Ab initio methods for nuclei

Lecture II

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Outline

- ⋆ Lecture I: Nuclear Many-Body Theory
- ⋆ Lecture II: Nucleon Green's function and nuclear response at low to moderate momentum transfer
  - The two-point Green's function
  - Interaction effects: quasi particles and correlations
  - Spectral function
  - Dynamic response of interacting many-body systems
  - Limits of applicability of the non relativistic approximation
- ⋆ Lecture III: Electron and neutrino cross section in the impulse approximation and beyond

The two-point Green's function

- ⋆ Quantum mechanical amplitude associated with the propagation of a particle from $x \equiv (t, \mathbf{x})$ to $x' \equiv (t', \mathbf{x}')$
- ⋆ Consider uniform nuclear matter ($k \equiv (E, \mathbf{k})$)

$$G(x, x') = G(x - x') = \int d^4k \, \mathrm{e}^{ikx} \, G(k)$$

$$\begin{aligned} G(\mathbf{k}, E) &= \langle 0 | a^\dagger_{\mathbf{k}} \frac{1}{E + (H - E_0) - i\eta} a_{\mathbf{k}} | 0 \rangle + \langle 0 | a_{\mathbf{k}} \frac{1}{E - (H - E_0) + i\eta} a^\dagger_{\mathbf{k}} | 0 \rangle \\ &= G_h(\mathbf{k}, E) + G_p(\mathbf{k}, E) \ , \end{aligned}$$

- ⋆ G_h and G_p describe the propagation of *hole* and *particle* states, respectively, and $\eta = 0^+$

Non interacting system

- Consider the hole spectral function, and insert a complete set of states

$$G_h(\mathbf{k}, E) = \sum_n \langle 0|a^\dagger_{\mathbf{k}}|n\rangle \frac{1}{E + E_n - E_0 - i\eta} \langle n|a_{\mathbf{k}}|0\rangle \ .$$

- In the non interacting system

$$\langle n|a_{\mathbf{k}}|0\rangle \to \theta(k_F - |\mathbf{k}|) \quad , \quad E_n \to E_0 - e^0_k \quad , \quad e^0_k = \mathbf{k}^2/2m$$

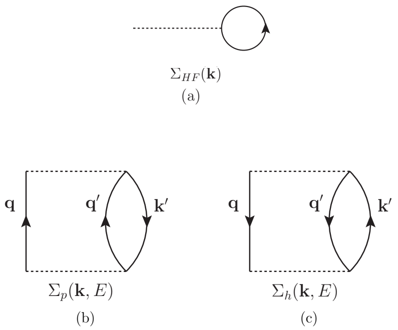
$$\mathcal{G}_h(\mathbf{k}, E) = \frac{\theta(k_F - |\mathbf{k}|)}{E - e^0_k - i\eta} \ .$$

Interacting system

★ In the presence of interactions

$$e^0_k \to e^0_k + \Sigma(\mathbf{k}, E) = e^0_k + \mathrm{Re}\Sigma(\mathbf{k}, E) + i\mathrm{Im}\Sigma(\mathbf{k}, E)$$

★ Self energy



★ Green's function

$$G_h(\mathbf{k}, E) = \frac{1}{E - e^0_k - \Sigma(\mathbf{k}, E)}$$

★ $\Sigma(\mathbf{k}, E)$ can be computed using eithr CFB or G-matrix perturbation theory

Quasiparticles and beyond

- ⋆ The identification of single particle properties in interacting many-body systems is a non trivial issue, addressed by Landau's theory of normal Fermi liquids
- ⋆ According to Landau there is a one-to-one correspondence between the elementary excitations of a Fermi liquid, dubbed quasiparticles, and those of the non interacting Fermi gas. Quasiparticle states of momentum $\mathbf{k}$ are specified by their energy, e_k and lifetime τ_k .

$$G_h(\mathbf{k},E) = \frac{Z_k}{E - e_k - i\tau_k^{-1}} + G_h^B(\mathbf{k},E)$$

$$e_k = e_k^0 + \Sigma(\mathbf{k},e_k) \quad , \quad \tau_k^{-1} = Z_k \mathrm{Im}\Sigma(\mathbf{k},e_k) \quad , \quad Z_k = \left[1 - \frac{\partial}{\partial E}\mathrm{Re}\ \Sigma(k,E)\right]^{-1}_{E=e_k}$$

