Outline:

Lecture 1

- Basics (this may overlap with other speakers' talks)
- The Relativistic Fermi Gas (RFG)
- Superscaling

Lecture 2

- More sophisticated model of inclusive scattering
- 2p-2h Meson –Exchange Currents (MEC)
- Comparisons with data
  - Inclusive electron scattering
  - Inclusive charge-changing neutrino reactions

Lecture 3

- Semi-inclusive semi-leptonic electroweak processes
- Reduction to t- and u-channel inclusive processes

Semi-Inclusive CC Neutrino Reactions

Using standard nuclear physics notation such reactions would be denoted $X(\nu_\ell, \ell^- x)$ and $X(\bar{\nu}_\ell, \ell^+ x)$, where $\ell = e$, μ, or τ. Here x can be any kinematically allowed particle, for instance, γ, a nucleon $N = p$ or n, a deuteron d or triton t, ^{3}He, α, fission fragment, π, K, and so on. The target X may be a nucleus or the proton itself. All of these possibilities are contained in the formalism to follow which is drawn from O. Moreno, T. W. Donnelly, J. W. Van Orden and W. P. Ford, *Phys. Rev.* **D90** (2014) 013014. One should be clear that this notation indicates what is presumed to be detected, not what is actually in the final state. For example, if $x = p$, this means that for sure one proton is in the final state; however, depending on the kinematics chosen for the reaction, there may be many open channels, a proton and a daughter nucleus in some discrete state, two protons and a different nucleus in some discrete state, a proton and a neutron and yet another nucleus in some discrete state, *etc.* The semi-inclusive cross section is then the sum/integral over all unobserved particles, excepting only the one that is presumed to be detected, in this example a proton. At a level lower, one has the inclusive cross section where all particles for all open channels are to be summed/integrated, as discussed above.

MDVF

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In the rest of the paper, to make things more specific and to explore the case of most present interest in the quasielastic regime (CCQE), we focus on the specific case of a nuclear target where a nucleon is the particle that is presumed to be detected ($x = N$). Nevertheless it should be clear that simply by changing the names of the particles involved all of the developments can immediately be used in any other semi-inclusive study. Accordingly we now consider reactions of the type ${}^{A}_{Z}X(\nu_\ell, \ell^- p)^{A-1}{}_{Z}Y$, ${}^{A}_{Z}X(\bar{\nu}_\ell, \ell^+ n)^{A-1}_{Z-1}Y$, ${}^{A}_{Z}X(\nu_\ell, \ell^- n)^{A-1}_{Z+1}Y$ and ${}^{A}_{Z}X(\bar{\nu}_\ell, \ell^+ p)^{A-1}_{Z-2}Y$. These are to be viewed in context with semi-inclusive electron scattering reactions ${}^{A}_{Z}X(e, e'p)^{A-1}_{Z-1}Y$ and ${}^{A}_{Z}X(e, e'n)^{A-1}{}_{Z}Y$. In the initial state one has some nucleus X in its ground state with mass number A and charge Z, while in the final state one has a nuclear system Y with mass number $A - 1$ and the charges indicated above. The latter daughter nucleus is not presumed to be in its ground state in general (although this is one possibility when the system is stable to nucleon emission) and may be in some discrete excited state (if any exist), may be a granddaughter nucleus plus two nucleons, and so on. All open channels are to be considered and we only require that the mass number and charge be as indicated, together with the kinematical information to be discussed in the following section.

Note also that of the four neutrino and antineutrino reactions given above, the first two are in some sense "natural" in that the reactions in the CCQE regime are at least dominated by the basic reactions on nucleons in the target nucleus, namely, $\nu_\ell + n \rightarrow \ell^- + p$ and $\bar{\nu}_\ell + p \rightarrow \ell^+ + n$, respectively. However, the third and fourth reactions can occur in nuclei. On the one hand, the final states involved are complex interacting many-body states, involving in general coupled channels whenever kinematically allowed. There may be several nucleons in the final state and it is possible that one with the "wrong" flavor is the one detected.

Note also that of the four neutrino and antineutrino reactions given above, the first two are in some sense "natural" in that the reactions in the CCQE regime are at least dominated by the basic reactions on nucleons in the target nucleus, namely, $\nu_\ell + n \to \ell^- + p$ and $\bar{\nu}_\ell + p \to \ell^+ + n$, respectively. However, the third and fourth reactions can occur in nuclei. On the one hand, the final states involved are complex interacting many-body states, involving in general coupled channels whenever kinematically allowed. There may be several nucleons in the final state and it is possible that one with the "wrong" flavor is the one detected.

In fact, for some situations there may be no bound state of the final nucleus reached and one for sure has nucleons of both flavors in the final state. On the other hand, while one certainly has one-body electroweak current operators (those that act on a single nucleon), it is also clear that two-body meson exchange currents (MEC) are also present. For instance, an important contribution to MEC at quasielastic kinematics are diagrams where two nucleons interact with an exchanged $W^\pm$, going through a virtual Δ which in turn exchanges a pion between the two nucleons, leaving two nucleons in the final state. Take for example the third reaction above: if the two initial nucleons are an nn pair in the nuclear ground state, one can absorb the exchanged W^+, go through a Δ^+, exchange a π^+, and have an np pair in the final state where the neutron is the particle detected in the third reaction (and the proton may be the one detected in the first reaction).

In the developments presented below the formalism is general enough to allow for MEC, no assumption is required about which specific reaction is being considered and only when applying these ideas with particular modeling are the details required. All of the developments are kept relativistic, *i.e.,* no non-relativistic approximations are made, with one exception which will be discussed later in this paper. All of the formalism may then be used regardless of the energy scale, whether at relatively low energies or, what is more typical, at high energies.

The cross section takes on its characteristic form involving the contraction of two second-rank Lorentz tensors, $d\sigma \sim \eta_{\mu\nu} W^{\mu\nu}$, corresponding to the leptonic and the hadronic contributions which are thus factorized and dealt with independently. The leptonic tensor is defined as

$$\eta_{\mu\nu} \equiv 2mm' \overline{\sum_{if}} j_\mu^* j_\nu, .$$

Its hadronic counterpart is

$$W^{\mu\nu} \equiv \overline{\sum_{if}} J_{fi}^{\mu *}(\mathbf{q}) J_{fi}^{\nu}(\mathbf{q}),$$

where the operations $\overline{\sum}_{if}$ in the two cases correspond to sums and averages over the appropriate sets of leptonic quantum numbers (the helicities, in fact) or hadron quantum numbers (helicities or spins, *etc.*) and integration over all unobserved particles in the final state of the $A-1$ system for hadrons.

It proves useful to decompose both leptonic and hadronic tensors into pieces which are symmetric (s) or antisymmetric (a) under index interchange $\mu \leftrightarrow \nu$, since in contracting them no symmetric-antisymmetric cross-terms are allowed. Both tensors can thus be decomposed as $\eta_{\mu\nu} = \eta^s_{\mu\nu} + \eta^a_{\mu\nu}$ and $W^{\mu\nu} = W^{\mu\nu}_s + W^{\mu\nu}_a$, where the terms are defined as

$$\eta^s_{\mu\nu} = \frac{1}{2}(\eta_{\mu\nu} + \eta_{\nu\mu}) \qquad \eta^a_{\mu\nu} = \frac{1}{2}(\eta_{\mu\nu} - \eta_{\nu\mu})$$
$$W^{\mu\nu}_s = \frac{1}{2}(W^{\mu\nu} + W^{\nu\mu}) \qquad W^{\mu\nu}_a = \frac{1}{2}(W^{\mu\nu} - W^{\nu\mu}).$$

Clearly one has that $\eta^s_{\mu\mu} = \eta_{\mu\mu}$ and $W^{\mu\mu}_s = W^{\mu\mu}$, whereas $\eta^a_{\mu\mu} = W^{\mu\mu}_a = 0$ (no summation over μ implied in these expressions). In addition, since each tensor is proportional to the bilinear combinations of the electroweak currents in the forms $\eta_{\mu\nu} \sim j^*_\mu j_\nu$ and $W^{\mu\nu} \sim J^{\mu *} J^\nu$, one has that $\eta^*_{\mu\nu} = \eta_{\nu\mu}$ and $W^{\mu\nu *} = W^{\nu\mu}$, and thus that

$$\eta^s_{\mu\nu} = \mathrm{Re}\eta_{\mu\nu} \qquad \eta^a_{\mu\nu} = i\mathrm{Im}\eta_{\mu\nu}$$
$$W^{\mu\nu}_s = \mathrm{Re}W^{\mu\nu} \qquad W^{\mu\nu}_a = i\mathrm{Im}W^{\mu\nu}.$$

Following the standard conventions established for electron scattering (including discussions of polarization degrees of freedom: see T. W. Donnelly and A. S. Raskin, Ann. Phys. **169** (1986) 247 and A. S. Raskin and T. W. Donnelly, Ann. Phys. **191** (1989) 78), let us begin by defining the following (real) symmetric (no prime) and antisymmetric (prime) hadronic response functions:

Following the standard conventions established for electron scattering (including discussions of polarization degrees of freedom: see T. W. Donnelly and A. S. Raskin, Ann. Phys. **169** (1986) 247 and A. S. Raskin and T. W. Donnelly, Ann. Phys. **191** (1989) 78), let us begin by defining the following (real) symmetric (no prime) and antisymmetric (prime) hadronic response functions:

$$
\begin{aligned}
W^{CC} &\equiv \mathrm{Re}W^{00} & W^{T'} &\equiv -2\mathrm{Im}W^{12} \\
W^{CL} &\equiv 2\mathrm{Re}W^{03} & W^{TC'} &\equiv -2\sqrt{2}\mathrm{Im}W^{02} \\
W^{LL} &\equiv \mathrm{Re}W^{33} & W^{TL'} &\equiv -2\sqrt{2}\mathrm{Im}W^{32} \\
W^{T} &\equiv \mathrm{Re}W^{22} + \mathrm{Re}W^{11} & W^{\underline{CL}'} &\equiv \mathrm{Im}W^{03} \\
W^{TT} &\equiv \mathrm{Re}W^{22} - \mathrm{Re}W^{11} & W^{\underline{TC}'} &\equiv 2\sqrt{2}\mathrm{Im}W^{01} \\
W^{TC} &\equiv 2\sqrt{2}\mathrm{Re}W^{01} & W^{\underline{TL}'} &\equiv 2\sqrt{2}\mathrm{Im}W^{31} \\
W^{TL} &\equiv 2\sqrt{2}\mathrm{Re}W^{31} & & \\
W^{\underline{TT}} &\equiv 2\mathrm{Re}W^{12} & & \\
W^{\underline{TC}} &\equiv 2\sqrt{2}\mathrm{Re}W^{02} & & \\
W^{\underline{TL}} &\equiv 2\sqrt{2}\mathrm{Re}W^{32} & &
\end{aligned}
$$

Here C refers to charge (the $\mu = 0$) projection, L refers to longitudinal (momentum transfer direction, $\mu = 3$) projection and T refers to transverse ($\mu = 1, 2$) projections. Spherical tensor components can easily be deduced from these (see MDVF).

