

Outline:

Lecture 1

Basics (this may overlap with other speakers' talks)
The Relativistic Fermi Gas (RFG)
Superscaling

Lecture 2

More sophisticated model of inclusive scattering
2p-2h Meson –Exchange Currents (MEC)
Comparisons with data
 Inclusive electron scattering
 Inclusive charge-changing neutrino reactions

Other models?

... first, consider a non-relativistic shell model,

showing, instead of the longitudinal response R_L , the scaled result f_L (where the single-nucleon response has been divided out), and plotting versus ψ' the scaling variable rather than the energy transfer ω

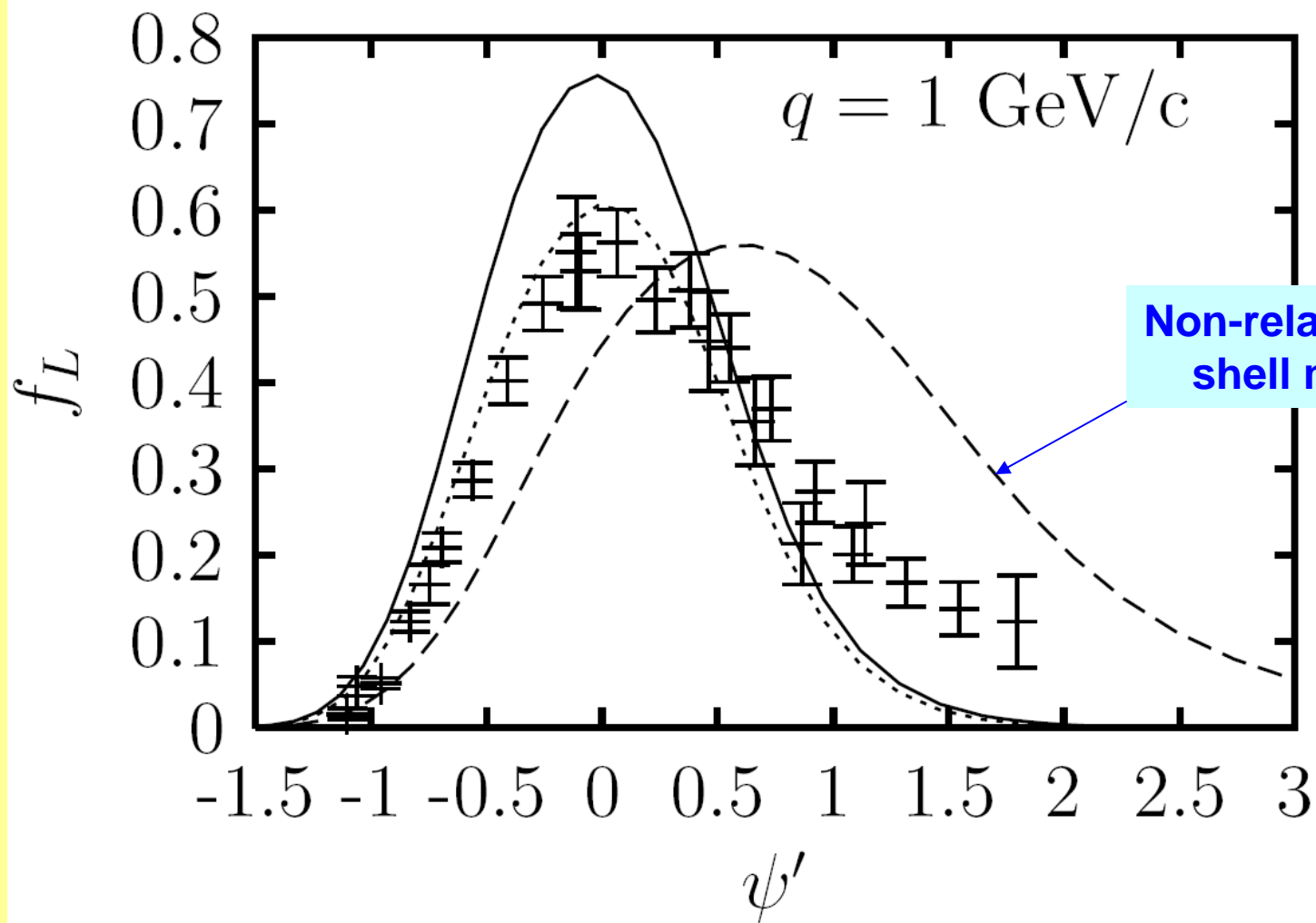
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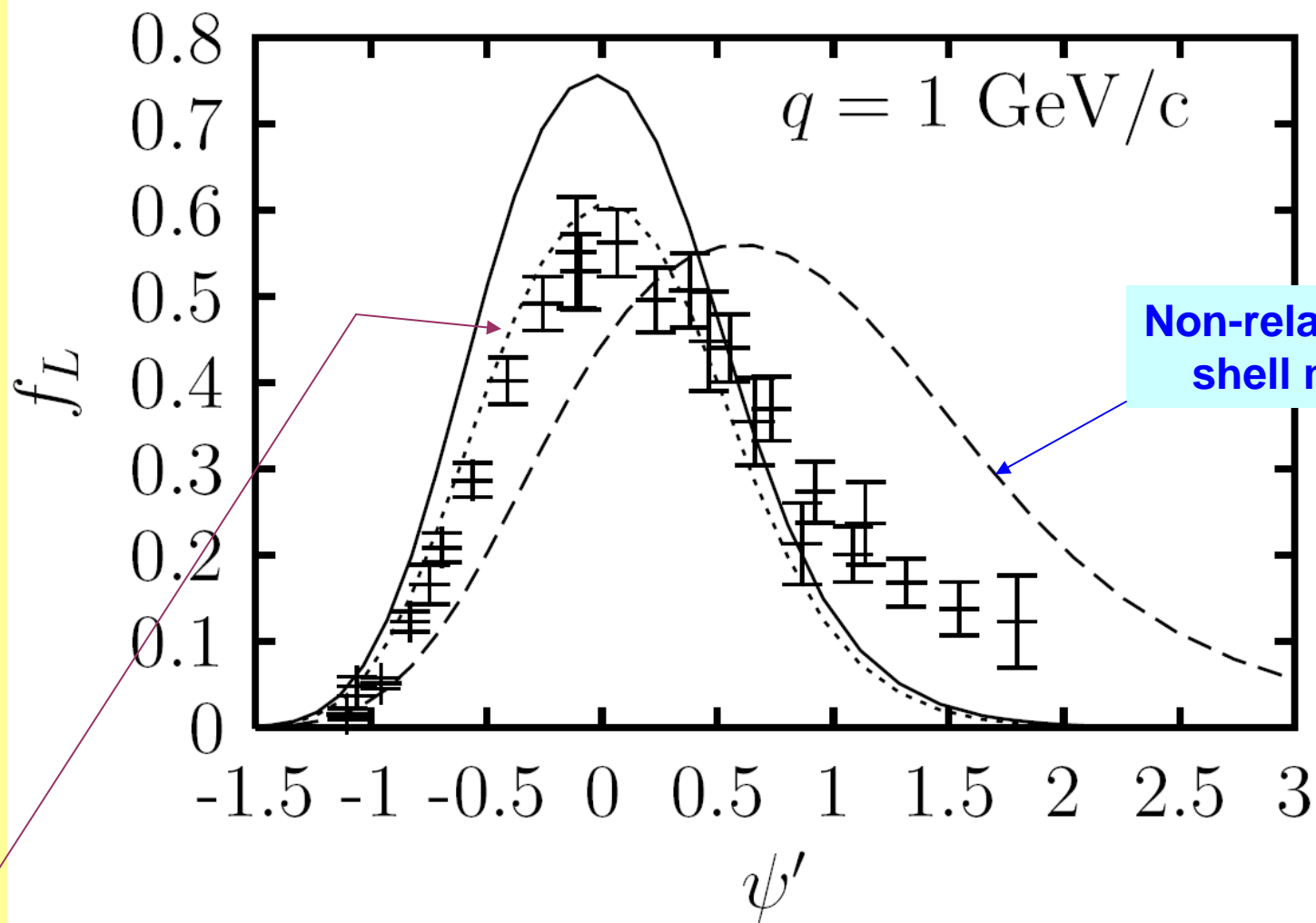
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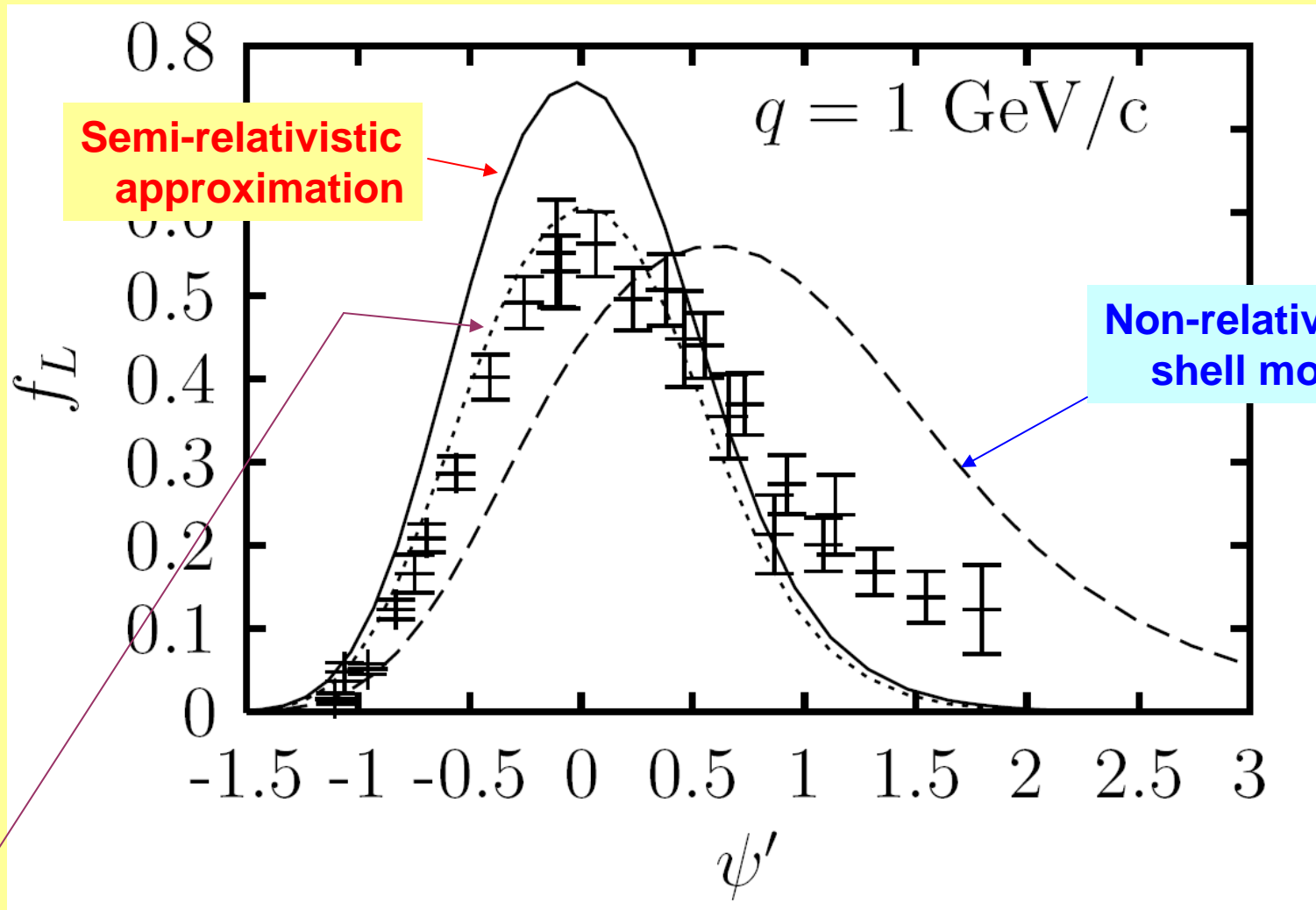
Important: the longitudinal response has only very small contributions from meson production and meson-exchange currents, and therefore provides a fair test of the one-body QE cross section. The electron scattering response scales (data shown).







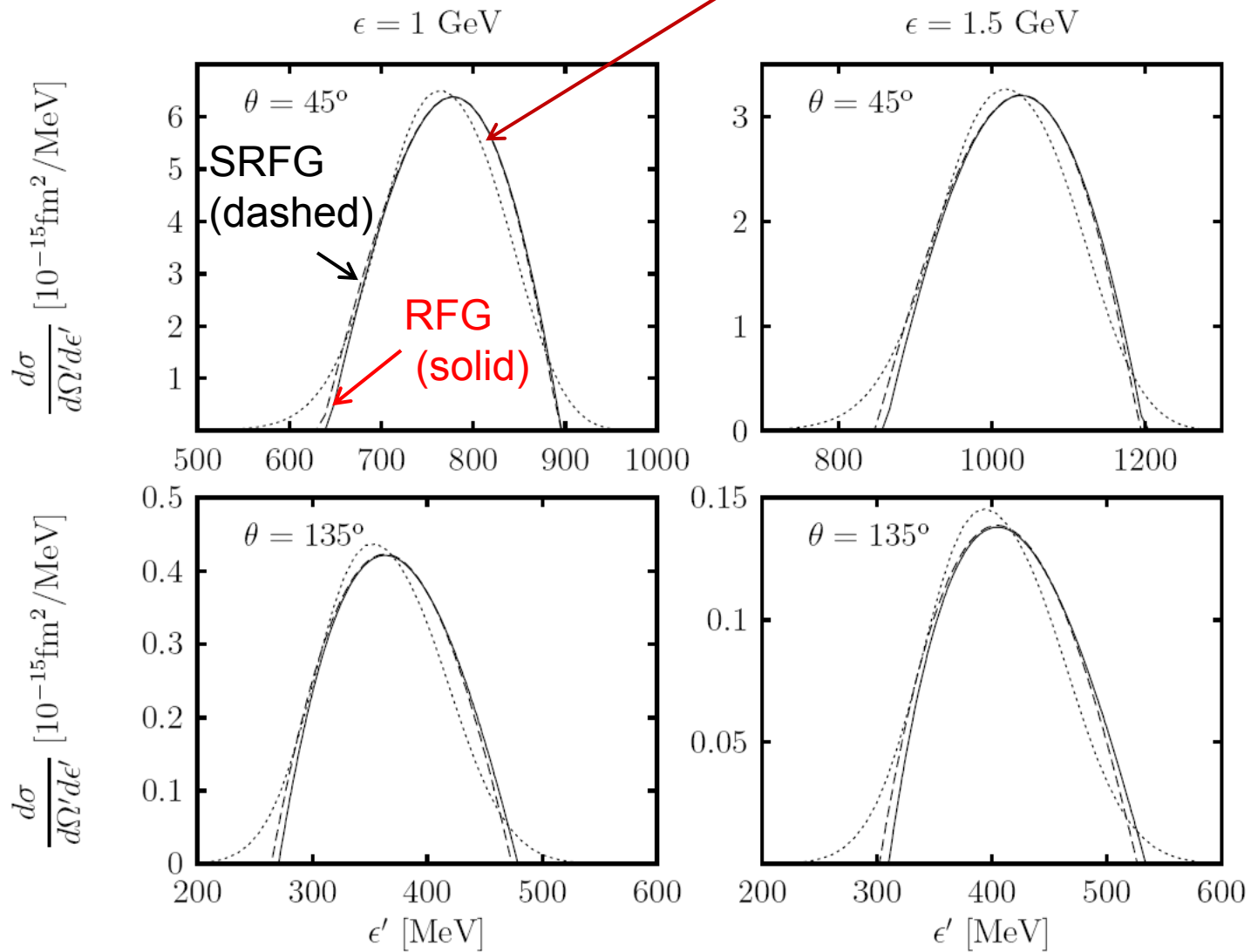
Non-relativistic current operators (no boost effects),
but with relativistic kinematics

