Understanding J/ψ Production
40 years after its discovery

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Based on works done with Z.-B. Kang, Y.-Q. Ma, G. Nayak,
G. Sterman, H. Zhang, …

Loopfest XIII
New York City College of Technology, Brooklyn, NY, June 18-20, 2014

November revolution (1974)

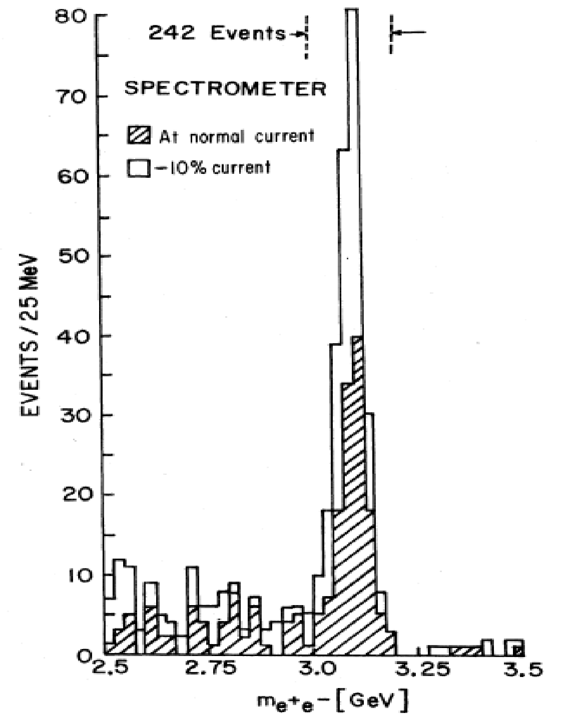
VOLUME 33, NUMBER 23 PHYSICAL REVIEW LETTERS 2 DECEMBER 19

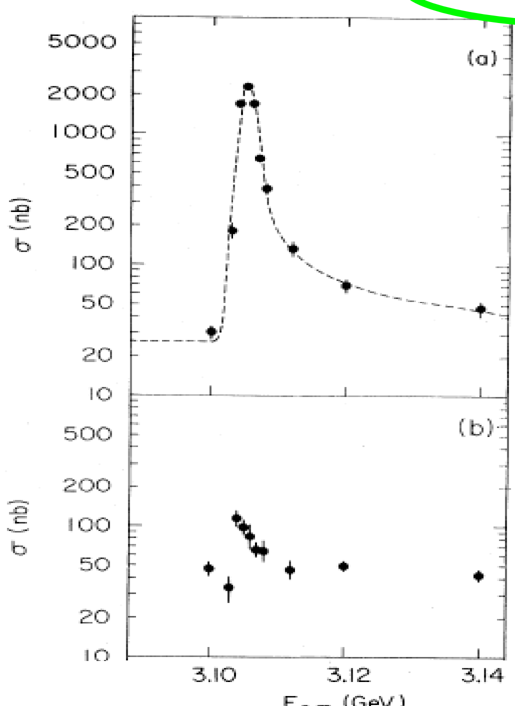
Experimental Observation of a Heavy Particle J†

J. J. Aubert, U. Becker, P. J. Biggs, J. Burger, M. Chen, G. Everhart, P. Goldhagen, J. Leong, T. McCorriston, T. G. Rhoades, M. Rohde, Samuel C. C. Ting, and Sau Lan Wu
Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

and

Y. Y. Lee
Brookhaven National Laboratory, Upton, New York 11973
(Received 12 November 1974)





November, 1974

Discovery of a Narrow Resonance in e^+e^- Annihilation*

J.-E. Augustin,† A. M. Boyarski, M. Breidenbach, F. Bulos, J. T. Dakin, G. J. Feldman, G. E. Fischer, D. Fryberger, G. Hanson, B. Jean-Marie,† R. R. Larsen, V. Lüth, H. L. Lynch, D. Lyon, C. C. Morehouse, J. M. Paterson, M. L. Perl, B. Richter, P. Rapidis, R. F. Schwitters, W. M. Tanenbaum, and F. Vannucci‡
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

and

G. S. Abrams, D. Briggs, W. Chinowsky, C. E. Friedberg, G. Goldhaber, R. J. Hollebeek, J. A. Kadyk, B. Lulu, F. Pierre,§ G. H. Trilling, J. S. Whitaker, J. Wiss, and J. E. Zipse
Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, California 94720
(Received 13 November 1974)

Question

Do we really know

How a heavy quarkonium was produced

in high energy collisions?

A long history for the production

- Color singlet model: 1975 –

  Einhorn, Ellis (1975),
  Chang (1980),
  Berger and Jone (1981), …

  Only the pair with right quantum numbers

  Effectively No free parameter!

- Color evaporation model: 1977 –

  Fritsch (1977), Halzen (1977), …

  All pairs with mass less than open flavor heavy meson threshold

  One parameter per quarkonium state

- NRQCD model: 1986 –

  Caswell, Lapage (1986)
  Bodwin, Braaten, Lepage (1995)
  QWG review: 2004, 2010

  All pairs with various probabilities – NRQCD matrix elements

  Infinite parameters – organized in powers of v and α_s

- QCD factorization approach: 2005 –

  Nayak, Qiu, Sterman (2005), …
  Kang, Qiu, Sterman (2010), …

  $P_T >> M_H$: M_H/P_T power expansion + α_s – expansion

  Unknown, but universal, fragmentation functions – evolution

- Soft-Collinear Effective Theory + NRQCD: 2012 –

  Fleming, Leibovich, Mehen, …

NRQCD – most successful so far

- **NRQCD factorization:**

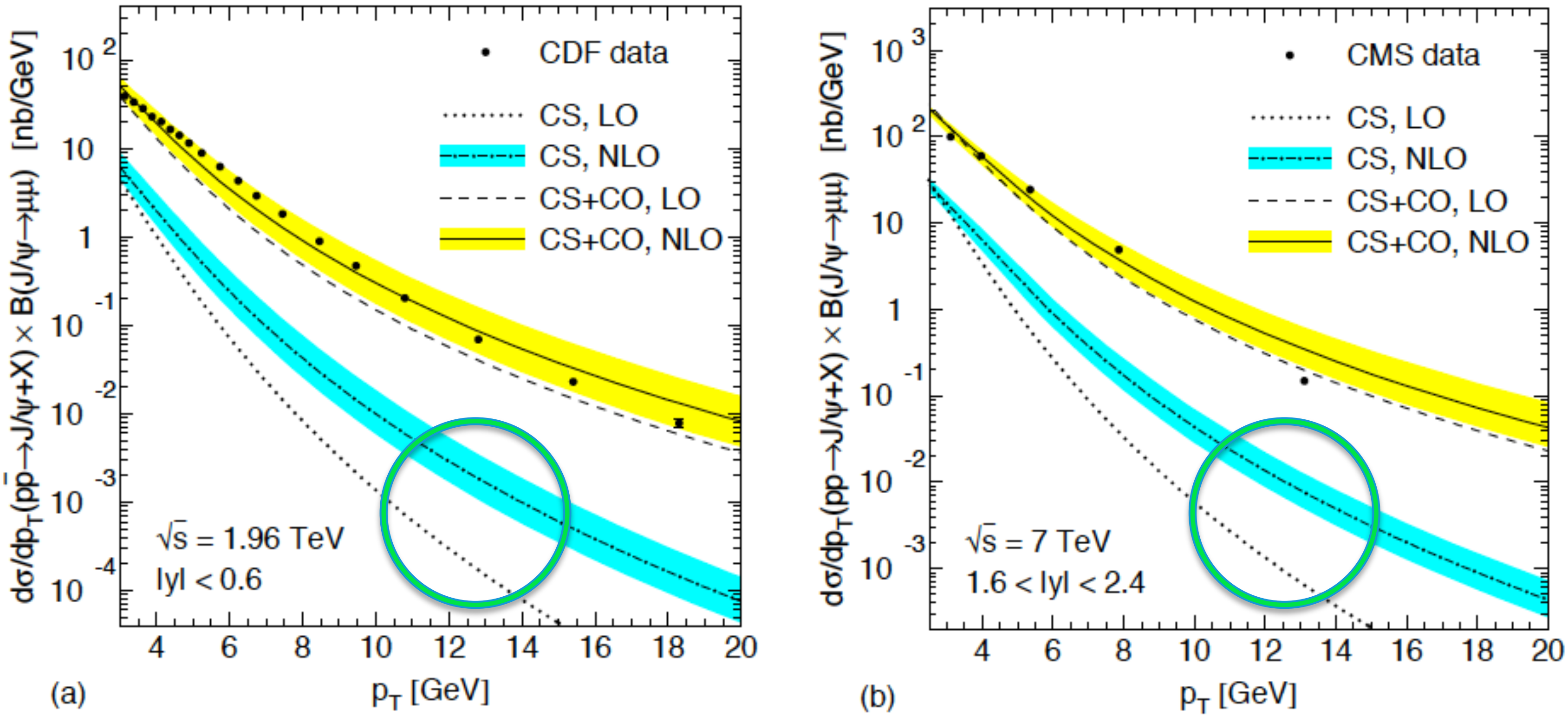
$$d\sigma_{A+B\to H+X} = \sum_n d\sigma_{A+B\to Q\bar{Q}(n)+X} \langle \mathcal{O}^H(n) \rangle$$