Equivalently to the hadronic case, the corresponding symmetric (no prime) and antisymmetric (prime) leptonic quantities may be defined:

$$
\begin{aligned}
v_0\widehat{V}_{CC} &\equiv \mathrm{Re}\eta_{00} & v_0\widehat{V}_{\underline{TL}} &\equiv \tfrac{1}{\sqrt{2}}\mathrm{Re}\eta_{32} \\
v_0\widehat{V}_{CL} &\equiv \mathrm{Re}\eta_{03} & v_0\widehat{V}_{T'} &\equiv \mathrm{Im}\eta_{12} \\
v_0\widehat{V}_{LL} &\equiv \mathrm{Re}\eta_{33} & v_0\widehat{V}_{TC'} &\equiv \tfrac{1}{\sqrt{2}}\mathrm{Im}\eta_{02} \\
v_0\widehat{V}_{T} &\equiv \tfrac{1}{2}(\mathrm{Re}\eta_{22}+\mathrm{Re}\eta_{11}) & v_0\widehat{V}_{TL'} &\equiv \tfrac{1}{\sqrt{2}}\mathrm{Im}\eta_{32} \\
v_0\widehat{V}_{TT} &\equiv \tfrac{1}{2}(\mathrm{Re}:\eta_{22}-\mathrm{Re}\eta_{11}) & v_0\widehat{V}_{\underline{CL'}} &\equiv -\mathrm{Im}\eta_{03} \\
v_0\widehat{V}_{TC} &\equiv \tfrac{1}{\sqrt{2}}\mathrm{Re}\eta_{01} & v_0\widehat{V}_{\underline{TC'}} &\equiv -\tfrac{1}{\sqrt{2}}\mathrm{Im}\eta_{01} \\
v_0\widehat{V}_{TL} &\equiv \tfrac{1}{\sqrt{2}}\mathrm{Re}\eta_{31} & v_0\widehat{V}_{\underline{TL'}} &\equiv -\tfrac{1}{\sqrt{2}}\mathrm{Im}\eta_{31} \\
v_0\widehat{V}_{\underline{TT}} &\equiv \mathrm{Re}\eta_{12} \\
v_0\widehat{V}_{\underline{TC}} &\equiv \tfrac{1}{\sqrt{2}}\mathrm{Re}\eta_{02} \\
v_0\widehat{V}_{\underline{TL}} &\equiv \tfrac{1}{\sqrt{2}}\mathrm{Re}\eta_{32}
\end{aligned}
$$

where the overall factor v_0 is defined as

$$v_0 \equiv (\varepsilon+\varepsilon')^2 - q^2.$$

The results found here are completely general; they are simply a convenient rewriting of the original components of the leptonic and hadronic tensors where the projections along the momentum transfer direction (L) and transverse to it provide the organizing principle.

Leptonic Tensor

The general leptonic tensor involving neutrinos and negatively charged leptons may be written in the following way — later it is straightforward to extend the results to include antineutrinos and positively charged leptons, and in fact to neutral current neutrino scattering or to any V-A electroweak leptonic tensor:

$$\begin{aligned}\eta_{\mu\nu}(K',K) &= mm'\sum_{s,s'}\bar{u}(K,s)(a_V\gamma_\mu + a_A\gamma_\mu\gamma_5)u(K',s') \\ &\quad \times\bar{u}(K',s')(a_V\gamma_\nu + a_A\gamma_\nu\gamma_5)u(K,s)\end{aligned}$$

which includes sum over final spin states and average over initial spin states. In the standard model the charged-current vector and axial coupling constants take the values $a_V = 1$ and $a_A = -1$, which yields the usual form of the vertex $\gamma_\mu : (1-\gamma_5)$. Upon eliminating the spinors using traces one finds:

$$\begin{aligned}\eta_{\mu\nu}(K',K) &\equiv \frac{1}{4}\left\{\mathrm{Tr}[a_V\gamma_\mu + a_A\gamma_\mu\gamma_5](K'')[a_V\gamma_\nu + a_A\gamma_\nu\gamma_5](\not{K}+m)\right\} \\ &= \frac{1}{4}\Big\{a_V^2\,\mathrm{Tr}\left[\gamma_\mu(K'')\gamma_\nu(\not{K}+m)\right]_{(1)} + a_A^2\,\mathrm{Tr}\left[\gamma_\mu\gamma_5(K'')\gamma_\nu\gamma_5(\not{K}+m)\right]_{(2)} \\ &\quad + a_V a_A\left(\mathrm{Tr}\left[\gamma_\mu(K'')\gamma_\nu\gamma_5(\not{K}+m)\right]_{(3)} + \mathrm{Tr}\left[\gamma_\mu\gamma_5(K'')\gamma_\nu(\not{K}+m)\right]_{(4)}\right)\Big\}.\end{aligned}$$

As above we introduce the following definitions (note: there are some slight differences in the notation used here and in the discussions of inclusive scattering):

$$\nu \equiv \frac{\omega}{q}$$

$$\rho \equiv \frac{|Q^2|}{q^2} = 1 - \nu^2 \qquad \rho' \equiv \frac{q}{\varepsilon + \varepsilon'}$$

$$\delta \equiv \frac{m}{\sqrt{|Q^2|}} \qquad \delta' \equiv \frac{m'}{\sqrt{|Q^2|}}$$

$$\tan^2 \widetilde{\theta}/2 = \frac{|Q^2|}{v_0} = \frac{\rho\rho'^2}{1-\rho'^2}.$$

In terms of the angle $\widetilde{\theta}$ the quantities Q^2 and v_0 can be written as

$$Q^2 = -4\varepsilon\varepsilon' \sin^2 \widetilde{\theta}/2$$

$$v_0 = 4\varepsilon\varepsilon' \cos^2 \widetilde{\theta}/2.$$

Upon performing the traces (see MDVF for details) one finds

$$
\begin{aligned}
\widehat{V}_{CC} &= \frac{1}{2}\left\{\left(a_V^2+a_A^2\right)-\left[a_V^2\left(\delta-\delta'\right)^2+a_A^2\left(\delta+\delta'\right)^2\right]\tan^2\widetilde{\theta}/2\right\} \\
\widehat{V}_{CL} &= -\frac{1}{2}\left(a_V^2+a_A^2\right)\left[\nu-\frac{1}{\rho'}\left(\delta^2-\delta'^2\right)\tan^2\widetilde{\theta}/2\right] \\
\widehat{V}_{LL} &= \frac{1}{2}\left\{\left(a_V^2+a_A^2\right)\left[\nu^2-\frac{1}{\rho'}\left(2\nu-\rho\rho'\left(\delta^2-\delta'^2\right)\right)\left(\delta^2-\delta'^2\right)\tan^2\widetilde{\theta}/2\right]\right. \\
&\quad \left.+\left[a_V^2\left(\delta-\delta'\right)^2+a_A^2\left(\delta+\delta'\right)^2\right]\tan^2\widetilde{\theta}/2\right\} \\
\widehat{V}_{T} &= \frac{1}{2}\left(a_V^2+a_A^2\right)\left\{\left[\frac{1}{2}\rho+\tan^2\widetilde{\theta}/2\right]\right. \\
&\quad \left.+\left(\frac{\nu}{\rho'}\left(\delta^2-\delta'^2\right)-\frac{1}{2}\rho\left(\delta^2-\delta'^2\right)^2\right)\tan^2\widetilde{\theta}/2\right\} \\
&\quad -\left(a_V^2-a_A^2\right)\delta\delta'\tan^2\widetilde{\theta}/2 \\
\widehat{V}_{TT} &= \frac{1}{2}\left(a_V^2+a_A^2\right)\left\{-\frac{1}{2}\rho\right. \\
&\quad \left.+\left[\left(\delta^2+\delta'^2\right)-\frac{\nu}{\rho'}\left(\delta^2-\delta'^2\right)+\frac{1}{2}\rho\left(\delta^2-\delta'^2\right)^2\right]\tan^2\widetilde{\theta}/2\right\} \\
\widehat{V}_{TC} &= -\frac{1}{2}\left(a_V^2+a_A^2\right)\frac{1}{\rho'}\tan\widetilde{\theta}/2 \\
&\quad \times\left(\frac{1}{2}-\frac{1}{\rho}\left[\left(\delta^2+\delta'^2\right)-\frac{\nu}{\rho'}\left(\delta^2-\delta'^2\right)+\frac{1}{2}\rho\left(\delta^2-\delta'^2\right)^2\right]\tan^2\widetilde{\theta}/2\right)^{1/2} \\
\widehat{V}_{TL} &= -\left(\nu-\rho\rho'\left(\delta^2-\delta'^2\right)\right)\widehat{V}_{TC} \\
\widehat{V}_{T'} &= a_V a_A\frac{1}{\rho'}\left(1+\nu\rho'\left(\delta^2-\delta'^2\right)\right)\tan^2\widetilde{\theta}/2 \\
\widehat{V}_{TC'} &= -a_V a_A\tan\widetilde{\theta}/2 \\
&\quad \times\left[\frac{1}{2}-\frac{1}{\rho}\left[\left(\delta^2+\delta'^2\right)-\frac{\nu}{\rho'}\left(\delta^2-\delta'^2\right)+\frac{1}{2}\rho\left(\delta^2-\delta'^2\right)^2\right]\tan^2\widetilde{\theta}/2\right]^{1/2} \\
\widehat{V}_{TL'} &= -\nu\widehat{V}_{TC'} \\
\widehat{V}_{\underline{TT}} &= \widehat{V}_{\underline{TC}}=\widehat{V}_{\underline{TL}}=\widehat{V}_{\underline{TC'}}=\widehat{V}_{\underline{TL'}}=\widehat{V}_{\underline{CL'}}=0
\end{aligned}
$$

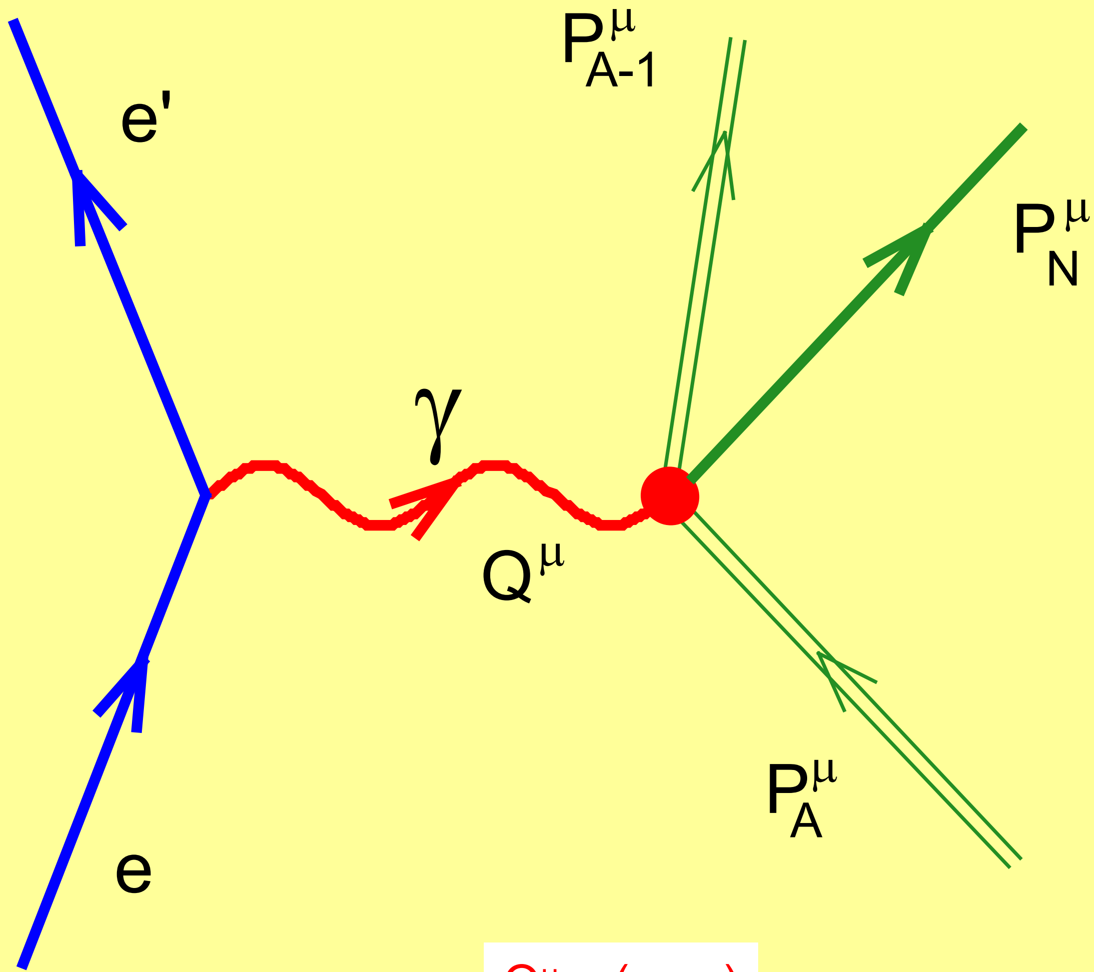
Within these 16 factors, 10 of them are symmetric and 6 are antisymmetric. Under the conditions in this work 6 of them vanish, namely the ones with underlined subscript.

