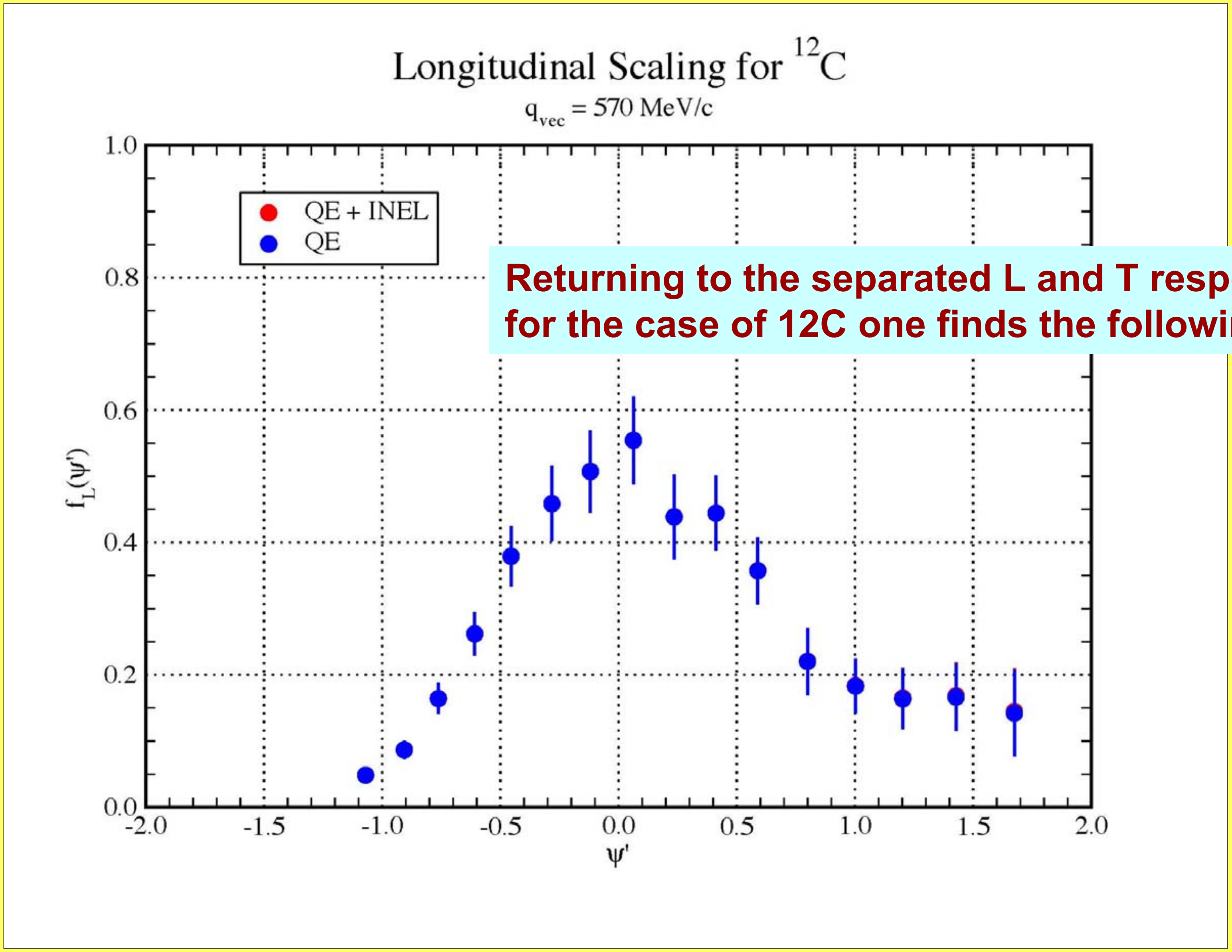
… of course, if the test in (4) fails, one should not expect to have very good predictions for neutrino reactions, as is the case for simplistic models such as the RFG


Quasielastic Scattering from 12C
p_inc = 680 MeV/c, θ = 60 deg, Saclay Data
q_QE = 606 MeV/c
d²σ/dω/dΩ [nb]/[sr]/[MeV]
ω [MeV]
Experimental Data
Quasielastic
Inelastic
MEC
Total
3.00
2.50
2.00
1.50
1.00
0.50
0.00
0
100
200
300
400
500
600