✧ **4 leading channels in v**

$${}^3S_1^{[1]},\ {}^1S_0^{[8]},\ {}^3S_1^{[8]},\ {}^3P_J^{[8]}$$

✧ **Full NLO in α_s**

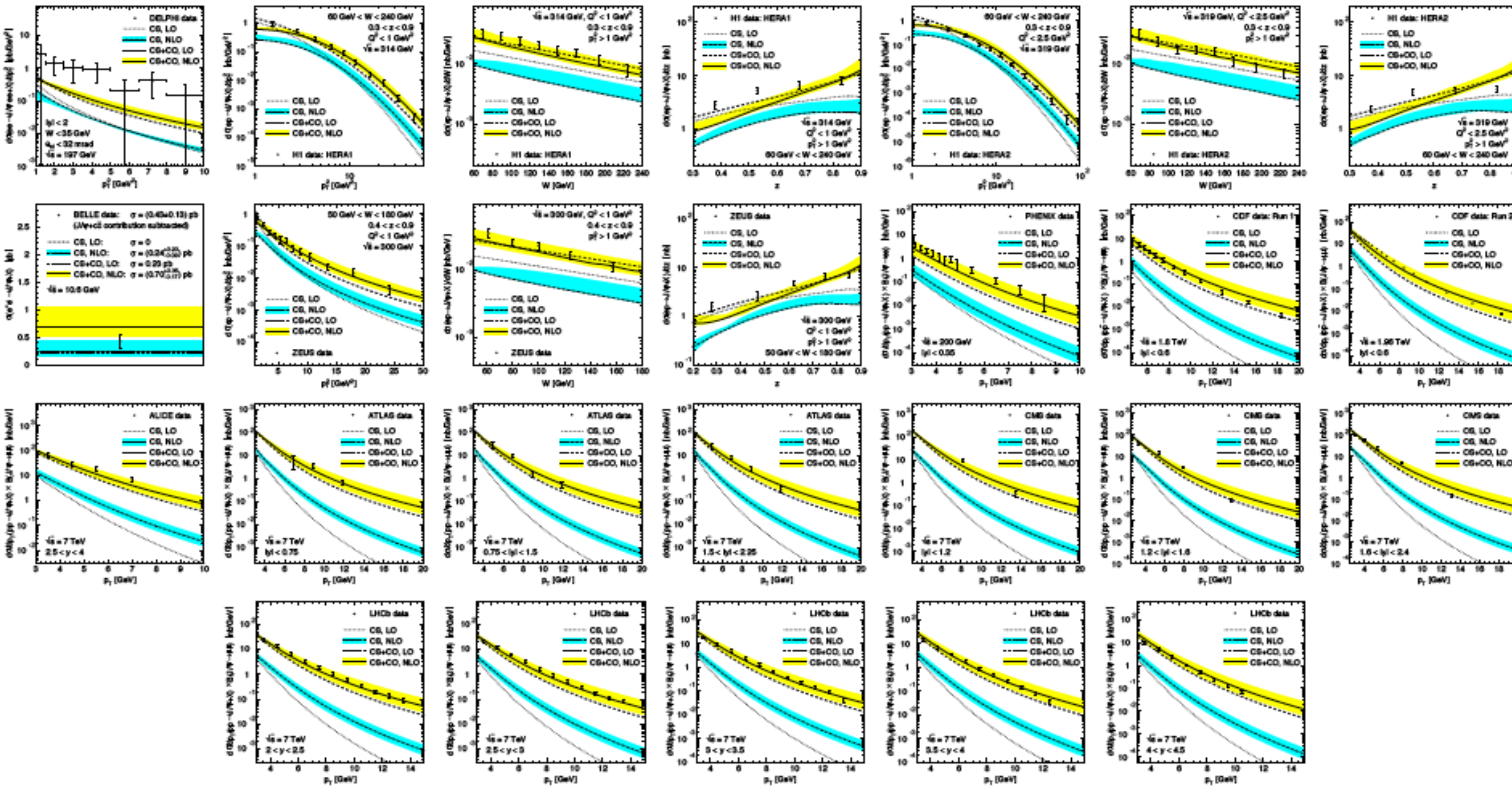
- **Phenomenology:**



- **Fine details – shape – high at large p_T?**

PRL 106, 022003 (2011)

NRQCD – global analysis



194 data points from 10 experiments, fix singlet $\langle O[^3S_1^{[1]}]\rangle = 1.32\ \mathrm{GeV}^3$

$\langle O[^1S_0^{[8]}]\rangle = (4.97 \pm 0.44)\cdot 10^{-2}\ \mathrm{GeV}^3$ $\langle O[^3S_1^{[8]}]\rangle = (2.24 \pm 0.59)\cdot 10^{-3}\ \mathrm{GeV}^3$

$\langle O[^3P_0^{[8]}]\rangle = (-1.61 \pm 0.20)\cdot 10^{-2}\ \mathrm{GeV}^5$

$\chi^2/d.o.f. = 857/194 = 4.42$

Butenschoen and Kniehl, arXiv: 1105.0820

Anomalies and surprises

- **Theory – the state of arts – NLO:**
  - ✧ **Very difficult to calculate, no analytical expression**

    ➡ **hard to obtain a clear physical picture on how various states of heavy quark pair are actually produced?**

  - ✧ **For some channels, NLO corrections are orders larger than LO**

    ➡ **questions whether higher order contributions are negligible, while it is extremely difficult, if not impossible, to go beyond the NLO**

- **Comparison with data:**
  - ✧ **Quarkonium polarization – "ultimate" test of NRQCD!**

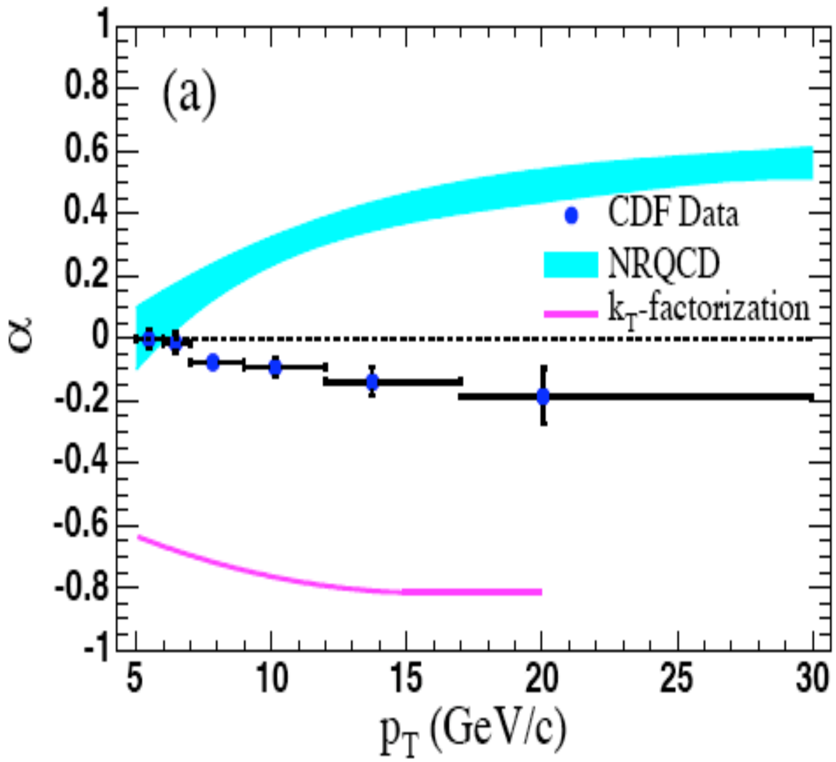
    ➡ **Clear mismatch between theory predictions and data**