These reduce to the following expressions in the extreme relativistic limit (ERL), defined as $\widehat{V}_K \xrightarrow{ERL} \frac{1}{2}\left(a_V^2 + a_A^2\right) v_K$ for the symmetric ones (no prime) and as $\widehat{V}_{K'} \xrightarrow{ERL} a_V a_A : v_{K'}$ for the antisymmetric ones (prime):

$$
\begin{aligned}
v_{CC} &= 1 \\
v_{CL} &= -\nu \\
v_{LL} &= \nu^2 \\
v_{T:} &= \frac{1}{2}\rho + \tan^2\theta/2 \\
v_{TT} &= -\frac{1}{2}\rho \\
v_{TC} &= -\frac{1}{\sqrt{2}\rho'}\tan\theta/2 \\
v_{TL} &= -\nu v_{TC} \\
v_{T'} &= \tan\theta/2\sqrt{\rho + \tan^2\theta/2} \\
v_{TC'} &= -\frac{1}{\sqrt{2}}\tan\theta/2 \\
v_{TL'} &= -\nu v_{TC'}
\end{aligned}
$$

Finally, one can easily complete the leptonic developments by going to the start and replacing the u-spinor by v-spinors so that the leptonic tensor for anti-particles can be obtained. The final result is that upon contracting the leptonic and hadronic tensors the VV and AA terms are as above, while the VA interference changes sign.

Begin by assuming that QE scattering is dominated by (e,e´N):



$Q^\mu = (\omega, \mathbf{q})$

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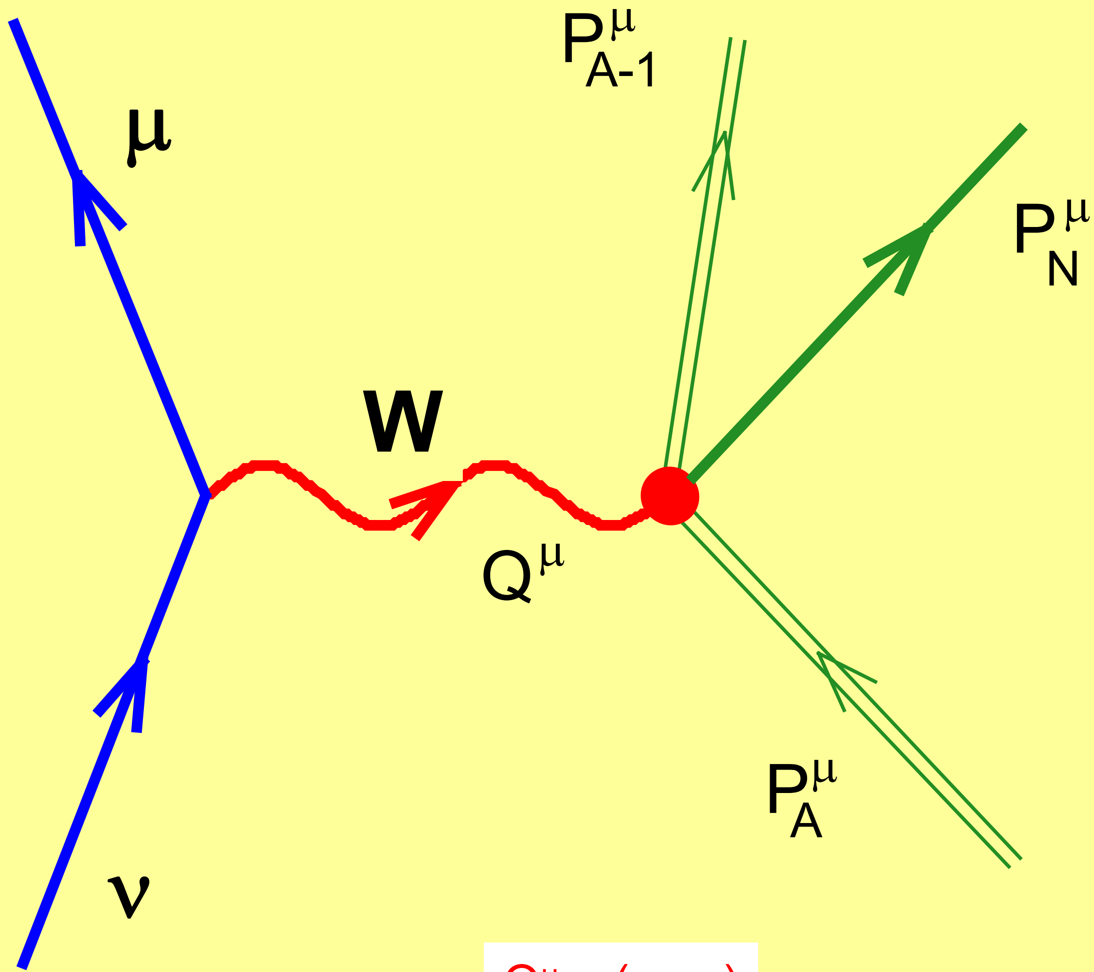
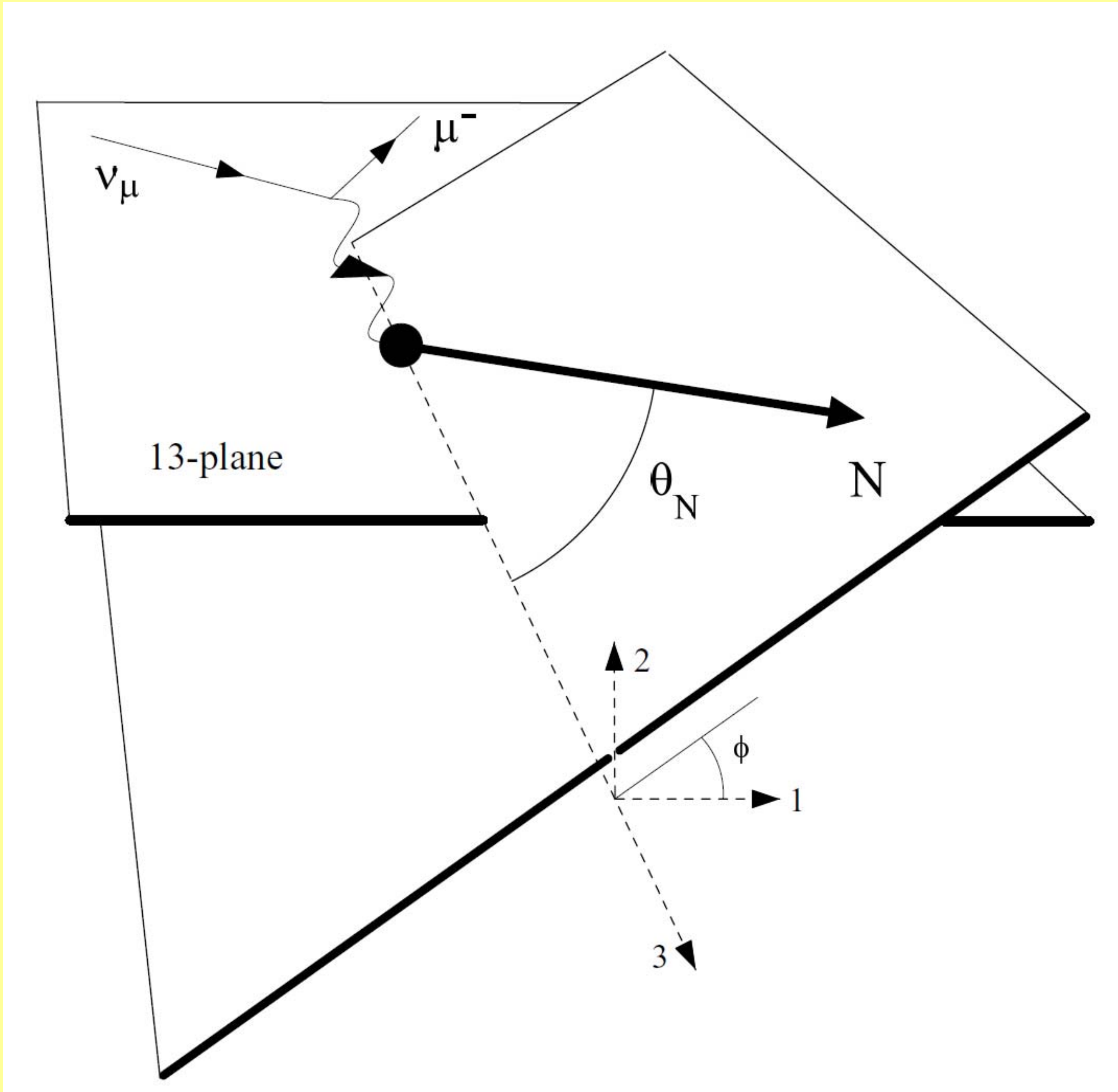


$Q^\mu = (\omega, \mathbf{q})$

… or by (ν,μN)



ν_μ
μ^-
13-plane
θ_N
N
2
ϕ
1
3

The daughter nucleus has 4-momentum

$$P^{\mu}_{A-1} = (E_{A-1}, \mathbf{p}_{A-1}) = Q^{\mu} + P^{\mu}_{A} - P^{\mu}_{N}$$

In the lab. system we define the **missing momentum**

$$p = |\mathbf{p}| \equiv |\mathbf{p}_N - \mathbf{q}| = |\mathbf{p}_{A-1}|$$

and an "excitation energy" (essentially **missing energy** – separation energy)

$$\mathcal{E}(p) \equiv \sqrt{(M_{A-1})^2 + p^2} - \sqrt{(M^0_{A-1})^2 + p^2}$$

where

$$M^0_{A-1} = M^0_A - m_N + E_s$$

with E_s the separation energy and M^0_{A-1} the daughter rest mass

Energy conservation gives

$$M_A^0 + \omega = E_N + E_{A-1}$$
$$= \sqrt{m_N{}^2 + p_N{}^2} + E_{A-1}^0 + \mathcal{E}$$
$$= \sqrt{m_N{}^2 + (\mathbf{q} + \mathbf{p})^2} + \sqrt{(M_{A-1}^0)^2 + p^2} + \mathcal{E}$$

which can be turned around to yield an expression for the excitation energy:

$$\mathcal{E} = M_A^0 + \omega - \sqrt{(M_{A-1}^0)^2 + p^2} - \sqrt{m_N{}^2 + q^2 + p^2 + 2pq\cos\theta}$$