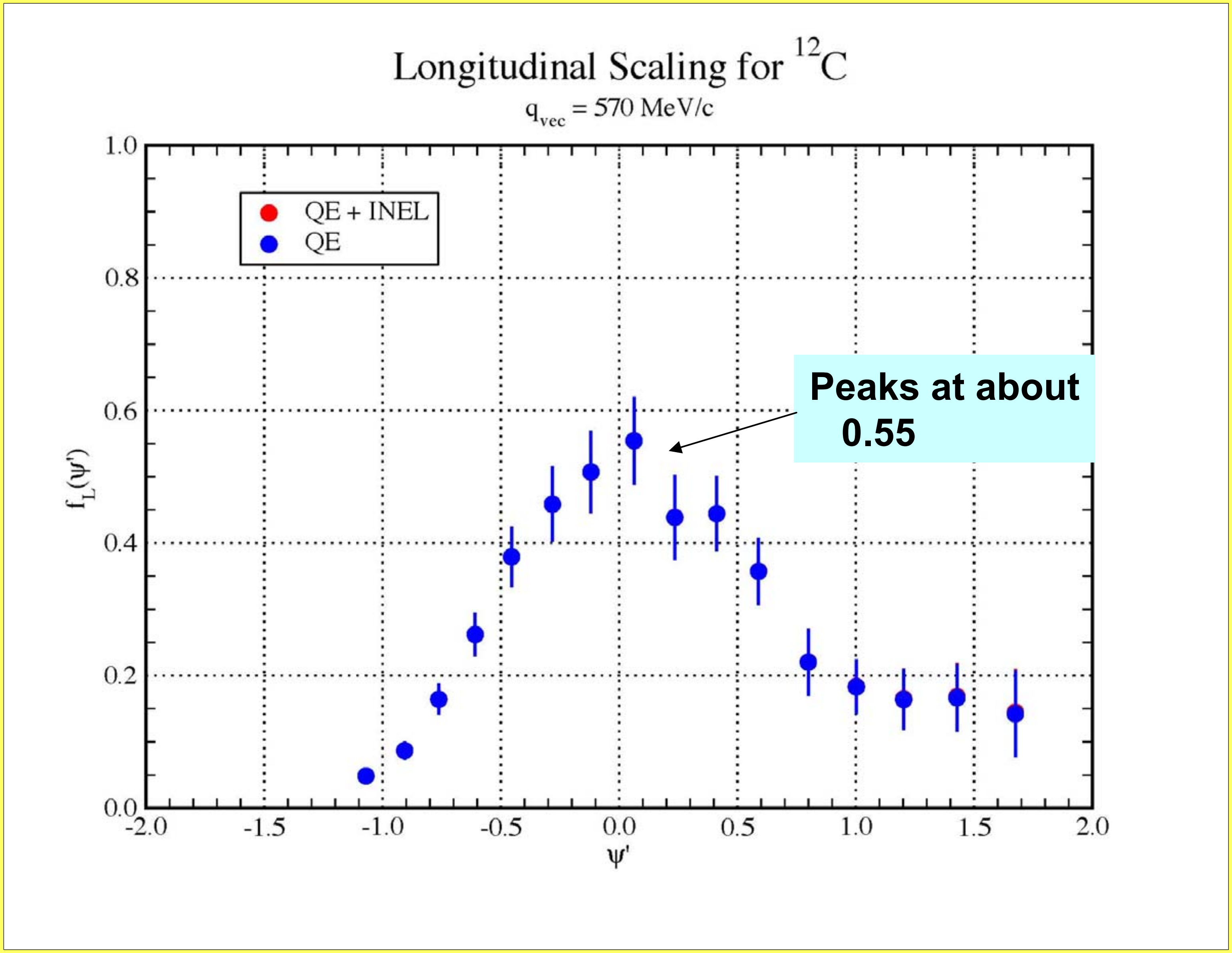

Quasielastic Scattering from 12C
p_inc = 3595 MeV/c, θ = 16 deg, SLAC Data
q_QE = 1045 MeV/c
d2σ/dω/dΩ [nb]/[sr]/[MeV]
ω [MeV]
0.0
1.0
2.0
3.0
4.0
0
500
1000
1500
2000
Experimental Data
Quasielastic
Inelastic
MEC
Total