- ⋆ $G_h^B(\mathbf{k},E)$ is a smooth contribution associated with multiparticle-multihole excitations

Spectral representation

- The interpretation of the Green's function is best understood in the spectral representation

$$G(\mathbf{k}, E) = \int_{-\infty}^{\mu} dE' \, \frac{P_h(\mathbf{k}, E')}{E - E' - i\eta} + \int_{\mu}^{+\infty} \frac{P_p(\mathbf{k}, E')}{E - E' + i\eta} \ ,$$

where $\mu = e_{k_F}$ is the *chemical potential*

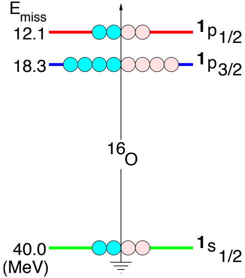
- The hole (particle) spectral function yields the probability of removing from (adding to) the system a particle with momentum $\mathbf{k}$ and energy E

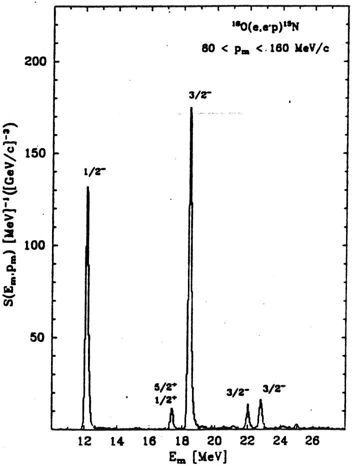
$$P_h(\mathbf{k}, E) = \frac{1}{\pi} \mathrm{Im} \, G_h(\mathbf{k}, \mu - E) = \sum_n |\langle n | a_{\mathbf{k}} | 0 \rangle|^2 \delta(E - E_0 + E_n) \ ,$$

$$P_p(\mathbf{k}, E) = -\frac{1}{\pi} \mathrm{Im} \, G_p(\mathbf{k}, \mu + E) = \sum_n |\langle n | a_{\mathbf{k}} | 0 \rangle|^2 \delta(E - E_n + E_0) \ ,$$

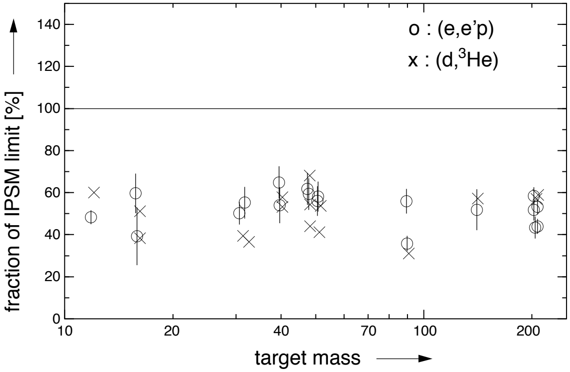
- ⋆ The spectral lines corresponding to the shell model states clearly seen in the missing energy spectra of measured by

$$e + \mathrm{A} \to e' + p + X$$

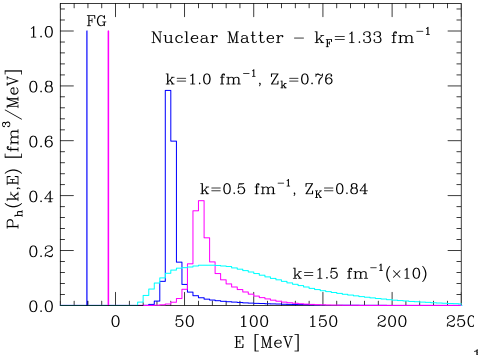


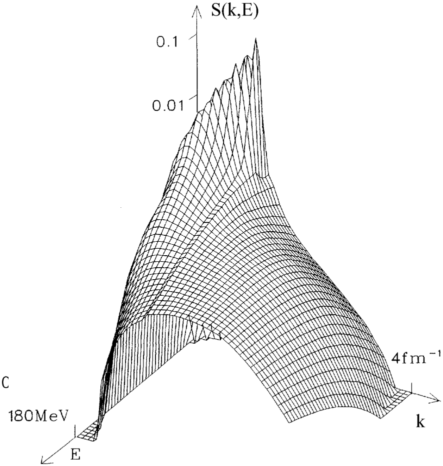


★ The spectroscopic factors (i.e. the residues of the Green's function at the quasiparticle poles, obtained integrating the spectra in the region of the correponding peak) turn out to be significantly below the shell model prediction, independently of A