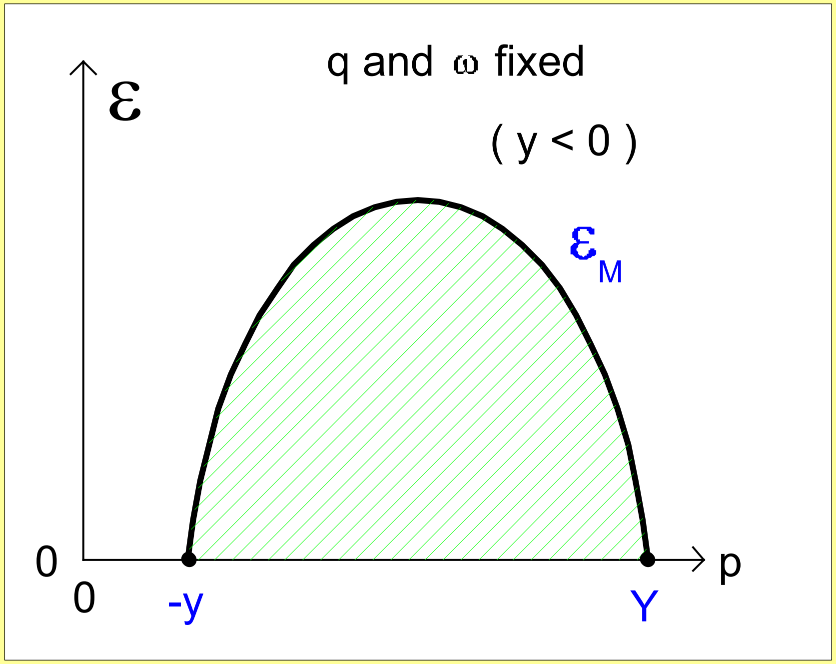
One can let the angle between *p* and *q* vary over all values and impose the constraints

$$p \geq 0$$

$$\varepsilon \geq 0$$

to find the allowed region in the missing-energy, missing-momentum plane. When

$$\omega < \omega_{QE} = \left|Q^2\right| / 2m_N$$ one finds



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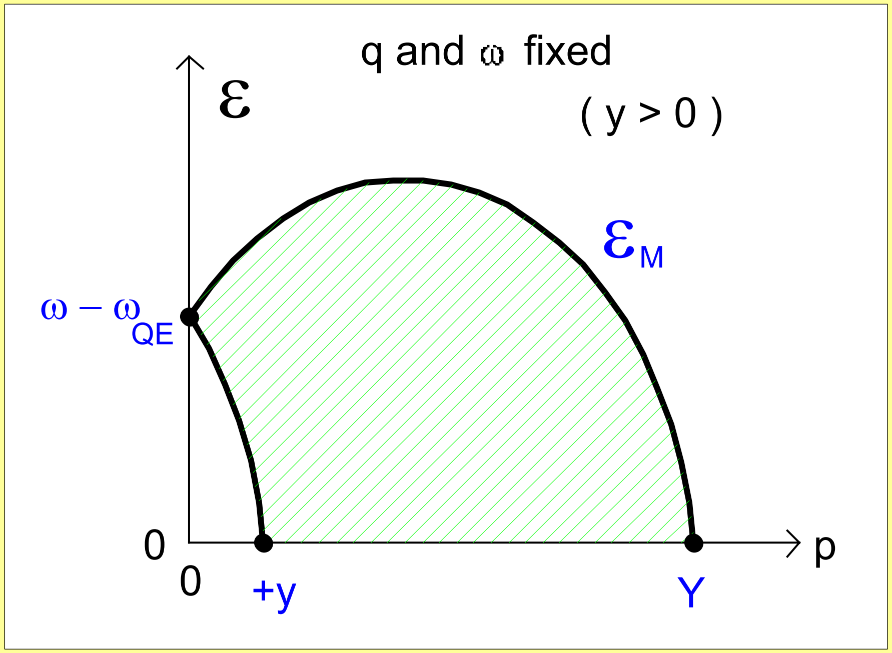
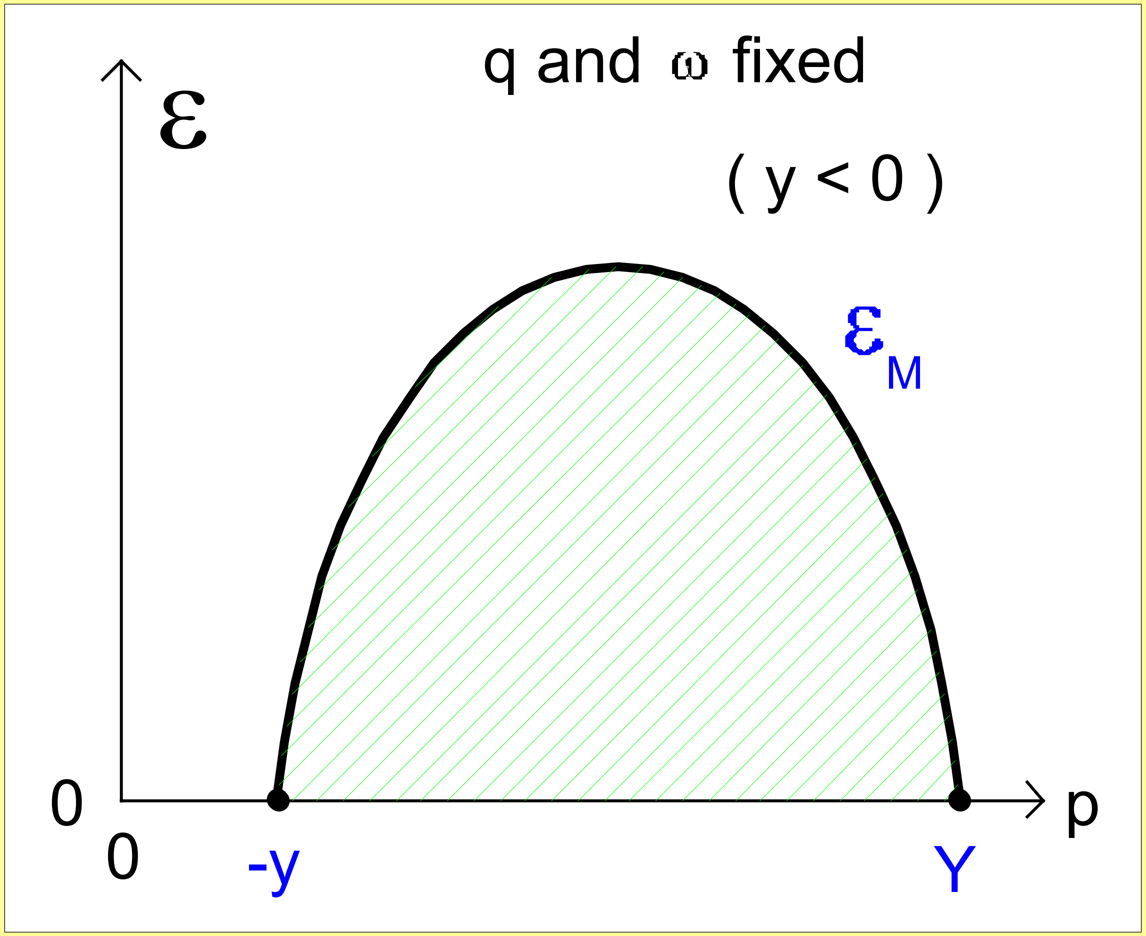
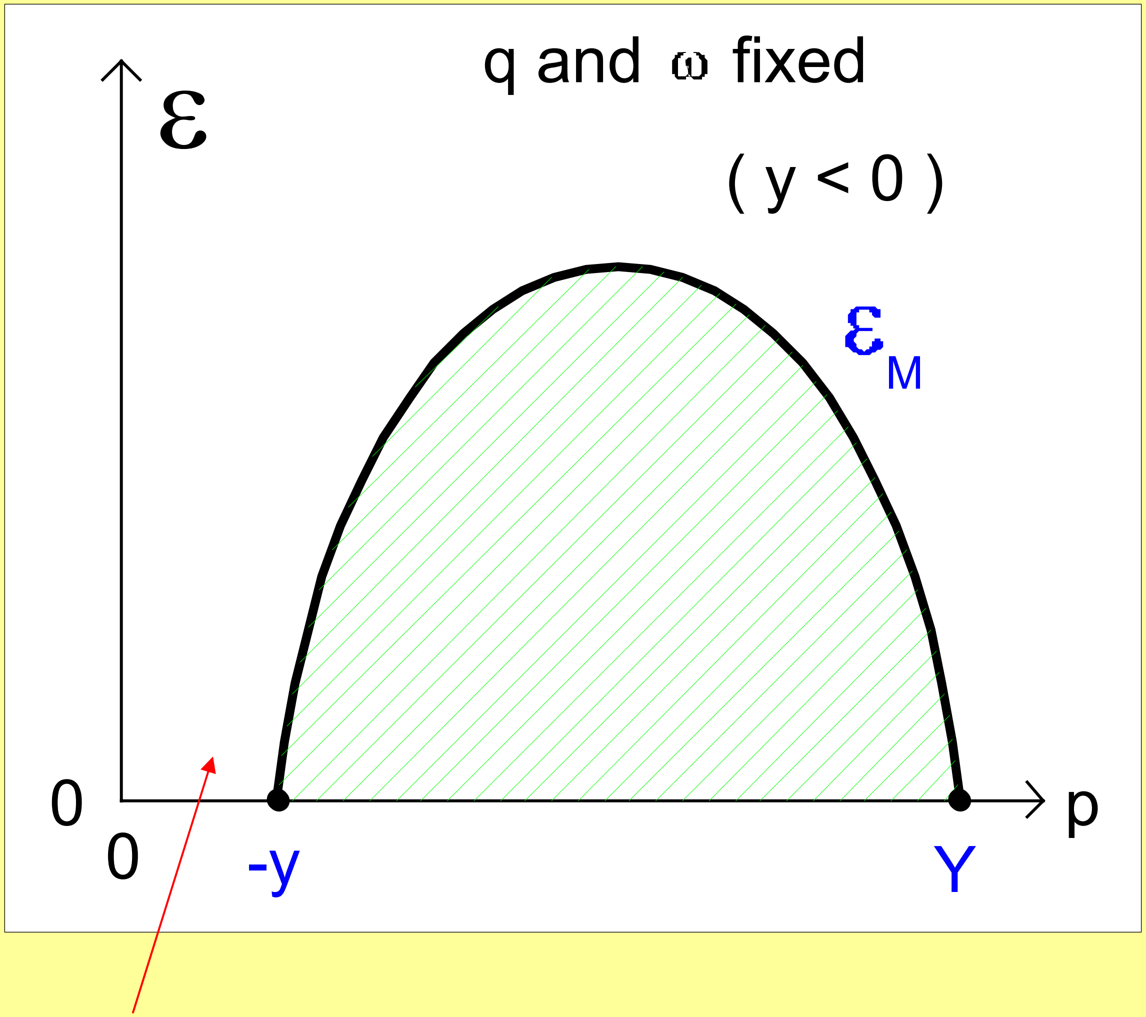
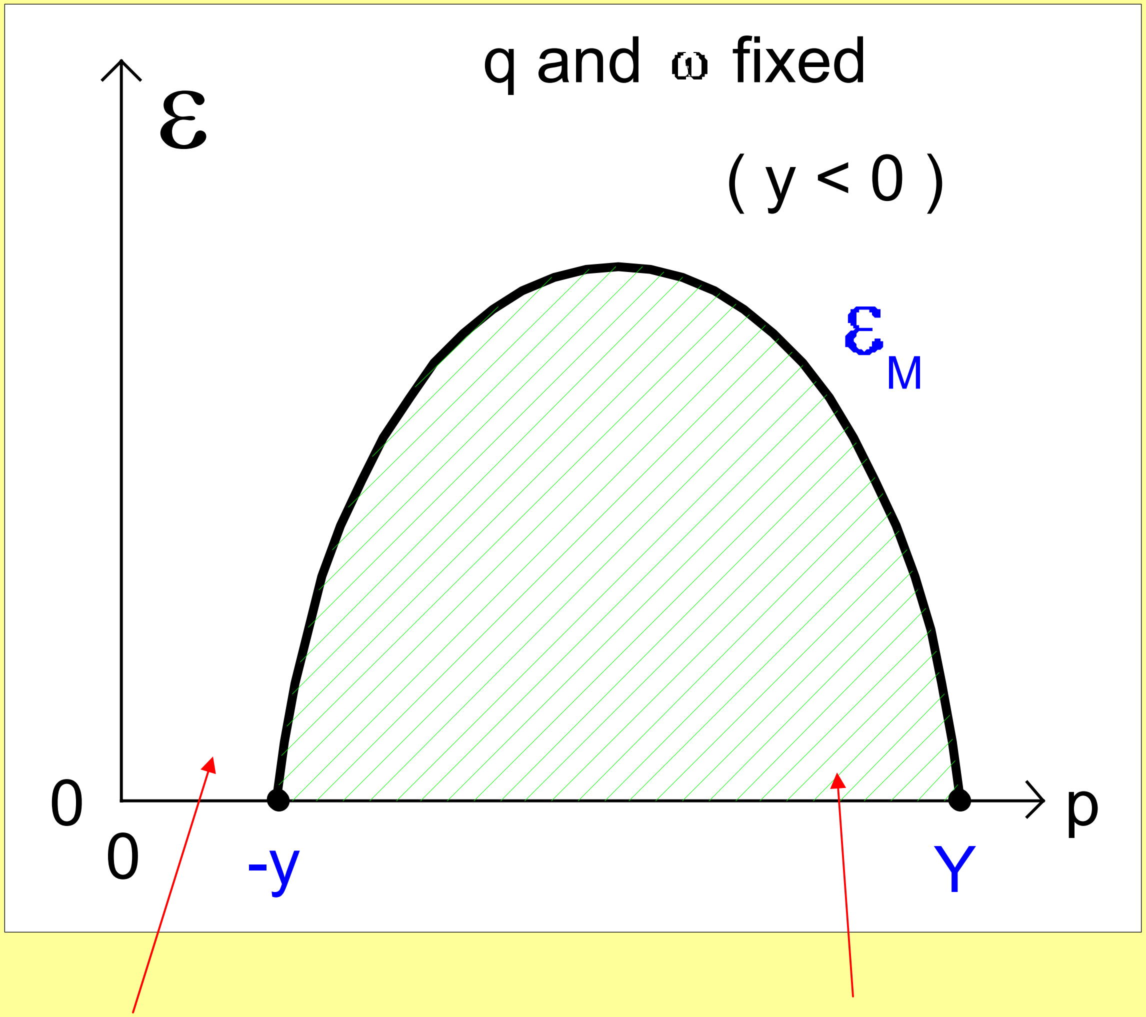