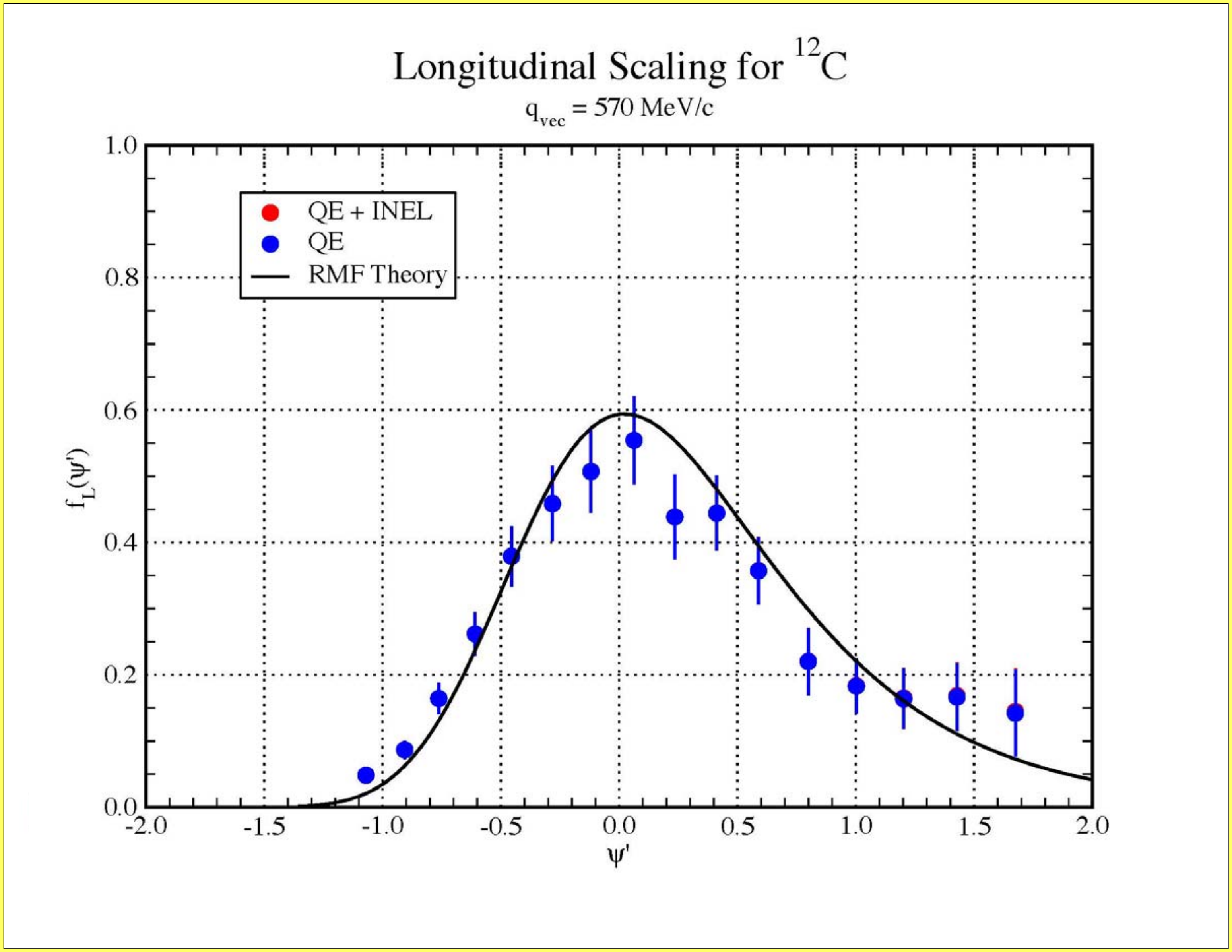
Quasielastic Scattering from ^{12}C
p_{inc} = 4045 MeV/c, θ = 23 deg, JLab Data
Experimental Data
Quasielastic
Inelastic
MEC
Total
q_{QE} = 1736 MeV/c
$d^2\sigma/d\omega/d\Omega$ [nb]/[sr]/[MeV]
ω [MeV]
0.00
0.02
0.04
0.06
0.08
0.10
0
200
400
600
800
1000
1200
1400