  - ✧ **Universality of NRQCD matrix elements – predictive power!**

    ➡ **Clear tension between different data sets, e+e-, ep, pp, …**

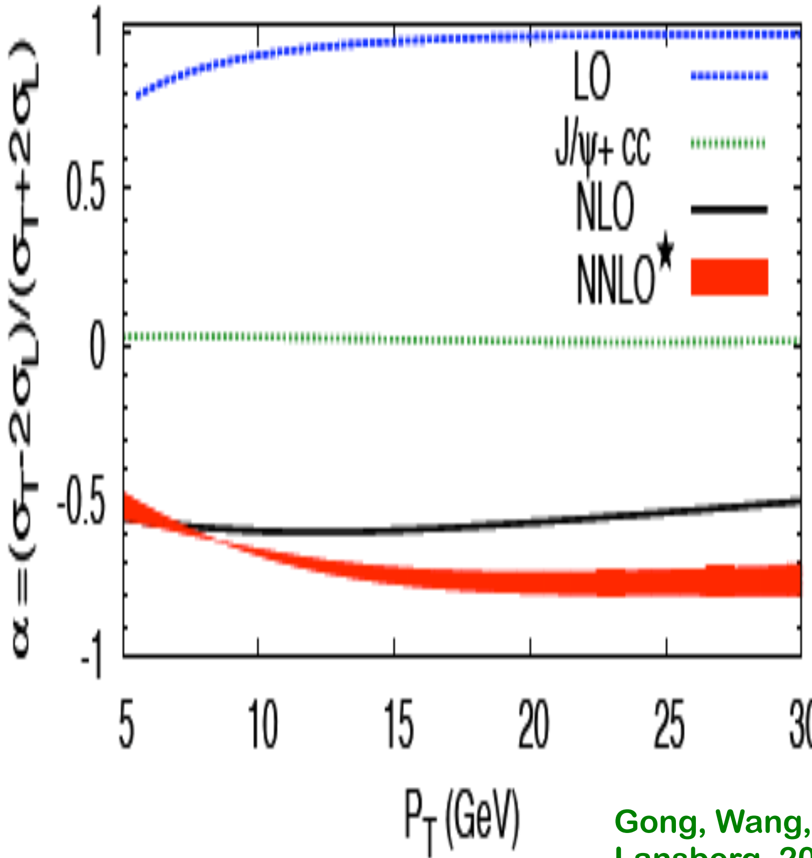
Theory predictions on J/ψ polarization

NRQCD



Cho & Wise, Beneke & Rothstein, 1995, …

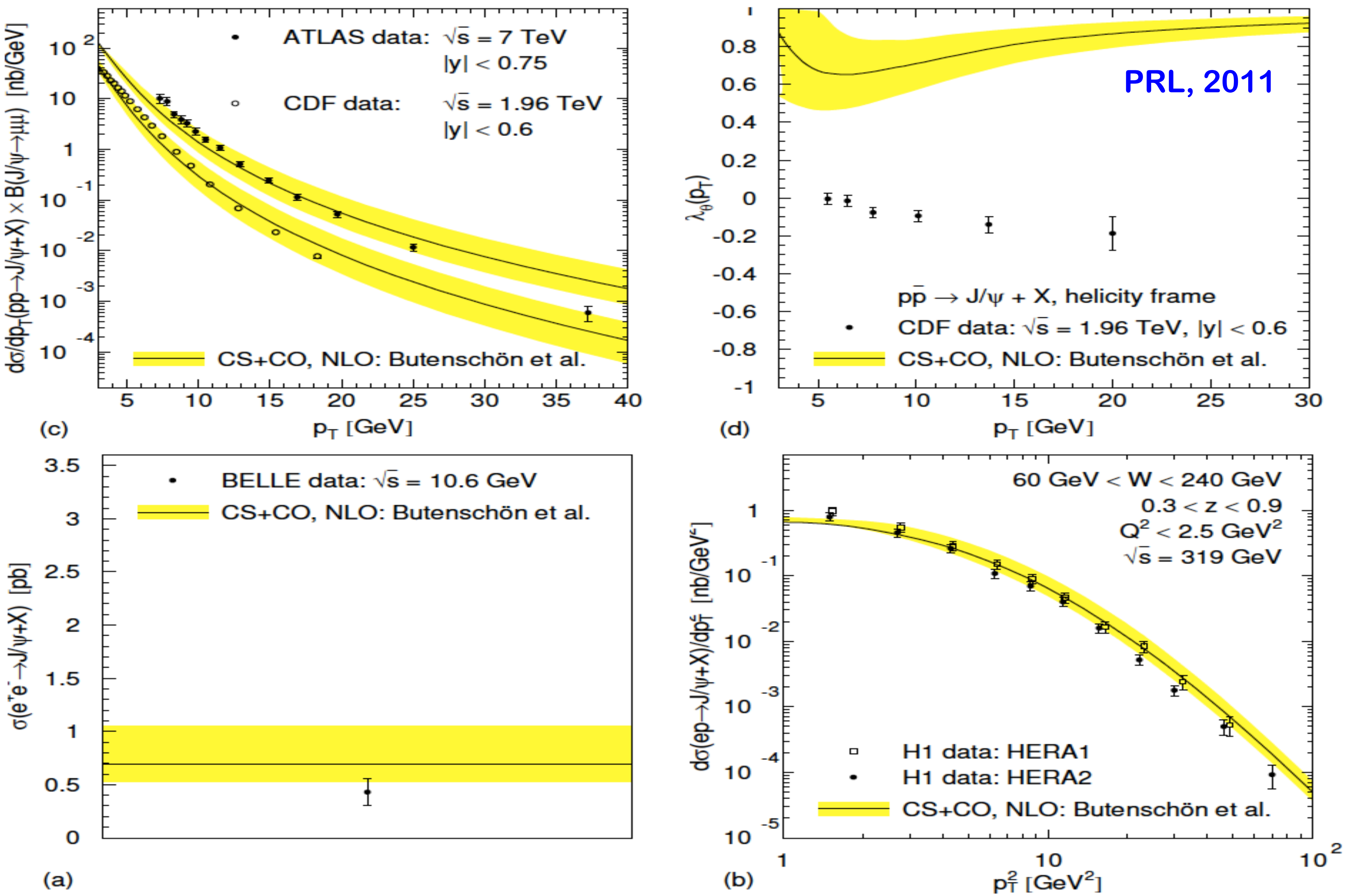
CSM



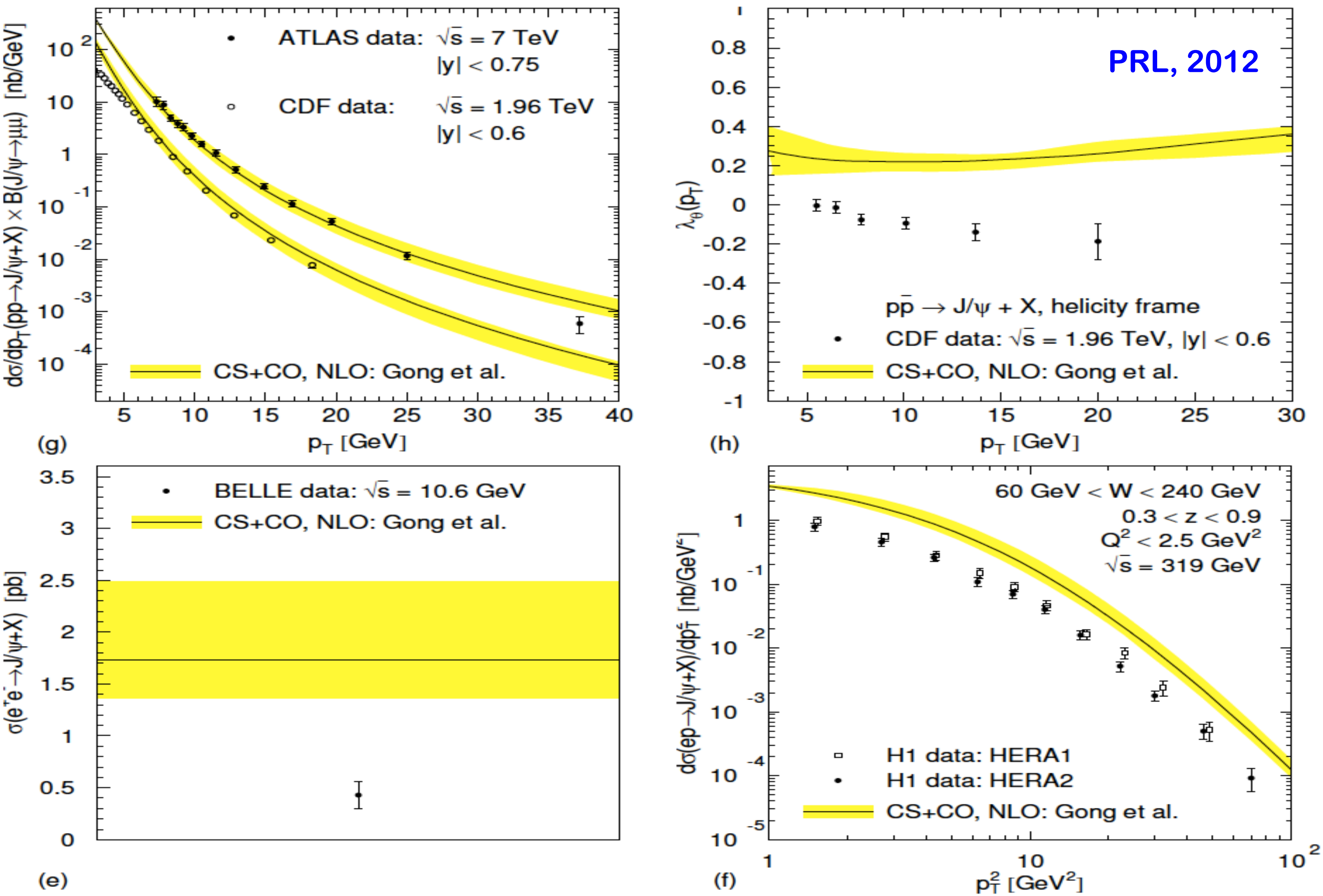
Gong, Wang, 2008
Lansberg, 2009

- ✧ **NRQCD:** Dominated by color octet – NLO is not a huge effect
- ✧ **CSM:** Huge NLO – change of polarization?