q and ω fixed
(y < 0)
ε
ε_M
0
0
-y
Y
p



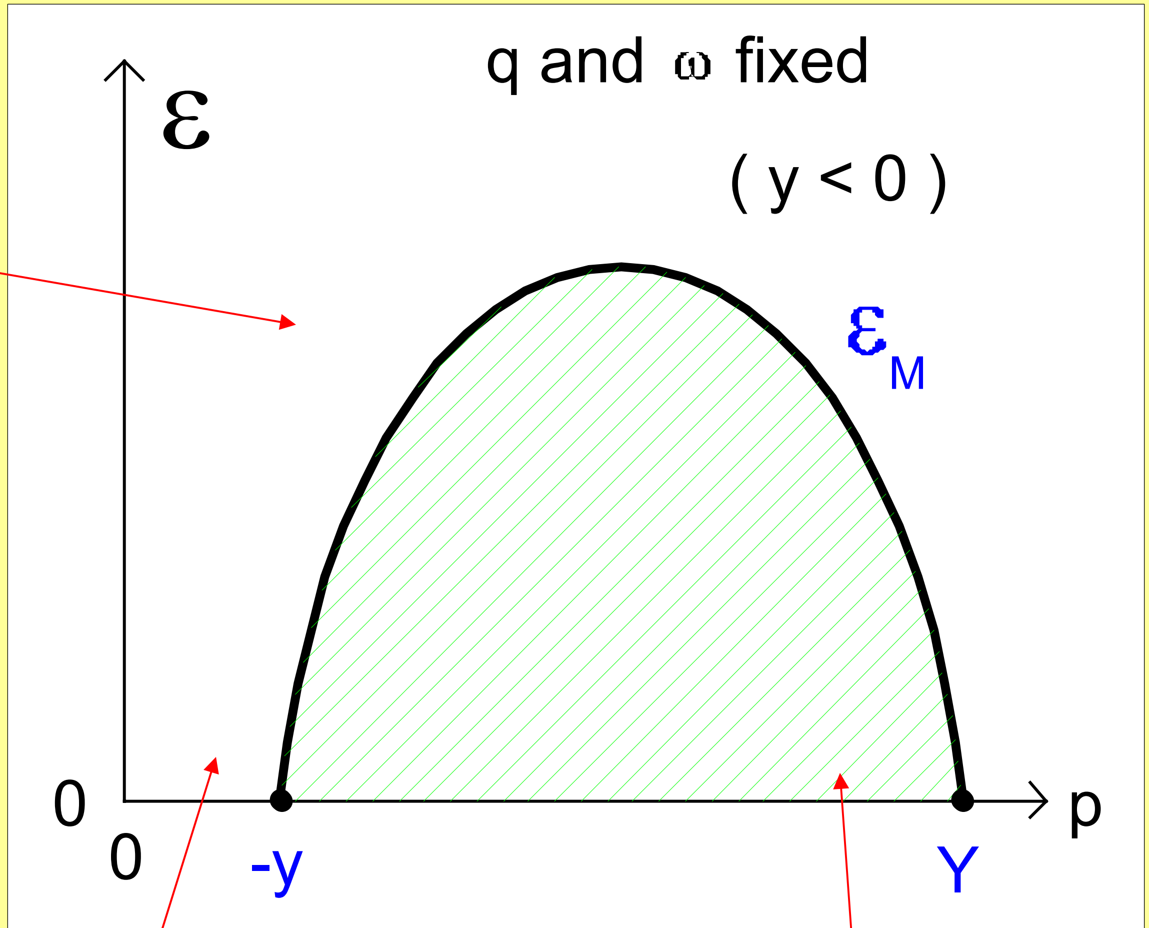
The semi-inclusive cross section is
typically largest at small **p** and ε



The semi-inclusive cross section is typically largest at small **p** and ε

… and is very small at large **p** and small ε

For given **y<0** the region at small **p**, but high $\mathcal{E}$ is inaccessible

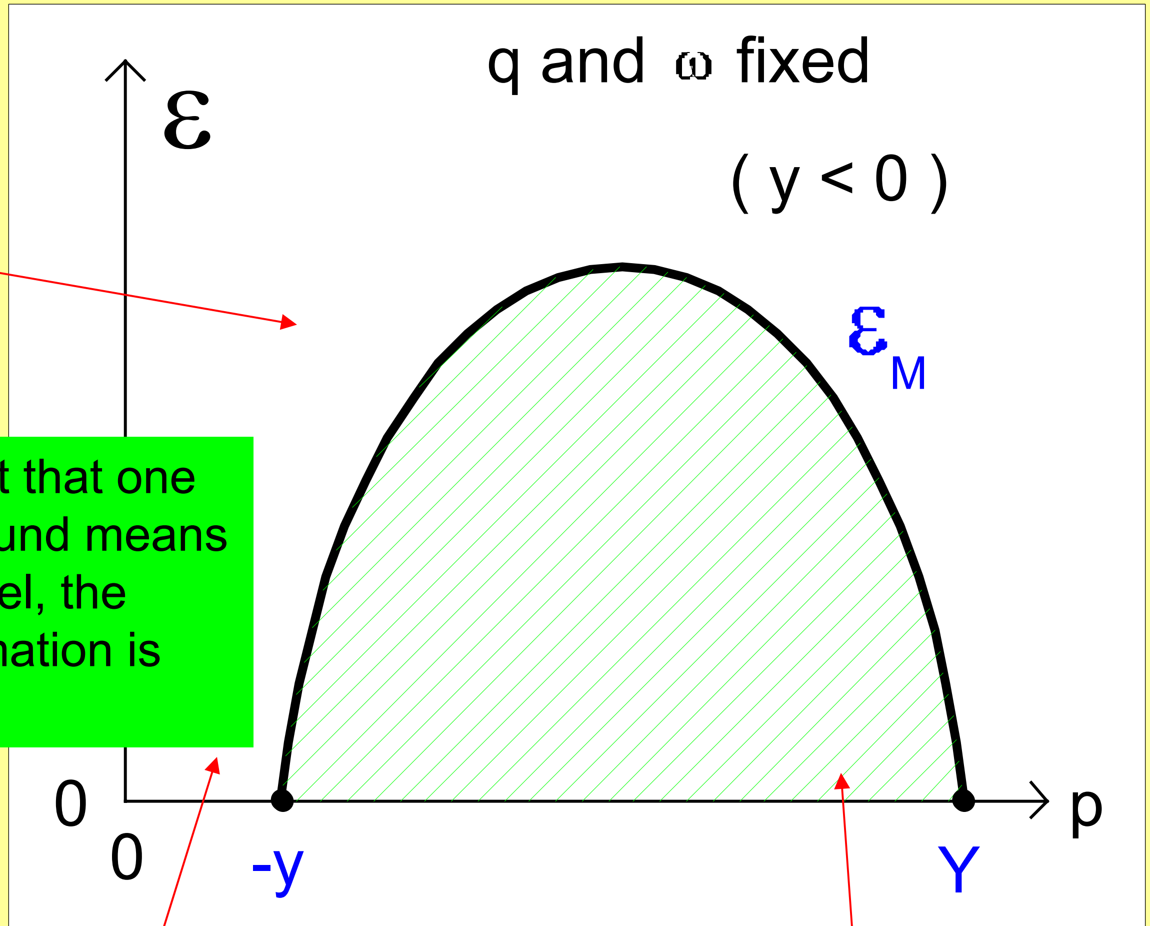


The semi-inclusive cross section is typically largest at small **p** and $\mathcal{E}$

… and is very small at large **p** and small $\mathcal{E}$

For given **y<0** the region at small **p**, but high ε is inaccessible

Note that the fact that one has an upper bound means that, at some level, the closure approximation is in error