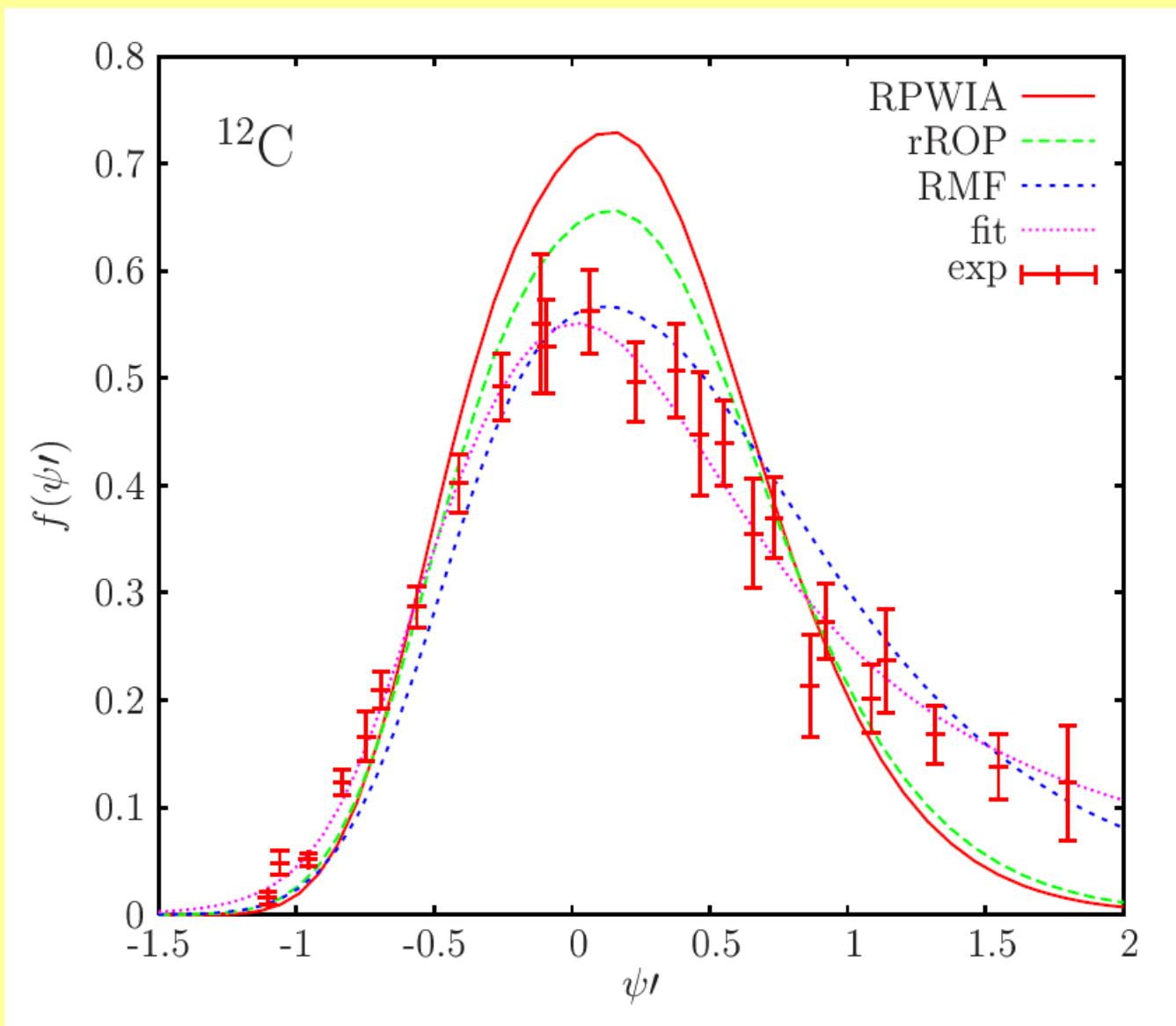


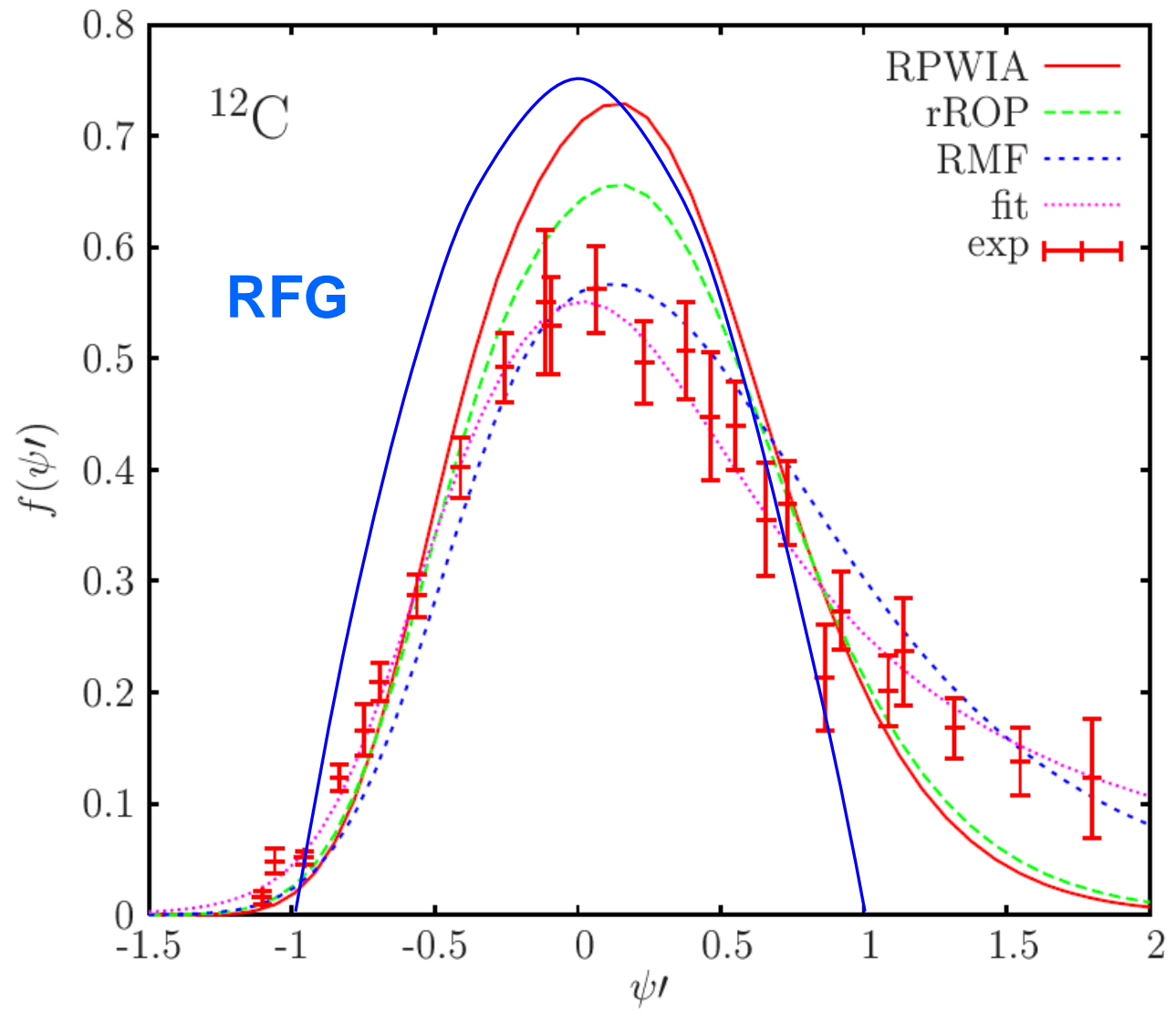
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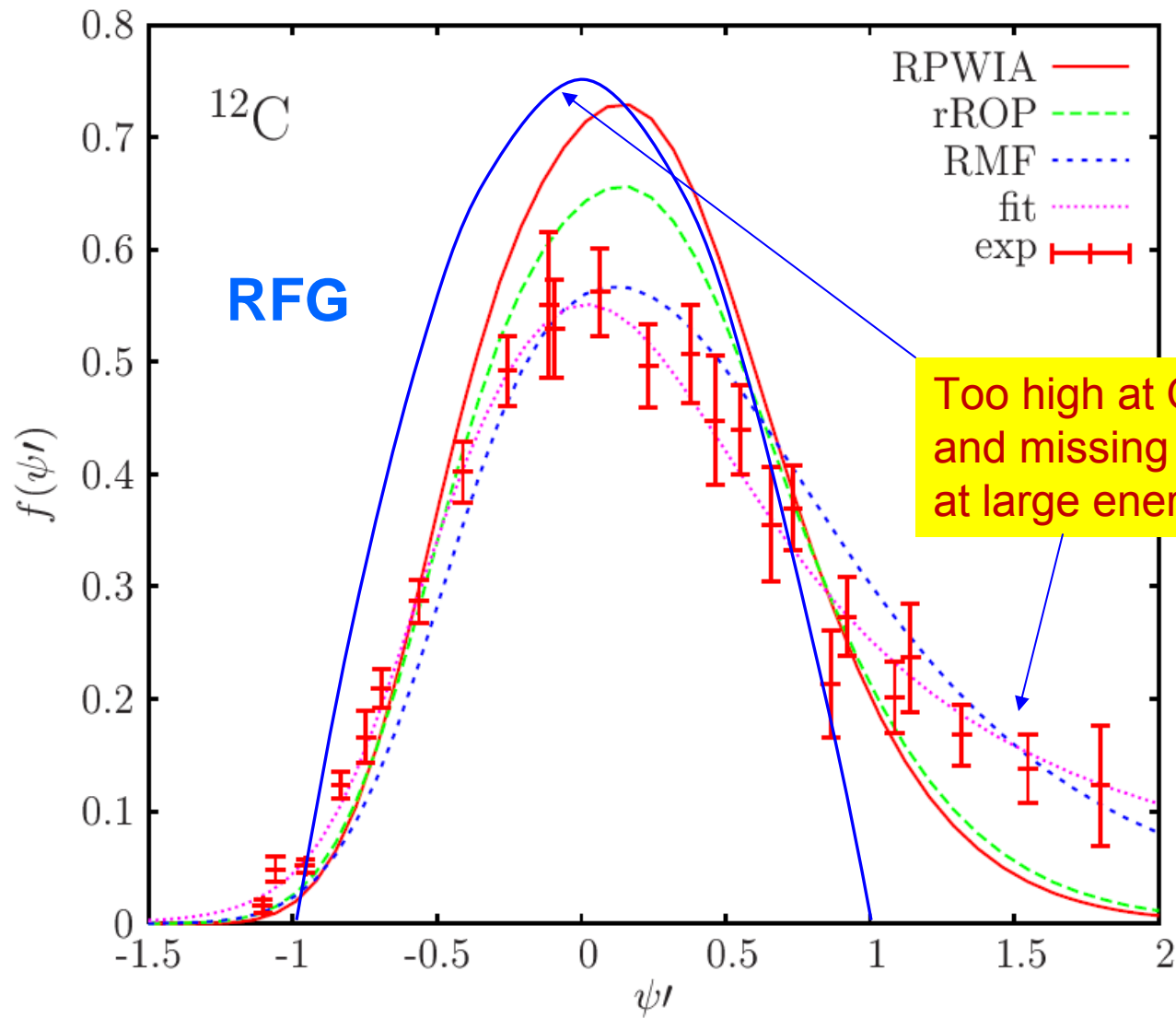
CC Neutrino Reactions (^{12}C)

Continuum Shell Model (dotted)

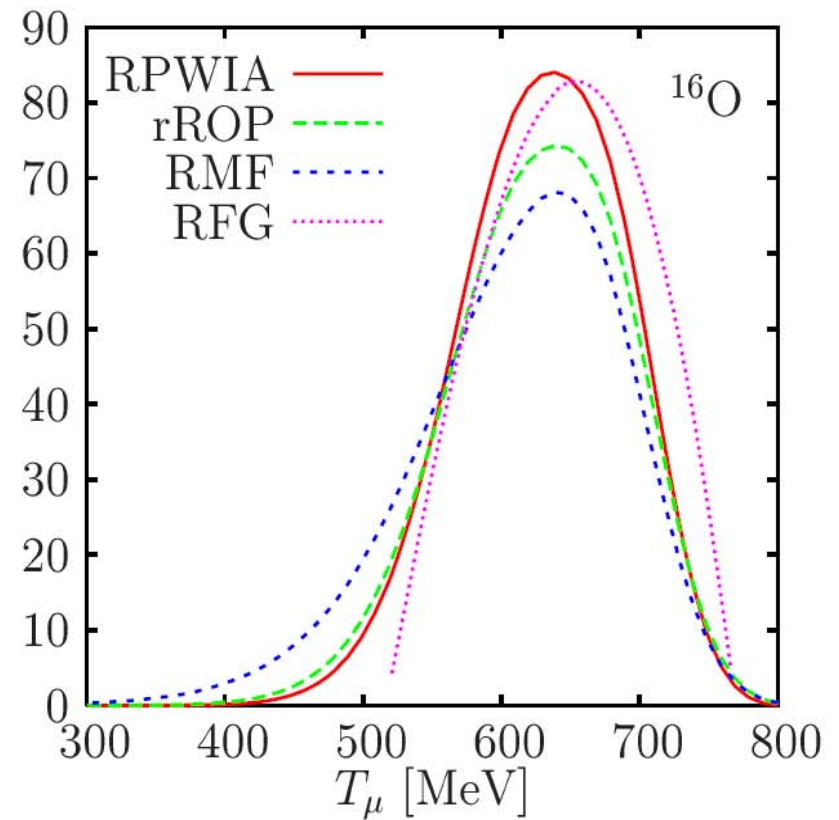
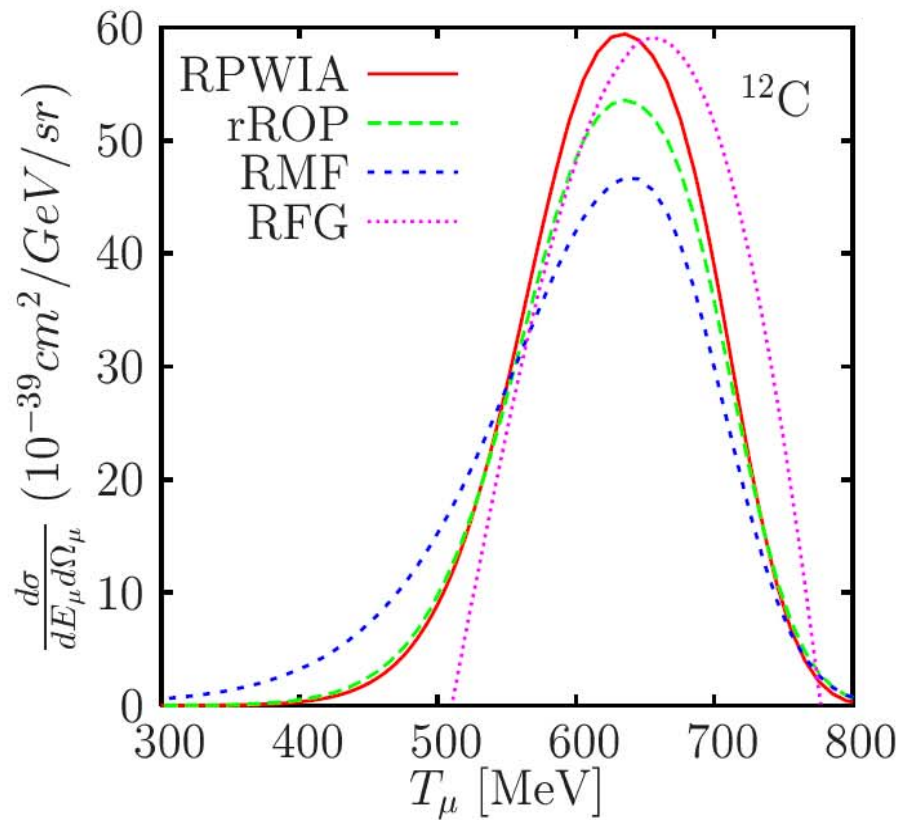








CC Neutrino Reactions (MiniBooNE conditions)



Summary so far:

2. Relativistic effects from kinematics and boost factors are essential.
3. Interaction contributions in both initial and final states are significant and naïve models such as the RFG fail to reproduce the data, while for inclusive scattering RMF theory is much better.

