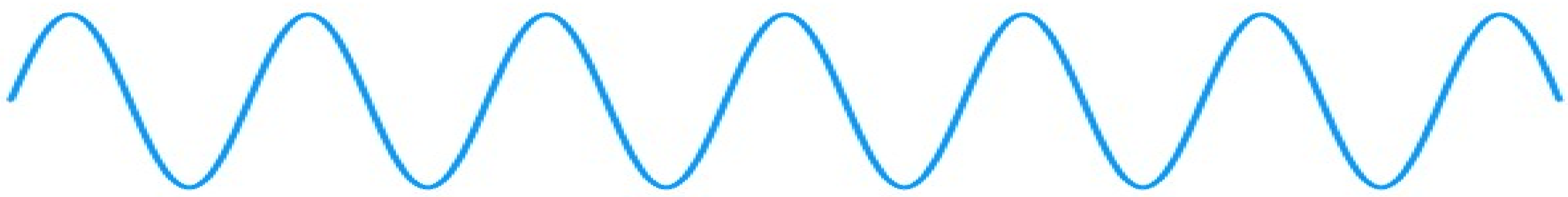
Slip Stacking:
Simultaneous Independent Longitudinal Focusing

by Jeffrey Eldred

New Perspectives 06/09/14

Basic Longitudinal Focusing

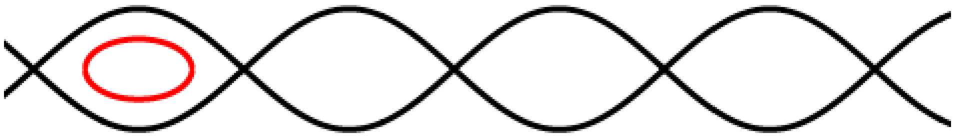
- RF accelerating cavities maintain a resonanting E&M wave to interact with the particle beam.
- Particles are either accelerated or decelerated depending on their arrival time.



- The particle phase is focused by the potential, which is identical in form to a simple gravity pendulum.

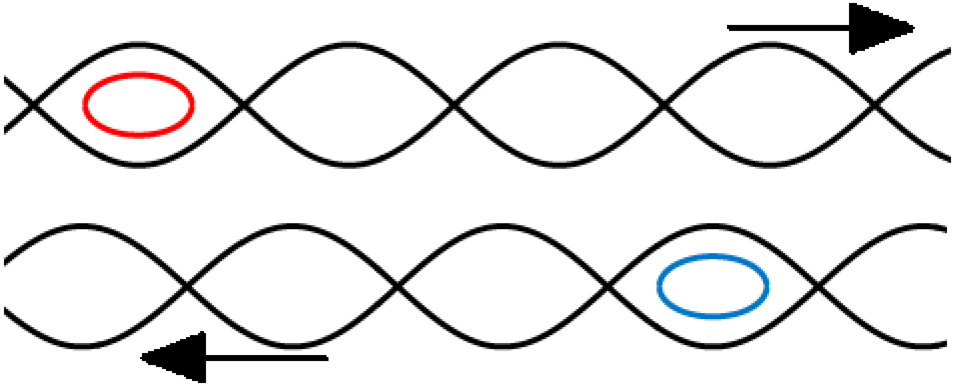
Phase-Space for a single RF

- Phase-space plot for a single RF cavity:



Phase-space for Slip-stacking RF

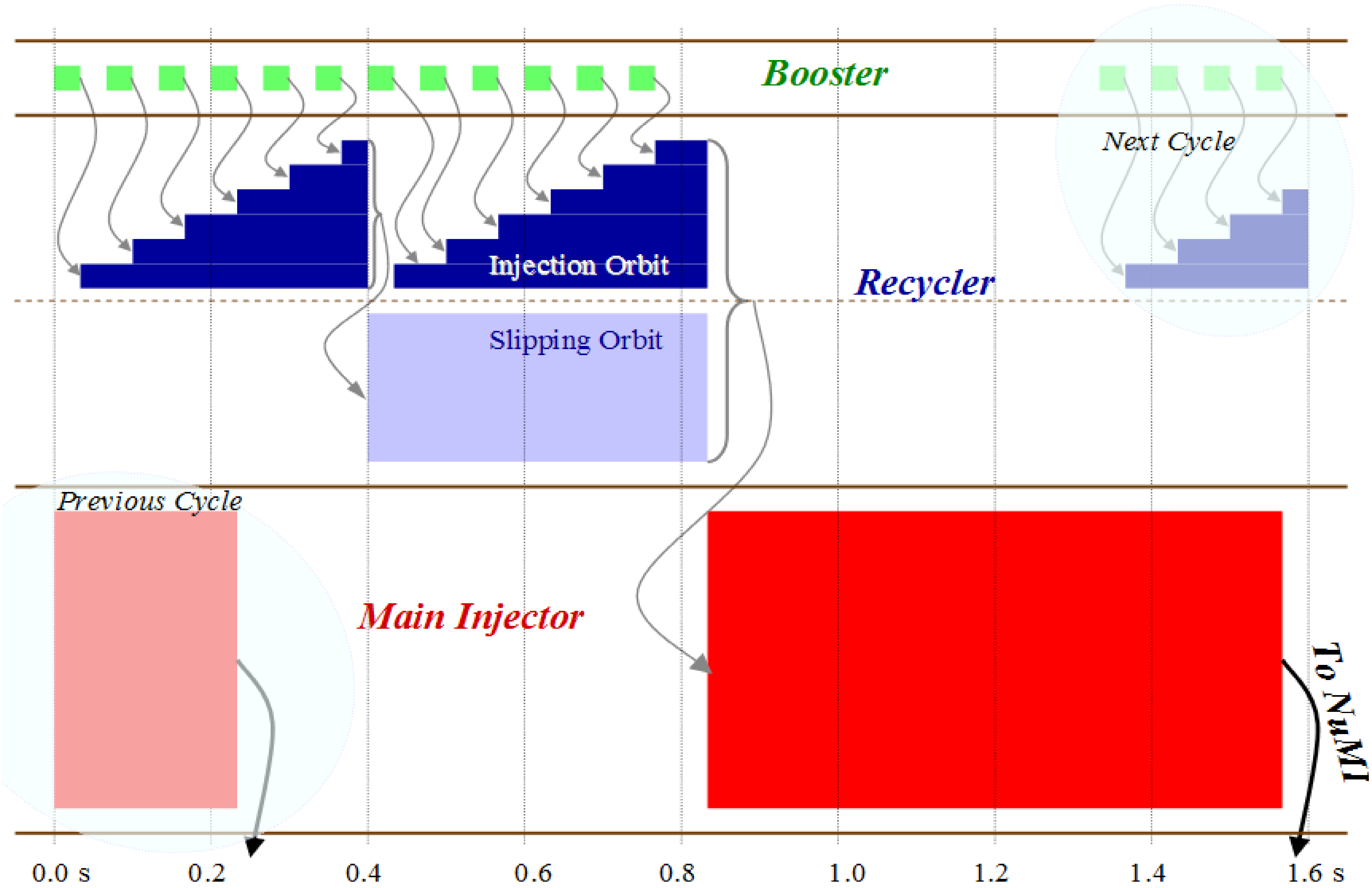
- Two RF cavities at near but different frequencies.
- Phase-plot for slip-stacking:



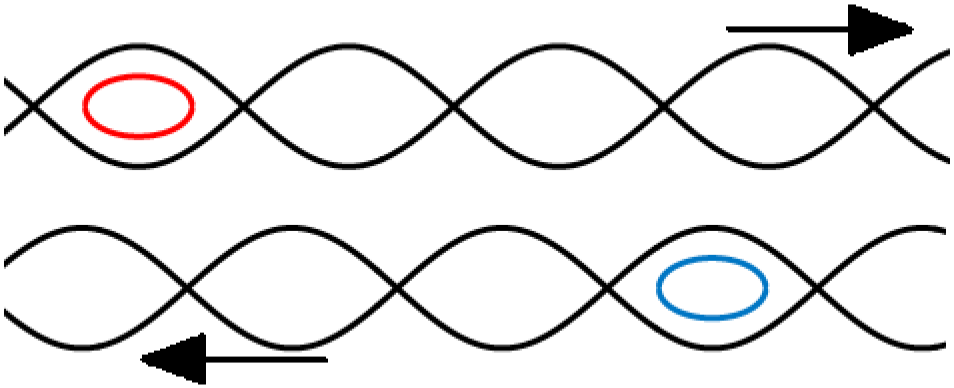
Slip Stacking at Fermilab

- Slip-stacking allows us to accumulate twice as many particles and double the MI proton intensity.
- Fermilab has implemented slip-stacking since 2004.
- Fermilab is the only accelerator complex to use slip-stacking operationally.

Recycler-MI Slip-Stacking Cycle



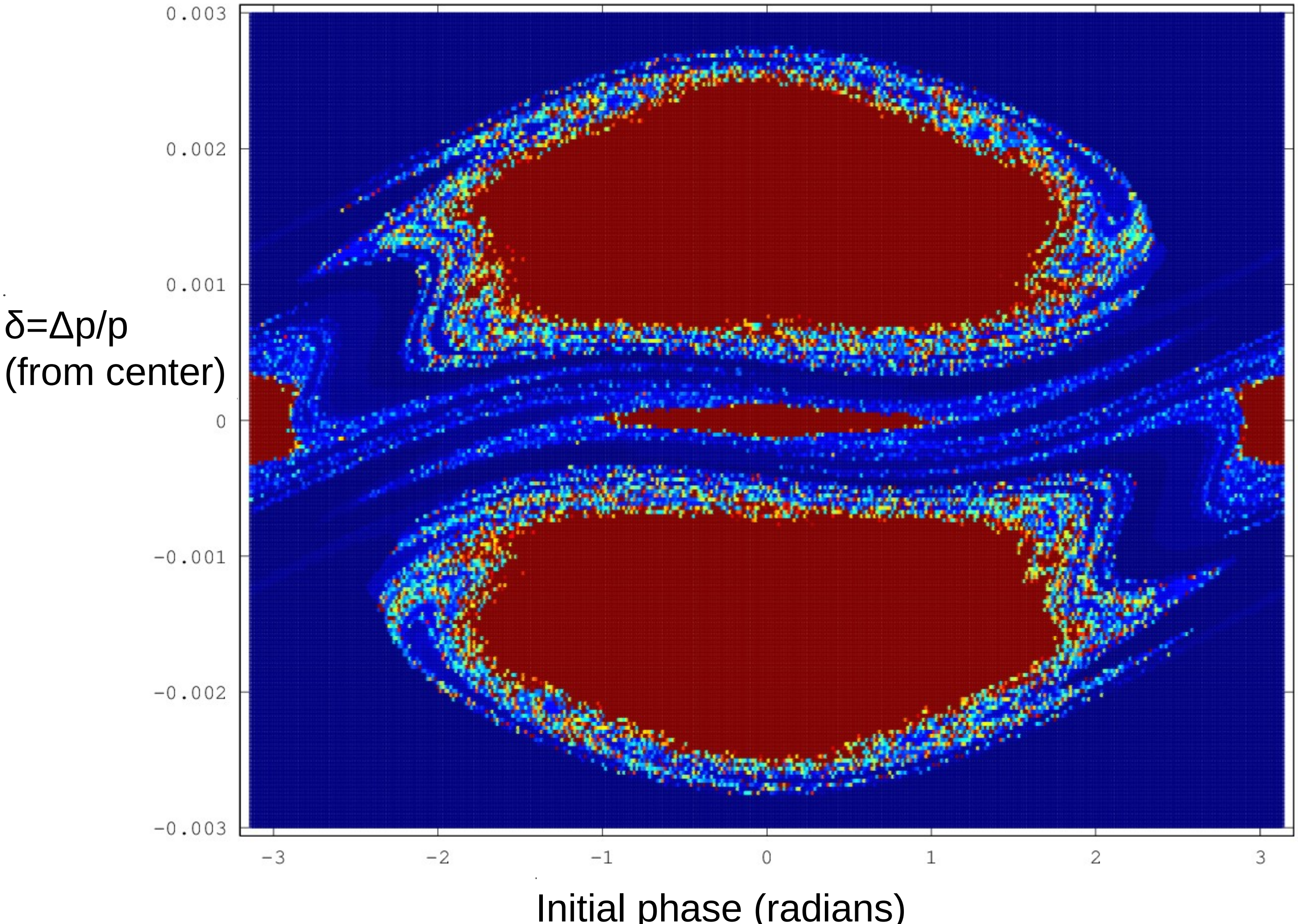
Dynamic Stability of Slip-stacking



- Its not as simple as this.
  - The second RF frequency interferes with the first RF frequency.
  - Finding the phase-space boundary of stable slip-stacking is a nonlinear time-dependent problem.

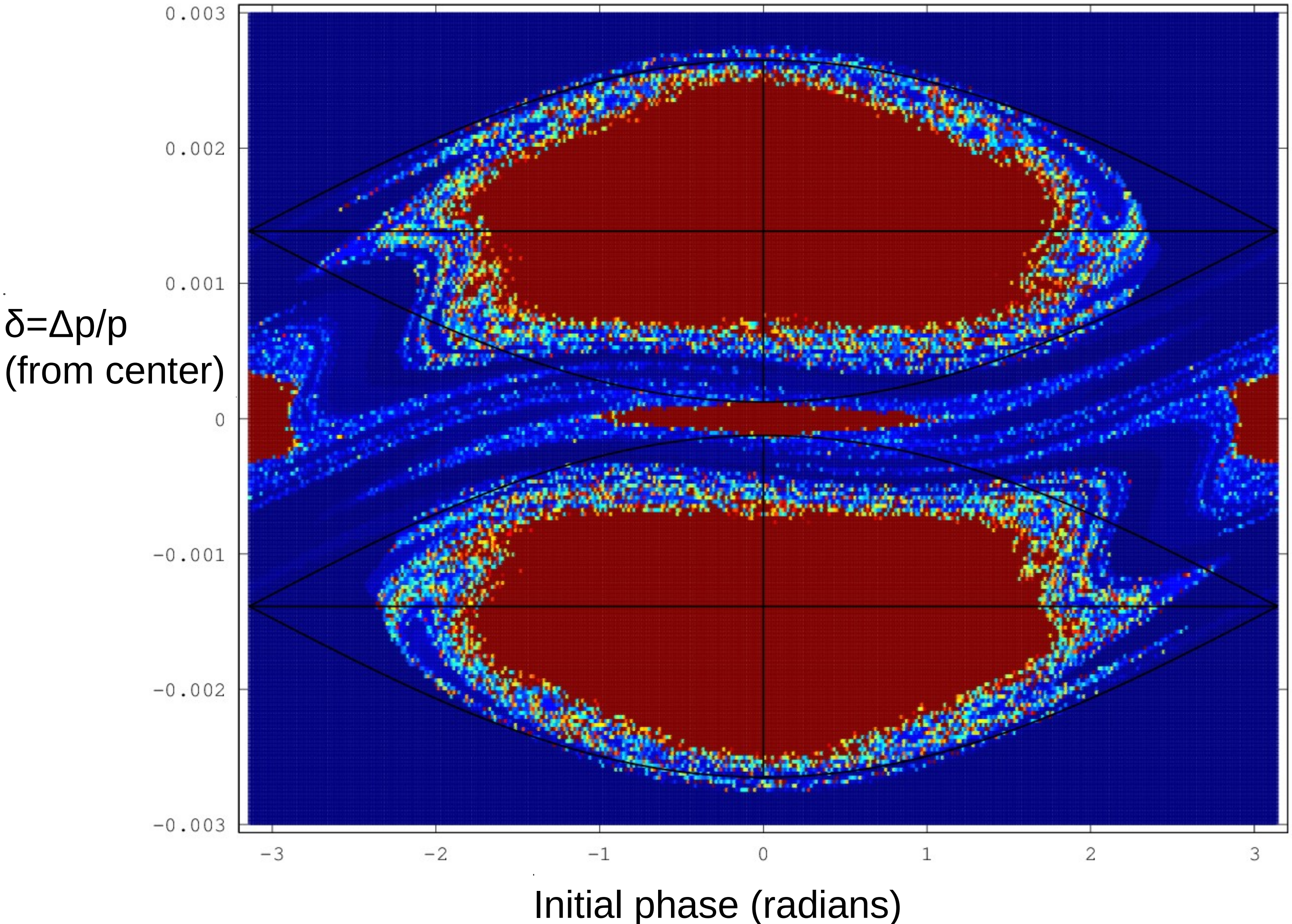
Numerical Result for Stable Area

Stability of Initial Positions (RF phase difference **0**)

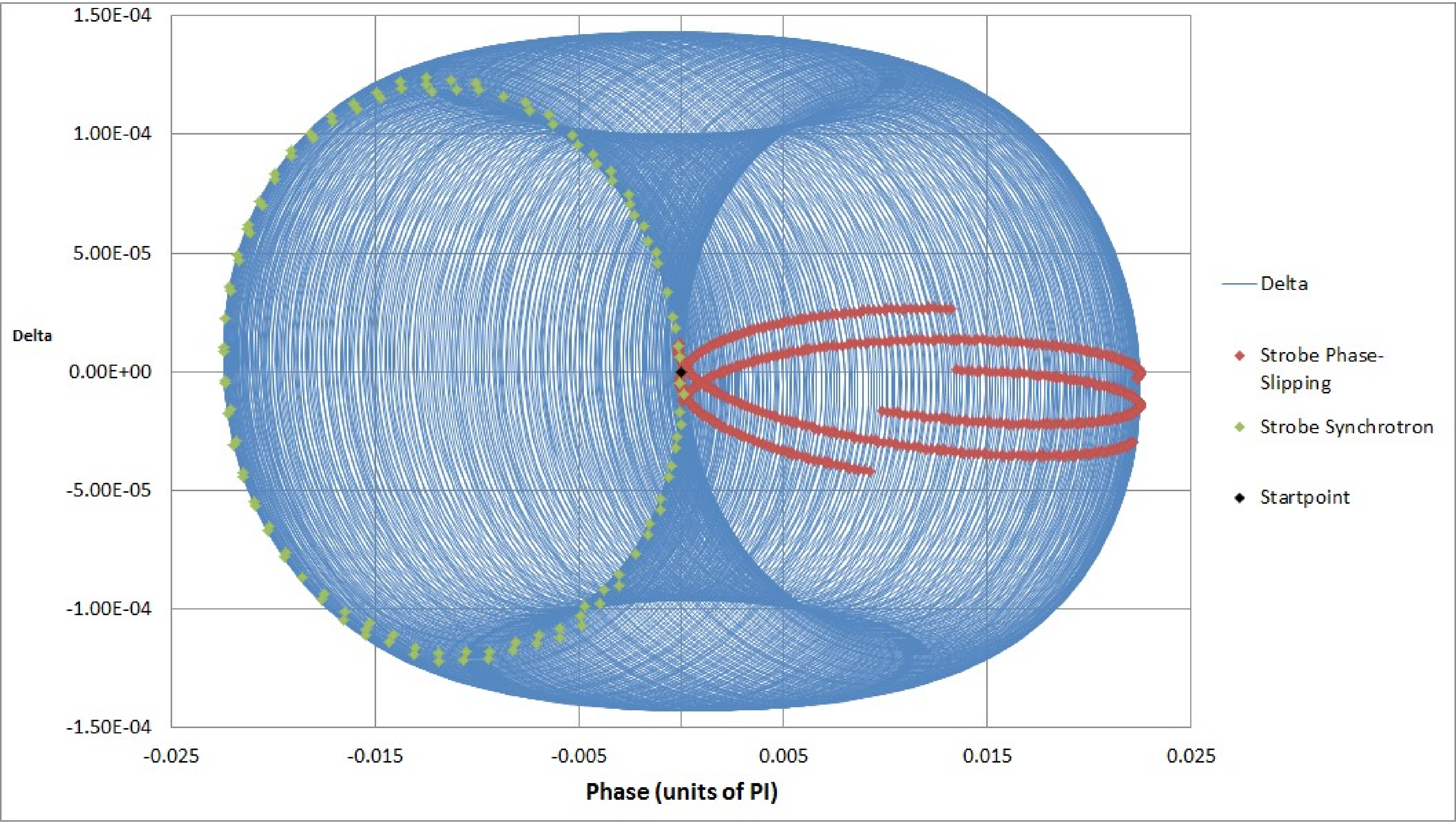


What do the buckets look like?

Stability of Initial Positions (RF phase difference **0**)

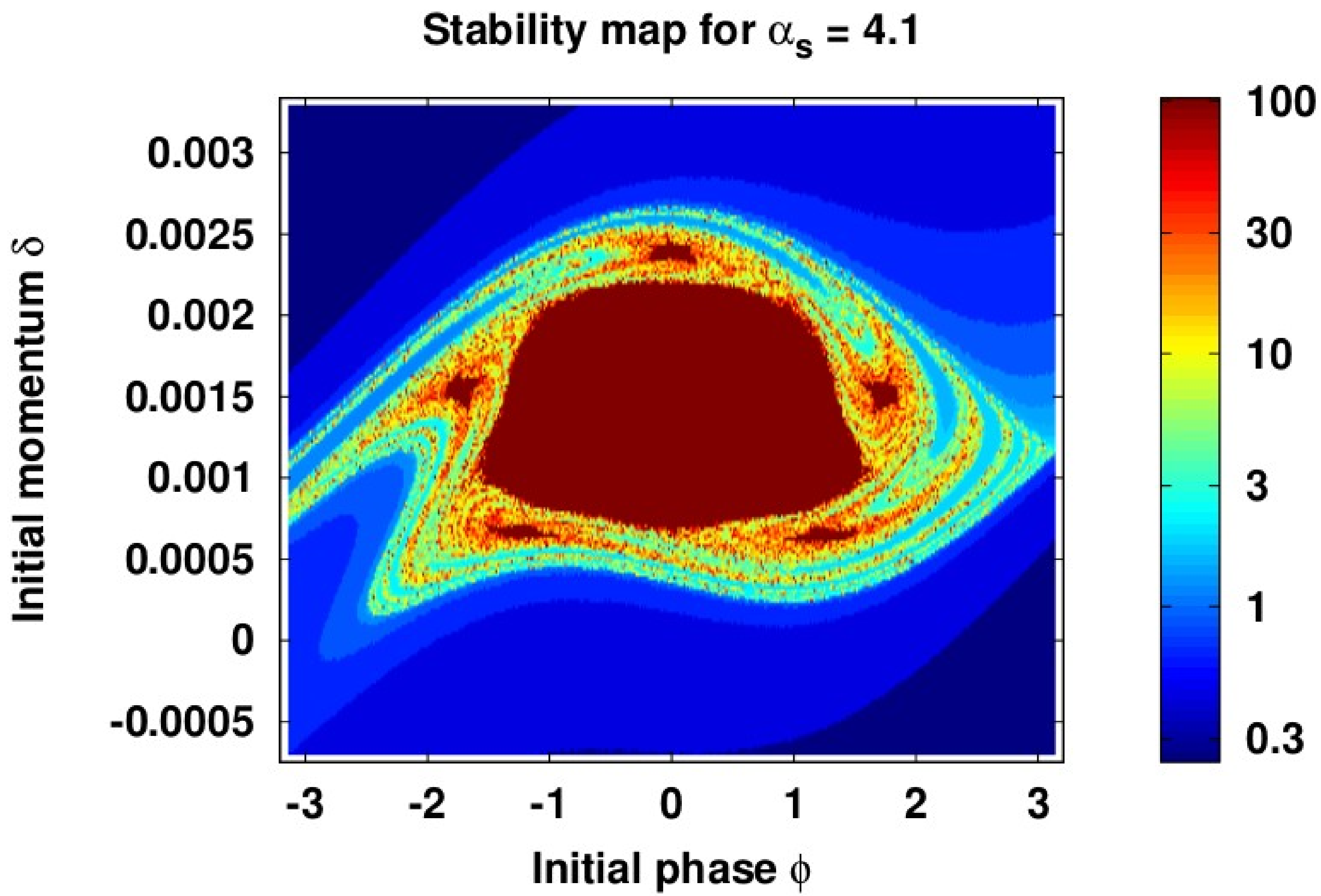


Phase-Space path (naïve center)



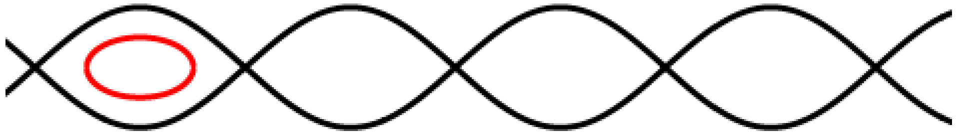
Parametric Resonances

$$m\omega_s(1+\sigma) = n\omega_p \qquad \ddot{\phi} \propto \rho^m \alpha_s^{-2(n-1)} \sin(m\psi) + \ldots \text{ bounded terms}$$



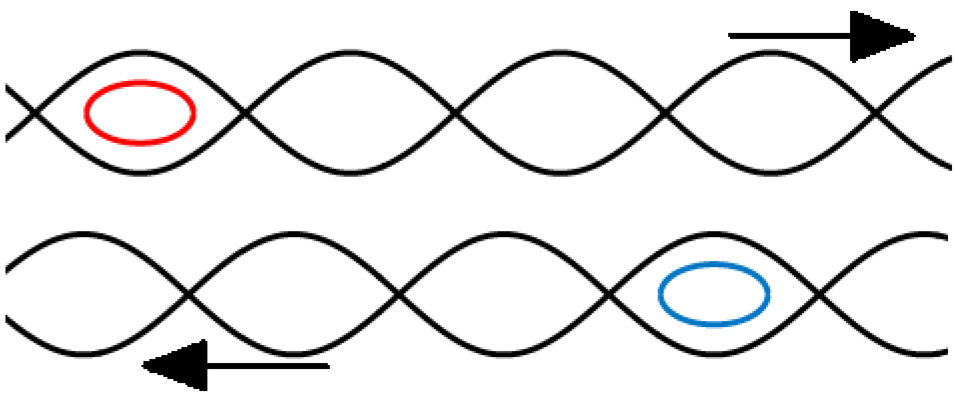
RFs and Pendulums

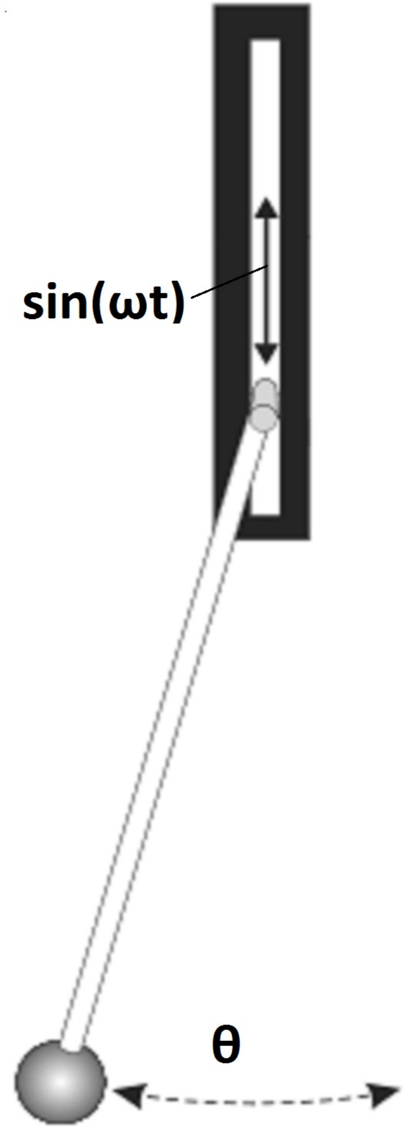
A single RF cavity is isomorphic to a simple gravity pendulum



RFs and Pendulums

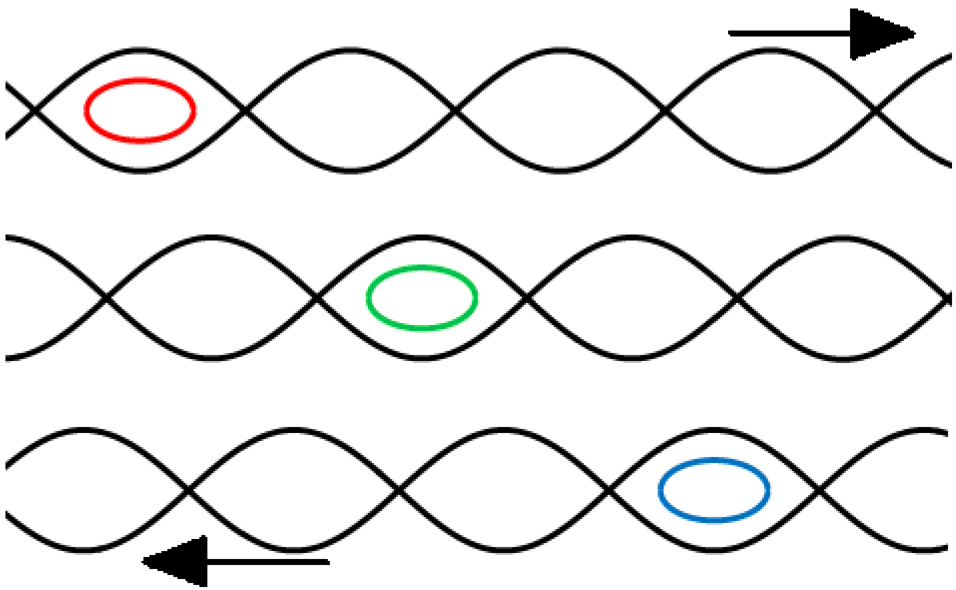
Two RF cavities (slip-stacking) is isomorphic to a driven pendulum





RFs and Pendulums

Three RF cavities
is isomorphic to
a driven pendulum with gravity



Application to Standing Wave Traps

- Standing wave traps are a sinesoidal potential
- Optical lattices used in AMO physics.
- Acoustic levitation techniques for fabrication.
- Two standing wave traps moving with respect to each other make a slip-stacking potential:
  - Trap-Accumulation
  - Controlled Collisions

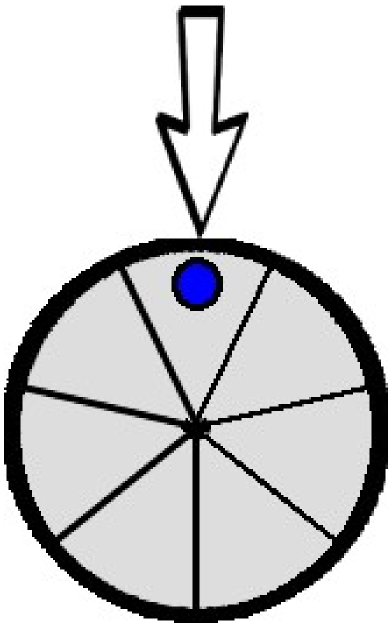
$$\alpha_s = v\sqrt{\frac{M}{2V_H}} > 5$$

Thank you for listening!

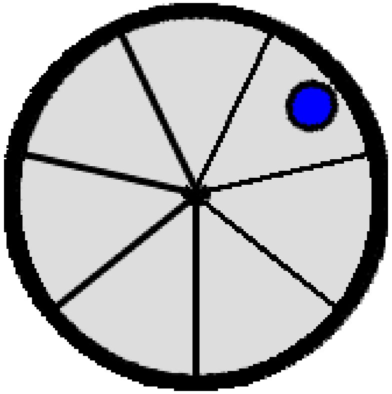
...Any Questions?

Backup Slides

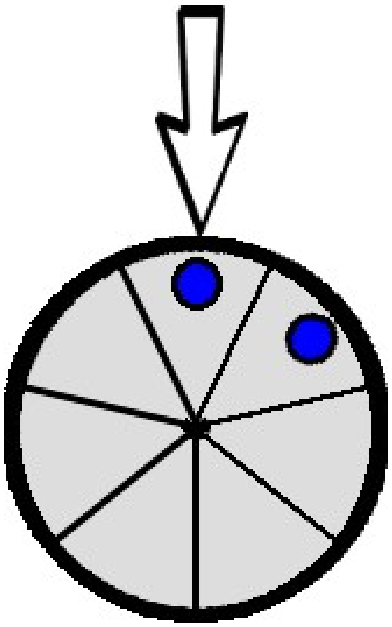
- A “batch” is injected from the Booster into 1/7 of the Recycler.



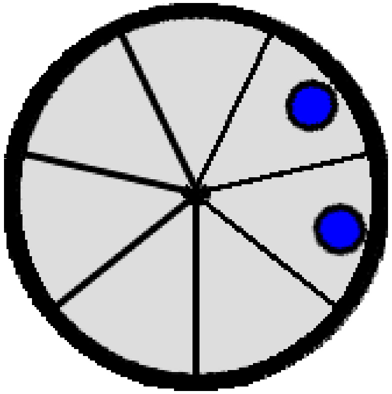
- The first batch is stored in the Recycler while the second batch is prepared in the Booster.



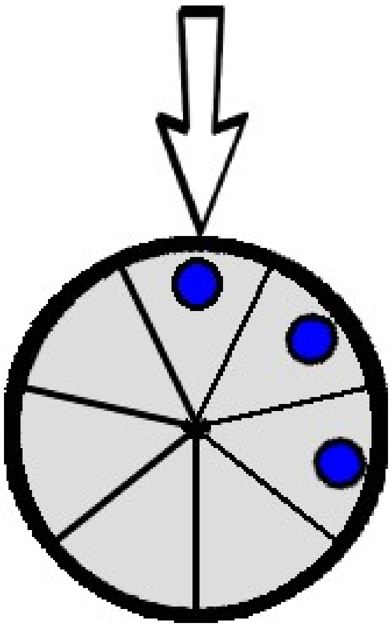
- The timing works out so that the second batch is injected immediately behind the first.
- Called “Boxcar Stacking”.



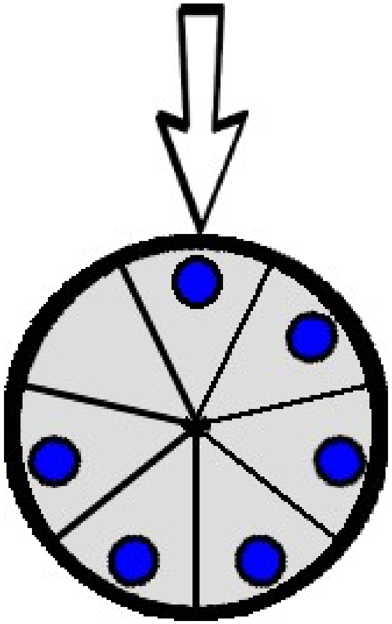
- Now two batches are stored...



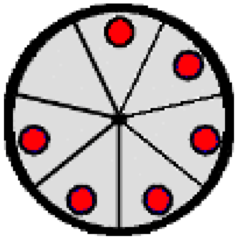
- And a third batch is injected…



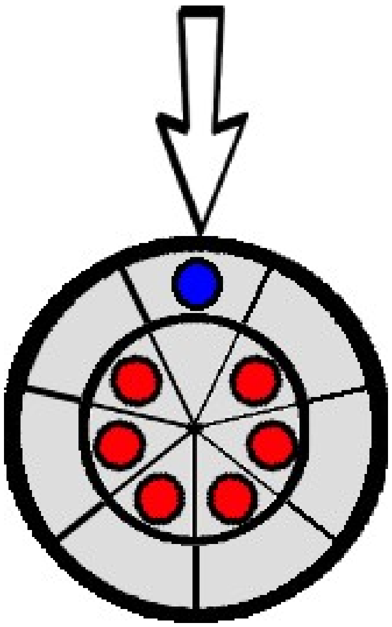
- This process repeats until 6/7 of the Recycler is filled.



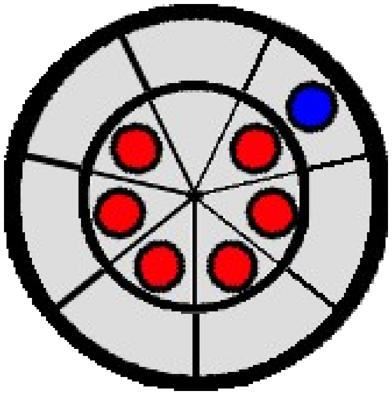
- The RF cavity is gradually lowered in frequency so that these 6 batches are now in a lower momentum orbit.



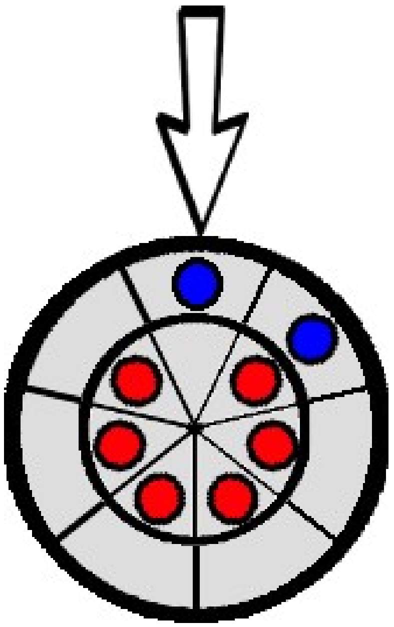
- Another batch can be injected in that 1/7 gap without kicking out any beam.



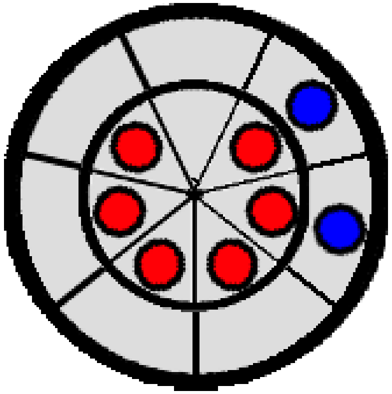
- The batches slip past each other and can occupy the same azimuthal space.
- Because the shifted batch is slower, the gap lines up again for the next injection.



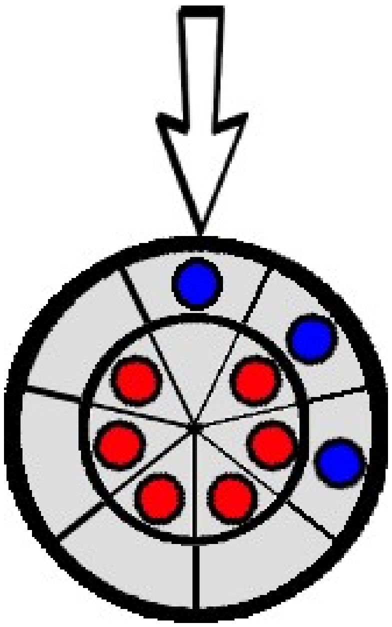
- The eight batch is injected immediately behind the seventh batch without kicking out the first six.



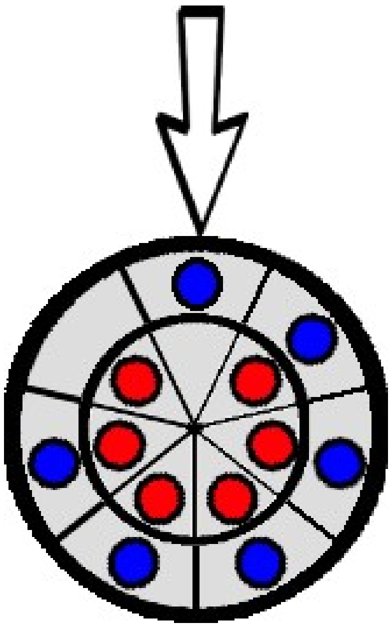
- Those batches can be stored and slipped as well.



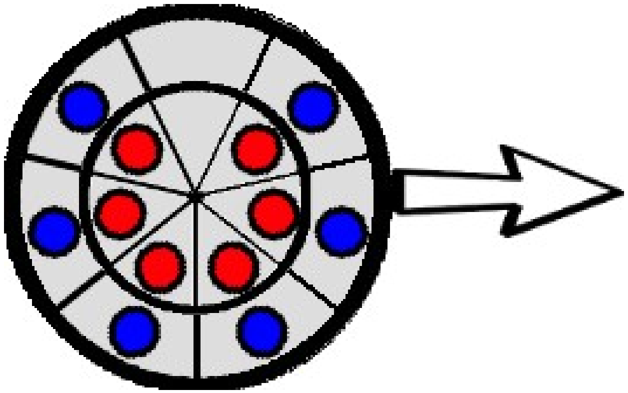
- And the ninth injection can proceed smoothly.



- The process continues until there are a total of twelve batches, six in each momentum orbit.



- All batches are ejected to the Main Injector.
- The Recycler can begin to fill again while the MI ramps.
- The extra 1/7 azimuthal space is used for the kicker.



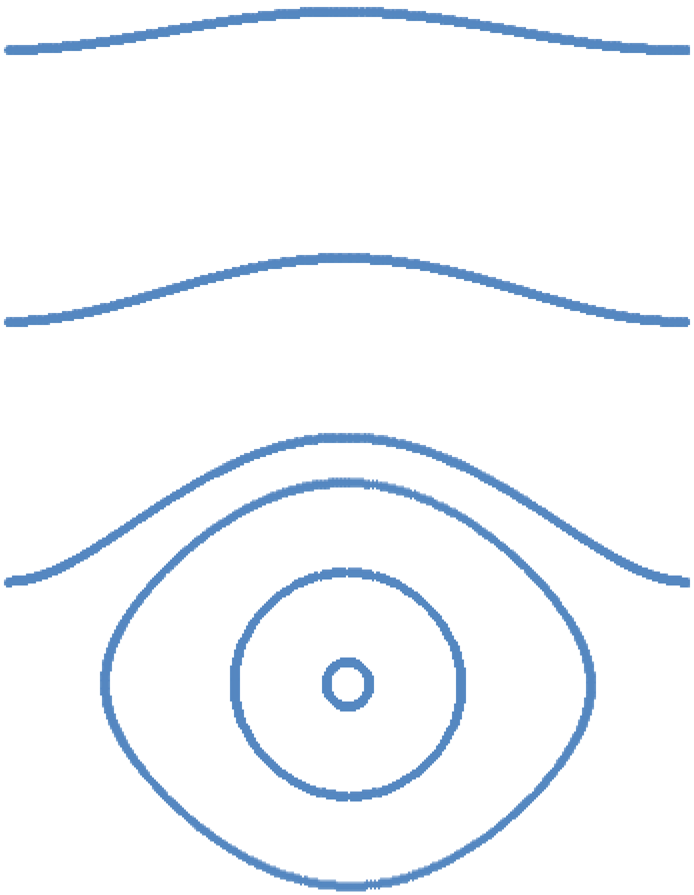
Booster Cycle Rate determines Slipping Rate

- The two momentum orbits must slip one batch (84 buckets) in time for each booster injection (15 Hz).
- Phase-slipping frequency 84*15 Hz = 1260 Hz.
- 20 Hz Booster would mean 1680 Hz.

Forces from RF Cavities

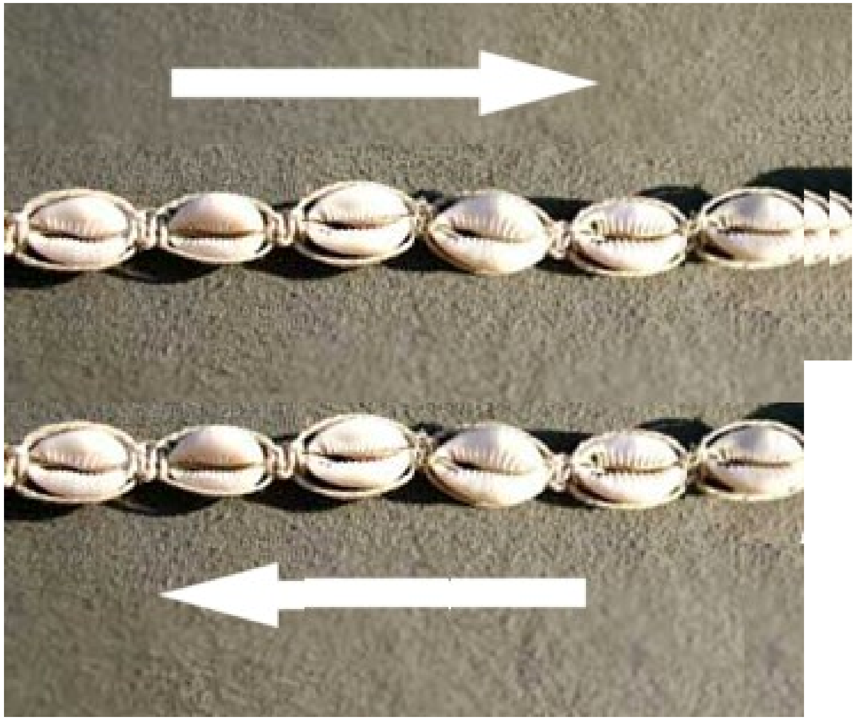
Single Bucket Synchrotron Motion
(still no slip stacking yet)

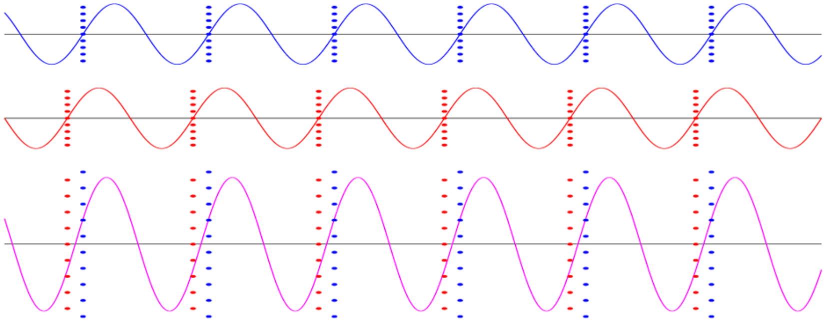
- Particles are accelerated/decelerated based on their phase relative to the RF-cavity.
- Particles can be classified as either in the bucket or not in the bucket.
- In the bucket:
  - Average phase and momentum is the synchronous point.
- Above/below the bucket:
  - The average particle momentum is not synchronous.
  - The particle phase is unbounded.