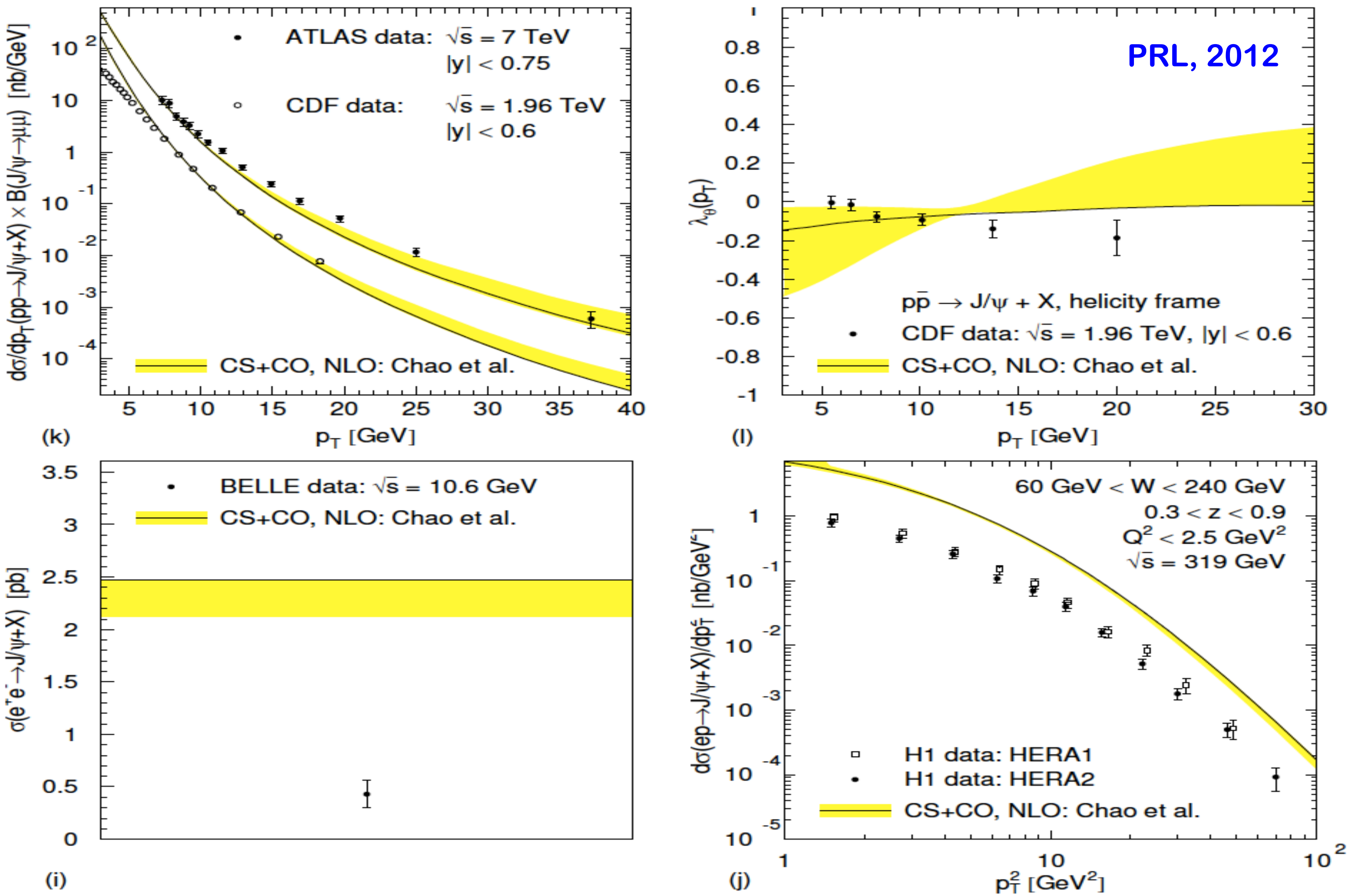
NLO theory fits – Butenschoen et al.



NLO theory fits – Gong et al.



NLO theory fits – Chao et al.

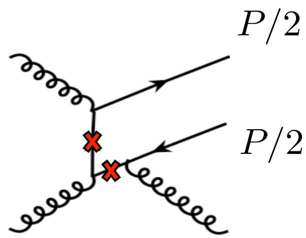


Why high orders in CSM are so large?

Kang, Qiu and Sterman, 2011

- **LO in α_s but higher power in $1/p_T$:**

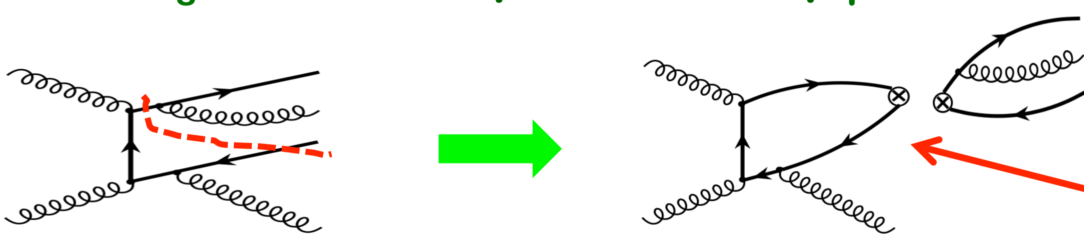
LO in α_s:



$$\hat{\sigma}^{\rm LO} \propto \frac{\alpha_s^3(p_T)}{p_T^8}$$

CSM and NRQCD spin-1 projection NNLP in $1/p_T$!

- **NLO in α_s but lower power in $1/p_T$:**

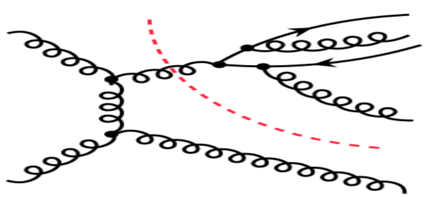


Relativistic projection to all "spin states"

$$\hat{\sigma}^{\rm NLO} \to \frac{\alpha_s^3(p_T)}{p_T^6} \otimes \alpha_s(\mu)\log(\mu^2/\mu_0^2)$$

$$\mu_0 \gtrsim 2m_Q$$

- **NNLO in α_s but leading power in $1/p_T$:**



$$\hat{\sigma}^{\rm NNLP} \to \frac{\alpha_s^2(p_T)}{p_T^4} \otimes \alpha_s^3(\mu)\log^m(\mu^2/\mu_0^2)$$

Leading order in α_s-expansion =\= leading power in $1/p_T$-expansion!

QCD factorization approach

❑ Factorization formalism:

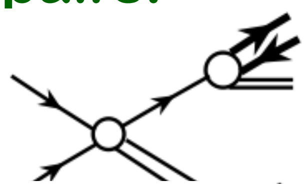
Nayak, Qiu, and Sterman, 2005
Kang, Qiu and Sterman, 2010

$$
\begin{aligned}
d\sigma_{A+B\to H+X}(p_T) &= \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q) \\
&\boxed{\begin{aligned} + \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \\ \otimes\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)\end{aligned}} \\
&+ \mathcal{O}(m_Q^4/p_T^4)
\end{aligned}
$$

❑ Production of the pairs:

$$\hat{p}_Q = \frac{1+\zeta}{2z}\,\hat{p}\,, \quad \hat{p}_{\bar{Q}} = \frac{1-\zeta}{2z}\,\hat{p}$$

✧ at $1/m_Q$:

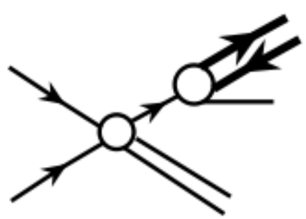


$$D_{i\to H}(z, m_Q, \mu_0)$$

✧ at $1/P_T$:

$$d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(P_{[Q\bar{Q}]}(\kappa), \mu)$$

✧ between: [$1/m_Q$, $1/P_T$]



$$\frac{d}{d\ln(\mu)} D_{i\to H}(z, m_Q, \mu) = \ldots$$

$$+ \frac{m_Q^2}{\mu^2}\,\Gamma(z) \otimes D_{[Q\bar{Q}(\kappa)]\to H}(\{z_i\}, m_Q, \mu)$$

Evolution of fragmentation functions

Kang, Ma, Qiu and Sterman, 2013

- **Independence of the factorization scale:**

$$\frac{d}{d\ln(\mu)}\sigma_{A+B\to HX}(P_T)=0$$

  - **at Leading power in $1/P_T$:**

DGALP evolution

$$\frac{d}{d\ln\mu^2}D_{H/f}(z,m_Q,\mu)=\sum_j\frac{\alpha_s}{2\pi}\gamma_{f\to j}(z)\otimes D_{H/j}(z,m_Q,\mu)$$

  - **next-to-leading power in $1/P_T$ – New non-linear evolution!**

$$\frac{d}{d\ln\mu^2}D_{H/f}(z,m_Q,\mu)=\sum_j\frac{\alpha_s}{2\pi}\gamma_{f\to j}(z)\otimes D_{H/j}(z,m_Q,\mu)$$
$$+\frac{1}{\mu^2}\sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s^2}{(2\pi)^2}\Gamma_{f\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta')\otimes\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu)$$

$$\frac{d}{d\ln\mu^2}\mathcal{D}_{H/[Q\bar{Q}(c)]}(z,\zeta,\zeta',m_Q,\mu)=\sum_{[Q\bar{Q}(\kappa)]}\frac{\alpha_s}{2\pi}K_{[Q\bar{Q}(c)]\to[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta')\otimes\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q,\mu)$$

- **Evolution kernels are perturbative:**

  - **Set mass:** $m_Q\to 0$ **with a caution**

Predictive power and status

- **Calculation of short-distance hard parts in pQCD:**

  Power series in α_s, without large logarithms

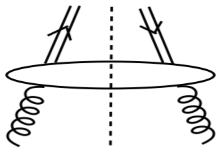
  LO is now available for all partonic channels — Kang, Ma, Qiu and Sterman, 2013

- **Calculation of evolution kernels in pQCD:**

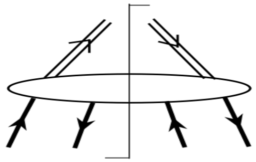
  Power series in α_s, without large logarithms — Kang, Ma, Qiu and Sterman, 2013

  LO is now available for both mixing kernels and pair evolution kernels of all spin states of heavy quark pairs

- **Input FFs at μ_0 – non-perturbative, but, universal**



$D_{H/f}(z, m_Q, \mu_0)$



$\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta, \zeta', m_Q, \mu_0)$

- **Physics of the input scale: $\mu_0 \sim 2m_Q$ – a parameter:**

  Evolution stops when $\log\left[\frac{\mu_0^2}{(4m_Q^2)}\right] \sim \left[\frac{4m_Q^2}{\mu_0^2}\right]$

  Different quarkonium states require different input distributions!