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3. Interaction contributions in both initial and final states are significant and naïve models such as the RFG fail to reproduce the data, while for inclusive scattering RMF theory is much better.

... additionally, so far only one-body currents have been discussed and one should account for the contributions of two-body Meson-Exchange Currents (MEC), especially those that contain an intermediate Δ and π exchange

2p-2h Meson_Exchange Currents (MEC)

Basic references:

Original early non-relativistic studies

J. W. Van Orden, T. W. Donnelly, T. deForest, Jr. and W. C. Hermans,
Phys. Lett. **76B** (1978) 393.

J. W. Van Orden and T. W. Donnelly, *Ann. Phys. (N.Y.)* **131** (1980) 451
together with Wally Van Orden's Ph.D. thesis (Stanford, 1978).

2p-2h Meson_Exchange Currents (MEC)

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J. W. Van Orden and T. W. Donnelly, *Ann. Phys. (N.Y.)* **131** (1980) 451
together with Wally Van Orden's Ph.D. thesis (Stanford, 1978).

More recent relativistic work

M. J. Dekker, P. J. Brussaard and J. A. Tjon, *Phys. Rev.* **C49** (1994) 2650.

A. De Pace, M. Nardi, W. M. Alberico, T. W. Donnelly and A. Molinari,
Nucl. Phys. **A726** (2003) 303 and **A741** (2004) 249.

+ recent work (J. E. Amaro *et al.*)

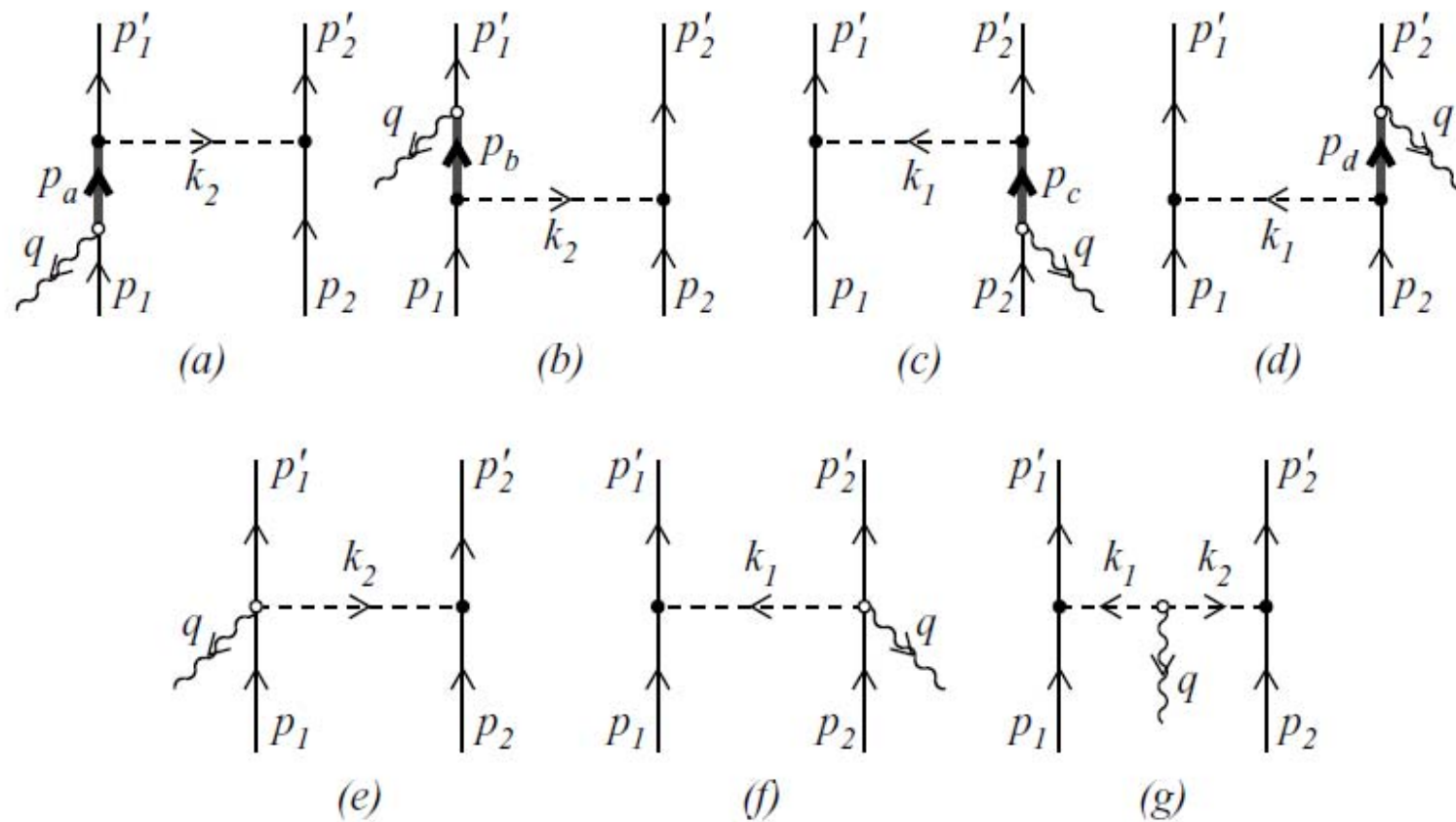


FIG. 1: The two-body meson exchange currents in free-space. The thick lines in the diagrams (a) to (d) represent the Δ propagation.

As mentioned earlier, our computation will

be confined to the 2p-2h transverse response function which for the RFG reads [27]:

$$\begin{aligned}
 R_T(\mathbf{q}, \omega) = & \frac{1}{2} \frac{V}{(2\pi)^9} \int \frac{d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}'_1}{16E_{\mathbf{p}'_1} E_{\mathbf{p}'_2} E_{\mathbf{p}_1} E_{\mathbf{p}_2}} \theta(|\mathbf{p}'_1| - k_F) \theta(|\mathbf{p}'_2| - k_F) \theta(k_f - |\mathbf{p}_1|) \theta(k_F - |\mathbf{p}_2|) \\
 & \times \delta[\omega - (E_{\mathbf{p}'_1} + E_{\mathbf{p}'_2} - E_{\mathbf{p}_1} - E_{\mathbf{p}_2})] \sum_{\sigma\tau} \sum_{i,j=1}^3 V^4 \left(\delta_{ij} - \frac{q_i q_j}{q^2} \right) \\
 & \times \left[J_i^\dagger(\mathbf{p}'_1, \mathbf{p}_1, \mathbf{p}'_2, \mathbf{p}_2) J_j(\mathbf{p}'_1, \mathbf{p}_1, \mathbf{p}'_2, \mathbf{p}_2) \right. \\
 & \quad \left. - J_i^\dagger(\mathbf{p}'_1, \mathbf{p}_1, \mathbf{p}'_2, \mathbf{p}_2) J_j(\mathbf{p}'_1, \mathbf{p}_2, \mathbf{p}'_2, \mathbf{p}_1) \right],
 \end{aligned}$$

where the two terms in the last factor correspond to direct and exchange contributions. In Eq. (12) \mathbf{p}'_1 and \mathbf{p}'_2 (\mathbf{p}_1 and \mathbf{p}_2) are the three-momenta of the on-shell particles (holes) taking part in the process and $E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + m_N^2}$.

one needs the space components of the two-body MEC. As is well-known there are three of them, namely the pion-in-flight

$$\mathbf{J}_f^\mu(p'_1, p_1, p'_2, p_2) = -i \frac{1}{V^2} \frac{f_{\pi NN}^2 f_{\gamma\pi\pi}}{\mu_\pi^2} (\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)})_3 \Pi(k_1)_{(1)} \Pi(k_2)_{(2)} (k_2 - k_1)^\mu, \quad (13a)$$

the seagull

$$\mathbf{J}_s^\mu(p'_1, p_1, p'_2, p_2) = -i \frac{1}{V^2} \frac{f_{\pi NN} f_{\gamma\pi NN}}{\mu_\pi^2} (\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)})_3 \left[\Pi(k_2)_{(2)} (\gamma^\mu \gamma^5)_{(1)} - \Pi(k_1)_{(1)} (\gamma^\mu \gamma^5)_{(2)} \right] \quad (13b)$$

and the Δ current (derived here using the Peccei Lagrangian)

$$\begin{aligned} \mathbf{J}_\Delta^\mu(p'_1, p_1, p'_2, p_2) = & -\frac{1}{V^2} \frac{f_{\pi NN} f_{\pi N\Delta} f_{\gamma N\Delta}}{2m_N \mu_\pi^2} \left\{ \left[\left(\frac{2}{3} \tau_3^{(2)} - \frac{i}{3} (\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)})_3 \right) \left(j_{(a)}^\mu(p_a, k_2, q) \gamma^5 \right)_{(1)} \right. \right. \\ & \left. \left. + \left(\frac{2}{3} \tau_3^{(2)} + \frac{i}{3} (\boldsymbol{\tau}^{(1)} \times \boldsymbol{\tau}^{(2)})_3 \right) \left(\gamma^5 j_{(b)}^\mu(p_b, k_2, q) \right)_{(1)} \right] \Pi(k_2)_{(2)} + (1 \leftrightarrow 2) \right\}. \end{aligned} \quad (13c)$$

In the above $k_1 = p'_1 - p_1$ and $k_2 = p'_2 - p_2$ are the momenta of the pions entering into each of the 2p-2h MEC diagrams (the four-momentum carried by the virtual photon is then $q = -k_1 - k_2$) and μ_π is the pion mass.

Also one has

$$\Pi(k)_{(i)} = \frac{(\not{k}\gamma^5)_{(i)}}{k^2 - \mu_\pi^2},$$

the index (i) distinguishing between the two interacting nucleons,

$$j_{(a)\mu}(p, k, q) = (4k_\beta - \not{k}\gamma_\beta)S^{\beta\gamma}(p, m_\Delta)\frac{1}{2}(-\gamma_\mu\not{q}\gamma_\gamma + q_\mu\gamma_\gamma)$$

and

$$j_{(b)\mu}(p, k, q) = \frac{1}{2}(-\gamma_\beta\not{q}\gamma_\mu + q_\mu\gamma_\beta)S^{\beta\gamma}(p, m_\Delta)(4k_\gamma - \gamma_\gamma\not{k}),$$

where $p_a \equiv p_1 - q$, $p_b \equiv p'_1 + q$, m_Δ is the Δ mass and $S^{\beta\gamma}$ the Rarita-Schwinger propagator.

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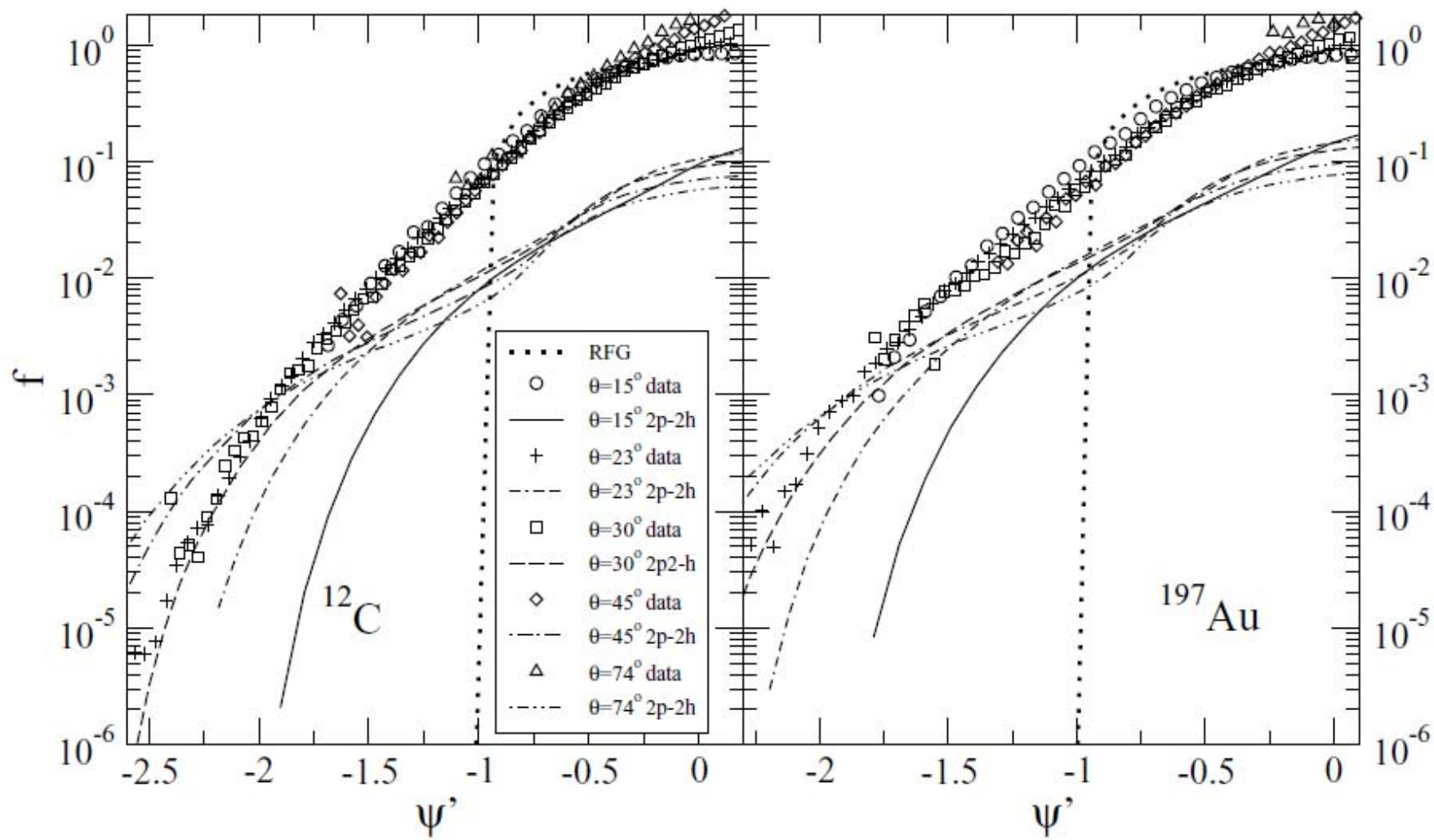
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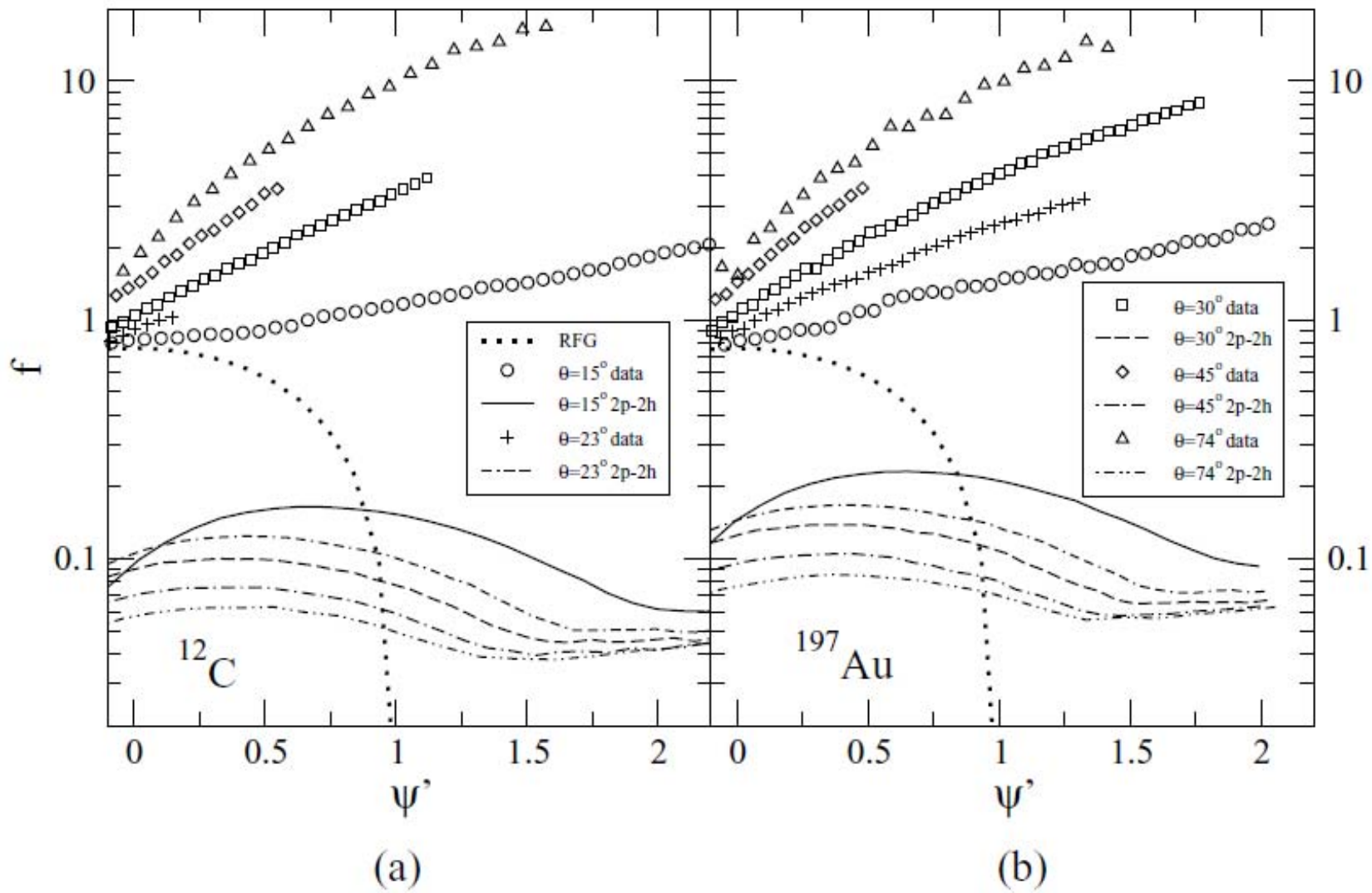
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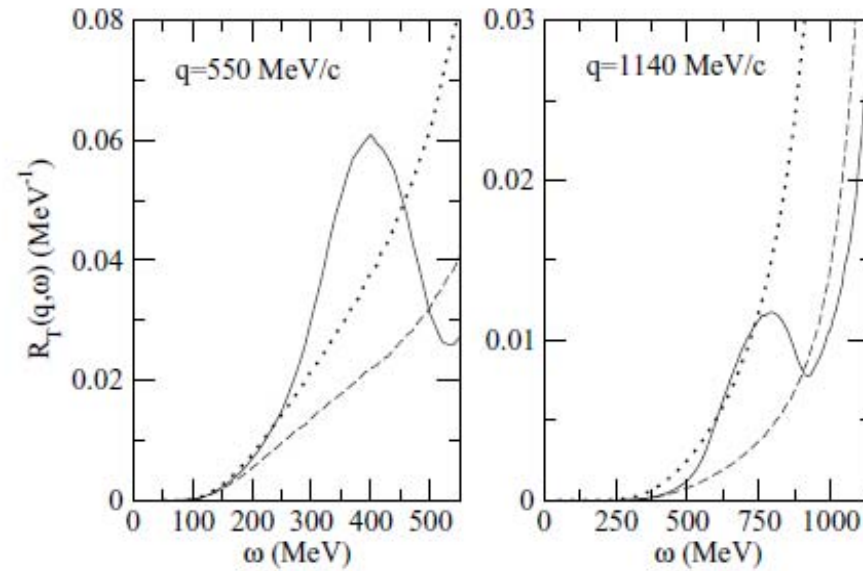
... and many of these are 7-dimensional integrals which must be done numerically