Independent Buckets Approximation

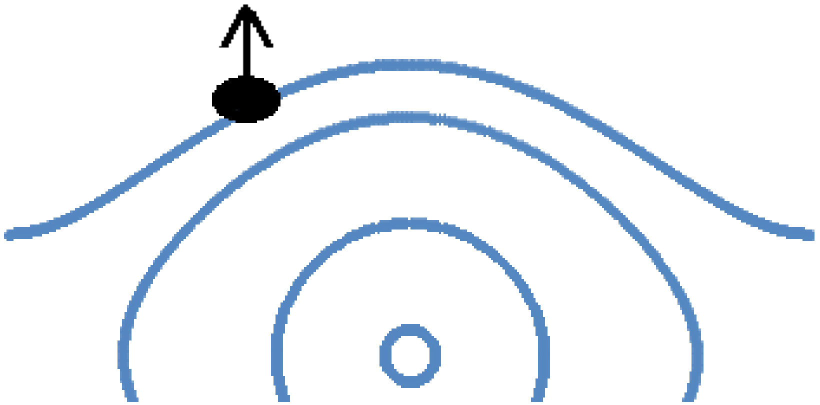
- In this approximation:
  - The string of separatrices slip past each other without affecting the other.
  - Each particle only sees the nearest RF cavity.
- The justification is that the force from the other RF cavity averages out.

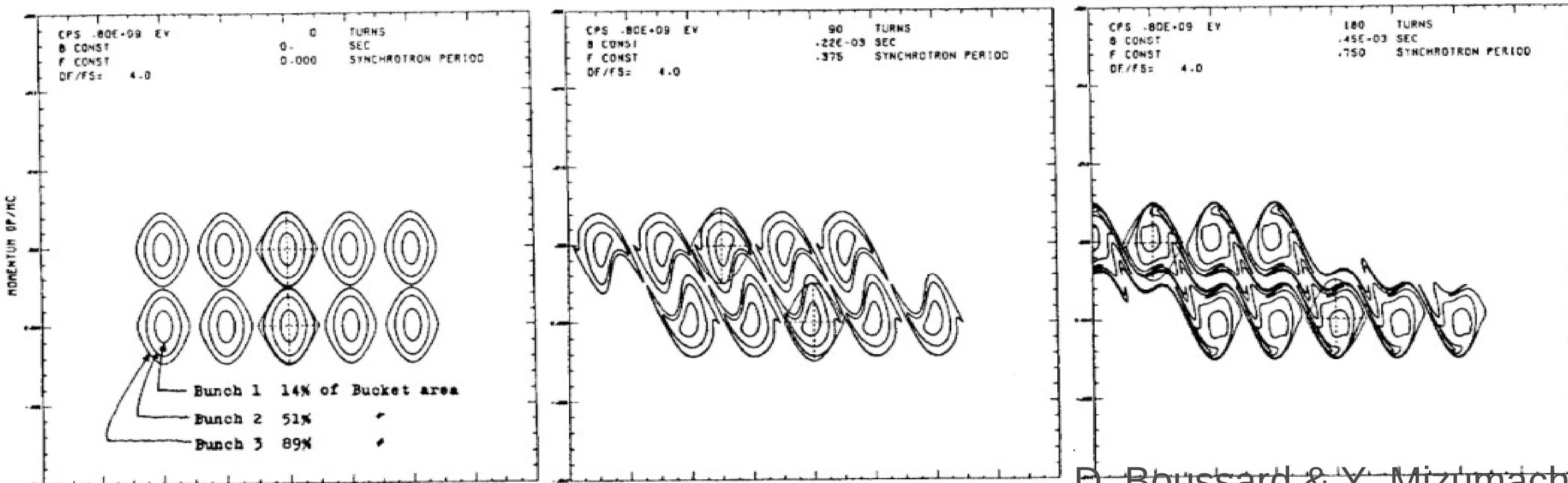




Bucket Oscillation & Deformation

- Each bucket oscillates as it slips pasts the other.
- Each bucket's shape has a time dependent deformation.

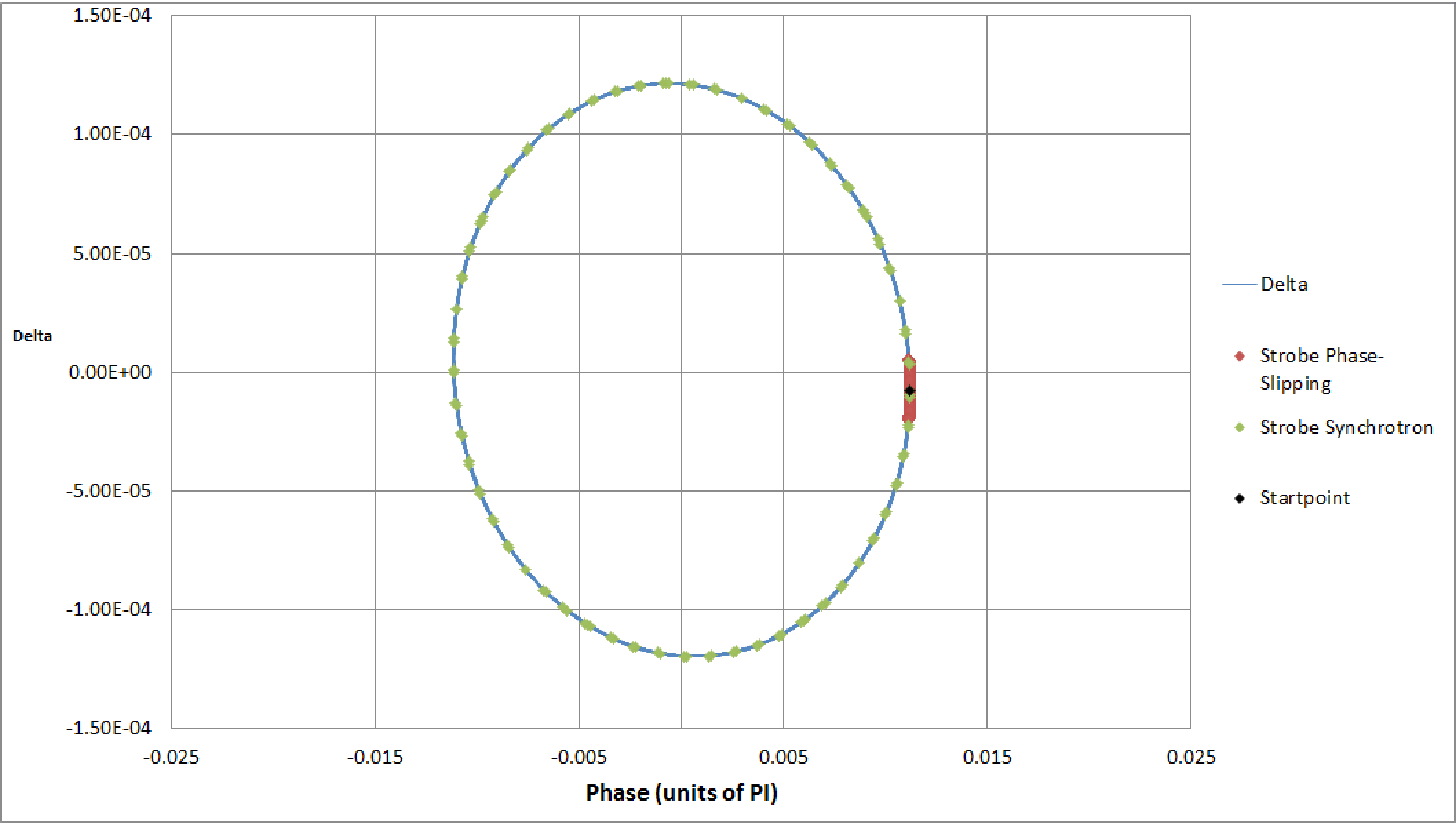




D. Boussard & Y. Mizumachi

Trajectories in a Slip-stacking Bucket

Phase-Space path (quasi-synchronous)



Slip-stacking Parameter

- The slip-stacking parameter is defined by:

$$\alpha_s = \frac{\omega_p}{\omega_s}$$

Phase-slipping frequency

Synchrotron frequency

- All non-trivial dynamics of slip-stacking are contained in this parameter.
- Two slip-stacking system with a different combination of parameters (η,E,h,ω_{rev},V, etc.) but the same slip-stacking parameter $\alpha_{s,}$ are related by a rescaling.
  - – an isomorphism

The bucket center oscillation

- Start with the slip-stacking single-particle mapping, from within the top RF bucket:

$$\dot{\phi} = 2\pi f_{rev} h\eta\delta, \;\; \dot{\delta} = f_{rev} V_\delta [\sin(\phi) + \sin(\phi - \omega_p t)]$$

- The Hamiltonian is explicitly time-dependent:

$$H(t) = \pi f_{rev} h\eta\delta^2 f_{rev} + \cos\phi\,[1 + \cos(\omega_p t)] \left[1 + \tan\phi \tan\left(\frac{\omega_p t}{2}\right)\right]$$

- Expressing this as a single 2nd order diff. eq.:

$$\ddot{\phi} = -\omega_s^2 [\sin(\phi) + \sin(\phi)\cos(\omega_p t) - \cos(\phi)\sin(\omega_p t)]$$

- Expression for small oscillations:

$$\ddot{\phi} = -\omega_s^2 [\phi + \phi\cos(\omega_p t) + \sin(\omega_p t)]$$

- And obtain perturbative solutions for the motion:

$$\begin{aligned}\phi = & A_1 \sin(\omega_p t) + A_2 \sin(2\omega_p t) \\ & + \rho \sin[(1+\sigma)\omega_s t + \psi] \\ & + B_{1,1} \sin[(1+\sigma)\omega_s t + \omega_p t + \psi] \\ & + B_{1,-1} \sin[(1+\sigma)\omega_s t - \omega_p t + \psi]\end{aligned}$$

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$$\phi = \boldsymbol{A_1 \sin(\omega_p t) + A_2 \sin(2\omega_p t)}$$

  - Motion of the bucket center
  - Not dependent on initial position
  - Multiples of ω_p

The bucket center oscillation

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- Synchrotron motion
- ρ and ψ from initial position
- Multiples of ω_s

The bucket center oscillation

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  - Interaction motion
  - Depends on initial position
  - Linear combination of ω_p and ω_s

The bucket center oscillation

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- Synchrotron frequency shift induced by slip-stacking.

$$\sigma \approx \frac{3}{4}\left(\frac{\omega_s}{\omega_p}\right)^4 = \frac{3}{4}\alpha_s^{-4}$$

Slip-stacking Bucket Area

Simulation

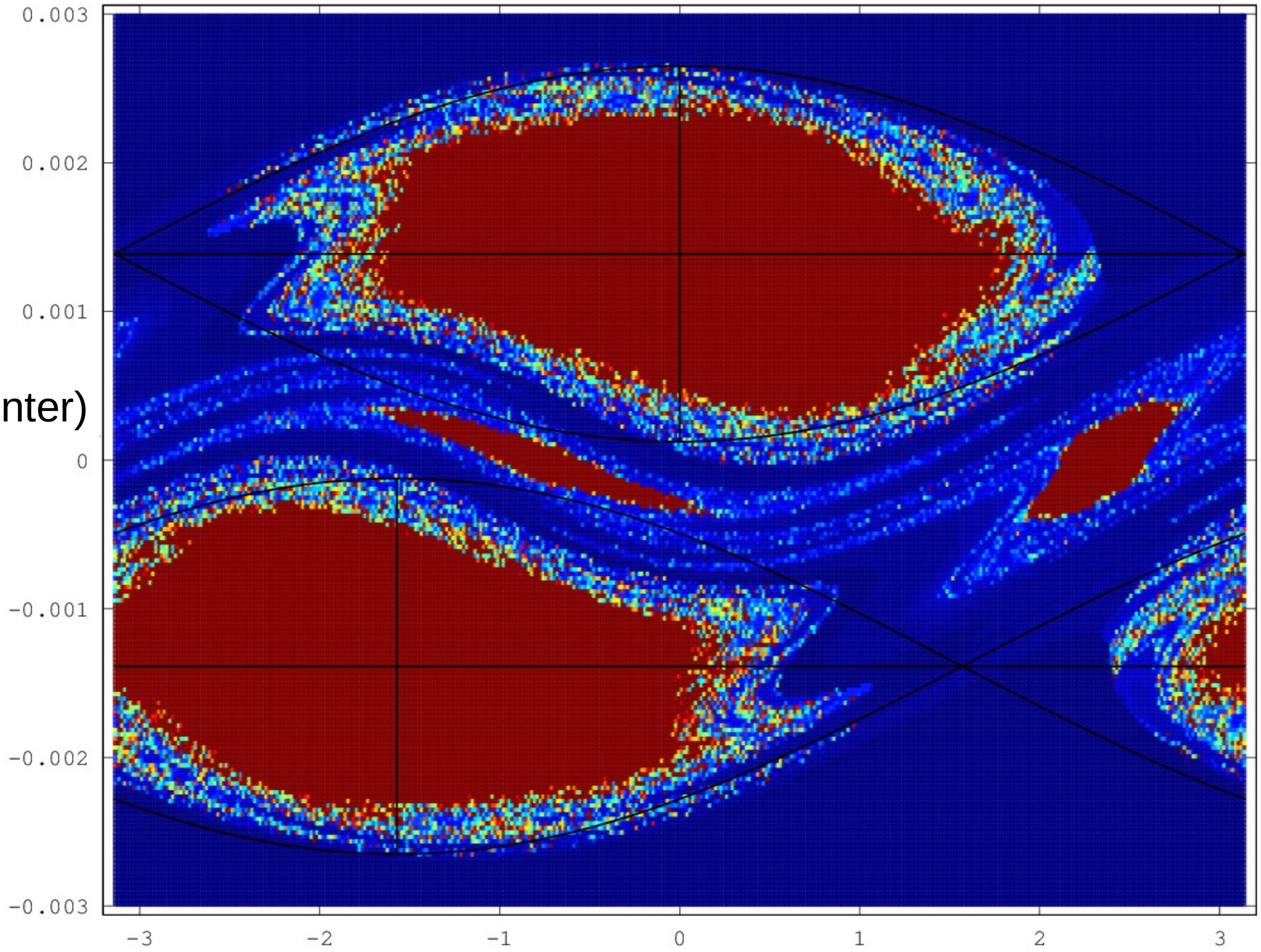
- Exact turn-by-turn mapping computed using Microsoft Excel or Matlab.

$$\Delta\delta = V[\sin(\phi) + \sin(\phi + \phi_D + \omega_p t)] \qquad \Delta\phi = 2\pi h \eta \delta$$

- Every particle trajectory handled independently.
  - No beamloading, space-charge, or intrabeam effects.

What do the buckets look like?

Stability of Initial Positions (RF phase difference **π/2**)

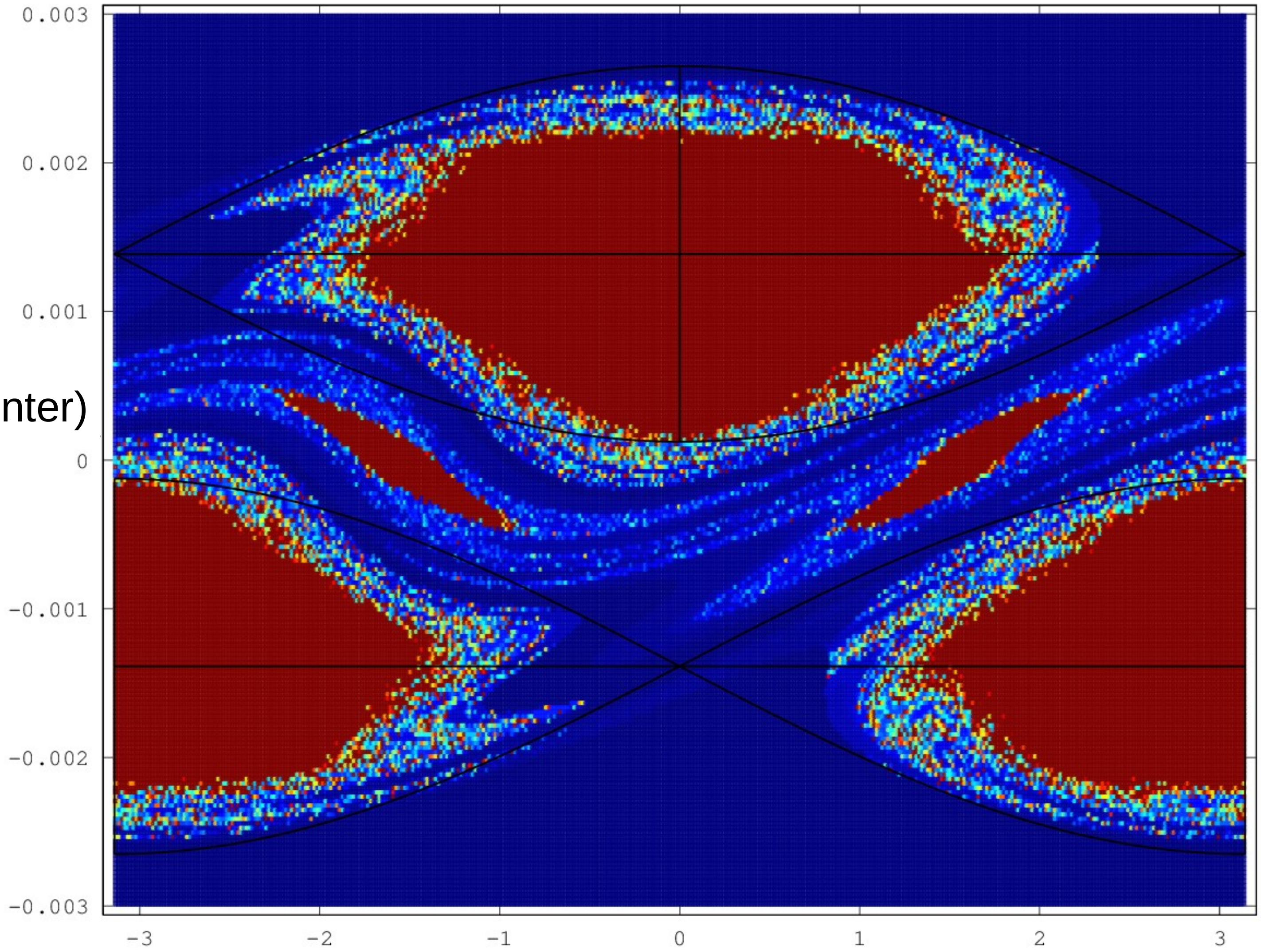


δ=Δp/p
(from center)

Initial phase (radians)

What do the buckets look like?

Stability of Initial Positions (RF phase difference **π**)

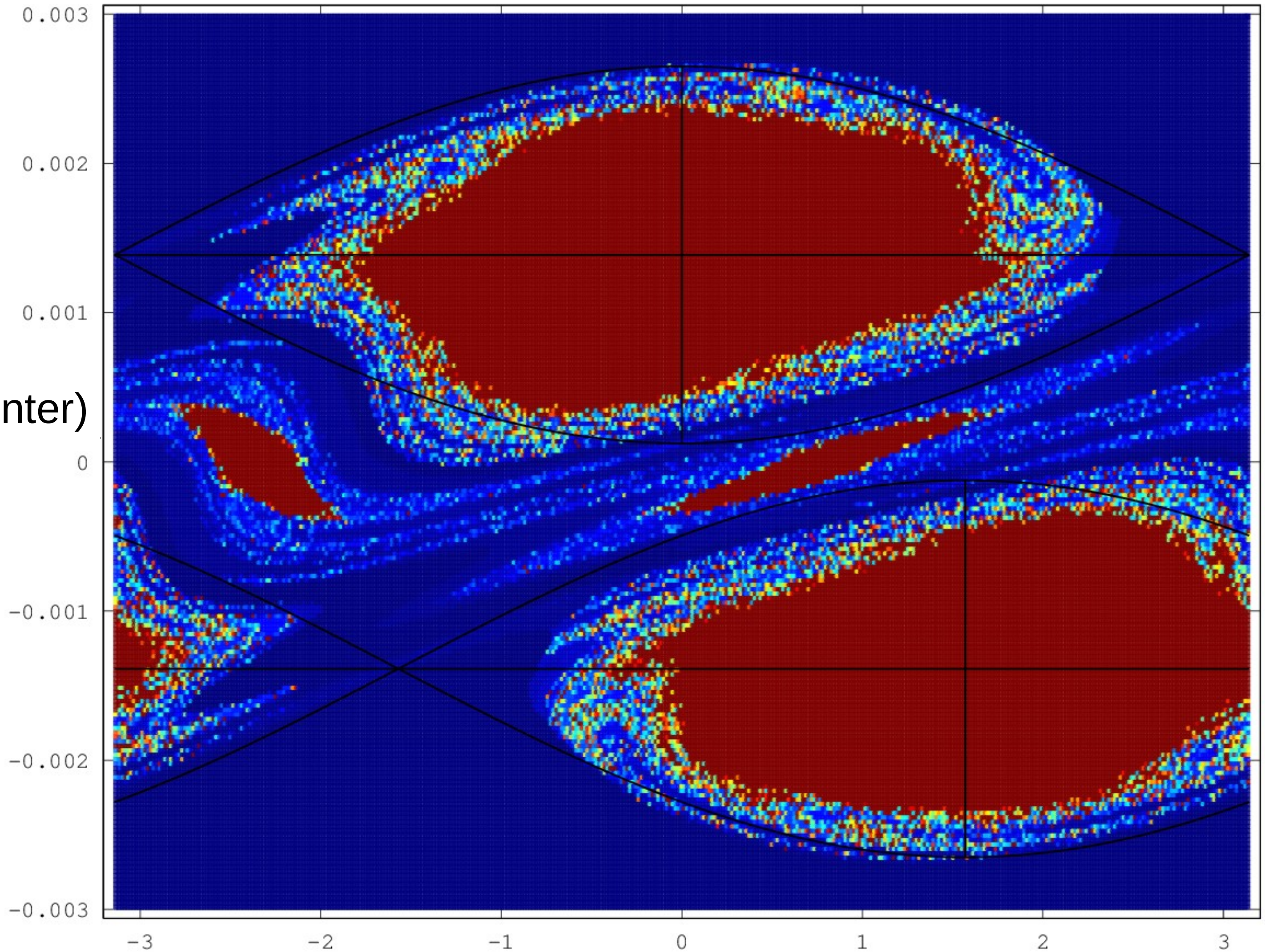


δ=Δp/p
(from center)

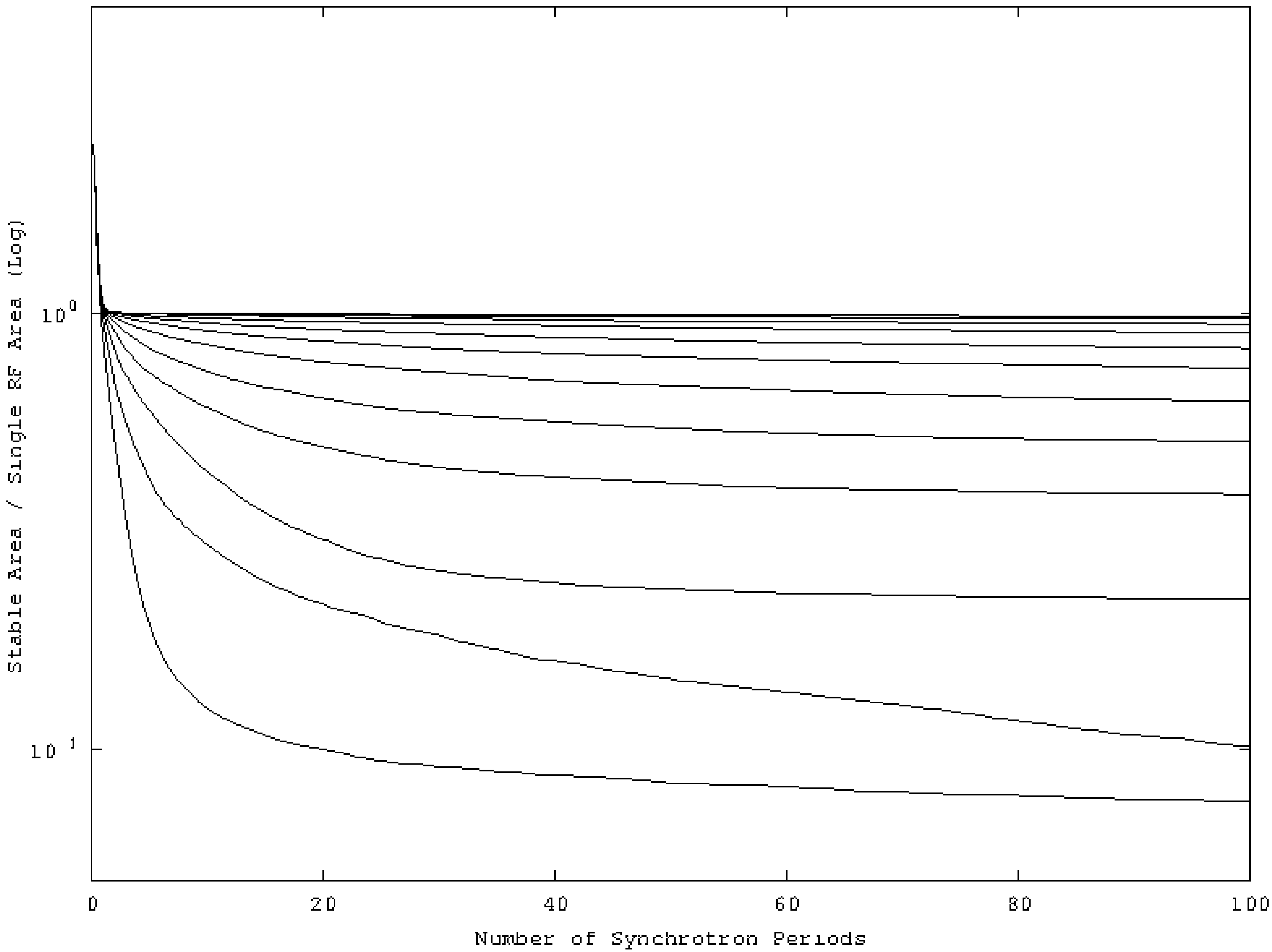
Initial phase (radians)

What do the buckets look like?

Stability of Initial Positions (RF phase difference **3π/2**)

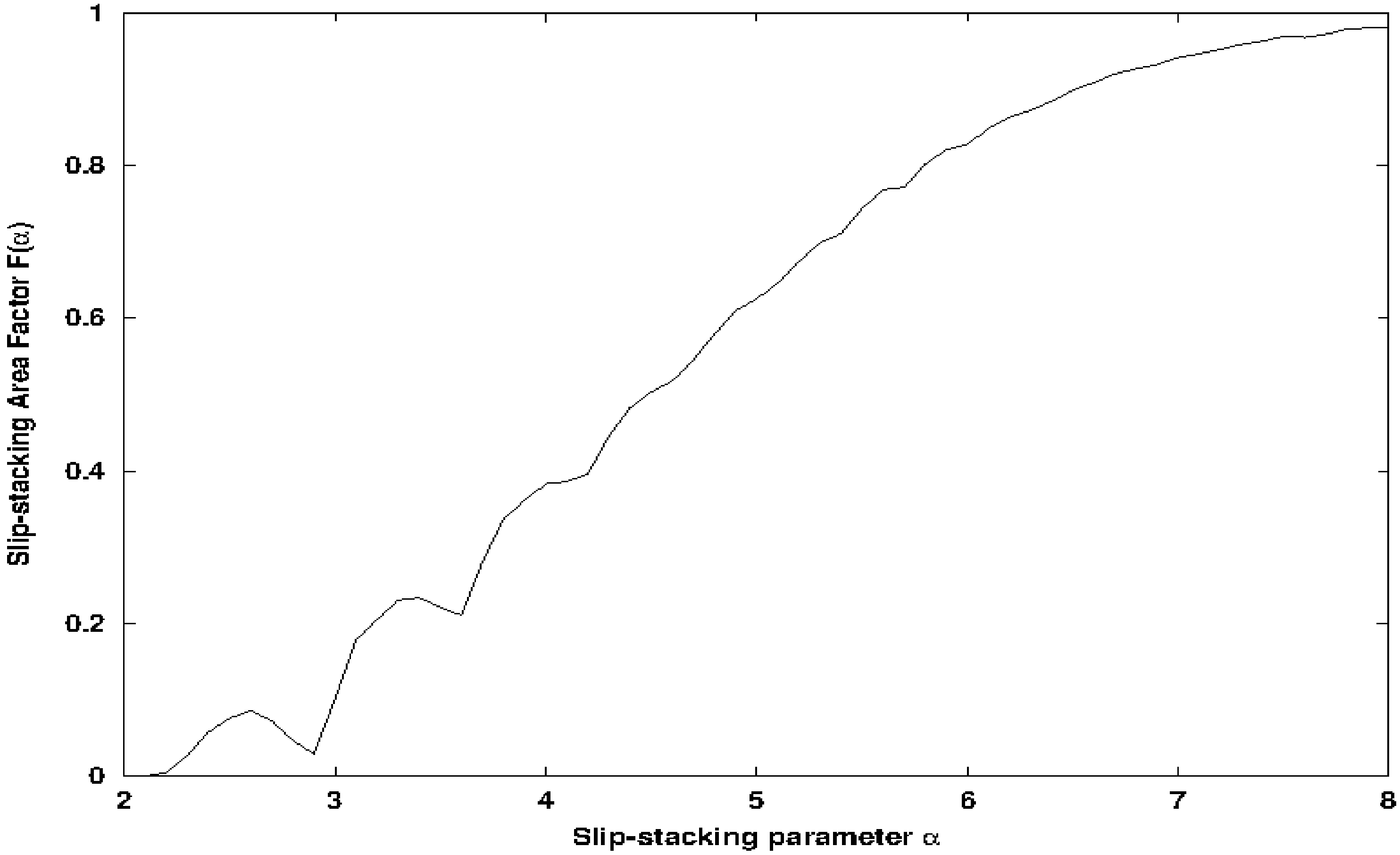


Metastable losses



Slip-stacking Area Factor

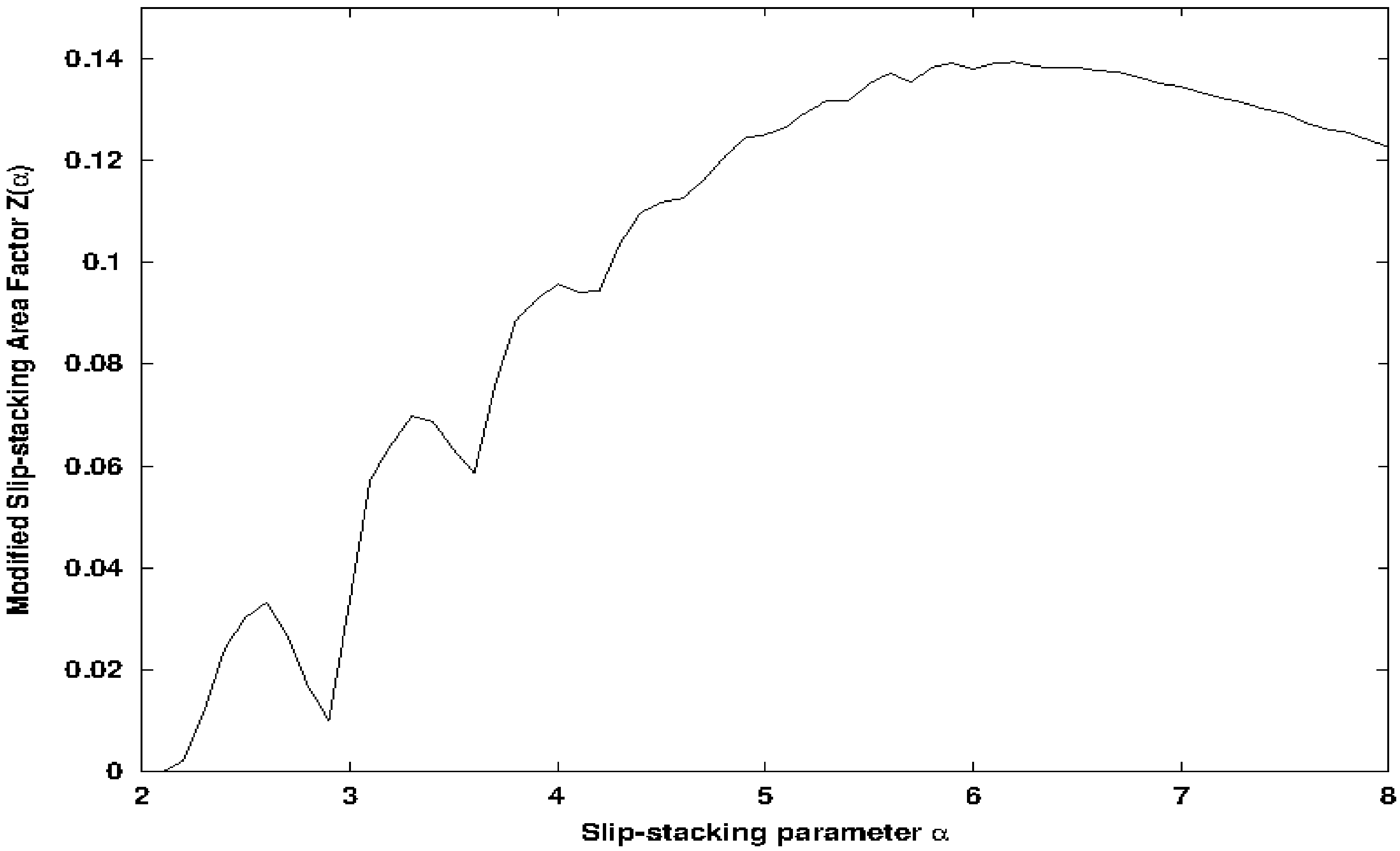
$$F(\alpha_s) = \mathcal{A}_s / \mathcal{A}_0 \qquad\qquad \mathcal{A}_s = \mathcal{A}_0 F(\alpha_s) = \frac{16}{h|\eta|} \frac{\omega_s}{\omega_{rev}} F(\alpha_s)$$



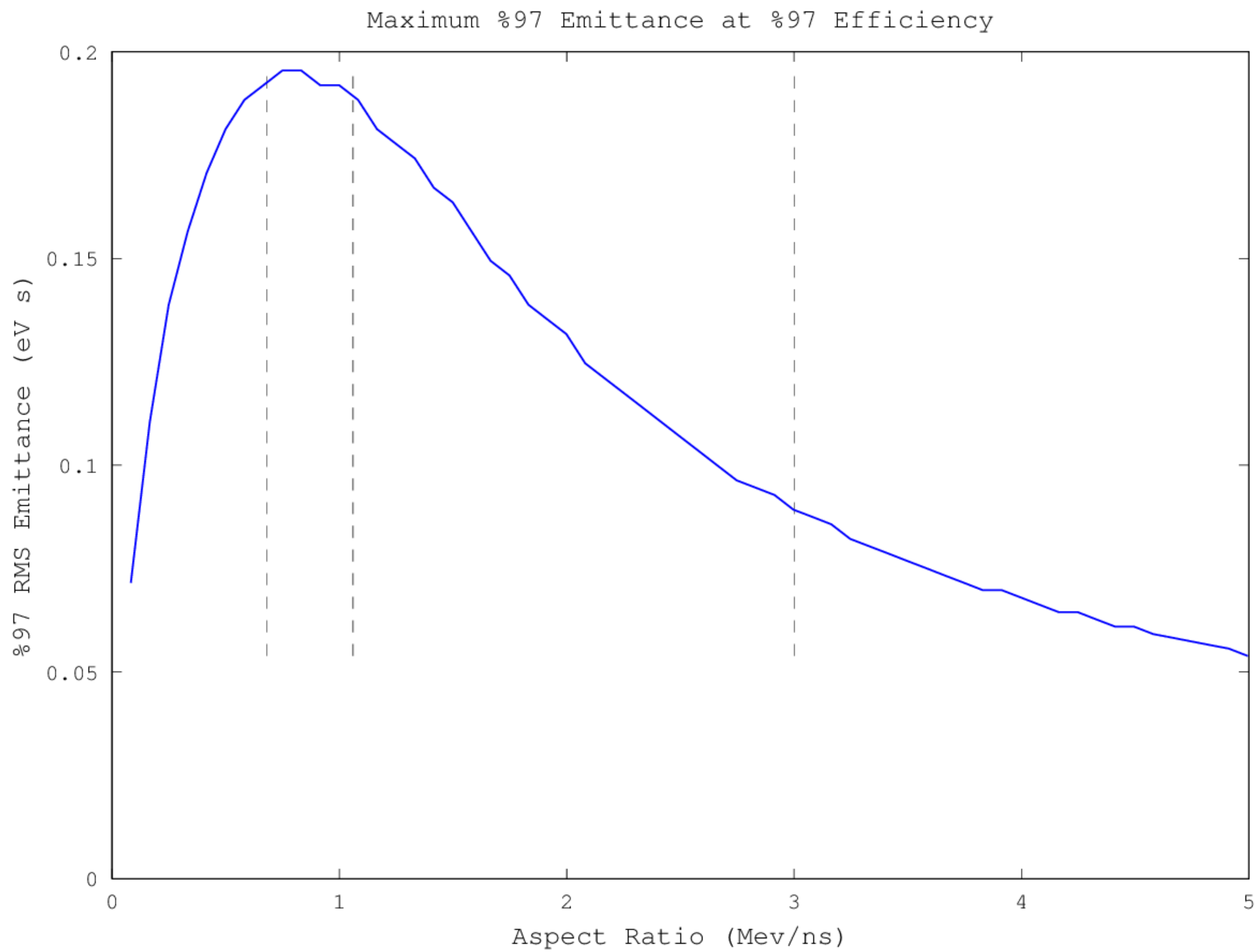
Modified Area Factor

$$Z(\alpha_s) = \frac{F(\alpha_s)}{\alpha_s}$$

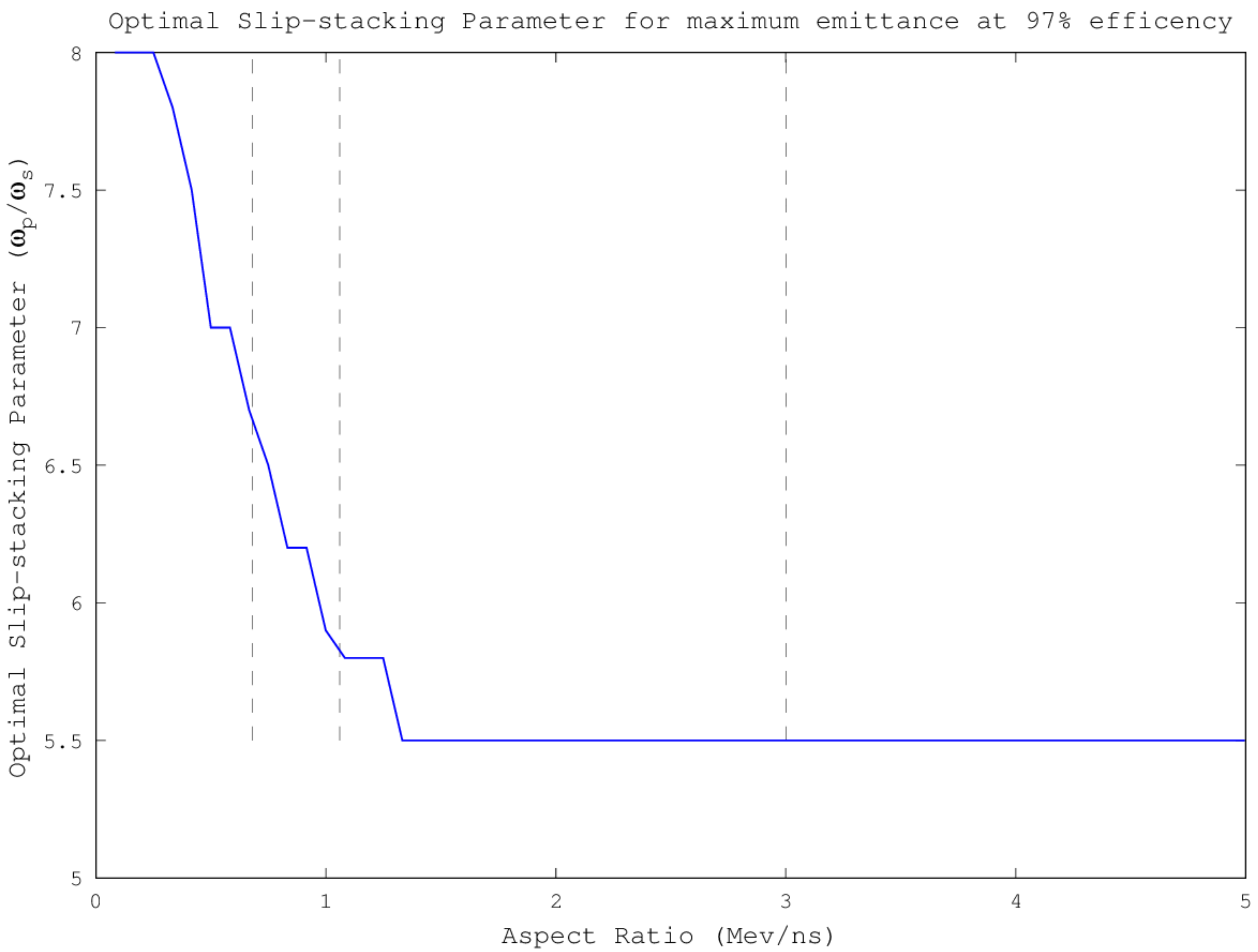
$$\mathcal{A}_s = \frac{16}{h|\eta|}\frac{\omega_p}{\omega_{rev}}\left(\frac{F(\alpha_s)}{\alpha_s}\right) = \frac{16}{h|\eta|}\frac{\omega_s}{\omega_{rev}}Z(\alpha_s)$$



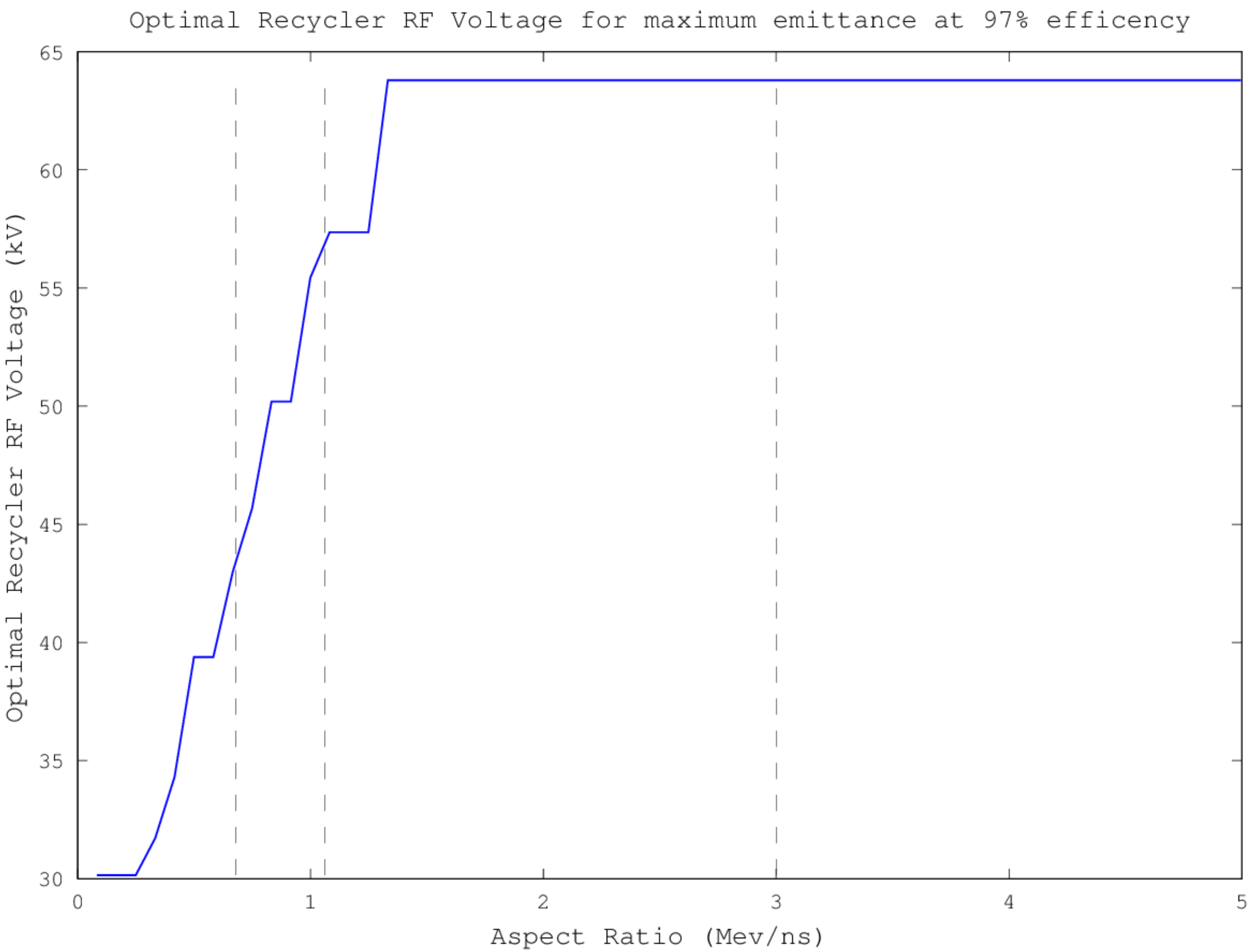
Emittance


Maximum %97 Emittance at %97 Efficiency
%97 RMS Emittance (eV s)
Aspect Ratio (Mev/ns)
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5