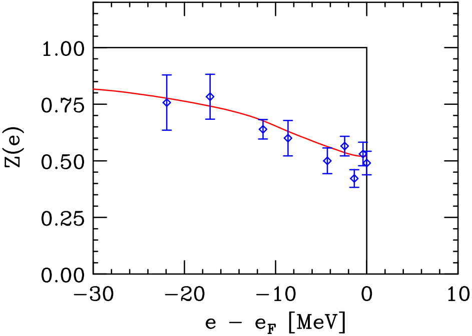
CBF calculation for isospin-symmetric nuclear matter





Spectroscopic factors of ^{208}Pb

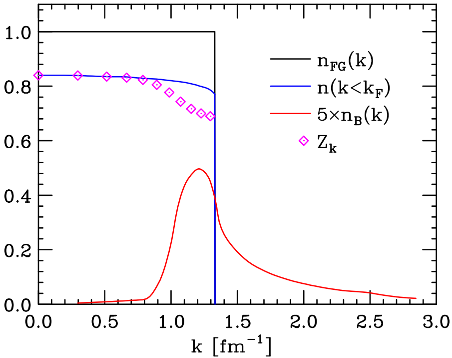
★ The deviation of the spectroscopic factors from unity provides an (indirect) measurement of correlation effects



Momentum distribution and spectroscopic factors of SNM

- In analogy with the spectral function, the momentum distribution can be split into quasi particle (pole) and and correlation (continuum) contributions

$$n(\mathbf{k}) = \int dE\, P(\mathbf{k}, E) = Z_k \theta(k_F - |\mathbf{k}|) + \int dE\, P_B(\mathbf{k}, E) = Z_k \theta(k_F - |\mathbf{k}|) + n_B(\mathbf{k})$$



The nuclear response function

- Consider scattering of a scalar probe, for simplicity

$$\frac{d\sigma}{d\Omega d\omega} \propto S(\mathbf{q},\omega) = \sum_n \langle 0|\rho^\dagger_{\mathbf{q}}|n\rangle\langle n|\rho_{\mathbf{q}}|0\rangle\delta(E_0+\omega-E_n)$$

$$\rho_{\mathbf{q}} = \sum_{\mathbf{k}} a^\dagger_{\mathbf{k}+\mathbf{q}} a_{\mathbf{k}}$$

$$H|0\rangle = E_0|0\rangle \quad , \quad H|n\rangle = E_n|n\rangle$$

- At $|\mathbf{q}| \lesssim 400$ MeV, exact calculations are feasible for $A \leq 4$ using integral transform techniques
- Accurate results for uniform nuclear matter have been also obtained (exploiting again translation invariance) CBF perturbation theory

Effects of interactions on the nuclear response

- In the absence of correlations, the only possible final states are one particle-one hole states
- For example, according to the Fermi gas model

$$M_n = \langle n| \sum_{\mathbf{k}} a^\dagger_{\mathbf{k}+\mathbf{q}} a_{\mathbf{k}} |0\rangle \rightarrow M_k = 1 \times \theta(k_F - |\mathbf{k}|)\theta(|\mathbf{k}+\mathbf{q}| - k_F)$$

$$S(\mathbf{q},\omega) = \sum_{\mathbf{k}} |M_k|^2 \delta(\omega + e_0(\mathbf{k}) - e_0(\mathbf{k}+\mathbf{q})) \quad , \quad e_0(\mathbf{k}) = \frac{\mathbf{k}^2}{2m}$$

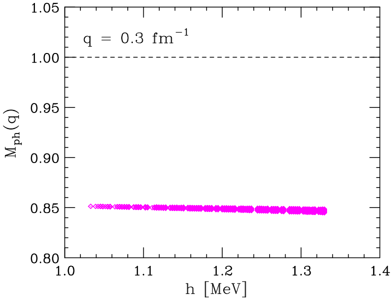
- Inclusion of interactions, through the replacement of Fermi gas states with CBF states, leads to a quenching of the transition matrix elements M_k and to a modification of the single particle spectrum $e_0(\mathbf{k})$

Correlations & interaction effects

★ Isospin symmetric nuclear natter at equilibrium density

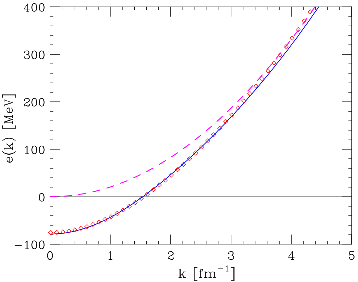
▹ Correlations

$$M_{ph} < 1$$



▹ Mean field

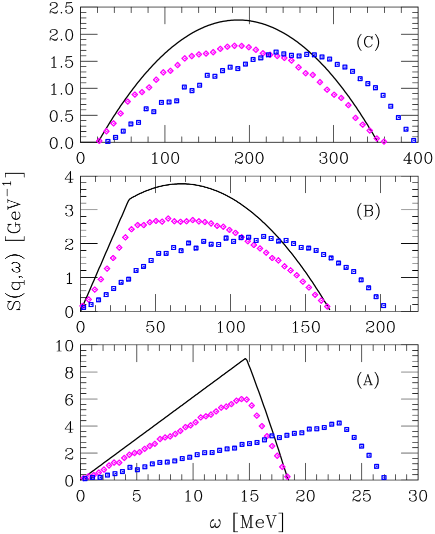
$$m \to m^{\star} = \left(\frac{1}{k} \frac{de}{dk} \right)^{-1}$$



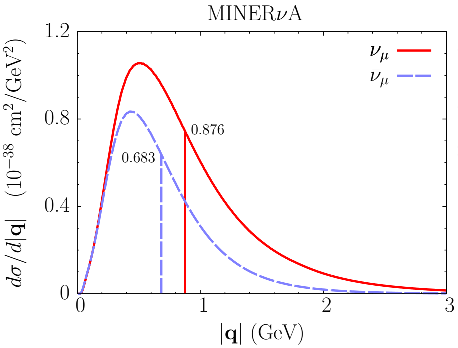
★ Note that $m^{\star}(k) \neq m$ *does not* measure correlation effects

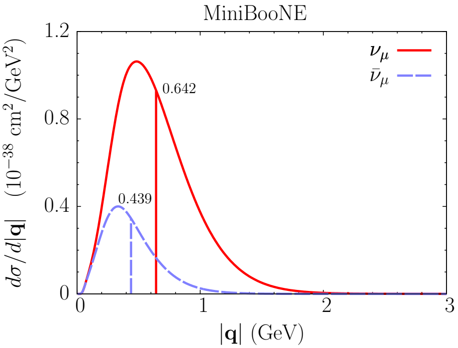
Correlation & interaction effects on the response

- ⋆ (A), (B), (C) → $|\mathbf{q}| = 0.3, 1.8, 3.0\ \mathrm{fm}^{-1}$



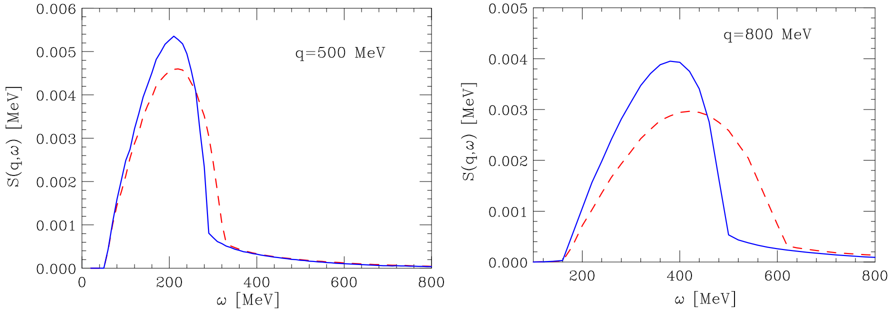
Why worry about relativity





- ⋆ $|\mathbf{q}|$-dependence of the CCQE cross section averaged with the Minerνa (left) and MiniBooNE (right) fluxes

Why worry about relativity (continued)



★ Difference between the nuclear response computed using relativistic (solid lines) and non relativistic (dashes) kinematics

Summary of Lecture II

- ⋆ The analysis of the two-point Green's function and the associated spectral functions allows for a clearcut identification of mean field and correlation effects
- ⋆ The role of correlation effects is unambiguously demonstrated by electron scattering data
- ⋆ Interaction effects are clearly visible in the dynamic response obtained from Nuclear Many Body Theory (NMBT)
- ⋆ The main limitation of the formalism of NMBT is the inability to describe the response in the region in which the non relativistic approximation breaks down, relevant to accelerator based neutrino experiments