(a) **JLab data at 4.045 GeV**

(b)





The relativistic transverse response function $R_T(q, \omega)$ at $q = 550 \text{ MeV}/c$ and $q = 1140 \text{ MeV}/c$ computed with various versions of the Δ propagator to illustrate the sensitivity to the latter: exact propagator (solid), static propagator (dashed) and constant propagator (dotted) (see text for the related definitions). In all instances $\bar{\epsilon}_2 = 70 \text{ MeV}$ and $k_F = 1.3 \text{ fm}^{-1}$.

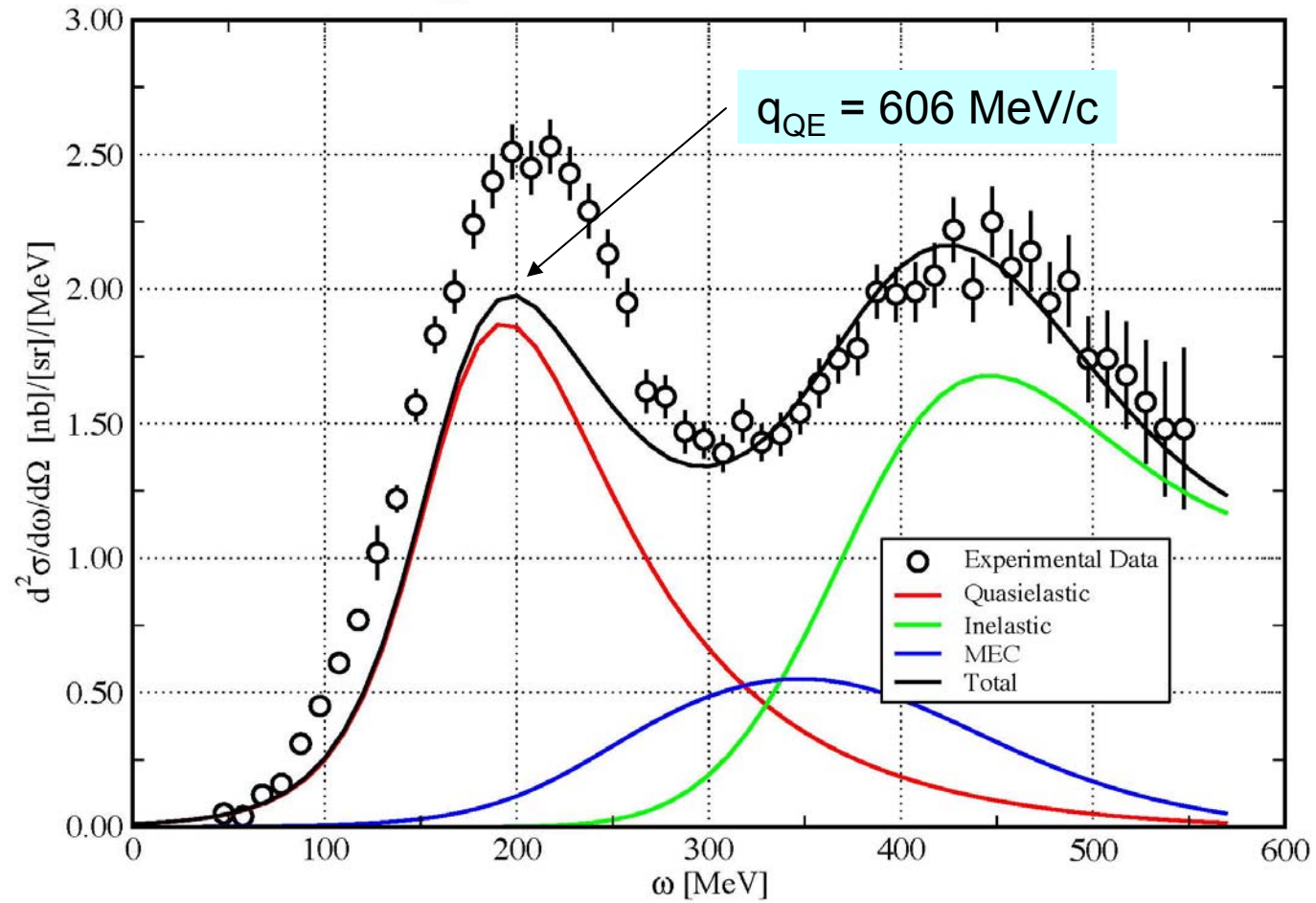
Two new efforts are in progress by the Sevilla-Granada-Torino-MIT collaboration (SuSA):

1. A new parametrization of the above MEC results has been obtained; this can be used in place of the full (computationally intensive) calculation
2. A different technique is being followed to avoid performing the traces; this should provide an independent evaluation of the **fully relativistic** MEC

Electron scattering

Quasielastic Scattering from ^{12}C

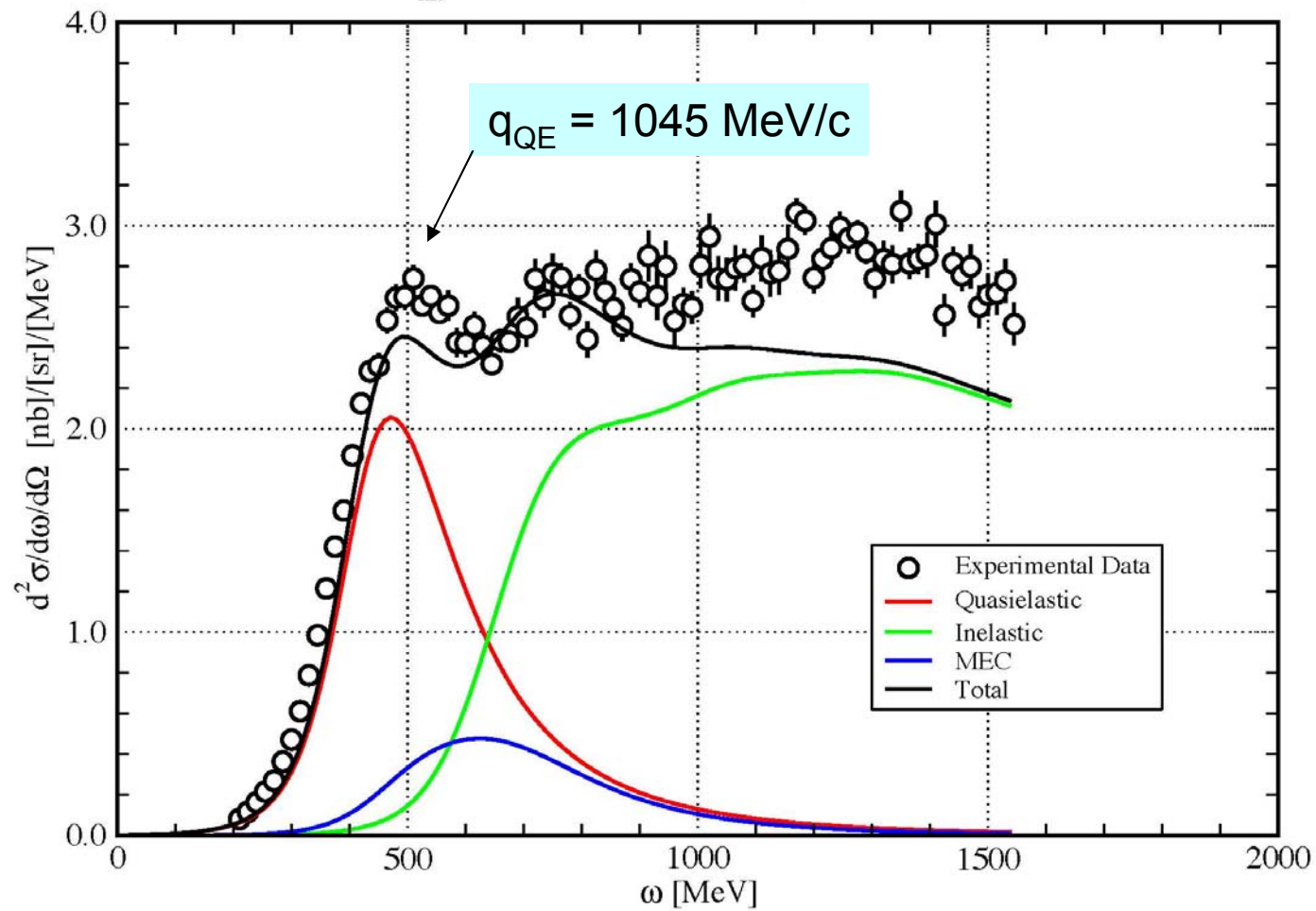
$p_{\text{inc}} = 680 \text{ MeV}/c$, $\theta = 60 \text{ deg}$, Saclay Data



Electron scattering

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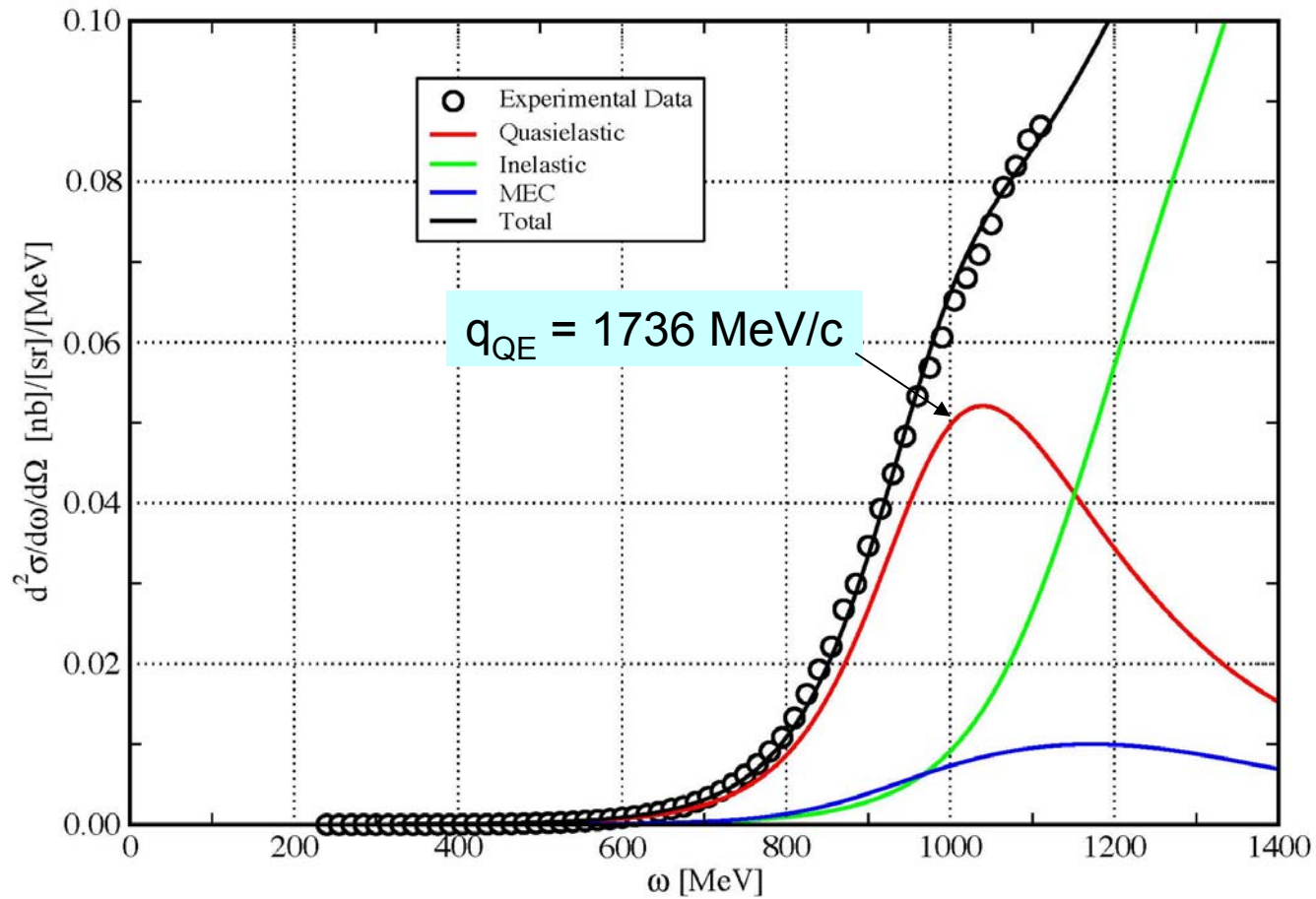
$p_{\text{inc}} = 3595 \text{ MeV}/c$, $\theta = 16 \text{ deg}$, SLAC Data



Electron scattering

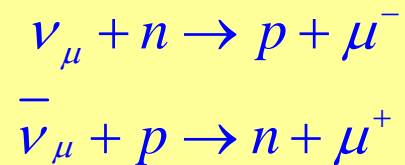
Quasielastic Scattering from ^{12}C

$p_{\text{inc}} = 4045 \text{ MeV}/c$, $\theta = 23 \text{ deg}$, JLab Data

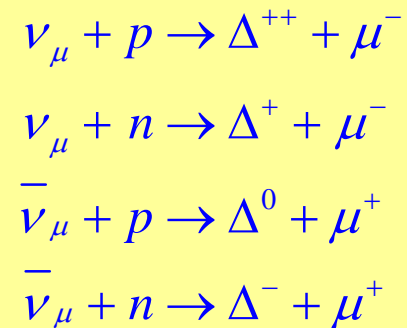


Just as for the electron scattering reactions in the QE and Δ regions, we **use the scaling functions determined above**, but now multiply by the corresponding **charge-changing neutrino reaction cross sections** for the Z protons and N neutrons in the nucleus.