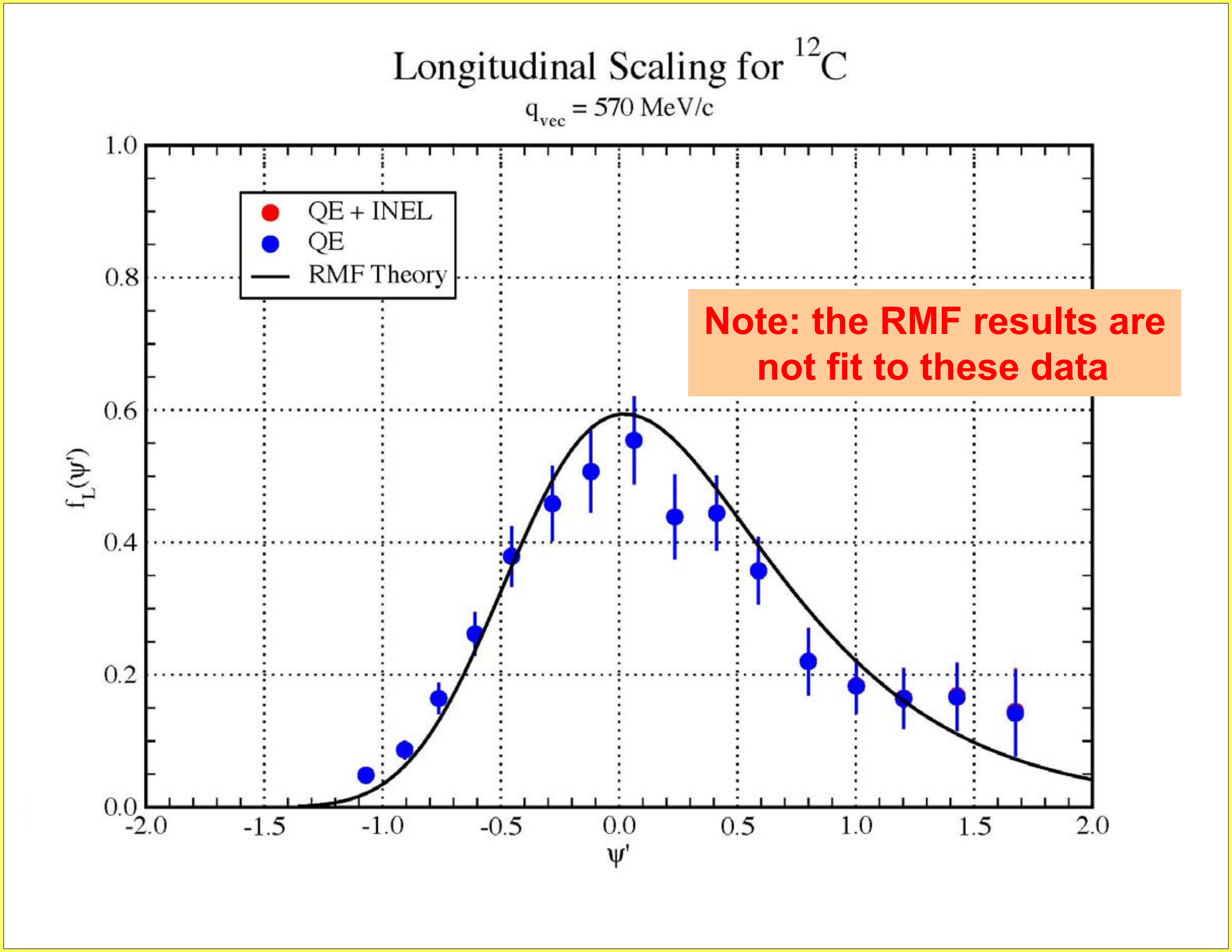


Returning to the separated L and T responses, for the case of 12C one finds the following:

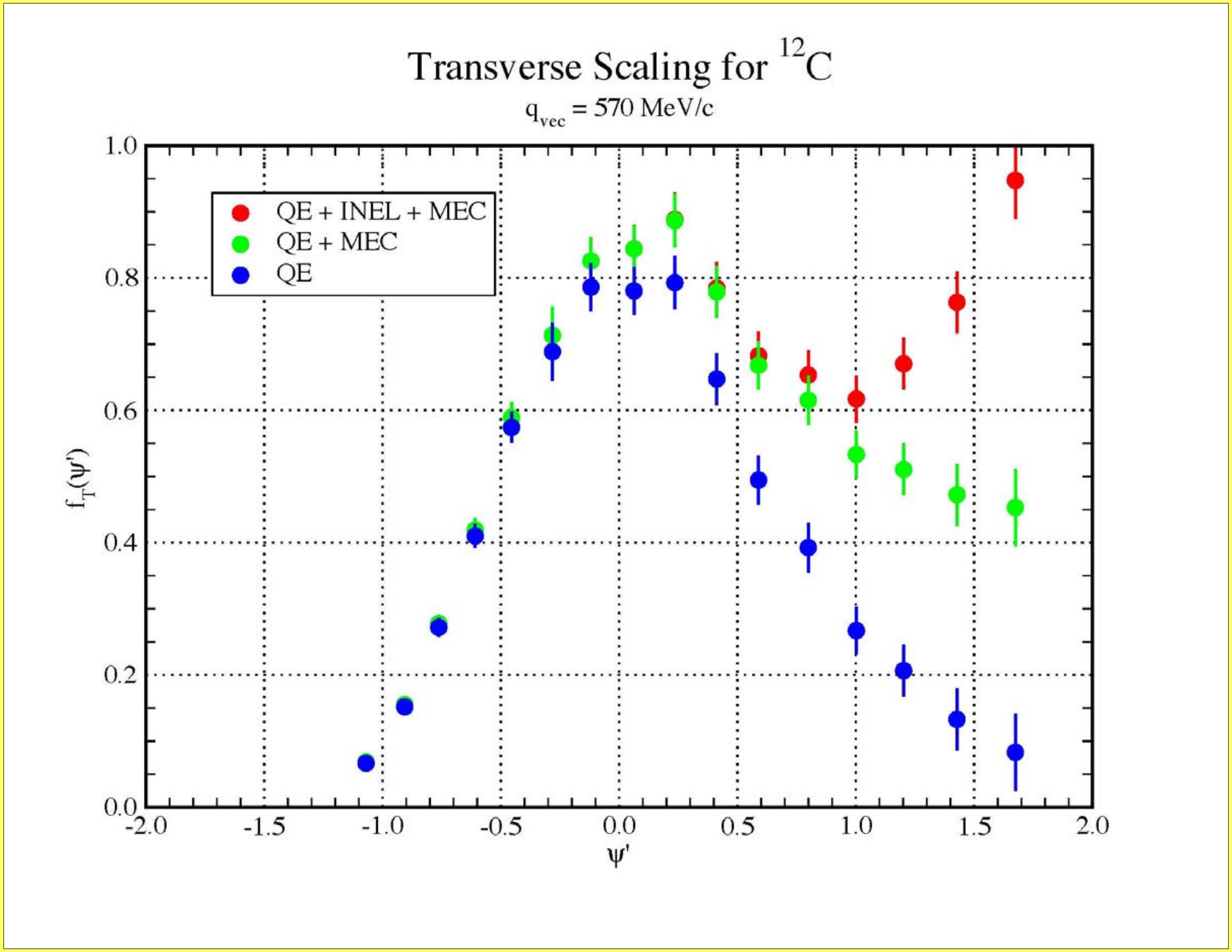


Longitudinal Scaling for ^{12}C
q_{vec} = 570 MeV/c
QE + INEL
QE
Peaks at about 0.55
$f_L(\psi')$
ψ'
1.0
0.8
0.6
0.4
0.2
0.0
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0