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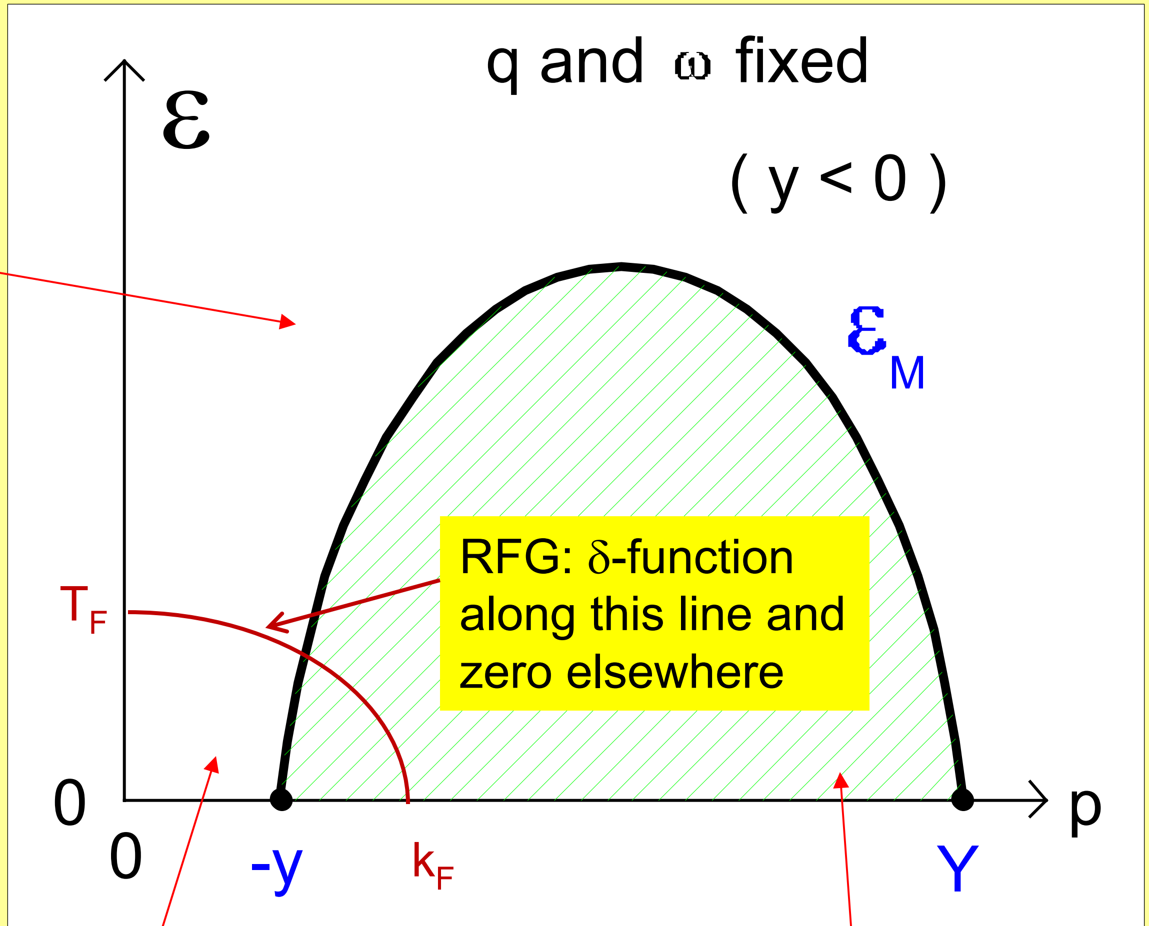

The semi-inclusive cross section is typically largest at small **p** and $\mathcal{E}$

… and is very small at large **p** and small $\mathcal{E}$

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And so, just because a specific model does well for **inclusive scattering** (which involves integrals over the regions shown above, summed over appropriate flavors of nucleons, and corrected for double-counting), that model may fail badly for **semi-inclusive scattering**: the strength in the missing energy/momentum plane, and hence the final-state nucleon kinematics, may be wrong. For example, the RFG is infinitely bad almost everywhere.

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And so, just because a specific model does well for **inclusive scattering** (which involves integrals over the regions shown above, summed over appropriate flavors of nucleons, and corrected for double-counting), that model may fail badly for **semi-inclusive scattering**: the strength in the missing energy/momentum plane, and hence the final-state nucleon kinematics, may be wrong. For example, the RFG is infinitely bad almost everywhere.

This means that adding on final-state interactions to a model that is only suited to inclusive scattering can incur significant errors; a realistic one-particle spectral function should be used for modeling semi-inclusive reactions. For reactions requiring the specification of two or more particles one must go beyond the existing spectral functions.

Semi-Inclusive Scattering of Leptons from Hadronic Systems: General Hadronic Tensor

To be specific here, let us consider reactions in which a lepton scatters from a nucleus and a nucleon is presumed to be detected in the final state, that is, in coincidence with the final-state lepton. Examples coulc be $(e, e'p)$ or $(\nu_\mu, \mu p)$ reactions. Following the ideas discussed previously for inclusive scattering and the discussions in O. Moreno, T. W. Donnelly, J. W. Van Orden and W. P. Ford, *Phys. Rev.* **D90** (2014) 013014 it is straightforward to construct the general form of the hadronic tensor using the available four-momenta. One has incoming momentum transfer Q^μ and the nuclear target momentum P_A^μ, as before. In the final state one has the momentum of the detected nucleon P_N^μ together with the residual nucleus' momentum which can be eliminated using four-momentum conservation: $P_{A-1}^\mu = Q^\mu + P_A^\mu - P_N^\mu$. Six invariants can be constructed:

$$I_1 \equiv Q^2 \qquad I_2 \equiv Q \cdot P_A \qquad I_3 \equiv Q \cdot P_N$$
$$I_4 \equiv P_A \cdot P_N \qquad I_5 \equiv P_A^2 = {M_A^0}^2 \qquad I_6 \equiv P_N^2 = m_N^2$$

of which the first four are dynamical variables, whereas the last two are fixed by the target nucleus and nucleon masses. Accordingly all invariant structure functions depend on the four dynamical invariants I_i, $i = 1, \ldots, 4$.

Accordingly all invariant structure functions depend on the four dynamical invariants I_i, $i = 1, \ldots, 4$. They can be expressed as:

$$\begin{aligned} I_1 &= \omega^2 - q^2 < 0 \\ I_2 &= M_A^0 \omega \\ I_3 &= \omega E_N - q p_N \cos\theta_N \\ I_4 &= M_A^0 E_N \end{aligned}$$

Next one can write symmetric and antisymmetric hadronic tensors as functions of the three independent four-momenta Q^μ, P_A^μ and P_N^μ. In fact, it proves to be more convenient to introduce projected four-momenta to replace the last two, namely,

$$\begin{aligned} U^\mu &\equiv \frac{1}{M_A^0}\left[P_A^\mu - \left(\frac{Q\cdot P_A}{Q^2}\right) Q^\mu\right] \\ V^\mu &\equiv \frac{1}{M_N}\left[P_N^\mu - \left(\frac{Q\cdot P_N}{Q^2}\right) Q^\mu\right], \end{aligned}$$

where then $Q\cdot U = Q\cdot V = 0$. Also, to keep the dimensions consistent in the developments below let us introduce a dimensionless four-momentum transfer

$$\widetilde{Q}^\mu \equiv \frac{Q^\mu}{\sqrt{|Q^2|}}.$$

The symmetric hadronic tensor may then be written

$$W_s^{\mu\nu} = X_1 g^{\mu\nu} + X_2 \widetilde{Q}^\mu \widetilde{Q}^\nu + X_3 U^\mu U^\nu + X_4 (\widetilde{Q}^\mu U^\nu + U^\mu \widetilde{Q}^\nu)$$
$$+ X_5 V^\mu V^\nu + X_6 (\widetilde{Q}^\mu V^\nu + V^\mu \widetilde{Q}^\nu) + X_7 (U^\mu V^\nu + V^\mu U^\nu) ,$$

where X_i, $i = 1 \ldots 7$ are invariant functions of the invariants discussed above. These seven types of terms arise from VV and AA contributions.

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where X_i, $i = 1 \ldots 7$ are invariant functions of the invariants discussed above. These seven types of terms arise from VV and AA contributions.

Likewise the antisymmetric tensor can be constructed from the basic four-momenta

$$W_a^{\mu\nu} = i \Big\{ Y_1 (\widetilde{Q}^\mu U^\nu - U^\mu \widetilde{Q}^\nu) + Y_2 (\widetilde{Q}^\mu V^\nu - V^\mu \widetilde{Q}^\nu) + Y_3 (U^\mu V^\nu - V^\mu U^\nu) + Z_1 \varepsilon^{\mu\nu\alpha\beta} \widetilde{Q}_\alpha U_\beta + Z_2 \varepsilon^{\mu\nu\alpha\beta} \widetilde{Q}_\alpha V_\beta + Z_3 \varepsilon^{\mu\nu\alpha\beta} U_\alpha V_\beta \Big\},$$

where again Y_i and Z_i, $i = 1 \ldots 3$ are invariant functions of the invariants above. The terms having no $\varepsilon^{\mu\nu\alpha\beta}$, namely the Y_i terms (as well as the X_i terms, as said above), arise from VV and AA contributions, whereas those with $\varepsilon^{\mu\nu\alpha\beta}$, namely the Z_i terms, come from VA interferences. Note that for inclusive scattering where one does not have V^μ as a building block only terms of the X_1, X_2, X_3, X_4, Y_1 and Z_1 type can occur.

For a conserved vector current (CVC) situation such as here for the VV terms or for purely polar-vector electron scattering the continuity equation in momentum space requires that

$$Q_\mu \left(W_s^{\mu\nu}\right)_{VV} = Q_\mu \left(W_a^{\mu\nu}\right)_{VV} = 0.$$

This contraction removes the terms with X_3, X_5, Y_3, Z_1, leaving the conditions

$$\left(-X_1^{VV} + X_2^{VV}\right)\widetilde{Q}^\nu + X_4^{VV} U^\nu + X_6^{VV} V^\nu = 0$$
$$Y_1^{VV} U^\nu + Y_2^{VV} V^\nu = 0,$$

where no terms with $\varepsilon^{\mu\nu\alpha\beta}$ can occur in a VV situation, *i.e.*, $Z_1^{VV} = Z_2^{VV} = Z_3^{VV} = 0$, as noted above. Since the basic four-momenta are linearly independent of each other the coefficients above must all be independently zero, namely $X_1^{VV} - X_2^{VV} = X_4^{VV} = X_6^{VV} = Y_1^{VV} = Y_2^{VV} = 0.$

Accordingly, one has

$$\begin{aligned}(W_s^{\mu\nu})_{VV} &= X_1^{VV}\left[g^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2}\right] + X_3^{VV} U^\mu U^\nu \\ &\quad + X_5^{VV} V^\mu V^\nu + X_7^{VV}\left(U^\mu V^\nu + V^\mu U^\nu\right) \\ (W_a^{\mu\nu})_{VV} &= Y_3^{VV}(U^\mu V^\nu - V^\mu U^\nu).\end{aligned}$$

For instance, in semi-inclusive electron scattering the symmetric terms lead to the standard L, T, TL and TT responses, while the antisymmetric term which becomes accessible with polarized electron scattering yields the TL' response, the so-called 5th response (see T. W. Donnelly and A. S. Raskin, Ann. Phys. **169** (1986) 247 and A. S. Raskin and T. W. Donnelly, Ann. Phys. **191** (1989) 78). For the other cases, the AA and VA responses, there is no further simplification in general.

The resulting number of contributions of each type is summarized below for semi-inclusive scattering.

$$
\begin{aligned}
X_1 &= X_1^{VV} + X_1^{AA} \\
X_2 &= X_1^{VV} + X_2^{AA} \\
X_3 &= X_3^{VV} + X_3^{AA} \\
X_4 &= X_4^{AA} \\
X_5 &= X_5^{VV} + X_5^{AA} \\
X_6 &= X_6^{AA} \\
X_7 &= X_7^{VV} + X_7^{AA}
\end{aligned}
\qquad
\begin{aligned}
Y_1 &= Y_1^{AA} \\
Y_2 &= Y_2^{AA} \\
Y_3 &= Y_3^{VV} + Y_3^{AA} \\
Z_1 &= Z_1^{VA} \\
Z_2 &= Z_2^{VA} \\
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X_5 &= X_5^{VV} + X_5^{AA} \\
X_6 &= X_6^{AA} \\
X_7 &= X_7^{VV} + X_7^{AA}
\end{aligned}
$$

$$
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Y_1 &= Y_1^{AA} \\
Y_2 &= Y_2^{AA} \\
Y_3 &= Y_3^{VV} + Y_3^{AA} \\
Z_1 &= Z_1^{VA} \\
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11 symmetric responses

7 antisymmetric responses

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Z_1 &= Z_1^{VA} \\
Z_2 &= Z_2^{VA} \\
Z_3 &= Z_3^{VA}
\end{aligned}
$$

11 symmetric responses

7 antisymmetric responses

… and these are all functions of 4 dynamical variables, rather than only 2 as in the inclusive case.