Non-perturbative input distributions

- **Sensitive to the properties of quarkonium produced:**

  Should, in principle, be extracted from experimental data

- **Large heavy quark mass and clear scale separation:**

  $\mu_0 \sim m_Q \gg m_Q\, v$ ⟹ Apply NRQCD to the FFs

  ✧ Single parton FFs – valid to two-loops:

  Nayak, Qiu and Sterman, 2005

$$D_{g\to J/\psi}(z,\mu_0,m_Q) \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{g\to[Q\bar{Q}(c)]}(z,\mu_0,m_Q)\langle \mathcal{O}_{[Q\bar{Q}(c)]}(0)\rangle|_{\rm NRQCD}$$

  Braaten, Yuan, 1994
  Ma, 1995, …
  Braaten, Chen, 1997
  Braaten, Lee, 2000,
  Ma, Qiu, Zhang, 2013
  …

  Complete LO+NLO for S, P states & NNLO for singlet S state

  ✧ Heavy quark pair FFs – valid to one-loop:

$$\mathcal{D}_{[Q\bar{Q}(\kappa)]\to J/\psi}(z,\zeta,\zeta',\mu_0,m_Q) \to \sum_{[Q\bar{Q}(c)]} \hat{d}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]}(z,\zeta,\zeta',\mu_0,m_Q)\langle \mathcal{O}_{[Q\bar{Q}(c)]}(0)\rangle_{\rm NRQCD}$$

  Kang, Ma, Qiu and Sterman, 2014

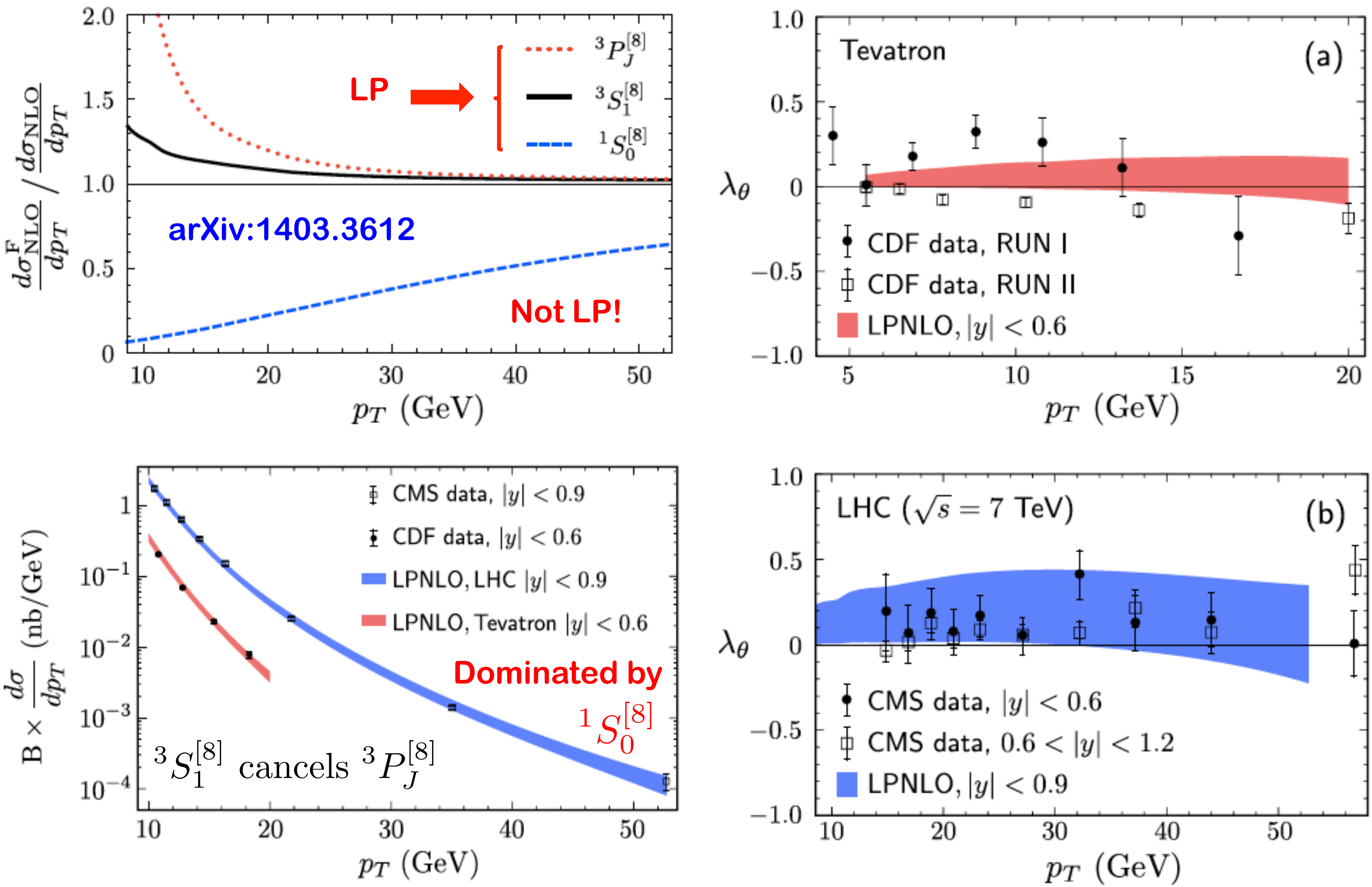
  Full LO+NLO for S, P states is now available

  Ma, Qiu, Zhang, 2013

- **No all-order proof of such factorization yet!**

  Reduce "many" unknown FFs to a few universal NRQCD matrix elements!

Leading power fragmentation – Bodwin et al.



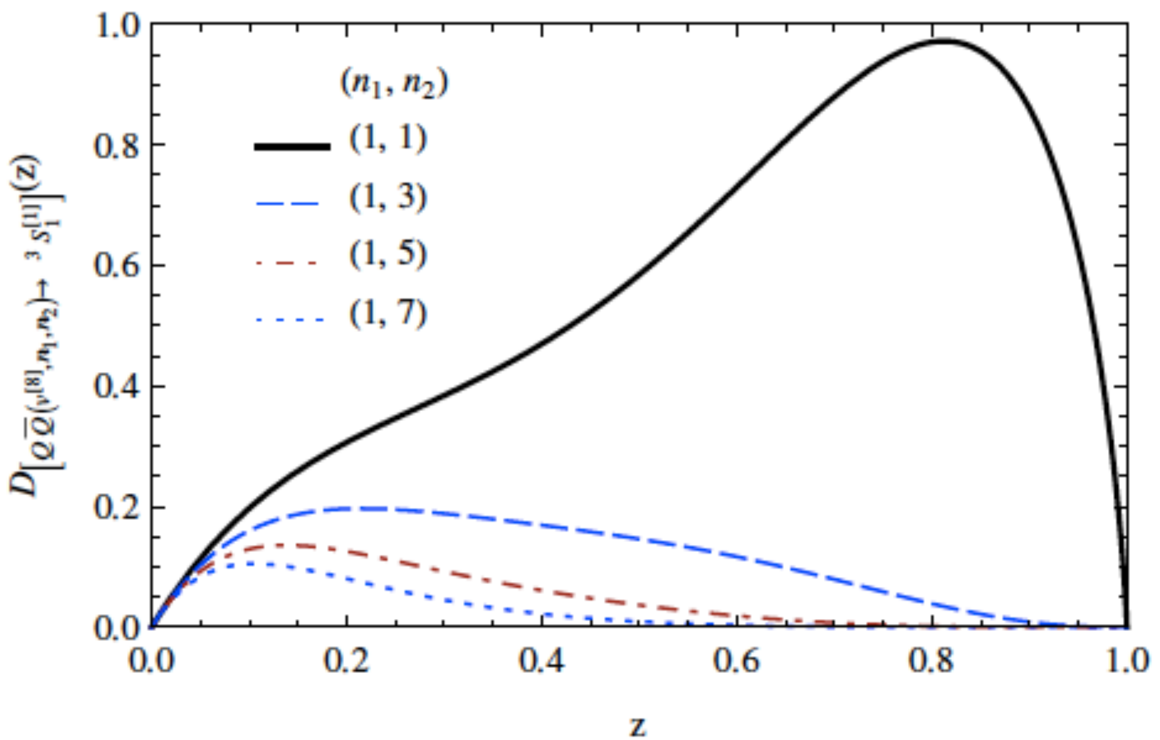
Next-to-leading power fragmentation – Ma et al.