Slip-stacking Parameter

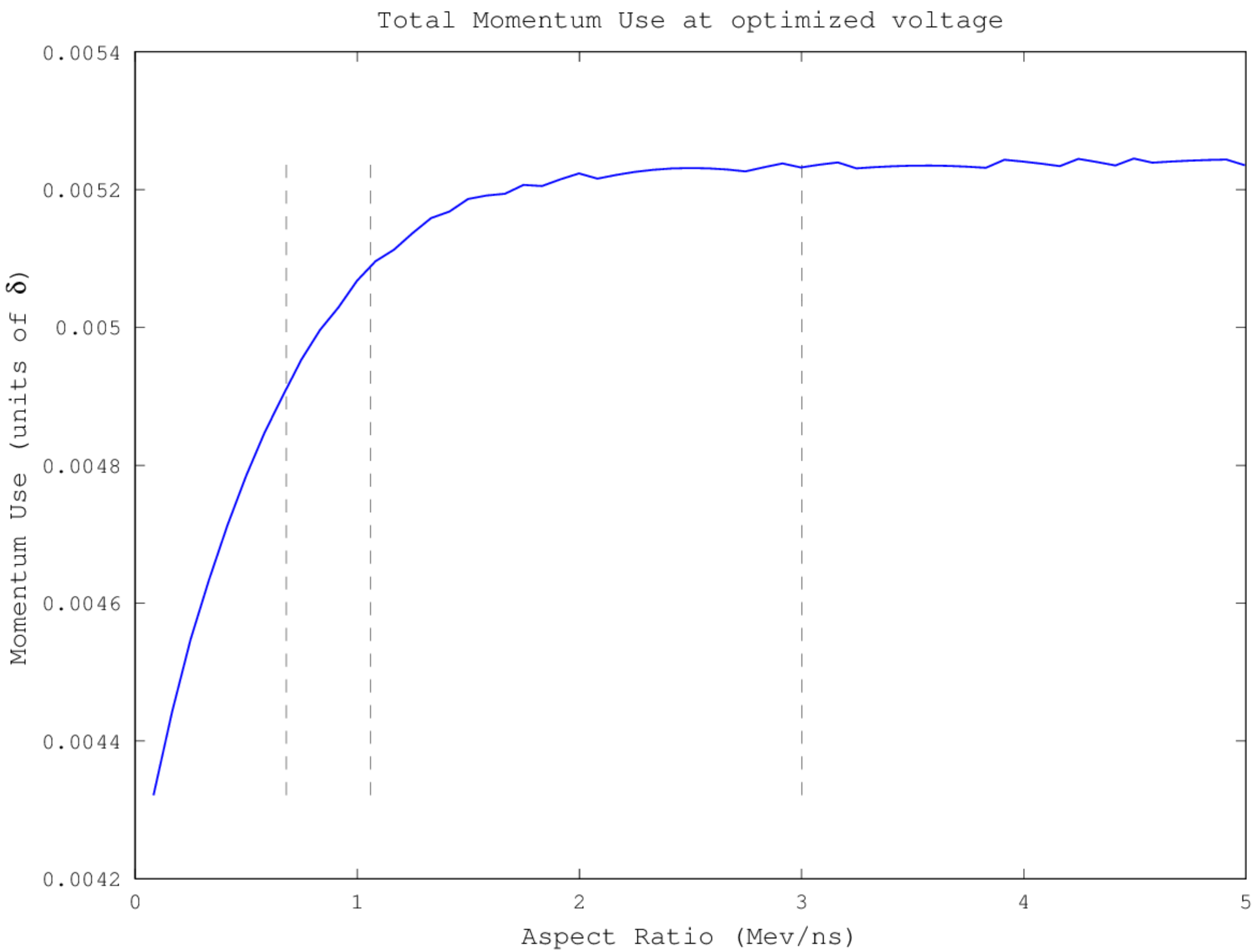


Voltage



Optimal Recycler RF Voltage for maximum emittance at 97% efficency
Optimal Recycler RF Voltage (kV)
65
60
55
50
45
40
35
30
0
1
2
3
4
5
Aspect Ratio (Mev/ns)

Momentum Usage



Connection to other subfields of physics

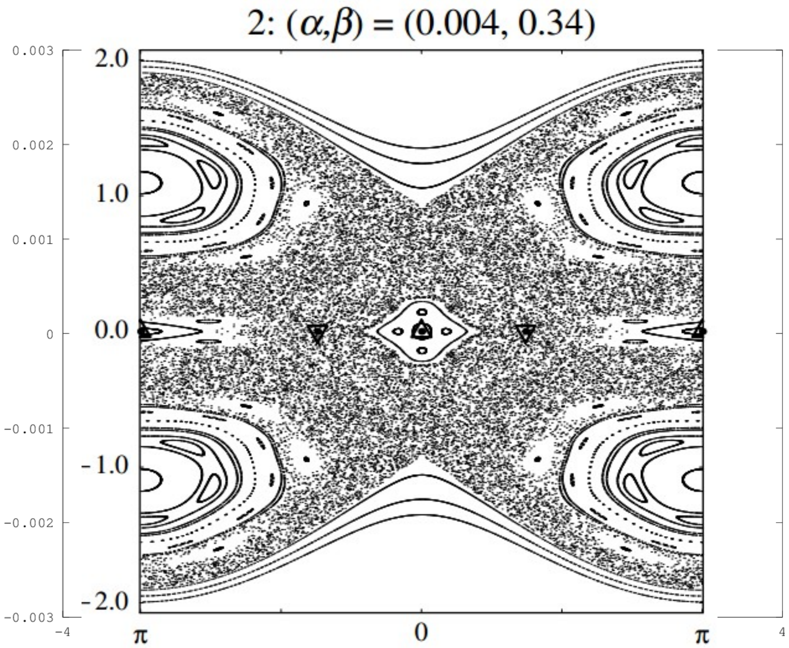
Physical Analogy to Driven Pendulum:

- The slip-stacking Hamiltonian is equivalent to a sinusoidally driven pendulum, confined to a 2D plane, in the absence of gravity & friction:

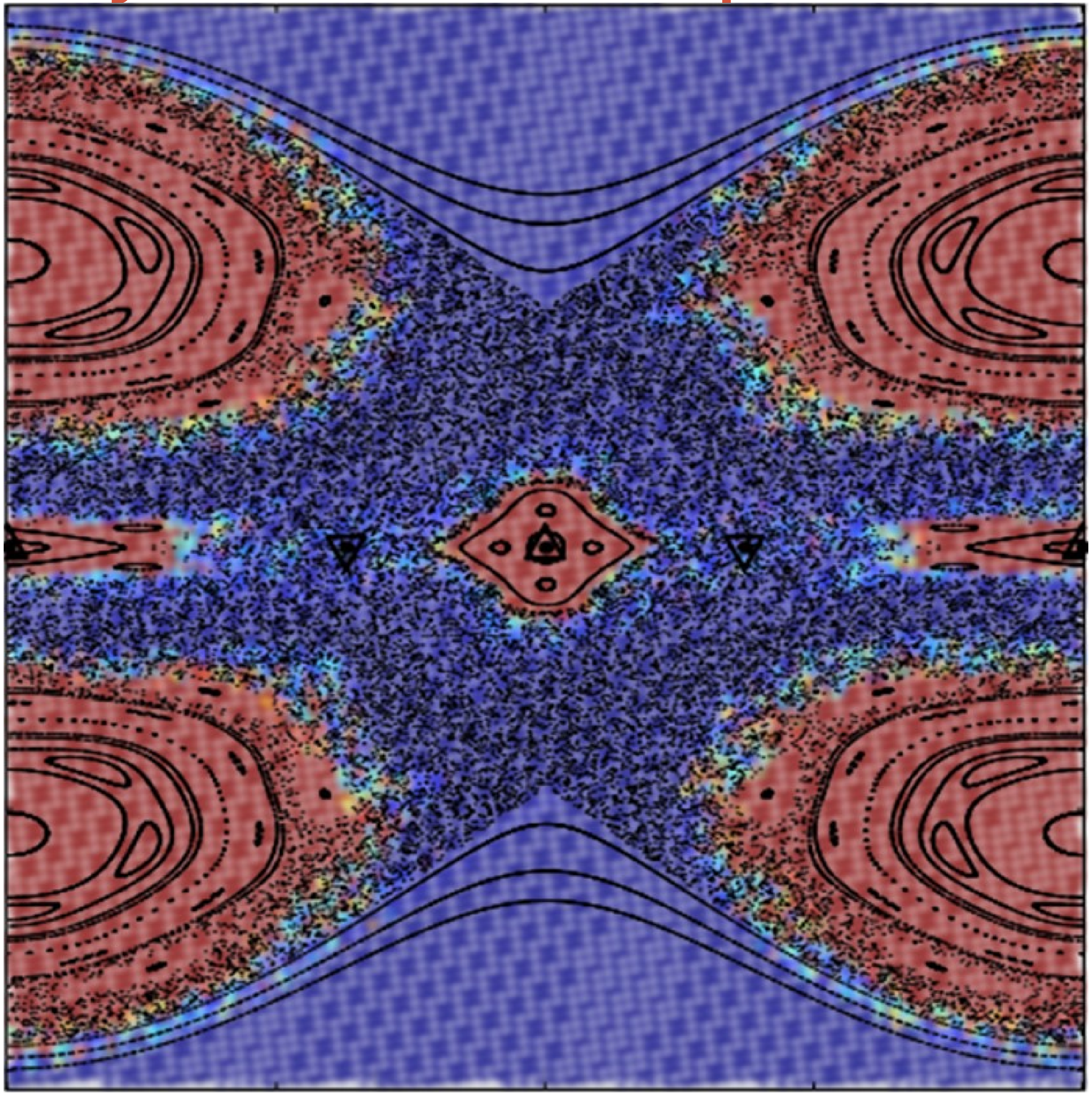
$$H = \frac{1}{2}\omega_{rev} h\eta\delta^2 + \frac{\omega_{rev} V}{2\pi}\cos(\phi_A)\cos(\phi_D - \frac{\omega_p}{2}t) \longrightarrow H = \frac{1}{2}I\dot{\theta}^2 + A\cos(\Omega t)\cos\theta$$

- "1½" Degree of Freedom – momentum, phase, time

H.W. Broer,
I. Hoveijn,
M. van Noort,
C. Simo, and
G. Vegter
(2004)



Stability & Stroboscope Overlay



Physical Analogy to Driven Pendulum:

- The slip-stacking Hamiltonian is equivalent to a sinusoidally driven pendulum, confined to a 2D plane, in the absence of gravity & friction:

$$H = \frac{1}{2}\omega_{rev} h \eta \delta^2 + \frac{\omega_{rev} V}{2\pi} \cos(\phi_A) \cos(\phi_D - \frac{\omega_p}{2} t) \longrightarrow H = \frac{1}{2} I \dot{\theta}^2 + A\cos(\Omega t)\cos\theta$$

- "1½" Degree of Freedom – momentum, phase, time

- They usually include a gravity term to their driven pendulum, which would represent a 3rd cavity
- Their "rotating solutions" are our buckets.

Application to Standing Wave Traps

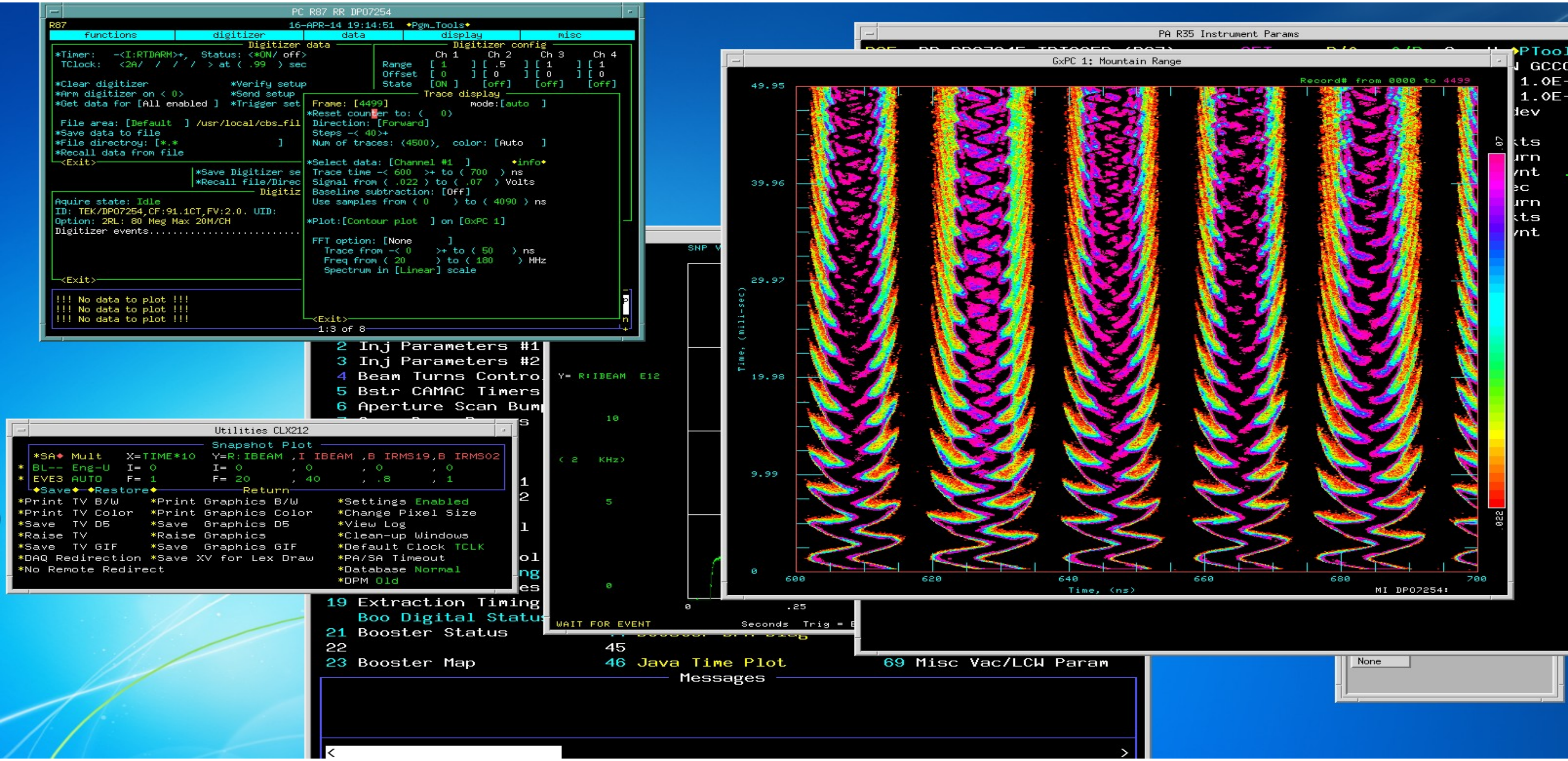
- Two counterpropagating waves make a standing wave pattern.
- Particles attracted to the nodes or anti-nodes of the standing waves are trapped in a sinesoidal potential.
- Optical lattices used in AMO physics.
- Acoustic levitation techniques for fabrication.
- A standing wave traps can move: $v = \lambda \Delta f$
- Two standing wave traps moving with respect to each other make a slip-stacking potential:
  - Trap-Accumulation
  - Controlled Collisions

$$\alpha_s = \lambda \Delta f \sqrt{\frac{M}{2V_H}} = v\sqrt{\frac{M}{2V_H}}$$

Open Areas of Research

Experimental Verification

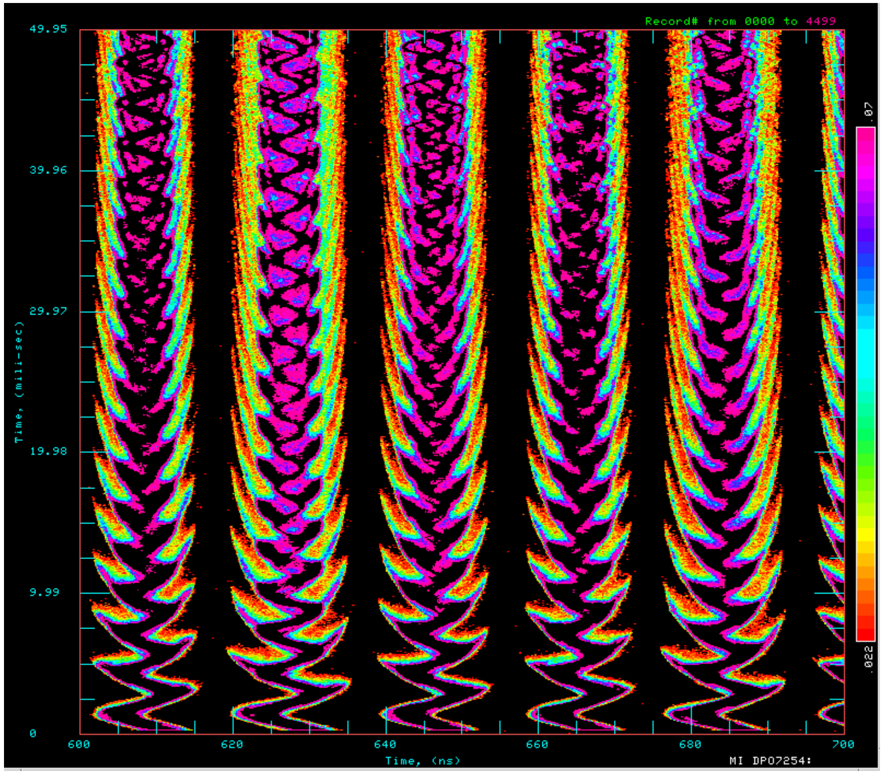
Ming-jen Yang and Phil Adamson helped me access and understand the Recycler Restive Wall Monitor oscilloscope.



- Measure the synchrotron oscillation during slip-stacking.

Experimental Verification & Support

- Observe matching and timing from Booster.
- Investigate whether losses for slip-stacking match those described in my analysis.
  - Stable phases.
  - Loss-rates.
- Propose improvements.



Continuing the research program

- We numerically mapped the stable area as a function of a single parameter – the slip-stacking parameter α_s.
- We could do the same sort of analysis including a second parameter that varies independently:
  - A running bucket.
  - A parameterization of direct space-charge.
  - A different voltage for each RF cavity.
  - A third RF cavity at the average frequency.
  - A harmonic cavity for average or each frequency.
  - A (synchrotron) cooling term.
- Make into jobs for a supercomputer to handle faster.
- Analytic work can handle more than two parameters.

Analysis of line distribution

- How does the slip-stacking process distort the distribution of particles in the bucket?
  - How does it look projected onto the time axis?
- What really happens in the chaotic region between buckets? Is there anyway to make sense of that?
- There are a couple of ways to explore this:
  - Direct simulation.
  - Analysis of experimental data.
  - Langrangian coherent structures.
- Applications:
  - Make better sense of experimental data.
  - Tomography program like TARDIS?
  - Useful rf manipulations?

Thank you for listening!

...Any Questions?

Backup Slides

Small Oscillation Coefficients

$$\ddot{\phi} = -\omega_s^2[\phi + \phi\cos(\omega_p t) + \sin(\omega_p t)]$$

$$\begin{aligned}\phi =& A_1\sin(\omega_p t) + A_2\sin(2\omega_p t)\\ &+ \rho\sin[(1+\sigma)\omega_s t + \psi]\\ &+ B_{1,1}\sin[(1+\sigma)\omega_s t + \omega_p t + \psi]\\ &+ B_{1,-1}\sin[(1+\sigma)\omega_s t - \omega_p t + \psi]\end{aligned}$$

$$\begin{aligned}A_1 &= -\frac{1}{\alpha_s^2 - 1}\\ A_2 &= \frac{1}{(2\alpha_s)^2 - 1}\left(\frac{A_1}{2}\right)\\ B_{1,\pm 1} &= \frac{\alpha_s^{-1}}{\alpha_s \pm 2}\left(\frac{\rho}{2}\right)\\ \sigma &= \frac{3}{4}\alpha_s^{-4}\end{aligned}$$

$$\begin{aligned}\Phi_0 &= \rho\sin(\psi)\\ \Delta_0 &= \rho\cos(\psi)\end{aligned}$$

$$\rho = \sqrt{\Phi_0^2 + \Delta_0^2}$$

$$\psi = \mathrm{sgn}(\Phi_0)\arccos\left(\frac{\Delta_0}{\rho}\right)$$

$$\alpha_s = \frac{\omega_p}{\omega_s}$$

$$\begin{aligned}\Phi_0 &= \frac{\alpha_s^2 - 4}{\alpha_s^2 - 3}\phi_0\\ \Delta_0 &= \frac{\alpha_s(\alpha_s^2 - 4)}{(\alpha_s^2 - 5) + \sigma\alpha_s^2}\left(\frac{2\pi f_{rev} h\eta}{\omega_p}\delta_0 - A_1 - 2A_2\right)\end{aligned}$$

Full Perturbative Solution

$$\ddot{\phi} = -\omega_s^2 \left\{ \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \phi^{2k+1} [1 + \cos(\omega_p t)] - \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} \phi^{2k} \sin(\omega_p t) \right\}$$

$$\phi = \sum_{n=1}^{\infty} A_n \sin(n\omega_p t) + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{m,n} \sin[m(1+\sigma)\omega_s t + n\omega_p t + m\psi] + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{m,-n} \sin[m(1+\sigma)\omega_s t - n\omega_p t + m\psi]$$

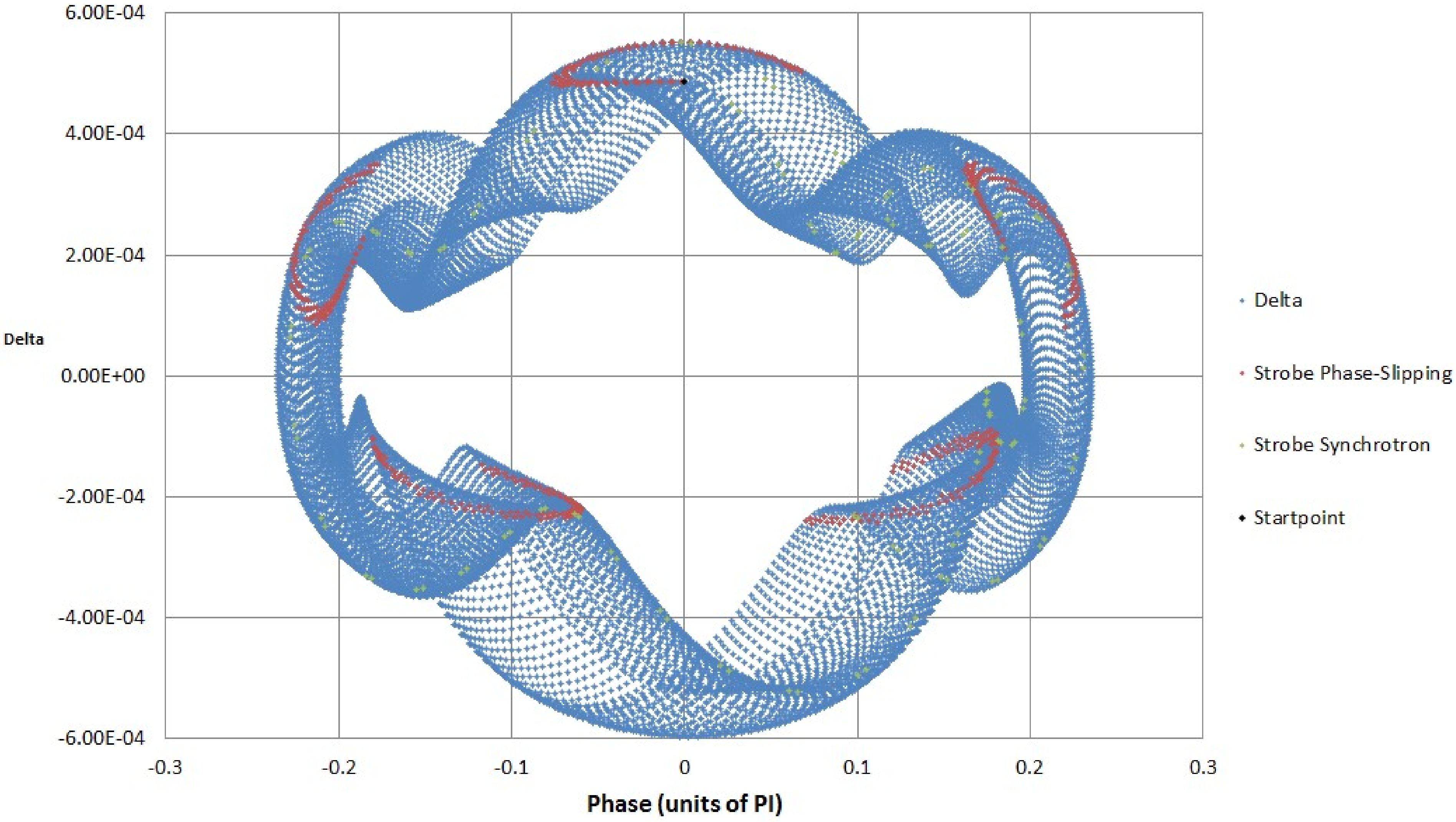
$$A_n \propto \alpha_s^{-2n}$$

$$B_{m,n} \propto \rho^m \alpha_s^{-2|n|}$$

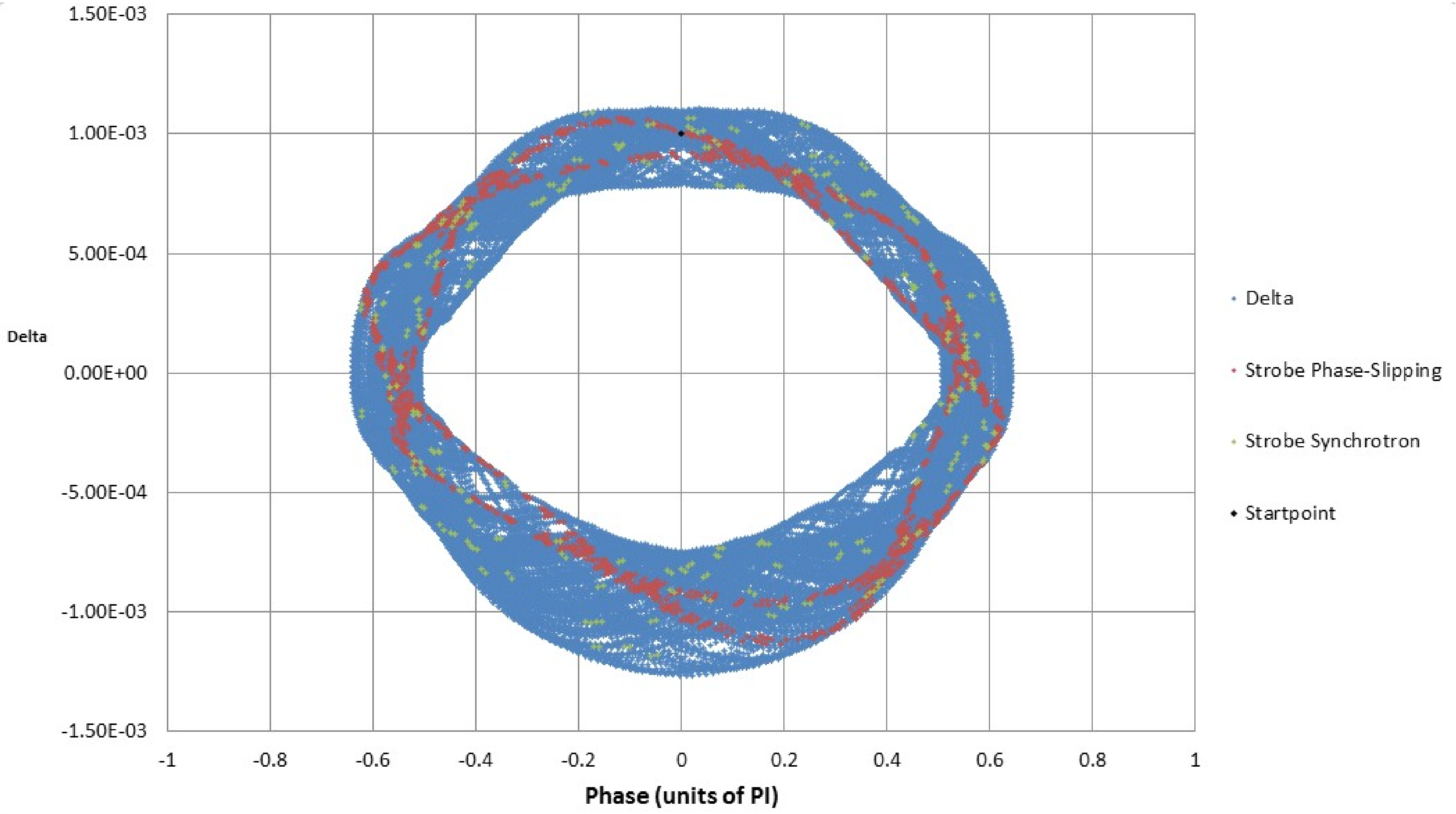
$$\text{except m even}, B_{m,0} \propto \rho^m \alpha_s^{-2}$$

$$\alpha_s = \frac{\omega_p}{\omega_s}$$

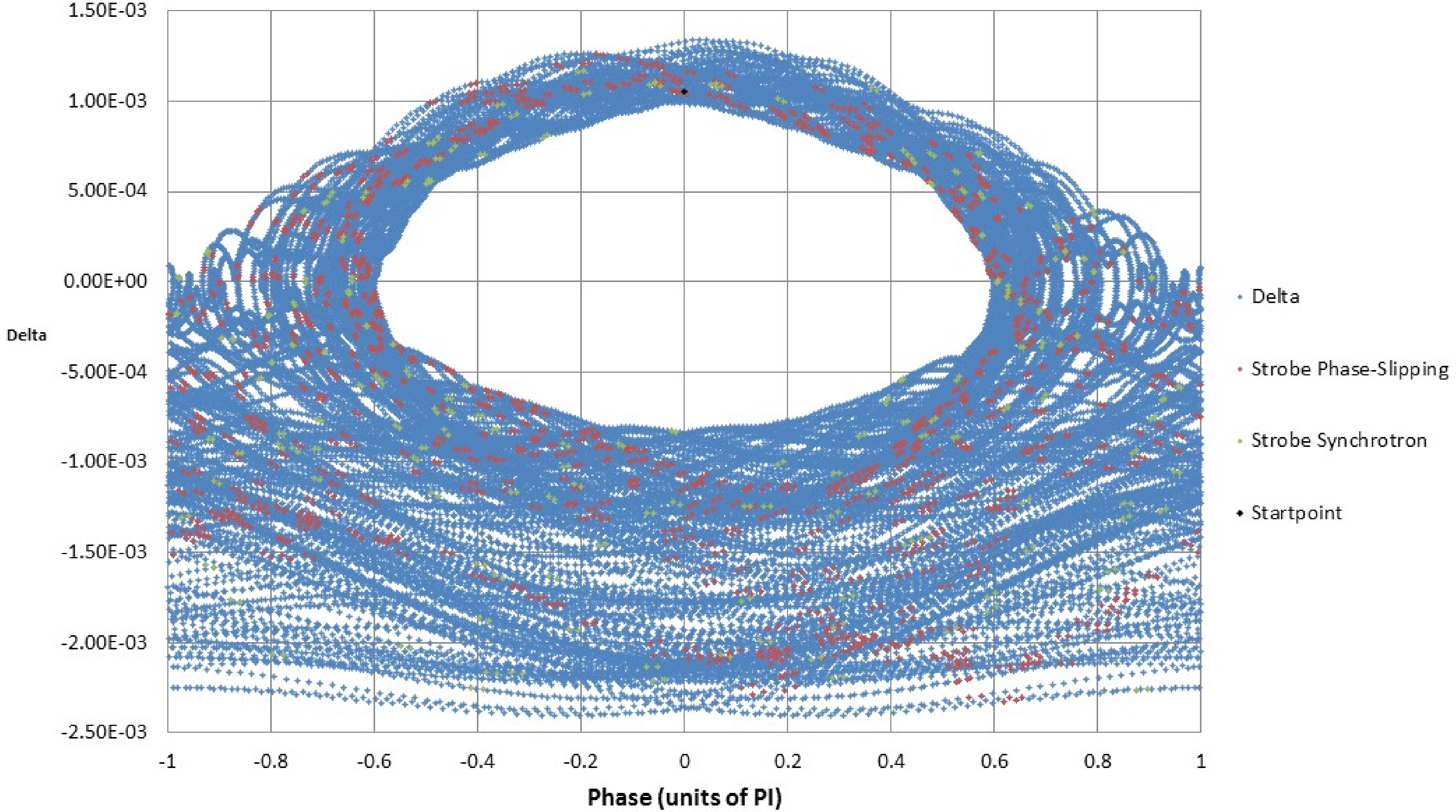
Fifth Harmonic Resonance



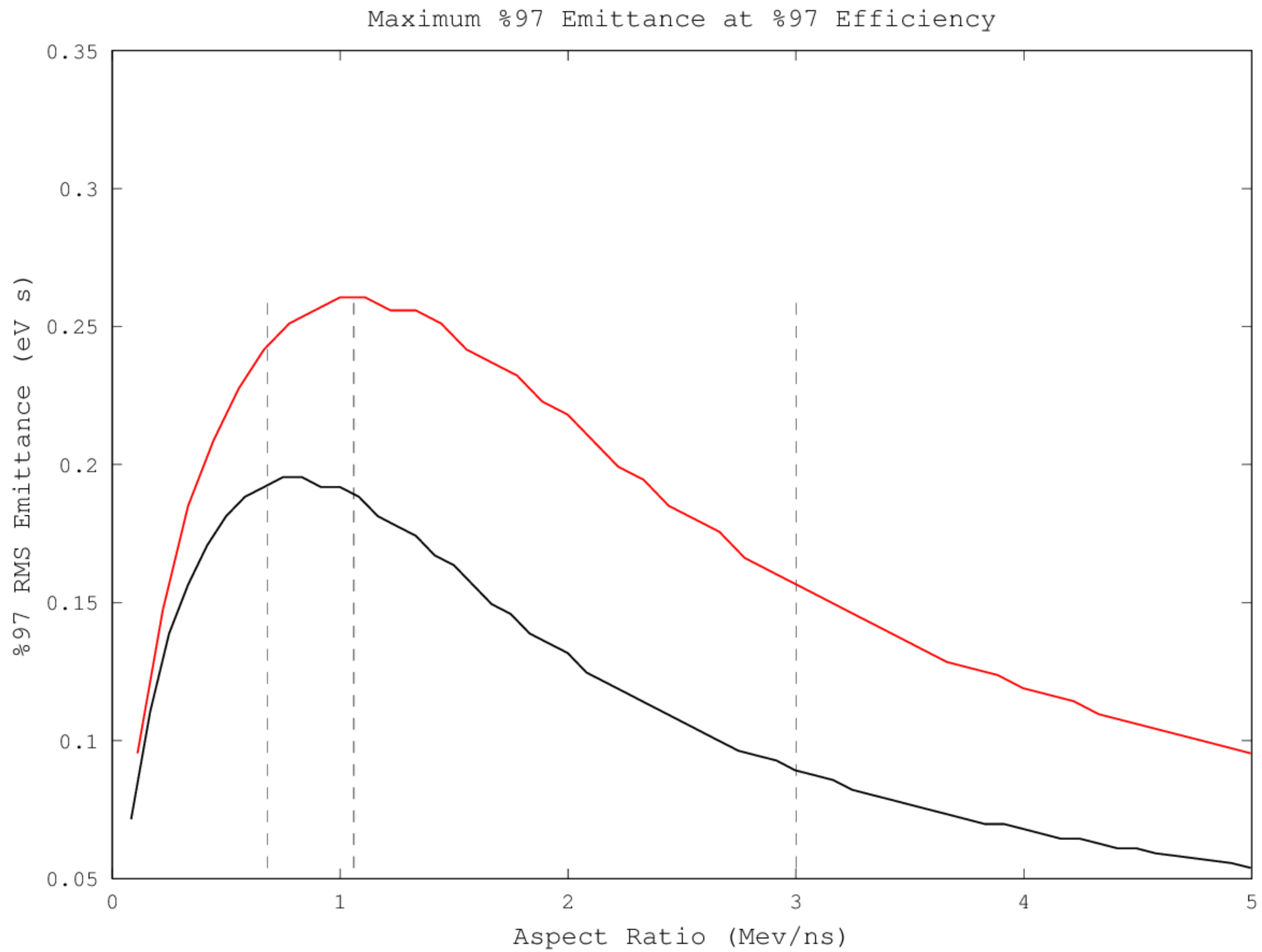
Sixth Harmonic Resonance



“Slipping” Phase-Space Trajectory

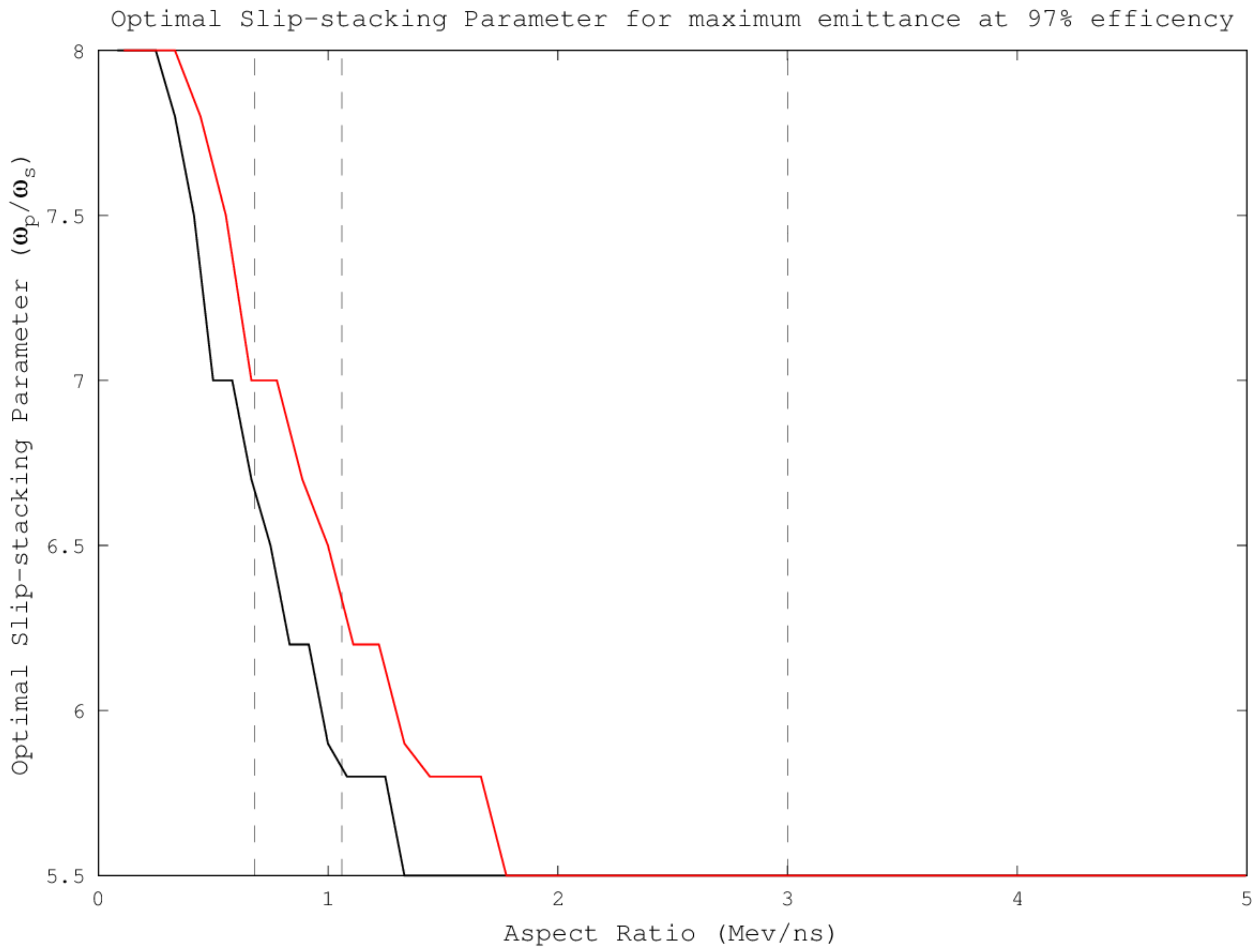


Emittance

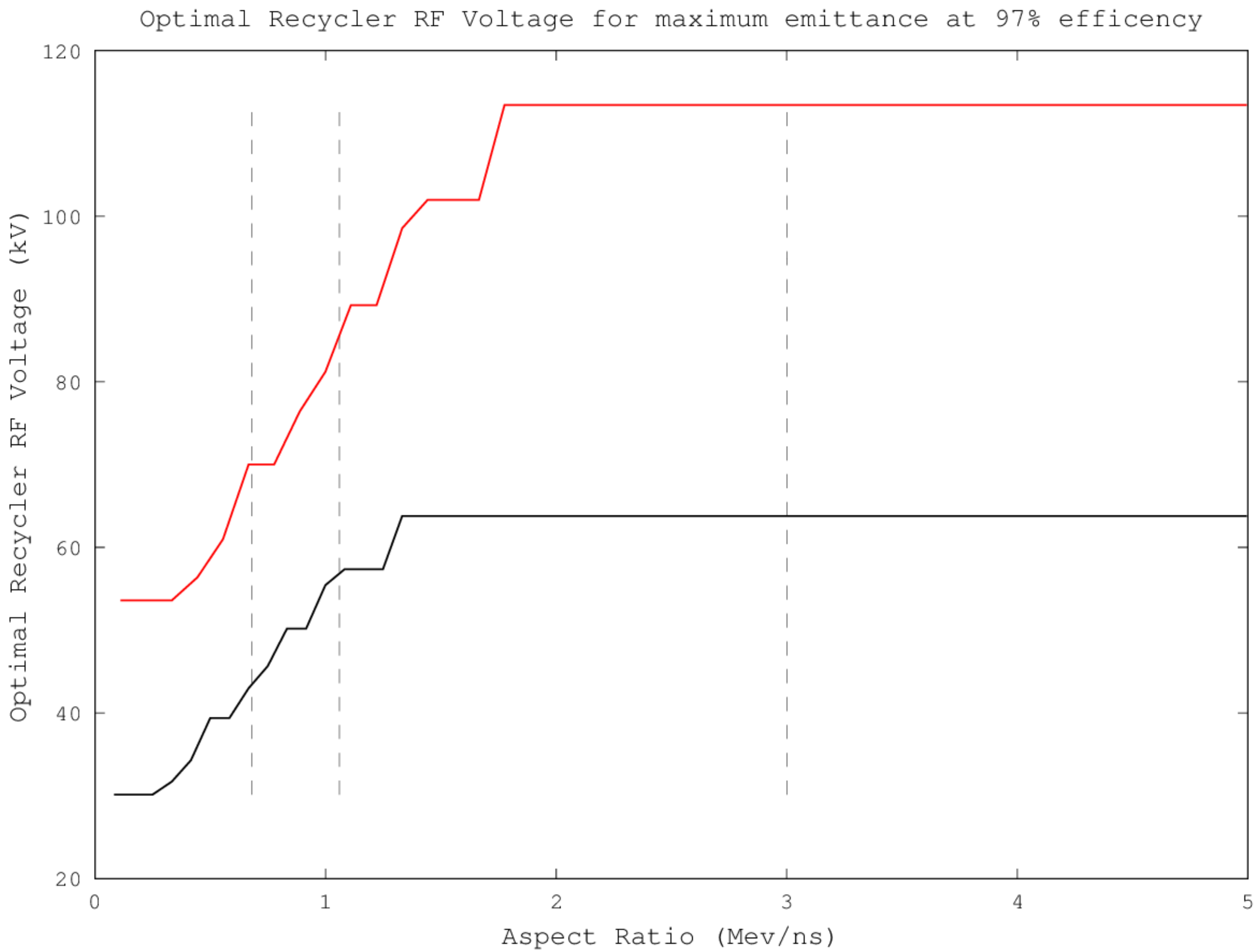


Maximum %97 Emittance at %97 Efficiency
0.35
0.3
0.25
0.2
0.15
0.1
0.05
%97 RMS Emittance (eV s)
0
1
2
3
4
5
Aspect Ratio (Mev/ns)