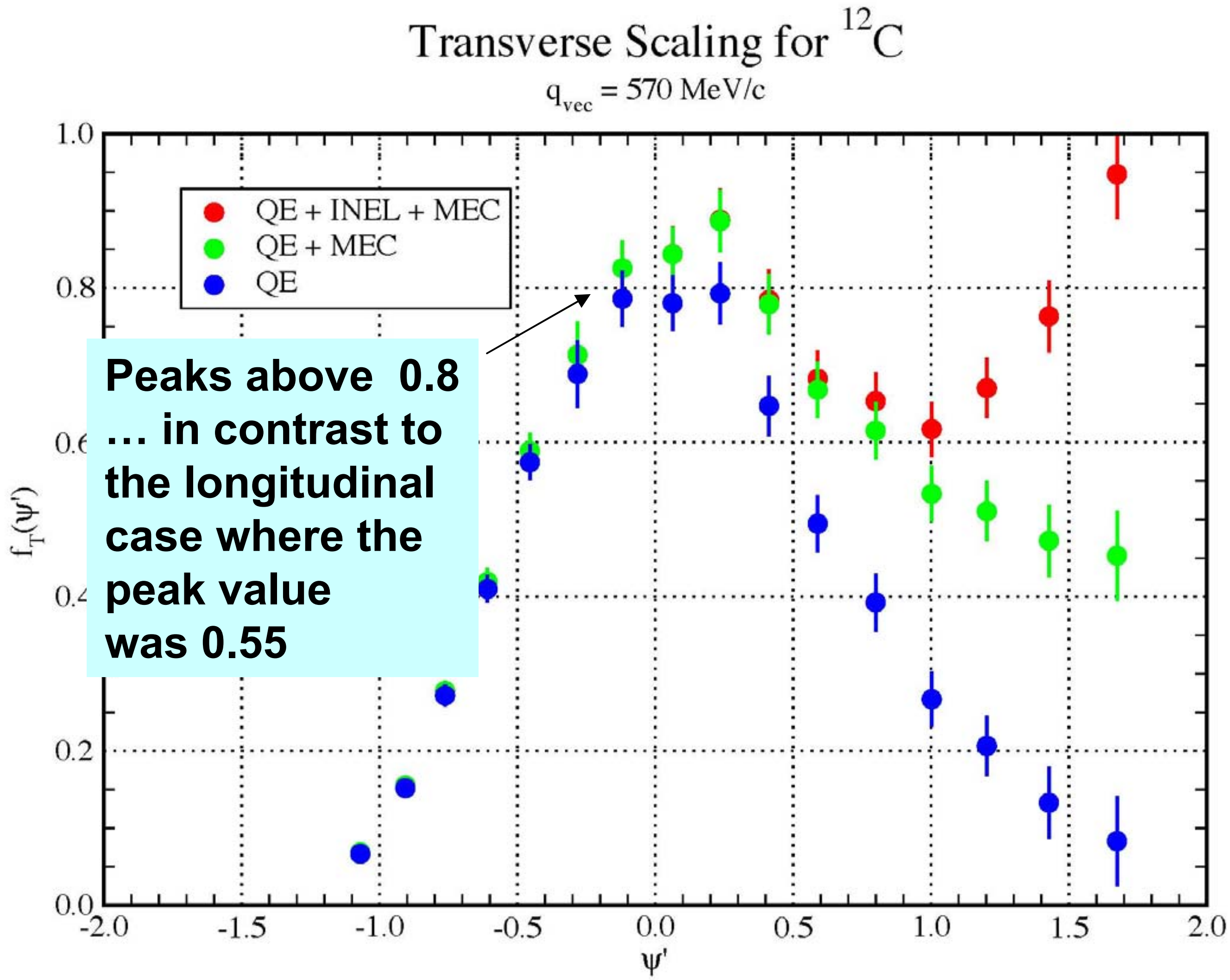

Longitudinal Scaling for 12C
q vec = 570 MeV/c
QE + INEL
QE
RMF Theory
1.0
0.8
0.6
0.4
0.2
0.0
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
f L(ψ')
ψ'



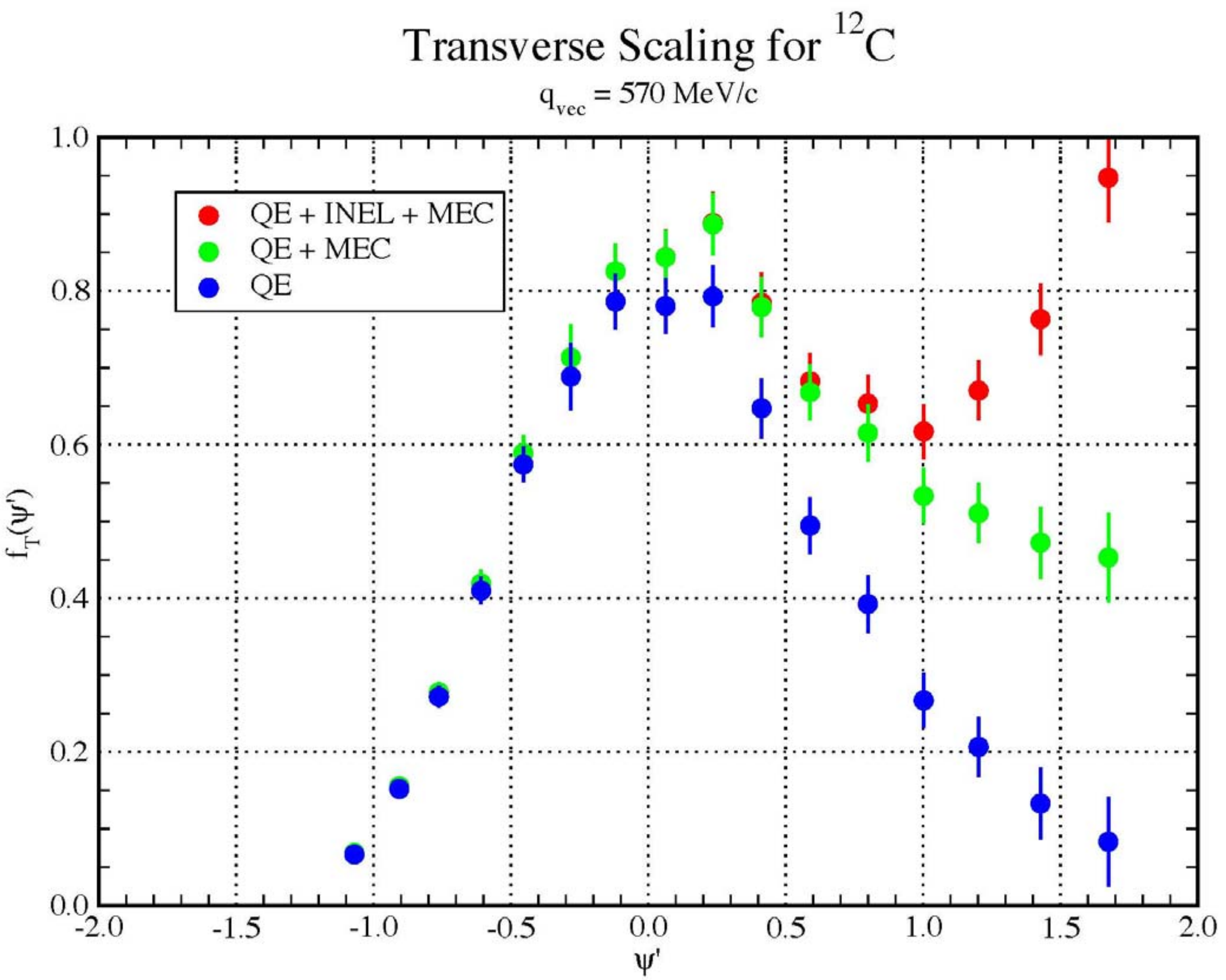
Note: the RMF results are not fit to these data