For the QE region we have the elementary reactions



While in the Δ region we have



... and so on.

Note that these reactions are **isovector** only, whereas electron scattering contains both isoscalar and isovector contributions (the transverse EM response is, in fact, predominantly isovector at high energy).

Thus, in going from electron scattering where the universal scaling function came from the L response (essentially 50% isoscalar and 50% isovector) to CC neutrino reactions we have had to invoke

Scaling of the 3rd Kind

where the isospin nature of the scaling functions is assumed to be universal.

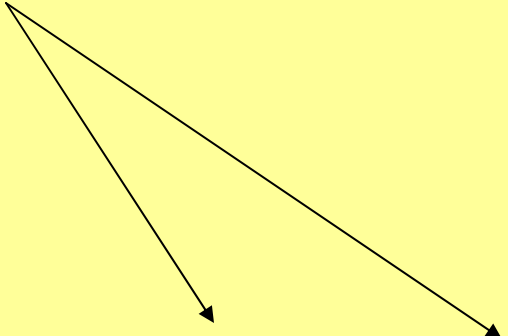
The nuclear response function may be decomposed into a generalization of the familiar Rosenbluth expression from studies of electron scattering (see above):

$$R_\chi = \left[\hat{V}_{CC} R_{CC} + 2\hat{V}_{CL} R_{CL} + \hat{V}_{LL} R_{LL} + \hat{V}_T R_T \right] + \chi \left[\hat{V}_{T'} R_{T'} \right]$$

$$R_K = \begin{cases} R_K^{VV} + R_K^{AA}, & K = CC, CL, LL, T \\ R_K^{VA}, & K = T' \end{cases}$$

↑
changes sign in
going from neutrinos
to anti-neutrinos

The cross section is dominantly **transverse** (T, T')


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... and has **VV**, AA and VA contributions

The **VV** response has the same (isovector) contributions as occur for electron scattering, including the **2p-2h MEC contributions**

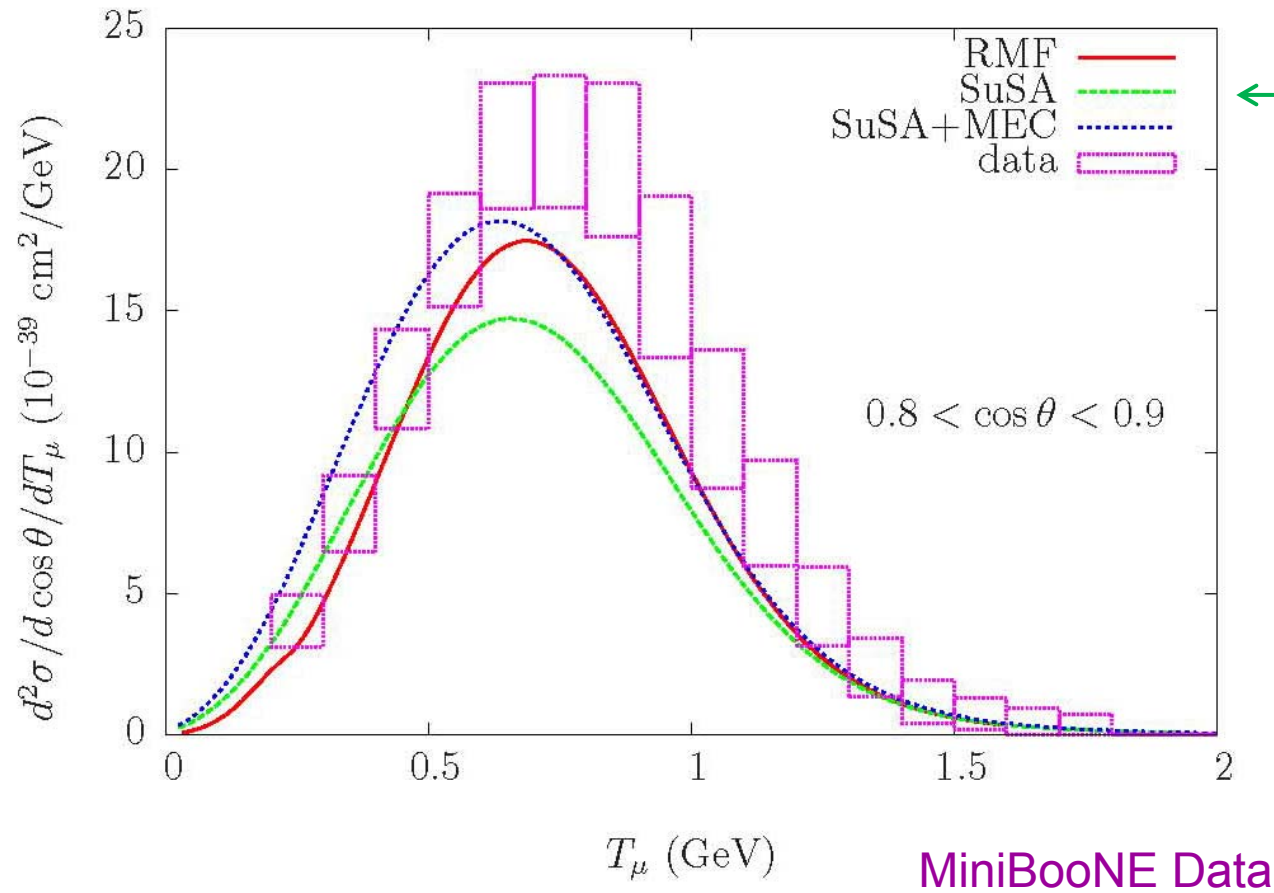
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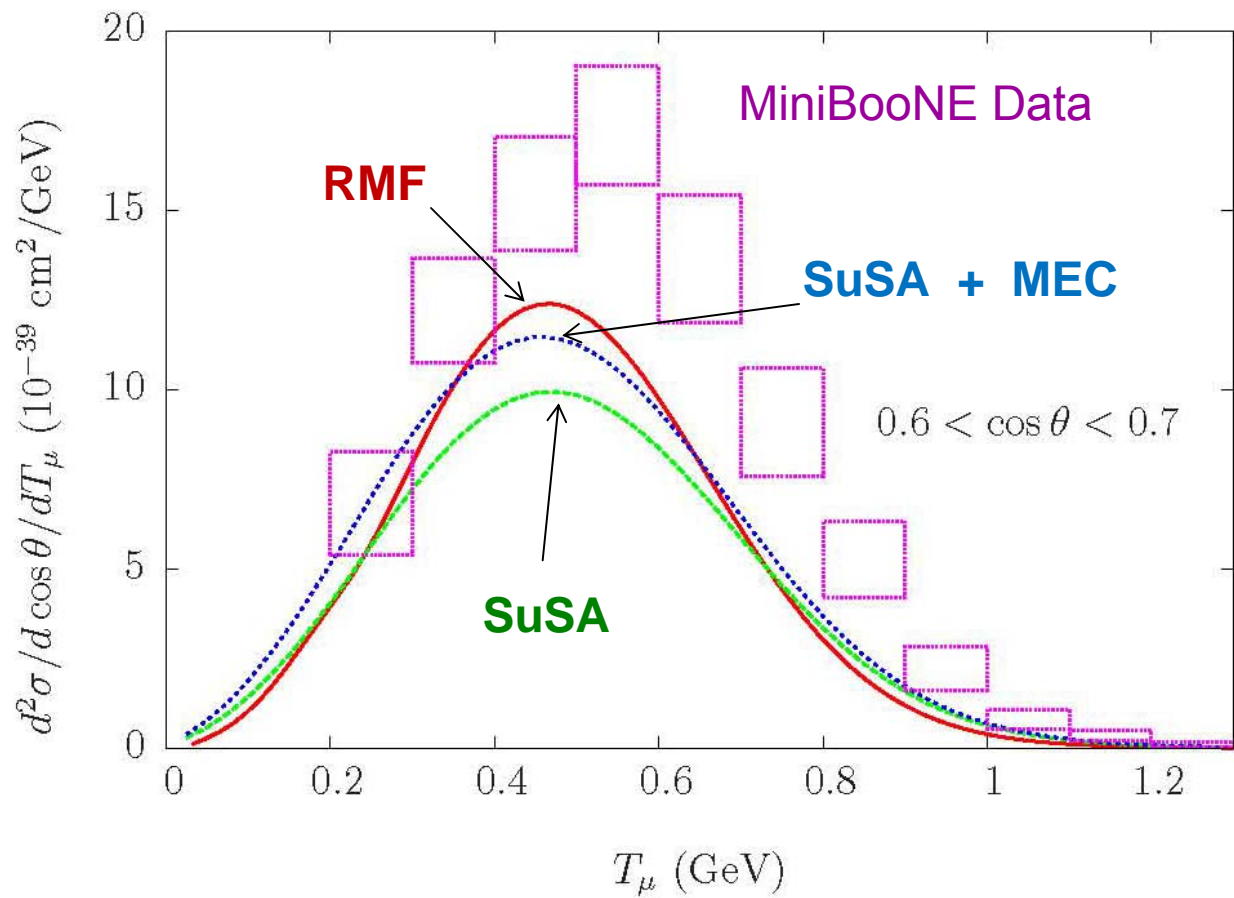
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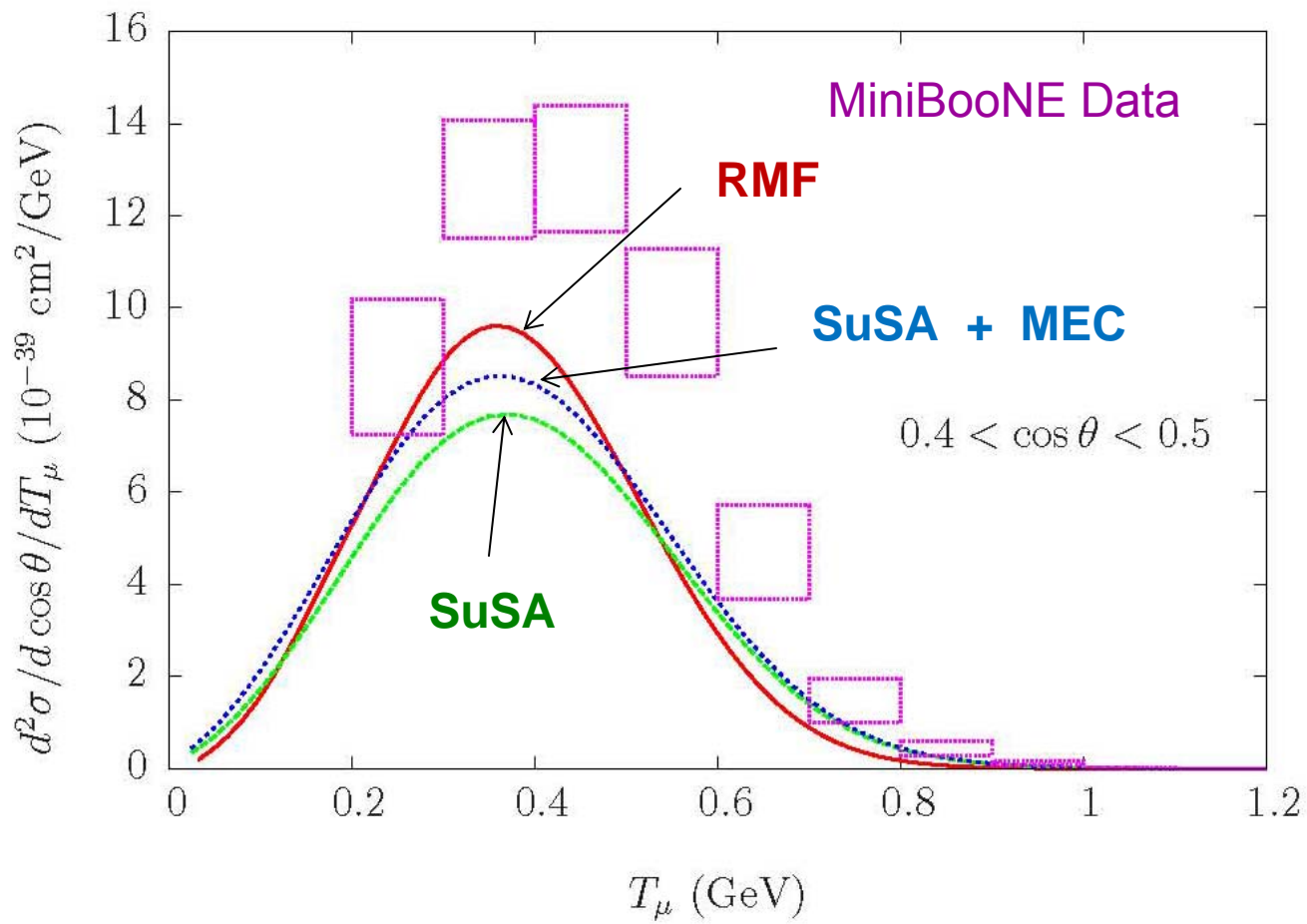
... and has VV, **AA** and **VA** contributions