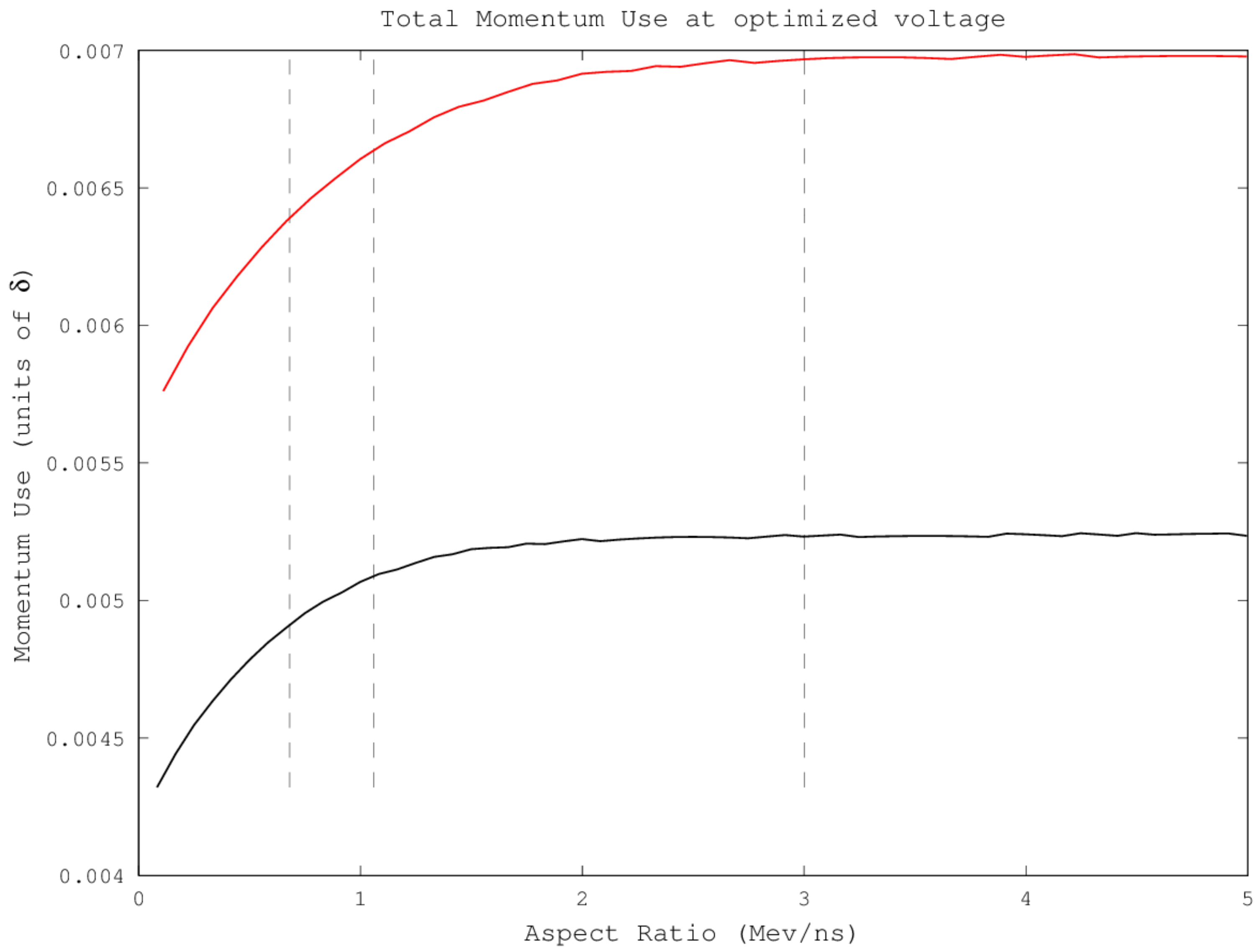
Slip-stacking Parameter



Voltage


Optimal Recycler RF Voltage for maximum emittance at 97% efficency
Optimal Recycler RF Voltage (kV)
Aspect Ratio (Mev/ns)
120
100
80
60
40
20
0
1
2
3
4
5

Momentum Usage

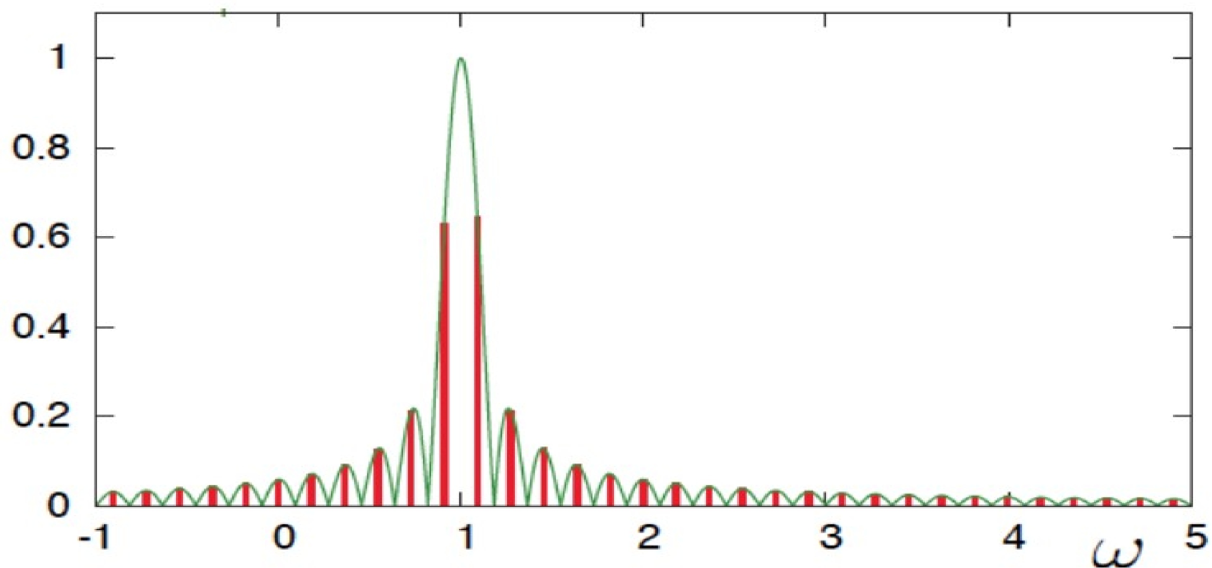


Principle Component Analysis (PCA)

- Input N data series of the same length.
- Finds the covariance of each data series with each other data series.
- Diagonalizes the covariance matrix.
- This takes linear combinations of the series to make principle components - features of the data series which vary together.
- Components and coefficients used to assemble components can then be analyzed.

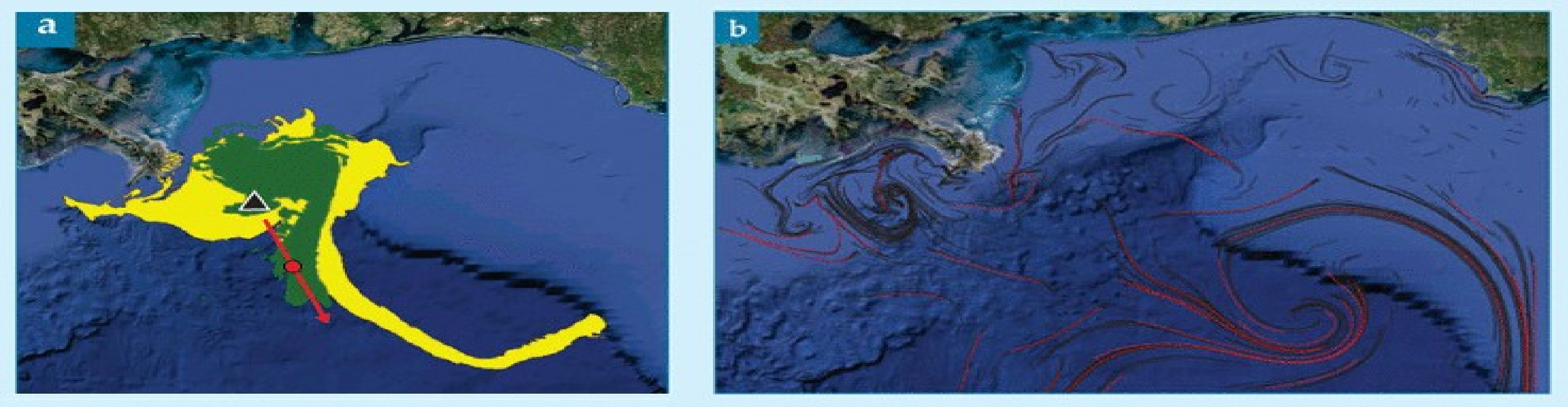
Numerical Analysis of Fundamental Frequencies (NAFF)

- Like a discrete Fourier transform, but continuous.
- Calculate inner product (covariance, convolution) of the data with a sine wave.
- Make a chart of the amplitude as a function of freq.
- Still limited in frequency resolution by the number of datapoints in the sample, but not as limited.

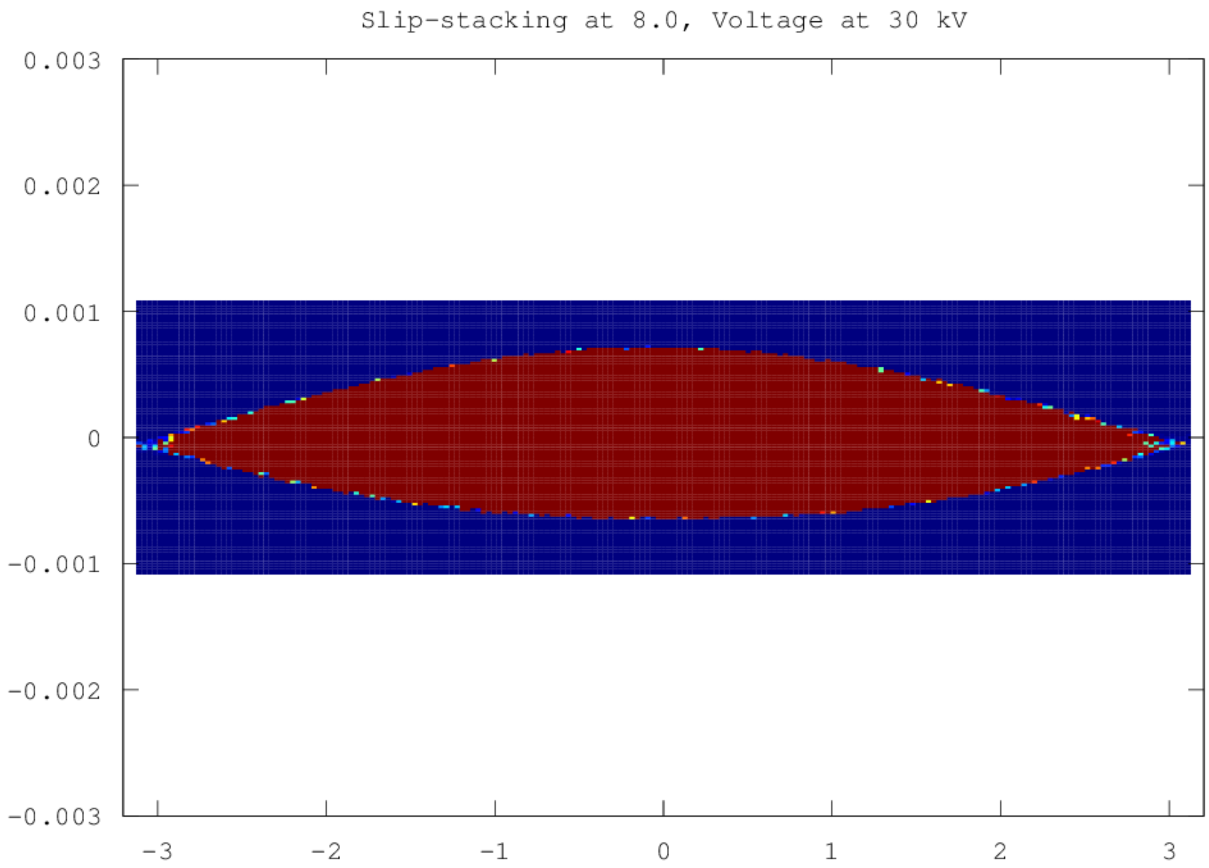


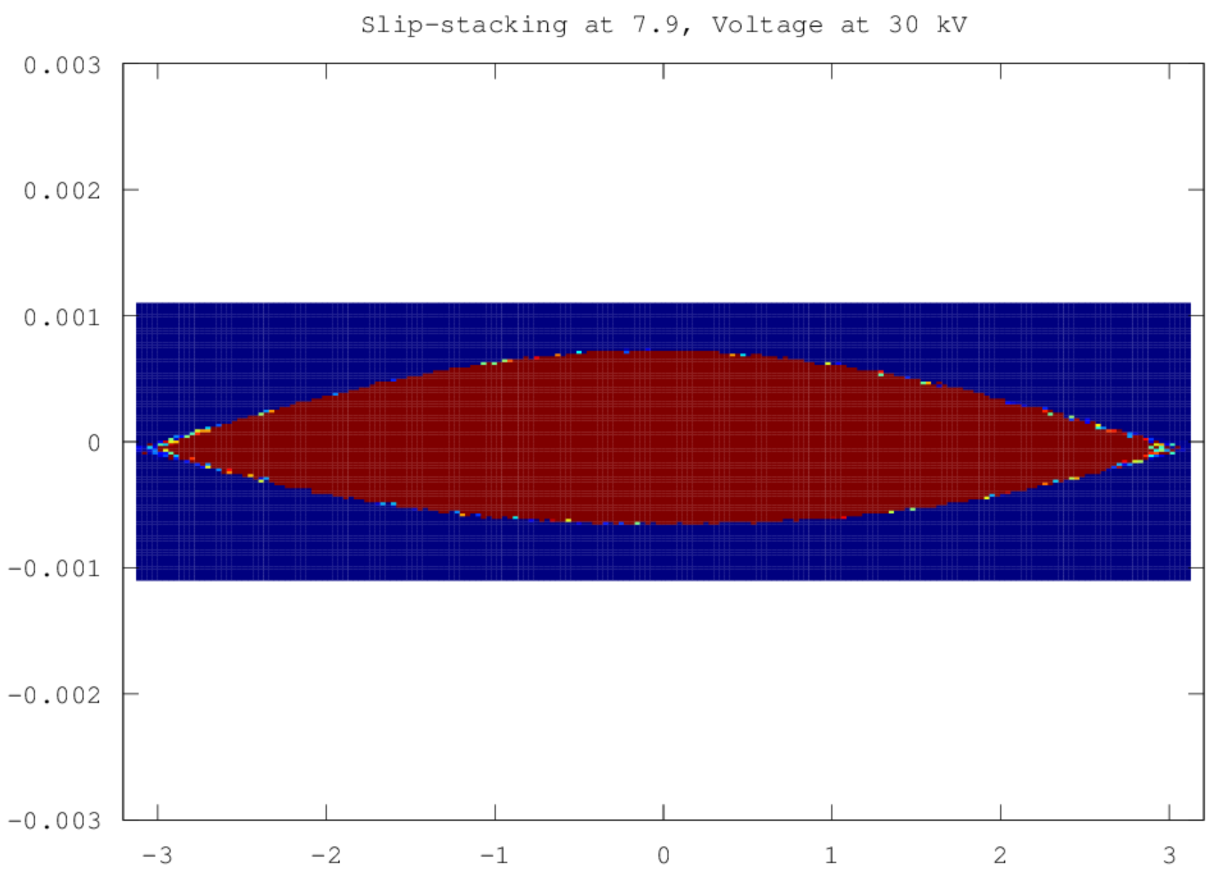
Langrangian Coherent Structures (LCS)

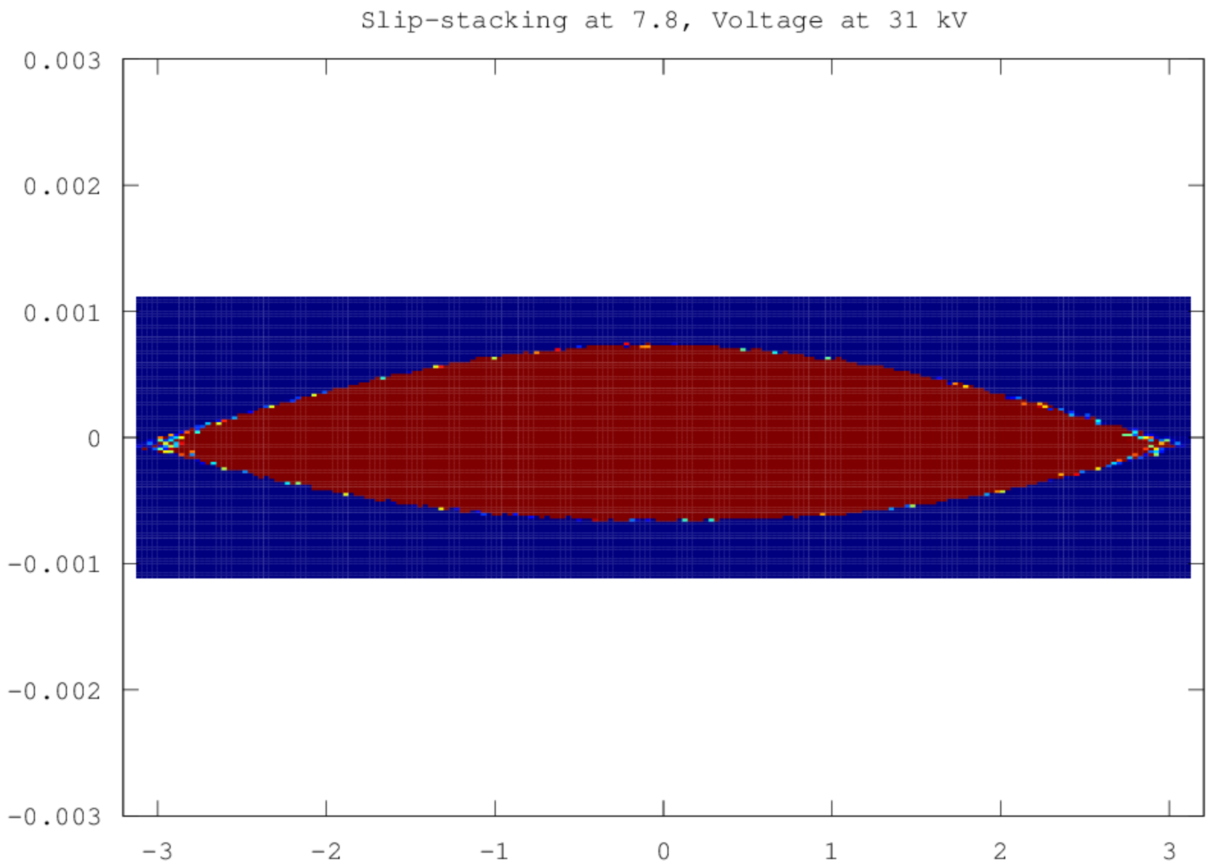
- A relatively new numerical technique designed to analyze turbulence in real-world problems.
- Organizes the phase-space by drawing trajectories (LCS strainlines) that have the property of either attracting or repeling nearby trajectories.



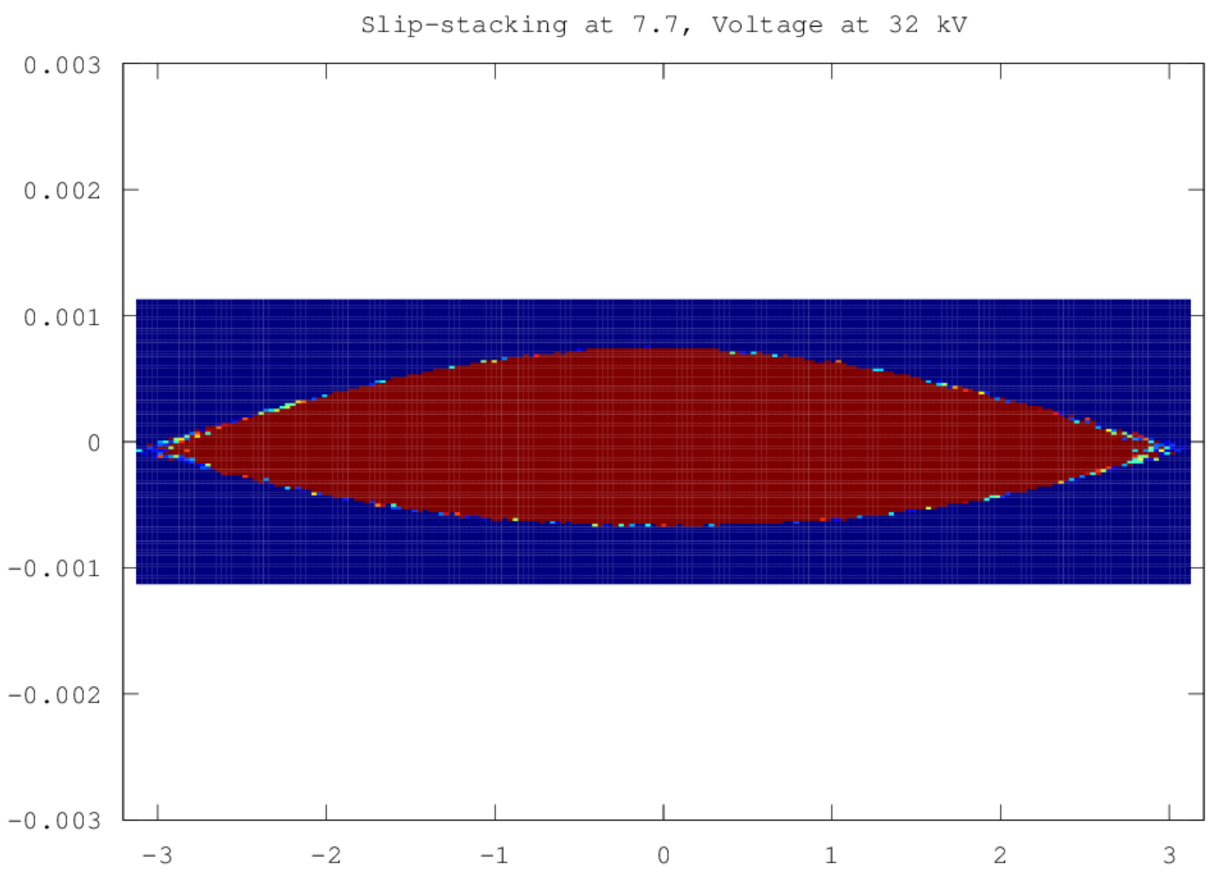
- There are many features matching what we want:
  - Unlike velocity fields which change in a rotating reference frame, LCSs are frame invariant.
  - LCS has an unfixed parameter representing the time-scale, we have a natural time-scale.


Slip-stacking at 8.0, Voltage at 30 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

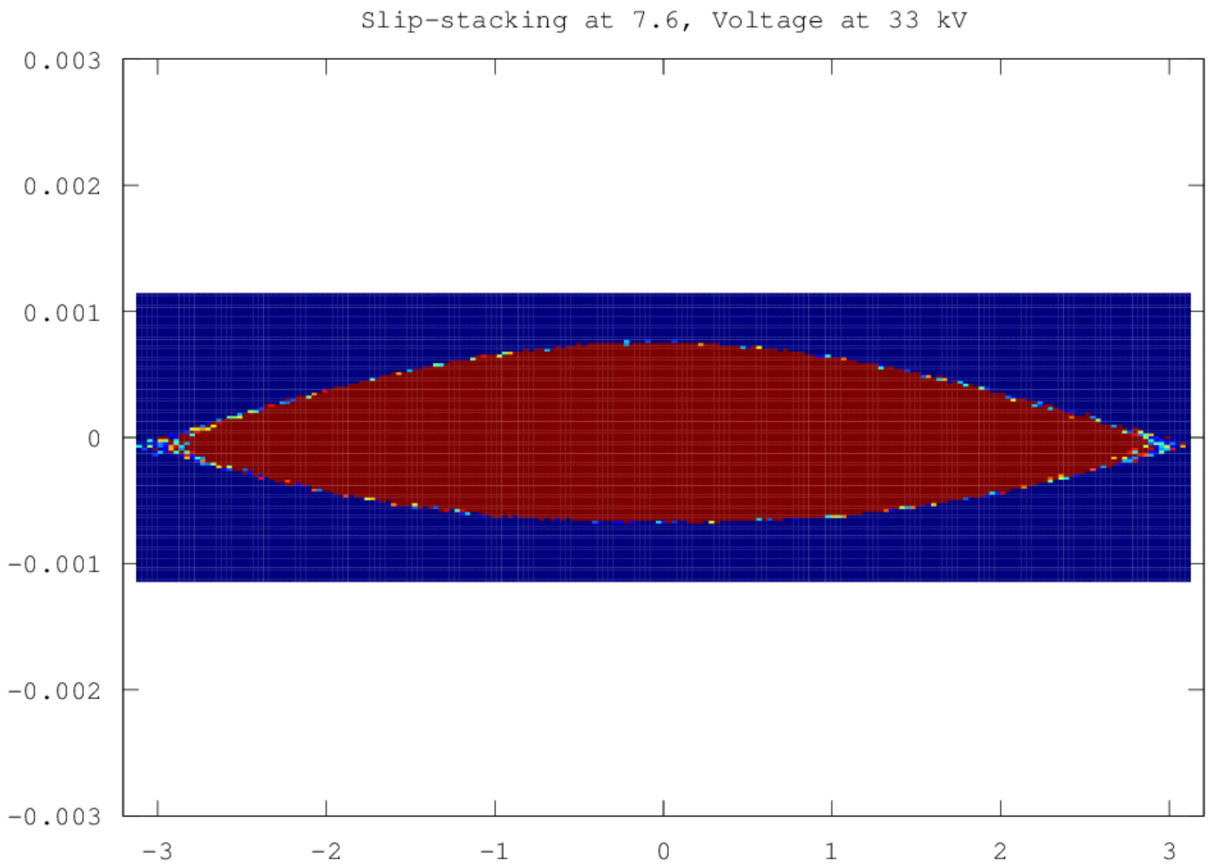

Slip-stacking at 7.9, Voltage at 30 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

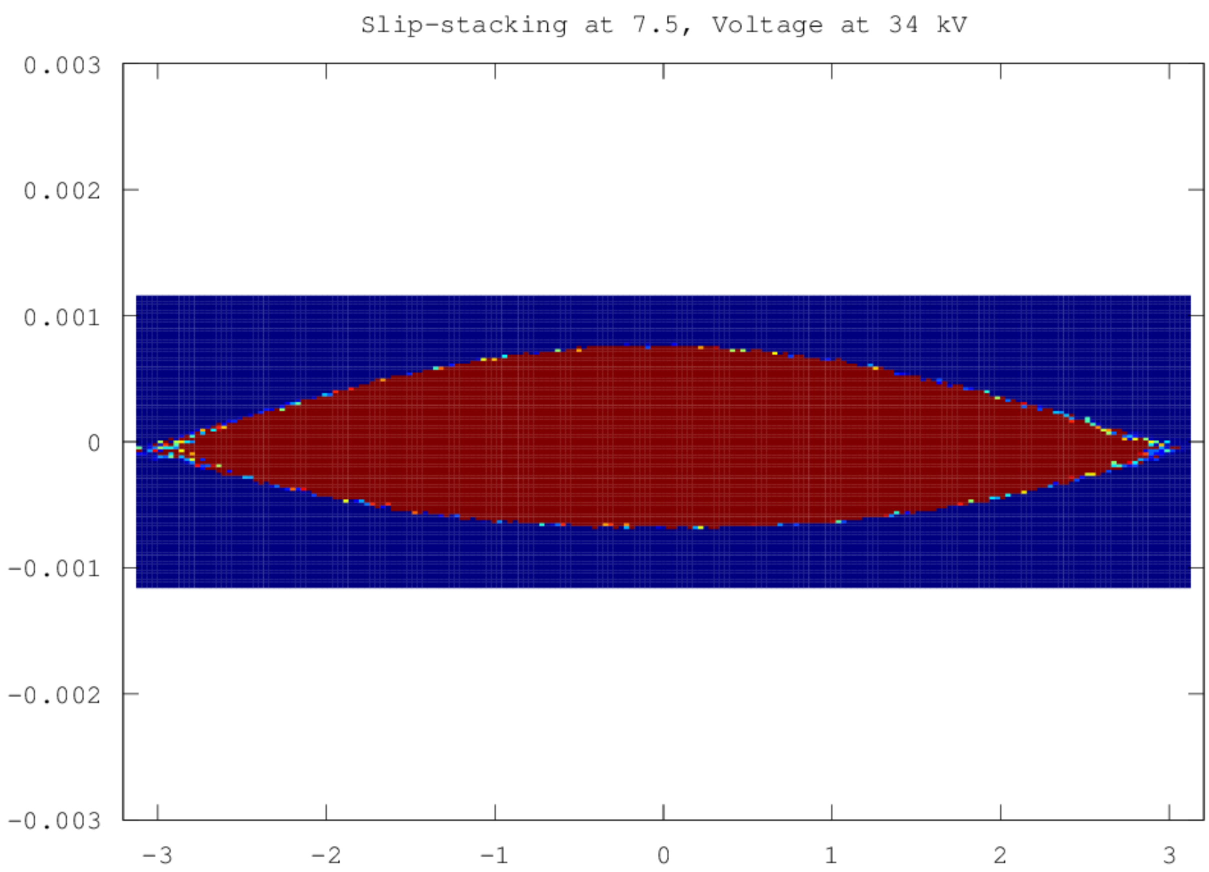


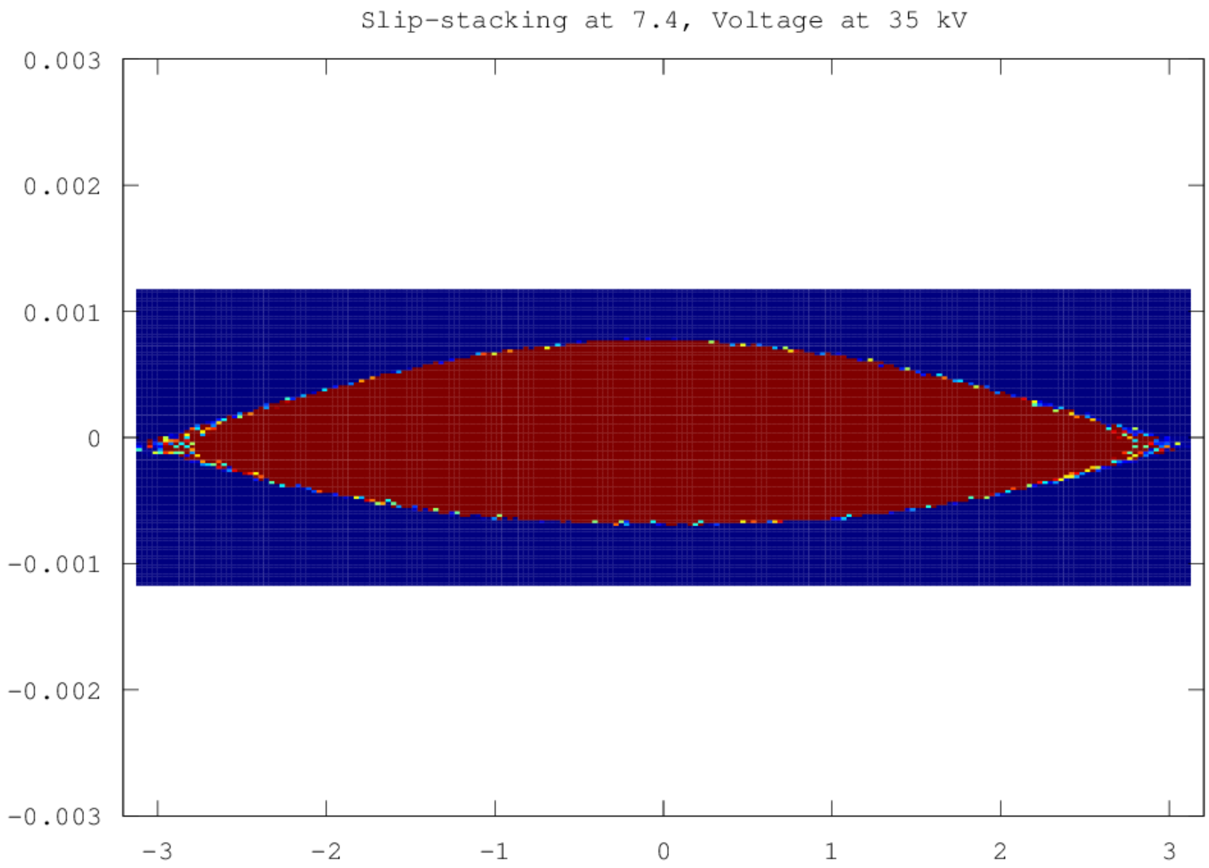
Slip-stacking at 7.8, Voltage at 31 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3



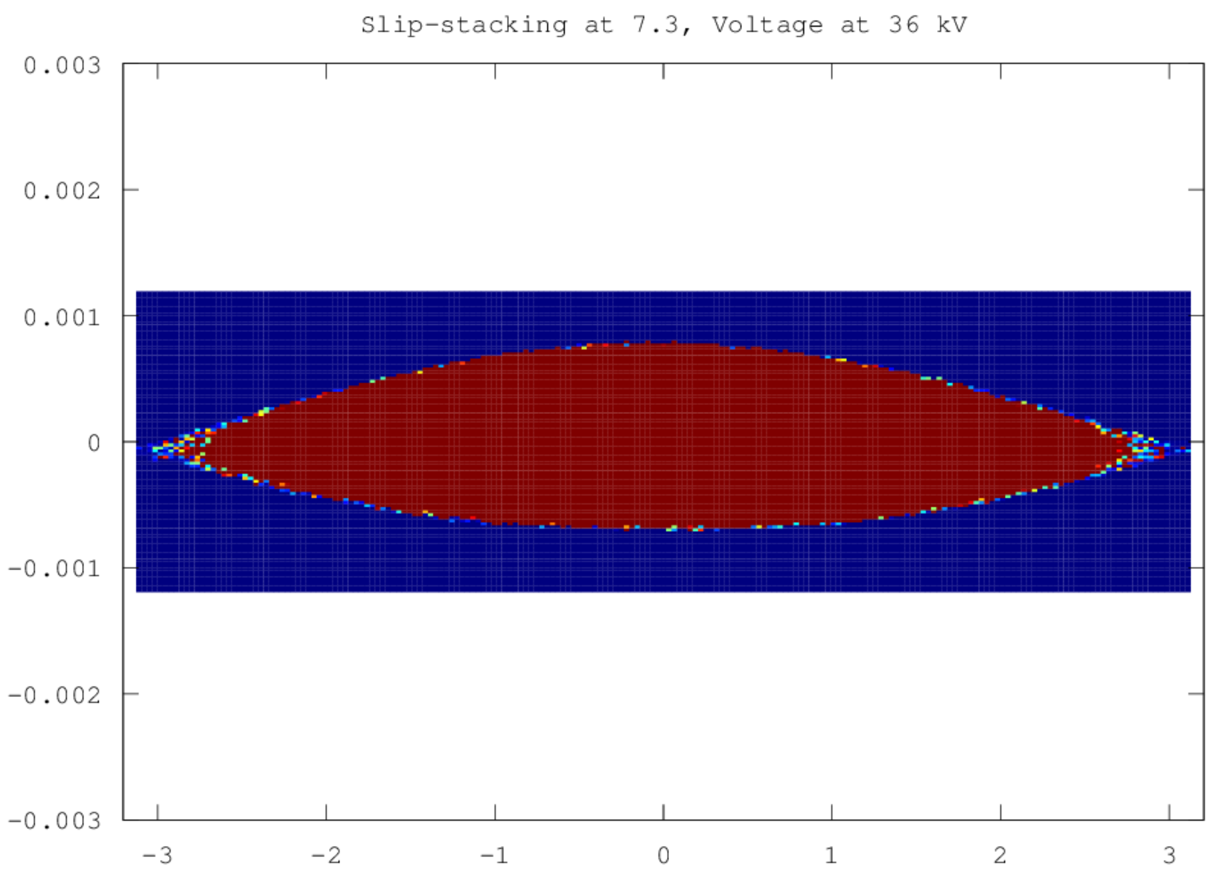
Slip-stacking at 7.7, Voltage at 32 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

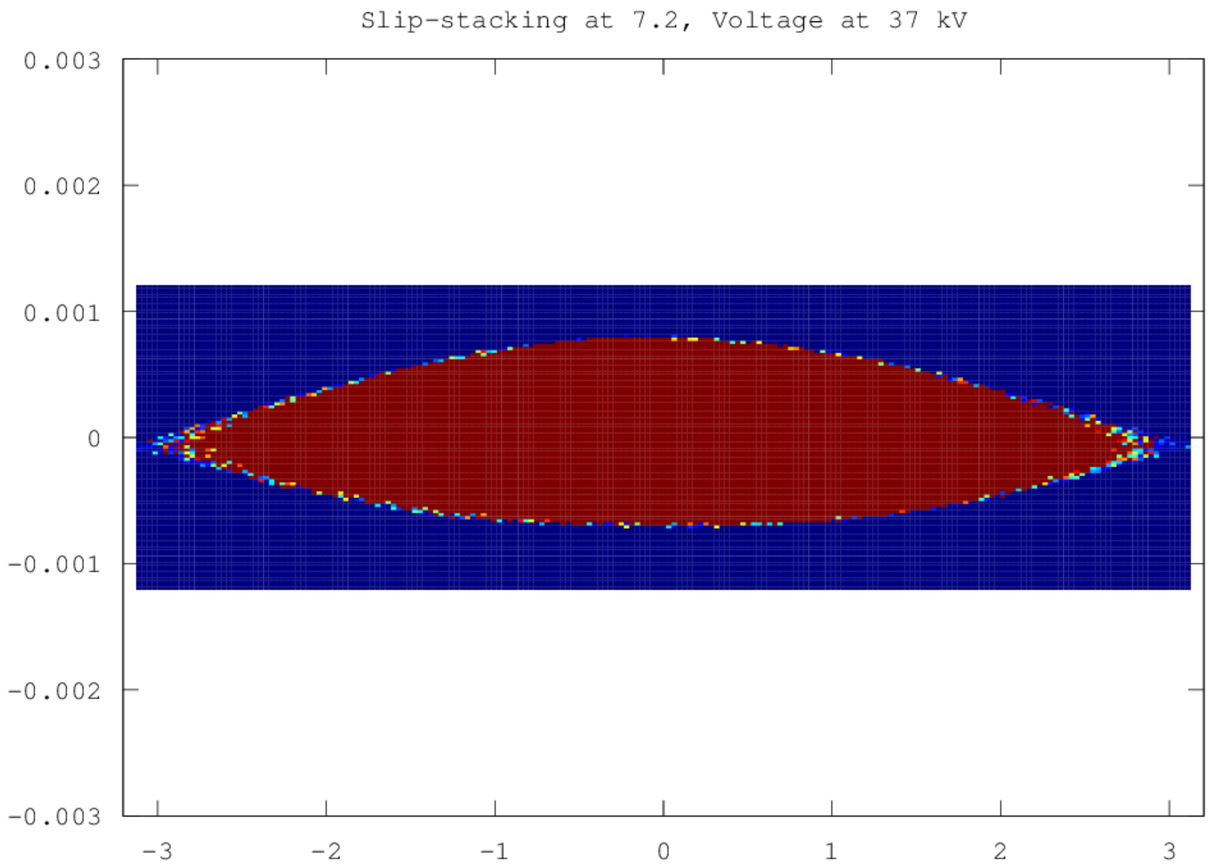

Slip-stacking at 7.6, Voltage at 33 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3


Slip-stacking at 7.5, Voltage at 34 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

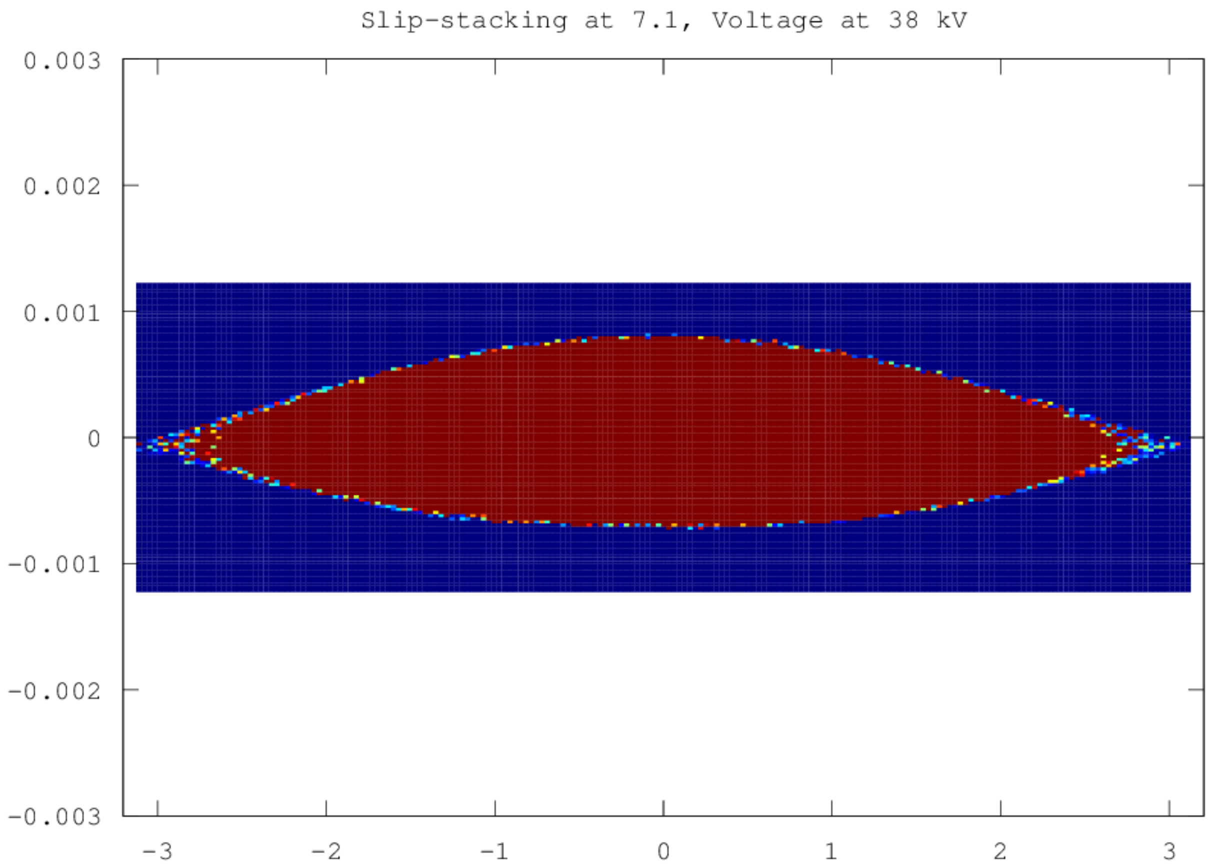


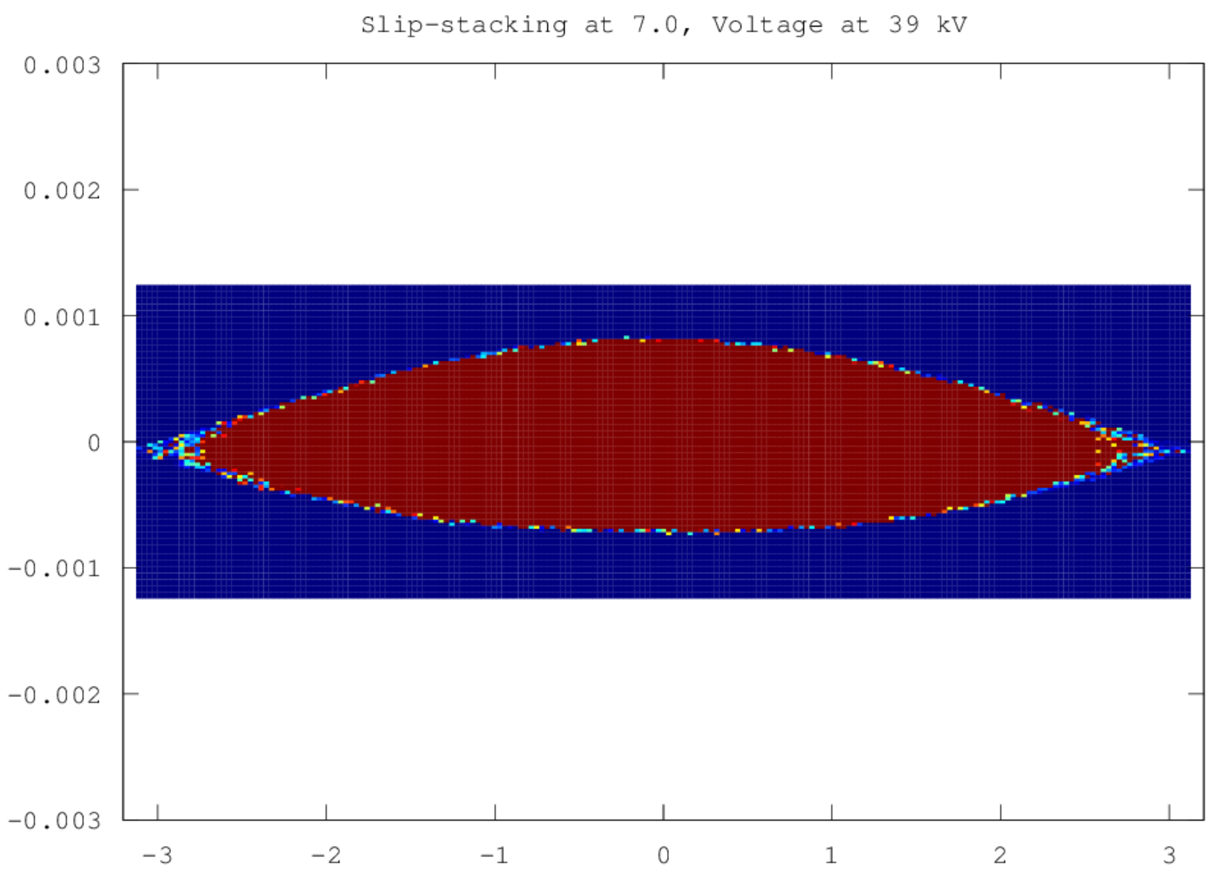
Slip-stacking at 7.4, Voltage at 35 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