Ma, Qiu, Zhang, 2013

- **Heavy quark pair FFs:**

$$\mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}(z,\zeta_1,\zeta_2,\mu_0;m_Q) = \sum_{[Q\bar{Q}(n)]} \left\{ \hat{d}^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z,\zeta_1,\zeta_2,\mu_0;m_Q,\mu_\Lambda) + \left(\frac{\alpha_s}{\pi}\right) \hat{d}^{(1)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z,\zeta_1,\zeta_2,\mu_0;m_Q,\mu_\Lambda) + O(\alpha_s^2) \right\} \times \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m_Q^{2L+1}}$$

- **Moment of the FFs:** $\mathcal{D}^{[n_1,n_2]}(z) \equiv \int_{-1}^{1} \frac{d\zeta_1\, d\zeta_2}{4} \zeta_1^{n_1} \zeta_2^{n_2} \mathcal{D}(z,\zeta_1,\zeta_2)$

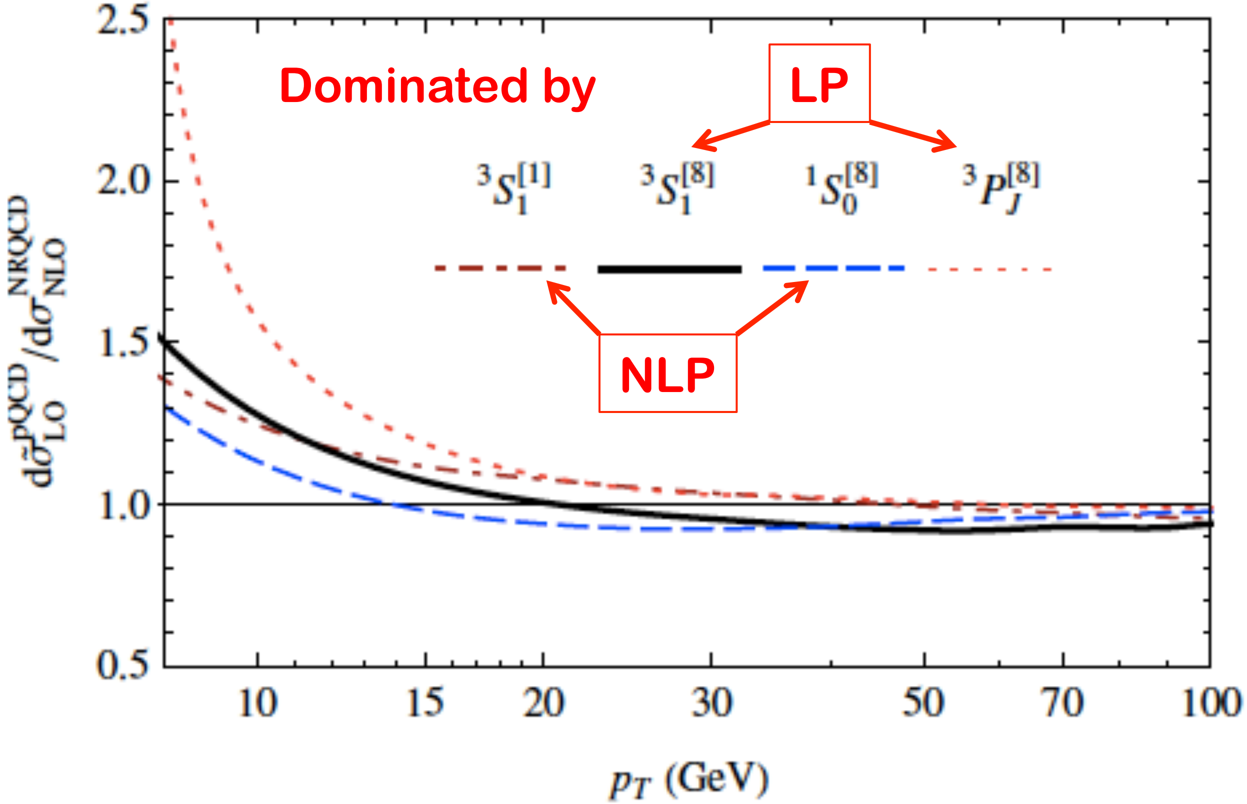


Next-to-leading power fragmentation – Ma et al.

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$

$$\boxed{+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta, \zeta', m_Q)}$$

- Channel-by-channel comparison:



independent of NRQCD matrix elements

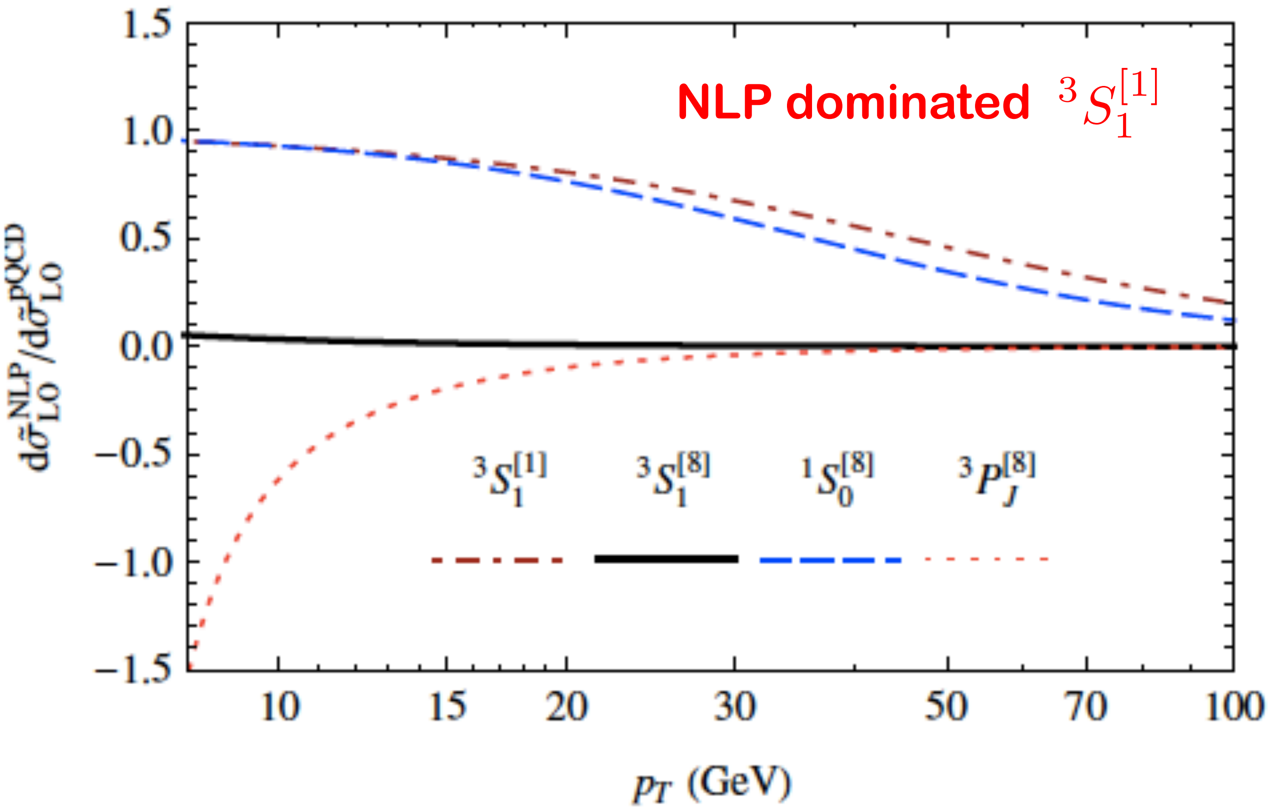
LO analytical results reproduce NLO NRQCD calculations (numerical)

Next-to-leading power fragmentation – Ma et al.

$$d\sigma_{A+B\to H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B\to f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$

$$\boxed{+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B\to[Q\bar{Q}(\kappa)]+X}(p(1\pm\zeta)/2z, p(1\pm\zeta')/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z,\zeta,\zeta',m_Q)}$$

- LP vs. NLP (both LO):



NLP dominated $^1S_0^{[8]}$ for wide pT

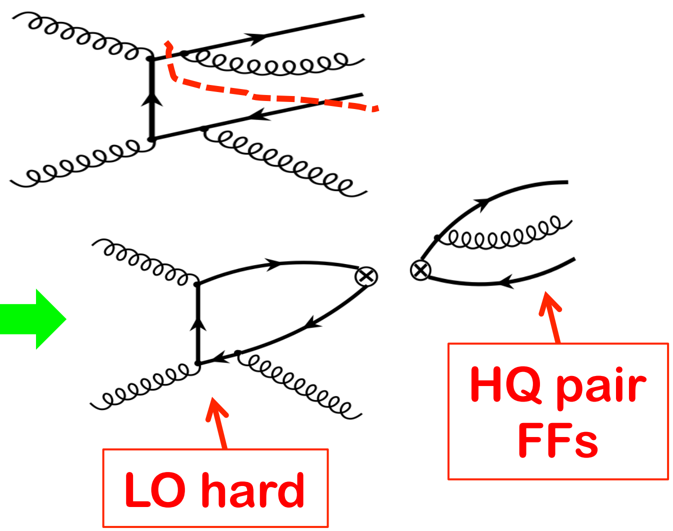
LP dominated $^3S_1^{[8]}$ and $^3P_J^{[8]}$

PT distribution is consistent with distribution of $^1S_0^{[8]}$

Next-to-leading power fragmentation – Ma et al.

Kang, Ma, Qiu and Sterman, 2014

- Color singlet as an example:

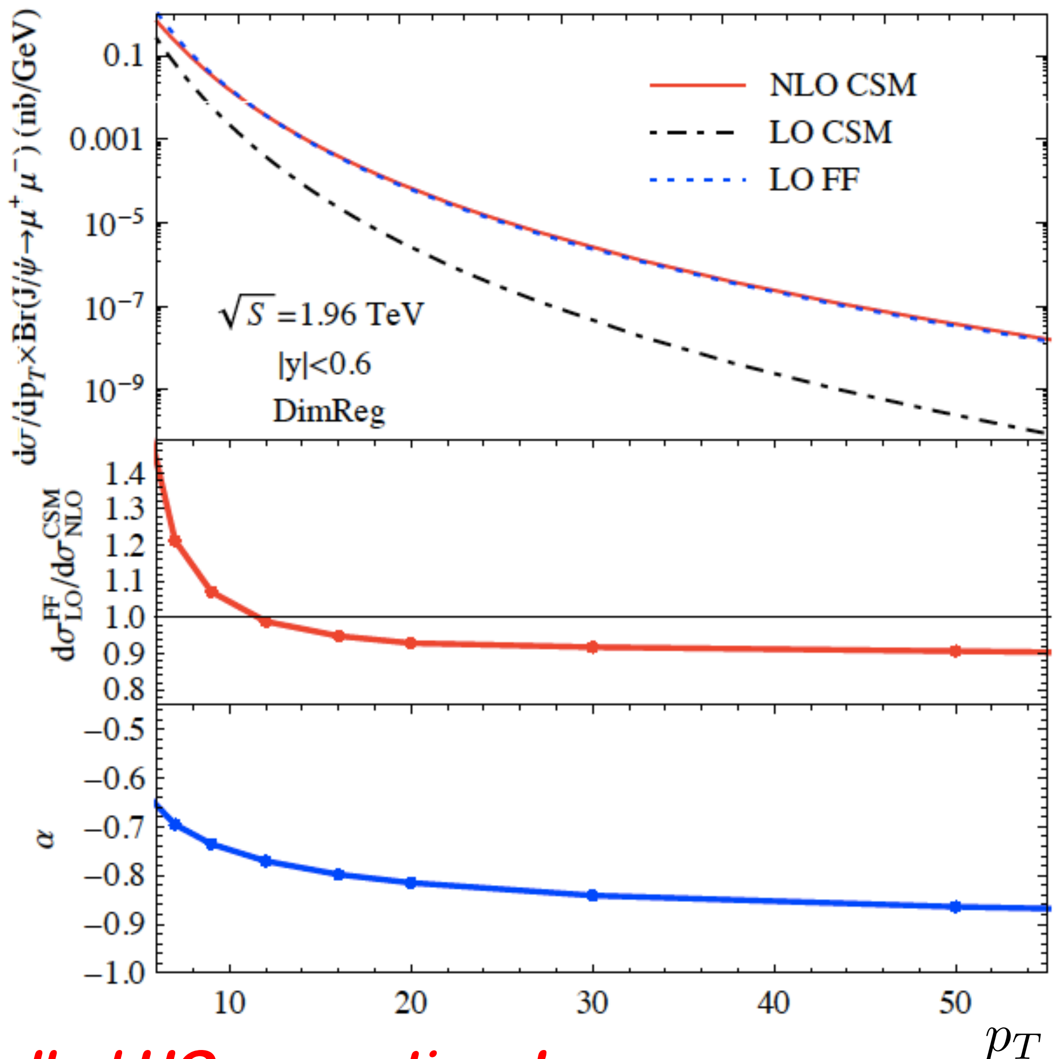


$$\hat{\sigma}^{\mathrm{NLO}} \to \frac{\alpha_s^3(p_T)}{p_T^6} \otimes \alpha_s(\mu)\log(\mu^2/\mu_0^2)$$

Reproduce NLO CSM

for p_T > 10 GeV!

Cross section + polarization



Factorization = better controlled HO corrections!

Summary

- It has been almost 40 years since the discovery of J/Ψ

- When $p_T >> m_Q$ at collider energies, earlier models calculations for the production of heavy quarkonia are not perturbatively stable

  LO in α_s-expansion may not be the LP term in $1/p_T$-expansion

- QCD factorization works for both LP and NLP (α_s for each power)

  - LP dominates: ${}^3S_1^{[8]}$ and ${}^3P_J^{[8]}$ channels
  - NLP dominates: ${}^1S_0^{[8]}$ and ${}^3S_1^{[1]}$ channels
  - From current data: ${}^3P_J^{[8]}$ likely to cancel ${}^3S_1^{[8]}$

    the production dominated by ${}^1S_0^{[8]}$

- A full global analysis, based on QCD factorization formalism including NLP and evolution, is needed!

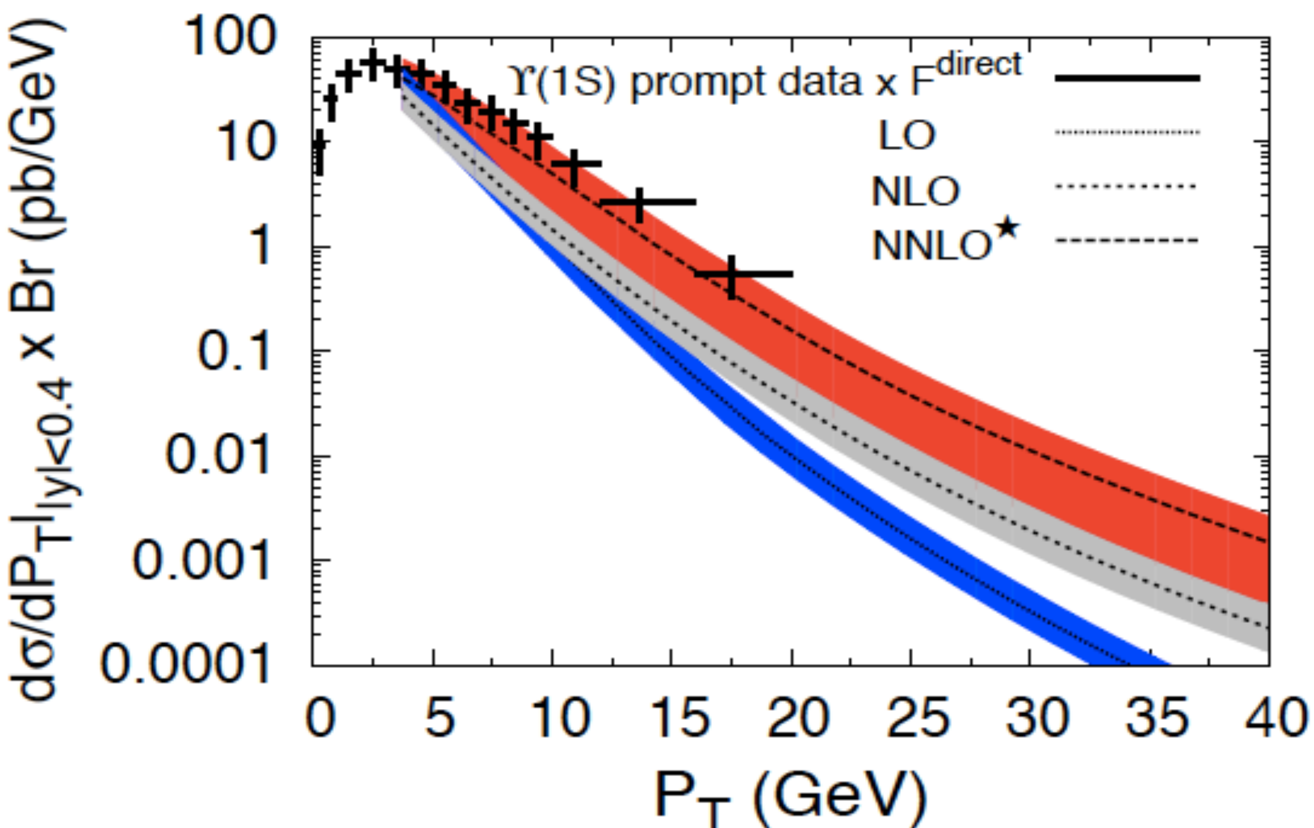
Thank you!

Backup slides

Color singlet model (CSM)

❑ **Effectively No parameter:**

Campbell, Maltoni, Tramontano (2007),
Artoisenet, Lansburg, Maltoni (2007),
Artoisenet, et al. (2008)