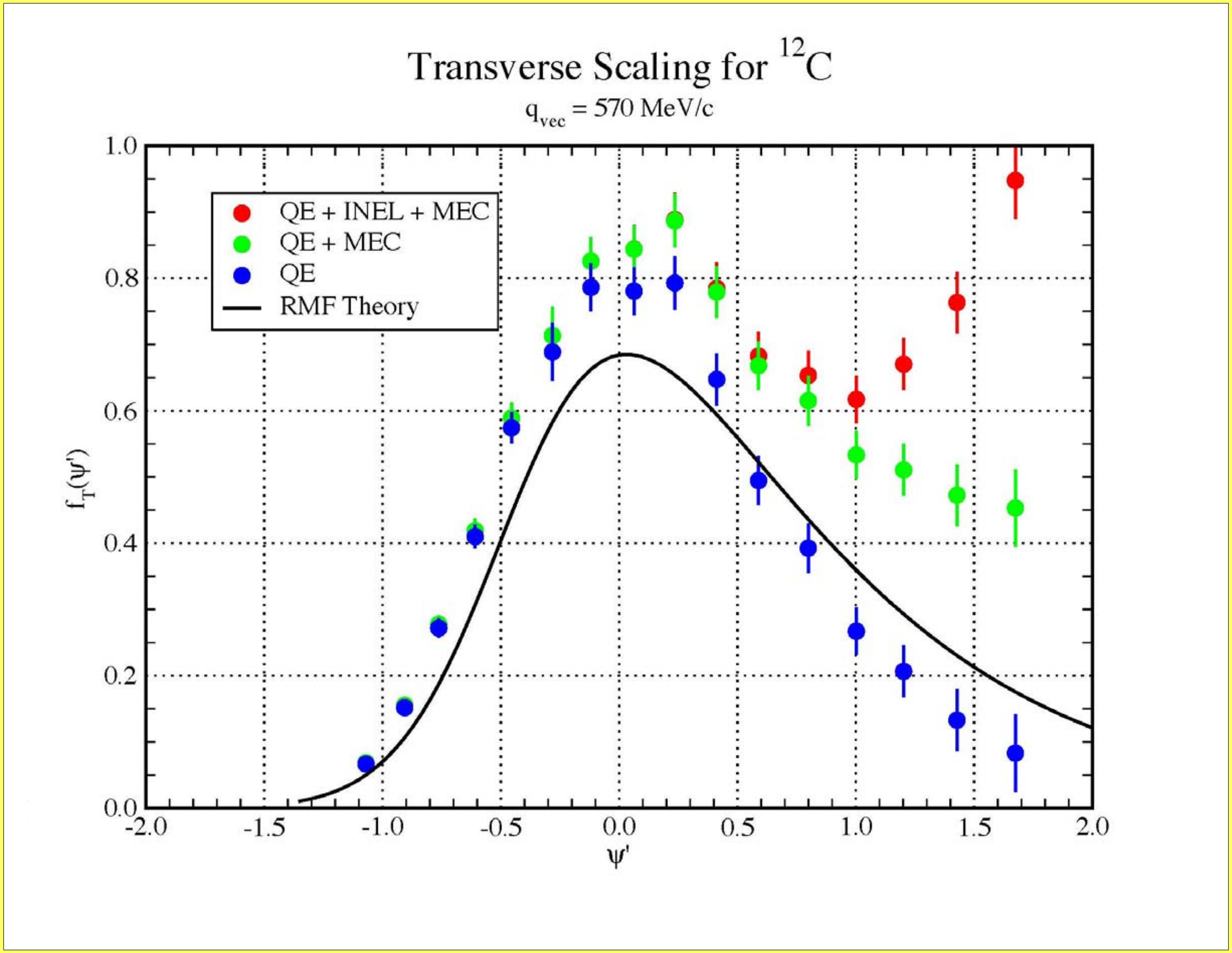
Transverse Scaling for ^{12}C
q_{vec} = 570 MeV/c
QE + INEL + MEC
QE + MEC
QE
$f_T(\psi')$
ψ'
1.0
0.8
0.6
0.4
0.2
0.0
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0