Upon using the kinematic variables in the laboratory system, together with the following definitions:

$$\eta_T \equiv \frac{p_N}{m_N} \sin\theta_N$$
$$H \equiv \frac{1}{m_N}\left[E_N - \nu p_N \cos\theta_N\right],$$

the hadronic response functions can be written as

$$
\begin{aligned}
W_s^{CC} &= \frac{1}{\rho^2}\left\{\rho^2 X_1 + \rho\nu^2 X_2 + X_3 + 2\sqrt{\rho}\nu X_4 \right.\\
&\left. + H^2 X_5 + 2\sqrt{\rho}\nu H X_6 + 2H X_7\right\}\\
W_s^{CL} &= \frac{2\nu}{\rho^2}\left\{\rho X_2 + X_3 + \sqrt{\rho}(\frac{1}{\nu}+\nu)X_4 \right.\\
&\left. + H^2 X_5 + \sqrt{\rho}(\frac{1}{\nu}+\nu) H X_6 + 2H X_7\right\}\\
W_s^{LL} &= \frac{1}{\rho^2}\left\{-\rho^2 X_1 + \rho X_2 + \nu^2 X_3 + 2\sqrt{\rho}\nu X_4 \right.\\
&\left. + \nu^2 H^2 X_5 + 2\sqrt{\rho}\nu H X_6 + 2\nu^2 H X_7\right\}\\
W_s^{T} &= -2X_1 + X_5\eta_T^2\\
W_s^{TT} &= -X_5\eta_T^2\cos 2\phi\\
W_s^{TC} &= \frac{2\sqrt{2}}{\rho}\eta_T\left\{H X_5 + \sqrt{\rho}\nu X_6 + X_7\right\}\cos\phi\\
W_s^{TL} &= \frac{2\sqrt{2}}{\rho}\eta_T\left\{\nu H X_5 + \sqrt{\rho} X_6 + \nu X_7\right\}\cos\phi\\
W_s^{\underline{TT}} &= X_5\eta_T^2\sin 2\phi\\
W_s^{\underline{TC}} &= \frac{2\sqrt{2}}{\rho}\eta_T\left\{H X_5 + \sqrt{\rho}\nu X_6 + X_7\right\}\sin\phi\\
W_s^{\underline{TL}} &= \frac{2\sqrt{2}}{\rho}\eta_T\left\{\nu H X_5 + \sqrt{\rho} X_6 + \nu X_7\right\}\sin\phi\\
W_a^{T'} &= \frac{1}{\sqrt{\rho}}\left\{Z_1 + H Z_2\right\}\\
W_a^{TC'} &= \frac{2\sqrt{2}}{\rho}\eta_T\left\{-\left(\sqrt{\rho}\nu Y_2 + Y_3\right)\sin\phi + \left(\sqrt{\rho} Z_2 + \nu Z_3\right)\cos\phi\right\}\\
W_a^{TL'} &= \frac{2\sqrt{2}}{\rho}\eta_T\left\{-\left(\sqrt{\rho} Y_2 + \nu Y_3\right)\sin\phi + \left(\sqrt{\rho}\nu Z_2 + Z_3\right)\cos\phi\right\}\\
W_a^{\underline{CL}'} &= -\frac{1}{\sqrt{\rho}}\left\{Y_1 + H Y_2\right\}\\
W_a^{\underline{TC}'} &= \frac{2\sqrt{2}}{\rho}\eta_T\left\{\left(\sqrt{\rho}\nu Y_2 + Y_3\right)\cos\phi + \left(\sqrt{\rho} Z_2 + \nu Z_3\right)\sin\phi\right\}\\
W_a^{\underline{TL}'} &= \frac{2\sqrt{2}}{\rho}\eta_T\left\{\left(\sqrt{\rho} Y_2 + \nu Y_3\right)\cos\phi + \left(\sqrt{\rho}\nu Z_2 + Z_3\right)\sin\phi\right\}
\end{aligned}
$$

Note how the explicit dependence on the azimuthal angle ϕ emerges: one has pairs of symmetric contributions, namely $TT \leftrightarrow \underline{TT}$, $TC \leftrightarrow \underline{TC}$, and $TL \leftrightarrow \underline{TL}$, where a cosine is replaced by a sine, as well as pairs of antisymmetric contributions, namely, $TC' \leftrightarrow \underline{TC'}$ and $TL' \leftrightarrow \underline{TL'}$, where a rotation is involved. Also note that, while these constitute the complete set of semi-inclusive responses, in fact none of the underlined cases enter when combined with the leptonic factors obtained above, since the latter are all zero.

Contraction of Tensors and Cross Section

The contraction of the leptonic and the hadronic tensors arises from the application of standard Feynman rules to the evaluation of the cross section of the process under study here; it is an invariant, taking the same form in the laboratory, in the center-of-momentum, or in any other system of reference. As mentioned above, the symmetric and the antisymmetric components of the leptonic and the hadronic tensors can be contracted separately, since no cross-terms are allowed:

$$v_0 \mathcal{F}_\chi^2 \equiv \eta_{\mu\nu} W^{\mu\nu} = \eta_{\mu\nu}^s W_s^{\mu\nu} + \chi \eta_{\mu\nu}^a W_a^{\mu\nu},$$

where $\chi = 1$ for incident neutrinos and $\chi = -1$ for antineutrinos.

One finds that

$$\begin{aligned}
\eta^s_{\mu\nu} W_s^{\mu\nu} &= \eta^s_{00} W_s^{00} + 2\eta^s_{03} W_s^{03} + \eta^s_{33} W_s^{33} + \eta^s_{11} W_s^{11} + \eta^s_{22} W_s^{22} \\
&\quad + 2\eta^s_{01} W_s^{01} + 2\eta^s_{31} W_s^{31} + 2\eta^s_{02} W_s^{02} + 2\eta^s_{32} W_s^{32} + 2\eta^s_{12} W_s^{12} \\
\eta^a_{\mu\nu} W_a^{\mu\nu} &= 2\eta^a_{03} W_a^{03} + 2\eta^a_{01} W_a^{01} + 2\eta^a_{31} W_a^{31} \\
&\quad + 2\eta^a_{02} W_a^{02} + 2\eta^a_{32} W_a^{32} + 2\eta^a_{12} W_a^{12},
\end{aligned}$$

One finds that

$$\begin{aligned}
\eta^s_{\mu\nu} W^{\mu\nu}_s &= \eta^s_{00} W^{00}_s + 2\eta^s_{03} W^{03}_s + \eta^s_{33} W^{33}_s + \eta^s_{11} W^{11}_s + \eta^s_{22} W^{22}_s \\
&\quad + 2\eta^s_{01} W^{01}_s + 2\eta^s_{31} W^{31}_s + 2\eta^s_{02} W^{02}_s + 2\eta^s_{32} W^{32}_s + 2\eta^s_{12} W^{12}_s \\
\eta^a_{\mu\nu} W^{\mu\nu}_a &= 2\eta^a_{03} W^{03}_a + 2\eta^a_{01} W^{01}_a + 2\eta^a_{31} W^{31}_a \\
&\quad + 2\eta^a_{02} W^{02}_a + 2\eta^a_{32} W^{32}_a + 2\eta^a_{12} W^{12}_a ,
\end{aligned}$$

which can be expressed as

$$\begin{aligned}
\eta^s_{\mu\nu} W^{\mu\nu}_s &= \mathrm{Re} : \eta_{00} \mathrm{Re} W^{00} + 2\mathrm{Re}\eta_{03} \mathrm{Re} W^{03} + \mathrm{Re}\eta_{33} \mathrm{Re} W^{33} \\
&\quad + \mathrm{Re}\eta_{11} \mathrm{Re} W^{11} + \mathrm{Re}\eta_{22} \mathrm{Re} W^{22} + 2\mathrm{Re}\eta_{01} \mathrm{Re} W^{01} \\
&\quad + 2\mathrm{Re}\eta_{31} \mathrm{Re} W^{31} + 2\mathrm{Re}\eta_{02} \mathrm{Re} W^{02} + 2\mathrm{Re}\eta_{32} \mathrm{Re} W^{32} \\
&\quad + 2\mathrm{Re}\eta_{12} \mathrm{Re} W^{12}
\end{aligned}$$

$$\begin{aligned}
-\eta^a_{\mu\nu} W^{\mu\nu}_a &= 2\mathrm{Im}\eta_{03} \mathrm{Im} W^{03} + 2\mathrm{Im}\eta_{01} \mathrm{Im} W^{01} + 2\mathrm{Im}\eta_{31} \mathrm{Im} W^{31} \\
&\quad + 2\mathrm{Im}\eta_{02} \mathrm{Im} W^{02} + 2\mathrm{Im}\eta_{32} \mathrm{Im} W^{32} + 2\mathrm{Im}\eta_{12} \mathrm{Im} W^{12} .
\end{aligned}$$

Finally, in terms of projections with respect to the momentum transfer direction the contractions read

$$\begin{aligned}\eta^s_{\mu\nu} W_s^{\mu\nu} &= v_0 \Big\{ \Big[\widehat{V}_{CC} W^{CC} + \widehat{V}_{CL} W^{CL} + \widehat{V}_{LL} W^{LL} \\ &\quad + \widehat{V}_{T} W^{T} + \widehat{V}_{TT} W^{TT} + \widehat{V}_{TC} W^{TC} + \widehat{V}_{TL} W^{TL} \Big] \\ &\quad + \Big[\widehat{V}_{\underline{TT}} W^{\underline{TT}} + \widehat{V}_{\underline{TC}} W^{\underline{TC}} + \widehat{V}_{\underline{TL}} W^{\underline{TL}} \Big] \Big\},\end{aligned}$$

$$\begin{aligned}\eta^a_{\mu\nu} W_a^{\mu\nu} &= v_0 \Big\{ \Big[\widehat{V}_{T'} W^{T'} + \widehat{V}_{TC'} W^{TC'} + \widehat{V}_{TL'} W^{TL'} \Big] \\ &\quad + \Big[\widehat{V}_{\underline{CL}'} W^{\underline{CL}'} + \widehat{V}_{\underline{TC}'} W^{\underline{TC}'} + \widehat{V}_{\underline{TL}'} W^{\underline{TL}'} \Big] \Big\},\end{aligned}$$

where the hadronic responses contain all the VV, AA, and VA terms applicable to each of them. In any of the above representations the symmetric contraction involves 10 terms and the antisymmetric one involves 6 terms, for an expected total of 16 terms.

From the tensor contractions above the matrix element of the process is:

$$|\mathcal{M}_\chi|^2 = \frac{G^2 \cos^2\theta_c v_0}{2mm'}\mathcal{F}_\chi^2,$$

where $G = 1.166 \times 10^{-5}$ GeV^{-2} is the coupling constant of the weak interaction, $\cos\theta_c = 0.974$ with θ_c the Cabibbo angle accounting for the misalignment between the strong and the weak hadronic eigenstates, v_0 was defined above, and, as said above, $\chi = +1$ for neutrino and $\chi = -1$ for antineutrino scattering.