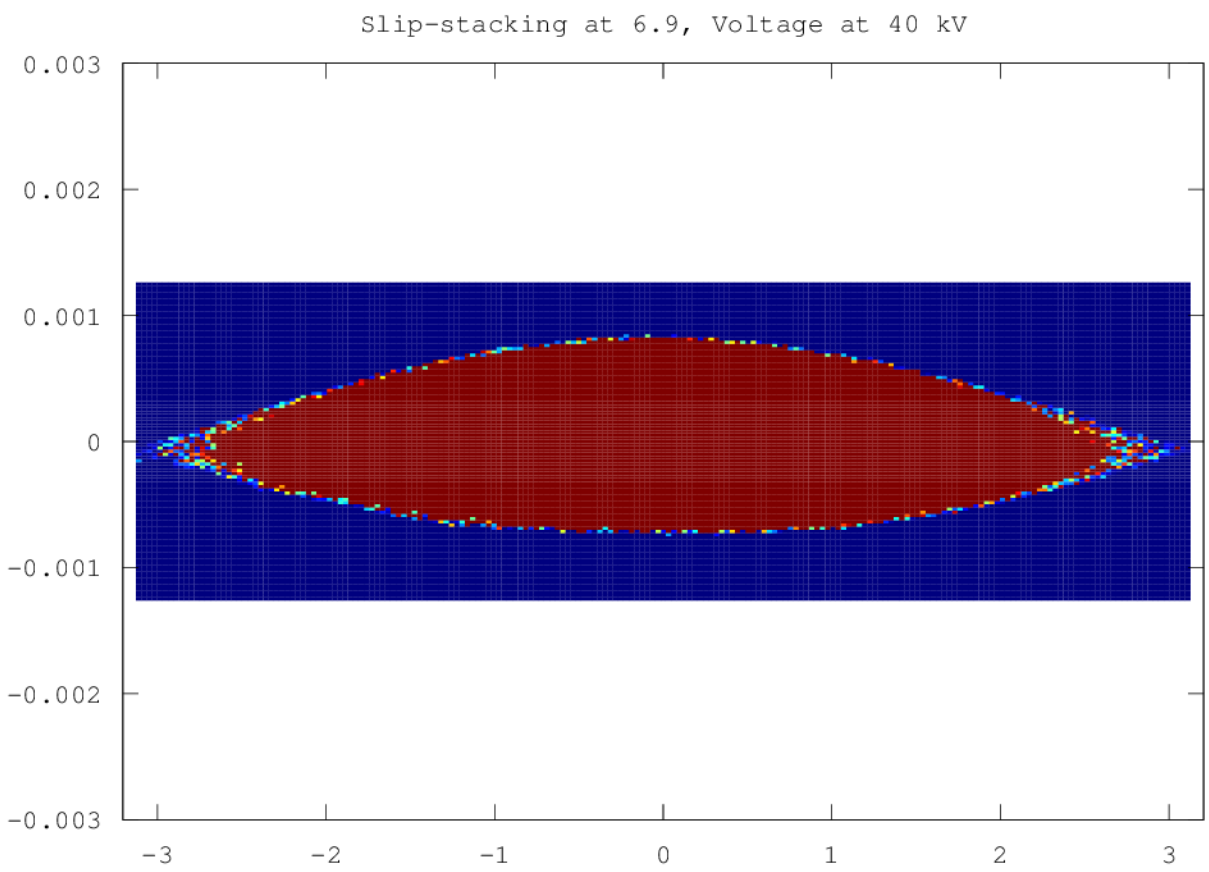

Slip-stacking at 7.3, Voltage at 36 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3



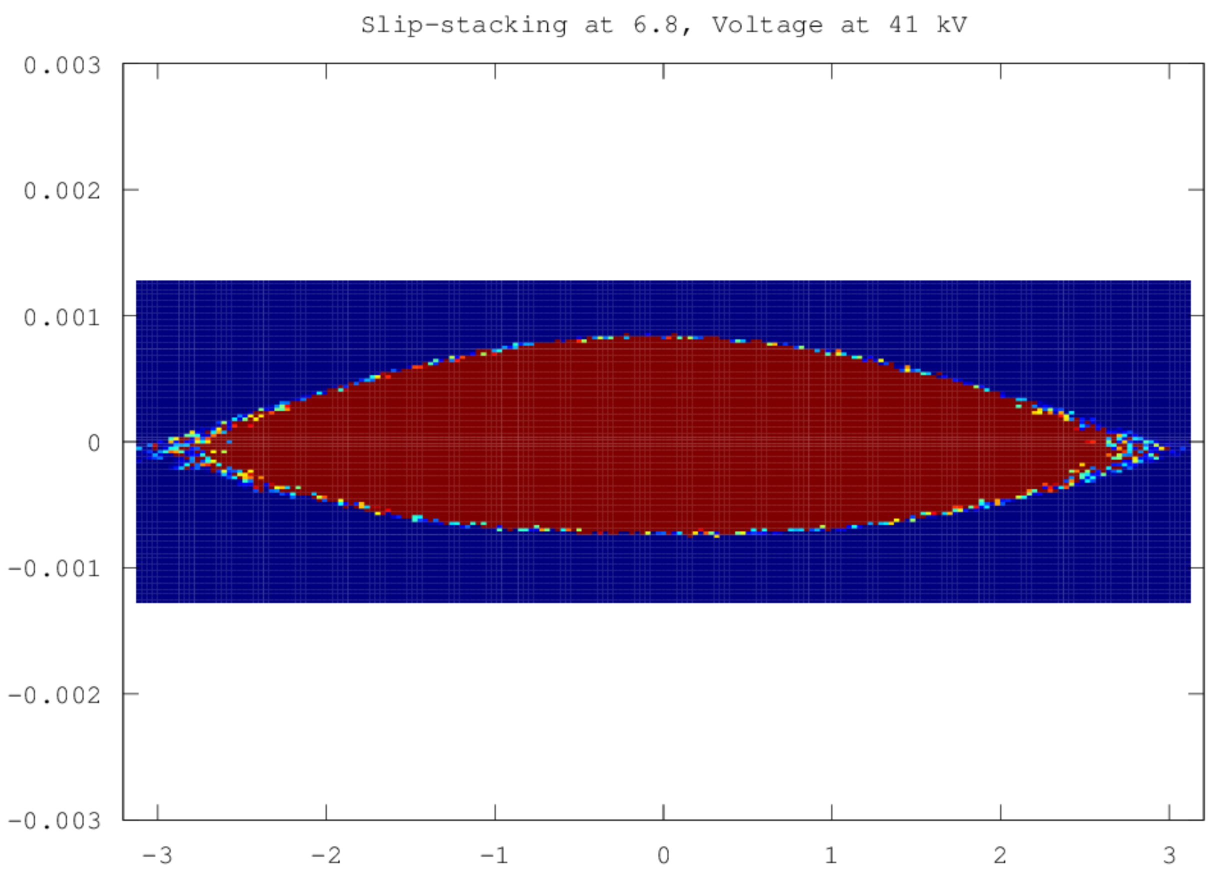
Slip-stacking at 7.2, Voltage at 37 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3


Slip-stacking at 7.1, Voltage at 38 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

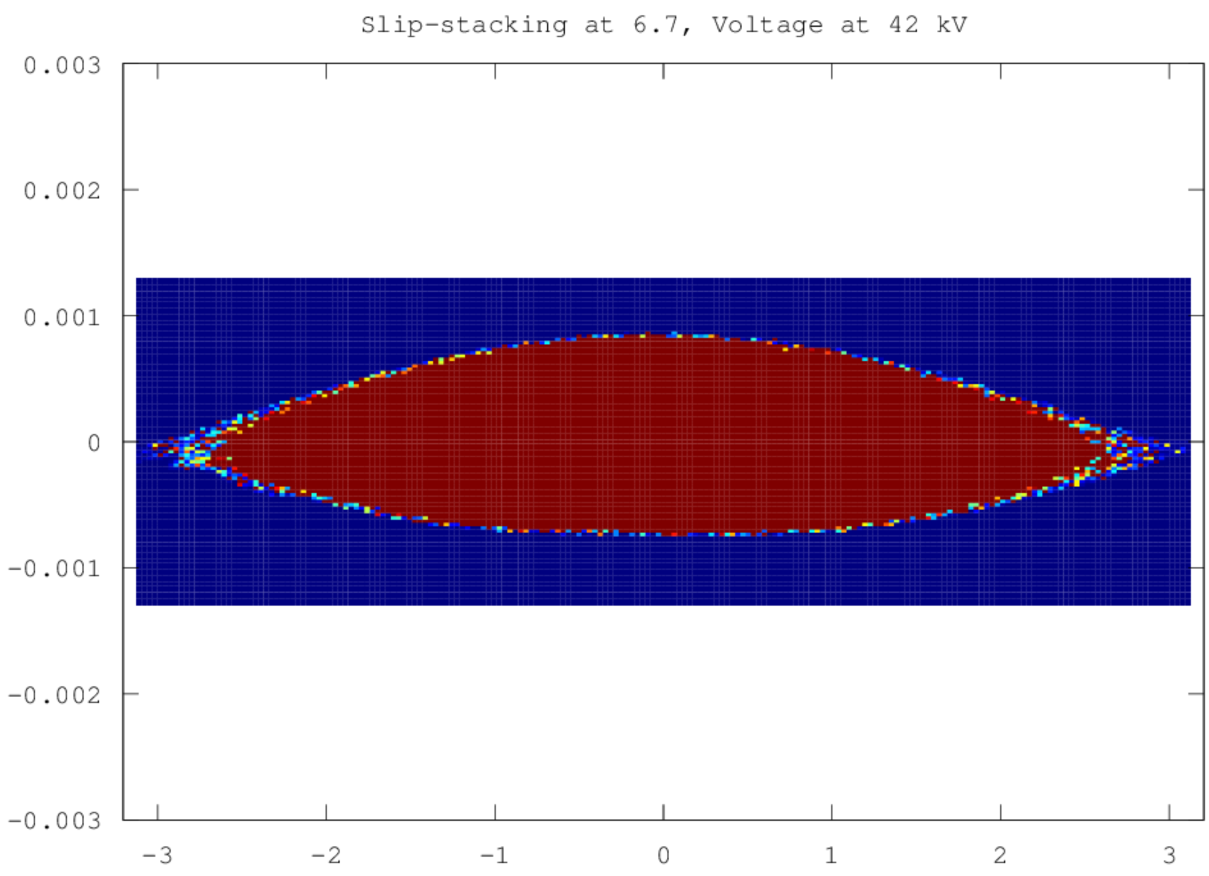

Slip-stacking at 7.0, Voltage at 39 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3



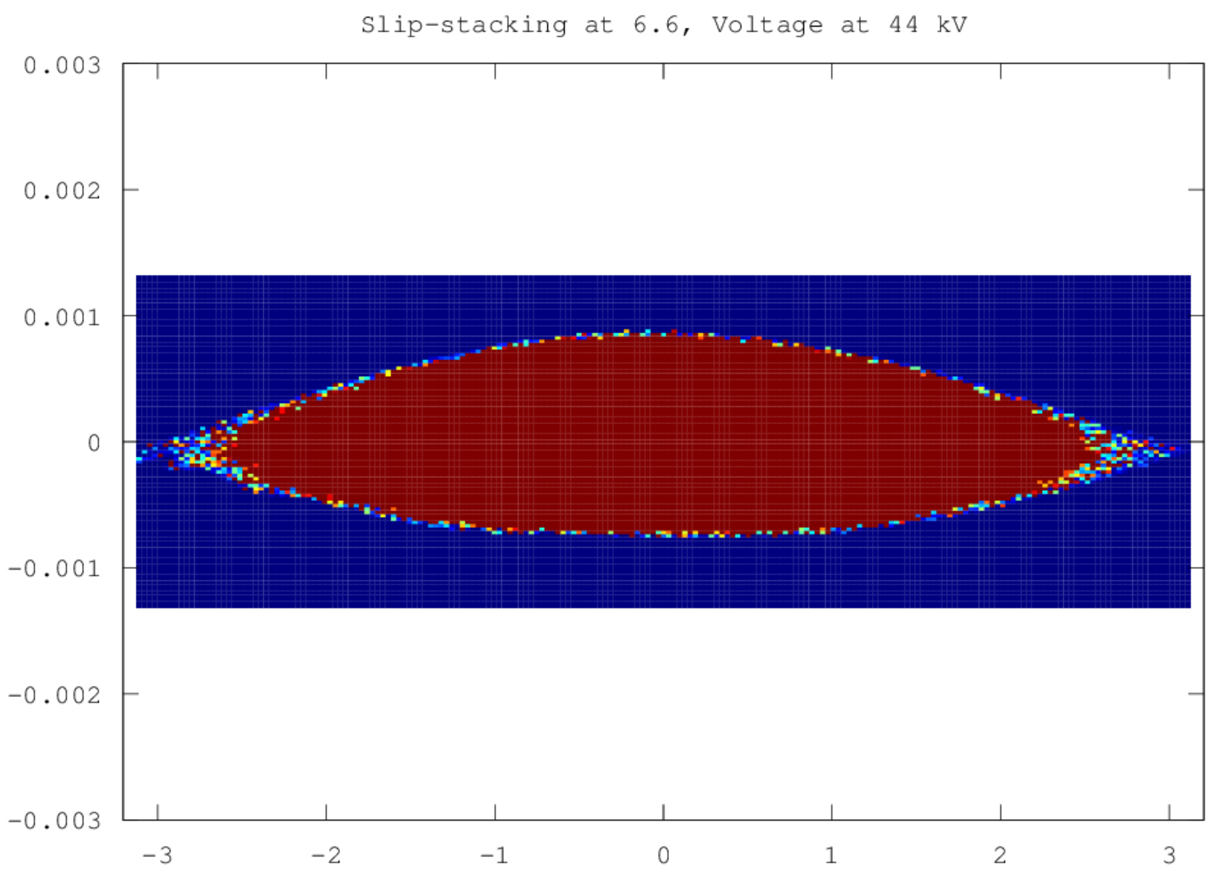
Slip-stacking at 6.9, Voltage at 40 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

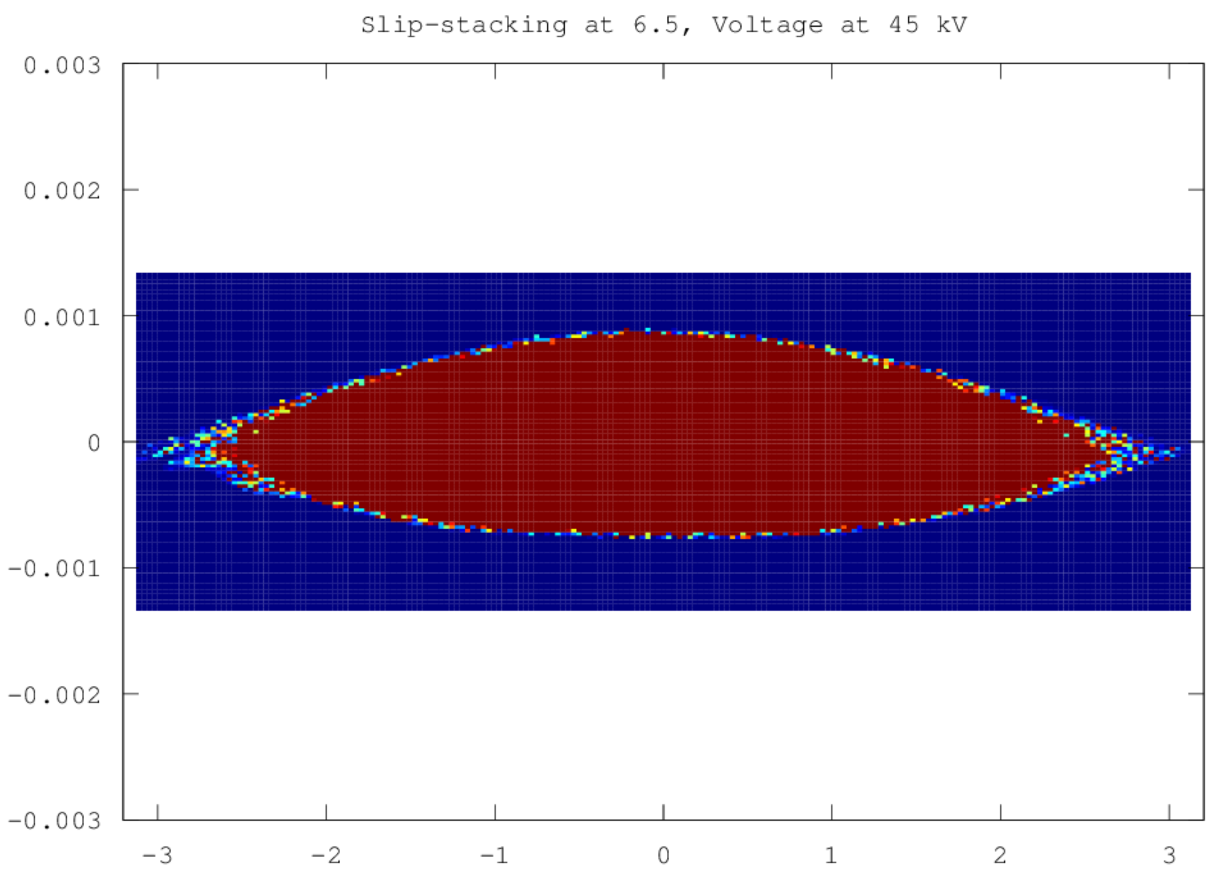


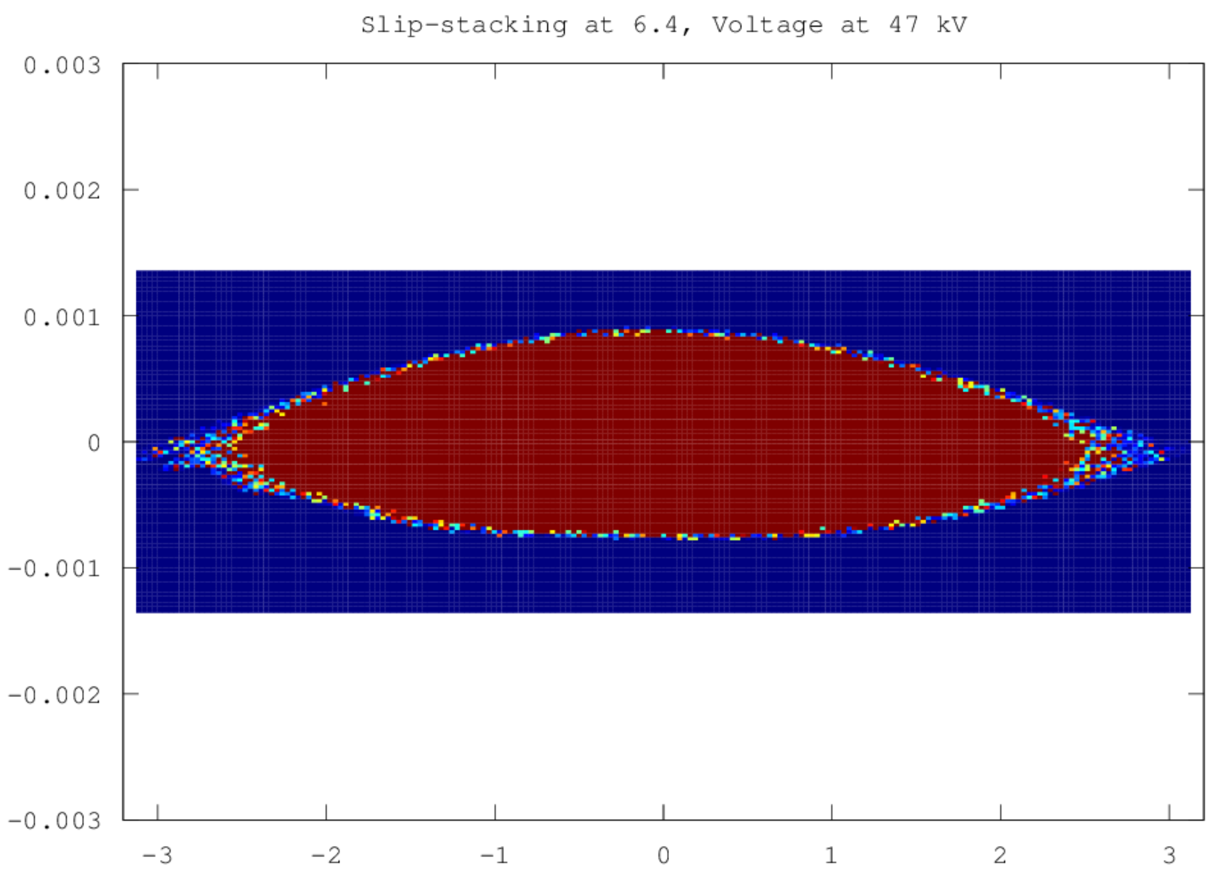
Slip-stacking at 6.8, Voltage at 41 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

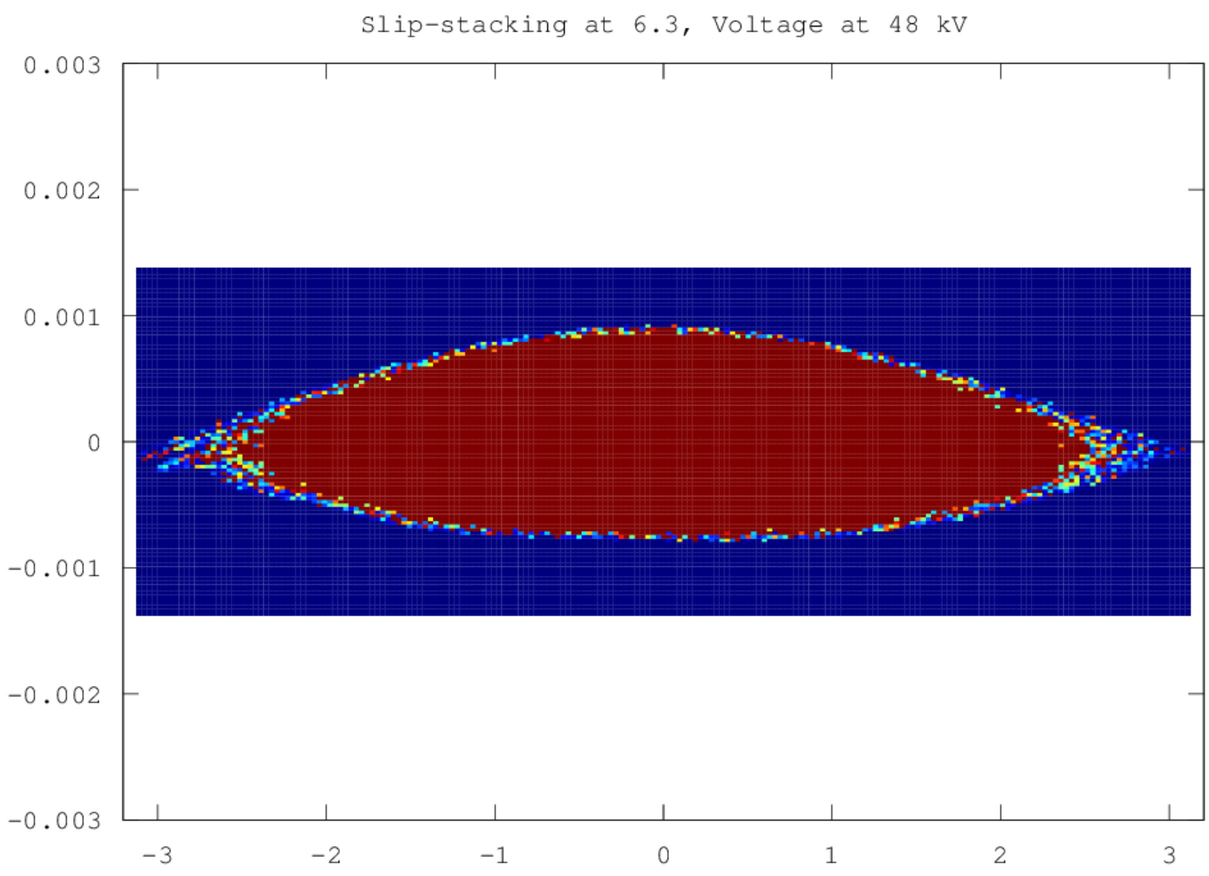


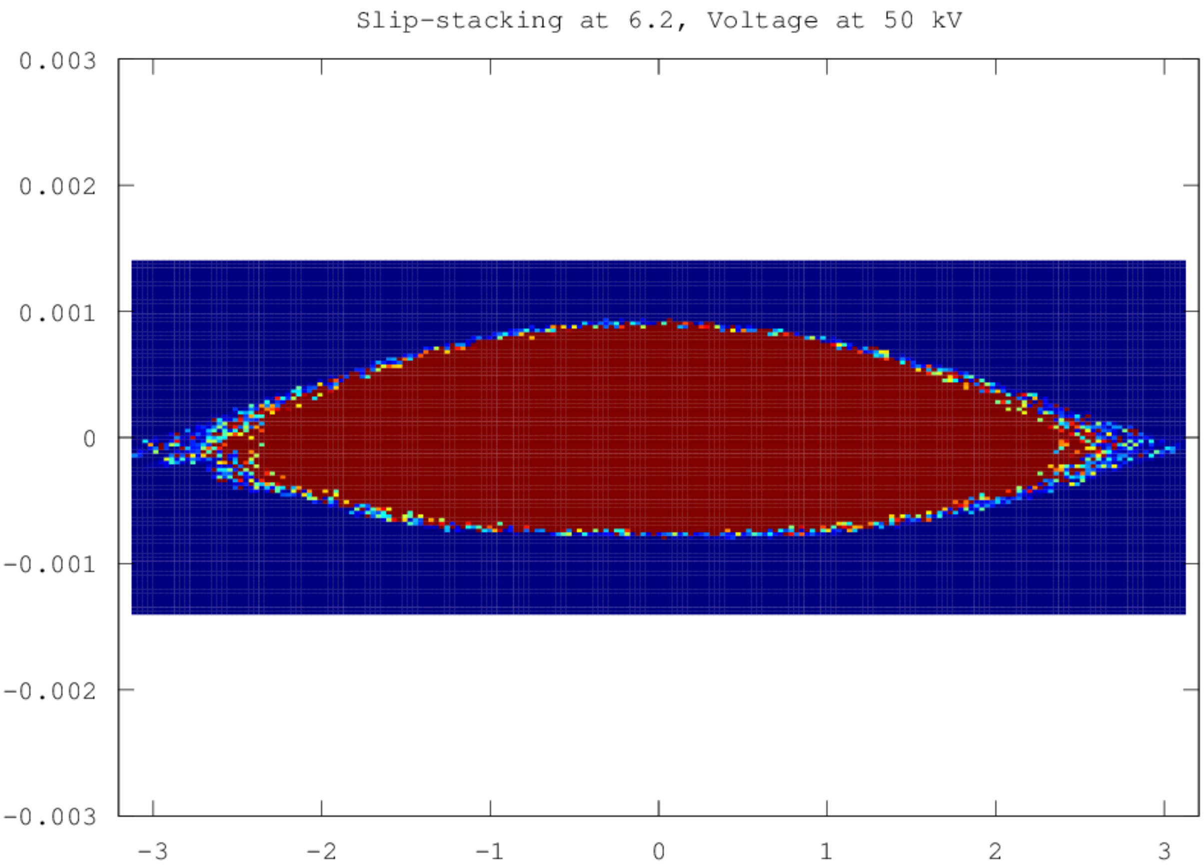
Slip-stacking at 6.7, Voltage at 42 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3


Slip-stacking at 6.6, Voltage at 44 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

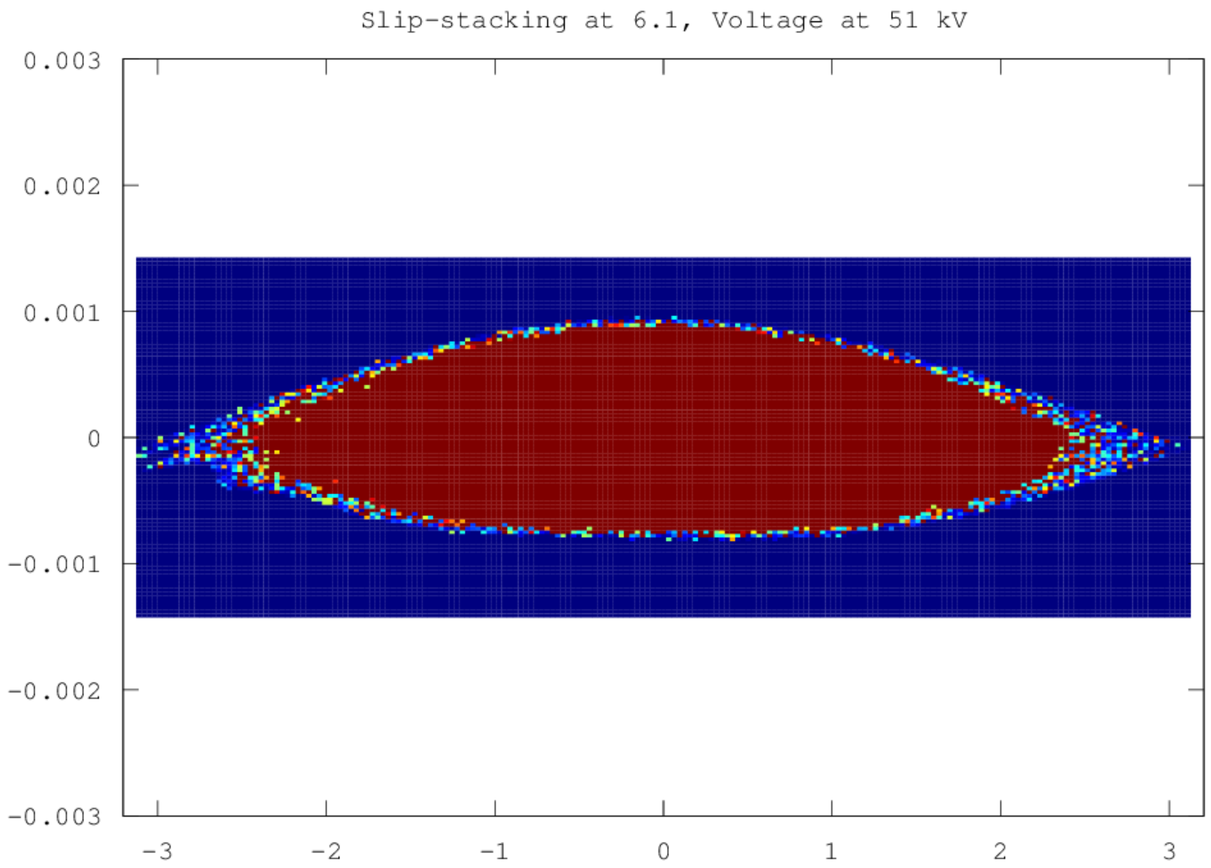

Slip-stacking at 6.5, Voltage at 45 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3


Slip-stacking at 6.4, Voltage at 47 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

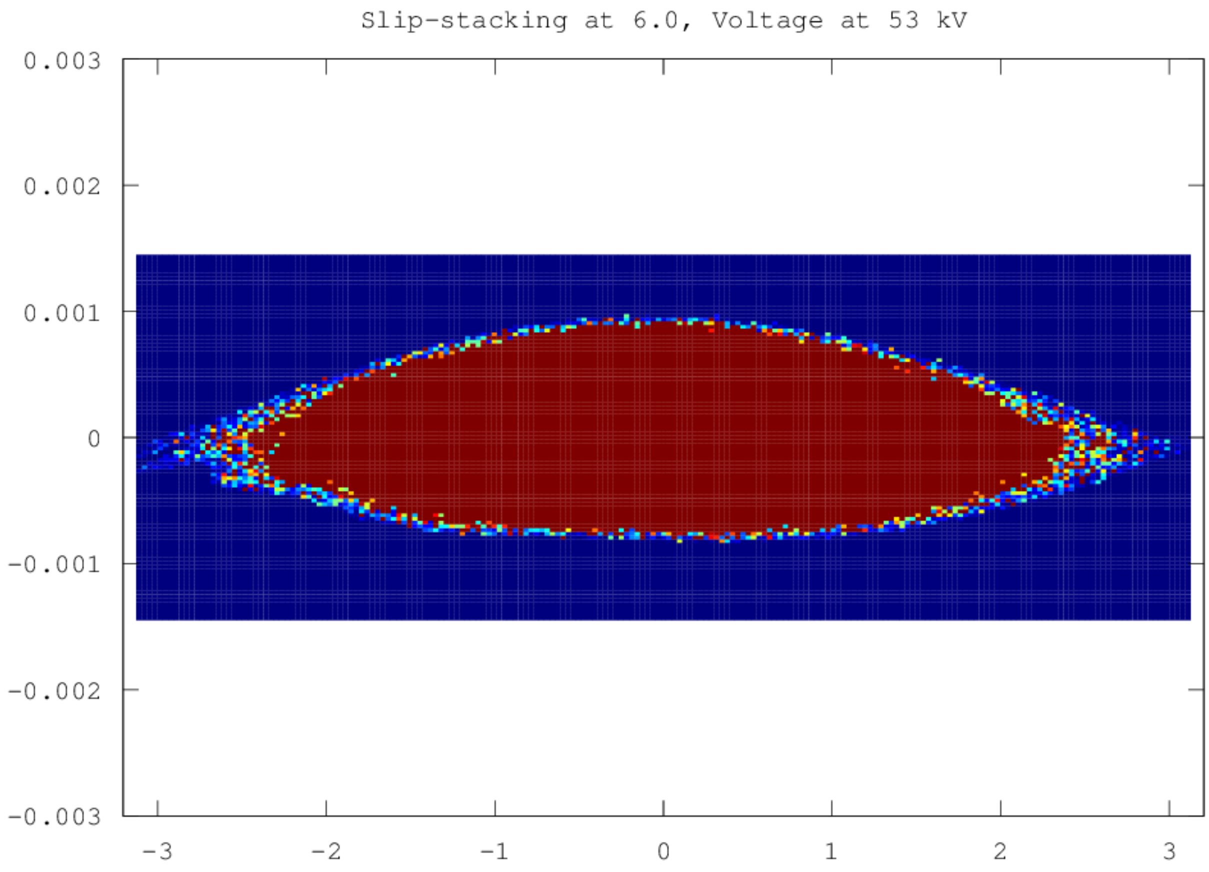

Slip-stacking at 6.3, Voltage at 48 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3



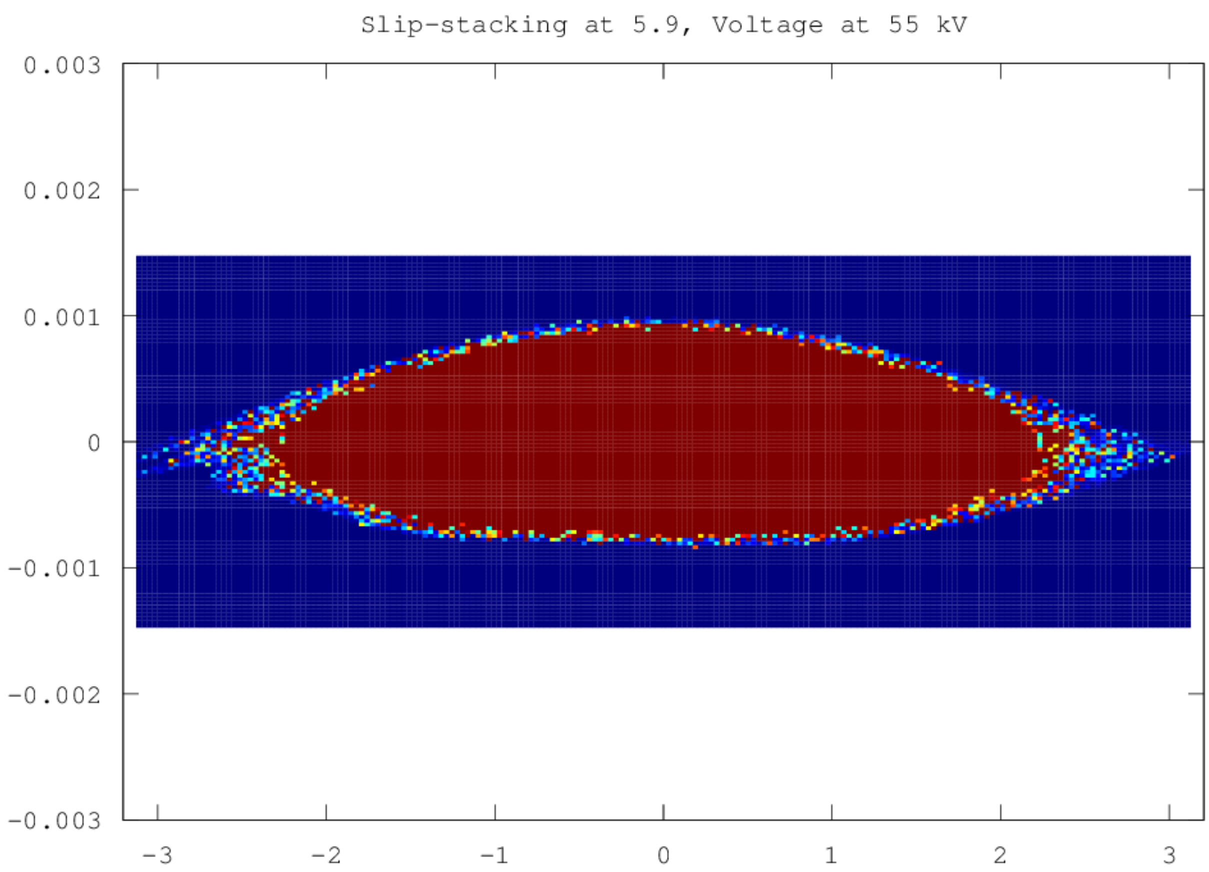
Slip-stacking at 6.2, Voltage at 50 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

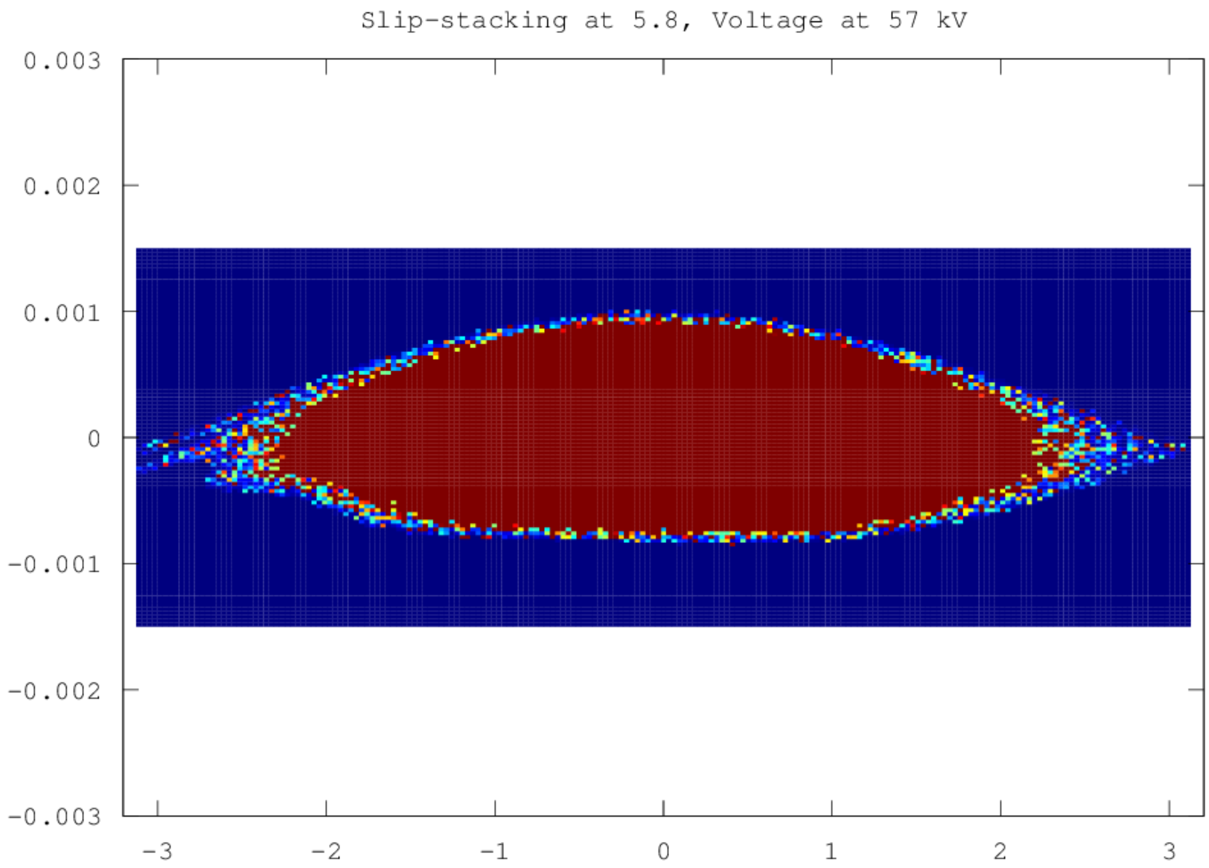


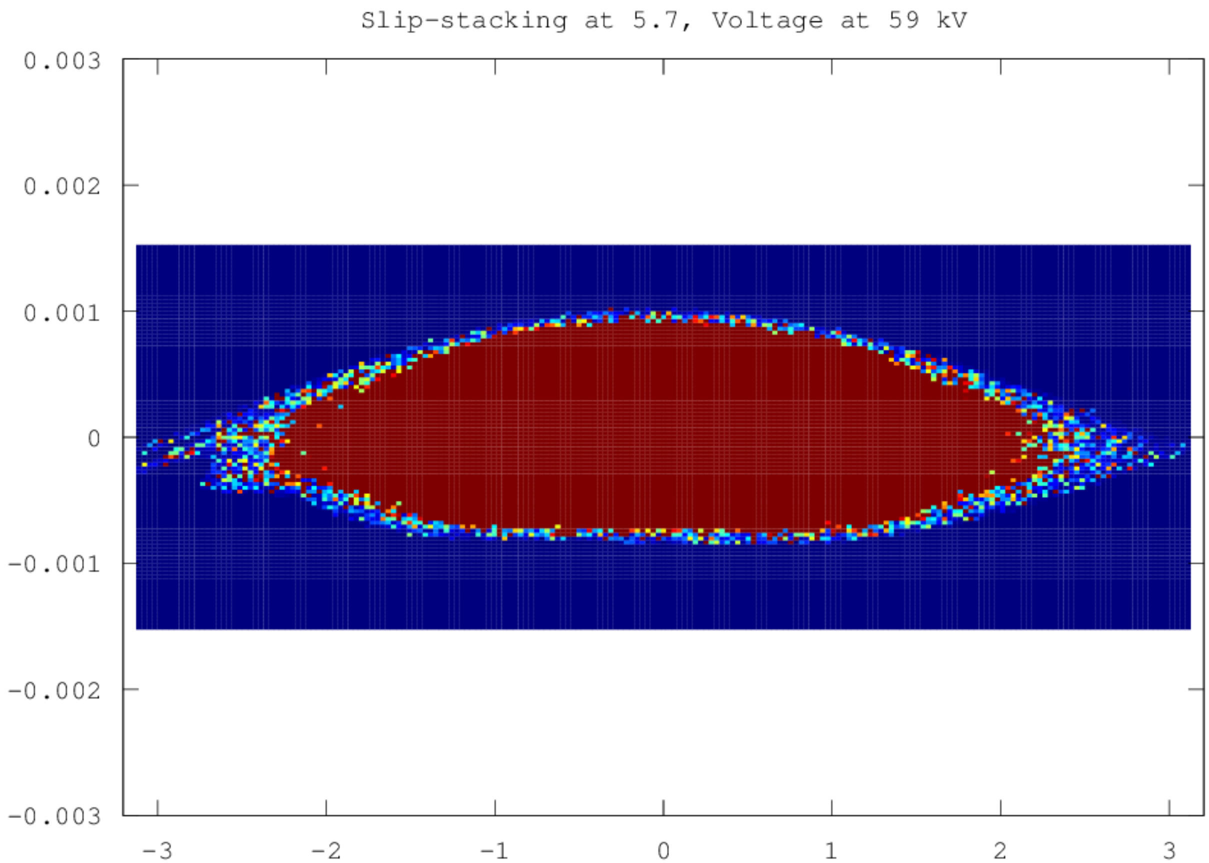
Slip-stacking at 6.1, Voltage at 51 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