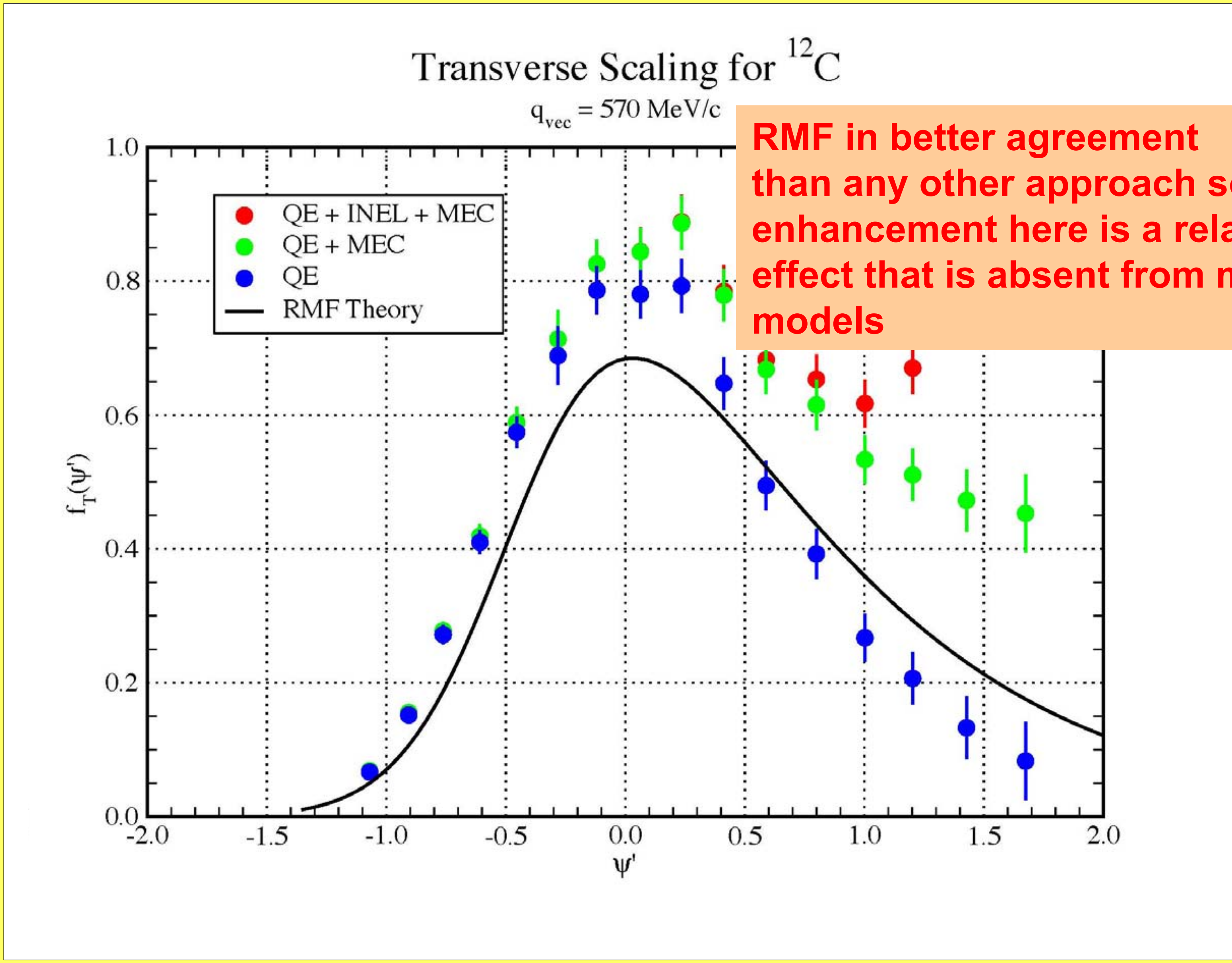


Peaks above 0.8 … in contrast to the longitudinal case where the peak value was 0.55



First clear evidence for violations of scaling of the 0th kind


Transverse Scaling for ^{12}C
q_{vec} = 570 MeV/c
QE + INEL + MEC
QE + MEC
QE
RMF Theory
$f_T(\psi')$
ψ'
1.0
0.8
0.6
0.4
0.2
0.0
-2.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0



RMF in better agreement than any other approach so far; enhancement here is a relativistic effect that is absent from most models