We then use the standard Feynman rules to evaluate the coincidence cross sections for the processes ${}^{A}X(\nu_\ell, \ell^- N)^{A-1}Y$ or ${}^{A}X(\bar{\nu}_\ell, \ell^+ N)^{A-1}Y$ in the laboratory system (see Raskin and Donnelly for the procedures for the analogous case of $(e, e'N)$ reactions). We obtain

$$d\sigma_\chi = \frac{G^2 \cos^2\theta_c}{2(2\pi)^5} \frac{m_N W_{A-1} v_0}{k\varepsilon' E_N E_{A-1}} \mathcal{F}_\chi^2 d^3\mathbf{k}' d^3\mathbf{p_N} d^3\mathbf{p_{A-1}} \delta^4(K + P_A - K' - P_{A-1} - P_N).$$

This form is exact in the cases where the $A-1$ system is in a bound ground state or a long-lived excited state. In other cases this form assumes that the wave function of the $A-1$ system can be factorized into center-of-mass and relative wave functions, which is not in general true for relativistic wave functions. However, since the momenta available to the $A-1$ system will generally be of the order of the Fermi momentum and the masses of the undetected fragments will tend to be large, the nuclear system will generally be treated non-relativistically and the factorization of the wave function will then be exact. Upon integration over the unobserved residual daughter nucleus momentum $\mathbf{p_{A-1}}$ and energy E_{A-1} one gets

$$\frac{d\sigma_\chi}{dk' d\Omega_{k'} d\Omega_{p_N}} = \frac{G^2 \cos^2\theta_c}{2(2\pi)^5} \frac{m_N W_{A-1}}{M_A^0} \frac{p_N k'^2 v_0}{k\varepsilon' F_{rec}} \mathcal{F}_\chi^2,$$

where W_{A-1} is defined so that $f \equiv 0$, with (from energy conservation)

$$f = \varepsilon + M_A^0 - \varepsilon' - \left(p_N^2 + m_N^2\right)^{1/2} - \left(q^2 + p_N^2 - 2qp_N\cos\theta_N + W_{A-1}^2\right)^{1/2}.$$

From the function f one obtains also the recoil factor F_{rec} as

$$F_{rec} = \frac{E_N E_{A-1}}{M_A^0 p_N} \left| \frac{\partial f}{\partial p_N} \right| = \left| 1 + \frac{\omega p_N - qE_N \cos\theta_N}{M_A^0 p_N} \right|.$$

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When integrating over the ejected nucleons to obtain the inclusive cross section one is obtaining the t-channel inclusive cross section

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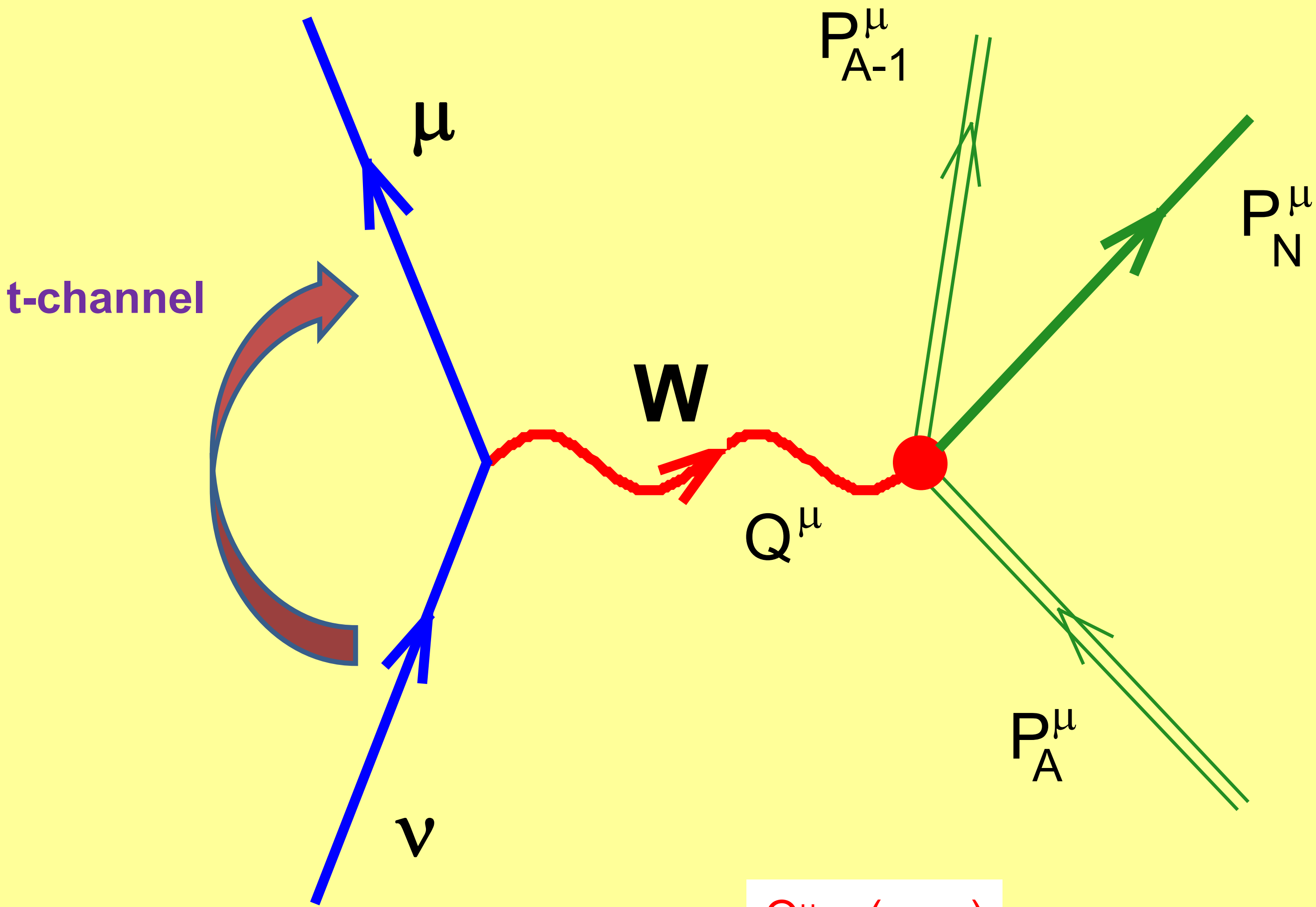
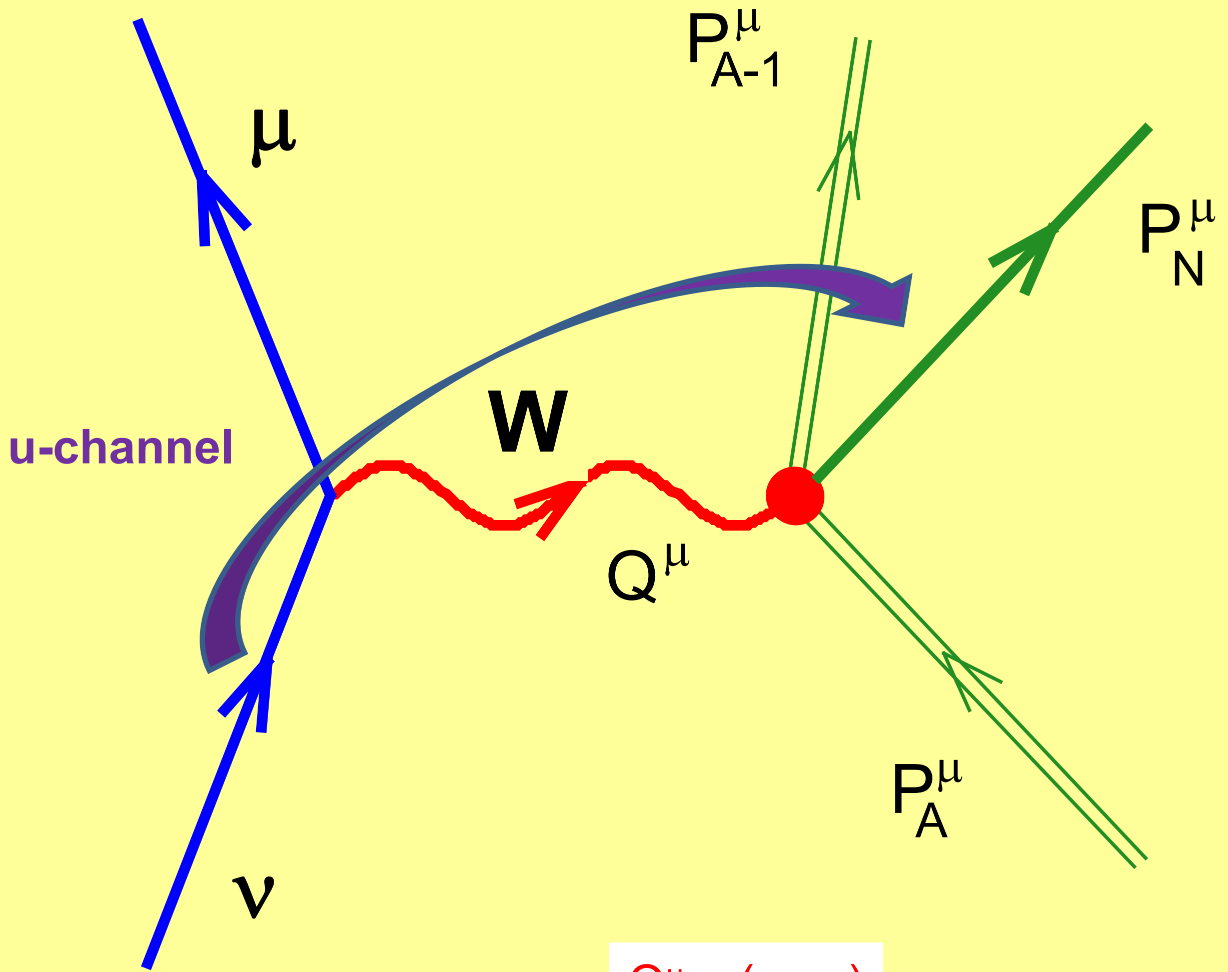


$Q^\mu = (\omega, \mathbf{q})$

… but, when integrating over the final scattered lepton, keeping the ejected nucleon, one obtains the u-channel inclusive cross section:



$Q^{\mu} = (\omega, \mathbf{q})$

In the latter case the interference responses do not integrate to give no contribution, but have non-zero contributions.

Thus, u-channel processes such as neutrino-in, proton out with no muon detected (CC) or no neutrino detected (NC) are not simply related to t-channel inclusive process in general.

See J. E. Amaro, M. B. Barbaro, J. A. Caballero and T. W. Donnelly, *Phys. Rev.* **C73** (2006) 035503 for more discussion of the NC case, including scaling in the u-channel.

Summary:

1. **Any model that does not succeed for electron scattering is very unlikely to be valid for neutrino reactions.**
2. **Relativistic effects from kinematics and boost factors are essential.**
3. **Interaction contributions in both initial and final states are significant and naïve models such as the RFG fail at the 25% level or so to reproduce the data, while for inclusive scattering RMF theory is much better.**
4. **MEC effects are significant (and should be modeled relativistically).**
5. **Inclusive "QE" model CC neutrino cross sections fall short of the MiniBooNE data, even when MEC effects are included, whereas for NOMAD kinematics they are much better.**
6. **While the models discussed here are not too bad for inclusive scattering, they are not suited to semi-inclusive scattering for all choices of missing energy/momentum.**
7. **For semi-inclusive reactions (detection of one final-state hadron) relativistic one-particle spectral functions are better, although they also involve approximations.**
8. **For reactions requiring detection of two or more particles one needs relativistic two-particle spectral functions!**


. . . thank you