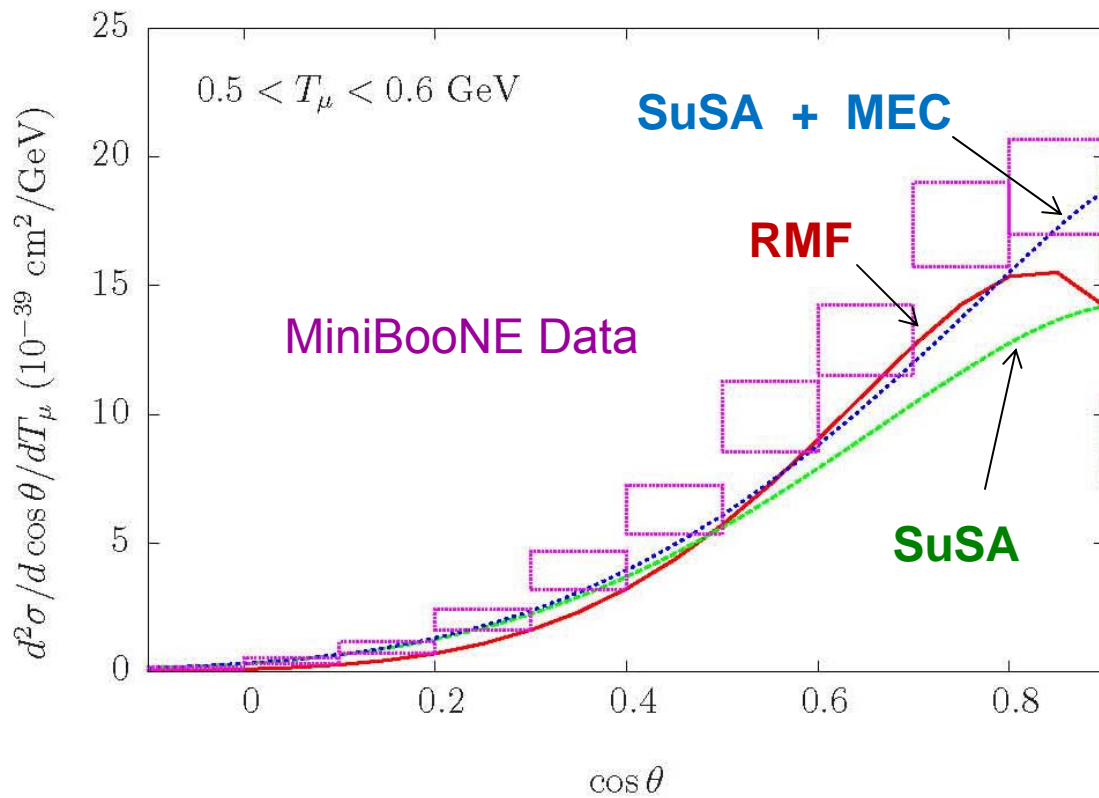
The VV response has the same (isovector) contributions as occur for electron scattering, including the **2p-2h MEC contributions**; however, the **transverse axial-vector matrix elements have no MEC pieces** in leading order and thus the **AA** and **VA** contributions do not contain the scaling violations from MEC

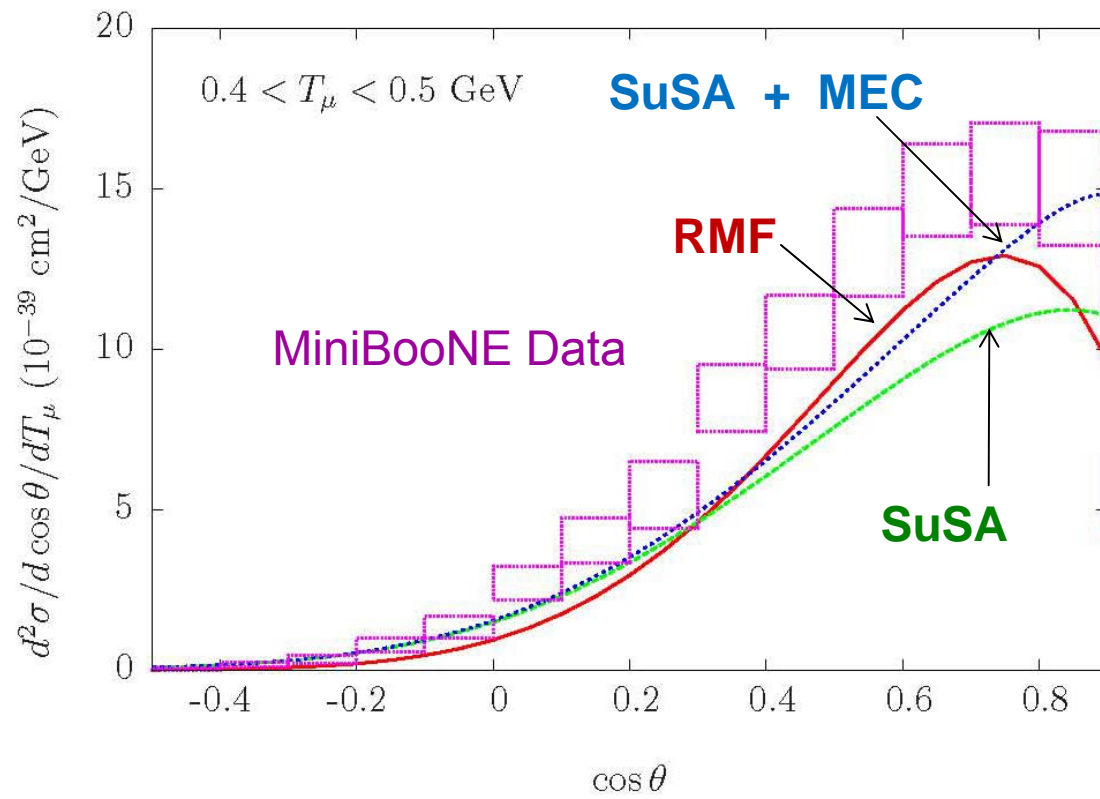


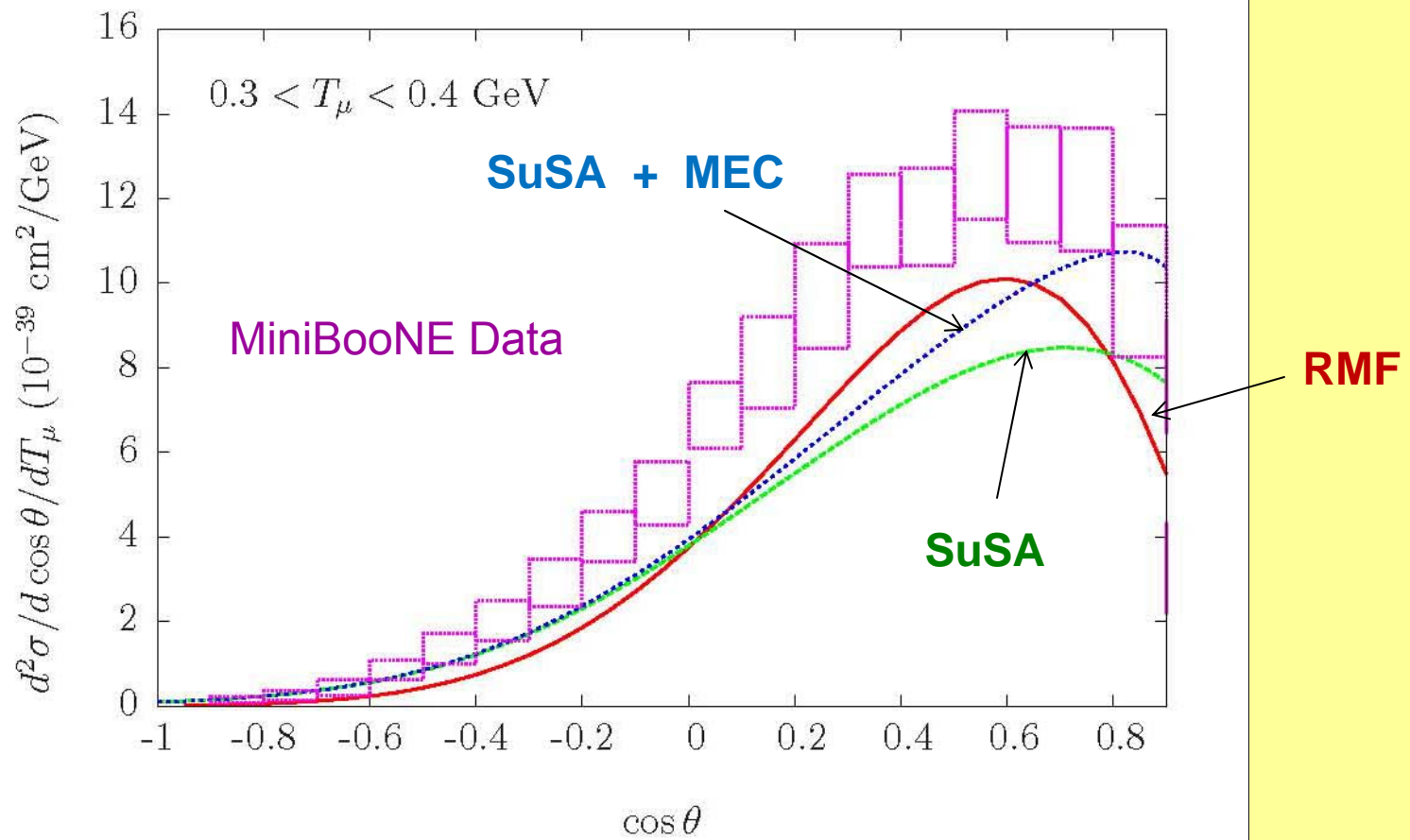
**SuperScaling
 Analysis
 (SuSA)**

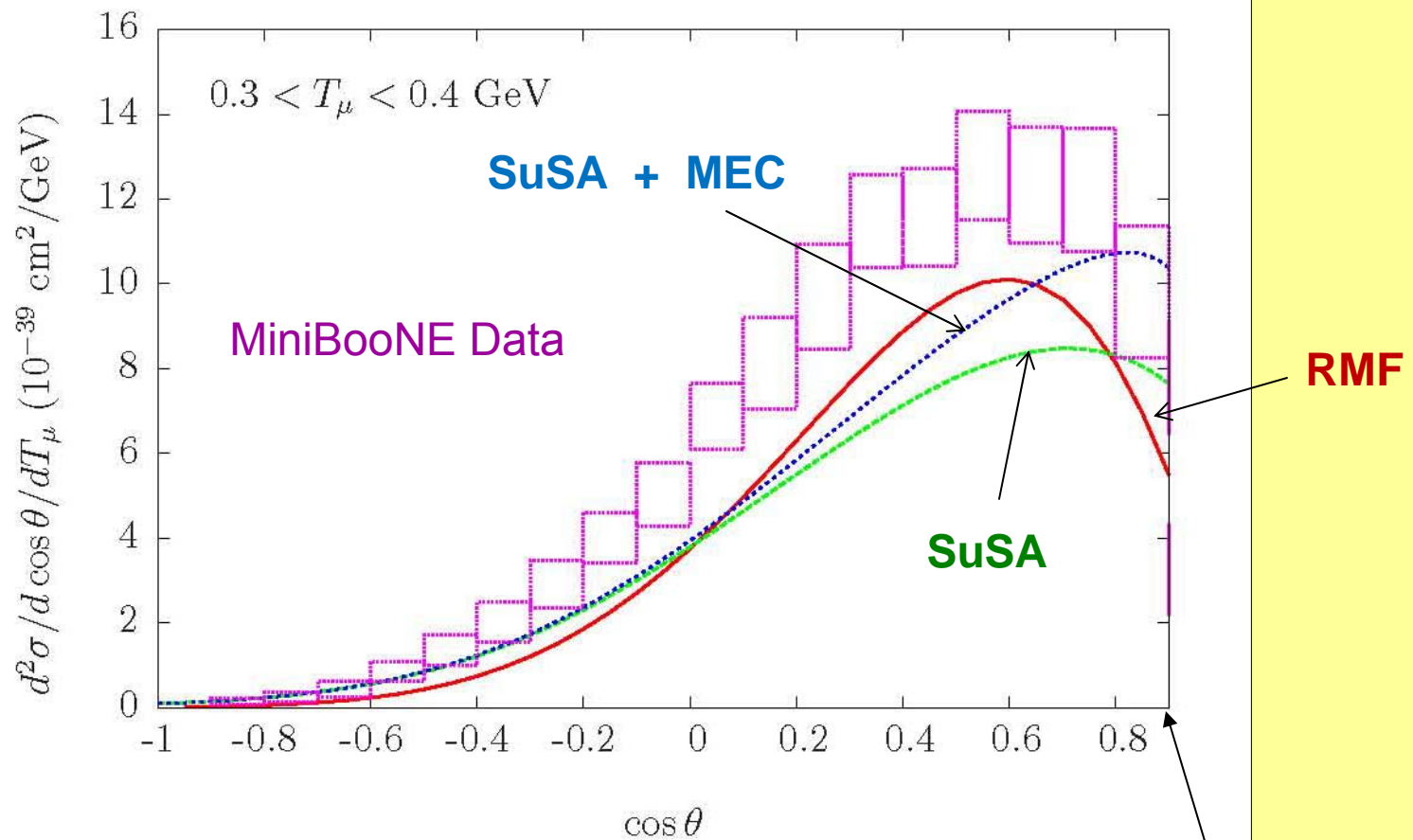










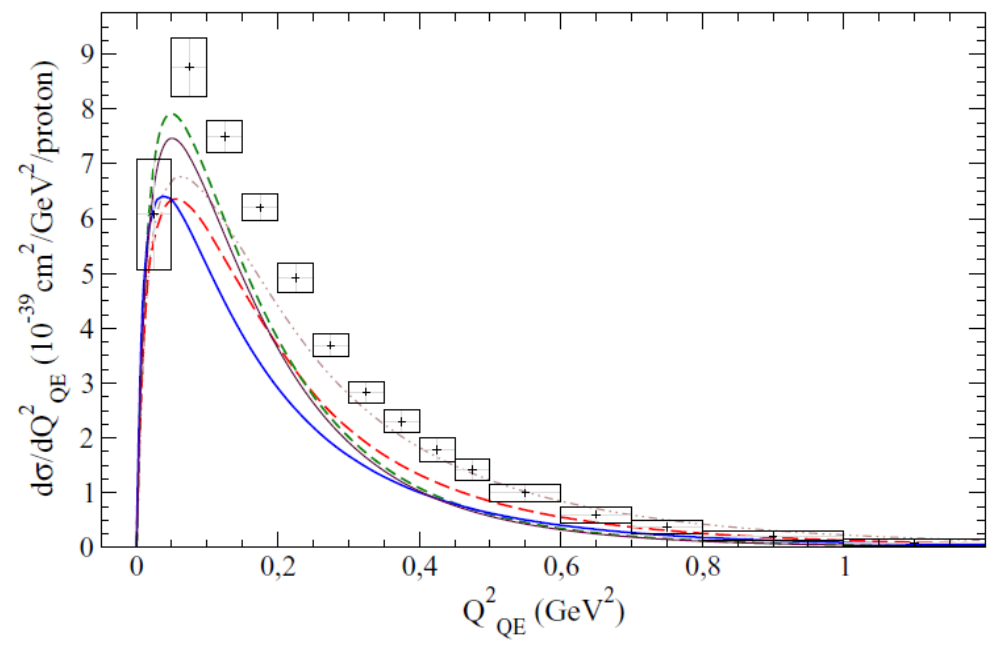
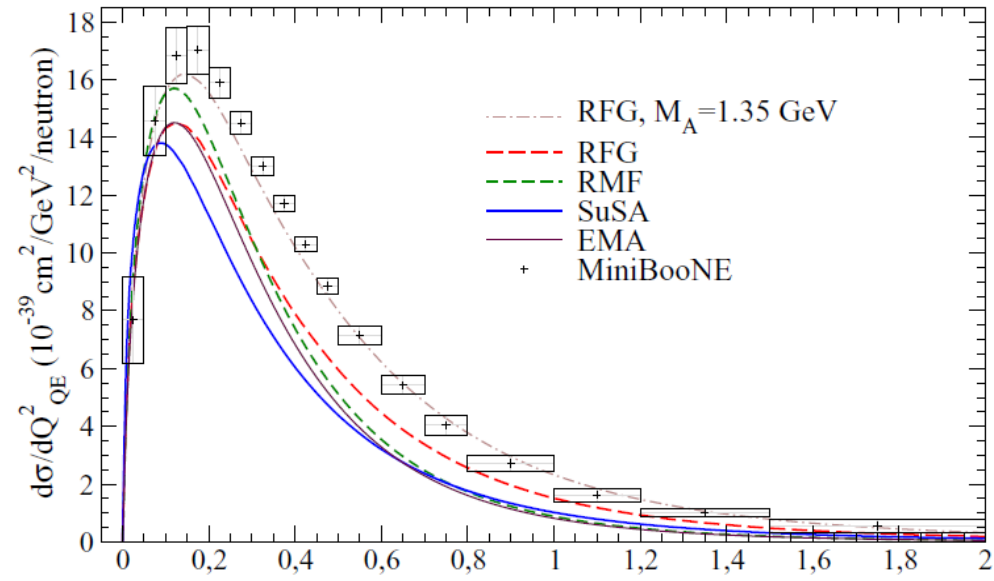


Note: results here cut off at 0.9, since between 0.9 and 1 roughly 1/2 of the cross section arises from excitations below 50 MeV

New work:

New studies of MiniBooNE and MINER ν A

G. D. Megias, M. V. Ivanov, R. Gonzalez-Jimenez, M. B. Barbaro,
J. A. Caballero, T. W. Donnelly and J. M. Udias,
Phys. Rev. **D89** (2014) 093002.



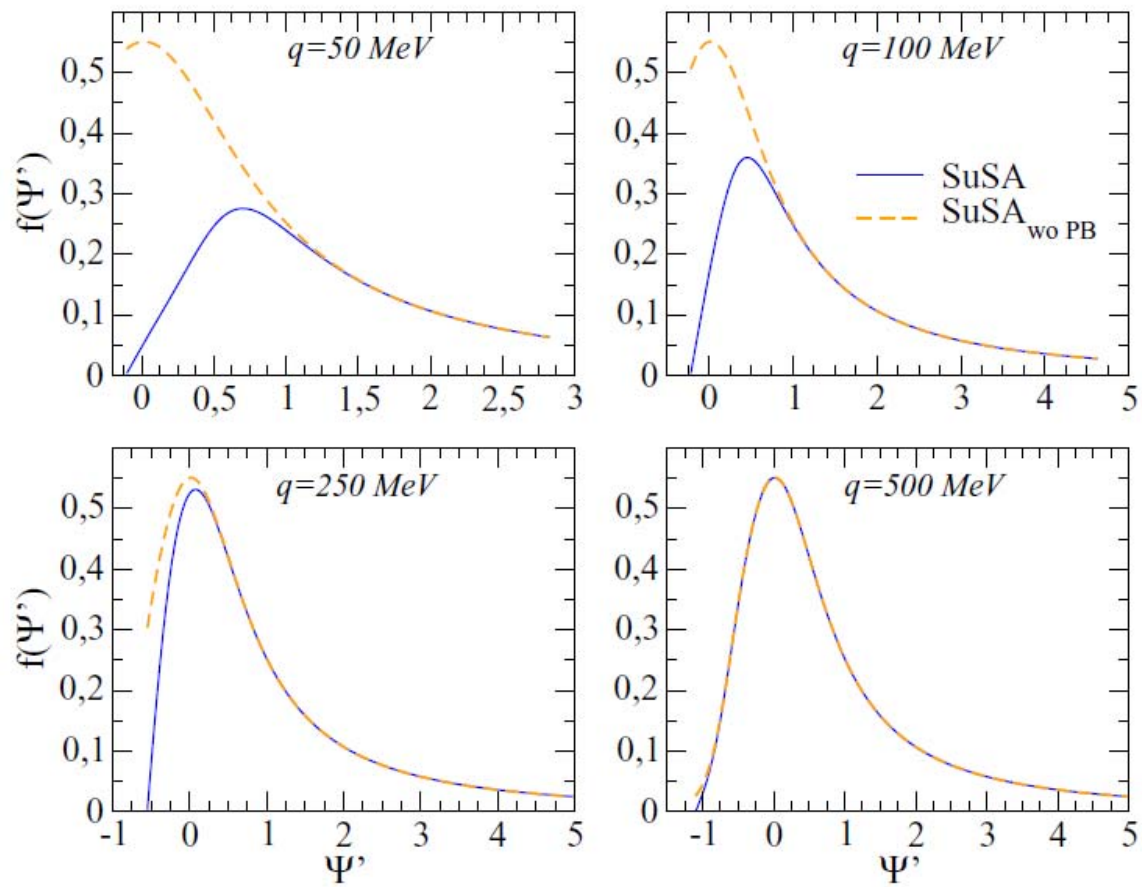
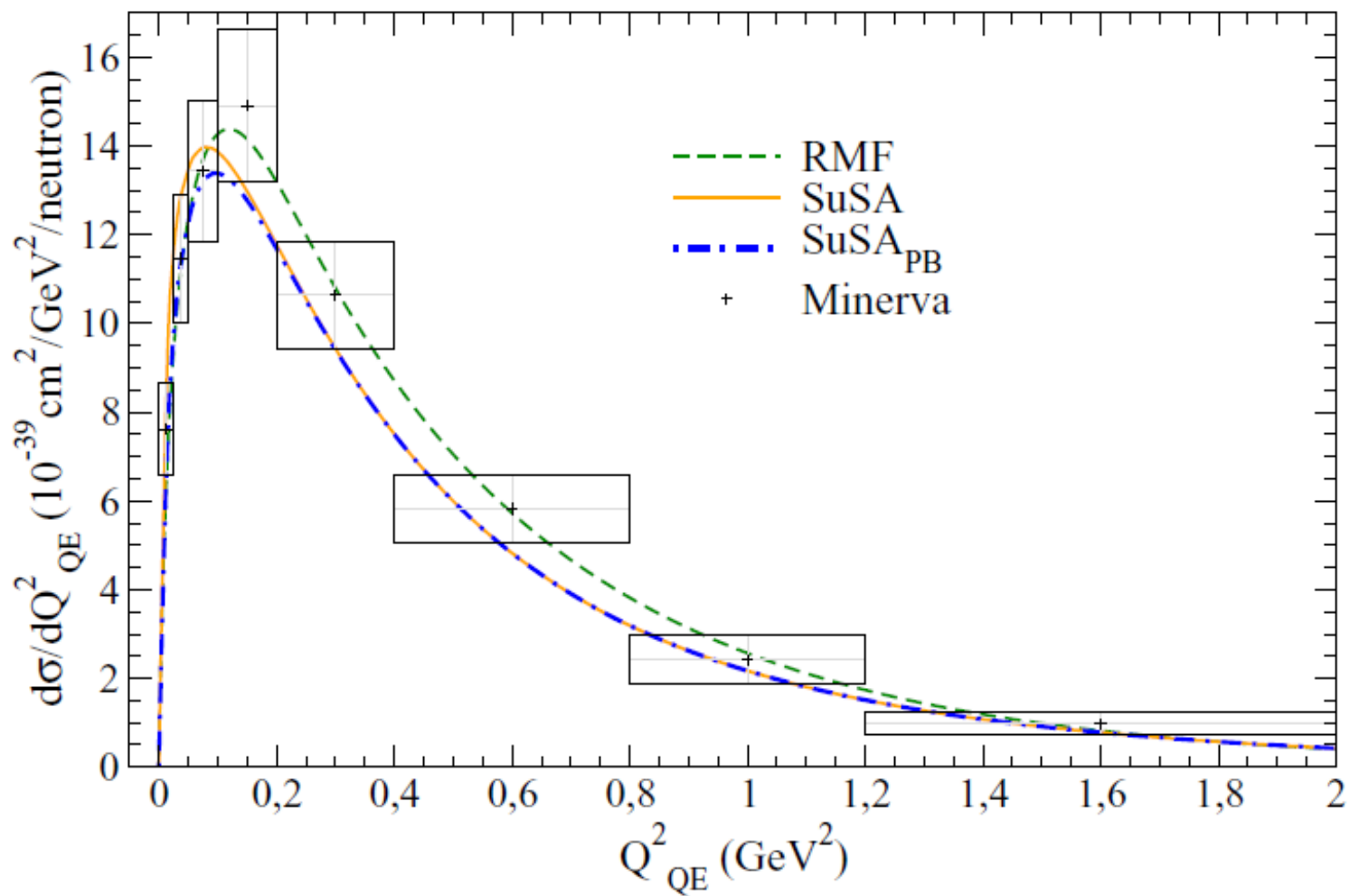
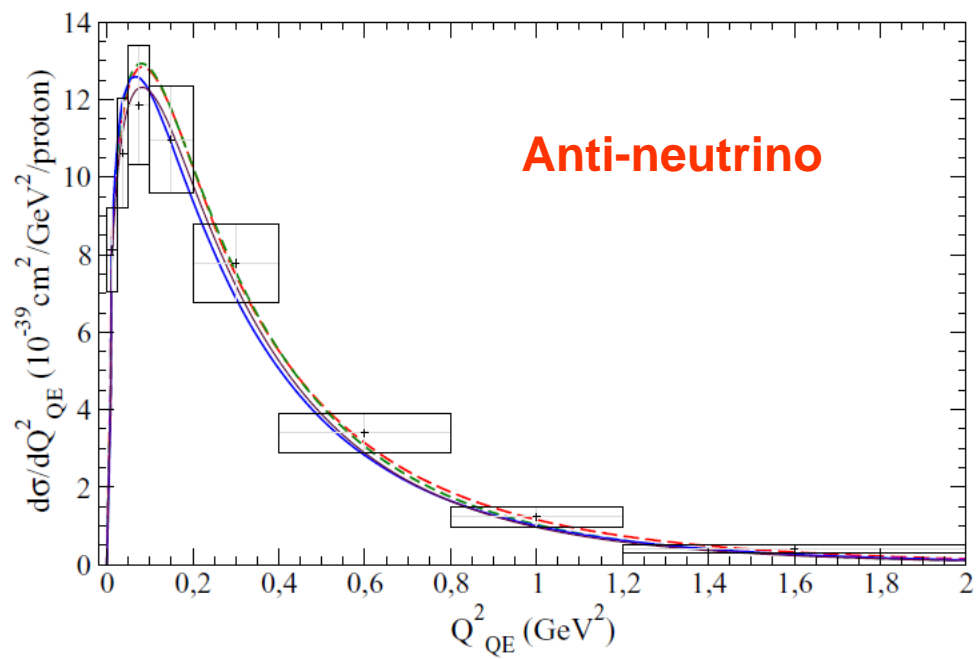
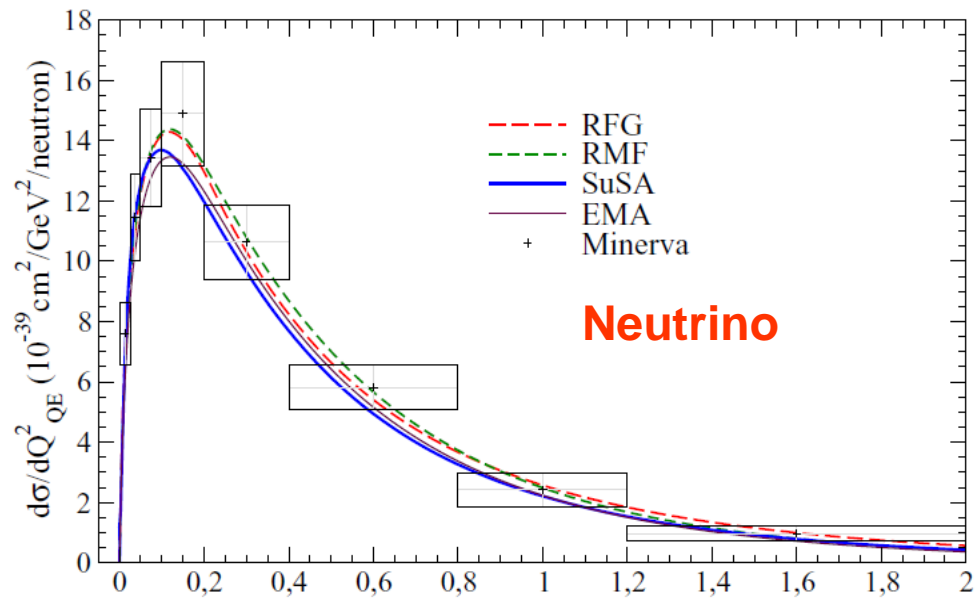
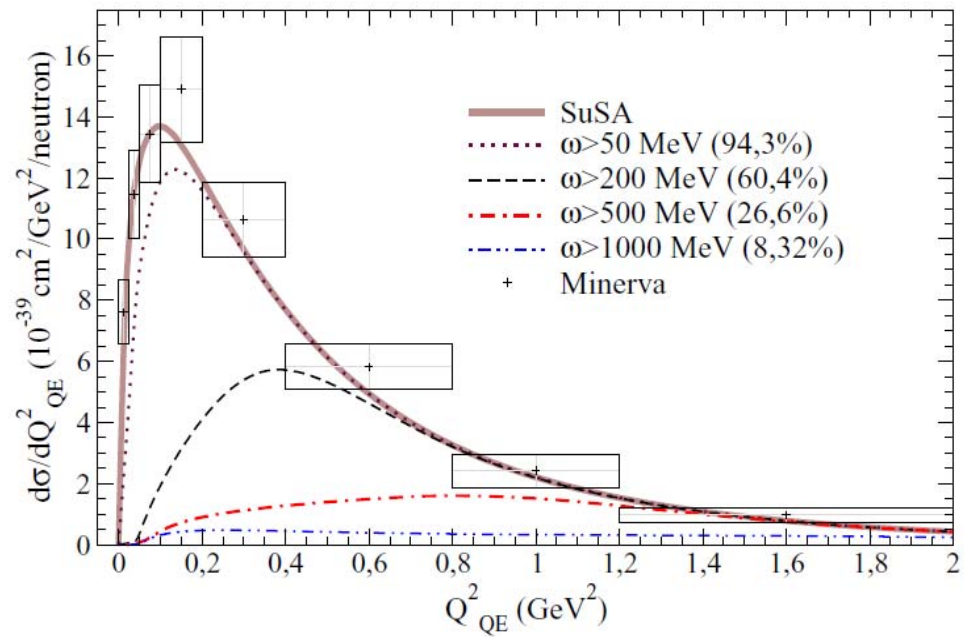
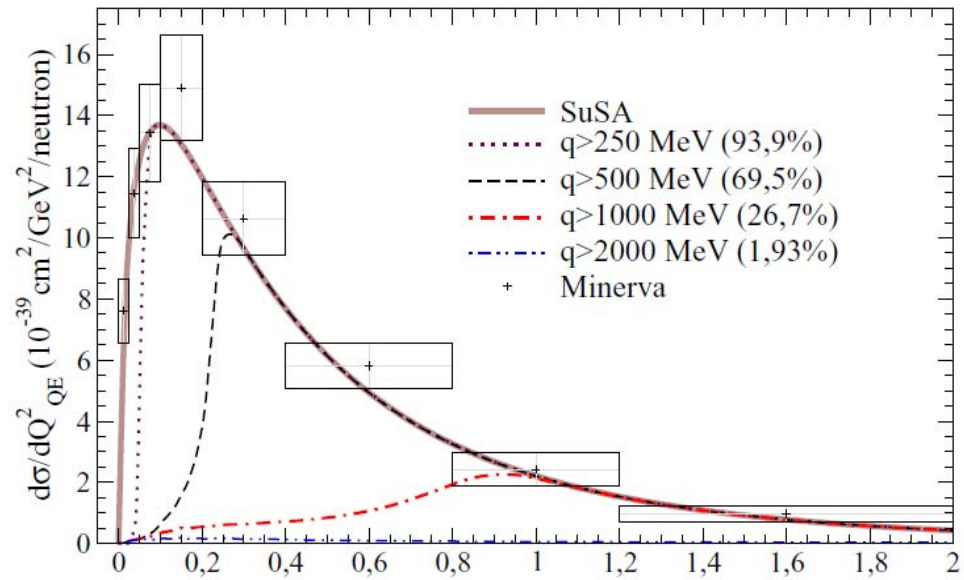
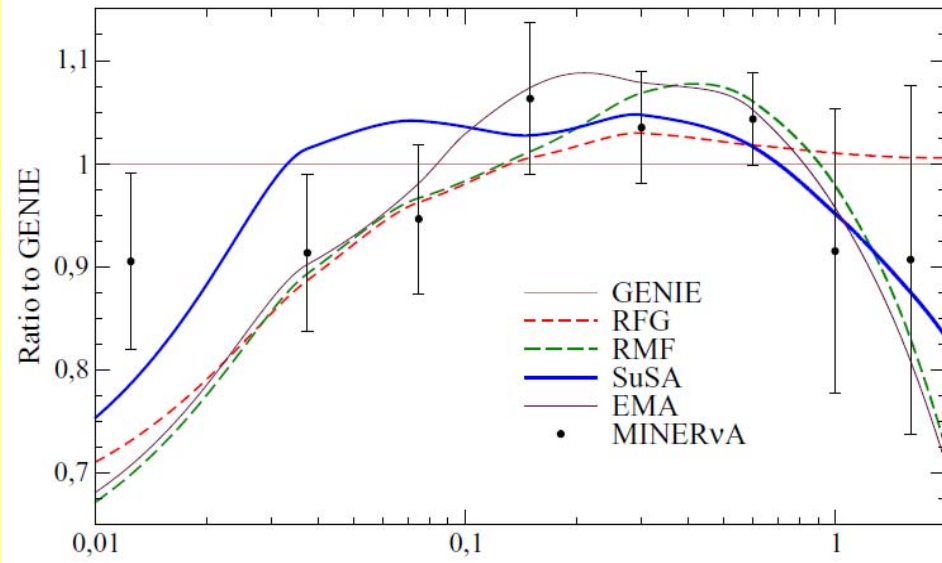


FIG. 1: Superscaling function vs. Ψ' at different q -fixed values and evaluated for the $SuSA$ model with ($SuSA$) and without ($SuSA_{woPB}$) Pauli blocking. .