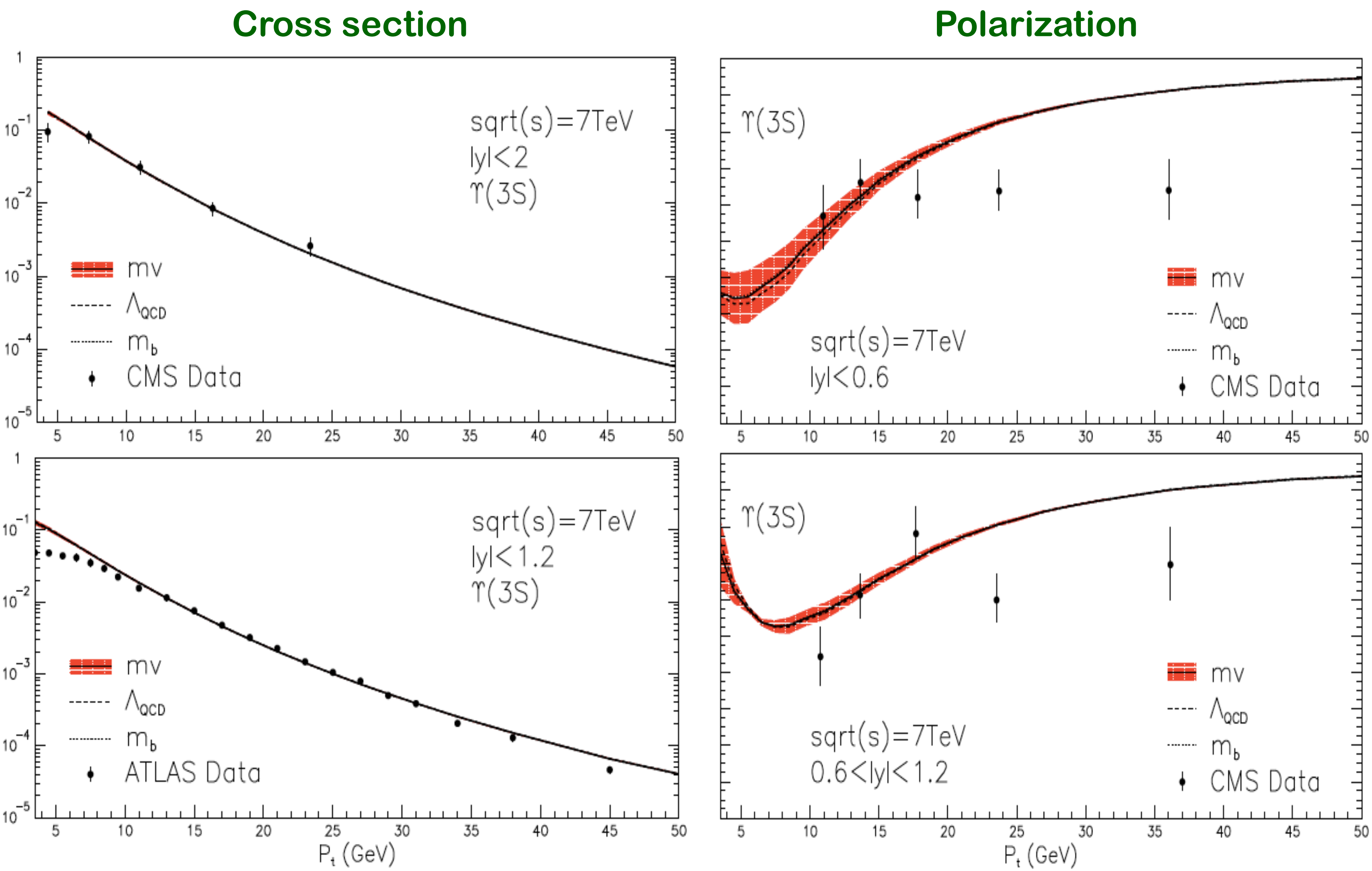


❑ **Questions:**

✧ **How reliable is the perturbative expansion?**

✧ **How to cure the IR singularities for P-wave quarkonia?**

NLO theory fits – Υ production



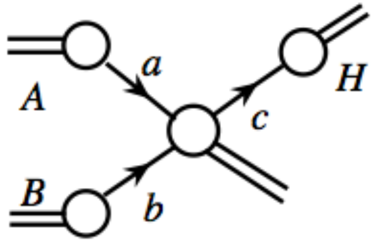
Gong et al. PRL, 2013

Why such power correction are important?

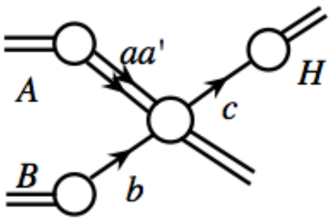
- **Leading power in hadronic collisions:**

Kang, Ma, Qiu and Sterman, 2013

$$d\sigma_{AB\to H} = \sum_{a,b,c} \phi_{a/A} \otimes \phi_{b/B} \otimes d\hat{\sigma}_{ab\to cX} \otimes D_{c\to H}$$



- **1st power corrections in hadronic collisions:**



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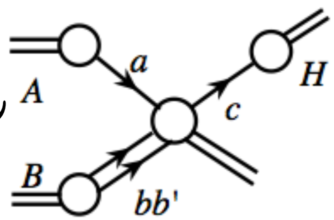

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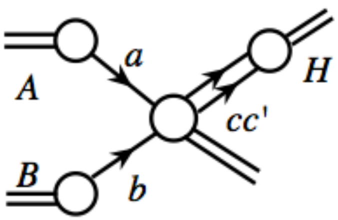
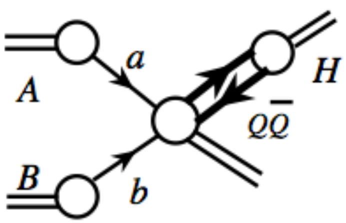


$$\sim \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_T^2}\right) \otimes D_{c\to H}$$

or $\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_T^2}\right) \otimes \mathcal{D}_{[ff]\to H}$

- **Dominated 1st power corrections:**



$$\sim \mathcal{O}\left(\frac{(2m_Q)^2}{P_T^2}\right) \otimes D^{(2)}_{[Q\bar{Q}]\to H}$$

Key: competition between $P_T^2 \gg (2m_Q)^2$ **and** $D^{(2)}_{[Q\bar{Q}]\to H} \gg D_{c\to H}$

Short-distance hard parts

Kang, Ma, Qiu and Sterman, 2013

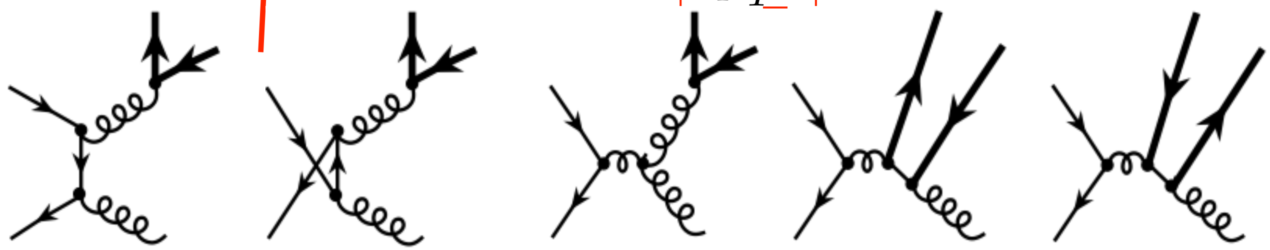
❑ Even tree-level needs subtraction:

$$\sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}(c)]g} = \hat{\sigma}^{(3)}_{q\bar{q}\to[Q\bar{Q}(\kappa)]g} \otimes D^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(c)]} + \hat{\sigma}^{(2)}_{q\bar{q}\to gg} \otimes D^{(1)}_{g\to[Q\bar{Q}(c)]}$$

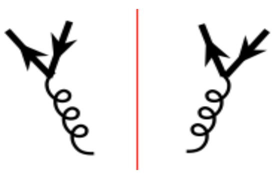
$$\Rightarrow \quad \hat{\sigma}^{(3)}_{q\bar{q}\to[Q\bar{Q}]g} = \sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}]g} - \sigma^{(2)}_{q\bar{q}\to g} \otimes D^{(1)}_{g\to[Q\bar{Q}]}$$

$$\boxed{\frac{\alpha_s^3(\mu)}{p_T^6}} \qquad \boxed{\frac{\alpha_s^2(\mu)}{p_T^4}} \qquad \boxed{\frac{\alpha_s(2m_Q)}{(2m_Q)^2}}$$

$\sigma^{(3)}_{q\bar{q}\to[Q\bar{Q}(c)]g}$:



$D^{(1)}_{g\to[Q\bar{Q}]}$:



$$\widetilde{\mathcal{P}}_{\mu\nu}(p) = \frac{1}{2}\left[-g_{\mu\nu} + \frac{p_\mu n_\nu + n_\mu p_\nu}{p\cdot n} - \frac{p^2}{(p\cdot n)^2} n_\mu n_\nu\right]$$

$$H^{(3)}_{q\bar{q}\to[Q\bar{Q}(a8)]g} = \frac{8\pi\alpha_s}{\hat{s}}\frac{\hat{t}^2+\hat{u}^2}{\hat{s}^2}\frac{1}{(1-\zeta^2)(1-\zeta'^2)}\frac{N^2-1}{N}\left[1+\zeta\zeta'-\frac{4}{N^2}\right]$$

Normalized to 2 → 2 amplitude square

Evolution kernels

Kang, Ma, Qiu and Sterman, 2013

❑ Evolution equation:

$$\kappa, \kappa' = v, a, t$$

$$\frac{\partial}{\partial \ln \mu^2} \mathcal{D}_{Q\bar{Q}[\kappa]\to J/\psi}(z_h, \zeta_1, \zeta_2, \mu^2)$$

$$= \frac{\alpha_s}{2\pi} \int_{z_h}^{1} \frac{dz}{z} \int_{-1}^{1} d\zeta_1' \int_{-1}^{1} d\zeta_2' P_{\kappa\to\kappa'}(\zeta_1, \zeta_2, \zeta_1', \zeta_2', z) \mathcal{D}_{Q\bar{Q}[\kappa']\to J/\psi}(z_h/z, \zeta_1', \zeta_2', \mu^2)$$

❑ Evolution kernels:

$$\frac{\partial}{\partial \ln \mu^2} \begin{pmatrix} \mathcal{D}_{Q\bar{Q}[v8]} \\ \mathcal{D}_{Q\bar{Q}[v1]} \\ \mathcal{D}_{Q\bar{Q}[a8]} \\ \mathcal{D}_{Q\bar{Q}[a1]} \\ \mathcal{D}_{Q\bar{Q}[t8]} \\ \mathcal{D}_{Q\bar{Q}[t1]} \end{pmatrix} = \frac{\alpha_s}{2\pi} \begin{pmatrix} \mathcal{K}_1 & \mathcal{T}_1 & \mathcal{K}_2 & \mathcal{T}_2 & 0 & 0 \\ \mathcal{R}_1 & \mathcal{S}_1 & \mathcal{R}_2 & 0 & 0 & 0 \\ \mathcal{K}_2 & \mathcal{T}_2 & \mathcal{K}_1 & \mathcal{T}_1 & 0 & 0 \\ \mathcal{R}_2 & 0 & \mathcal{R}_1 & \mathcal{S}_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{K}_1' & \mathcal{T}_1' \\ 0 & 0 & 0 & 0 & \mathcal{R}_1' & \mathcal{S}_1' \end{pmatrix} \otimes \begin{pmatrix} \mathcal{D}_{Q\bar{Q}[v8]} \\ \mathcal{D}_{Q\bar{Q}[v1]} \\ \mathcal{D}_{Q\bar{Q}[a8]} \\ \mathcal{D}_{Q\bar{Q}[a1]} \\ \mathcal{D}_{Q\bar{Q}[t8]} \\ \mathcal{D}_{Q\bar{Q}[t1]} \end{pmatrix}$$

Example: $\mathcal{K}_1 = P_{v8\to v8} = P_{a8\to a8}$

NOTE: Our results are consistent with those by Fleming et al. [arXiv: 1301.3822], but, a difference in logarithms