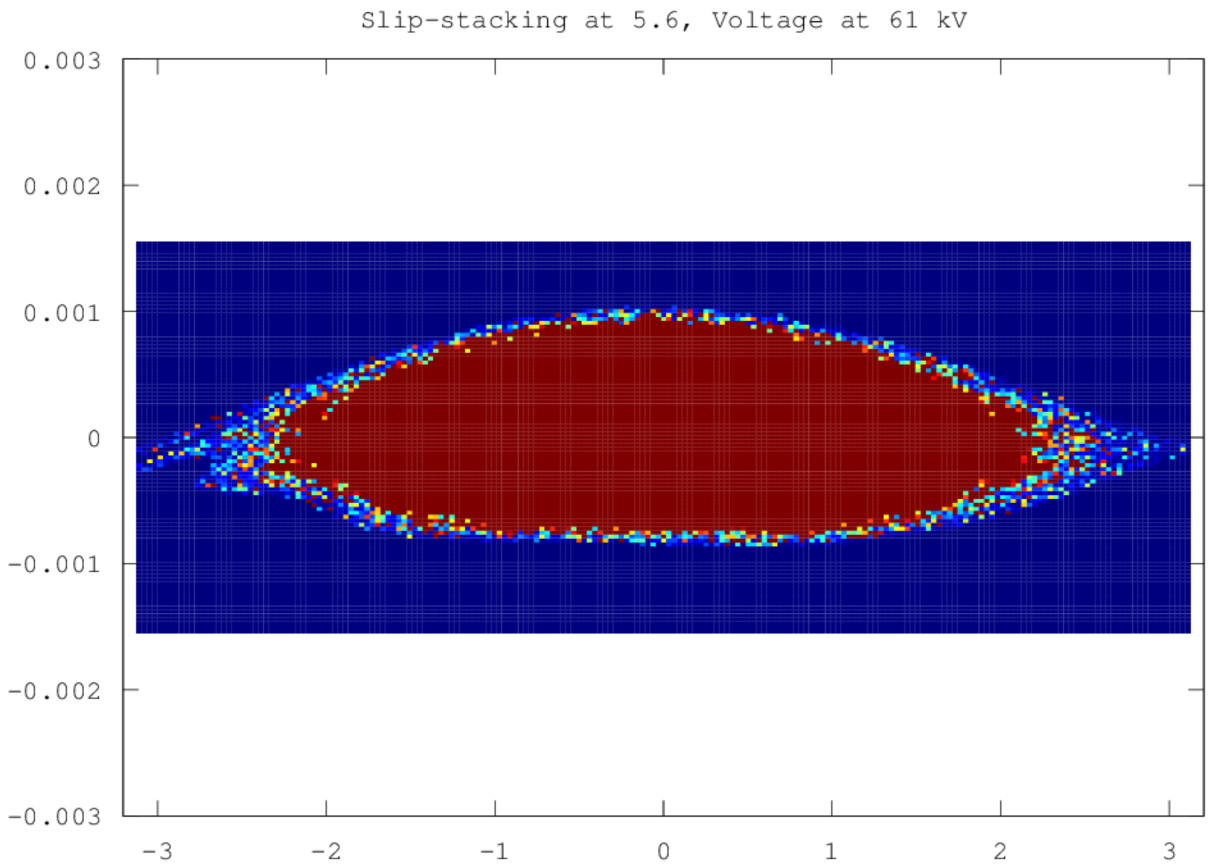


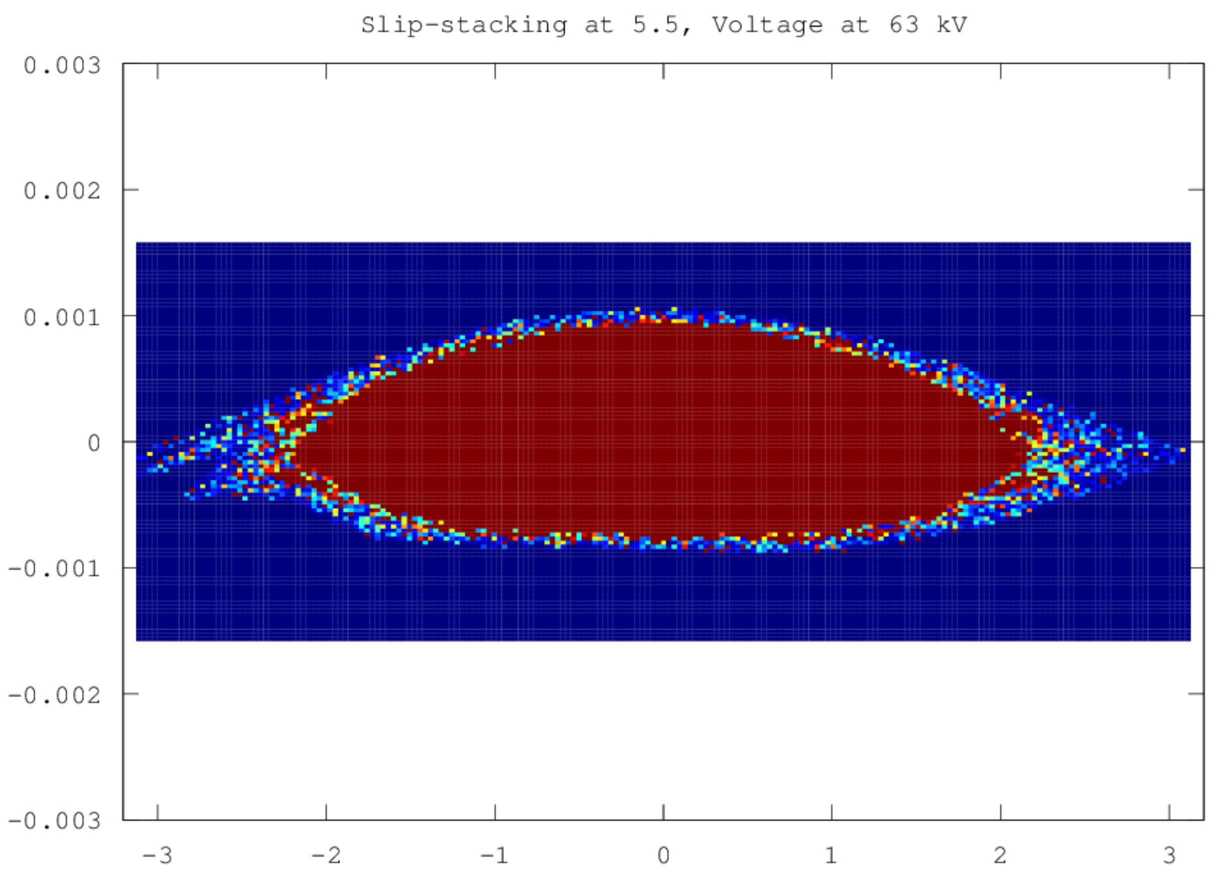
Slip-stacking at 6.0, Voltage at 53 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

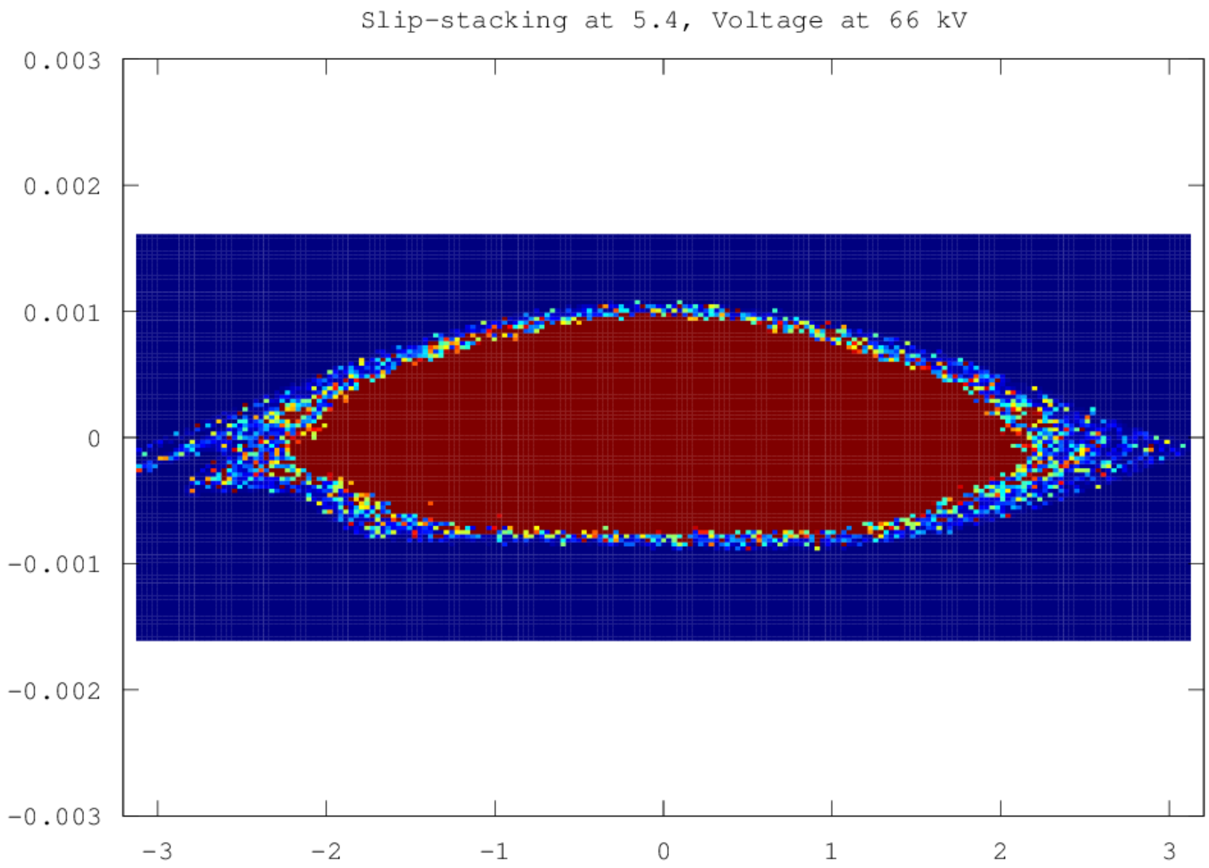

Slip-stacking at 5.9, Voltage at 55 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3


Slip-stacking at 5.8, Voltage at 57 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

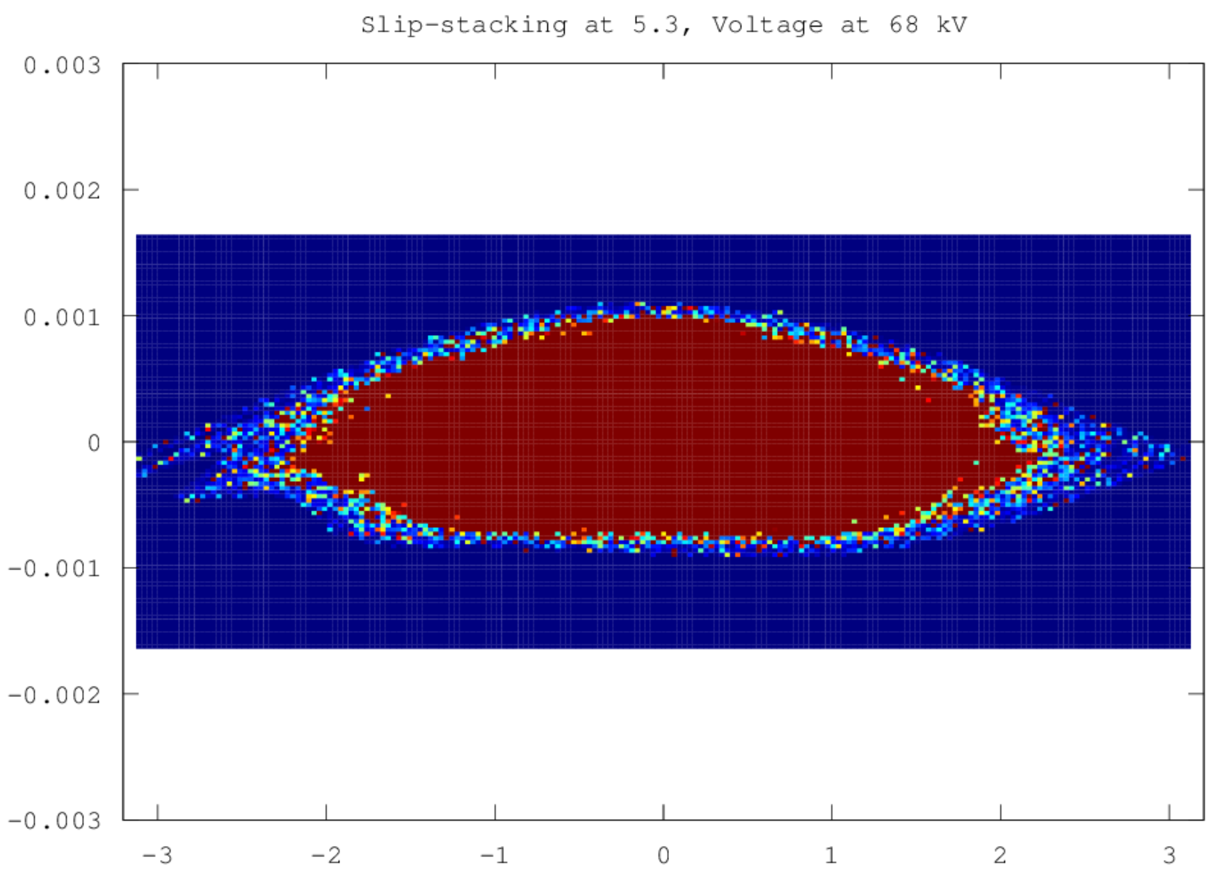

Slip-stacking at 5.7, Voltage at 59 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

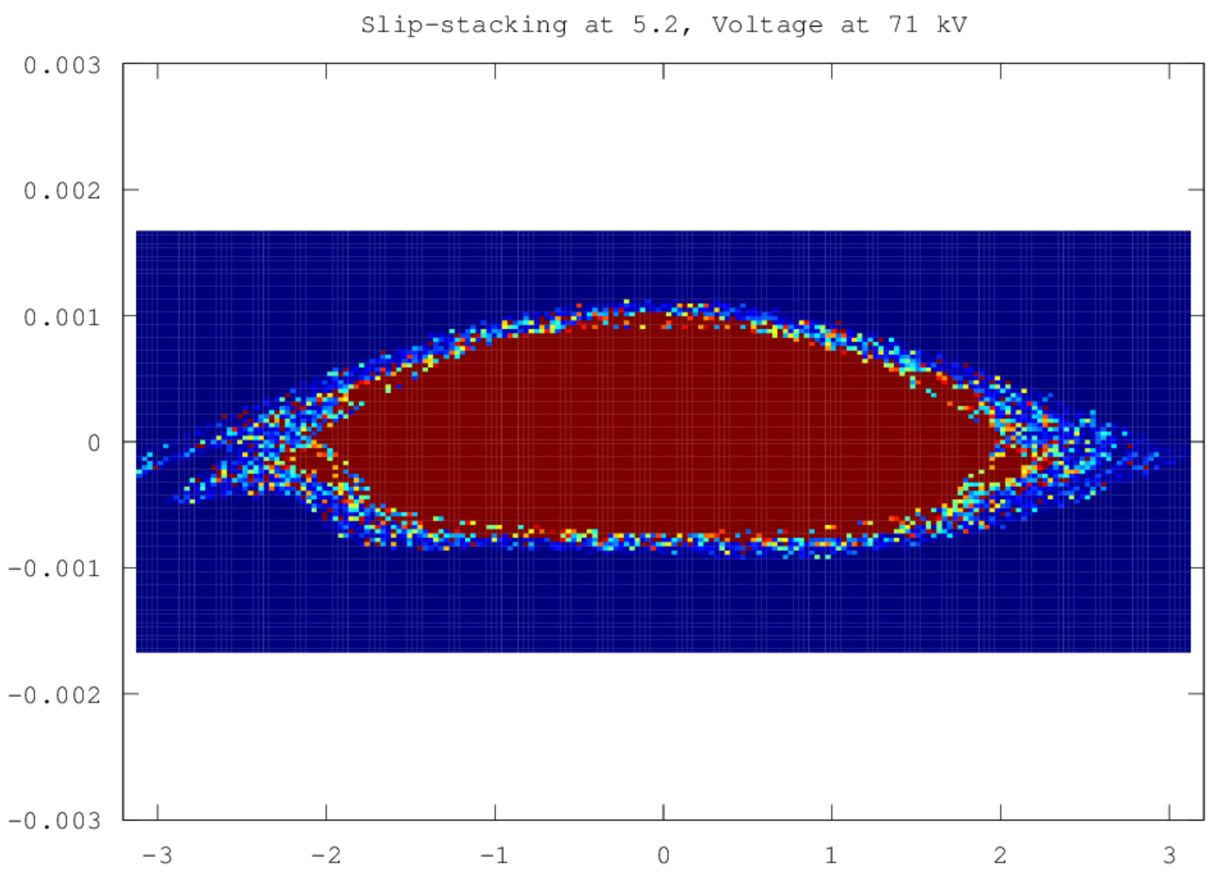

Slip-stacking at 5.6, Voltage at 61 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3


Slip-stacking at 5.5, Voltage at 63 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

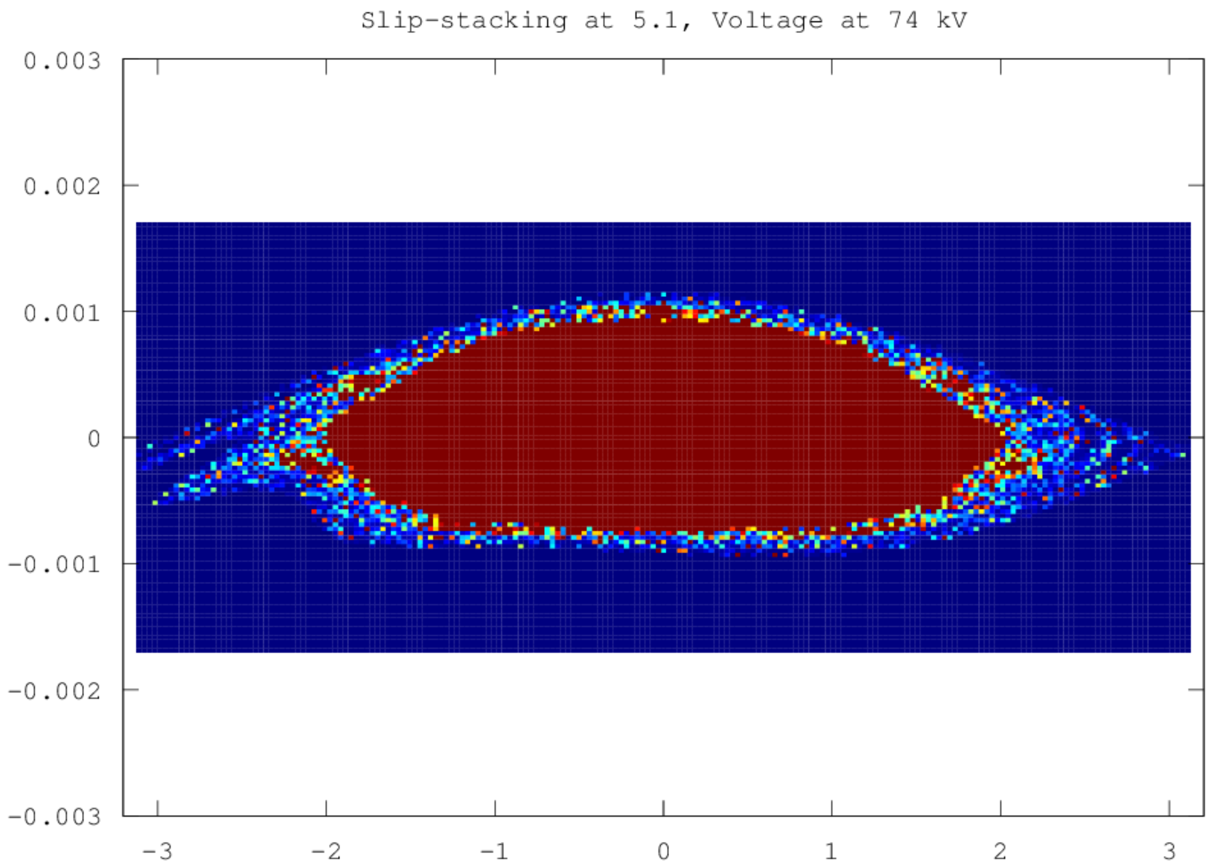


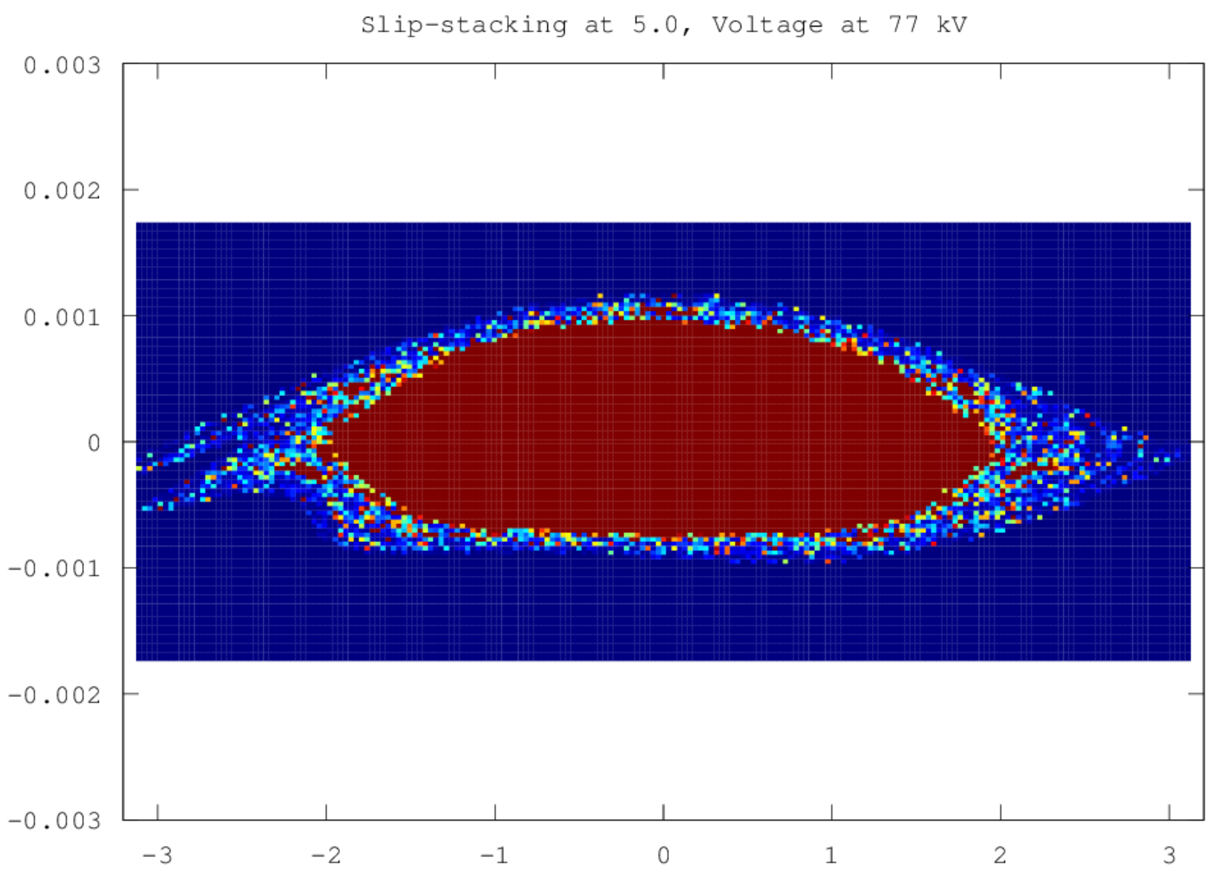
Slip-stacking at 5.4, Voltage at 66 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3


Slip-stacking at 5.3, Voltage at 68 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

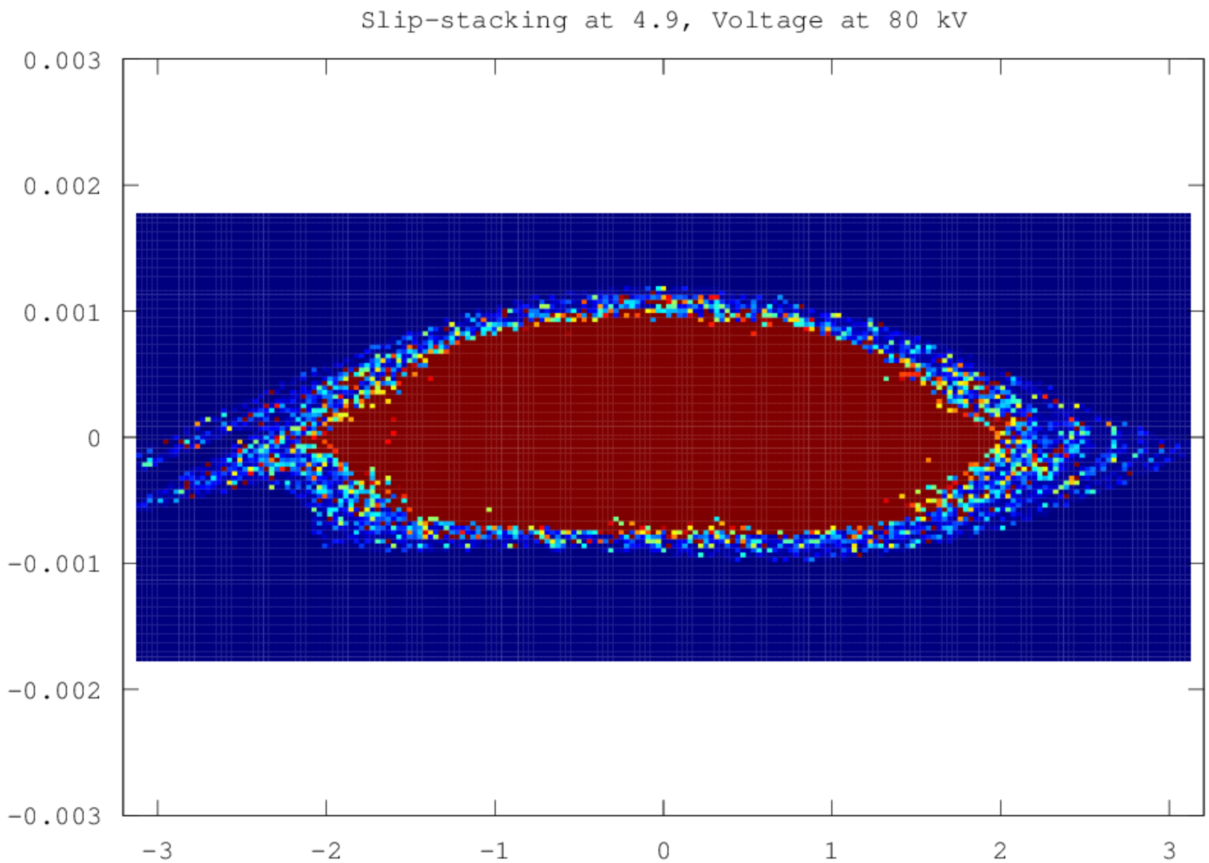


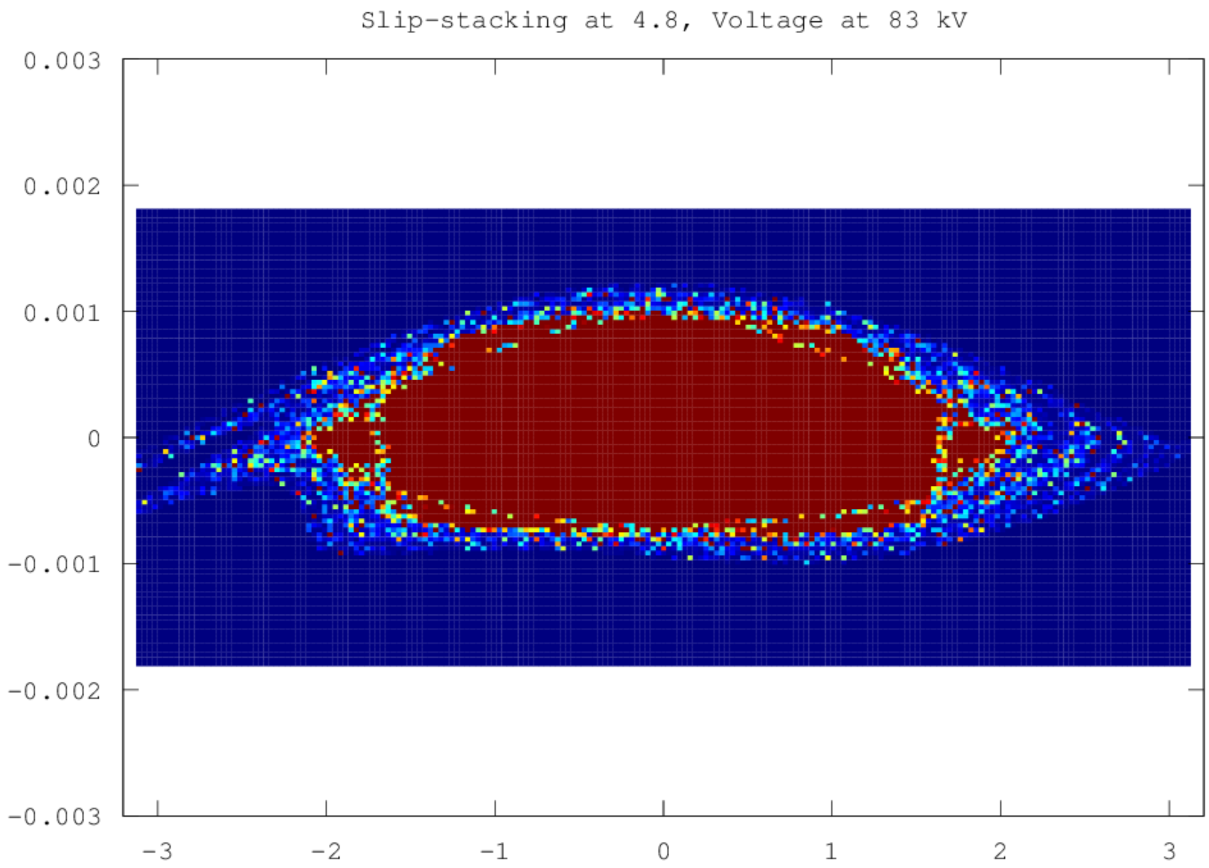
Slip-stacking at 5.2, Voltage at 71 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

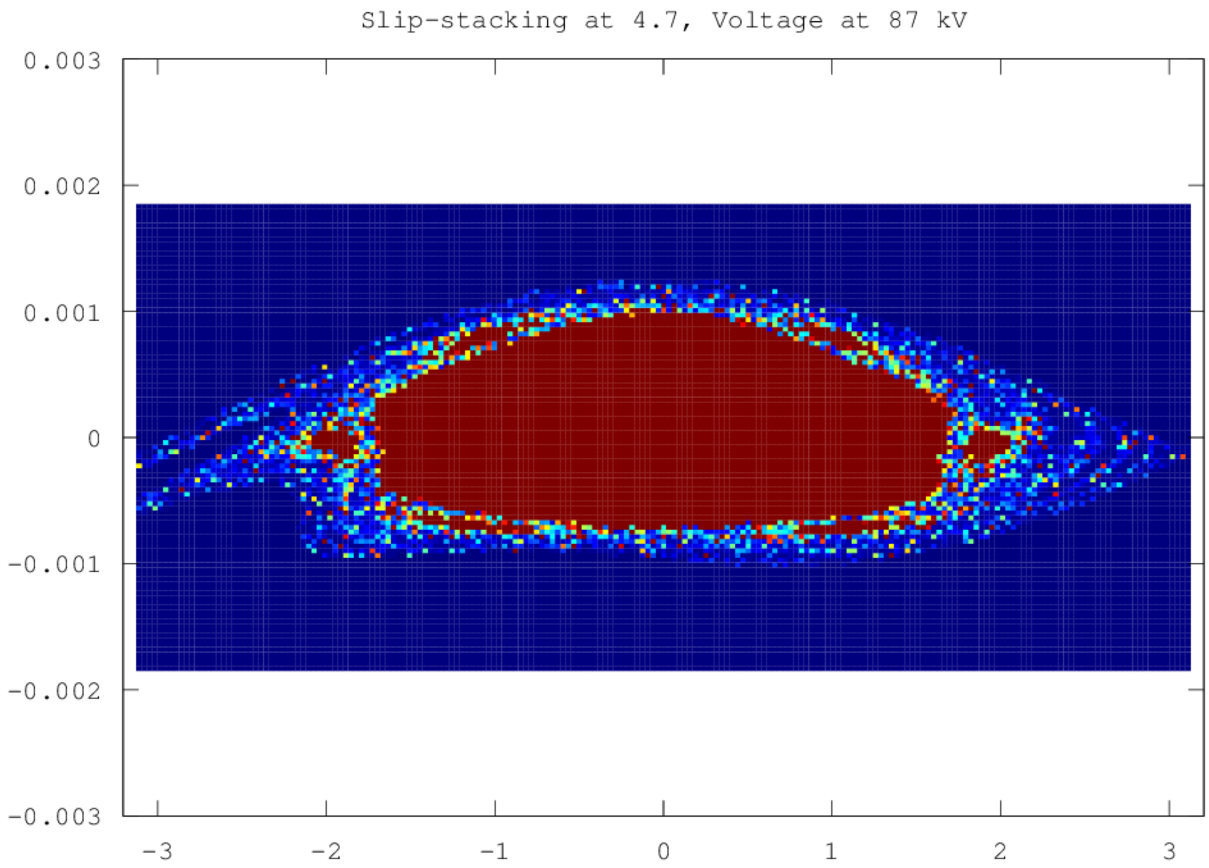

Slip-stacking at 5.1, Voltage at 74 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3



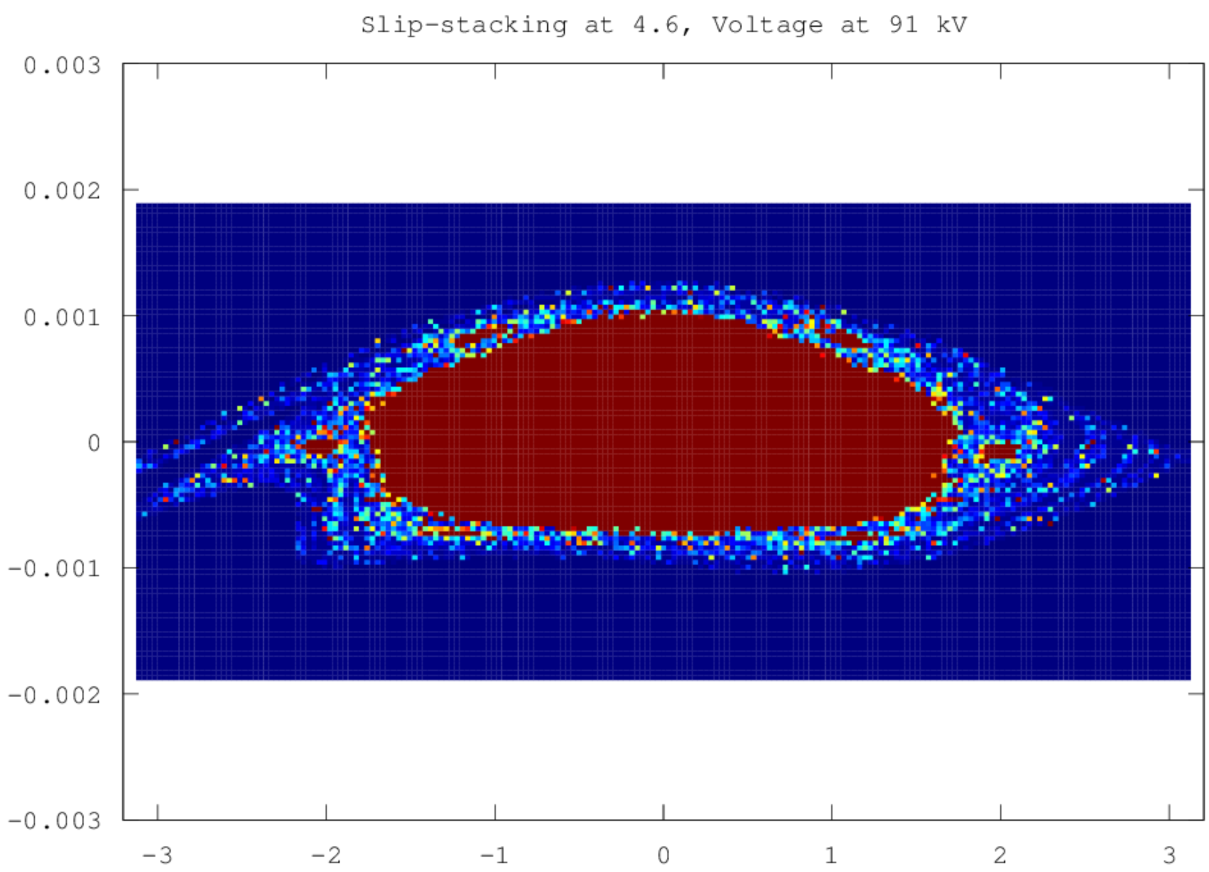
Slip-stacking at 5.0, Voltage at 77 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

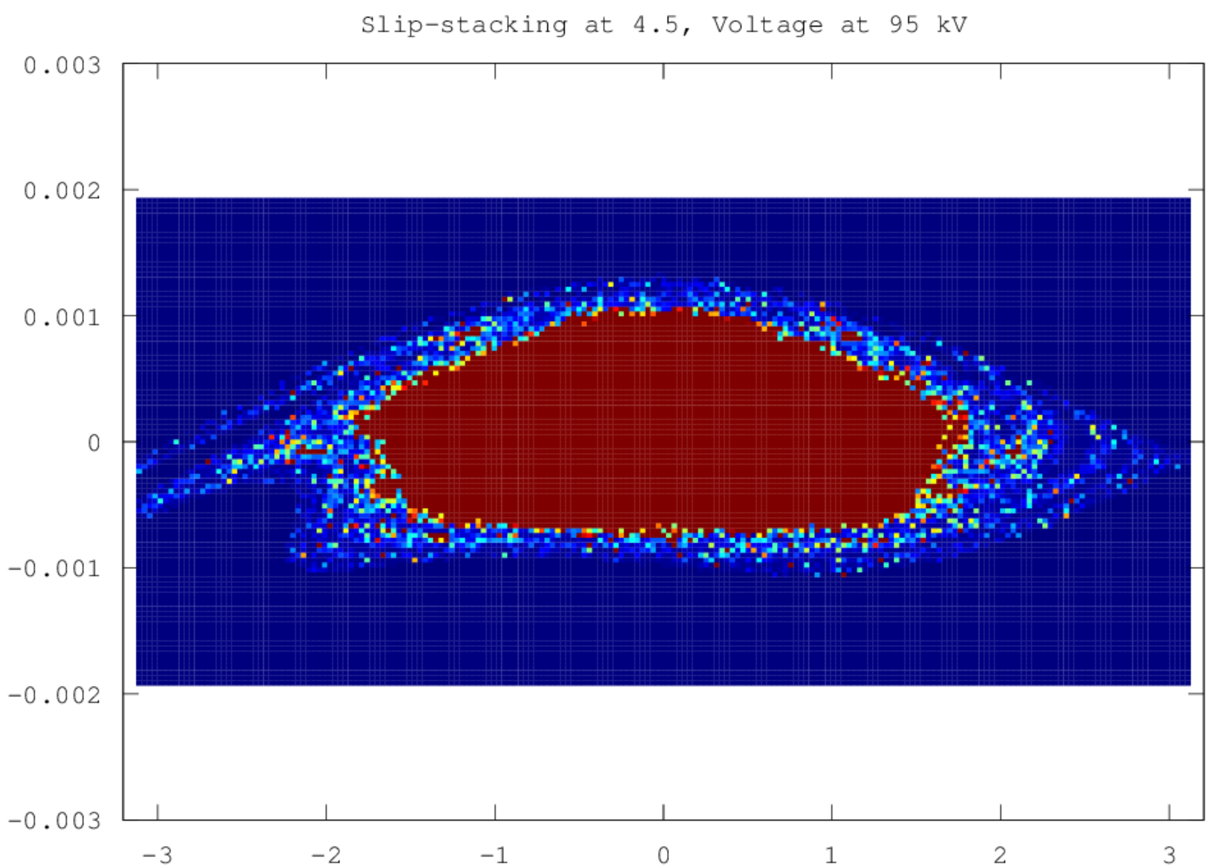

Slip-stacking at 4.9, Voltage at 80 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3


Slip-stacking at 4.8, Voltage at 83 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