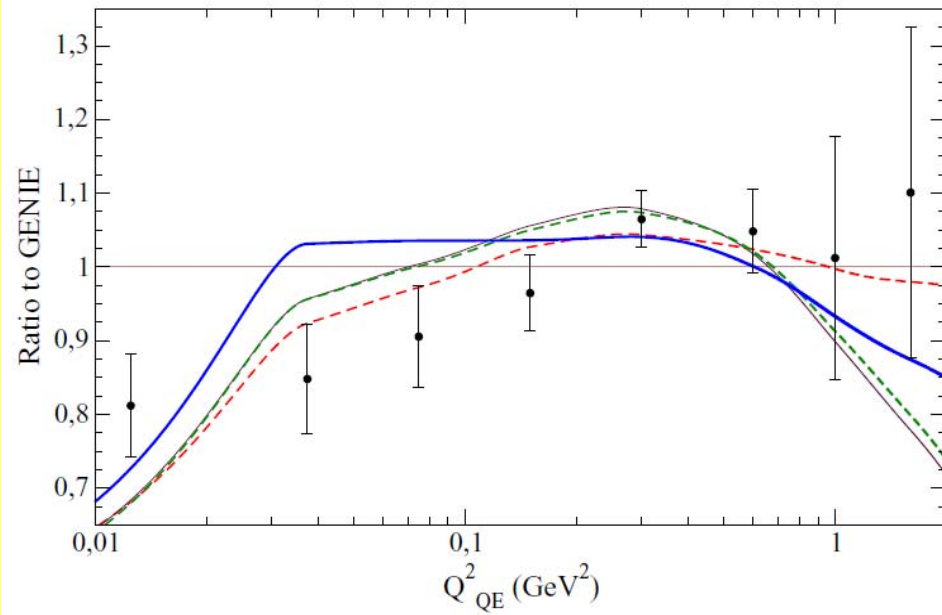








Neutrino



Anti-neutrino

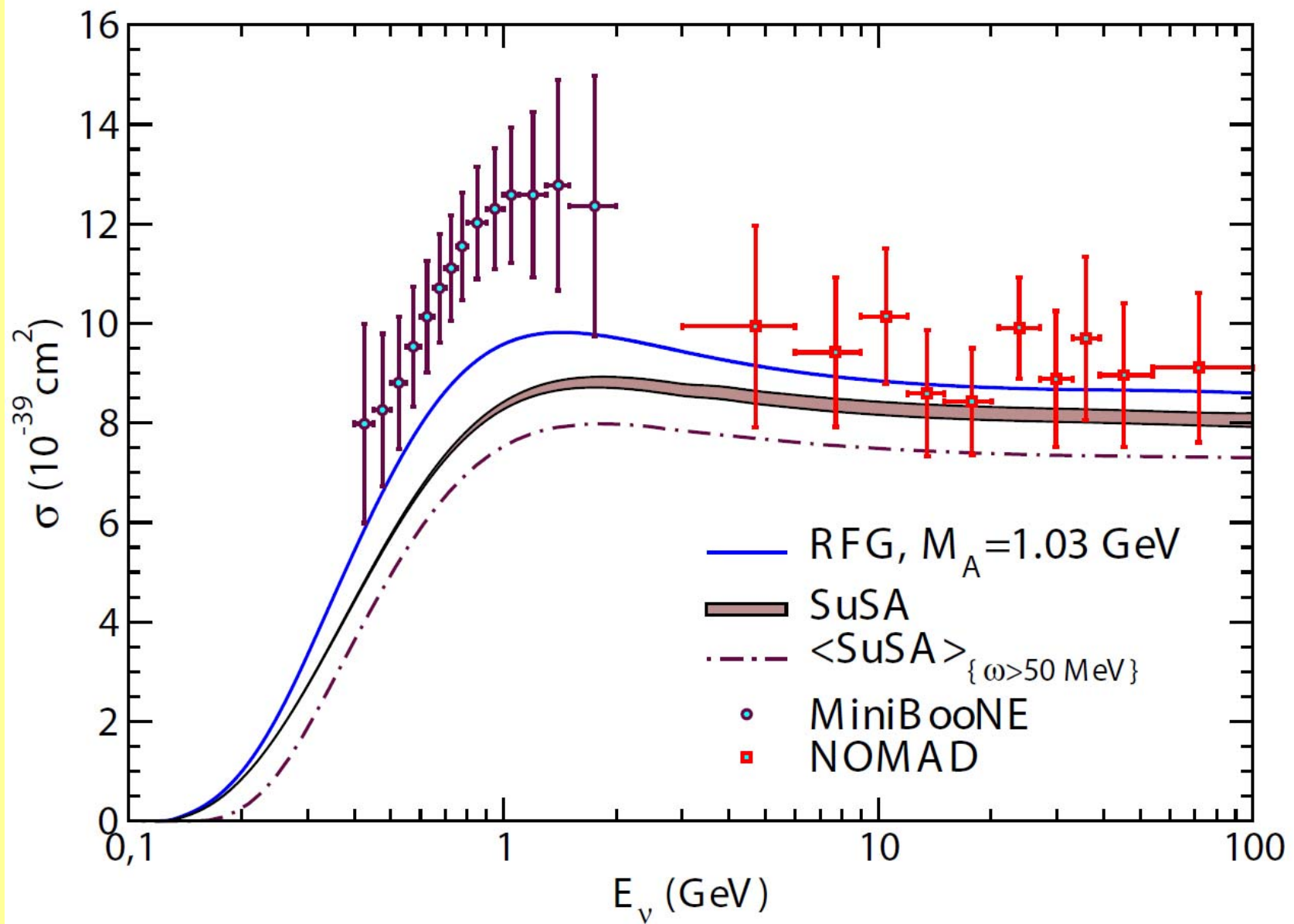
New work:

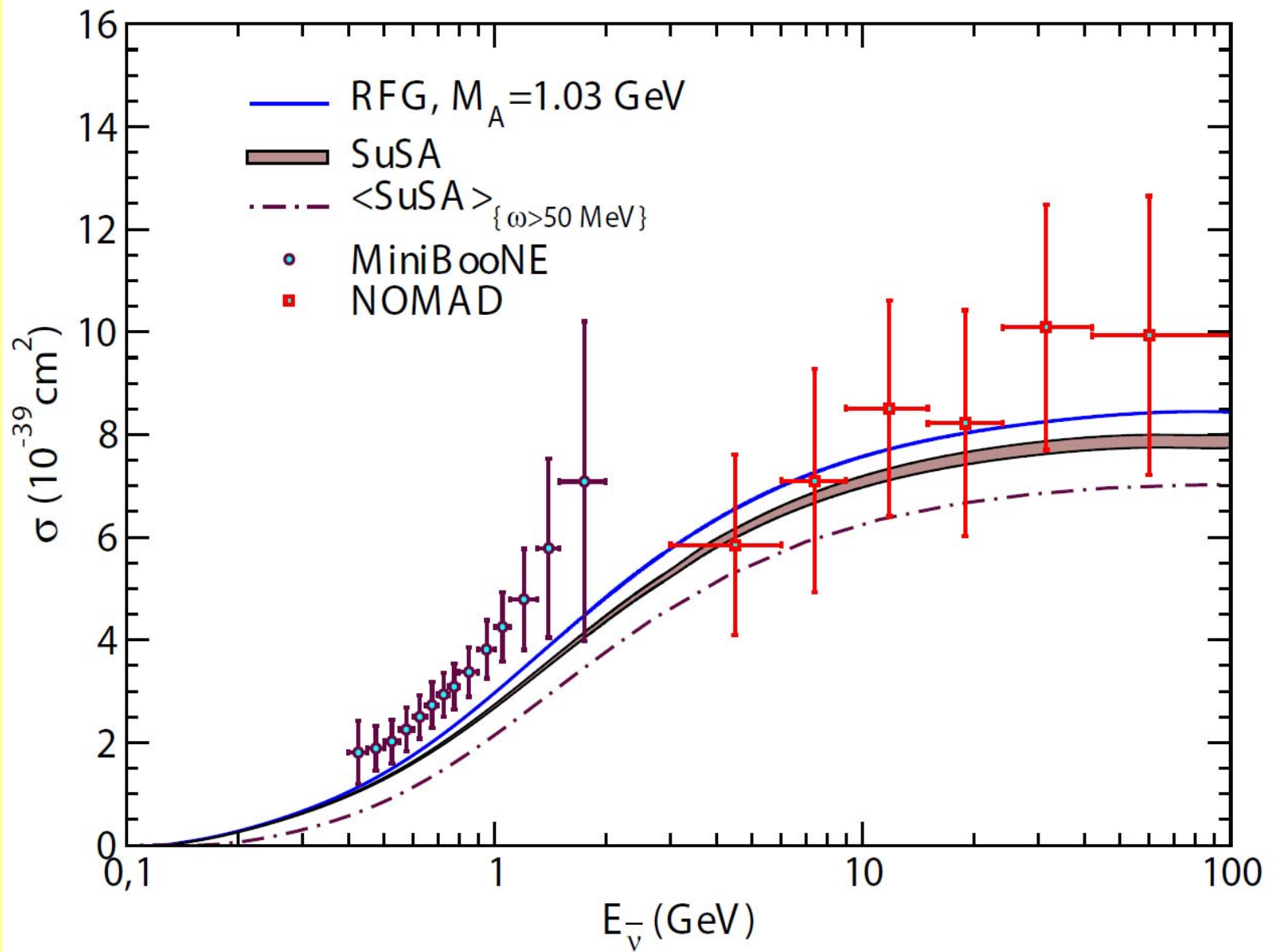
New studies of MiniBooNE and MINER ν A

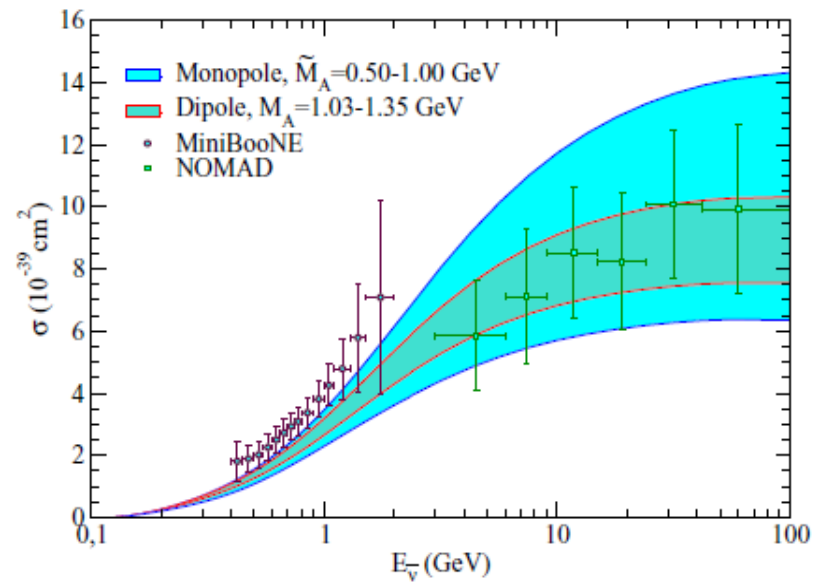
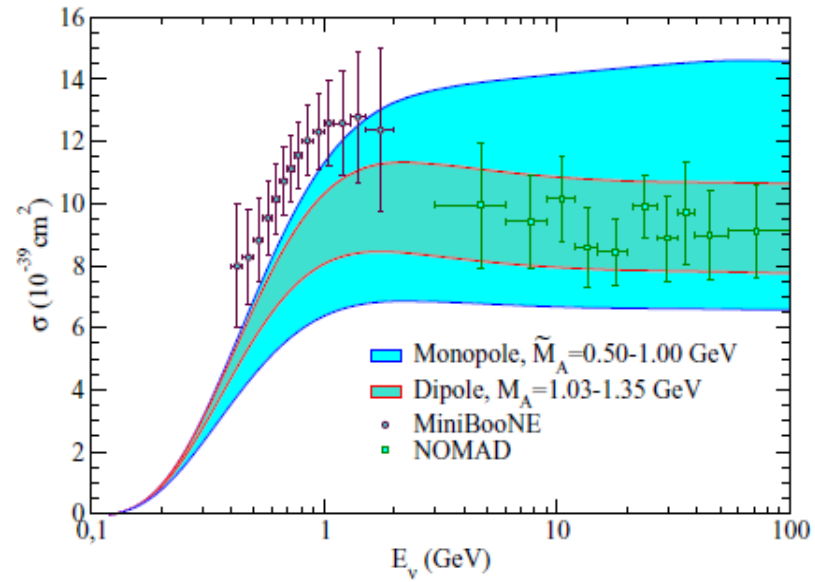
G. D. Megias, M. V. Ivanov, R. Gonzalez-Jimenez, M. B. Barbaro, J. A. Caballero, T. W. Donnelly and J. M. Udias, *Phys. Rev.* **D89** (2014) 093002.

Studies of NOMAD

J. E. Amaro, M. B. Barbaro, J. A. Caballero, G. D. Megias and T. W. Donnelly, *Phys. Lett.* **B725** (2013) 170.







Neutrino

Anti-neutrino

New work:

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SuSA ν 2

R. Gonzalez-Jimenez, G. D. Megias, M. B. Barbaro, J. A. Caballero and T. W. Donnelly, *Phys. Rev.* **C90** (2014) 035501.

Summary here:

4.MEC effects are significant (and should be modeled relativistically).

5.Interaction contributions in both initial and final states are significant and naïve models such as the RFG fail at the 25% level or so to reproduce the data, while for inclusive scattering RMF theory is much better.