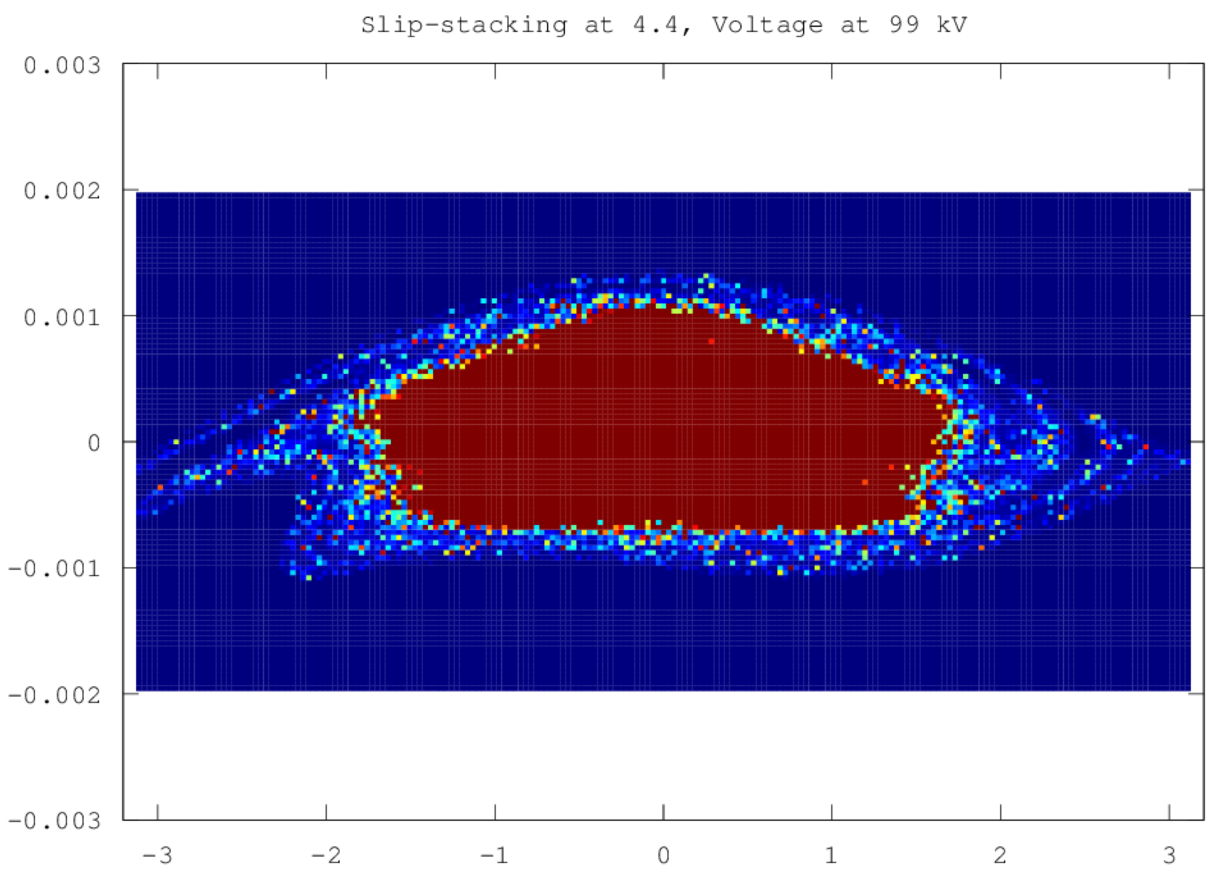


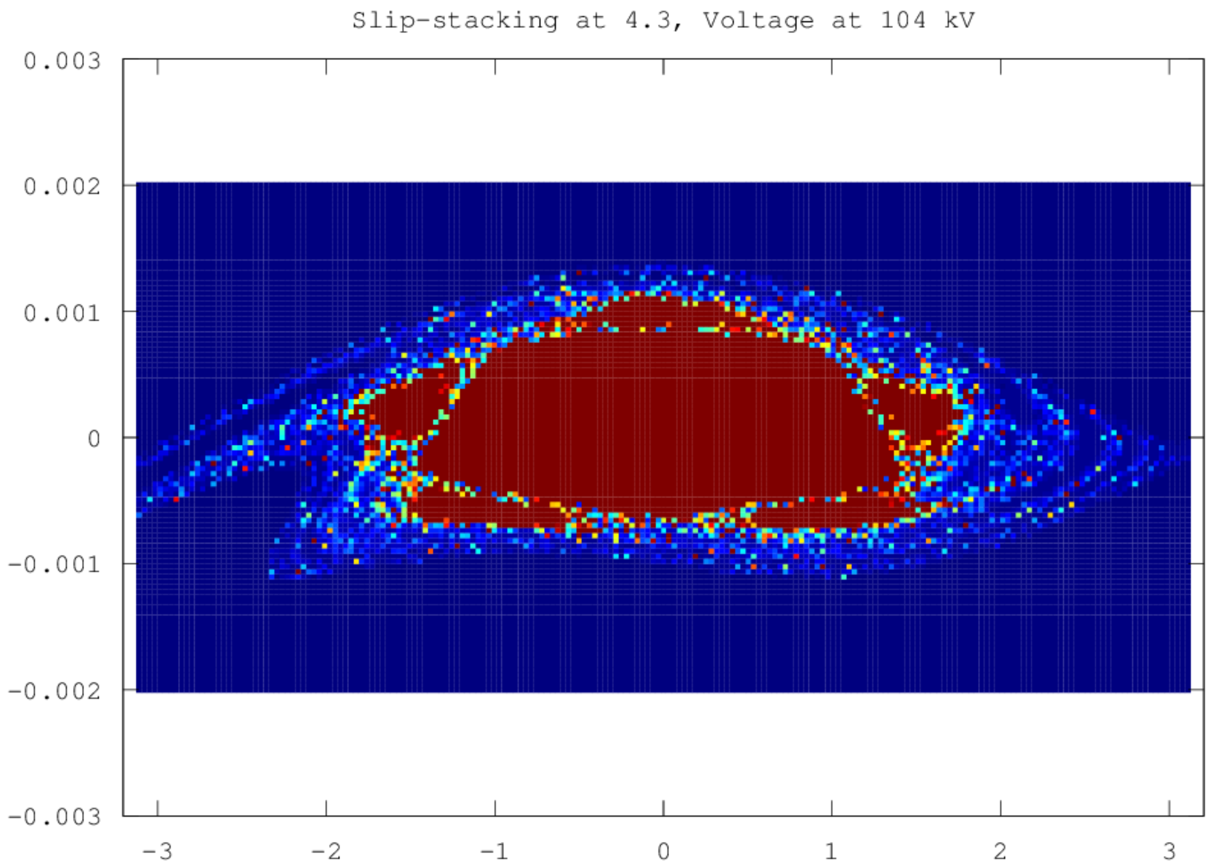
Slip-stacking at 4.7, Voltage at 87 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

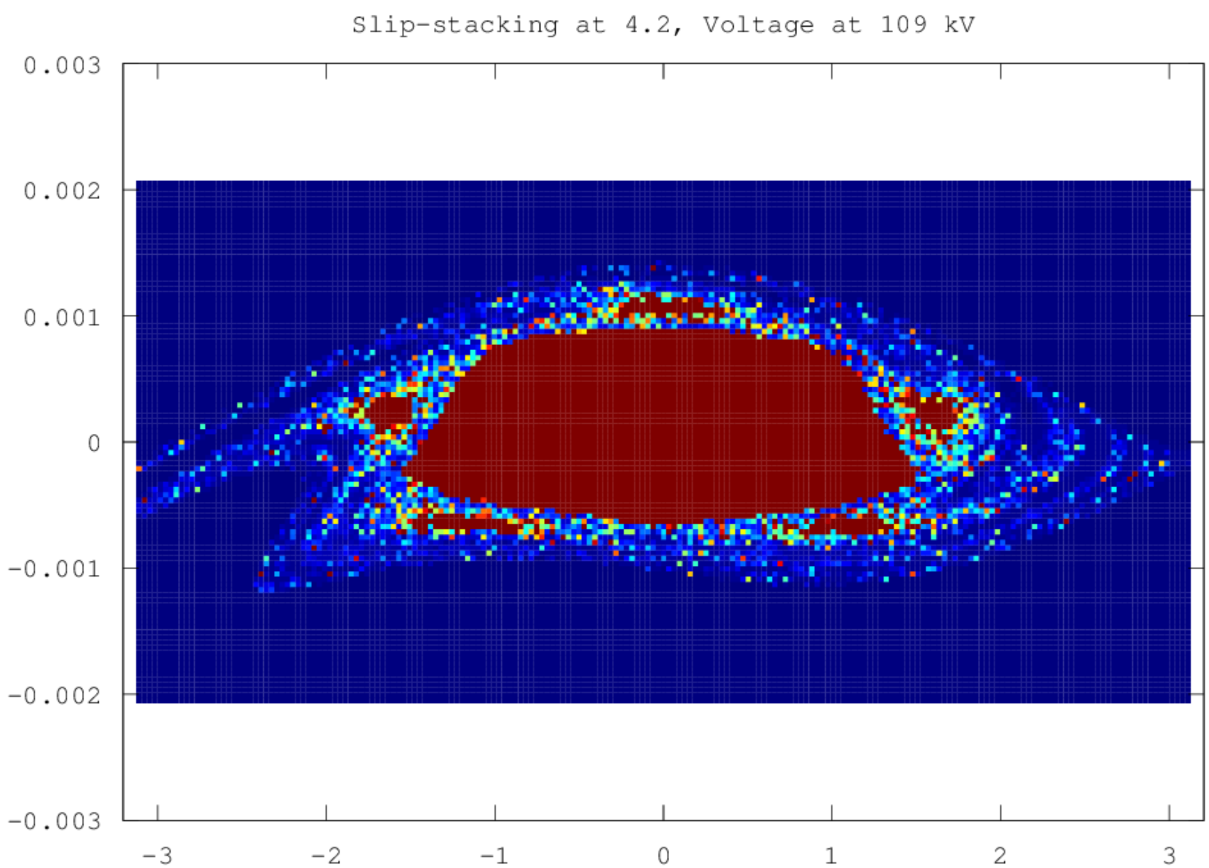

Slip-stacking at 4.6, Voltage at 91 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3



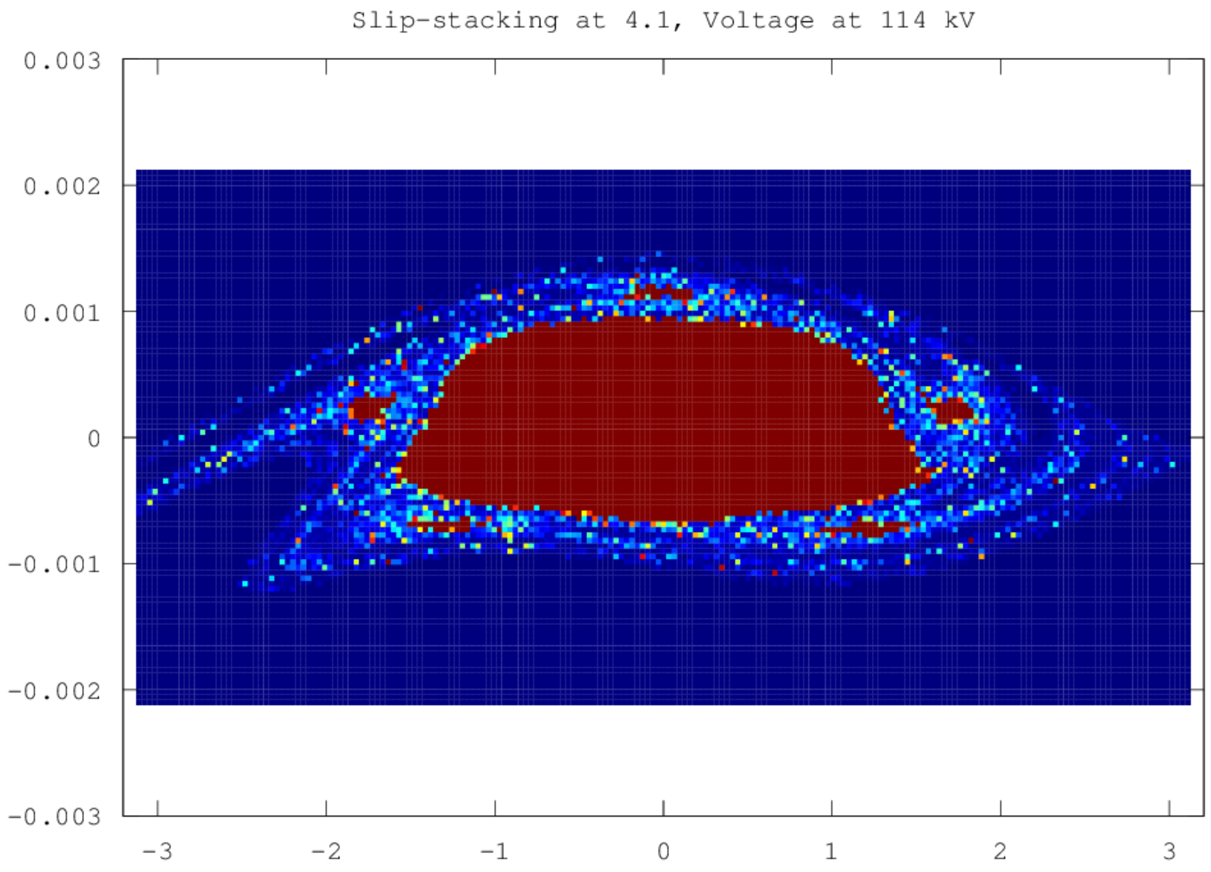
Slip-stacking at 4.5, Voltage at 95 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3


Slip-stacking at 4.4, Voltage at 99 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

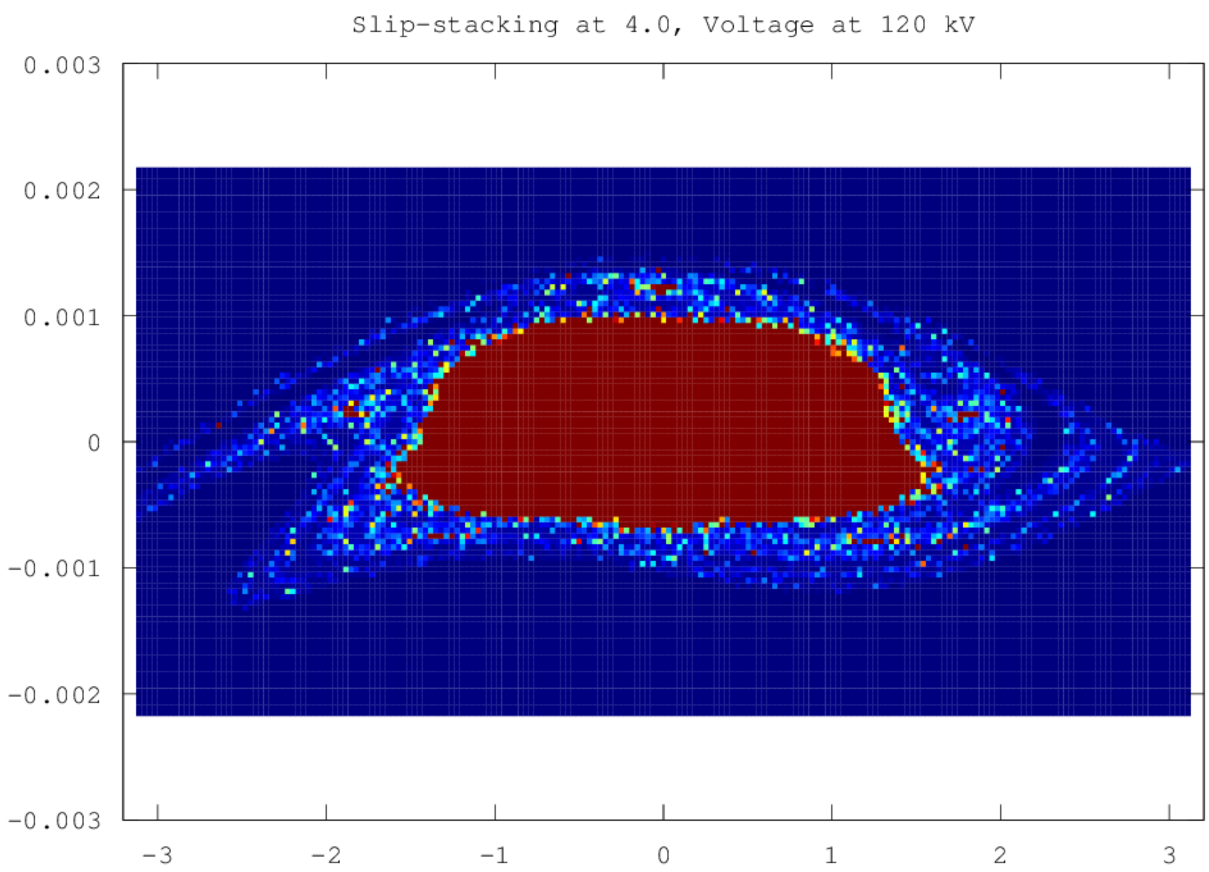

Slip-stacking at 4.3, Voltage at 104 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3



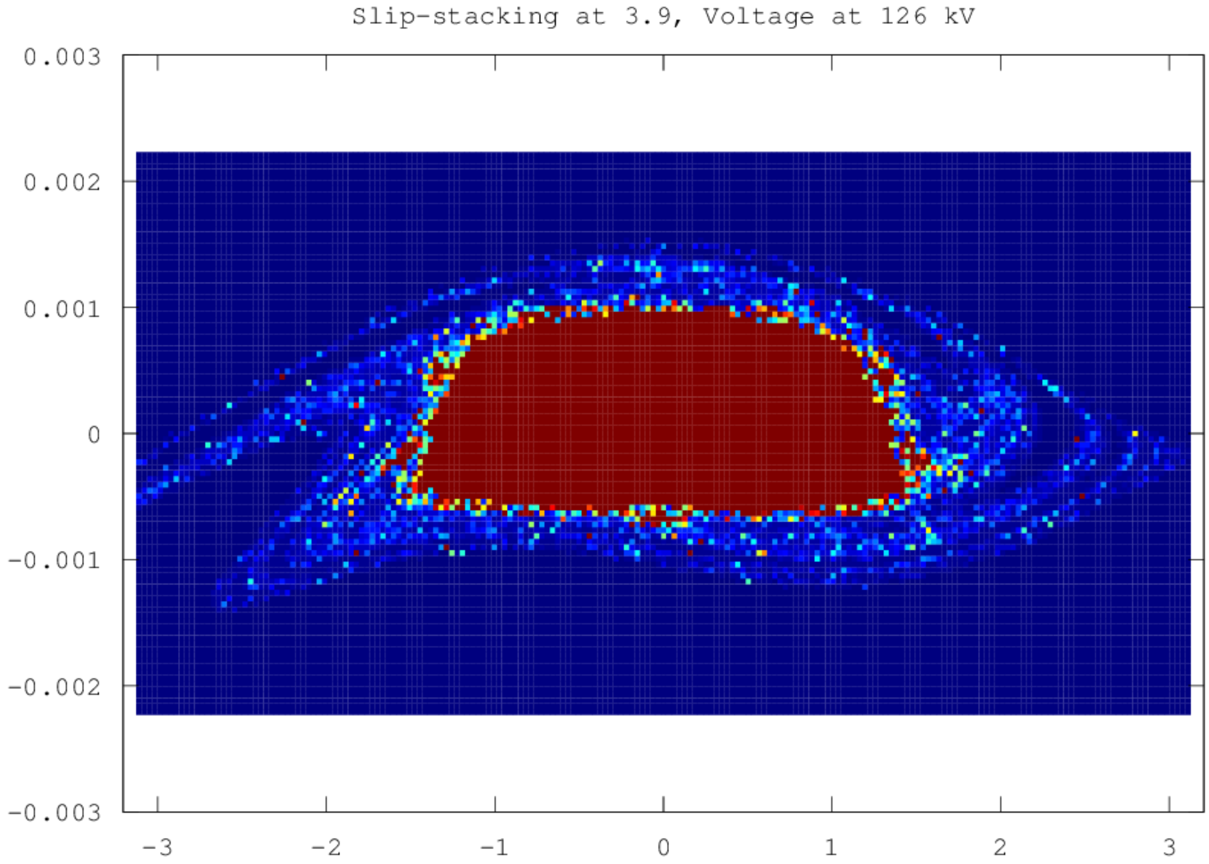
Slip-stacking at 4.2, Voltage at 109 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

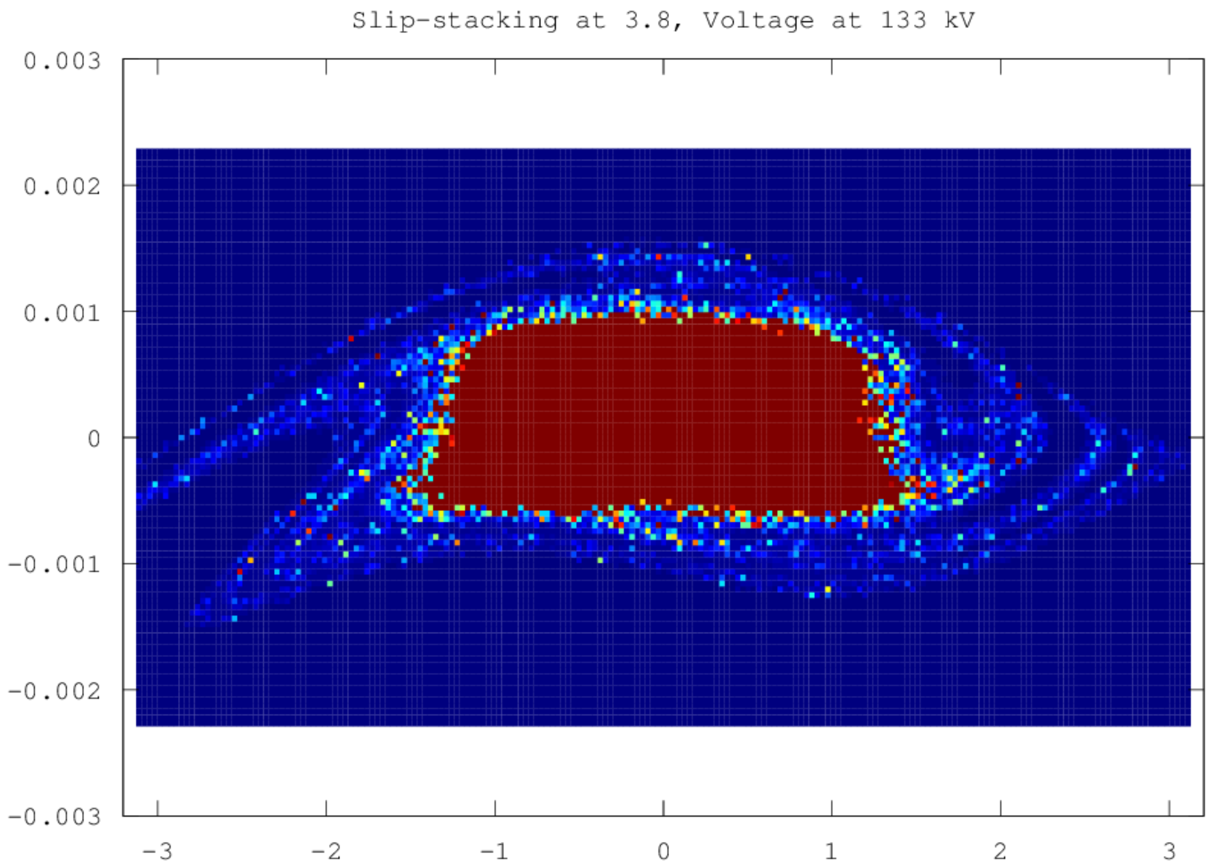


Slip-stacking at 4.1, Voltage at 114 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

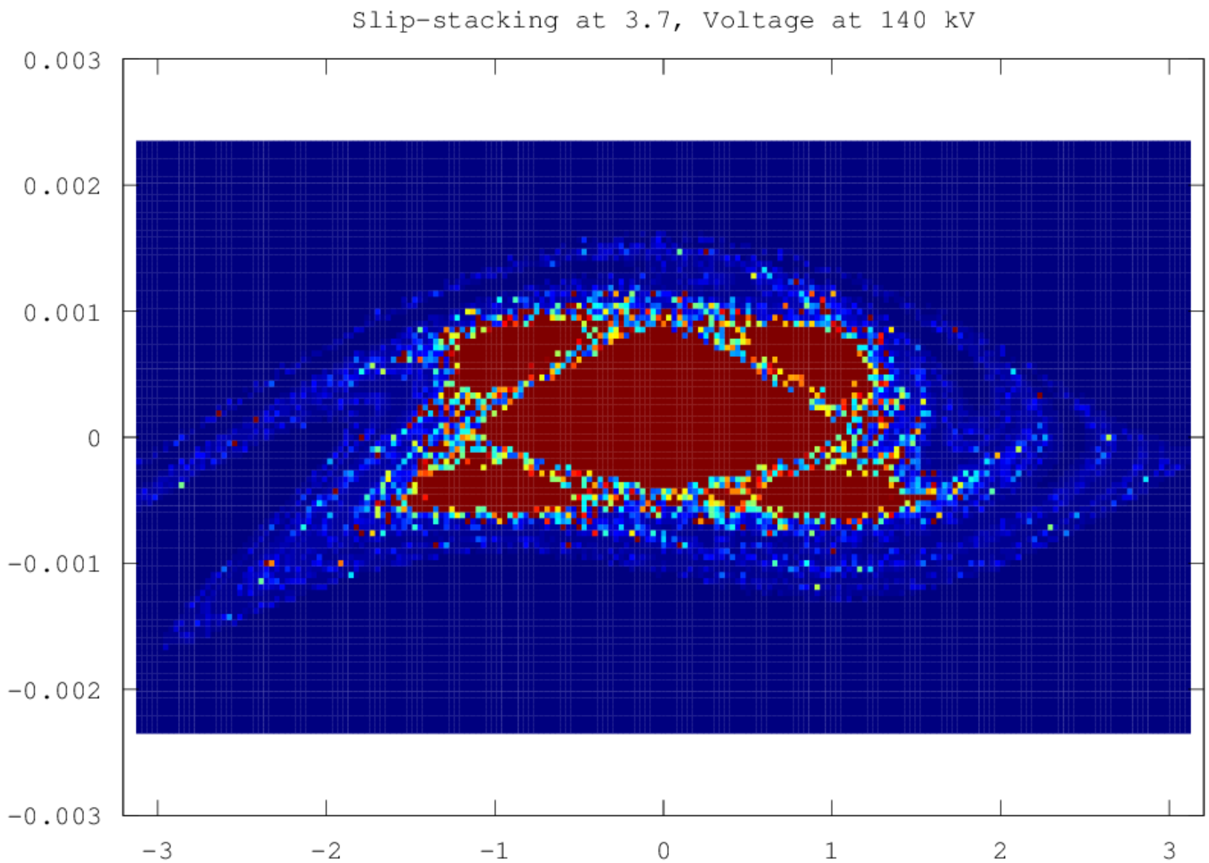


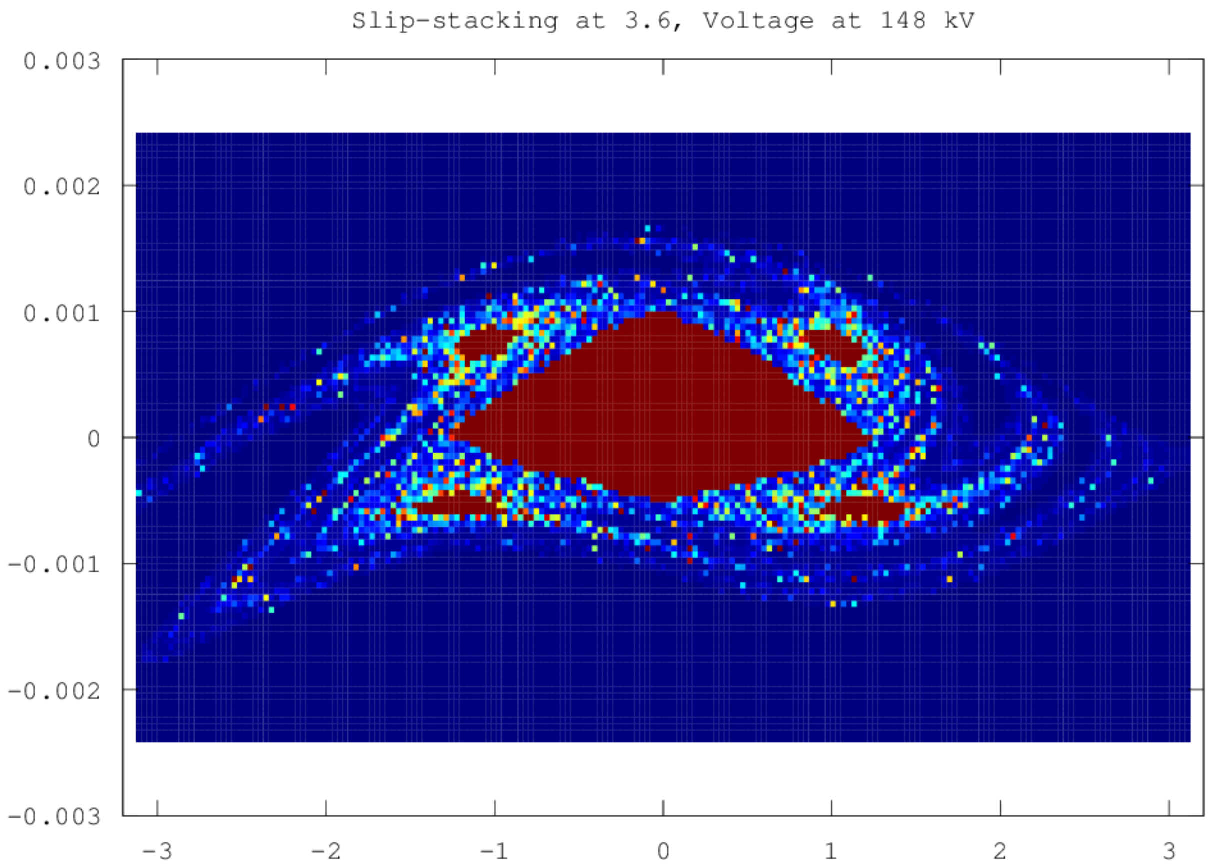
Slip-stacking at 4.0, Voltage at 120 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

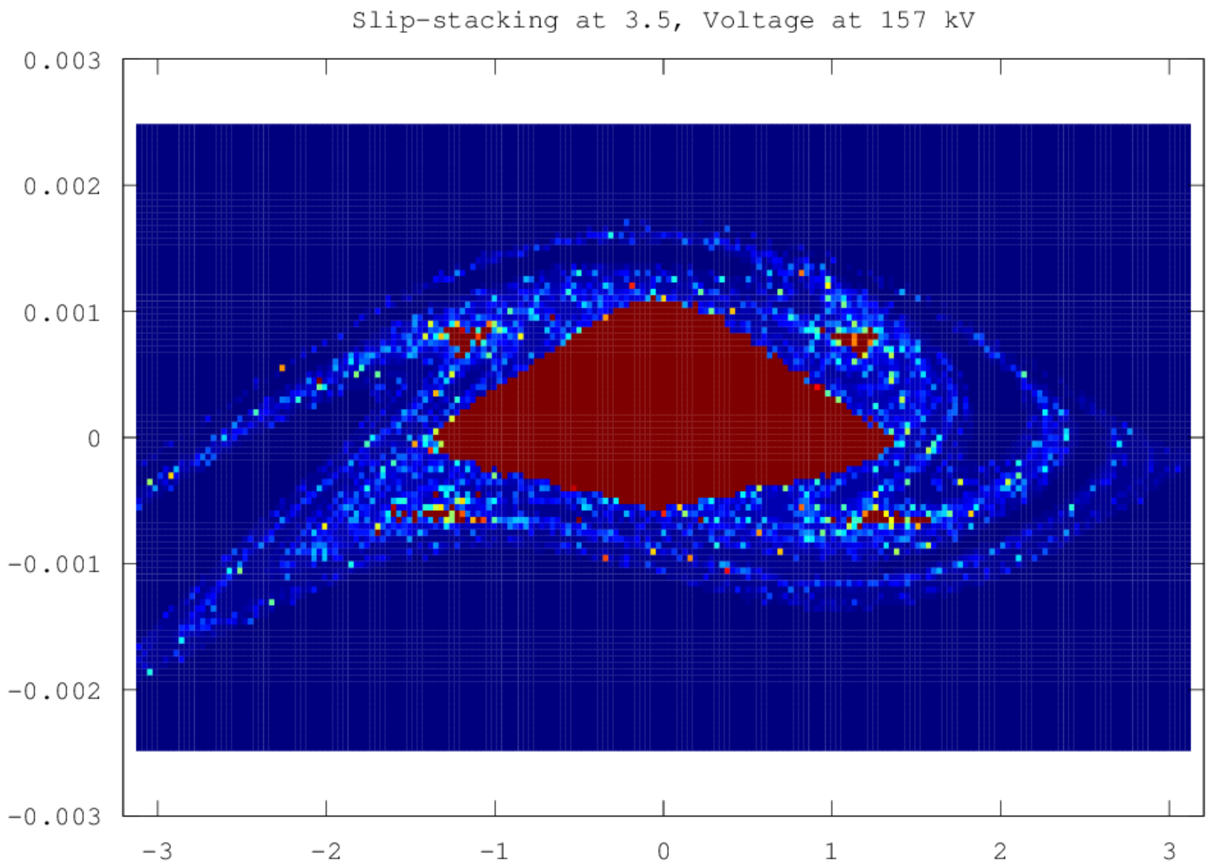

Slip-stacking at 3.9, Voltage at 126 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

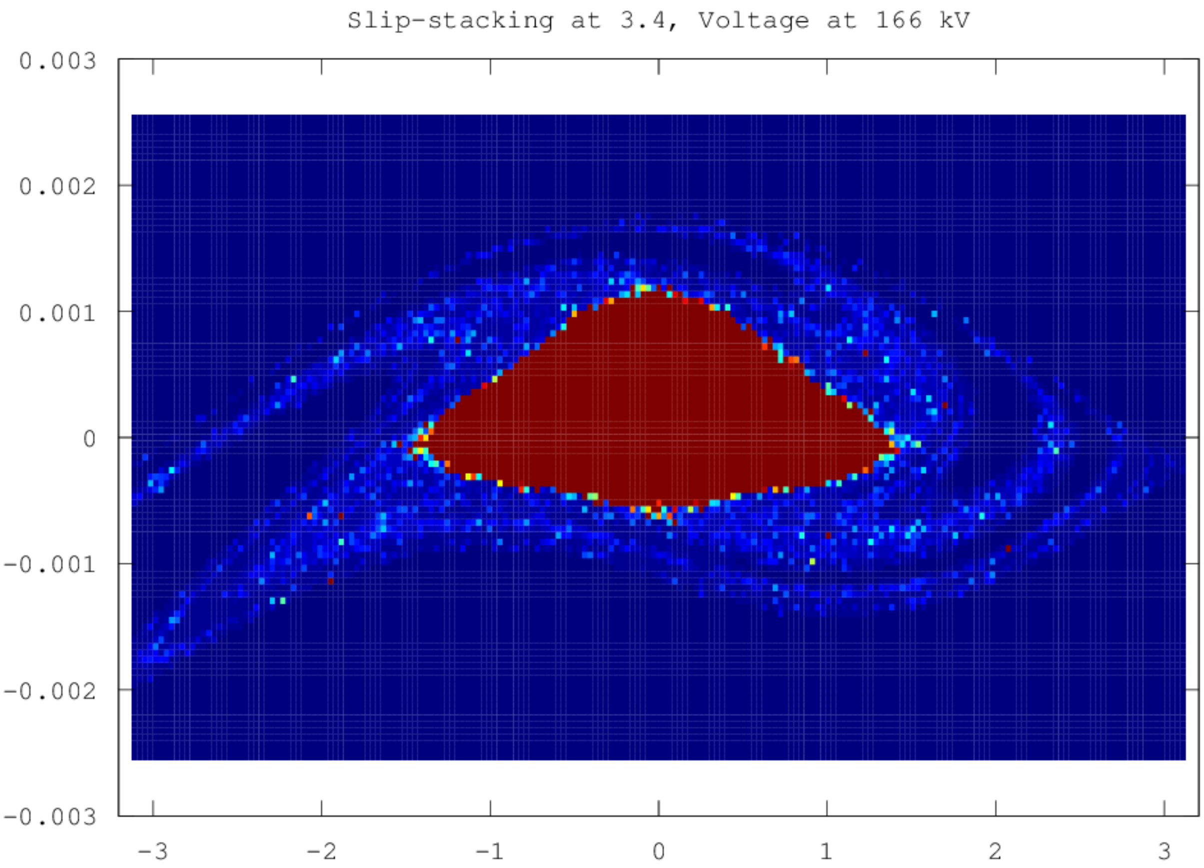


Slip-stacking at 3.8, Voltage at 133 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

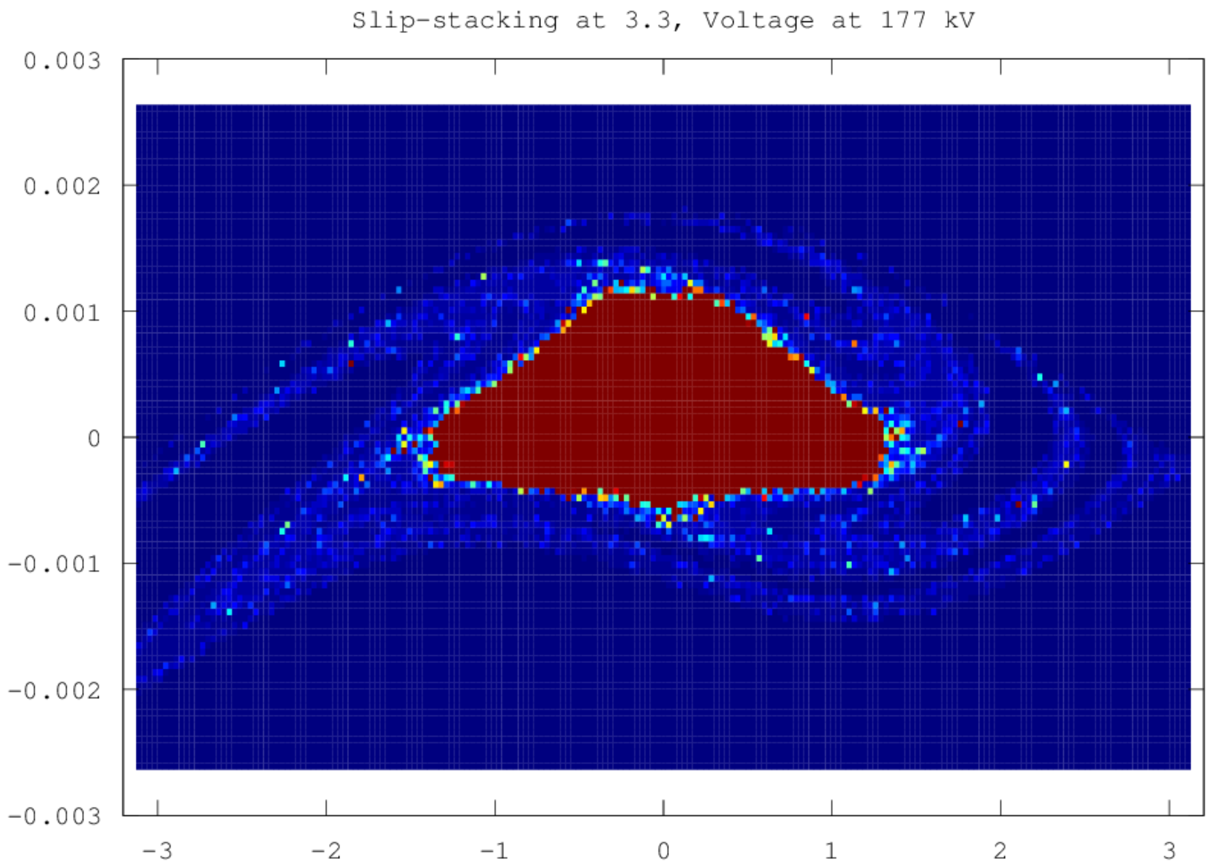

Slip-stacking at 3.7, Voltage at 140 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3


Slip-stacking at 3.6, Voltage at 148 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

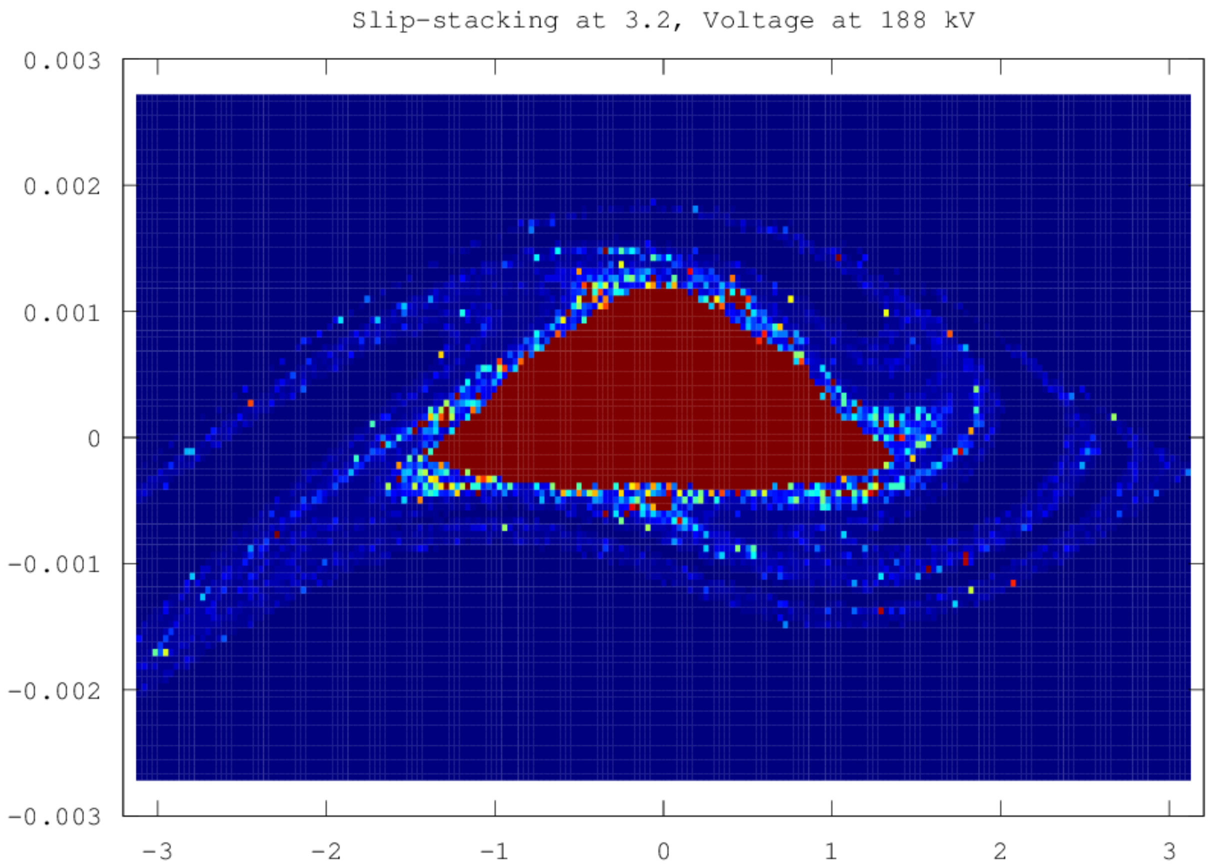

Slip-stacking at 3.5, Voltage at 157 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

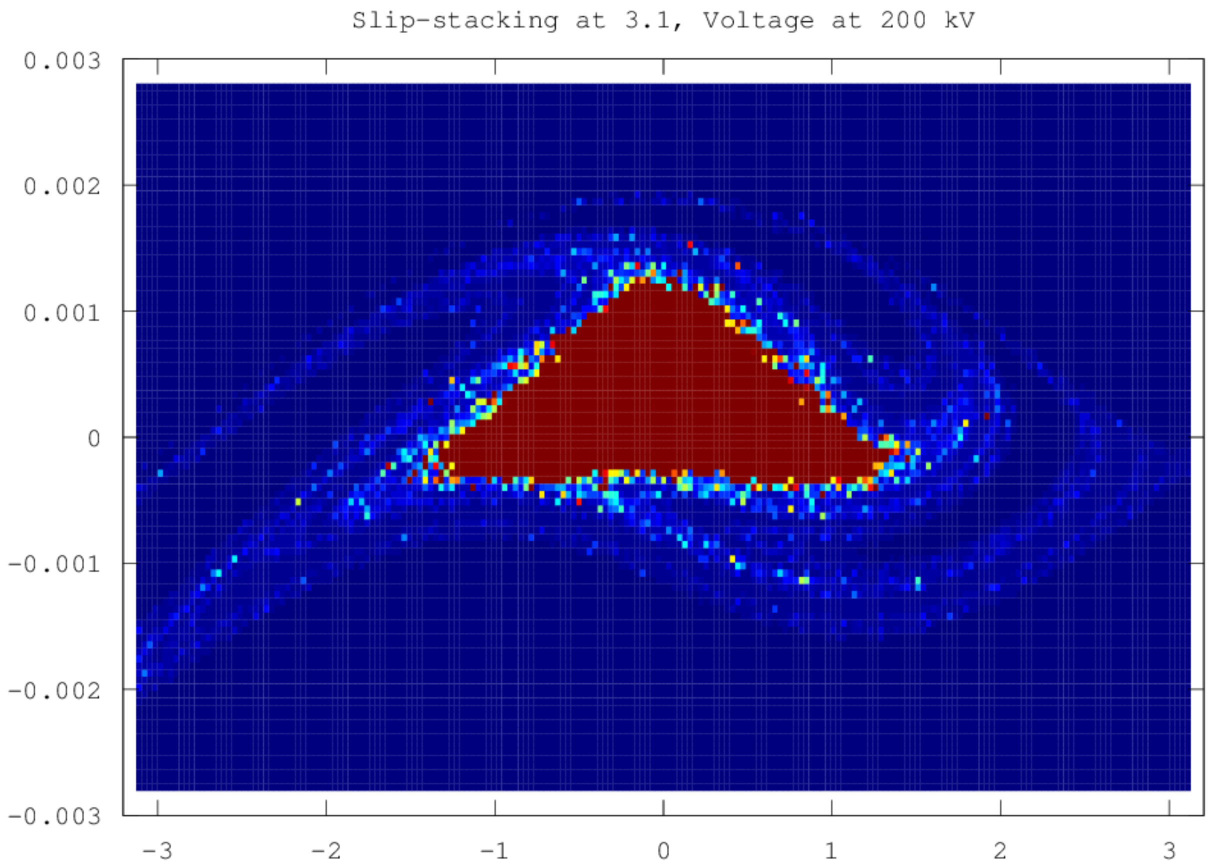


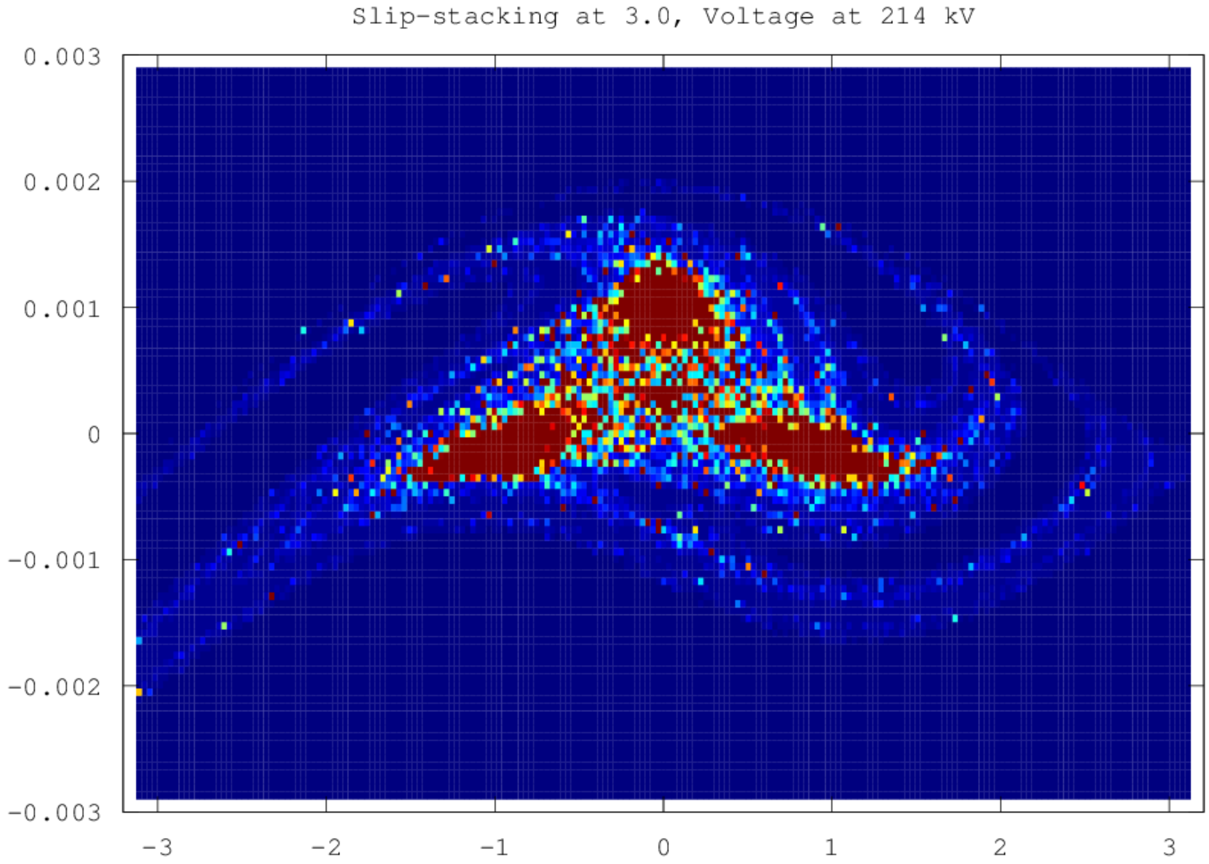
Slip-stacking at 3.4, Voltage at 166 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3



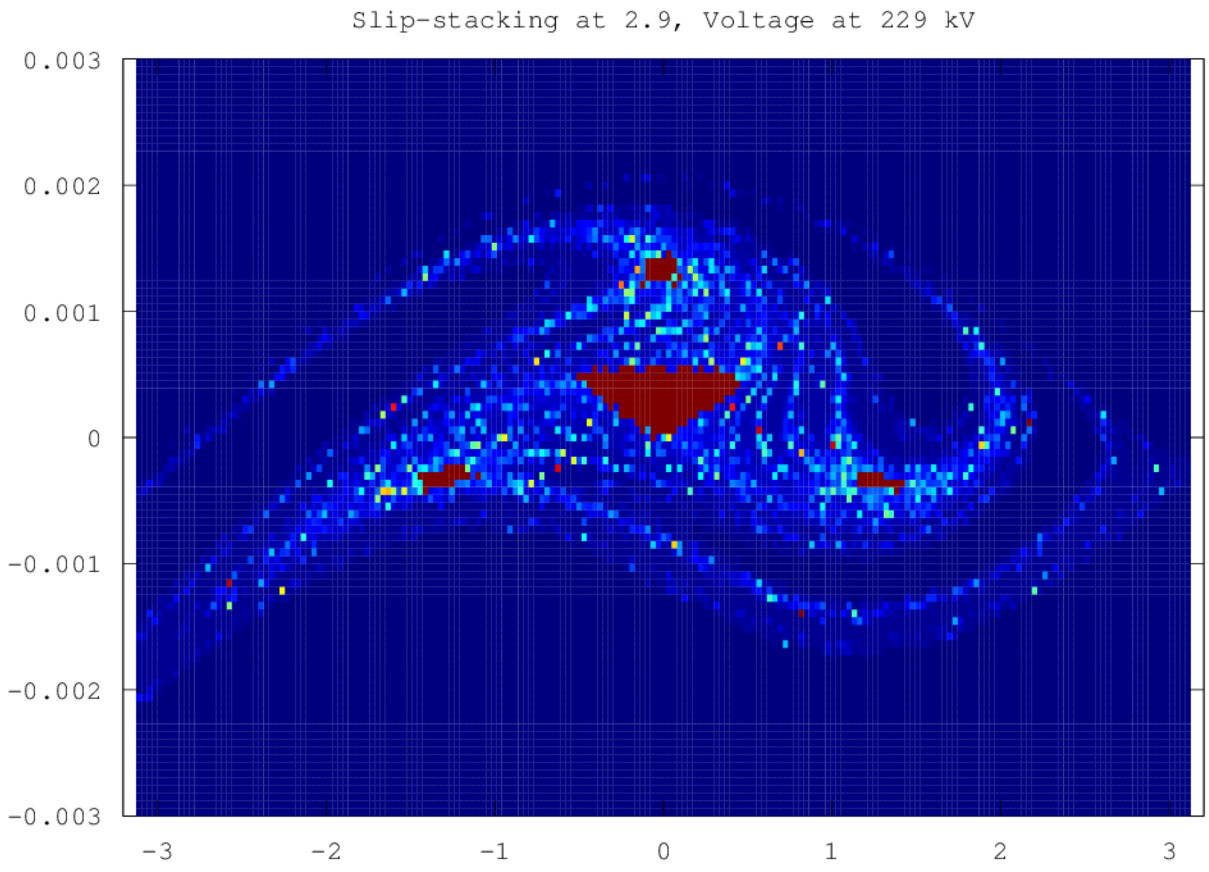
Slip-stacking at 3.3, Voltage at 177 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

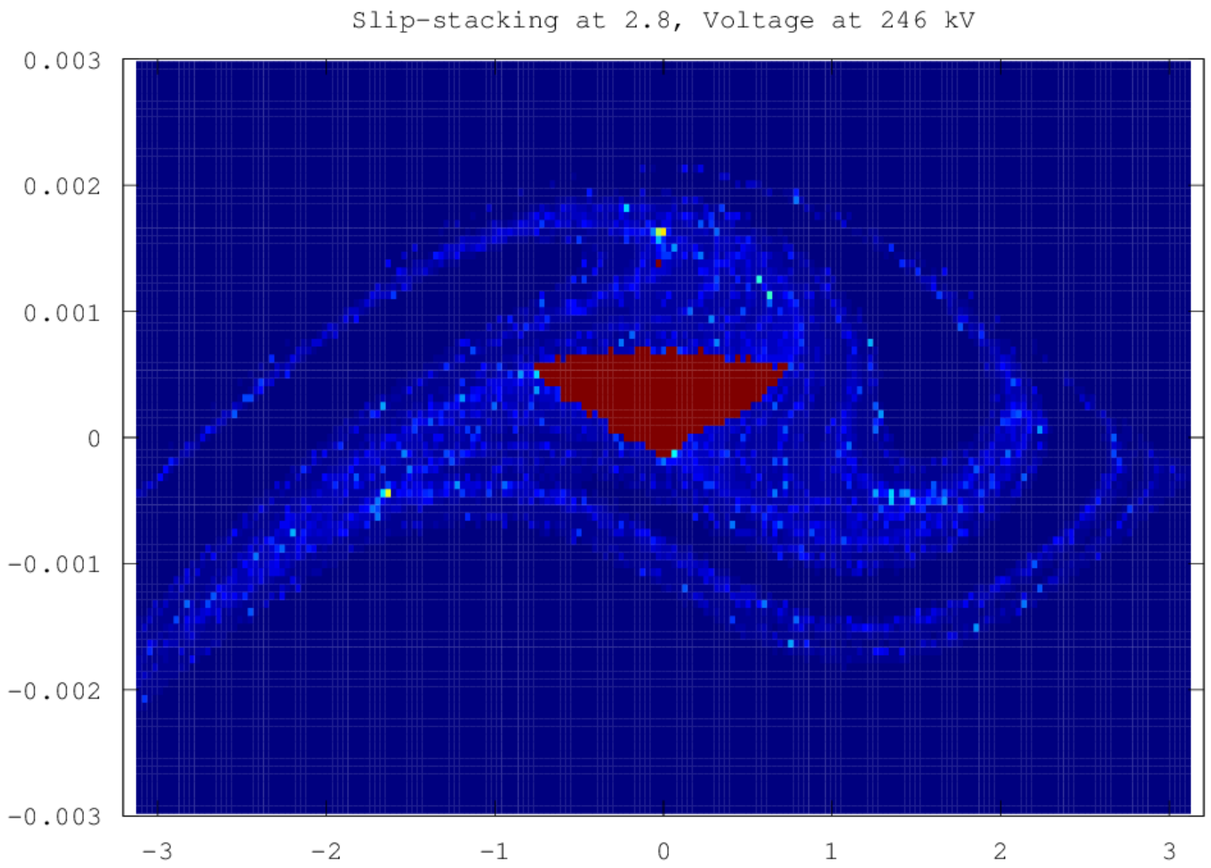

Slip-stacking at 3.2, Voltage at 188 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3


Slip-stacking at 3.1, Voltage at 200 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

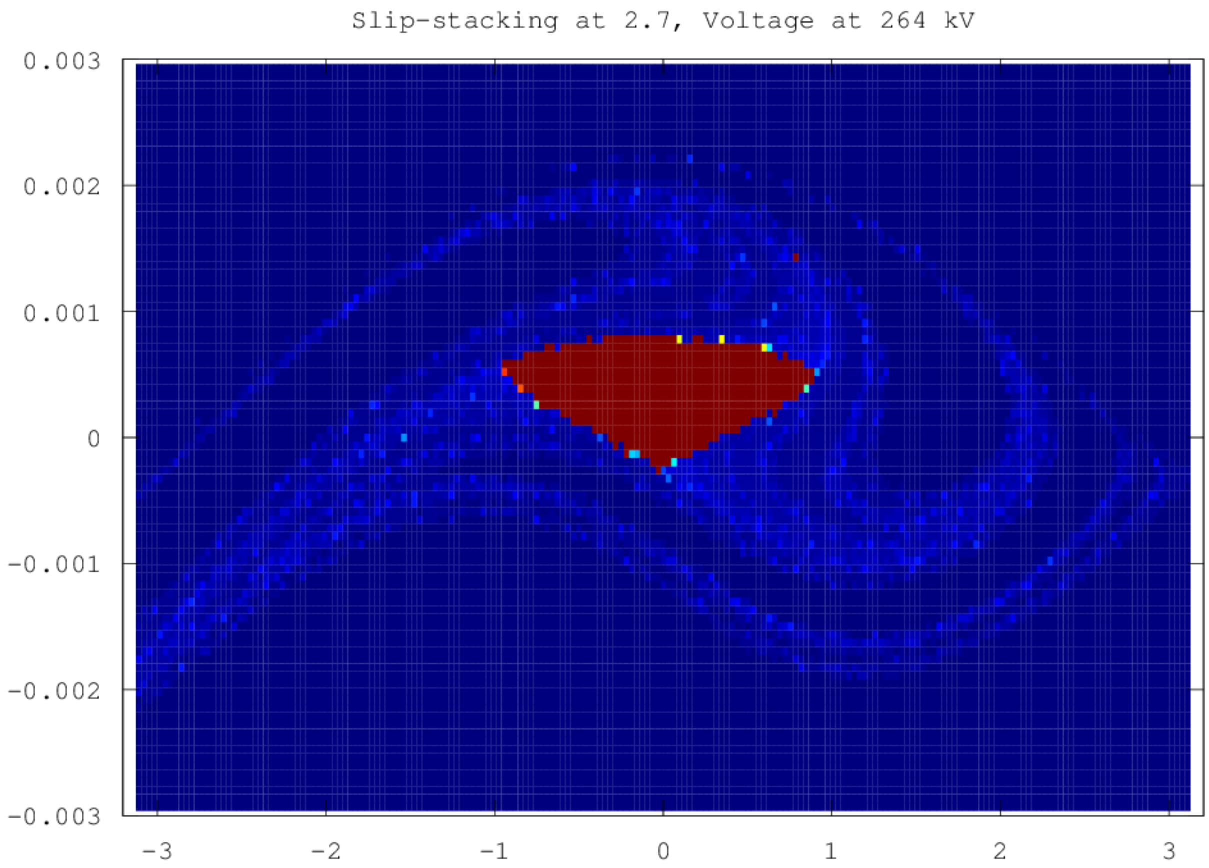


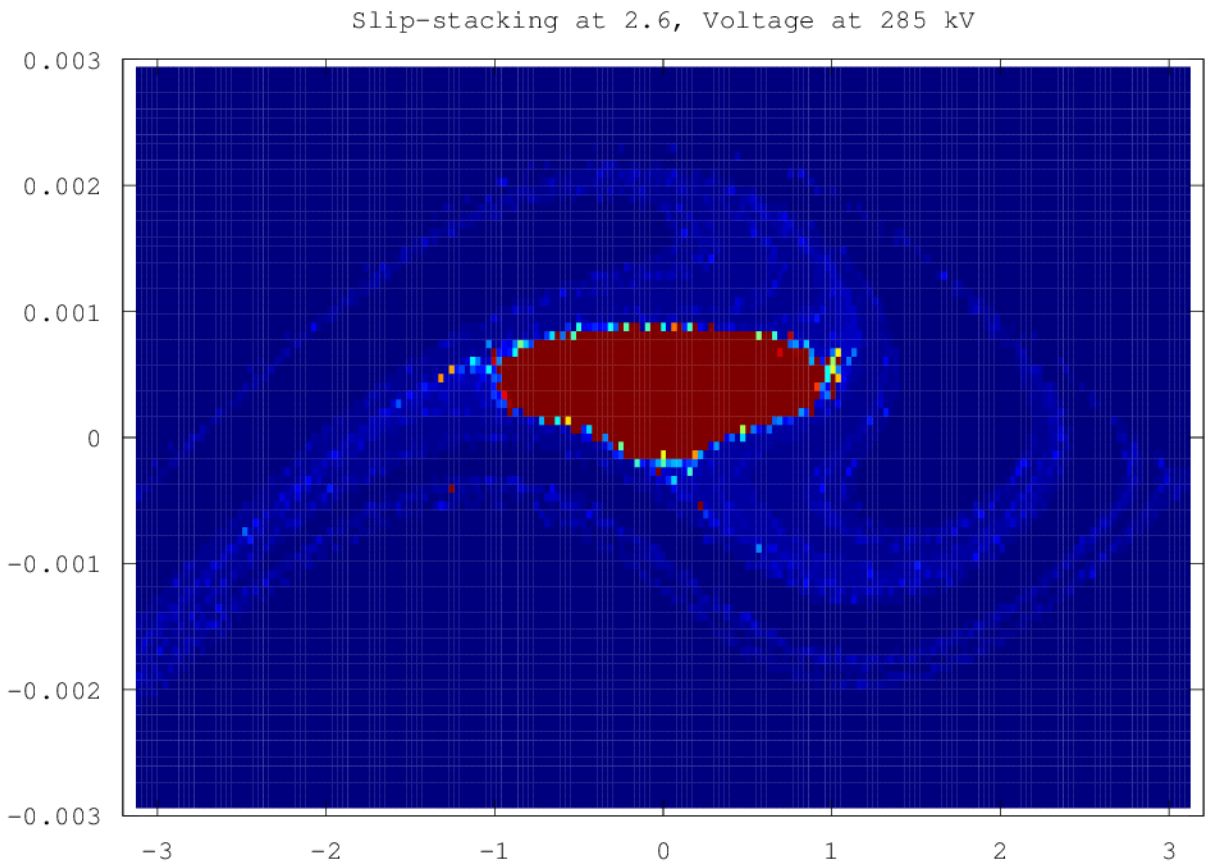
Slip-stacking at 3.0, Voltage at 214 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

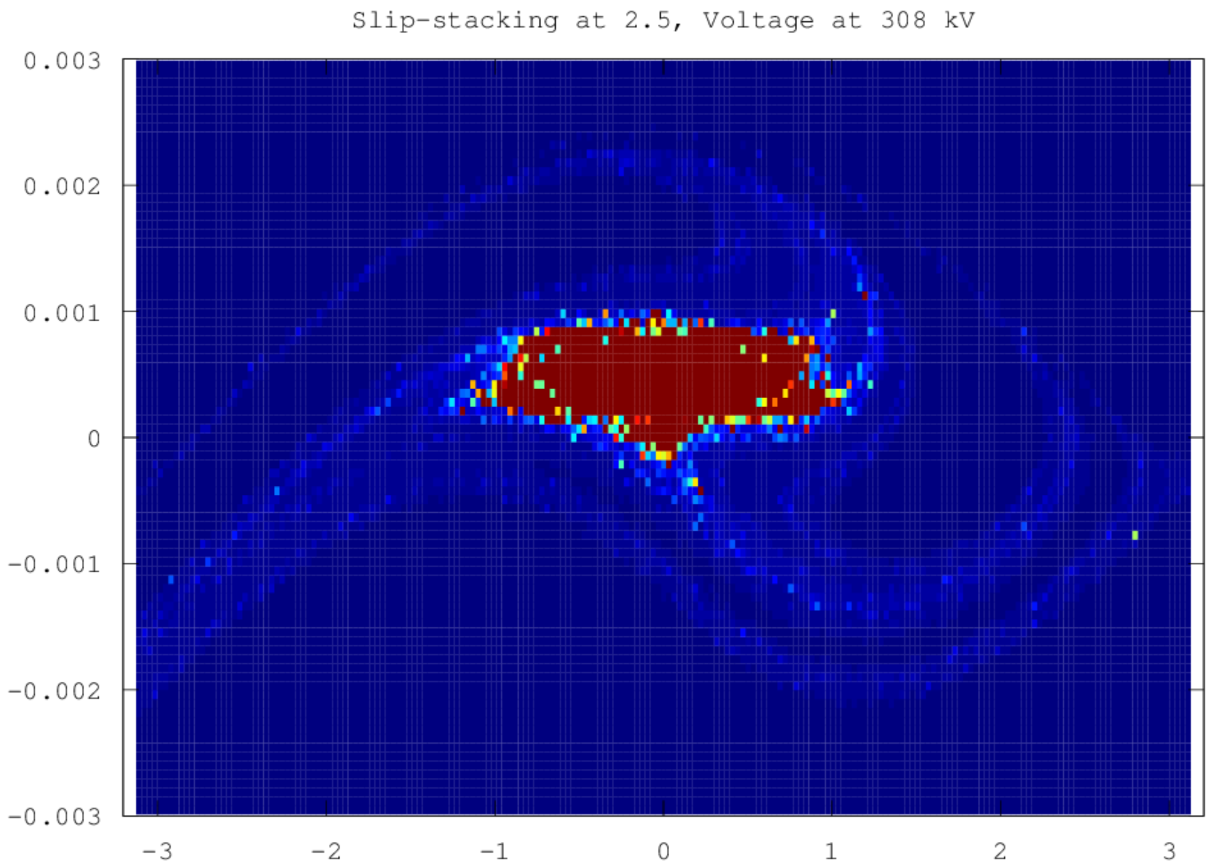

Slip-stacking at 2.9, Voltage at 229 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

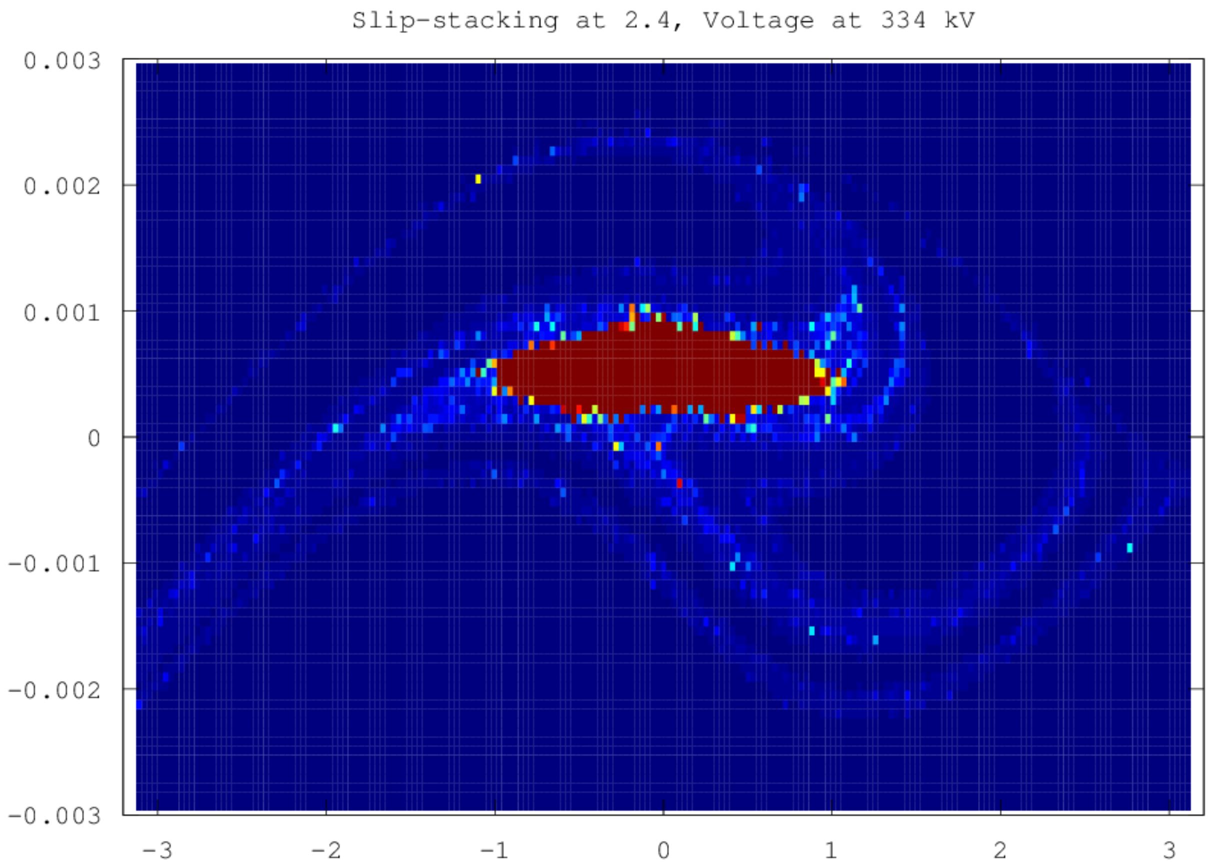


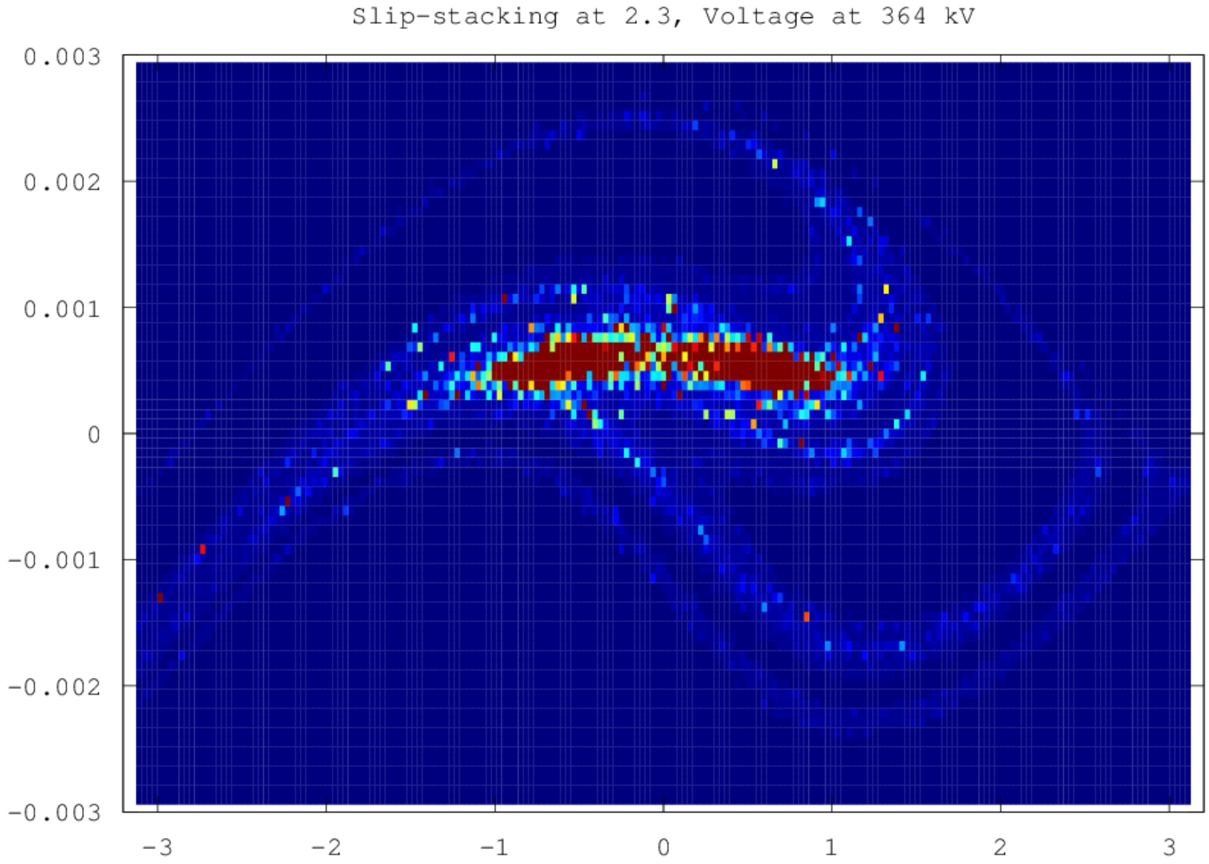
Slip-stacking at 2.8, Voltage at 246 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3


Slip-stacking at 2.7, Voltage at 264 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3


Slip-stacking at 2.6, Voltage at 285 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

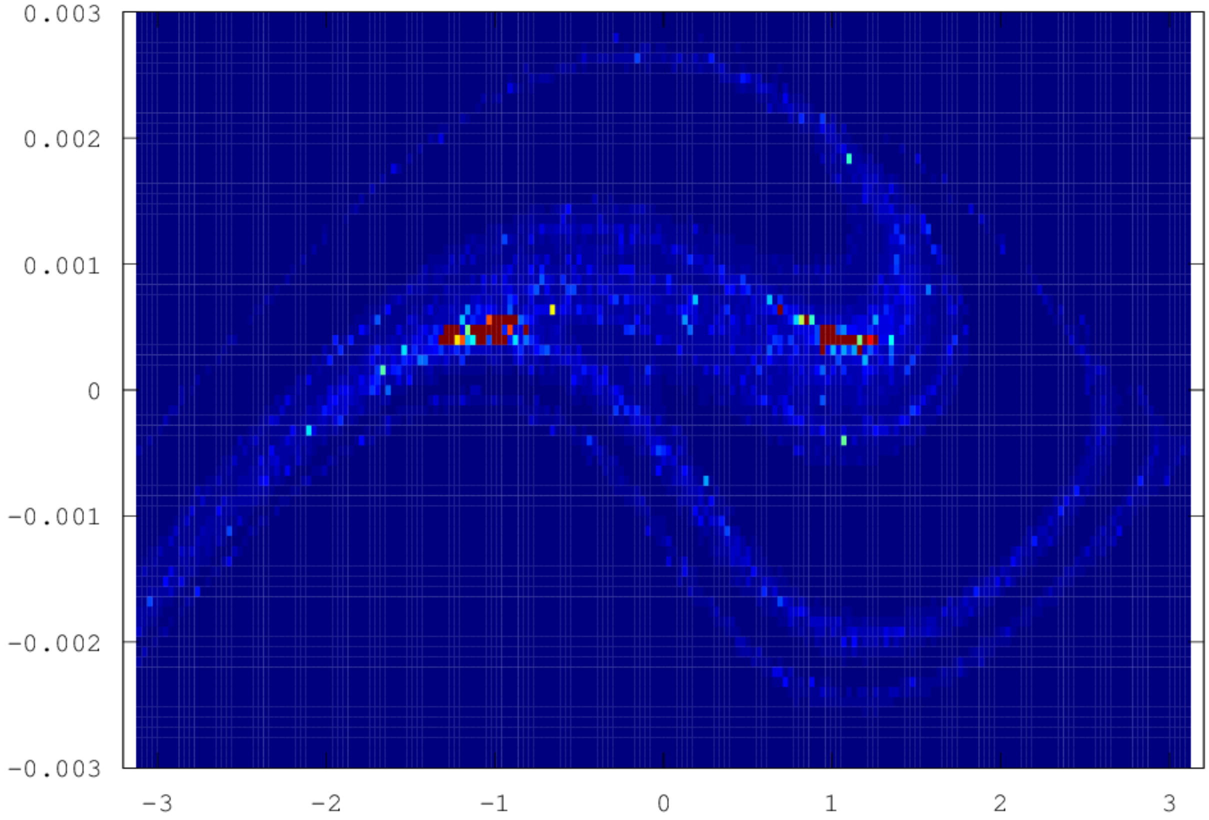

Slip-stacking at 2.5, Voltage at 308 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

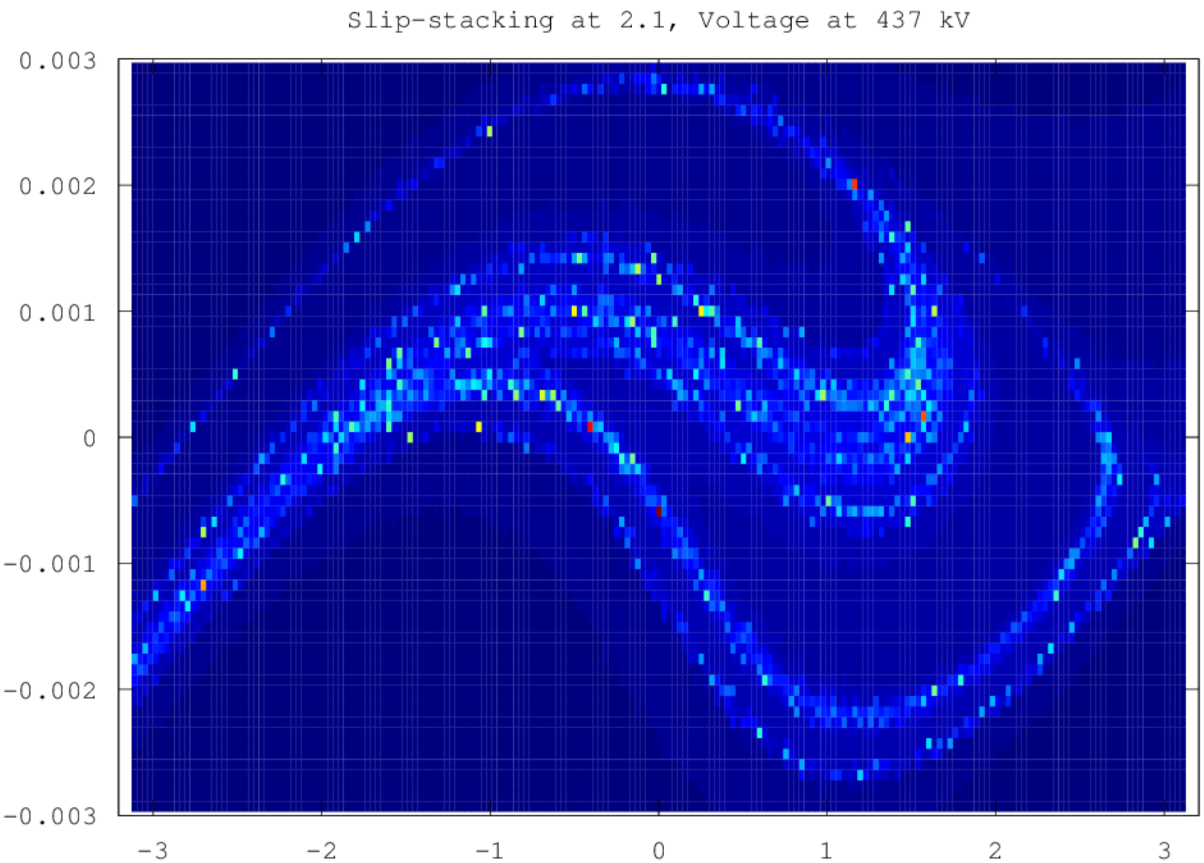

Slip-stacking at 2.4, Voltage at 334 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

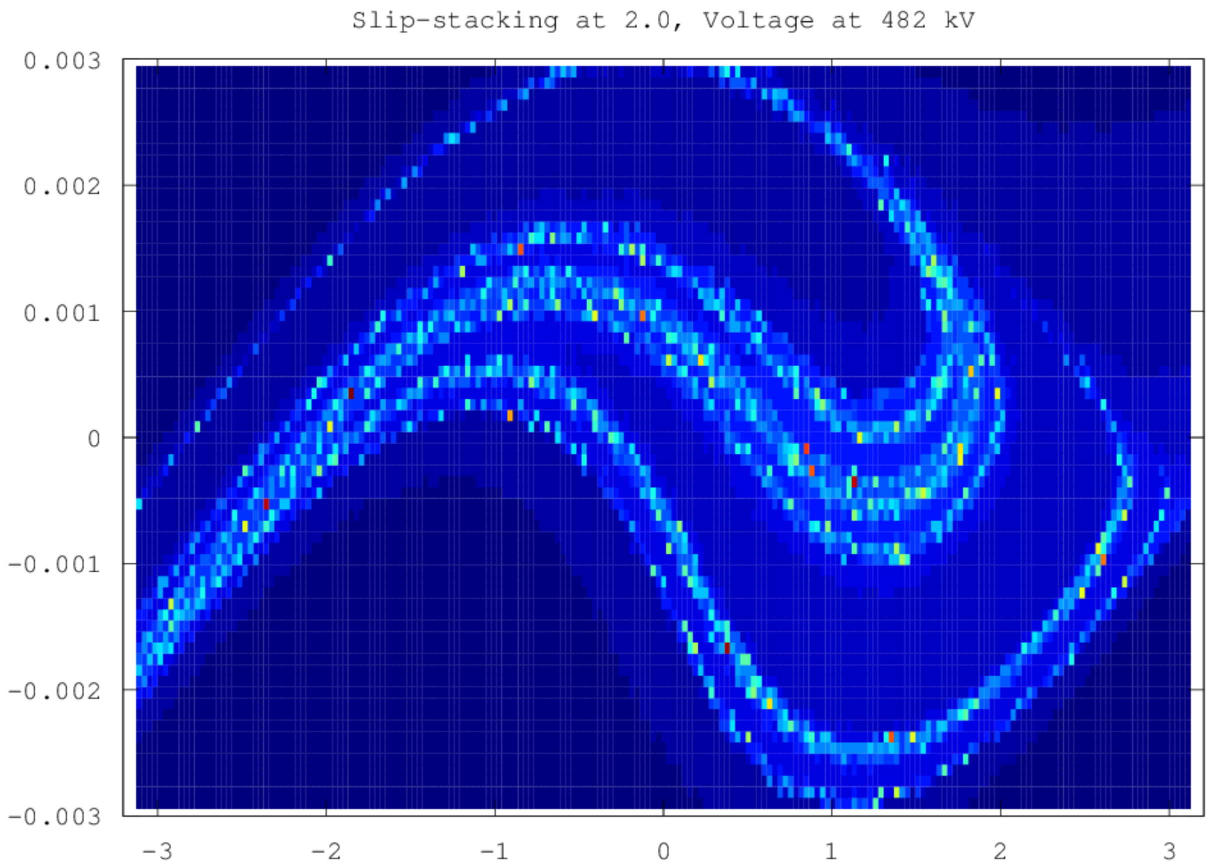


Slip-stacking at 2.3, Voltage at 364 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

Slip-stacking at 2.2, Voltage at 398 kV

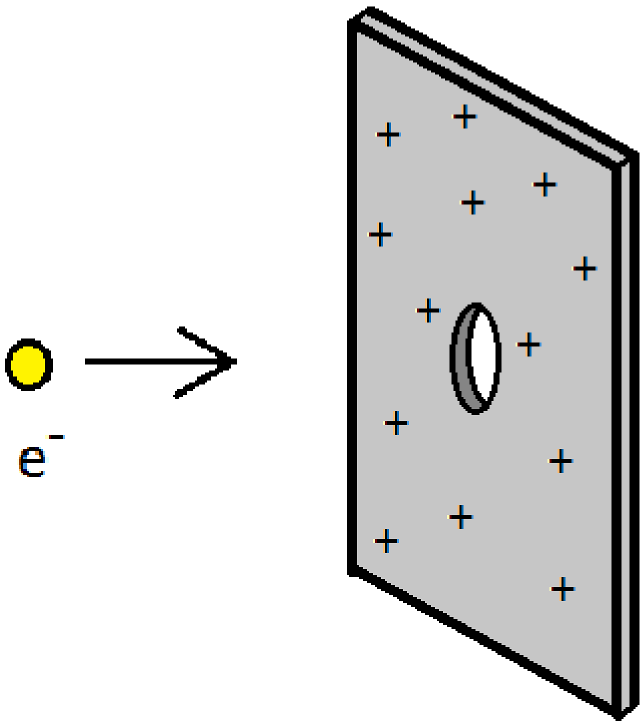



Slip-stacking at 2.1, Voltage at 437 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3


Slip-stacking at 2.0, Voltage at 482 kV
0.003
0.002
0.001
0
-0.001
-0.002
-0.003
-3
-2
-1
0
1
2
3

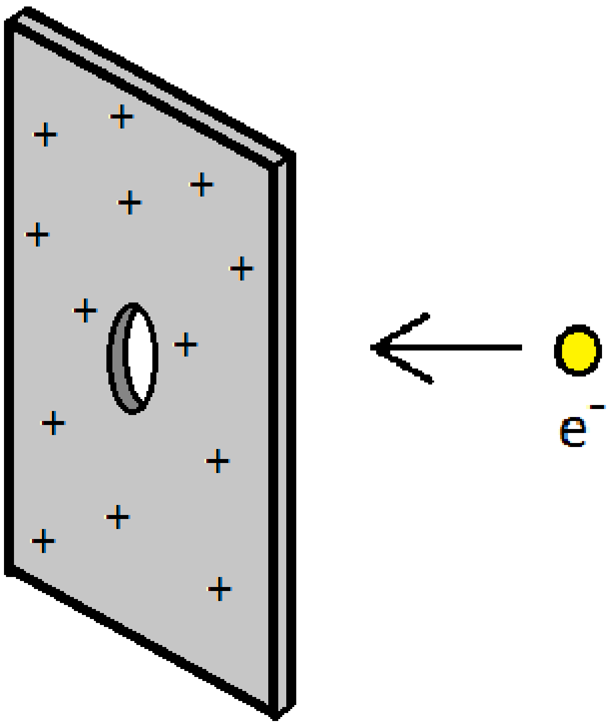
How To Accelerate Particles

One simple idea would be something like this:



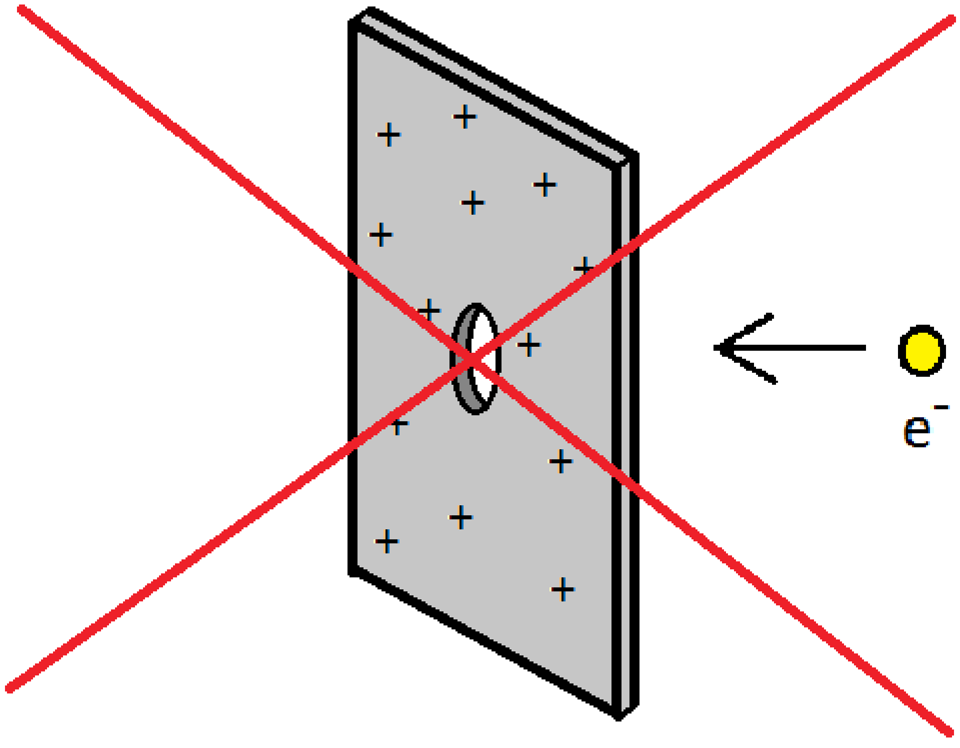
How To Accelerate Particles

But there would be no net acceleration:



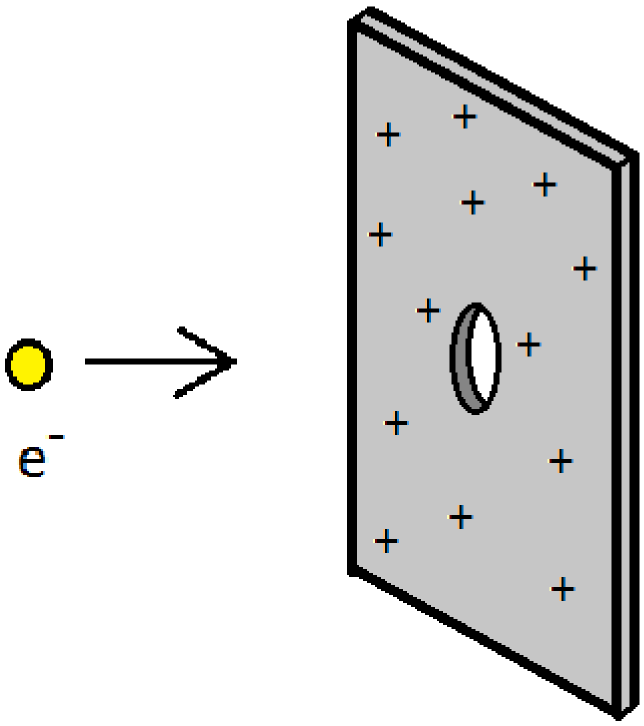
How To Accelerate Particles

So don't do that:



How To Accelerate Particles

You need a time-dependent field for net acceleration:



How To Accelerate Particles

You need a time-dependent field for net acceleration: